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Research Article

Extending the Lifetime of Sensor Networks through Adaptive Reclustering

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We analyze the lifetime of *clustered* sensor networks with decentralized binary detection under a physical layer quality-of-service (QoS) constraint, given by the maximum tolerable probability of decision error at the access point (AP). In order to properly model the network behavior, we consider four different distributions (exponential, uniform, Rayleigh, and lognormal) for the lifetime of a single sensor. We show the benefits, in terms of longer network lifetime, of *adaptive reclustering*. We also derive an analytical framework for the computation of the network lifetime and the penalty, in terms of *time delay* and *energy consumption*, brought by adaptive reclustering. On the other hand, *absence of reclustering* leads to a shorter network lifetime, and we show the impact of various clustering configurations under different QoS conditions. Our results show that the organization of sensors in a *few big clusters* is the winning strategy to maximize the network lifetime. Moreover, the observation of the phenomenon should be *frequent* in order to limit the penalties associated with the reclustering procedure. We also apply the developed framework to analyze the energy consumption associated with the proposed reclustering protocol, obtaining results in good agreement with the performance of realistic wireless sensor networks. Finally, we present simulation results on the lifetime of IEEE 802.15.4 wireless sensor networks, which enrich the proposed analytical framework and show that typical networking performance metrics (such as throughput and delay) are influenced by the sensor network lifetime.

1. INTRODUCTION

Distributed detection has been an active research field for a long time [1]. The increasing interest for sensor networks has spurred a significant scientific activity on distributed detection [2]. In the last years, an increasing number of civilian applications have been developed, especially for environmental monitoring [3, 4].

Several communication-theoretic-oriented approaches have been proposed to study decentralized detection [5]. In [6], the authors follow a Bayesian approach for the minimization of the probability of decision error at the access point (AP). Most of the proposed approaches are based on the assumption of *ideal* communication links between the sensors and the AP. However, in a realistic communication scenario, these links are likely to be *noisy* [7]. In [8], the presence of noisy communication links, modeled as binary symmetric channels (BSCs), is considered and a few techniques are proposed to make the system more robust against the noise.

The problem of extending the sensor network lifetime has also been studied extensively. In particular, the derivation of upper bounds for the sensor network lifetime has been exploited. In [9–17], various analyses are carried out according to the particular sensor network architecture and the definition of sensor network lifetime. In [18], a simple formula, independent of these parameters, is provided for the computation of the sensor network lifetime and a medium access control (MAC) protocol is proposed to maximize the sensor network lifetime. In [19], a distributed MAC protocol is designed in order to maximize the network lifetime. In [20], network lifetime maximization is considered as the main criterion for the design of sensor networks with data gathering. In [21], the authors consider a realistic sensor network with nodes equipped with TinyOS, an event-based operating system for networked sensor motes. In this scenario, the network lifetime is evaluated as a function of the average distance of the sensors from the central data collector. In [22], an analytical framework, based on the Chen-Stein method of Poisson approximation, is proposed in order to find the critical time at which isolated nodes, that is, nodes without neighbors in the network, begin to appear, due to the deaths of other nodes. Although this method is derived for generic networks where nodes are randomly deployed and can die in a random manner, this can also be applied to sensor networks. In [23], an analysis of network lifetime using IEEE 802.15.4 sensor networks [24] is proposed for applications in the medical field.

In this paper, we consider a scenario where sensors[1] are *clustered* and there are local fusion centers (FCs) associated with the clusters. This can be considered as an accurate model for realistic scenarios where sensors may form groups, depending on how they are placed and the environmental characteristics (some sensors might not communicate directly with the AP) or in order to reduce their transmission range (and, consequently, to save battery energy). All sensors observe a *common* binary phenomenon, but our approach can be extended to a scenario where the phenomenon may change from sensor to sensor [25]. Each of the FCs makes a decision based on the data collected from its sensors and sends its decision to the AP, which makes the final decision on the status of the phenomenon [26]. We suppose that the FCs can be power-supplied (i.e., they do not have energy limitations). However, the FCs will perform data aggregation on sensors' decisions in order to save as much bandwidth as possible. In [26], it is shown that *uniform clustering* leads to minimum performance degradation, in terms of probability of decision error at the AP, with respect to the case with the absence of clustering. In this paper, we propose a novel analysis of the lifetime of sensor networks with uniform clustering, considering a quality-of-service (QoS) condition given by the maximum tolerable probability of decision error at the AP. The analysis is carried out in two cases: (i) *ideal reclustering*, where the surviving sensors, after the death of a sensor, reconfigure themselves in uniform clusters, and (ii) *absence of reclustering*, where the initial cluster configuration remains fixed, regardless of the sequence of sensors' deaths. The impact, on system performance, of the number of sensors, the QoS condition, and the distribution of sensors' lifetime is evaluated in both the scenarios of interest. We show that in the absence of reclustering, the longest lifetime is guaranteed by an initial configuration characterized by the presence of *few big clusters*. We also derive an analytical framework to compute the network lifetime and the penalties, in terms of *time delay* and *energy consumption*, induced by ideal reclustering. Finally, simulation results of realistic IEEE 802.15.4 wireless sensor networks, in terms of throughput and delay, are presented to validate the theoretical results of our framework.

The structure of this paper is the following. In Section 2, communication-theoretic preliminaries on sensor networks with decentralized binary detection are given. In Section 3, we propose a simple approach for evaluating the sensor network lifetime under a physical-layer-oriented QoS condition. In Section 4, an analytical framework for the computation of the sensor network lifetime is derived. In Section 5, simple energetic considerations about the cost of reclustering are discussed. In Section 6, the impact of noisy communication links on the sensor network lifetime is evaluated. In Section 7, simulation results are presented. Finally, concluding remarks are given in Section 8.

[1] We point out that the term "sensor" will be used to denote a remote node which is equipped with a sensor. Obviously, this node has a wireless transceiver.

2. COMMUNICATION-THEORETIC PRELIMINARIES

We consider a network scenario where N sensors observe a *common binary phenomenon*. They are clustered into $n_c < N$ groups, and each of them can communicate with only one local FC. The FCs collect data from the sensors in their corresponding clusters and make local decisions on the status of the binary phenomenon. At this point, each local FC transmits its decision to the AP, which makes a final decision on the phenomenon status. A pictorial description of clustered sensor networks with $N = 16$ sensors is presented in Figure 1, where (a) *uniform* and (b) *nonuniform* topologies are shown. More precisely, in Figure 1(a), the 16 sensors are grouped into 4 identical clusters, whereas in Figure 1(b) there are one large cluster (with 10 sensors) and three small clusters (with 2 sensors each). In the rest of this paper, we will consider only scenarios with uniform clustering. This choice will be motivated further in the following.

The status of the common binary phenomenon under observation is characterized as follows:

$$H = \begin{cases} H_0 & \text{with probability } p_0, \\ H_1 & \text{with probability } 1 - p_0, \end{cases} \tag{1}$$

where $p_0 \triangleq P(H = H_0)$. The observed signal at the ith sensor can be expressed as

$$r_i = c_{\mathrm{E}} + n_i, \quad i = 1, \ldots, N, \tag{2}$$

where

$$c_{\mathrm{E}} \triangleq \begin{cases} 0 & \text{if } H = H_0, \\ s & \text{if } H = H_1. \end{cases} \tag{3}$$

Assuming that the noise samples $\{n_i\}$ are independent with the same Gaussian distribution $\mathcal{N}(0, \sigma^2)$, the *common* signal-to-noise ratio (SNR) at the sensors can be defined as follows:

$$\mathrm{SNR}_{\mathrm{sensor}} = \frac{\left[\mathbb{E}\{c_{\mathrm{E}} \mid H_1\} - \mathbb{E}\{c_{\mathrm{E}} \mid H_0\}\right]^2}{\sigma^2} = \frac{s^2}{\sigma^2}. \tag{4}$$

Each sensor makes a decision comparing the observation r_i with a threshold value τ_i and computes a local decision $u_i = U(r_i - \tau_i)$, where $U(\cdot)$ is the unit step function. In order to optimize the system performance, the thresholds $\{\tau_i\}$ need to be properly chosen. In this paper, we use a common threshold value τ for all sensors. While in a scenario with no clustering and ideal communication links between the sensors and the AP, the relation between τ and s has been obtained, through proper optimization, in [6]; in the presence of clusters and noisy communication links the decision

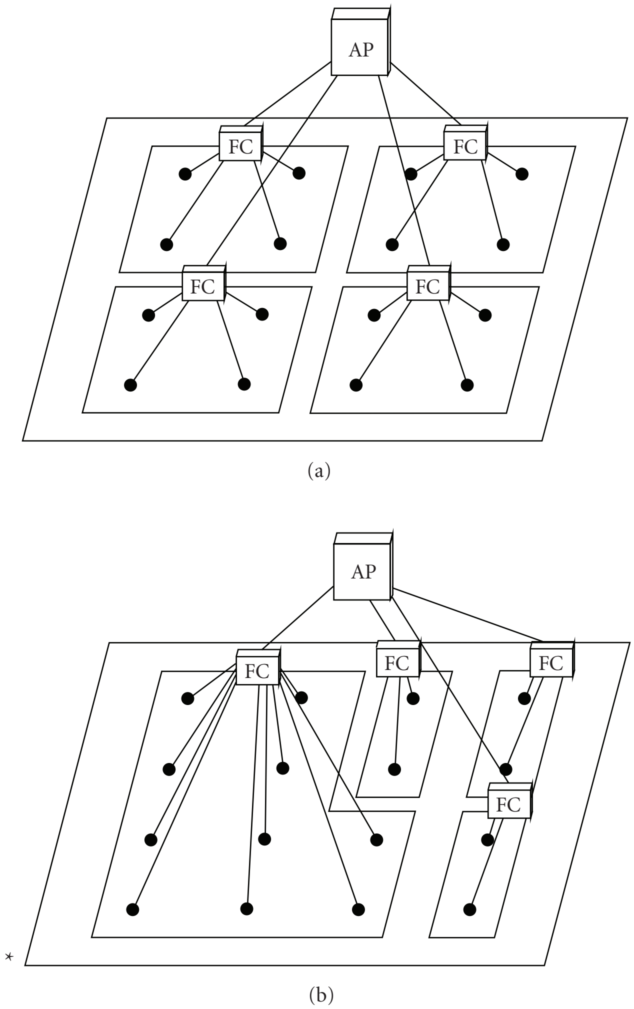


Figure 1: An example of clustered sensor networks with $N = 16$ sensors: (a) uniform clustering and (b) nonuniform clustering.

threshold τ needs to be optimized. This optimization is carried out in the derivation of all results presented in the following by minimizing the probability of decision error at the AP. This optimization is carried out by considering all possible values of τ in an interval $(\tau_{\min}, \tau_{\max})$, whose extremes are properly chosen ($\tau_{\min} = 0$ and $\tau_{\max} = s$). However, our results show that for practical values of the sensor SNR, $\tau \simeq s/2$ is the optimal choice for all configurations.

In a scenario with ideal communication links, the N sensors observe the common binary phenomenon H and send their decisions $\{u_i\}$ to the n_c FCs. Each of the n_c clusters contains d_c sensors, with $N = n_c \cdot d_c$. The jth FC ($j = 1, \ldots, n_c$) performs an information fusion, and computes a local decision according to the following majority-like rule [6]:

$$\hat{H}_j = \Gamma\left(u_1^{(j)}, \ldots, u_{d_c}^{(j)}\right) = \begin{cases} 0 & \text{if } \sum_{m=1}^{d_c} u_m^{(j)} < k, \\ 1 & \text{if } \sum_{m=1}^{d_c} u_m^{(j)} \geq k, \end{cases} \tag{5}$$

where k is the threshold[2] at the FCs and $u_i^{(j)}$ ($i = 1, \ldots, d_c$ and $j = 1, \ldots, n_c$) is the decision at the ith sensor in the jth cluster.

The decisions generated by the FCs are sent to the AP, which makes the following final decision:

$$\hat{H} = \Theta(\hat{H}_1, \ldots, \hat{H}_{n_c}) = \begin{cases} H_0 & \text{if } \sum_{m=1}^{n_c} \hat{H}_m < k_f, \\ H_1 & \text{if } \sum_{m=1}^{n_c} \hat{H}_m \geq k_f, \end{cases} \tag{6}$$

where k_f is the AP threshold. Using a combinatorial approach (based on the use of the repeated trials formula [27]), one can write the probability of decision error as [26]

$$\begin{aligned} P_e = {} & p_0 \text{bin}(k_f, n_c, n_c, i, \text{bin}(k, d_c, d_c, j, 1 - \Phi(\tau))) \\ & + (1 - p_0) \text{bin}(0, k_f - 1, n_c, i, \text{bin}(k, d_c, d_c, j, 1 - \Phi(\tau - s))), \end{aligned} \tag{7}$$

where $\Phi(x) \triangleq \int_{-\infty}^{x} (1/\sqrt{2\pi}) \exp(-y^2/2) \mathrm{d}y$ and $\text{bin}(a, b, n, z) \triangleq \sum_{i=a}^{b} \binom{n}{i} z^i (1-z)^{(n-i)}$, $0 \leq z \leq 1$. It can be shown that the probability of decision error (7) reduces to that derived in [8] if $n_c = d_c = 1$, that is, there is no clustering. The proposed approach can be straightforwardly extended to decentralized detection schemes with a generic number of decision levels, that is, schemes characterized by the presence of more than one layer of FCs between the sensors and the AP [28].

In general, one can assume that the communication links are *noisy*. In [8], a noisy link is modeled as a BSC with *crossover* probability p. In particular, we assume that only the links between the sensors and the FCs are noisy. The higher-level links in the network, that is, those between the FCs and the AP, are assumed ideal. In fact, in a realistic scenario, the network designer is likely to be able to control the placement of the FCs in the environment to be monitored. Therefore, the links between FCs and AP can be considered more reliable. We note that a BSC can model a large variety of communication channels and can be extended to account for more realistic communication constraints.

In order to apply the previous analytical approach to a scenario with noisy communication links, one can observe that only the terms $1 - \Phi(\tau)$ and $1 - \Phi(\tau - s)$ in (7) have to be properly modified, with respect to an ideal scenario, in order to take into account the presence of communication noise in the links between sensors and FCs. More precisely, these terms have to be replaced, respectively, by [8]

$$\begin{aligned} P_{c_0} &\triangleq [1 - \Phi(\tau)](1 - p) + \Phi(\tau) p, \\ P_{c_1} &\triangleq [1 - \Phi(\tau - s)](1 - p) + \Phi(\tau - s) p. \end{aligned} \tag{8}$$

In the following, in order to evaluate the impact of clustering on network lifetime, we will first investigate the network behavior in the case of ideal communication links. However, we

[2] The threshold k is the same for all the FCs, since the clusters are supposed to have the same dimension. An extension to the case of nonuniform clustering is provided in [26].

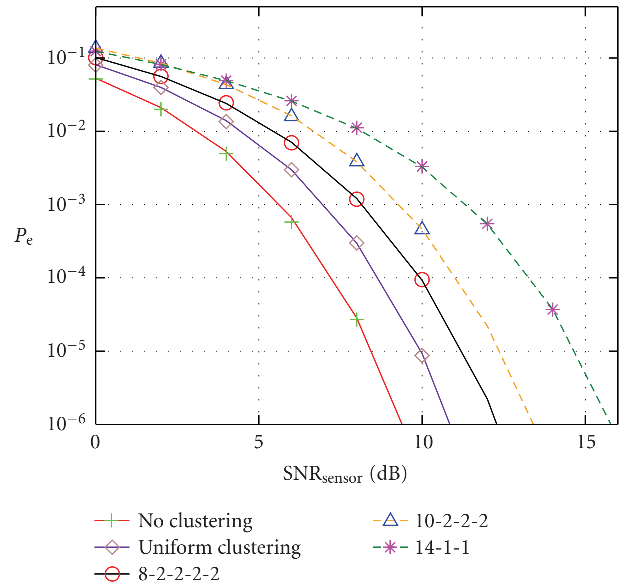


FIGURE 2: Probability of decision error, as a function of the sensor SNR, in a scenario with $N = 16$ sensors and equal a priori probabilities of the phenomenon ($p_0 = p_1 = 1/2$). Three different topologies are considered: (i) absence of clustering, (ii) uniform clustering, and (iii) nonuniform clustering (in this case, the specific configurations are indicated explicitly). Lines are associated with analytical results, whereas symbols are associated with simulation results.

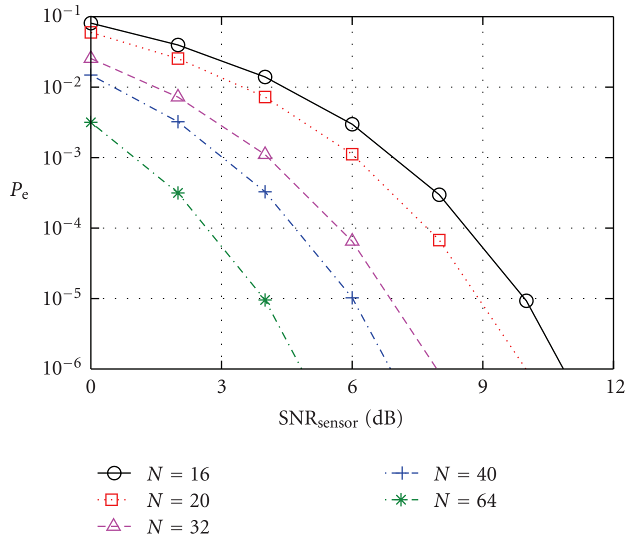


FIGURE 3: Probability of decision error, as a function of the sensor SNR, in a scenario with uniform clustering and equal a priori probabilities of the common binary phenomenon ($p_0 = p_1 = 1/2$). Different values of the number of sensors are considered.

will also extend our results to account to the presence of noisy communication links, evaluating their impact in Section 6.

In Figure 2, the probability of decision error is shown, as a function of the sensor SNR, in three possible scenarios with $N = 16$ sensors: (i) absence of clustering; (ii) uniform clustering; and (iii) nonuniform clustering. Both analytical (lines) and simulation (symbols) results are shown. As one can observe, there is excellent agreement between them—this is to be expected, since the analysis is *exact*. For nonuniform clustering, the derivation of the probability of decision error is similar to that outlined in this section. However, since the dimensions of the clusters are different, the derivation of the probability of decision error requires the use of a generalized version of the repeated trials formula [26]. All the topologies with uniform clustering, that is, 8-8 (2 clusters with 8 sensors each), 4-4-4-4 (4 clusters with 4 sensors each), and 2-2-2-2-2-2-2-2 (8 clusters with 2 sensors each), are characterized by the same performance curve. One can conclude that the performance does not depend, as long as clustering is uniform and the number of sensors N is given, on the particular distribution of the sensors among the clusters. In fact,

(i) in the presence of a few large clusters, the decisions from the FCs are already very reliable (before being fused at the AP);

(ii) in the presence of a large number of small clusters, the decisions from the FCs may not be very reliable, but the fusion operation allows to recover this lack of reliability.

In the presence of uniform clustering, the two effects (number of clusters and fusion at the AP) compensate with each other perfectly. For comparison, in Figure 2 the curves associated with no clustering and nonuniform clustering are also shown. For example, the label 10-2-2-2 denotes a sensor network with a 10-sensor cluster and three 2-sensor clusters (as shown in Figure 1(b)). The other labels have to be interpreted similarly. It is clear that the higher the nonuniformity degree is, the worse the performance is. On the other hand, uniform clustering leads to the minimum performance loss with respect to the case with the absence of clustering. Therefore, in the rest of this paper, we will consider only scenarios with uniform clustering. Based on the following derivation and the results in Figure 2, the reader can predict that the presence of nonuniform clustering will lead to a (possibly significant) network lifetime reduction.

In Figure 3, the probability of decision error is shown, as a function of the sensor SNR, for different values of the number of sensors N in a scenario with uniform clustering and equal a priori probabilities of the phenomenon ($p_0 = p_1 = 1/2$). In particular, the considered values for N are 16, 20, 32, 40, and 64. Observe that only one curve is associated with each value of N, since we have previously shown that the performance does not depend on the number of clusters (for a given N), as long as clustering is uniform. Obviously, the performance improves (i.e., the probability of decision error decreases) when the number of sensors in the network becomes larger. The results in Figure 3 will be used in Section 3 to compute the sensor network lifetime under a QoS condition on the maximum acceptable probability of decision error.

3. SENSOR NETWORK LIFETIME UNDER A PHYSICAL LAYER QOS CONDITION

In order to evaluate the sensor network lifetime, one needs first to define when the network has to be considered "alive." We assume that the network is "alive" until a given QoS condition is satisfied. Since the sensor network performance is characterized in terms of probability of decision error, the chosen QoS condition is the following:

$$P_{\mathrm{e}} \leq P_{\mathrm{e}}^{*}, \quad (9)$$

where P_{e}^{*} is the maximum tolerable probability of decision error at the AP. When a sensor in the network dies (e.g., there is a hardware failure or its battery exhausts), the probability of decision error increases since a lower number of sensors are alive (see, e.g., Figure 3). Moreover, the presence of a specific clustering configuration might make the process of network death faster. More precisely, the network dies when the desired QoS condition (9) is no longer satisfied, as a consequence of the death of a *critical sensor*. Therefore, the network lifetime corresponds to the lifetime of this critical sensor. Obviously, the criticality of a sensor's death depends on the particular sequence of previous sensors' deaths.

Based on the considerations in the previous paragraph, in order to estimate the *network* lifetime, one first needs to consider a reasonable model for the *sensor* lifetime. We denote by $F(t) \triangleq P\{T_{\mathrm{sensor}} \leq t\}$ the cumulative distribution function (CDF) of a sensor's lifetime T_{sensor} (the same for all sensors) and we consider the following four distributions as representative:

$$\begin{aligned}
\text{exponential:} \quad & F(t) = \left[1 - e^{-t/\mu}\right] U(t), \\
\text{uniform:} \quad & F(t) = \begin{cases} 0 & \text{if } t < 0, \\ \dfrac{t}{t_{\max}} & \text{if } 0 \leq t \leq t_{\max}, \\ 1 & \text{if } t > t_{\max}, \end{cases} \\
\text{Rayleigh:} \quad & F(t) = \left[1 - e^{-t^2/2\sigma_{\mathrm{ray}}^2}\right] U(t), \\
\text{lognormal:} \quad & F(t) = \left[\frac{1}{2} + \frac{1}{2}\mathrm{Erf}\left(\frac{\ln t - \zeta}{\sqrt{2\sigma_{\log}^2}}\right)\right] U(t),
\end{aligned} \quad (10)$$

where $\mathrm{Erf}(x) \triangleq (2/\sqrt{\pi}) \int_{-\infty}^{x} \exp(-y^2)\mathrm{d}y$ is the error function, $t_{\max}$ is a suitable maximum lifetime, and the time t is measured in arbitrary units (dimension (aU)). We have chosen the distributions in (10) as good models for a sensor lifetime. In fact, a realistic sensor should have a characteristic *average* value, whereas longer or shorter lifetimes should be less likely. Distributions like those in (10), with the exception of the uniform distribution (which is, however, interesting), comply with these characteristics.[3]

In order to obtain a "fair" comparison between different sensor lifetime distributions, we impose that the *average sensor lifetime* is the same for all the distributions in (10). Without loss of generality, we fix the average value of the exponential distribution (i.e., μ) and we impose that the other lifetime distributions have the same average value. After a few manipulations, one obtains that the parameters of the remaining distributions in (10) need to be set as follows:

$$\begin{aligned}
t_{\max} &= 2\mu, \\
\sigma_{\mathrm{ray}} &= \sqrt{\frac{2\mu^2}{\pi}}, \\
\zeta + \frac{\sigma_{\log}^2}{2} &= \ln\mu.
\end{aligned} \quad (11)$$

In particular, for a lognormal distribution (associated with the last equation in (11)), there are two free parameters: ζ and $\sigma_{\log}$. Therefore, one can set arbitrarily one of the two parameters, deriving the other consequently. In the following, various configurations for a lognormal distribution will be considered. We point out that a lognormal distribution allows to model, through proper choice of the parameters ζ and $\sigma_{\log}$, a large variety of realistic sensor lifetime distributions.

As mentioned in Section 2, we are interested in analyzing the network behavior when the QoS condition (9) is satisfied. More precisely, in the following subsections we evaluate the sensor network lifetime in scenarios with (A) ideal reclustering and (B) no reclustering. The obtained results are then commented.

3.1. Analysis with ideal reclustering

In the case of *ideal reclustering*, the network dynamically reconfigures its topology, immediately after a sensor's death, in order to recreate a uniform configuration. Obviously, the time needed for rearranging the network topology depends on the specific strategy chosen in order to reconfigure correctly (according to the updated network configuration) the connections between the sensors and the FCs and those between the FCs and the AP. In Section 4, a simple reconfiguration strategy will be proposed.

Given a maximum tolerable probability of decision error P_{e}^{*}, one can determine the lowest number of sensors, denoted as $N_{\min}$, required to satisfy the desired QoS condition. For instance, considering Figure 3 and fixing a maximum tolerable value P_{e}^{*}, one can observe that for decreasing numbers of sensors, at some point the actual probability of decision error P_{e} becomes higher than P_{e}^{*}. In other words, the probability of decision error is lower than P_{e}^{*} if *at least* $N_{\min}$ sensors are alive or, equivalently, until $N_{\mathrm{crit}} = N - N_{\min} + 1$ sensors

[3] We point out that the exponential distribution is typically considered to model the lifetime of a device [29, Chapter 8]. Another useful failure model is given by the Weibull distribution [29, Chapter 8]. However, considering the Rayleigh and lognormal distributions allow to model a large variety of scenarios as well. Further experimental investigation is needed to model accurately the lifetime of commercial sensors (in particular, large experimental test beds are required to obtain statistically reliable sensor lifetime distributions).

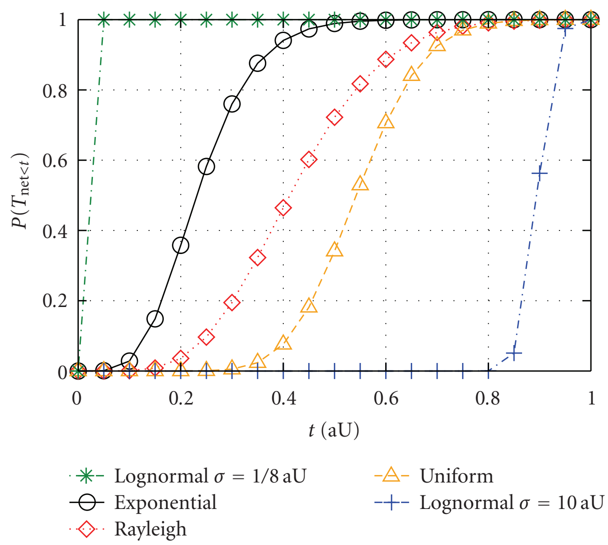


Figure 4: CDF of the network lifetime, as a function of time, in a scenario with $N = 32$ sensors, uniform clustering, *ideal reclustering*, and $\text{SNR}_{\text{sensor}} = 5$ dB. The QoS condition is set to $P_e^* = 10^{-3}$. All the distributions for the sensor lifetime in (10) are considered. Lines are associated with analysis, whereas symbols are associated with simulations.

die. Therefore, denoting as T_{net} the network lifetime, one can write

$$P(T_{\text{net}} \leq t) = P\{\text{at least } N_{\text{crit}} \text{ sensors have } T_{\text{sensor}} < t\}, \quad (12)$$

where T_{sensor} is the sensor lifetime (recall that this random variable has the same distribution for all sensors) with CDF $F(t)$. Since the lifetimes of different sensors are supposed independent, using the repeated trials formula, one obtains

$$P(T_{\text{net}} \leq t) = \sum_{i=N_{\text{crit}}}^{N} \binom{N}{N_{\text{crit}}} [F(t)]^i [1 - F(t)]^{N-i}. \quad (13)$$

In Figure 4, the CDF of the network lifetime is shown, as a function of time, in a scenario with $N = 32$ sensors grouped in uniform clusters. Ideal reclustering is considered. The sensor SNR is set to 5 dB and the maximum tolerable probability of decision error is $P_e^* = 10^{-3}$. In particular, we fix the average value of the exponential distribution to $\mu = 1$ aU, and consequently we derive the values for the parameters of the other distributions according to (11), obtaining $t_{\max} = 2$ aU (uniform distribution) and $\sigma_{\text{ray}} = 0.8$ aU (Rayleigh distribution). For the lognormal distribution, instead, we use two possible values for $\sigma_{\log}$ (10 and 1/8, resp.), and consequently two values for ζ (-50 aU and -0.008 aU, resp.). In Figure 4, both analytical (lines) and simulation (symbols) results are shown. As one can note, there is excellent agreement between them.

3.2. Absence of reclustering

In Section 3.1, we have analyzed the network evolution in an ideal scenario where the topology is dynamically reconfigured in response to a sensor death (e.g., because of the depletion of its battery or hardware failure). However, it might happen that the initial clustered configuration is fixed, that is, the connections between sensors, FCs, and AP cannot be modified after a sensor death. In this case, the following question is relevant: is there an optimum initial topology which leads to longest network lifetime? In order to answer this question, we will analyze the network evolution in scenarios where there is no reclustering. As in Section 3.1, the network is considered dead when the QoS condition (9) is no longer satisfied.

In the absence of ideal reclustering, an analytical performance evaluation is not feasible, that is, there does not exist a closed-form expression for the CDF of the network lifetime. In fact, the CDF depends on the particular network evolution, that is, it depends on how the sensors die among the clusters in the network. Therefore, each sequence of sensors' deaths is characterized by a specific lifetime, and one needs to resort to simulations in order to extrapolate an average statistical characterization. The simulations are performed according to the following steps.

(1) The lifetimes of all N sensors are generated according to the chosen distribution and the sensors are randomly assigned to the clusters.
(2) The sensors' lifetimes are ordered in an increasing manner.
(3) After a sensor death, the network topology is updated.
(4) The probability of decision error is computed in correspondence to the surviving topology determined at the previous point: if the QoS condition (9) is satisfied, then the evolution of the network continues from step 3, otherwise, step 5 applies.
(5) The network lifetime corresponds to the lifetime of the last dead sensor.

In Figure 5, the CDF of the network lifetime is shown, as a function of time, in a scenario with $N = 32$ sensors grouped, respectively, in 2, 4, and 8 clusters. The sensor SNR is set to 5 dB and the maximum tolerable probability of decision error is $P_e^* = 10^{-3}$. The distribution of a sensor lifetime is *exponential* (similar considerations can be carried out for the other distributions in (10)). For comparison, the curve associated with ideal reclustering is also shown. One can observe that the larger the number of clusters is, the worse the performance is, that is, the higher the probability of network death is. Moreover, the curve associated with 2 clusters is very close to that relative to ideal reclustering. In fact, in a scenario with only 2 clusters, the average number of sensors which die in each cluster is approximately the same, and consequently the topology remains approximatively uniform.

In Figure 6, the CDF of the network lifetime is shown, as a function of time, in a scenario with $N = 64$ sensors, uniform clustering, and considering, respectively, 2 clusters (solid lines) and 4 clusters (dashed lines). The operating

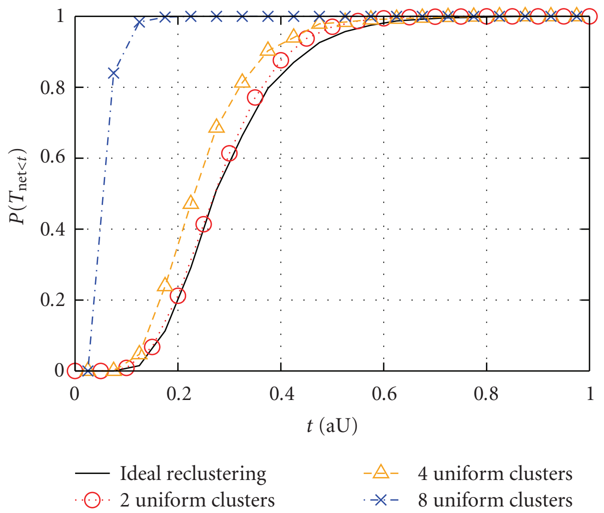


Figure 5: CDF of the network lifetime, as a function of time, in a scenario with $N = 32$ sensors, uniform clustering (with, resp., 2, 4, and 8 clusters), and *absence of reclustering* (simulation results). The sensor SNR is set to 5 dB and the maximum tolerable probability of decision error is $P_e^* = 10^{-3}$. For comparison, the curve associated with ideal reclustering (analytical results) is also shown. Each sensor has an exponential distribution.

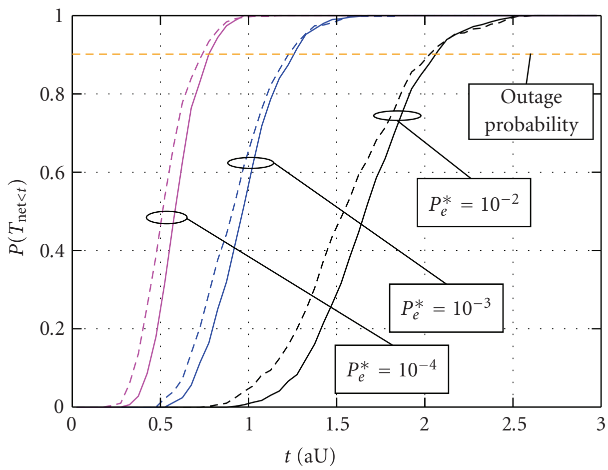


Figure 6: CDF of the network lifetime, as a function of time, in a scenario with $N = 64$ sensors, $\text{SNR}_{\text{sensor}} = 5$ dB, and *absence of reclustering* (simulation results). Three values for the maximum tolerable probability of decision error P_e^* are considered: (i) 10^{-2}, (ii) 10^{-3}, and (iii) 10^{-4}. Solid lines correspond to an initial topology with 2 clusters, whereas dashed lines are associated with an initial topology formed by 4 clusters. The distribution of the sensors' lifetime is exponential.

conditions are the same of those in Figure 5, and we consider three values for the maximum tolerable probability of decision error P_e^*: (i) 10^{-2}, (ii) 10^{-3}, and (iii) 10^{-4}, respectively. One can observe that similar to Figure 5, the higher the number of clusters in the network is, the shorter the network lifetime is. Moreover, the more stringent the QoS condition is (i.e., the lower P_e^* is), the shorter the network lifetime is (i.e., the higher the CDF is). This is to be expected, since if P_e^* is very low, then a relatively small number of sensors need to die in order to make the entire network die. Moreover, one can observe that the more stringent the QoS condition is (i.e., the lower is P_e^*), the steeper the CDF is, that is, the sensor network evolves rapidly (in a short interval) from life (i.e., full operating conditions) to death.

3.3. *Discussion*

In Table 1, the network lifetime corresponding to a CDF equal to 0.9 (i.e., an outage probability of 90%) is shown, assuming an *exponential* sensor lifetime (with $\mu = 1$ aU), for various clustering configurations and values of the maximum tolerable probability of decision error P_e^*. The number of sensors is $N = 64$. For comparison, the network lifetime with ideal reclustering is also shown. From the results in Table 1, the following observations can be carried out.

(i) For a small number of clusters (2 or 4), the lifetime reduction, with respect to a scenario with ideal reclustering, is negligible. This is to be expected from the results in Figures 5 and 6, and is due to the fact that the sensors die “more or less” uniformly in all clusters. When the number of clusters increases beyond 4, the network lifetime starts reducing appreciably. Therefore, our results show that in the *absence of ideal reclustering*, the winning strategy to prolong network lifetime is to *form few large clusters.*

(ii) The impact of the QoS condition is very strong. In fact, when the QoS condition becomes more stringent (i.e., P_e^* decreases), the network lifetime shortens, since a lower number of sensor deaths are sufficient to violate this condition. On the other hand, if the QoS condition is less stringent, then a larger number of sensors have to die in order to violate it.

(iii) The impact of the number of nodes on the network lifetime has not been directly analyzed. However, since the performance improves when the number of sensors increases (as shown in Figure 3), one can conclude that for a fixed QoS condition, a network with a larger number of sensors will satisfy the QoS condition for a longer time, and therefore the network lifetime will be prolonged. Equivalently, one can impose a stronger QoS condition (a lower value of P_e^*), still guaranteeing the same network lifetime.

4. ANALYTICAL COMPUTATION OF NETWORK LIFETIME

In Section 3, we have analyzed the network performance without taking into account the *cost* of reclustering. In this section, instead, we investigate, from an analytical viewpoint, the cost of the used reclustering protocol in terms of its impact on the sensor network lifetime. In order to evaluate the cost of reclustering, one first needs to detail a reclustering protocol. We note that we limit ourselves mainly (but not

Table 1: Sensor network lifetime corresponding to an outage probability equal to 90% for the scenarios considered in Figure 6. The lifetime of each sensor has an exponential distribution with $\mu = 1$ aU. All time values in the table entries are expressed in aU.

P_e^*	Ideal reclustering	No reclustering (2 clusters)	No reclustering (4 clusters)	No reclustering (8 clusters)
10^{-2}	2.1	2.1	2.0	1.68
10^{-3}	1.3	1.3	1.2	1.012
10^{-4}	0.78	0.78	0.74	0.625

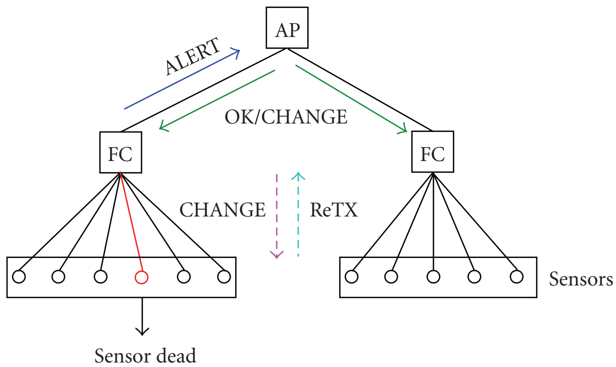


Figure 7: Message exchange in the proposed reclustering protocol. A network scenario with $N = 11$ sensors and two clusters (with 6 and 5 sensors, resp.) is considered. The control messages evolution follows the death of a sensor.

only) to scenarios with two (big) clusters, since they are associated with the minimum loss, in terms of probability of decision error at the AP, with respect to the scenario with the absence of clustering.

The reclustering protocol which will be used can be characterized as follows.

(1) When an FC senses that a sensor belonging to its cluster is dead, for example, when it does not receive packets from this sensor, it sends a control message, referred to as "ALERT," to the AP.

(2) Assuming that the AP is aware of the current network topology, when it receives an ALERT message, it decides if reclustering has to be carried out. If so, the optimized network topology is determined.

(3) If no reclustering is required, the AP sends to both FCs an "OK" message to confirm the current topology. On the other hand, if reclustering has to be carried out, another message, referred to as "CHANGE" and containing the new topology information, is sent to the FCs. In the latter case, the FCs send the CHANGE message also to sensors in order to allow them to communicate with the correct FC from then on.

(4) If reclustering has happened, the sensors retransmit their previous packet to the FCs according to the new topology and a new data fusion is carried out at the AP.

In Figure 7, the behavior of this simple protocol is pictured in an illustrative scenario with $N = 11$ sensors and two clusters (with 6 and 5 sensors, resp.). The control messages associated with solid lines are exchanged in the absence of reclustering, whereas the messages associated with dashed lines are exchanged in the presence of reclustering.

In order to derive a simple analytical framework for evaluating the sensor network lifetime, the following assumptions are expedient.

(a) The observation frequency, referred to as f_{obs}, is sufficiently low to allow regular transmissions from the sensors to the AP and, if necessary, the applicability of the reclustering protocol (this is reasonable for scenarios where the status of the observed phenomenon does not change rapidly).

(b) Transmissions between sensors and FCs and between FCs and AP are supposed instantaneous (this is reasonable, e.g., if FCs and AP are connected through wired links or very reliable wireless links).

(c) Data processing and topology reconfiguration are instantaneous (this is reasonable if the processing power at the AP is sufficiently high).

(d) There is perfect synchronization among all nodes in the network (this is a reasonable assumption if nodes are equipped with synchronization devices, e.g., global positioning system).

The proposed reclustering algorithm and the assumptions above might look too simplistic for a realistic wireless sensor network scenario. However, they allow to obtain significant insights about the cost, in terms of network lifetime, of adaptive reclustering.

We preliminary assume that the duration of a data packet transmission has no influence on the lifetime of a single sensor. A more accurate analysis, which takes properly into account the actual duration of a data transmission, will be proposed in Section 5. In this case, the network lifetime can be written as

$$D_{net} = \sum_{i=1}^{N_{crit}} T_{d,i}, \qquad (14)$$

where N_{crit} has been introduced in Section 3.1 and $T_{d,i}$ is the time interval between the $(i-1)$th sensor death and the ith sensor death. Obviously, $T_{d,1}$ is the time interval until the death of the first sensor and can be written as

$$T_{d,1} = \min_{j=1,\ldots,N} \{T_j\}, \qquad (15)$$

where T_j is the lifetime of the jth sensor. Since D_{net} is a random variable (RV), one could determine its statistics (e.g., the CDF). However, in order to concisely characterize the

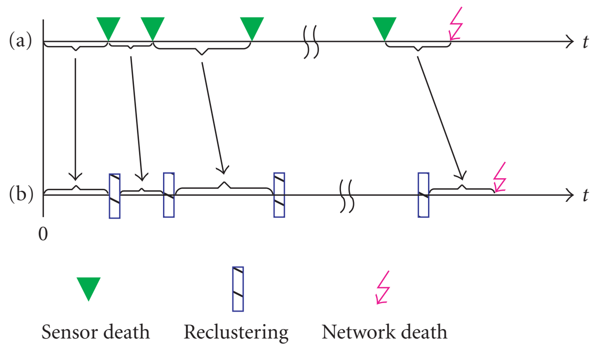


Figure 8: Pictorial description of the network time evolution. Two scenarios are considered: (a) absence of reclustering and (b) ideal reclustering.

impact of reclustering, it is of interest to evaluate its average value, that is,

$$\mathbb{E}[D_{\text{net}}] = \mathbb{E}\left[\sum_{i=1}^{N_{\text{crit}}} T_{\text{d},i}\right]. \tag{16}$$

In Figure 8, a pictorial description of the network evolution, as a function of time, is shown. Two scenarios are considered: (a) absence of reclustering and (b) ideal reclustering. In the figure, it is highlighted that the intervals between consecutive deaths are the same *regardless* of the presence/absence of reclustering. In the presence of reclustering, however, in correspondence to each death there is a network topology screening and, if necessary, reclustering.[4] In the following, we will evaluate the average network lifetime (16), following a theoretical approach, in both considered scenarios, that is, without reclustering and with ideal reclustering.

4.1. Absence of reclustering

In this case, N_{crit} and $\{T_{\text{d},i}\}$ in (16) are independent RVs. In fact, they depend on the sensors' lifetime distribution and the particular evolution (due to the nodes' deaths) of the network topology. Therefore, the sum in (16) is a stochastic sum. Using the conditional expectation theorem [27], one can write

$$\begin{aligned}\mathbb{E}\left[\sum_{i=1}^{N_{\text{crit}}} T_{\text{d},i}\right] &= \mathbb{E}_{N_{\text{crit}}}\left[\mathbb{E}_{\{T_{\text{d},i}\}}\left[\sum_{i=1}^{N_{\text{crit}}} T_{\text{d},i} \mid N_{\text{crit}}\right]\right]\\ &= \mathbb{E}_{N_{\text{crit}}}\Bigg[\underbrace{\sum_{i=1}^{N_{\text{crit}}} \mathbb{E}_{T_{\text{d},i}}[T_{\text{d},i}]}_{\triangleq f(N_{\text{crit}})}\Bigg]\\ &= \mathbb{E}[f(N_{\text{crit}})],\end{aligned} \tag{17}$$

where the fact that $\mathbb{E}_{T_{\text{d},i}}[T_{\text{d},i} \mid N_{\text{crit}}] = \mathbb{E}_{T_{\text{d},i}}[T_{\text{d},i}]$ (due to the independence between $T_{\text{d},i}$ and N_{crit}) has been used. By applying the fundamental theorem of probability [27], it follows that

$$\begin{aligned}\mathbb{E}[f(N_{\text{crit}})] &= \sum_{j=1}^{N} f(N_{\text{crit}} = j)P(N_{\text{crit}} = j)\\ &= \sum_{j=1}^{N} P(N_{\text{crit}} = j)\sum_{i=1}^{j}\mathbb{E}[T_{\text{d},i}].\end{aligned} \tag{18}$$

At this point, one needs to resort to simulations to compute the probabilities $\{P(N_{\text{crit}} = j)\}$. In fact, they strongly depend on the particular network evolution before its death. Numerical results will be presented in Section 4.4.

4.2. Ideal reclustering

In Section 3, we have shown that the presence of ideal reclustering leads to an upper bound on the network lifetime, that is, it tolerates the maximum number of sensors' deaths before the network dies. This bound can be analytically evaluated using (16) and replacing N_{crit} with the value n_{crit}^R defined as follows:

$$n_{\text{crit}}^R = \min_{n_{\text{crit}}=1,\ldots,N}\{P_{\text{e}}(\text{after } n_{\text{crit}} \text{ sensors' deaths}) \geq P_{\text{e}}^*\}. \tag{19}$$

The value of n_{crit}^R can be determined by numerical inversion of the QoS condition. Therefore, an upper bound for the network lifetime can be expressed as

$$\text{UB}_{D_{\text{net}}} \triangleq \mathbb{E}[D_{\text{net}} | N_{\text{crit}} = n_{\text{crit}}^R] = \sum_{i=1}^{n_{\text{crit}}^R}\mathbb{E}[T_{\text{d},i}]. \tag{20}$$

In this case, one can observe that the sum in (20) is deterministic, and therefore can be analytically evaluated through the computation of $\{\mathbb{E}[T_{\text{d},i}]\}$. Using (15), one obtains

$$\mathbb{E}[T_{\text{d},1}] = \mathbb{E}\left[\min_{i=1,\ldots,N}\{T_i\}\right]. \tag{21}$$

In the case of an *exponential* distribution with parameter $1/\mu$ (as considered in Section 3.2), after a few manipulations it follows that

$$\mathbb{E}[T_{\text{d},1}] = \frac{\mu}{N}. \tag{22}$$

In order to compute the average values of $\{T_{\text{d},i}\}$ ($i = 2,\ldots,N$), one has to observe that the probability density function (PDF) of $T_{\text{d},i}$ can be easily derived when the order statistics are independent and identically distributed (i.i.d.) with exponential distribution [30]. A simple derivation of the PDF of $T_{\text{d},i}$ ($i = 2,\ldots,N$) is provided in Appendix A. In this case, one can show that

$$\mathbb{E}[T_{\text{d},i}] = \mu\frac{N-i}{(N-i+1)^2}, \quad i = 2,\ldots,N. \tag{23}$$

[4] In Figure 8, we assume that the time spent in the case of no reclustering after a sensor death is the same as that in the case with reclustering. However, in general they might be different.

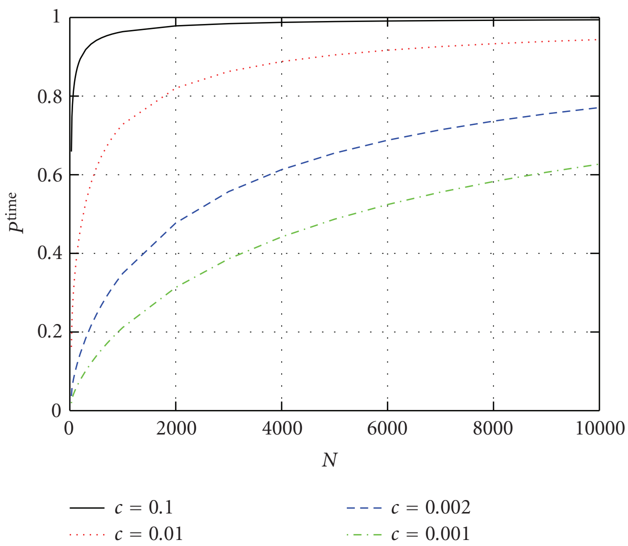


FIGURE 9: Time penalty, as a function of the number of sensors N, in a scenario with $\mu = 1$ aU. Four possible values of c are considered: (i) 0.1, (ii) 0.01, (iii) 0.002, and (iv) 0.001.

Substituting (22) and (23) in (20), it follows that

$$\text{UB}_{D_{\text{net}}} = \frac{\mu}{N} + \sum_{i=2}^{n_{\text{crit}}^R} \mu \frac{N-i}{(N-i+1)^2}. \quad (24)$$

Finally, one needs to evaluate the extra time required by the application of the reclustering procedure. We will refer to this quantity as T_R. Under the given assumptions and since the probability that reclustering has happened is equal to 1/2 (the derivation of this probability is summarized in Appendix B), T_R can be expressed as

$$T_R = (n_{\text{crit}}^R - 1)\, T_{\text{RECL}}, \quad (25)$$

where T_{RECL} represents the time required by a single reclustering operation.[5] The duration of this time interval cannot be a priori specified, since it depends on the dimensions of the OK, CHANGE, and ALERT messages, the data rate, and other network parameters. It is reasonable to assume that the longer the average sensor lifetime μ is, the shorter (proportionally) T_{RECL} should be. In other words, one could assume $T_{\text{RECL}} = c \cdot \mu$, where c is small if μ is large and vice versa. In general, c can be chosen to model accurately the situation of interest.

Finally, one can define a *time penalty* as the ratio between the time necessary for the application of the reclustering protocol and the total time, given by the sum of reclustering and "useful" times (i.e., the time spent for data transmission). It follows that

$$P^{\text{time}} = \frac{T_R}{T_R + \mathbb{E}[D_{\text{net}}]} = \frac{(n_{\text{crit}}^R - 1)\, T_{\text{RECL}}}{(n_{\text{crit}}^R - 1)\, T_{\text{RECL}} + \mu/N + \sum_{i=2}^{n_{\text{crit}}^R} \mu\left((N-i)/(N-i+1)^2\right)}. \quad (26)$$

After a few manipulations, one obtains

$$P^{\text{time}} = \frac{(n_{\text{crit}}^R - 1)c}{(n_{\text{crit}}^R - 1)c + 1/N + \sum_{i=2}^{n_{\text{crit}}^R}\left((N-i)/(N-i+1)^2\right)} \geq \frac{(n_{\text{crit}}^R - 1)c}{(n_{\text{crit}}^R - 1)c + 1/N + \sum_{i=N-n_{\text{crit}}^R}^{N-2}(1/i)}, \quad (27)$$

where we have used the fact that

$$\sum_{i=2}^{n_{\text{crit}}^R} \frac{N-i}{(N-i+1)^2} \leq \sum_{i=2}^{n_{\text{crit}}^R} \frac{1}{N-i}. \quad (28)$$

Our results show that the critical number of sensors' deaths is proportional to the number of sensors (as will be more clearly shown in Figure 11(b)), that is, $n_{\text{crit}}^R \simeq N - k^*$, where k^* is a proper constant which depends only on the value of P_e^* (but not on N). After a few mathematical passages, from (27) it follows that

$$P^{\text{time}} \gtrsim \frac{(N - k^* - 1)c}{(N - k^* - 1)c + 1/N + \ln(N-2) - \ln(k^* - 1)}, \quad (29)$$

where we have used the fact that $\sum_{i=1}^{m} 1/i \simeq \ln m + 0.577$ [31].

In Figure 9, P^{time} is shown, as a function of N, in the case with $\mu = 1$ aU. Four different values for c are considered: (i) 0.1, (ii) 0.01, (iii) 0.002, and (iv) 0.001. One can observe that when the number of sensors is large, the reclustering procedure is not effective, since it is associated with the maximum time penalty $P^{\text{time}} = 1$. From (29) and owing to the fact that k^* is approximately constant, one can analytically show that

$$\lim_{N \to \infty} P^{\text{time}} \simeq 1, \quad \forall c. \quad (30)$$

In other words, if the number of sensors is large, for a fixed value of c the proposed reclustering algorithm does not guarantee a limited time penalty. Similarly, one can show that

$$\lim_{c \to 0} P^{\text{time}} \simeq 0, \quad \forall N. \quad (31)$$

In other words, for a fixed number of nodes, the reclustering protocol is effective, using the algorithm proposed in Section 4, *provided that* the duration of a single reclustering operation is sufficiently short (e.g., very small control packets are used). Moreover, one can observe that the higher the number of sensors is, the weaker the impact of reclustering is. In fact, when N is (relatively) small, the slope of the penalty curve is higher than that for a (relatively) large number of

[5] The time duration T_{RECL} is assumed to be the same regardless of the fact that an actual reclusterization takes place. This is in agreement with the pictorial description in Figure 8.

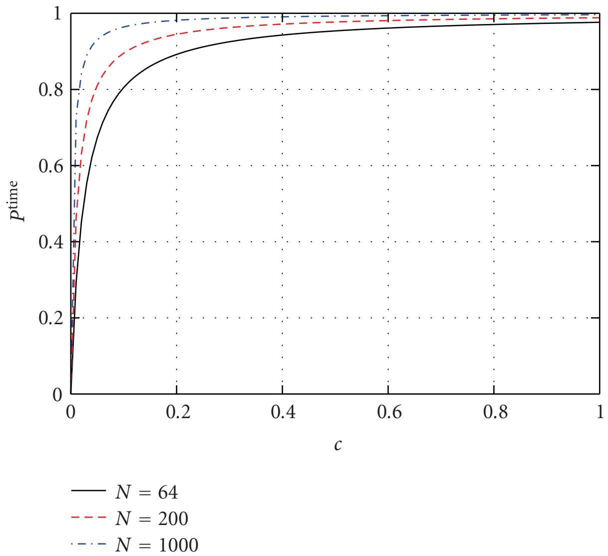


FIGURE 10: Time penalty, as a function of the fraction of reclustering time c, in a scenario with $\mu = 1$ aU. Three possible values of N are considered: (i) 64, (ii) 200, and (iii) 1000.

sensors. Therefore, this suggests that the proposed reclustering protocol is scalable for large values of N.

In Figure 10, the time penalty P^{time} is shown, as a function of c, in the case with $\mu = 1$ aU, considering three different values of the number of sensors N: (i) 64, (ii) 200, and (iii) 1000. Considerations similar to those made for Figure 9 can be carried out. In fact, the limiting behaviors (for $N \to \infty$ and $c \to 0$, resp.) of P^{time} are confirmed. Moreover, for a fixed value of c, one can observe that the distances between the curves are approximately the same. As previously observed, the protocol is scalable for increasing numbers of sensors. Finally, the protocol is effective when the time spent in reclustering operations is much shorter than the average sensor lifetime, that is, when $c \ll 1$.

4.3. Lower bound

Finally, we derive a simple lower bound on the network lifetime. This bound, for a fixed number of sensors, is obtained when all sensors' deaths occur in the same cluster. In this way, for a fixed topology, the highest possible probability of decision error is obtained at each instant, and consequently the corresponding network lifetime is the shortest possible. This bound can be expressed as

$$\text{LB}_{D_{\text{net}}} \triangleq \mathbb{E}[D_{\text{net}} \mid N_{\text{crit}} = n_{\text{crit}}^{\text{LB}}] = \frac{\mu}{N} + \sum_{i=2}^{n_{\text{crit}}^{\text{LB}}} \mu \frac{N-i}{(N-i+1)^2}. \tag{32}$$

Expression (32) for $\text{LB}_{D_{\text{net}}}$ is derived from (24) by replacing n_{crit}^{R} with $n_{\text{crit}}^{\text{LB}}$, which is obtained through simulations, since it depends on the network evolution. The value of $\text{LB}_{D_{\text{net}}}$ is smaller than that of $\text{UB}_{D_{\text{net}}}$, since $n_{\text{crit}}^{R} > n_{\text{crit}}^{\text{LB}}$. As previously mentioned, we consider an initial topology with two big clusters. In fact, this scenario allows to obtain the lowest probability of decision error at each instant, because the network topology is less unbalanced than in scenarios with a higher number of clusters, for example, 8. Therefore, evolution of the lower bound (32) in correspondence to a scenario with two clusters leads to the tightest possible lower bound with respect to a scenario with no reclustering.

4.4. Numerical results

In Figure 11, numerical results based on the application of the analytical framework derived in Sections 4.1, 4.2, and 4.3 are shown. In particular, (a) the average network lifetime $\mathbb{E}[D_{\text{net}}]$ and (b) the critical number of deaths N_{crit} are shown as functions of the number of sensors N. The average network lifetime in a scenario with no reclustering (for various numbers of clusters) is compared with the upper and lower bounds derived in Sections 4.2 and 4.3, respectively. The QoS condition is associated with $P_e^* = 10^{-3}$ and the sensor SNR is set to 5 dB. In order to compare these results with those in Section 3.2, the distribution of the sensors' lifetime is assumed to be exponential with $\mu = 1$ aU. From the results in Figure 11(a), one can observe that when the number of sensors increases, also the network lifetime becomes longer, since a larger number of sensors' deaths have to occur in order to violate the QoS condition. This is confirmed in Figure 11(b), where the critical number of sensors' deaths is shown as a function of the number of sensors. Moreover, as expected, the sensor network lifetime in the absence of reclustering is shorter than in the presence of ideal reclustering (with the proposed reclustering protocol), since the network topology becomes more and more nonuniform, and therefore the probability of decision error becomes higher and higher. As previously shown in Figure 5, when the initial number of clusters is equal to two, the network lifetime with no reclustering is very close to that corresponding to the application of the reclustering protocol. This is due to the fact that the sensors' deaths are, on average, equally distributed among the two clusters, that is, there is a sort of "natural" reclustering. Finally, one can observe that when the number of clusters in the initial topology increases (e.g., is equal to 8), the network lifetime drastically reduces for *low* values of the number of sensors, since it is more difficult to satisfy the QoS condition. However, it is interesting to observe that for sufficiently large values of N, the lifetime penalty incurred by the presence of a large number of clusters is negligible, suggesting that there may exist a minimum cluster dimension which guarantees acceptable performance. This is probably due to the fact that when the number of sensors is sufficiently large, the cluster dimension is also sufficiently large, and consequently its lifetime is longer. Therefore, the lifetime of the entire sensor network is longer, since the network topology is less unbalanced.

5. ENERGY BUDGET

The analysis of the reclustering cost provided in Section 4 is ideal, since it does not consider the energy spent by the nodes in the network. Although this assumption is reasonable for

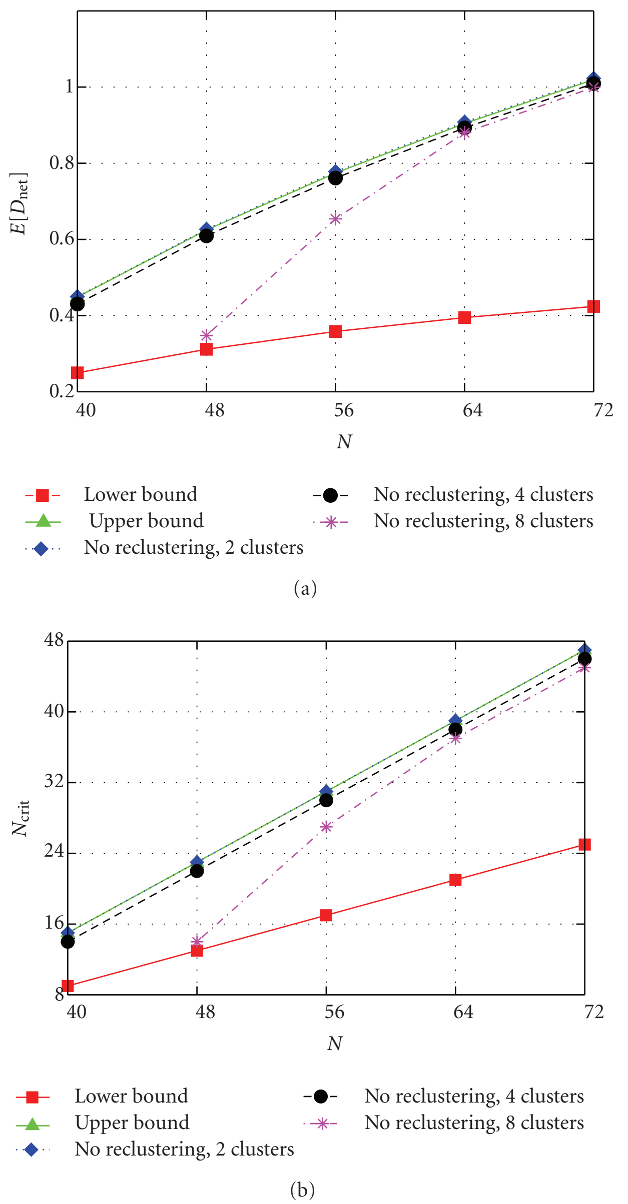


Figure 11: Sensor network performance using the proposed reclustering algorithm: (a) network lifetime and (b) critical number of deaths, as functions of the number of sensors. The performance in the absence of reclustering (with 2, 4, and 8 clusters, resp.) is compared with the proposed upper bound $\mathrm{UB}_{D_{\mathrm{net}}}$ and lower bound $\mathrm{LB}_{D_{\mathrm{net}}}$. The QoS condition is $P_e^* = 10^{-3}$ and the sensor SNR is set to 5 dB. The distribution of a sensor lifetime is exponential with $\mu = 1$.

the FCs and the AP,[6] this is not realistic for remote nodes (sensors) which need to rely on an energy-limited battery. Moreover, there exists a delay associated with a packet transmission. In this section, the realistic network energy consumption is evaluated in the presence of ideal reclustering, using the reclustering protocol proposed in Section 4. In order to analyze this energy consumption, we will refer to a commercial wireless sensor network with a communication protocol based on the IEEE 802.15.4 standard (also considered in Section 7) [24]. In particular, the analysis in Section 5.1 does not take into account the energy of the sensor battery, whereas in Section 5.2 we show the impact of an energy-limited battery at the sensors.

5.1. Analysis with infinite energy battery at the sensors

The energetic cost, for a single sensor, of the application of our reclustering algorithm can be written as

$$C_{\mathrm{tot}}^{\mathrm{en}} = P_t C_{\mathrm{tot}}^{\mathrm{time}}, \tag{33}$$

where $C_{\mathrm{tot}}^{\mathrm{en}}$ is the total cost in terms of energy spent by a sensor, P_t is the transmit power at each sensor, and $C_{\mathrm{tot}}^{\mathrm{time}}$ is the total time cost associated with packet transmission. In particular, the cost (in terms of both time and energy) has two components, associated with (i) *data* packet transmission and (ii) *control* packet transmission, respectively. The total time cost can be written as

$$C_{\mathrm{tot}}^{\mathrm{time}} = C_{\mathrm{data}}^{\mathrm{time}} + C_R^{\mathrm{time}}, \tag{34}$$

where $C_{\mathrm{data}}^{\mathrm{time}}$ and C_R^{time} are the time costs for transmissions of data and control packets (due to reclustering), respectively.

(i) We first evaluate the time cost for transmission of control packets. Assuming that the FCs and the AP are power-supplied, the cost associated with the reclustering protocol is given only by sensors' retransmissions.[7] Therefore, it is obtained that

$$\begin{aligned} C_R^{\mathrm{time}} &= \sum_{i=1}^{n_{\mathrm{crit}}^R - 1} P_R \left[C_{\mathrm{rx}}^{\mathrm{time}} + C_{\mathrm{retx}}^{\mathrm{time}} \right] \\ &= \left[P_R \left(C_{\mathrm{rx}}^{\mathrm{time}} + C_{\mathrm{retx}}^{\mathrm{time}} \right) \right] \left(n_{\mathrm{crit}}^R - 1 \right). \end{aligned} \tag{35}$$

The terms appearing in the final expression in (35) can be characterized as follows.

(a) P_R denotes the probability that reclustering has happened. It is equal to 1/2, as previously discussed in Section 4.2 (see Appendix B), and does not depend on the particular reclustering event.

(b) $C_{\mathrm{rx}}^{\mathrm{time}}$ denotes time necessary at the sensors to receive the CHANGE control packet from the FCs. This term is equal to L_{cont}/R_b, where L_{cont} is the length of a control packet (dimension (b/pck)) and R_b is the data rate (dimension (b/s)).

(c) $C_{\mathrm{retx}}^{\mathrm{time}}$ denotes time necessary at the sensors to retransmit their previous decisions to the FCs. This term is equal to L_{data}/R_b, where L_{data} is the length of a data packet (dimension (b/pck)).

[6] In fact, they may be placed by the network designer so that they can be power-supplied.

[7] The proposed analysis can be extended, however, taking into account possible energy consumption at the FCs and AP.

Therefore, the time cost for control packets can be expressed as follows:

$$C_R^{\text{time}} = \frac{1}{2}\left[\frac{L_{\text{cont}} + L_{\text{data}}}{R_b}\right](n_{\text{crit}}^R - 1). \tag{36}$$

(ii) The time used to transmit "useful" data, instead, can be expressed, following the derivation in Section 4.2, as

$$C_{\text{data}}^{\text{time}} = \sum_{i=1}^{n_{\text{crit}}^R}\{\text{number of transmissions in interval } i\} \times \{\text{time cost per packet}\}, \tag{37}$$

where an average number of packets have to be considered in each interval (in fact, the number of packets is a random variable—see Section 4). The previous equation can be easily rewritten as

$$C_{\text{data}}^{\text{time}} = \frac{L_{\text{data}}}{R_b} f_{\text{obs}} \sum_{i=1}^{n_{\text{crit}}^R} \mathbb{E}[T_{\text{d},i}], \tag{38}$$

where $\mathbb{E}[T_{\text{d},i}]$ ($i = 1, \ldots, n_{\text{crit}}^R$) can be computed according to (22) and (23).

Substituting (36) and (38) in (34) and (33), the total energetic cost can be written as

$$C_{\text{tot}}^{\text{en}} = P_t\left\{\frac{L_{\text{data}}}{R_b} f_{\text{obs}} \sum_{i=1}^{n_{\text{crit}}^R} \mathbb{E}[T_{\text{d},i}] + \frac{1}{2}\left[\frac{L_{\text{cont}} + L_{\text{data}}}{R_b}\right](n_{\text{crit}}^R - 1)\right\}. \tag{39}$$

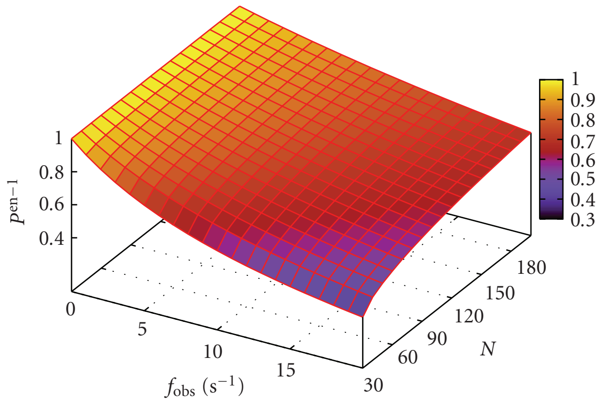
Expression (39) for the energetic cost represents the total energy spent by any of the $N - n_{\text{crit}}^R$ surviving sensors after the network death. Obviously, this energetic cost represents a worst case, since there are n_{crit}^R nodes (i.e., those which die while the network is still alive) which spend a lower amount of energy in their shorter lifetimes. An average cost per sensor can be easily computed using the same approach proposed above. In Appendix C, the following expression for the average energy cost is derived:

$$\begin{aligned}\overline{C}_{\text{tot}}^{\text{en}} &= P_t\left(\overline{C}_R^{\text{time}} + \overline{C}_{\text{data}}^{\text{time}}\right)\\ &= P_t \frac{L_{\text{data}} f_{\text{obs}}}{R_b N} \sum_{i=1}^{n_{\text{crit}}^R}\left((N - n_{\text{crit}}^R)\mathbb{E}[T_{\text{d},i}] + \sum_{j=1}^{i}\mathbb{E}[T_{\text{d},j}]\right)\\ &\quad + P_t \frac{(n_{\text{crit}}^R - 1)(L_{\text{data}} + L_{\text{cont}})}{4R_b}.\end{aligned} \tag{40}$$

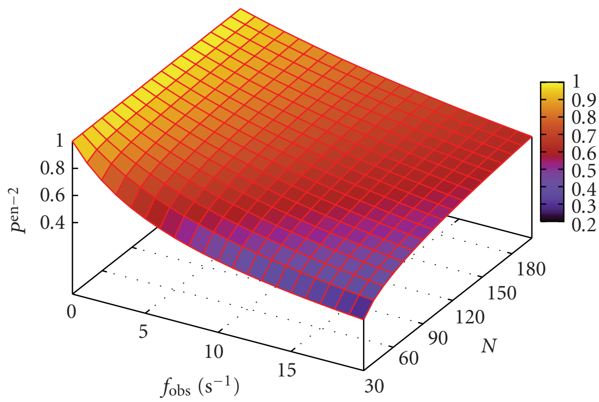
Similar to (26), we define the following *energy penalties*:

$$P^{\text{en}-1} \triangleq \frac{C_R^{\text{en}}}{C_{\text{tot}}^{\text{en}}} = \frac{C_R^{\text{time}}}{C_R^{\text{time}} + C_{\text{data}}^{\text{time}}}, \tag{41}$$

$$P^{\text{en}-2} \triangleq \frac{\overline{C}_R^{\text{en}}}{\overline{C}_{\text{tot}}^{\text{en}}} = \frac{\overline{C}_R^{\text{time}}}{\overline{C}_R^{\text{time}} + \overline{C}_{\text{data}}^{\text{time}}}, \tag{42}$$



(a)



(b)

FIGURE 12: Energy penalty, associated with the reclustering protocol, as a function of both the observation frequency f_{obs} and the number of sensors N. Two possible cases are considered: (a) *maximum* penalty (associated with a sensor which survives until the end) and (b) *average* penalty (among all the sensors in the network).

where $P^{\text{en}-1}$ is the *worst-case* penalty (associated with a sensor which survives until the end) and $P^{\text{en}-2}$ is the *average-case* penalty (associated with the average energetic costs among all sensors in the network). As mentioned at the end of Section 4.2, the energy penalties (41) and (42) take into account, with respect to (26), realistic network parameters, such as L_{data}, f_{obs}, R_b, and P_t.

In Figure 12, the energy penalty is shown, as a function of the number of sensors N and the observation frequency f_{obs}, in the two cases previously highlighted: (a) *worst-case* energy consumption (obtained by using expression (41)) and (b) *average-case* energy consumption (obtained by using expression (42)).[8] In order to compare the results in Figure 12 with the results given in the previous sections, we have set $P_e^* = 10^{-3}$ and $\text{SNR}_{\text{sensor}} = 5$ dB. Realistic values for the

[8] Note that the two figures seem similar. However, one should observe that the legends of the colors (on the right-hand side of the figures) are different.

network parameters, provided by the ZigBee standard, correspond to $P_t = 1$ mW, $R_b = 250$ Kb/s, $L_{\text{data}} = 1024$ b/pck, and $L_{\text{cont}} = 80$ b/pck.[9] One can note that for low values of the observation frequency (*rare observations*), the performance worsens since the network spends more time in reclustering than in transmitting useful data. For a fixed value of the number of sensors N, the following limits hold:

$$\lim_{f_{\text{obs}} \to 0} P^{\text{en}-1} = \frac{C_R^{\text{en}}}{C_R^{\text{en}}} = 1, \qquad \lim_{f_{\text{obs}} \to 0} P^{\text{en}-2} = \frac{\overline{C}_R^{\text{en}}}{\overline{C}_R^{\text{en}}} = 1. \quad (43)$$

Besides, one can observe that for increasing values of the observation frequency (*frequent observations*), the performance is better. In fact, for a fixed number of sensors, there is a larger number of data transmissions from the sensors to the AP and the value of D_R^{en} becomes increasingly negligible with respect to the value of $D_{\text{data}}^{\text{en}}$. Analytically, one can write

$$\lim_{f_{\text{obs}} \to \infty} P^{\text{en}-1} = \frac{1}{C_{\text{data}}^{\text{en}}} = 0, \qquad \lim_{f_{\text{obs}} \to \infty} P^{\text{en}-2} = \frac{1}{\overline{C}_{\text{data}}^{\text{en}}} = 0. \quad (44)$$

Note that a high value of the observation frequency might not be admissible. In fact, in Section 4 we have supposed that the inverse of the observation frequency is much smaller than the time necessary to complete a transmission to the AP and, eventually, the reclustering protocol (hypothesis (a) in Section 4).

5.2. *Analysis with energy-limited battery at the sensors*

In the previous derivations, the proposed framework and the presented results have used arbitrary time units. However, it is of interest to map these arbitrary time units into realistic units. In order to do so, we assume that a node is equipped with a limited-energy battery with initial energy E_{battery} (dimension (J)). When a sensor battery energy exhausts, the sensor dies, and consequently the network is closer to breaking the QoS condition. The average sensor lifetime (dimension (s)) can be expressed as

$$\mathbb{E}[T_{\text{sensor}}] = \frac{E_{\text{battery}}}{\overline{P}}, \quad (45)$$

where $\overline{P}$ is the average power depleted at the node (dimension (W)). In a realistic wireless sensor network (e.g., ZigBee wireless sensor networks [24]), four *states* are admissible at the node: (1) *transmission*, (2) *reception*, (3) *idle*, and (4) *sleep*. In this case, the average power depleted at the node is given by

$$\overline{P} = \sum_{i=1}^{4} P_i p_i, \quad (46)$$

where P_i and p_i ($i = 1, 2, 3, 4$) are, respectively, the power consumption in the ith state and the probability that the sensor is in the ith state—note that $P_1 = P_t$. Typically, in a ZigBee wireless sensor network, $P_4 \ll 1$ and $p_2 \ll p_3, p_1$ [33]. Therefore, the average depleted power in (46) can be written as

$$\overline{P} \simeq P_1 p_1 + P_2 p_2, \quad (47)$$

where $p_2 = 1 - p_1$ and $P_1 = P_2 = P_t$ [33]. Therefore, the average consumed power in (46) becomes

$$\overline{P} = P_t, \quad (48)$$

and it follows that

$$\mathbb{E}[T_{\text{sensor}}] = \frac{E_{\text{battery}}}{P_t}. \quad (49)$$

Using the value of $\mathbb{E}[T_{\text{sensor}}]$ given in (49) for the computation of $C_{\text{tot}}^{\text{time}}$ according to the framework derived in Section 5.1, the lifetime of a realistic ZigBee wireless sensor network, with the parameters used to derive the results in Figure 12, can be obtained. The sensor network lifetime values, associated with different battery energies at the sensors (typical for practical applications), are summarized in Table 2. In particular, a scenario with $N = 64$ sensors, $P_t = 1$ mW, and $f_{\text{obs}} = 20\ s^{-1}$ is considered. One can observe that the theoretical results given in Section 4.4 are confirmed also in a more realistic ZigBee wireless sensor network. However, note that for $N = 64$ sensors, the network lifetime in the ideal scenario is shorter than $\mathbb{E}[T_{\text{sensor}}]$, whereas it is longer in a realistic scenario. This behavior is due to the fact that our theoretical framework does not consider the delay associated with packet transmissions, as considered, instead, in the performance analysis for a ZigBee network.

Table 2: Sensor network lifetime for a realistic ZigBee wireless sensor network in a scenario with $N = 64$ sensors, $P_t = 1$ mW, and $f_{\text{obs}} = 20\ s^{-1}$. The ZigBee parameters are the same considered in Figure 12. Different values of the battery energy at a sensor are considered.

Battery energy E_{battery} (kJ)	Average sensor lifetime $\mathbb{E}[T_{\text{sensor}}]$ (days)	Sensor network lifetime $C_{\text{tot}}^{\text{time}}$ (days)
12.96 (400 mAh, 9 V)	150	196
19.44 (600 mAh, 9 V)	224	294
31.68	365	480
32.4 (1 Ah, 9 V)	375	491

[9] In our analysis, we use the maximum possible data rate allowed by the ZigBee standard, that is, $R_b = 250$ Kb/s. However, our experimental results show that only a maximum value $R_b = 25$ Kb/s can be achieved by practical sensor networks [32]. Moreover, the length of data packets include header (80 bits) and payload (994 bits) lengths and is the maximum allowed by the standard.

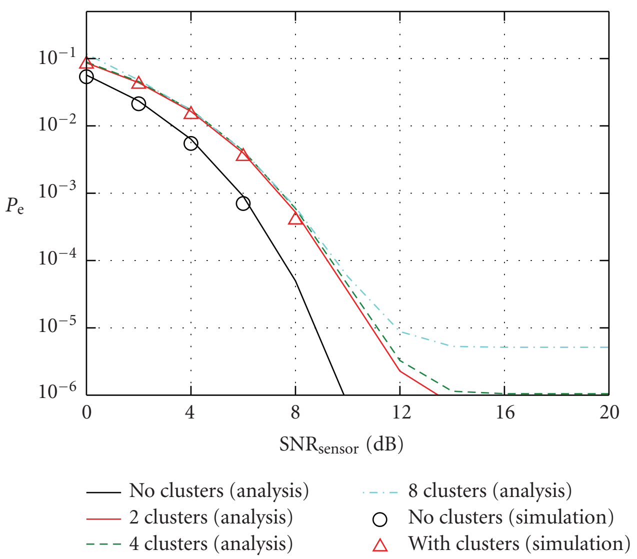


FIGURE 13: Probability of decision error, as a function of the sensor SNR, in a scenario with $N = 16$ sensors, uniform clustering, and equal a priori probabilities of the common binary phenomenon (i.e., $p_0 = p_1 = 1/2$). Communication links are noisy with $p = 10^{-2}$.

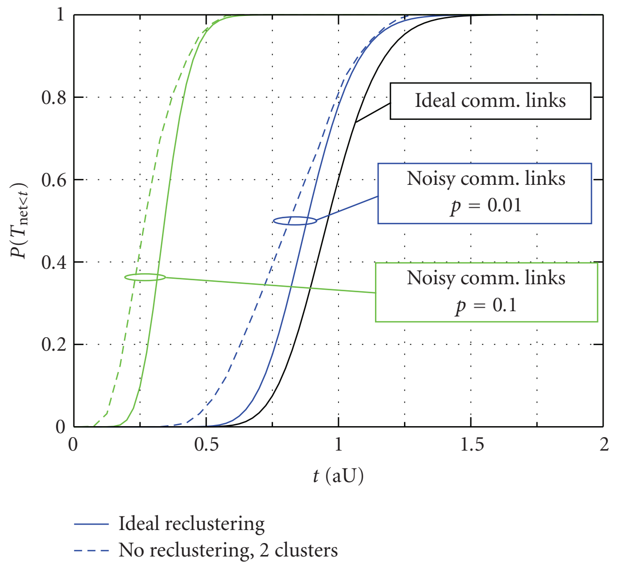


FIGURE 14: CDF of the network lifetime, as a function of time, in a scenario with $N = 64$ sensors, uniform clustering, and noisy communication links. Two possible values for the crossover probability are considered: (i) $p = 0.1$ and (ii) $p = 0.001$. The sensor SNR is set to 5 dB and the maximum tolerable probability of decision error is $P_e^* = 10^{-3}$. For comparison, the curve relative to ideal communication links is also shown. The distribution of a sensor lifetime is exponential.

6. NOISY COMMUNICATION LINKS

The analysis of the sensor network lifetime proposed in Section 4 is quite general and, in particular, no assumption has been made on the communication links. However, the results presented in Section 4.4 are obtained under the assumption of *ideal* communication links. In this section, we extend the previous derivation presenting numerical results for a scenario with noisy communication links.

As previously mentioned in Section 2, a BSC model with crossover probability p can be used to model a noisy communication link. The probability of decision error in a scenario with noisy communication links is shown, as a function of the sensor SNR, in Figure 13 [26, 28]. In this case, a network with $N = 16$ sensors, uniform clustering, and equal a priori probabilities of the common binary phenomenon (i.e., $p_0 = p_1 = 1/2$) is considered. The crossover probability of the BSC is $p = 10^{-2}$. Two main differences, with respect to a scenario with ideal communication links, can be observed.

(i) For a given value of the sensor SNR, the presence of noisy communication links leads to a performance loss (i.e., higher probability of decision error).

(ii) A probability of decision error floor can be visualized for high values of the sensor SNR.

These differences between the scenarios with ideal communication links and those with noisy communication links imply that the network lifetime will be shorter, since the QoS condition will be satisfied for a shorter time. Moreover, the presence of a probability of decision error floor implies that for a given value of the sensor SNR, the QoS condition might never be satisfied. These considerations suggest that the QoS condition and the operating sensor SNR, for a given value of the number of sensors N, have to be properly chosen.

In Figure 14, the CDF of the network lifetime is shown, as a function of time,[10] in a scenario with $N = 64$ sensors, uniform clustering, and noisy communication links. Two possible values for the crossover probability are considered: (i) $p = 0.1$ and (ii) $p = 0.001$. For comparison, the curve associated with ideal communication links is also shown. The distribution of a sensor lifetime is exponential. The sensor SNR is set to 5 dB and the maximum tolerable probability of decision error is $P_e^* = 10^{-3}$. One can observe that the higher the noise intensity in the communication links is, the higher the CDF of the network lifetime is. In fact, in this case the transfer of information from the sensors to the AP is less reliable, and consequently the probability of decision error becomes higher and higher and the QoS condition can be guaranteed for a shorter time. As in a scenario with ideal communication links, the presence of reclustering prolongs the network lifetime with respect to a scenario with no reclustering. Obviously, for a given reclustering strategy, a scenario with ideal communication links corresponds to a longer network lifetime, since the probability of decision error is the lowest possible.

[10] We recall that the time is measured, here, in arbitrary units. For more realistic scenarios, see the considerations at the end of Section 5.

7. THROUGHPUT AND DELAY WITH VARYING SENSOR NETWORK LIFETIMES

In this section, we evaluate the performance of a realistic ZigBee wireless sensor network subject to nodes' failures. In order to carry out this analysis, we resort to simulations using Opnet Modeler 11.5 [34] and a built-in model for IEEE 802.15.4 networks, provided by the National Institute for Standards and Technology (NIST) [35]. Since the NIST model only supports one-hop communications between the sensors and the AP, in this section we analyze the network performance (in terms of number of transmitted packets, throughput, and delay) in scenarios with no clustering (and, therefore, no reclustering). The goal of this section is to show the impact of different QoS conditions (given in terms of the required percentage of nodes' deaths which makes the network die) on different network performance indicators (e.g., throughput and delay). For the sake of simplicity, in this section we consider only scenarios with *no clustering*, since the performance in the presence of relaying is analyzed in [36]. As discussed in Section 2, the performance of sensor networks with no clustering can be considered, from a network lifetime viewpoint, as a lower bound, since the probability of decision error is lower than in scenarios with clustering. In the simulations, the following parameters are considered: $R_b = 250$ Kb/s, $L_{\text{data}} = 994$ b/pck, and $g = 0.236$ second, where g is the packet interarrival time at the sensors. Moreover, no transmission of acknowledgement packets is considered from the AP to the remote nodes. In all presented results, four QoS conditions will be considered: (i) network death corresponds to 100% of sensors' deaths (i.e., the network survives until there is a single sensor alive), (ii) network death corresponds to 70% of sensors' deaths, (iii) network death corresponds to 50% of sensors' deaths, and (iv) network death corresponds to 20% of sensors' deaths.

In Figure 15, the number of transmitted packets is shown, as a function of the number of sensors N, for two possible distributions of a single sensor lifetime: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines). First, one has to observe that the curves associated with a uniform distribution for the sensors' lifetime are higher than those associated with an exponential distribution. This is in agreement with the results presented in Figure 4, since in a scenario with uniform distribution of the sensors' lifetime there are more surviving nodes towards the end of network activity period. Consequently, a larger number of transmissions between sensors and AP are possible. Then, the more stringent the QoS condition is, the smaller the number of transmissions is since the sensor network lifetime is shorter, as previously discussed in Section 3.3.

In Figure 16, the throughput is shown, as a function of the number of sensors N, for two possible distributions of a single sensor lifetime: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines). The throughput is computed as

$$S = \frac{\text{number of received packets}}{\text{number of transmitted packets}}. \tag{50}$$

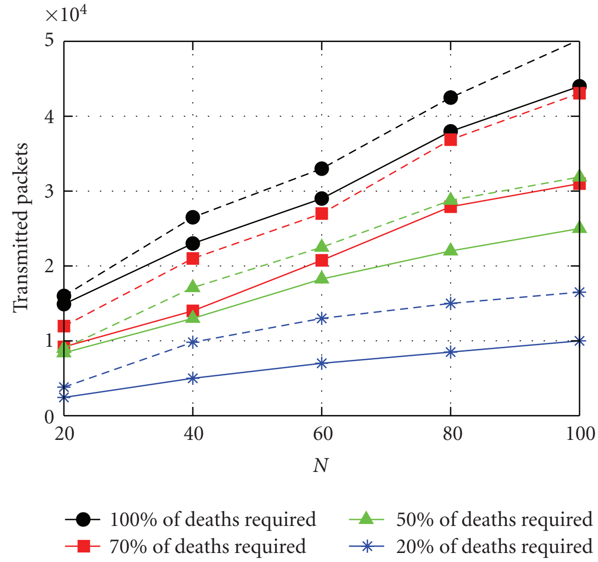


FIGURE 15: Number of transmitted packets, as a function of the number of sensors N, in a ZigBee wireless sensor network with nodes' failures. Two possible distributions for a single sensor lifetime are considered: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines).

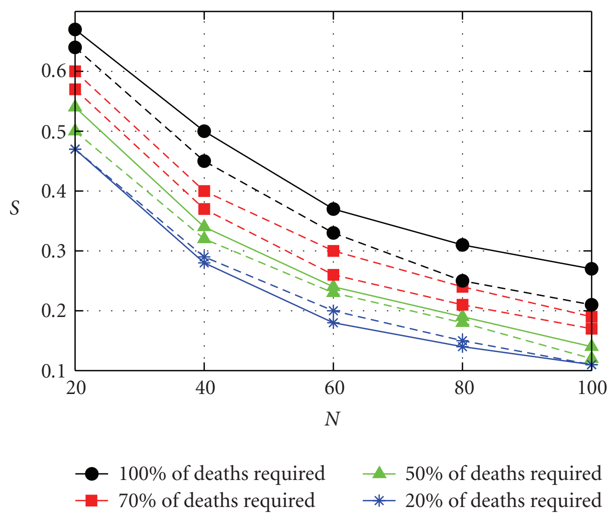


FIGURE 16: Throughput, as a function of the number of sensors N, in a ZigBee wireless sensor network with nodes' failures. Two possible distributions for a single sensor lifetime are considered: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines).

Similar to Figure 15, one can observe that the more stringent the QoS condition is, the lower the throughput is. In fact, a smaller number of transmissions are possible (since the network lifetime is shorter) and a larger number of collisions happen, because there are a large number of sensors which try to transmit to the AP and a larger number of packets are lost. Moreover, a scenario with uniform distribution of the sensors' lifetime has a lower throughput with respect to

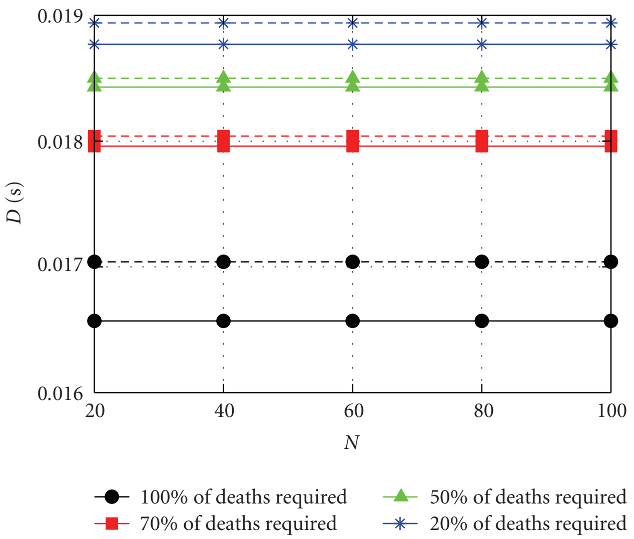


Figure 17: Average MAC delay D, as a function of the number of sensors N, in a ZigBee wireless sensor network with nodes' failures. Two possible distributions for a single sensor lifetime are considered: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines).

a scenario with exponential distribution, since more packets are lost due to the collisions.

In Figure 17, the average MAC delay[11] over all the received packets D is shown, as a function of the number of sensors N, for two possible distributions of a single sensor lifetime: (a) exponential with $\mu = 300$ seconds (solid lines) and (b) uniform with $t_{\max} = 600$ seconds (dashed lines). Similar to what happens for the throughput in Figure 16, a larger number of collisions also cause a higher delay in receiving the packets. Therefore, scenarios with a uniform distribution of the sensors' lifetimes are characterized by a higher delay with respect to scenarios with an exponential distribution. In this case as well, however, the more stringent the QoS condition is, the higher the average MAC delay is. Finally, the average MAC delay does not depend on the number of sensors, for a fixed QoS condition, since the number of surviving sensors is (almost) the same, and therefore the average delay in the packet transmissions is constant.

8. CONCLUDING REMARKS

In this paper, we have presented a framework to analyze the *network lifetime* of clustered sensor networks subject to a physical-layer-oriented QoS condition, given by the maximum tolerable probability of decision error at the AP. First, we have considered a model for the *sensor lifetime*, using a few distributions which may be representative of a realistic sensor lifetime. In the presence of ideal reclustering, the network lifetime is the longest possible. On the other hand, in the presence of a fixed clustered configuration, our results show that the number of clusters has a strong impact on the network lifetime. More precisely, the network lifetime is maximized if there are a *few large clusters* (at most four). In all cases, the QoS condition has a strong impact on the network lifetime: the more stringent this condition is, the shorter the network lifetime is. We have also evaluated the cost associated with the reclustering procedure, from both *time delay* and *energy consumption* perspectives. Our results show that reclustering is not useful when phenomenon observations are *rare*, since the network spends more time in transferring control messages than useful data. The impact of noisy communication links, modeled as BSCs, on the network lifetime has also been investigated, showing that the higher the noise level is, the shorter the network lifetime is. However, in this scenario as well, reclustering can prolong the network lifetime. Finally, we have presented a simulation-based analysis of realistic IEEE 802.15.4 wireless sensor networks. Our results show that typical network performance indicators (such as throughput and delay) are influenced by the network lifetime.

[11] The average MAC delay corresponds to the delay averaged over all packets which are correctly received at the MAC level during the Opnet simulations.

APPENDICES

A. ANALYTICAL COMPUTATION OF THE PDFS OF THE NUMBER OF TRANSMISSIONS WITH EXPONENTIAL SENSORS' LIFETIME

The PDF of the time interval $T_{d,1}$ until the first death of a sensor, denoted as $g_1(t)$, can be written as [30]

$$g_1(t) = N\left[1 - F(t)\right]^{N-1} f(t), \tag{A.1}$$

where $F(t)$ and $f(t)$ are, respectively, the CDF and the PDF of a single sensor lifetime. By using the proper expressions for $F(t)$ and $f(t)$ in the case of exponential distribution ($F(t) = (1 - \exp(-t/\mu))U(t)$ and $f(t) = 1/\mu \exp(-t/\mu)U(t)$), after a few manipulations, one obtains

$$g_1(t) = \frac{N}{\mu}\exp\left\{-\frac{t}{\mu}N\right\}U(t). \tag{A.2}$$

In general, the PDF of $W_{i,j} \triangleq T_j - T_i$ $(1 \le i < j \le N)$ can be computed as [30]

$$\begin{aligned} g_{i,j}(w) &= \frac{N!}{(i-1)!(j-i-1)!(N-j)!}\int_0^{\infty}\left[F(t)\right]^{i-1} \\ &\quad \times\left[F(t+w)-F(t)\right]^{j-i-1}\left[1-F(t+w)\right]^{N-j} \\ &\quad \times f(t)f(t+w)\mathrm{d}t \quad 0 \le w < \infty \\ &= \frac{N!}{(i-1)!(j-i-1)!(N-j)!}\frac{1}{\mu^2}\left[1-e^{-w/\mu}\right]^{j-i-1} \\ &\quad \times e^{-(w/\mu)(n-j+1)}\int_0^{\infty}\left[1-e^{-t/\mu}\right]^{\alpha}e^{-\beta(t/\mu)}\mathrm{d}t, \quad 0 \le w < \infty, \end{aligned} \tag{A.3}$$

where $\alpha \triangleq i-1$ and $\beta \triangleq N-i+1$. After a few manipulations, it follows that

$$\int_0^\infty [1-e^{-t/\mu}]^\alpha e^{-\beta(t/\mu)} dt = \mu \frac{\alpha(\alpha+1)\cdots(\alpha+\beta-2)\beta!}{\alpha+\beta}. \tag{A.4}$$

By using (A.4) in (A.3), one obtains

$$g_{i,j}(w) = \frac{(N-i)!}{(j-i-1)!(N-j)!} \frac{1}{\mu} [1-e^{-w/\mu}]^{j-i-1} e^{-w/\mu}. \tag{A.5}$$

Since $T_{d,i} = W_{i-1,i}$ $(i = 2,\dots,N)$, from (A.5) the PDF of $T_{d,i}$ $(i = 2,\dots,N)$ can be expressed as

$$g_i(t) = \frac{N-i}{\mu} \exp\left\{-\frac{t}{\mu}(N-i+1)\right\} U(t). \tag{A.6}$$

B. ANALYTICAL COMPUTATION OF THE PROBABILITY OF RECLUSTERING

In clustered scenarios with two (big) clusters and no reclustering, the probability that reclustering has happened at time instant t can be written as

$$P_R(t) = P(|d_{c,1}(t) - d_{c,2}(t)| > 1), \tag{B.1}$$

where $d_{c,k}(t)$ $(k = 1,2)$ is the number of sensors in the kth cluster at the generic instant t. Using the total probability theorem [27], expression (B.1) can be rewritten as

$$P_R(t) = P(\mathcal{D}_1 \mid \mathcal{E}_{2,t})P(\mathcal{E}_{2,t}) + P(\mathcal{D}_2 \mid \mathcal{E}_{1,t})P(\mathcal{E}_{1,t}), \tag{B.2}$$

where $\mathcal{D}_i$ $(i = 1,2)$ represents the event "death in the ith cluster,"[12] whereas $\mathcal{E}_{j,t}$ $(j = 1,2)$ represents the event "$d_{c,j}(t) = d_{c,l}(t) - 1$," $l \neq j$. By considering all possible combinations, one can easily write

$$\begin{aligned} P(\mathcal{D}_1 \mid \mathcal{E}_{2,t}) &= \frac{d_{c,1}(t)}{d_{c,1}(t) + d_{c,2}(t)}, \\ P(\mathcal{D}_2 \mid \mathcal{E}_{1,t}) &= \frac{d_{c,2}(t)}{d_{c,1(t)+d_{c,2}(t)}}. \end{aligned} \tag{B.3}$$

The probability $P(\mathcal{E}_{j,t})$ $(j = 1,2)$ can be equivalently written as the probability that at the time instant t', a sensor dies in cluster j, given the fact that the two clusters have the same dimension. The time instant t' is a generic time instant before the death of the sensor in the cluster j. By considering all possible combinations, one obtains that

$$P(\mathcal{E}_{1,t}) = P(\mathcal{E}_{2,t}) = \frac{1}{2}, \quad \forall t. \tag{B.4}$$

Substituting (B.3) and (B.4) in (B.1), it finally follows that

$$P_R(t) = \frac{1}{2}, \quad \forall t. \tag{B.5}$$

[12] Note that since sensors' lifetimes are independent, the events $\{D_i\}$ do not depend on t.

C. ANALYTICAL COMPUTATION OF THE AVERAGE ENERGY COST

The time costs for data and control messages at each node at the jth death can be written, similarly to (36) and (38), as

$$\begin{aligned} C^{\text{time}}_{\text{data},j} &= \frac{L_{\text{data}}}{R_b N} f_{\text{obs}} \sum_{i=1}^{j} \mathbb{E}[T_{d,i}], \\ C^{\text{time}}_{R,j} &= \frac{1}{2}\left(\frac{L_{\text{cont}} + L_{\text{data}}}{R_b}\right)\frac{j-1}{N}. \end{aligned} \tag{C.1}$$

By summing over all possible values of j (i.e., till the network death), one obtains the average time costs (data and control, resp.) per node:

$$\begin{aligned} C^{\text{time}}_{\text{data, av}} &= \frac{L_{\text{data}}}{R_b N} f_{\text{obs}} \sum_{j=1}^{n^R_{\text{crit}}} \sum_{i=1}^{j} \mathbb{E}[T_{d,i}], \\ C^{\text{time}}_{\text{R, av}} &= \frac{1}{2}\left(\frac{L_{\text{cont}} + L_{\text{data}}}{R_b N}\right) \sum_{j=1}^{n^R_{\text{crit}}} (j-1). \end{aligned} \tag{C.2}$$

The time costs associated with the worst case, that is, the energy spent by the $N - n^R_{\text{crit}}$ surviving nodes after the network death, can be, instead, written as

$$\begin{aligned} C^{\text{time}}_{\text{data, max}} &= \left(\frac{L_{\text{data}}}{R_b} f_{\text{obs}} \sum_{i=1}^{n^R_{\text{crit}}} \mathbb{E}[N_{d,i}]\right) \frac{N - n^R_{\text{crit}}}{N}, \\ C^{\text{time}}_{\text{R, max}} &= \frac{1}{2}\left(\frac{L_{\text{cont}} + L_{\text{data}}}{R_b}\right)(n^R_{\text{crit}} - 1)\frac{N - n^R_{\text{crit}}}{N}, \end{aligned} \tag{C.3}$$

where the multiplicative factor $(N - n^R_{\text{crit}})/N$ is a normalization factor required for the computation of the average costs over all nodes in the network. Note that proper corrective terms are used in the previous equations, with respect to those in Section 5, in order to take into account the correct number of nodes associated with a given energy consumption. The total average energy cost associated with the reclustering is given by

$$\begin{aligned} \overline{C}^{\text{en}}_{\text{tot}} &= P_t\left(C^{\text{time}}_{\text{data, max}} + C^{\text{time}}_{\text{R, max}} + C^{\text{time}}_{\text{R, av}} + C^{\text{time}}_{\text{data, av}}\right) \\ &= P_t \frac{L_{\text{data}} f_{\text{obs}}}{R_b N} \sum_{i=1}^{n^R_{\text{crit}}} \left((N - n^R_{\text{crit}})\mathbb{E}[T_{d,i}] + \sum_{j=1}^{i} \mathbb{E}[T_{d,j}]\right) \\ &\quad + P_t \frac{(n^R_{\text{crit}} - 1)(L_{\text{data}} + L_{\text{cont}})}{4R_b}. \end{aligned} \tag{C.4}$$

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