



PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: October 21, 2011

REVISED: February 10, 2012

ACCEPTED: March 24, 2012

PUBLISHED: April 10, 2012

## The 4D composite Higgs

Stefania De Curtis,<sup>a,c</sup> Michele Redi,<sup>a,b</sup> and Andrea Tesi<sup>c</sup>

<sup>a</sup>*INFN, Sezione di Firenze,*

*Via G. Sansone, 1; I-50019 Sesto Fiorentino, Italy*

<sup>b</sup>*CERN, Theory Division,*

*CH-1211, Geneva 23, Switzerland*

<sup>c</sup>*Università degli Studi di Firenze, Dipartimento di Fisica e Astronomia,*

*Via G. Sansone, 1; I-50019 Sesto Fiorentino,*

*E-mail:* [stefania.decurtis@fi.infn.it](mailto:stefania.decurtis@fi.infn.it), [michele.redi@cern.ch](mailto:michele.redi@cern.ch),  
[andrea.tesi@yahoo.it](mailto:andrea.tesi@yahoo.it)

**ABSTRACT:** We propose a four dimensional description of Composite Higgs Models which represents a complete framework for the physics of the Higgs as a pseudo-Nambu-Goldstone boson. Our setup captures all the relevant features of 5D models and more in general of composite Higgs models with partial compositeness. We focus on the minimal scenario where we include a single multiplet of resonances of the composite sector, as these will be the only degrees of freedom which might be accessible at the LHC. This turns out to be sufficient to compute the effective potential and derive phenomenological consequences of the theory. Moreover our simplified approach is well adapted to simulate these models at the LHC. We also consider the impact of non-minimal terms in the effective lagrangian which do not descend from a 5D theory and could be of phenomenological relevance, for example contributing to the  $S$ -parameter.

**KEYWORDS:** Beyond Standard Model, Technicolor and Composite Models, Higgs Physics

**ARXIV EPRINT:** [1110.1613](https://arxiv.org/abs/1110.1613)

---

**Contents**

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Resonances in <math>G/H</math></b>	<b>3</b>
2.1	Spin 1	3
2.2	Spin $\frac{1}{2}$	5
2.3	Other terms	6
2.4	Relation to other works	7
2.4.1	5D models	7
2.4.2	SILH and discrete models	8
<b>3</b>	<b>Minimal 4D composite Higgs</b>	<b>9</b>
3.1	Gauge sector	11
3.2	Fermions	12
<b>4</b>	<b>Higgs potential</b>	<b>14</b>
<b>5</b>	<b>Non-minimal interactions</b>	<b>18</b>
<b>6</b>	<b>Conclusions</b>	<b>19</b>
<b>A</b>	<b>Self-energies</b>	<b>19</b>
A.1	Spin-1	20
A.2	Spin-1/2	21
A.2.1	CHM <sub>5</sub>	21
A.2.2	CHM <sub>10</sub>	22
<b>B</b>	<b>Effective potential</b>	<b>23</b>
B.1	Gauge contribution	23
B.2	Fermionic contribution	24
<b>C</b>	<b>Interaction vertices <math>\rho\pi\pi</math></b>	<b>25</b>
C.1	$\rho \rightarrow W_L^+ W_L^-, Z_L h$ decay widths	26

---

**1 Introduction**

If the hierarchy problem is solved by strong dynamics we expect resonances to appear around the TeV scale which will be hopefully within the reach of the LHC. Producing these states would allow to determine precious informations about the symmetries and dynamics of the strong sector. For example in technicolor theories the electro-weak symmetry is spontaneously broken by strong dynamics according to the pattern  $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$  and the dynamics of the techni-resonances plays an important role.

Due to the notorious difficulties of theories without a Higgs, a logical possibility is that the Higgs doublet itself is a composite state, as in this case the scale of strong dynamics could be larger, alleviating phenomenological problems. This can be most naturally realized if the Higgs is a Goldstone Boson (GB) associated to the spontaneous breaking of a symmetry  $G$  to  $H$ , an idea which goes back to the '80s [1–4]. One important ingredient of modern realizations is the mechanism of partial compositeness where the SM fermions and gauge fields mix to the composite states analogously to the photon- $\rho$  mixing in QCD. The simplest example based on the symmetry pattern  $SO(5)/SO(4)$  was considered in [5] in the context of Randall-Sundrum scenarios. These models are also dual to strongly coupled 4D theories through the AdS/CFT correspondence [6, 7].

In this paper we provide a general 4D effective description of Composite Higgs Models (CHM) with partial compositeness (see [8] for a review and refs.). Our starting point will be the  $\sigma$ -model  $G/H$  describing the low energy interactions of GBs. Spin-1 resonances non-linearly realizing the symmetry can be very simply included by introducing an additional  $\sigma$ -model  $G_L \times G_R/G_{L+R}$  and gauging the diagonal subgroup of  $G_R$  and  $G$ . Proceeding in the same way a tower of resonances can be included at will. Fermionic resonances can be treated in a similar way. Differently from other constructions we find it natural and economic to include complete  $G$  multiplets. Couplings to the elementary fields are simply accounted by adding kinetic terms for the sources associated to the operators of the composite sector.

The motivation of our work is two-fold. Given that the 5D theories under consideration are effective theories with a cut-off, it is natural to write an effective 4D lagrangian containing only the degrees of freedom below the cut-off: up to cut-off dependent effects the two theories will coincide. This is particularly useful because in phenomenologically relevant models only very few resonances lie below the cut-off. The same idea was considered in [9] in the context of 5D models for QCD. Moreover only the lowest resonances have a chance to be produced at the LHC, so for collider applications it is useful to write a truncation of the theory to the first multiplet of resonances. One practical advantage is that in this limit very simple, explicit formulas can be derived which are well suited for simulating the model at the LHC. Our philosophy is here similar to the one of [10]. In this ref. an effective lagrangian capturing the essential features of partial compositeness was considered but the full GB structure of the theory was not incorporated. This was also done recently in [11] with a different construction from ours.

Since our framework is based on general symmetry principles it is also interesting to consider a fully 4D point of view and treat our effective theory with one set of  $G$  multiplets on its own. This can be interesting because terms not obtained from the 5D theory can be added to the action compatibly with the symmetries which could be relevant phenomenologically. We will show in particular that they contribute to the  $S$ -parameter allowing to reduce it for appropriate choice of the parameters.

The paper is organized as follows. In section 2 we develop the general formalism to include an arbitrary tower of spin-1 and spin-1/2 resonances in a theory with global symmetry  $G$  spontaneously broken to  $H$ . Various limits are considered which reproduce in particular higher dimensional theories and the CCWZ construction [12, 13]. In section 3 we

apply our tools to the Minimal Composite Higgs  $SO(5)/SO(4)$  focusing on the case where a single  $SO(5)$  multiplet of resonances is included. All the results can be conveniently expressed in terms of 2-point functions of the composite sector which are collected in appendix A. The Higgs potential is discussed in section 4 and appendix B. In section 5 we study the effect of non-minimal terms in the effective action. Conclusions are in section 6. Couplings and decay widths of spin-1 resonances can be found in appendix C.

## 2 Resonances in $G/H$

We wish to describe scenarios where Higgs is GB of some strongly coupled sector with global symmetry  $G$  spontaneously broken to a subgroup  $H$ . In the simplest realization [5], on which we will focus later on, the coset structure is  $SO(5)/SO(4)$  which delivers a single Higgs doublet. Our discussion however is general, relying solely on the symmetries of the theory, and could be applied for example to extended Higgs sectors studied in [14, 15].

To lowest order in derivatives the dynamics of the GBs is determined by the symmetries. In general, starting from the GB matrix,<sup>1</sup>

$$U(\Pi) = e^{\frac{i\Pi^{\hat{a}}T^{\hat{a}}}{f}}, \tag{2.1}$$

which transforms under the action of the group  $G$  as

$$U(\Pi') = gU(\Pi)h^\dagger(\Pi, g) \quad g \in G, \quad h(\Pi, g) \in H, \tag{2.2}$$

one constructs the Maurer-Cartan form  $U^\dagger \partial_\mu U = iE_\mu^a T^a + iD_\mu^{\hat{a}} T^{\hat{a}}$ , where hatted and unhatted indexes correspond to broken and unbroken generators respectively. To leading order the low energy GB lagrangian reads,

$$\frac{f^2}{2} D_\mu^{\hat{a}} D^{\mu\hat{a}}, \tag{2.3}$$

while the connection  $E_\mu^a$  allows to write general  $G$  invariant couplings to matter fields. In the scenarios of interest, upon gauging the SM gauge interaction and introducing couplings to the elementary SM fermions, eq. (2.3) describes the low energy dynamics of CHM with partial compositeness. The GB structure has important consequences on the couplings of the Higgs which was discussed in a model independent fashion in [16]. In particular this encodes the relevant low energy consequences of 5D models such as [5].

To add resonances to this picture we extend the construction of [17], which was formulated in the context of QCD. See also [18, 19] for a related construction.

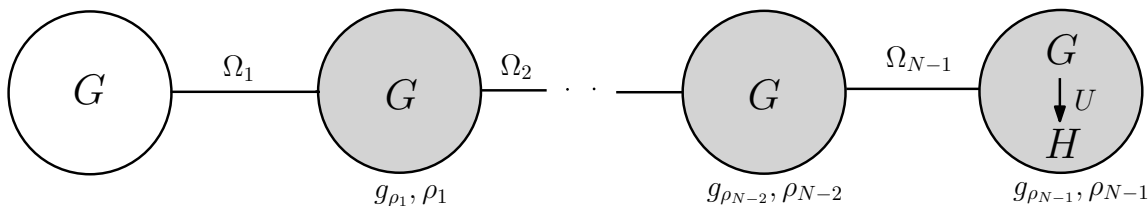
### 2.1 Spin 1

Starting from the GB lagrangian (2.3), a simple way to add vector resonances compatibly with the symmetries is to introduce a second  $\sigma$ -model field  $\Omega$  of  $G_L \times G_R/G_{L+R}$ . This can be parametrized by  $G$  valued matrices transforming as

$$\Omega \rightarrow g_L \Omega g_R^\dagger. \tag{2.4}$$

---

<sup>1</sup>To fix our normalizations the generators are defined with  $\text{Tr}[T^A T^B] = 2\delta^{AB}$ .



**Figure 1.** *Moose* diagram of the theory. The white site on the left corresponds to the global  $G$  symmetry, while the grey ones are gauged.

A lagrangian manifestly invariant under a global  $G'$  symmetry spontaneously broken to  $H$  containing spin 1 resonances ( $\rho_\mu$ ) is obtained by gauging the diagonal subgroup of  $G_R$  and  $G$  in eq. (2.3),

$$\mathcal{L}_{2\text{-site}} = \frac{f_1^2}{4} \text{Tr} |D_\mu \Omega|^2 + \frac{f_2^2}{2} \mathcal{D}_\mu^{\hat{a}} \mathcal{D}^{\mu \hat{a}} - \frac{1}{4g_\rho^2} \rho_{\mu\nu}^A \rho^{A\mu\nu}, \tag{2.5}$$

where

$$D_\mu \Omega = \partial_\mu \Omega - iA_\mu \Omega + i\Omega \rho_\mu, \tag{2.6}$$

$\mathcal{D}^{\hat{a}}$  is obtained by gauging the global symmetry of the  $\sigma$ -model (2.3), and  $\rho_{\mu\nu}^A$  is the field strength of  $\rho_\mu^A$ . We have also included in the covariant derivative (2.6) the gauging of  $G_L$ . From the point of view of the composite sector this is a global symmetry so that  $A_\mu$  is not a dynamical field and should be treated as an external classical source which is zero in the vacuum. These sources are useful to compute correlation functions of the global currents the theory: the path integral as a function of  $A$  is the generating functional of the conserved currents. Moreover, as we will see in the next section, the coupling to external fields (corresponding to SM fermions and gauge fields) is simply accounted by adding kinetic terms for the sources of the various fields. One can then determine the effects of the composite sector on the SM fields in terms of the correlation functions of the composite sector.

This procedure can be generalized in an obvious way to add a tower of resonances by adding  $\sigma$ -model fields  $\Omega_n$  and gauging nearest neighbor diagonal groups. One finds the moose lagrangian depicted in figure 1,

$$\begin{aligned} \mathcal{L}_{N\text{-sites}} &= \sum_{n=1}^{N-1} \frac{f_n^2}{4} \text{Tr} |D_\mu \Omega_n|^2 + \frac{f_N^2}{2} \mathcal{D}_\mu^{\hat{a}} \mathcal{D}^{\mu \hat{a}} - \sum_{n=1}^{N-1} \frac{1}{4g_{\rho_n}^2} \rho_{n,\mu\nu}^A \rho_n^{A\mu\nu} \\ D^\mu \Omega_n &= \partial^\mu \Omega_n - i\rho_{n-1}^\mu \Omega_n + i\Omega_n \rho_n^\mu, \quad n = 1, \dots, N-1 \end{aligned} \tag{2.7}$$

where  $\rho_n^\mu \in \text{Adj}[G_n]$  and  $\rho_0^\mu = A^\mu$ . The spectrum now contains  $N-1$   $G$  multiplets of massive spin-1 resonances. The breaking  $G/H$  is triggered by the last  $\sigma$ -model. The GBs associated to the breaking  $G/H$  can be identified with the combination,

$$U' \equiv \left( \prod_{n=1}^{N-1} \Omega_n \right) U. \tag{2.8}$$

To make manifest the particle content of the theory it is useful to choose a gauge where the GBs do not mix with the heavy gauge bosons. This is given by

$$\Omega_n = \exp i \frac{f}{f_n^2} \Pi, \quad n = 1, \dots, N \quad (2.9)$$

with  $U = \Omega_N$ . The scale  $f$  can be conveniently chosen to be the decay constant of the GBs in the theory. From the normalization of the kinetic terms one quickly derives,

$$\sum_{n=1}^N \frac{1}{f_n^2} = \frac{1}{f^2}. \quad (2.10)$$

## 2.2 Spin $\frac{1}{2}$

Fermions can be treated in a similar way with one subtlety. The simplest option is to assume that the composite sector contains Dirac fermions arising from the strong dynamics. Each SM chiral fermion will be associated to one (or more)  $G$  multiplets of the strong sector. We will then add Dirac fermions  $\Psi_n$  in a representation  $G_n$  (same at each site), in general reducible.

As for the gauge field we can write a lagrangian with nearest neighbor interactions. For the states associated to the left-handed SM chiralities we will consider,

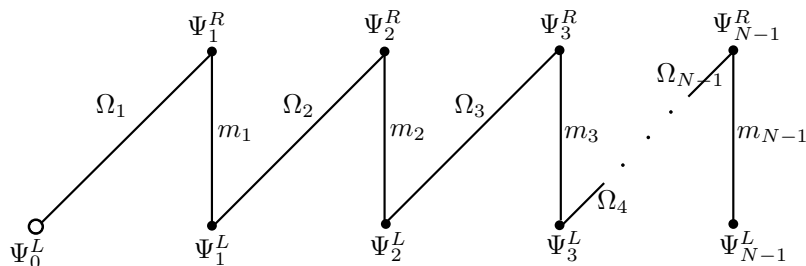
$$\begin{aligned} \mathcal{L}_{\text{fermions}} &= \sum_{n=1}^{N-1} \bar{\Psi}_n^{(r)} \left[ i \not{D}^{\rho_n} - m_n^{(r)} \right] \Psi_n^{(r)} + \sum_{n=1}^{N-1} \Delta_n^{(r)} \left( \bar{\Psi}_{r,L}^{n-1} \Omega_n \Psi_{r,R}^n + \text{h.c.} \right), \\ D^\mu \Psi_n^{(r)} &= \partial^\mu \Psi_n^{(r)} - i \rho_n^\mu \Psi_n^{(r)}, \end{aligned} \quad (2.11)$$

while for right-handed fermions  $L \leftrightarrow R$ . The index  $r$  refers to the irreducible representations of  $G$  and summation is understood. The hopping terms written above are not the most general ones, connecting only the left chirality of the massive fermion at site  $n$  to the right chirality of the fermion at site  $n + 1$ , as shown in figure 2. This choice can be motivated as follows. As in the gauge sector the field on the first site  $\Psi_0^{(r)}$  is a classical source. Differently however, we would like the associated operator to be chiral so that only one chirality of  $\Psi_0^{(r)}$  is identified as the source. This suggests to interpret at each site the left (right) fields as sources for the fields on the next site, while the right-handed (left-handed) fermions as independent dynamical fields. This leads to the lagrangian above.

Following the same logic, on the last site we write the most general zero derivatives lagrangian compatible with the spontaneously broken symmetry and with the  $LR$  structure. As explained in [15] this is achieved by writing all the possible  $H$  invariant terms constructed with GBs,

$$\mathcal{L}_{\frac{G}{H}} = m_\Psi \sum_{r,s} \bar{\Psi}_L^{(r),N-1} U(\Pi) P_A^{rs} U(\Pi)^\dagger \Psi_R^{(s),N-1} + \text{h.c.} \quad (2.12)$$

where  $P_A^{rs}$  is a projector over the irreducible representations,  $r$  refers to the fields associated to the left sources and  $s$  to the ones associated to the right-sources. This formalism also allows to introduce composite massless fermions in a representation  $H$  of the symmetry which will interact with the fields on the last site by means of the GBs.



**Figure 2.** Nearest neighbor interactions in the fermion sector. For fields associated to left-handed sources, the left chirality at site  $n$  couples to the right chirality at site  $n + 1$ .

### 2.3 Other terms

The effective lagrangians (2.7), (2.11) and (2.12) are not the most general compatible with the symmetries of the theory and this provides powerful restrictions on the spectrum and the dynamics of the theory. Consider the case with only one multiplet of resonances. To the gauge lagrangian in eq. (2.5) we can add the term,

$$\frac{f_0^2}{2} \tilde{D}^{\mu\hat{a}} \tilde{D}_{\mu\hat{a}}, \tag{2.13}$$

where  $\tilde{D}^{\mu\hat{a}}$  is now obtained from the Maurer-Cartan form for  $\Omega U$ . The addition of this term changes the GBs decay constant to

$$f^2 = f_0^2 + \frac{f_1^2 f_2^2}{f_1^2 + f_2^2}, \tag{2.14}$$

while leaving the masses of composite gauge bosons unchanged. One important consequence is that the coupling of spin-1 resonances to GBs is also modified.

While this term violates the logic of nearest neighbor interactions, there are several reasons to include it. In a phenomenological approach to QCD it is needed to better reproduce the data [20]. In that case one finds that the best agreement is for  $f_0^2 \approx -f_\pi^2$ . Moreover if we imagine starting from a theory with many sites, we could integrate out the heavier states and write an effective lagrangian for the lighter ones. This term would then be induced at tree level. Finally we expect these terms, appropriately suppressed, to be generated by quantum loops.

From a 5D point of view (see below) the term above corresponds to a non-local interaction. This should be suppressed if the 5D theory is weakly coupled. Indeed one can see that such term, if large, tends to ruin the partial unitarization of the scattering of GBs that one achieves in a theory with nearest neighbor interactions (see below). However phenomenologically interesting theories require strong coupling, so that just few resonances have a weakly coupled description. In this situation it is interesting to consider phenomenological consequences of (2.13). As we will see in section 5 this term could be relevant for the decay of the resonances and contributes to the  $S$  parameter. Analogous terms could also be introduced for the fermions but we will not consider their effects in this paper.

**2.4 Relation to other works**

**2.4.1 5D models**

The reader will likely not have missed that the lagrangians (2.7) and (2.11) resemble the discretization of a 5D theory. Our approach indeed is reminiscent of dimensional deconstruction [21, 22]. We wish to make the connection manifest. The discussion will follow the one in [9] in the context of extra-dimensional models for QCD. Let us first consider the gauge sector. The continuum limit is obtained by taking the number of sites  $N \rightarrow \infty$  with

$$\begin{aligned}
 f_n^2 &= \frac{a[(z_{n-1} + z_n)/2] 2 N}{g_5^2 L}, \\
 g_n^2 &= \frac{g_5^2 N}{a(z_n) L},
 \end{aligned}
 \tag{2.15}$$

where  $g_5$  and  $L$  are identified respectively the extra-dimensional gauge coupling and interval length.  $a(z)$  is a smooth function and  $z_n = z_0 + nL/N$ . One can check that the lagrangian (2.7) reproduces a gauge theory in a space with metric  $g_{MN} = a(z)^2 \eta_{MN}$ . Choosing couplings and decay constants an arbitrary metric can be obtained.

This picture however is only true at the classical level. Quantum mechanically the 5D gauge theory is a non-renormalizable effective theory meaningful only at energies below the 5D cut-off,

$$\Lambda_5 \sim \frac{16\pi^2}{g_5^2}.
 \tag{2.16}$$

As a consequence only the resonances below this energy scale are weakly coupled and can be trusted within the effective description while the heavier states parametrize cut-off dependent effects. The 4D theory is also an effective theory where the cut-off originates from the non-linearities of the  $\sigma$ -models. In a theory with just GBs the maximum cut-off would be  $4\pi f$  (corresponding to the scale where the scattering of GB becomes strongly coupled) which is in general smaller than  $\Lambda_5$ . In our lagrangian with nearest neighbor interactions the local cut-off is given by  $4\pi f_n$ . From (2.10)  $f_n$  grows with the number of links. The cut-off is then larger than the naive cut-off and indeed one can show that the 5D one can be reproduced. The reason for this is that the resonances partially unitarize the scattering of GBs allowing the theory to be reliable up to energies parametrically higher than  $4\pi f$ . In practice phenomenologically interesting theories demand the effective coupling of the higher dimensional theory to be rather strong so that just a few resonances will be captured in an effective description. In this sense the notion of an extra-dimension becomes rather ethereal as there is no regime where five-dimensional physics is recovered. In this situation it is natural to think of this theory as a 4D theory with just a few states.

Let us turn to the spontaneous breaking of the symmetry. In the language of the 5D theory the last site corresponds to the IR brane where the symmetry  $G$  is spontaneously broken to  $H$ . This is often achieved imposing Neumann or Dirichlet boundary conditions for the gauge fields associated to the broken or unbroken generators respectively. In the language of the deconstructed theory this corresponds to setting  $f_N \rightarrow \infty$ . Such an infinite



VEV however is unlikely to be realized physically. A more physical picture is that some dynamics is responsible for the breaking of the symmetry in the IR producing a finite  $f_N$ . We will leave  $f_N$  as a free parameter in what follows.

In the fermion sector the discretization of the 5D theory is less trivial. Usually theories obtained by naive discretization suffer from fermion doubling and need to be regulated by adding term of higher order in derivatives relevant for the deconstructed theory and irrelevant for continuum theory (see for example [23]). One can show that our fermionic action (2.11) has the correct continuum limit and this is in fact another motivation for our choice.

### 2.4.2 SILH and discrete models

In the “Strongly Interacting Light Higgs” (SILH) [16], the effective lagrangian for the Higgs as a GB was studied, focusing on the low energy consequences of the symmetry. It can be seen that the leading order lagrangian in an expansion  $E/m_\rho$  corresponds to our setup with one site (no composite resonances) and kinetic term for the sources corresponding to the SM fields. In the model with two or more sites integrating out the heavy resonances will generate higher order terms in  $E/m_\rho$  of phenomenological importance in the low energy action, reproducing sub-leading terms with the appropriate power counting described in [16]. Our setup can then be considered as the extension of SILH with resonances integrated in. This is of course mandatory to study the production of these states.

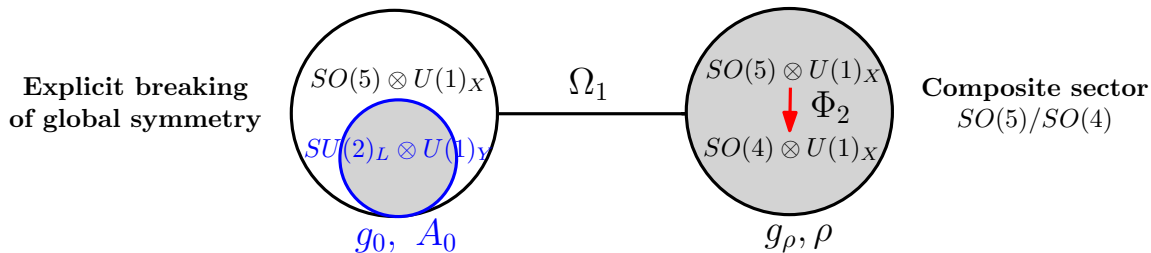
A standard way to introduce spin 1 resonances into this picture is through the CCWZ formalism, see [24] for a recent study within  $SO(5)/SO(4)$ . For spin 1 resonances in the adjoint of  $H$  this can be realized by considering gauge fields of  $H$  with a lagrangian,

$$\mathcal{L} = \frac{f^2}{2} D_\mu^{\hat{a}} D^{\mu\hat{a}} - \frac{1}{4g_\rho^2} \rho_{\mu\nu}^a \rho^{\mu\nu a} + \frac{f'^2}{2} (\rho_\mu^a - E_\mu^a)^2. \tag{2.17}$$

Note that spin 1 resonances in a different representation of  $H$  cannot be coupled in this way, since gauge fields are necessarily in the adjoint of the group. As we will see, the resonances in  $G/H$  can play an important role. For matter fields instead any  $H$  invariant lagrangian can be lifted to a  $G$  invariant one with the aid of the GBs.

In our construction fields are complete  $G$  multiplets so that we treat resonances in  $H$  and  $G/H$  on the same ground. This is quite natural in QCD where mesons fill complete representations of the chiral symmetry. We can however recover the CCWZ lagrangian with only  $H$  resonances starting from the model with one  $G$  multiplet and taking  $f_2 \rightarrow \infty$  in eq. (2.5). In this limit the spin-1 resonances in  $G/H$  become infinitely heavy and one can easily check that one obtains the lagrangian (2.17). The non-nearest neighbor interaction, introduced in section 2.3, is necessary to reproduce the most general lagrangian (2.17) where  $f$  and  $f'$  are different.

For the case of two sites with symmetry structure  $SO(5)/SO(4)$ , on which we will focus in the rest of the paper, our construction is philosophically similar to ref. [10], where however the non-linear GB structure was not included. Roughly the lagrangian in [10] can be understood as the truncation of the GB lagrangian to the lowest dimension operators. One difference is however that the Higgs necessarily couples to the elementary fields in our case. The extension of ref. [10] to the GB Higgs was also considered in ref. [11] where a



**Figure 3.** 2-site model  $SO(5)/SO(4)$ : gauge sector. The first site is the elementary sector, the second the composite sector with  $SO(5) \times U(1)_X$  heavy multiplets.

discrete model to describe composite resonances was proposed based on  $G_L \times G_R/G_{L+R}$   $\sigma$ -models.<sup>2</sup> In that work the spontaneous breaking  $G/H$  was realized by gauging a subgroup  $H$  of the  $G_R$  symmetry of the last  $\sigma$ -model  $G_L \times G_R/G_{L+R}$ . While this breaks explicitly the symmetry  $G_R$  on the last site, it can be seen that the action describes the spontaneous breaking  $G/H$  and contains incomplete  $G$  multiplets. A different choice was also considered for the fermionic fields. In our construction we find it natural to start from the  $\sigma$ -model  $G/H$  and add complete  $G$  multiplets of resonances to this picture as explained above, effectively extending [16]. For the gauge sector the setup of [11] with  $N$  sites can be recovered starting from our  $N$ -sites lagrangian in the limit  $f_N \rightarrow \infty$ .

### 3 Minimal 4D composite Higgs

In the previous section we discussed the features of the composite sector relevant to build CHM where the Higgs is a GB. We will now apply this framework to the Minimal Composite Higgs of ref. [5]. As in that work, we find it very useful to express our results in terms of the correlation functions of the composite sector which we collect in appendix A. We will focus on the case with two sites depicted figure 3, even though the formulas in terms of the correlation functions are general. There are two main reasons for this choice. Most importantly the 2-site lagrangian contains only the states which might be accessible at the LHC capturing the relevant features of all CHM with partial compositeness. Secondly this truncation allows to obtain extremely simple formulas depending on few parameters which are ready made to simulate the scenario at the LHC.

The symmetry structure of the theory is  $SO(5)/SO(4)$  which delivers 4 GBs in the vector representation of  $SO(4)$ , appropriate for the Higgs boson. The breaking can be parametrized by the VEV of a vector of  $SO(5)$  so that the gauge lagrangian (2.5) can be written explicitly as,<sup>3</sup>

$$\mathcal{L}_{\text{gauge}} = \frac{f_1^2}{4} \text{Tr} |D_\mu \Omega_1|^2 + \frac{f_2^2}{2} (D_\mu \Phi_2) (D^\mu \Phi_2)^T - \frac{1}{4g_\rho^2} \rho_{\mu\nu}^A \rho^{A\mu\nu}, \quad (3.1)$$

<sup>2</sup>A two site description can be also found in [25]. Our gauge sector is similar to the one described in section 6.2 of that work with the scale  $F$  kept finite. A “Little-Higgs” inspired 2-site model is described also in [26].

<sup>3</sup>We follow the conventions of [5]. In particular we have redefined the generators so that in the vector representation  $\text{Tr}[T^A T^B] = \delta^{AB}$ . This is convenient as the  $SU(2)_L$  subgroup has then standard structure constants  $\epsilon_{ijk}$ .

where  $|\Phi_2|^2 = 1$ .

The GBs can be identified as explained in section 2.1. Introducing the GB matrix,

$$\Pi = \sqrt{2}h\hat{T}^{\hat{a}} = -i \begin{pmatrix} 0_4 & \mathbf{h} \\ -\mathbf{h}^T & 0 \end{pmatrix} \quad (3.2)$$

and with the parametrization (2.9) we find

$$\Omega_n = \mathbf{1} + i\frac{s_n}{h}\Pi + \frac{c_n - 1}{h^2}\Pi^2, \quad s_n \equiv \sin(fh/f_n^2), \quad c_n \equiv \cos(fh/f_n^2), \quad h \equiv \sqrt{h\hat{a}h\hat{a}} \quad (3.3)$$

with  $n = 1, 2$ . We define  $\Phi_2 = \phi_0\Omega_2^T$ , where  $\phi_0^i = \delta^{i5}$ . Following eq. (2.8), the GBs are parametrized by,

$$\Phi = \Phi_2\Omega_1^T = \phi_0 e^{-i\frac{\Pi}{f}} = \frac{1}{h} \sin\frac{h}{f} \begin{pmatrix} h_1 & h_2 & h_3 & h_4 & h \cot\frac{h}{f} \end{pmatrix}. \quad (3.4)$$

To reproduce hyper-charge assignments of the SM fermions we need to extend the symmetry of the composite sector to  $\text{SO}(5) \times \text{U}(1)_X$  where the latter symmetry is unbroken in the vacuum. The hyper-charge is identified with the combination

$$Y = T_{3R} + X. \quad (3.5)$$

In the framework of partial compositeness the composite sector should also possess an  $\text{SU}(3)$  global symmetry, corresponding to color. However this does not play a role in what follows so we will omit it (except for taking into account the color factors).

Having discussed the composite sector, the next step is to introduce SM fermions and gauge fields. Practically this amounts to adding kinetic terms for the sources of the operators of the composite sector to which the SM field couple and setting to zero non-dynamical fields. To gauge the SM gauge symmetry we then introduce the kinetic terms,

$$\mathcal{L}_{\text{gauge}}^{el} = -\frac{1}{4g_0^2}F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{4g_{0Y}^2}Y_{\mu\nu} Y^{\mu\nu} \quad (3.6)$$

for the sources associated to the  $\text{SU}(2)_L$  and  $\text{U}(1)_Y$  symmetries. Formally non-dynamical fields correspond to infinite elementary kinetic terms.

For fermions it works in a similar way. We add kinetic terms for the elementary SM fermions (we will focus here only on the third generation quarks),

$$\mathcal{L}_{\text{fermions}}^{el} = \frac{1}{y_{q_L}^2} \bar{q}_L^{el} i \not{D}^{el} q_L^{el} + \frac{1}{y_{t_R}^2} \bar{t}_R^{el} i \not{D}^{el} t_R^{el} + \frac{1}{y_{b_R}^2} \bar{b}_R^{el} i \not{D}^{el} b_R^{el}, \quad (3.7)$$

where  $\not{D}^{el}$  corresponds to the covariant derivative with respect to the elementary fields. There exists also the possibility that one chirality, typically the right-handed top, is part of the strong sector in which case no elementary component exists but we will not consider it here. To determine the model we just need to specify to which sources the elementary fields correspond. This in general depends on the fermionic representation of the composite sector.

The elementary lagrangian explicitly breaks the global symmetry of the theory so that the GBs become approximate. In particular the VEV becomes physical and the vacuum alignment determines the electro-weak VEV. The main consequences are the generation of mass terms for  $W^\pm$  and  $Z$  and Yukawa couplings for SM fermion. The explicit breaking also generates a potential for Higgs at 1-loop which we investigate in section 4.

As an example we consider in detail the model where SM fermions couple to **5** reps of  $SO(5)$  as in [27]. This is a realistic model which allows to satisfy precision electro-weak measurements as well as generating successful electro-weak symmetry breaking (EWSB). A similar analysis can be done for other models and it is essentially determined once the representation of the composite sector are fixed.

We consider first the gauge sector which is model independent.

### 3.1 Gauge sector

Composite spin 1 resonances fill an adjoint representation of  $SO(5) \times U(1)_X$ . Before coupling to the elementary gauge fields the spectrum has an unbroken  $SO(4) \times U(1)_X$  symmetry,

$$\begin{aligned} m_\rho^2 &= \frac{g_\rho^2 f_1^2}{2}, \\ m_{a_1}^2 &= \frac{g_\rho^2 (f_1^2 + f_2^2)}{2}, \\ m_{\rho_X}^2 &= \frac{g_{\rho_X}^2 f_X^2}{2}, \end{aligned} \tag{3.8}$$

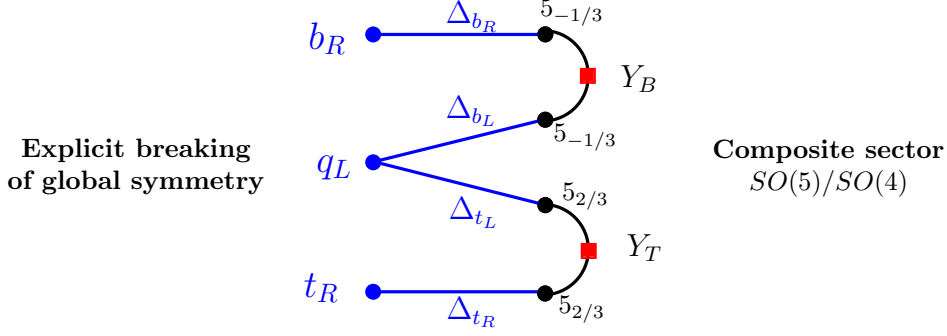
where in analogy with QCD we denoted the masses of the resonances in  $H$  and  $G/H$  as  $m_\rho$  and  $m_{a_1}$  respectively (see [28] for a related discussion). The mass and coupling of the resonance associated to  $U(1)_X$  can be different from  $m_\rho$  so  $g_{\rho_X}$  and  $f_X$  are in general independent parameters. For simplicity we will take  $f_X = f_1$  in what follows. These formulas continue to hold also when  $f_0 \neq 0$ . While from eq. (2.14) for  $f_0 = 0$  the sign of  $f_2^2$  is strictly positive, it could in principle be negative for  $f_0 \neq 0$  leading to coset resonances lighter than  $H$  resonances.

The kinetic terms for the SM gauge fields breaks the  $SO(4)$  degeneracy. The fields decompose into  $SU(2)_L \times U(1)_Y$  multiplets with masses given by the zeros of eqs. (A.5). The mass of composite states coupled to the elementary fields are in general heavier than their  $SO(4)$  partners. For example for  $SU(2)_L$  we have one massless triplet of gauge fields ( $W^{aL}$ ) and one heavy triplet of fields with mass:

$$m_{\rho_{aL}} = \frac{m_\rho}{\cos \theta_L}, \quad \tan \theta_L = \frac{g_0}{g_\rho}. \tag{3.9}$$

From the correlation functions in appendix A we can identify the low energy parameters of the theory. The electro-weak gauge couplings are given by,

$$\begin{aligned} \frac{1}{g^2} &= -\Pi'_0(0) = \frac{1}{g_0^2} + \frac{1}{g_\rho^2}, \\ \frac{1}{g'^2} &= -\Pi'_Y(0) = \frac{1}{g_{0Y}^2} + \frac{1}{g_\rho^2} + \frac{1}{g_{\rho_X}^2}, \end{aligned} \tag{3.10}$$



**Figure 4.** 2-site model  $SO(5)/SO(4)$ : fermionic sector. The spontaneous breaking of  $SO(5)$  is achieved through Yukawa couplings (2.12) in the composite sector, here drawn as red squares.

while the electro-weak VEV is

$$v = f \sin \frac{\langle h \rangle}{f}. \quad (3.11)$$

The interaction vertices and decay of the resonances are studied in appendix C.

### 3.2 Fermions

As an example we consider the first model of ref. [27] where SM fermions couple to fermionic operators in the  $\mathbf{5}$  of  $SO(5)$ . This model, known as  $CHM_5$ , is a realistic scenario compatible with precision electro-weak measurements. Analogous results can be derived for any other fermionic choice, see appendix A for the model with  $\mathbf{10}$  of  $SO(5)$  ( $CHM_{10}$ ). For simplicity we will consider only the third generation quarks, which will be relevant for the computation of the potential in the next section.

Under  $SU(2)_L \times SU(2)_R$  the vector representation of  $SO(5)$  decomposes as a  $(\mathbf{2}, \mathbf{2})$  and a  $(\mathbf{1}, \mathbf{1})$ . The SM left doublet can be embedded in a  $(\mathbf{2}, \mathbf{2})_{2/3} \in \Psi_T$  as,

$$\mathbf{5}_{2/3} = (\mathbf{2}, \mathbf{2})_{2/3} \oplus (\mathbf{1}, \mathbf{1})_{2/3}, \quad (\mathbf{2}, \mathbf{2})_{2/3} = \begin{pmatrix} T & T_{\frac{5}{3}} \\ B & T_{\frac{2}{3}} \end{pmatrix} \quad (3.12)$$

while  $t_R$  can be coupled to a singlet in a different  $\mathbf{5}$  rep,  $\Psi_{\tilde{T}}$ . For the bottom sector  $b_R$  is coupled to the singlet in a  $\mathbf{5}_{-1/3}$  ( $\Psi_{\tilde{B}}$ ) so to generate the bottom Yukawa it is also necessary (by  $U(1)_X$  symmetry) to couple the SM doublet to a second doublet in  $\mathbf{5}_{-1/3}$  ( $\Psi_B$ ) which contains

$$\mathbf{5}_{-1/3} = (\mathbf{2}, \mathbf{2})_{-1/3} \oplus (\mathbf{1}, \mathbf{1})_{-1/3}, \quad (\mathbf{2}, \mathbf{2})_{-1/3} = \begin{pmatrix} B_{-\frac{1}{3}} & T' \\ B_{-\frac{4}{3}} & B' \end{pmatrix}. \quad (3.13)$$

To summarize, the spectrum contains  $\mathbf{5}_{2/3}$  ( $\Psi_T, \Psi_{\tilde{T}}$ ) and  $\mathbf{5}_{-1/3}$  ( $\Psi_B, \Psi_{\tilde{B}}$ ) reps, associated respectively to the top and bottom sectors. This choice of representations, while minimal, has special phenomenological virtues suppressing dangerous corrections to the couplings of the  $Z$  which otherwise would be problematic phenomenologically [29].

Following section 2, the effective lagrangian for the composite states and elementary fields reads<sup>4</sup>

$$\begin{aligned}
 \mathcal{L}^{\text{CHM}_5} = & \mathcal{L}_{\text{fermions}}^{el} \\
 & + \Delta \bar{q}_L^{el} \Omega_1 \Psi_T + \Delta \bar{t}_R^{el} \Omega_1 \Psi_{\tilde{T}} + \text{h.c.} \\
 & + \bar{\Psi}_T (i \not{D}^\rho - m_T) \Psi_T + \bar{\Psi}_{\tilde{T}} (i \not{D}^\rho - m_{\tilde{T}}) \Psi_{\tilde{T}} \\
 & - Y_T \bar{\Psi}_{T,L} \Phi_2^T \Phi_2 \Psi_{\tilde{T},R} - m_{Y_T} \bar{\Psi}_{T,L} \Psi_{\tilde{T},R} + \text{h.c.} \\
 & + (T \rightarrow B),
 \end{aligned} \tag{3.14}$$

where  $\Delta$  is introduced for dimensional reasons. As explained in section 2.2 we do not include all the possible terms in the composite sector allowed by the symmetries but only the ones with the required  $LR$  structure. This is actually the minimal choice to generate the SM Yukawas. The  $\text{SO}(5)$  invariant proportional to  $m_{Y_T}$  is needed to describe the most general embedding of the elementary fields.

To leading order the mass of the top is given by,

$$m_t \sim \frac{v}{\sqrt{2}} \frac{\Delta_{t_L}}{m_T} \frac{\Delta_{t_R}}{m_{\tilde{T}}} \frac{Y_T}{f} \tag{3.15}$$

where we defined,

$$\Delta_{t_L} = y_{t_L} \Delta, \quad \Delta_{t_R} = y_{t_R} \Delta. \tag{3.16}$$

It is natural to interpret  $Y_T/f$  as the coupling of the composite sector and  $\Delta_{t_L}/m_T$ ,  $\Delta_{t_R}/m_{\tilde{T}}$  as the elementary-composite mixings. We should mention that the approximation above, excellent for the light quarks, is not always sufficient for the top, the exact expression used in our numerical computations is reported in eq. (A.13).

The mass spectrum is in general most easily extracted from the fermionic correlators in appendix A. In the top sector breaking of  $\text{SO}(4)$  symmetry due to the mixings is sizable so the spectrum is only (approximately)  $\text{SU}(2)_L \times \text{U}(1)_Y$  symmetric. Before EWSB the spectrum of heavy fermions is given by,

- two  $\mathbf{2}_{1/6}$ , zeros of  $\Pi_0^q$ ,
- two  $\mathbf{2}_{7/6}$ , poles of  $\hat{\Pi}_0^{qL}$ ,
- two  $\mathbf{1}_{2/3}$ , zeros of  $\Pi_0^g$ ,

where form factors are defined in (A.11), (A.12). Rather than writing the explicit formulas, computable as explained, in the next section we will present the fermionic spectrum as a function of the Higgs mass. Here we only anticipate that top partners inside  $\mathbf{2}_{7/6}$  can be very light, in the region of large elementary-composite mixings (3.16),

$$m_{\mathbf{2}_{7/6}}^2 = \frac{1}{2} \left[ m_T^2 + m_{\tilde{T}}^2 + m_{Y_T}^2 \pm \sqrt{(m_T^2 - m_{\tilde{T}}^2)^2 + m_{Y_T}^2 (m_{Y_T}^2 + 2(m_T^2 + m_{\tilde{T}}^2))} \right]. \tag{3.17}$$

---

<sup>4</sup>To introduce the Yukawas of the down sector we define the source of the quark doublet as  $\Psi_L = \cos \varphi \Psi_T^0 + \sin \varphi \Psi_B^0$  and the orthogonal combination  $\Psi_L^\perp = -\sin \varphi \Psi_T^0 + \cos \varphi \Psi_B^0$ . We then identify  $y_{t_L} = y_{q_L} \cos \varphi$  and  $y_{b_L} = y_{q_L} \sin \varphi$ . In order to reproduce  $m_b \ll m_t$  it is natural to take  $\sin \varphi \ll 1$  so that the mixing in the down sector is small and can be neglected for some purposes such as computation of the potential. Moreover this choice guarantees that the shift of the coupling of  $b_L$  to the  $Z$  is small.

## 4 Higgs potential

The coupling to the SM elementary fields explicitly breaks the global symmetry of the composite sector. As a consequence the Higgs is only approximately a GB and a potential is generated starting at 1-loop. Since the breaking is proportional to the mixings the potential is normally dominated by the contributions associated to the top quark. In the low energy theory with just GBs the contributions to the potential are formally divergent. More precisely since the lagrangian contains SM interactions one obtains the same quadratic divergences as in the SM plus additional ones which originate from non-renormalizable interactions, for example Higgs dependent kinetic terms. We expect that these divergences will be regulated by the new states in the theory so that the physical cut-off entering the loops will be the compositeness scale, roughly the mass of the lowest resonances running in the loop. This is exactly what happens in QCD for the electro-magnetic splitting of the pions, see [8]. The same intuition is also correct here.

In the 5D constructions the Higgs effective potential can be calculated at 1-loop and is finite. This property can be traced to the locality of the theory: since the divergences originate from short distance physics, they cannot be generated as there is no local gauge invariant operator compatible with the symmetries which contributes to the Higgs mass. The operator which contributes to the Higgs mass is non-local, being associated to the Wilson line of a gauge field across the fifth dimension. From this argument it is natural to suspect that the same property will survive in a discretized theory with nearest neighbor interactions if a sufficient number of sites is included. Interestingly in the model under consideration with a single multiplet of resonances the 1-loop potential is finite, both for the gauge and the fermionic contributions. The same effect was also found in [11] in a slightly different setup with an extra layer of resonances. In that paper a general argument explaining the appearance of divergent terms was also given. Let us mention that exact finiteness is spoiled in the fermionic sector if terms without  $LR$  structure (but compatible with the symmetries) are added to the action (3.14). The divergence in this case is logarithmic and can be estimated using as cut-off the compositeness scale. At any rate exact finiteness of the potential will be always spoiled as we increase the number of loops so it will always be necessary to treat these computation in an effective field theory sense.

We consider in detail the potential of the  $\text{CHM}_5$  model described in the previous section. To leading order the functional form of the potential is fixed by the structure of the symmetry breaking effects [27],

$$V(h) \approx \alpha s_h^2 - \beta s_h^2 c_h^2, \quad (4.1)$$

where  $s_h = \sin h/f$ . The coefficients can be expressed as integrals of the correlations functions of the strong sector, see appendix B for all the details. The presence of resonances in our setup renders the integrals finite with the individual terms saturated by the scale associated to the resonances.

In absence of tuning the natural location of the minimum of the potential is at  $\langle h \rangle = f$  or at 0. In order to achieve successful EWSB ( $v < f$ ) a tuning of the fundamental parameters of the lagrangian must be performed so that the coefficient of the quadratic

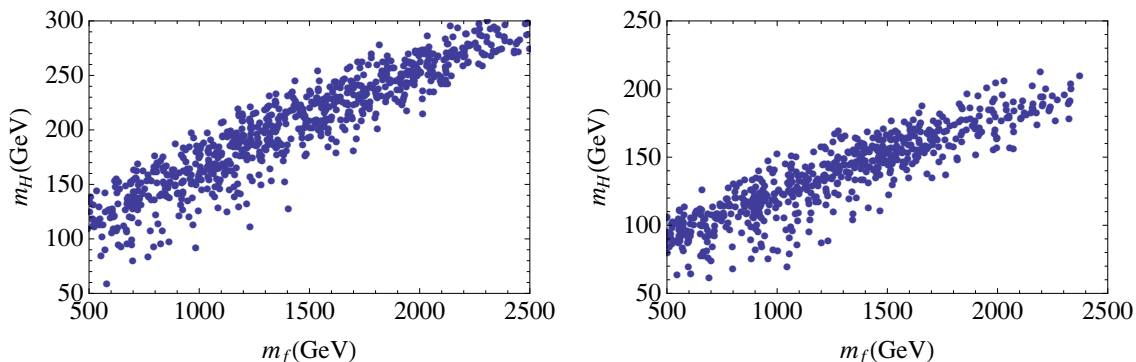
term in the expansion of (4.1) is smaller than its natural size and is of course negative. In this specific model this requires,

$$\frac{\Delta_{t_L}}{m_T} \sim \frac{\Delta_{t_R}}{m_{\tilde{T}}}, \quad (4.2)$$

i.e. similar mixings for left and right chirality. Once the tuning is performed the Higgs mass is given by,

$$\sin \frac{\langle h \rangle}{f} = \sqrt{\frac{\beta - \alpha}{2\beta}}, \quad m_H^2 = \frac{2}{f^2} \frac{\beta^2 - \alpha^2}{\beta}. \quad (4.3)$$

To determine the Higgs mass as a function of the resonance masses we have performed a scan over the parameters of the model demanding that the correct electro-weak VEV and top mass are generated. In figure 5 we present the results for the unconstrained scan over the six fermionic parameters  $\Delta_{t_L}$ ,  $\Delta_{t_R}$ ,  $m_T$ ,  $m_{\tilde{T}}$ ,  $m_{Y_T}$  and  $Y_T$  for  $f = 500$  GeV and  $f = 800$  GeV. In general we find a strong correlation between Higgs mass and the mass of the lightest resonances, which are typically but not always the  $\mathbf{2}_{7/6}$  states. In particular if the Higgs is light and the tuning is not large the resonances must be light. This is in agreement with 5D models [27, 30]. As we increase  $f$  a Higgs above 115 GeV requires heavy top partners. In the low mass region we find that the contribution of gauge fields is not necessarily negligible so it is included.

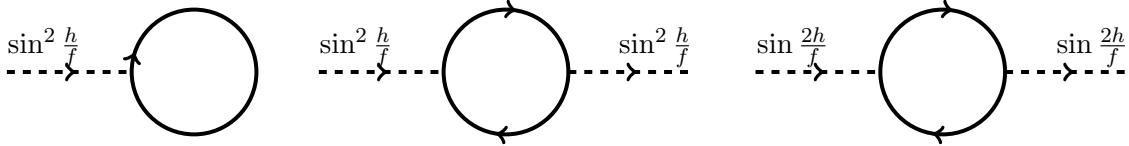


**Figure 5.** Masses of the lightest fermionic partners as a function of the Higgs mass for  $m_t \in [165, 175]$  GeV. On the left  $f = 500$  GeV and on the right  $f = 800$  GeV. The six fermionic parameters are varied between .5 and 3 TeV. The gauge contribution corresponds to  $f_1 = f_2 = \sqrt{2}f$  and  $m_\rho = 2.5$  TeV.

Sharper predictions can be obtained making assumptions on the parameters. In figure 7 we present two scans for different choices of mixings and the IR parameters  $Y_T$  and  $m_{Y_T}$ . In the first figure with  $Y_T \sim -m_{Y_T}$  and large mixings the lightest fermionic partners are the  $\mathbf{2}_{7/6}$  states. In the second figure we consider the case  $m_{Y_T} \ll Y_T$  and smaller mixings, finding that the singlet  $\mathbf{1}_{2/3}$  is the lightest state.

The Higgs potential can be understood from naturalness arguments, see [27] and [11] for related discussions. The low energy lagrangian for the Higgs and SM quarks can be easily extracted from the correlators of the composite sector in appendix A and is given





**Figure 6.** Feynman diagrams contributing to the Higgs potential. The first two originate from Higgs dependent kinetic terms, while the third is obtained from Yukawa interactions.

in eq. (A.8). Its non-linear structure is fixed by the global symmetries. To lowest order in derivatives we have Yukawa interactions,

$$\mathcal{L}_{\text{Yuk}} = y_t f \frac{s_h c_h}{h} (\bar{q}_L H^c t_R + \text{h.c.}) \quad (4.4)$$

and field dependent kinetic terms for the fermions,

$$\mathcal{L}_{\text{kin}} = \frac{y_{t_L}^2}{2y_T^2} s_h^2 \bar{t}_L \not{D} t_L + \frac{y_{t_R}^2}{y_T^2} c_h^2 \bar{t}_R \not{D} t_R, \quad (4.5)$$

where  $y_{T,\tilde{T}}^{-2} = \widehat{\Pi}_1^{q_L, u_R}(0)$  in (A.11).

Performing a loop of fermions as in figure 6, we obtain a contribution to the Higgs potential which is divergent within the low energy effective theory. We expect the divergence to be cut-off by the compositeness scale which for the top loops will be the mass of the lightest top partners. From the loop associated to the Yukawa interactions we find,

$$V(h)_{\text{Yuk}} \sim N_c \frac{y_t^2}{4\pi^2} m_f^2 f^2 s_h^2 c_h^2 \quad (4.6)$$

where  $N_c = 3$  is the QCD color factor. From the kinetic terms we obtain to leading order in  $y_{t_L, t_R}$ ,

$$V(h)_{\text{kin}}^{(1)} \sim N_c \frac{2y_{t_R}^2 - y_{t_L}^2}{32\pi^2} \frac{m_f^4}{y_T^2} s_h^2. \quad (4.7)$$

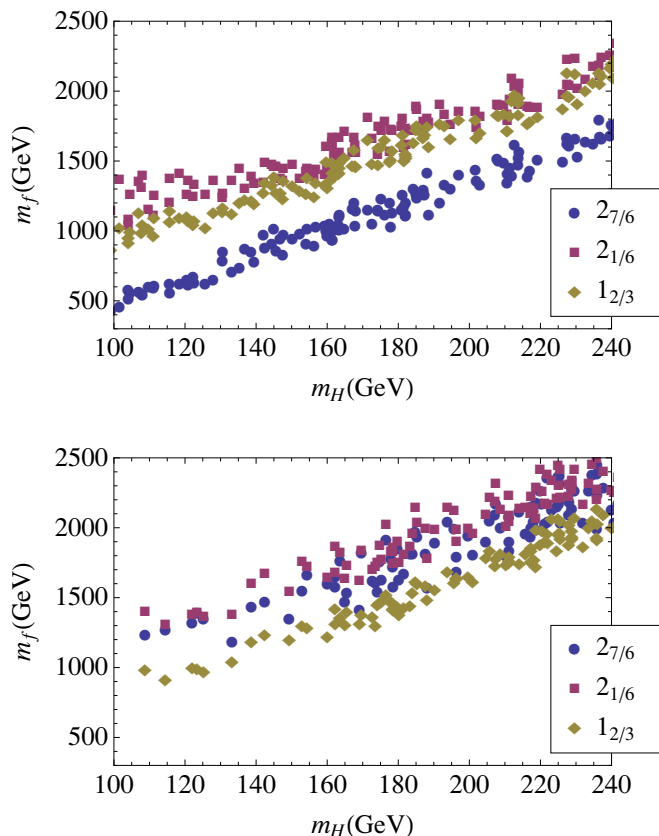
There are also sub-leading contributions from the second diagram in figure 6,

$$V(h)_{\text{kin}}^{(2)} \sim N_c \frac{y_{t_{L,R}}^4}{16\pi^2} \frac{m_f^4}{y_T^4} s_h^4. \quad (4.8)$$

To obtain a Higgs VEV  $\langle h \rangle < v$  we need to tune the different contributions. Let us focus on the case  $y_{t_L} \sim y_{t_R}$ . The leading contributions are the ones proportional to  $y_{t_{L,R}}^2$ , however they tend to cancel for (4.2). Due to the different functional dependence of  $V(h)_{\text{Yuk}}$  and  $V(h)_{\text{kin}}^{(2)}$  we can obtain a small Higgs VEV by tuning these terms versus  $V(h)_{\text{kin}}^{(1)}$ . The quartic of the potential is then determined by the sub-leading contributions (4.6), (4.7). We estimate,

$$m_H \sim 0.3 y_t \frac{m_f}{f} v \quad (4.9)$$

in rough agreement with figure 5. In particular we see very clearly that, for a given value of  $f$ , the mass of the Higgs depends linearly on the mass of the top partners. For a small



**Figure 7.** Masses of lightest fermionic excitations as a function of the Higgs mass for  $m_t \in [165, 175]$  GeV and  $f = 500$  GeV. In the upper plot the mixings are  $1.2 \leq \Delta_{t_L}/m_T \leq 1.8$ ,  $0.7 \leq \Delta_{t_R}/m_{\bar{T}} \leq 1.3$  and the IR parameters  $0.5 \leq Y_T(\text{TeV}) \leq 3$ ,  $-1.2Y_T \leq m_{Y_T} \leq -0.8Y_T$ . In the lower plot the mixings are  $0.6 \leq \Delta_{t_L}/m_T \leq 0.9$ ,  $0.35 \leq \Delta_{t_R}/m_{\bar{T}} \leq 0.7$  and the IR parameters  $0.5 \leq Y_T(\text{TeV}) \leq 3$ ,  $-0.5 \leq m_{Y_T}(\text{TeV}) \leq 0.5$ .

value of  $f$ , which is required if the tuning is small, the presence of a light Higgs would then imply light top partners. Present bounds, with some model dependence, require  $m_f > 500$  GeV [31, 32]. Within this model top partners could be soon within the reach of the LHC if the Higgs is light.

One interesting fact is that the contribution of gauge fields, normally sub-leading, is not always negligible in the region of light fermionic partners. Along the same lines as above we can estimate the contribution of gauge loops as (see (B.3) for the exact expression),

$$V(h)_{\text{gauge}} \sim \frac{9}{4} \frac{g_0^2}{16\pi^2} \frac{m_\rho^4}{g_\rho^2} \sin^2 \frac{h}{f}, \quad (4.10)$$

where  $m_\rho$  is the mass of the spin-1 resonances. Contrary to the fermionic resonances  $m_\rho$  cannot be very low because of precision electro-weak tests, in particular contributions to the  $S$  parameter. As a consequence the gauge contribution can become important if the top partners are light,  $m_f \ll m_\rho$ . Moreover the gauge contribution is strictly positive so that it might off-set the instability of the potential at the origin. In general we find that the contribution of gauge loops tends to reduce the number of points with successful EWSB.

## 5 Non-minimal interactions

In this section we wish to briefly discuss the effect of adding the non-nearest neighbor interaction of section 2.3 to the 2-site lagrangian. The role of this term in Higgsless theories [33, 34] based on the pattern  $SO(4)/SO(3)$  was recently considered in [35] and a related discussion for  $SO(5)/SO(4)$  appears in [24]. The term in eq. (2.13) can be written explicitly as

$$\frac{f_0^2}{2}(D_\mu\Phi)(D^\mu\Phi)^T, \tag{5.1}$$

where  $\Phi = \Phi_2\Omega_1^T$  as in eq. (3.4). This contributes to the kinetic term of the GBs but not to the mass of the composite resonances in eq. (3.8). Importantly it changes the relation between the gauge coupling of the resonances and their coupling to GBs. For  $SO(4)$  resonances, in the limit  $f_2 \rightarrow \infty$ , one finds,<sup>5</sup>

$$g_{\rho\pi\pi} = \frac{f^2 - f_0^2}{2f^2} g_\rho \tag{5.2}$$

while for coset resonances there is no coupling to two GBs by  $SO(4)$  symmetry. From this expression, using eq. (3.8), one obtains

$$m_\rho^2 = 2\frac{f^2}{f^2 - f_0^2} g_{\rho\pi\pi}^2 f^2 \tag{5.3}$$

which is the analog of the KRSF relation in QCD. The modified coupling of the  $\rho$  to the Higgs can have important phenomenological effects. First of all it modifies the decay of the resonances into  $W_L^+W_L^-$  and  $Z_L h$  which together with third generation quarks are the main decay channel. Since the decay goes like  $g_{\rho\pi\pi}^2$  this term can easily generate an order one change in the decay, see appendix C for more details.

Importantly the term (5.1) also contributes to the  $S$ -parameter. This can be readily computed in terms of the two-point functions of the currents of the strong sector [5],

$$S = 4\pi\Pi_1'(0)\frac{v^2}{f^2}. \tag{5.4}$$

Including the term proportional to  $f_0^2$  amounts to adding a constant in  $\Pi_1$  so that the dependence of  $S$  on  $f_0$  (for fixed resonance masses) appears through the redefinition of  $f$  in eq. (2.14). The result can be written as,

$$S = 4\pi v^2 \left( \frac{1}{m_\rho^2} + \frac{1}{m_{a_1}^2} \right) \frac{f^2 - f_0^2}{f^2}. \tag{5.5}$$

We note however that the  $S$  parameter could be suppressed if  $f_0^2$  is sizable and positive.<sup>6</sup> In particular  $S$  would be zero if  $f_0 = f$ . In this limit, which can be realized taking  $f_2 = 0$

---

<sup>5</sup>We define  $g_{\rho\pi\pi}$  from the interaction  $g_{\rho\pi\pi} f^{\hat{a}\hat{b}\hat{c}} \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} \rho_\mu^{\hat{c}}$ . Equivalent formulas to eqs. (5.2), (5.3) were recently obtained in [24] in the CCWZ language. Note that there are corrections to these equations for finite  $f_2$ , see appendix C.

<sup>6</sup>We should mention that in QCD the sign of the analogous term is negative which would correspond to an enhancement of the  $S$ -parameter. It would be interesting to know whether the sign can be robustly fixed in a strongly coupled theory not QCD-like.

keeping the spectrum finite, GBs and resonances are decoupled from each other, the latter becoming degenerate  $SO(5)$  multiplets. A similar effect was noticed in ref. [36], in the context of Higgsless theories. Indeed our setup with symmetry structure  $SO(4)/SO(3)$  can be used as 2-site model for Higgsless and in this case the contribution to  $S$  would be given by eq. (5.5) with  $f = v$ . From the point of view of the effective theory  $S$  could even be negative. This can be achieved if  $f_0 > f$  which corresponds to having the mass of the coset resonances lighter than  $SO(4)$  resonances. While these extreme values of  $f_0$  might be questionable, since the resonances not even partially unitarize the scattering of GBs, for phenomenological purposes even a mild suppression can be important.

## 6 Conclusions

In this note we have presented a simplified 4D description of Composite Higgs Models with partial compositeness. Our formalism allows to introduce an arbitrary tower of resonances, bosonic and fermionic, in a theory where the Higgs is a GB. As a particular case one can reproduce the physics of higher dimensional models.

For phenomenological applications we focused on the minimal model with only the lowest lying resonances which provides a self-contained framework reproducing all the relevant features of Composite Higgs Models at the LHC. For example our theoretical setup already allows to compute the Higgs potential with a minimal number of degrees of freedom. We believe that this is the most natural and economic setup, being dictated by the symmetries of theory. As an example we considered in detail the  $CHM_5$  scenario of [27] with results comparable to the 5D scenario for similar choices of parameters.

While our construction was inspired by 5D models one can consider the truncation of the theory to the lowest resonances on its own. Based on the symmetries of the theory more terms can be added to the lagrangian which do not follow from the discretization of the 5D theory. In particular we have shown that, as for the  $\rho$  in QCD, these terms modify the coupling of the resonances to the GBs and could be of phenomenological relevance changing their decays and possibly reducing the  $S$ -parameter.

Our work is the starting point for a study of the model at LHC. The simplified lagrangian allows to capture all the effects of compositeness and is simple enough to be implemented on an event generator. We hope to return to this in future work.

## Acknowledgments

We would like to thank Roberto Contino, Leandro da Rold and Andrea Wulzer for useful discussions. AT would like to thank CERN for partial support and hospitality.

## A Self-energies

In this appendix we provide explicit expressions for the two-point functions of the strong sector in the 2-site picture. We follow closely ref. [5] where this technique was introduced. The great advantage of this approach is that the relevant informations can be extracted

from the correlators of the composite sector computed for zero Higgs VEV (since the composite sector does not depend on it). We analyze the gauge and fermionic sectors of the theory presenting the formulas for CHM<sub>5</sub> and CHM<sub>10</sub> models of [27].

### A.1 Spin-1

In the background of a constant GB Higgs the most general quadratic lagrangian for the vector sources invariant under  $\text{SO}(5) \times \text{U}(1)_X$  takes the form,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} P_{\mu\nu}^T \left[ \widehat{\Pi}_0^X(p^2) X^\mu X^\nu + \widehat{\Pi}_0(p^2) \text{Tr}[A^\mu A^\nu] + \widehat{\Pi}_1(p^2) \Phi A^\mu A^\nu \Phi^T \right], \quad (\text{A.1})$$

where  $P_{\mu\nu}^T = \eta_{\mu\nu} - p_\mu p_\nu / p^2$ ,  $A^A, X \in \text{Adj}[\text{SO}(5) \times \text{U}(1)_X]$  and  $\Phi$  is defined in eq. (3.4). To determine the form factors we need to express the lagrangian (3.1) as a function of  $A^{a,\widehat{a}}$  and  $X$  by integrating out the fields  $\rho^\mu$  according to their equations of motion. The result, computed for  $h = 0$ , is:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} P_{\mu\nu}^T \left[ \Pi_a(p^2) A_a^\mu A_a^\nu + \Pi_{\widehat{a}}(p^2) A_{\widehat{a}}^\mu A_{\widehat{a}}^\nu + \Pi_X(p^2) X^\mu X^\nu \right] \quad (\text{A.2})$$

where,<sup>7</sup>

$$\begin{aligned} \Pi_a(p^2) &= \frac{m_\rho^2 p^2}{g_\rho^2 (p^2 - m_\rho^2)}, \\ \Pi_{\widehat{a}}(p^2) &= \frac{m_\rho^2 (p^2 - (m_{a_1}^2 - m_\rho^2))}{g_\rho^2 (p^2 - m_{a_1}^2)}, \\ \Pi_X(p^2) &= \frac{m_{\rho_X}^2 p^2}{g_{\rho_X}^2 (p^2 - m_{\rho_X}^2)}. \end{aligned} \quad (\text{A.3})$$

The form factors in (A.1) are identified as,

$$\widehat{\Pi}_0(p^2) = \Pi_a(p^2), \quad \widehat{\Pi}_0^X(p^2) = \Pi_X(p^2), \quad \widehat{\Pi}_1(p^2) = 2 [\Pi_{\widehat{a}}(p^2) - \Pi_a(p^2)]. \quad (\text{A.4})$$

To this we have to add the kinetic terms for the elementary gauge fields.<sup>8</sup> Therefore we define,

$$\begin{aligned} \Pi_0(p^2) &= -\frac{p^2}{g_0^2} + \widehat{\Pi}_0(p^2) = -\frac{p^2}{g_0^2} + \frac{m_\rho^2 p^2}{g_\rho^2 (p^2 - m_\rho^2)}, \\ \Pi_Y(p^2) &= -\frac{p^2}{g_{0Y}^2} + \widehat{\Pi}_0(p^2) + \widehat{\Pi}_0^X(p^2) = -\frac{p^2}{g_{0Y}^2} + \frac{m_\rho^2 p^2}{g_\rho^2 (p^2 - m_\rho^2)} + \frac{m_{\rho_X}^2 p^2}{g_{\rho_X}^2 (p^2 - m_{\rho_X}^2)}, \\ \Pi_1(p^2) &= \widehat{\Pi}_1(p^2) = -\frac{2m_\rho^4 (m_\rho^2 - m_{a_1}^2)}{g_\rho^2 (p^2 - m_{a_1}^2) (p^2 - m_\rho^2)}. \end{aligned} \quad (\text{A.5})$$

<sup>7</sup>The formulas here and below are well defined for euclidean momenta and the appropriate analytic continuation to Lorentzian signature is understood.

<sup>8</sup>These terms are not necessary in the 5D realization of the model since a kinetic term for the zero modes will be generated by the volume of the extra-dimension.

Setting to zero the non dynamical fields we obtain the coupling of the Higgs to the SM gauge fields,

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{gauge}} = & \frac{1}{2} P_{\mu\nu}^T \left[ \left( \Pi_0(p^2) + \frac{\sin^2(h/f)}{4} \Pi_1(p^2) \right) A_{aL}^\mu A_{aL}^\nu \right. \\ & \left. + \left( \Pi_Y(p^2) + \frac{\sin^2(h/f)}{4} \Pi_1(p^2) \right) Y^\mu Y^\nu + 2 \sin^2(h/f) \Pi_1(p^2) \widehat{H}^\dagger T_L^a Y \widehat{H} A_{aL}^\mu Y^\nu \right], \end{aligned} \quad (\text{A.6})$$

where  $A_\mu^{aL}, Y_\mu$  are  $\text{SU}(2)_L \times \text{U}(1)_Y$  gauge fields and  $\widehat{H} = \frac{1}{h} \begin{pmatrix} h^2 - ih^1 \\ h^4 - ih^3 \end{pmatrix}$ . Expanding in  $p^2$  we extract the  $W$  and  $Z$  masses:

$$\begin{aligned} m_W^2 &= -\frac{1}{4} \frac{\Pi_1(0)}{\Pi_0'(0)} \sin^2 \frac{\langle h \rangle}{f} = \frac{g^2}{4} f^2 \sin^2 \frac{\langle h \rangle}{f}, \\ m_Z^2 &= -\frac{1}{4} \left[ \frac{\Pi_1(0)}{\Pi_0'(0)} + \frac{\Pi_1(0)}{\Pi_Y'(0)} \right] \sin^2 \frac{\langle h \rangle}{f} = \frac{g^2 + g'^2}{4} f^2 \sin^2 \frac{\langle h \rangle}{f}. \end{aligned} \quad (\text{A.7})$$

from which the definition of  $v$  in (3.11) follows.

## A.2 Spin-1/2

The effective action for SM fermions for  $\text{CHM}_5$  and  $\text{CHM}_{10}$  takes the form [27],

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{q}_L \not{p} \left[ \Pi_0^q(p^2) + \frac{s_h^2}{2} \left( \Pi_1^{q1}(p^2) \widehat{H}^c \widehat{H}^{c\dagger} + \Pi_1^{q2}(p^2) \widehat{H} \widehat{H}^\dagger \right) \right] q_L \\ & + \bar{u}_R \not{p} \left( \Pi_0^u(p^2) + \frac{s_h^2}{2} \Pi_1^u(p^2) \right) u_R + \bar{d}_R \not{p} \left( \Pi_0^d(p^2) + \frac{s_h^2}{2} \Pi_1^d(p^2) \right) d_R \\ & + \frac{s_h c_h}{\sqrt{2}} M_1^u(p^2) \bar{q}_L \widehat{H}^c u_R + \frac{s_h c_h}{\sqrt{2}} M_1^d(p^2) \bar{q}_L \widehat{H} d_R + \text{h.c.} \end{aligned} \quad (\text{A.8})$$

We find that the form factors can be expressed in both models in terms of the following functions,

$$\begin{aligned} \widehat{\Pi}[m_1, m_2, m_3] &= \frac{(m_2^2 + m_3^2 - p^2) \Delta^2}{p^4 - p^2(m_1^2 + m_2^2 + m_3^2) + m_1^2 m_2^2}, \\ \widehat{M}[m_1, m_2, m_3] &= -\frac{m_1 m_2 m_3 \Delta^2}{p^4 - p^2(m_1^2 + m_2^2 + m_3^2) + m_1^2 m_2^2}. \end{aligned} \quad (\text{A.9})$$

### A.2.1 $\text{CHM}_5$

To determine the form factors in (A.8) we write the  $\text{SO}(5) \times \text{U}(1)_X$  invariant action for the sources  $\Psi_{qL}$  and  $\Psi_{uR}$  (neglecting the bottom sector),

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{CHM}_5} = & \bar{\Psi}_{qL}^i \not{p} \left( \delta^{ij} \widehat{\Pi}_0^{qL}(p^2) + \Phi^i \Phi^j \widehat{\Pi}_1^{qL}(p^2) \right) \Psi_{qL}^j + \bar{\Psi}_{uR}^i \not{p} \left( \delta^{ij} \widehat{\Pi}_0^{uR}(p^2) + \Phi^i \Phi^j \widehat{\Pi}_1^{uR}(p^2) \right) \Psi_{uR}^j \\ & + \bar{\Psi}_{qL}^i \left( \delta^{ij} \widehat{M}_0^u(p^2) + \Phi^i \Phi^j \widehat{M}_1^u(p^2) \right) \Psi_{uR}^j + \text{h.c.} \end{aligned} \quad (\text{A.10})$$

The form factors can be obtained integrating out the composite fermions in (2.11). In terms of the building blocks (A.9) one derives,

$$\begin{aligned}
 \widehat{\Pi}_0^{qL} &= \widehat{\Pi}[m_T, m_{\widetilde{T}}, m_{Y_T}], & \widehat{\Pi}_1^{qL} &= \widehat{\Pi}[m_T, m_{\widetilde{T}}, m_{Y_T} + Y_T] - \widehat{\Pi}[m_T, m_{\widetilde{T}}, m_{Y_T}], \\
 \widehat{\Pi}_0^{uR} &= \widehat{\Pi}[m_{\widetilde{T}}, m_T, m_{Y_T}], & \widehat{\Pi}_1^{uR} &= \widehat{\Pi}[m_{\widetilde{T}}, m_T, m_{Y_T} + Y_T] - \widehat{\Pi}[m_{\widetilde{T}}, m_T, m_{Y_T}], \\
 \widehat{M}_0^u &= \widehat{M}[m_T, m_{\widetilde{T}}, m_{Y_T}], & \widehat{M}_1^u &= \widehat{\Pi}[m_T, m_{\widetilde{T}}, m_{Y_T} + Y_T] - \widehat{\Pi}[m_T, m_{\widetilde{T}}, m_{Y_T}].
 \end{aligned}
 \tag{A.11}$$

The matching with eq. (A.8) is obtained for:

$$\begin{aligned}
 \Pi_0^q &= \frac{1}{y_{tL}^2} + \widehat{\Pi}_0^{qL}, & \Pi_1^{q1} &= \widehat{\Pi}_1^{qL}, & M_1^u &= \widehat{M}_1^u, \\
 \Pi_0^u &= \frac{1}{y_{tR}^2} + \widehat{\Pi}_0^{uR} + \widehat{\Pi}_1^{uR}, & \Pi_1^u &= -2\widehat{\Pi}_1^{uR}
 \end{aligned}
 \tag{A.12}$$

where we have added the kinetic terms for the elementary fields. In our CHM<sub>5</sub>,  $\Pi_1^{q2}$  in (A.8) is negligible due to the choice  $\Delta_{bL} \ll \Delta_{tL}$ .

From eq. (A.8) we extract the top mass,

$$\begin{aligned}
 m_t &\approx \frac{s_h c_h}{\sqrt{2}} \frac{M_1^u(0)}{\sqrt{\Pi_0^q(0)\Pi_0^u(0)}} \\
 &\approx \frac{v}{\sqrt{2}} \cdot \frac{1}{\sqrt{\left(1 + \frac{(m_{\widetilde{T}}^2 + m_{Y_T}^2)\Delta_{tL}^2}{m_{\widetilde{T}}^2 m_T^2}\right) \left(1 + \frac{(m_T^2 + (m_{Y_T} + Y_T)^2)\Delta_{tR}^2}{m_{\widetilde{T}}^2 m_T^2}\right)}} \cdot \frac{\Delta_{tL}}{m_T} \frac{\Delta_{tR}}{m_{\widetilde{T}}} \frac{Y_T}{f}
 \end{aligned}
 \tag{A.13}$$

used for the numerical computations in section 4.

### A.2.2 CHM<sub>10</sub>

Here we briefly present the relevant formulas for the CHM<sub>10</sub> model of [27]. We refer to this paper for all the details. In this model the SM quark are embedded in three distinct **10** reps with  $X = 2/3$ . Under  $SU(2)_L \times SU(2)_R$  the representation decomposes as  $\mathbf{10} = (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$ . Differently from the CHM<sub>5</sub> case, SM singlet  $t_R$  and  $b_R$  couple to a  $(\mathbf{1}, \mathbf{3})$  with  $T_R^3 = 0, -1$  respectively.

The 2-site CHM<sub>10</sub> lagrangian reads,

$$\begin{aligned}
 \mathcal{L}^{\text{CHM}_{10}} &= \mathcal{L}_{\text{fermions}}^{el} \\
 &+ \Delta \text{Tr} [\bar{q}_L \Omega_1 \Psi_T] + \Delta \text{Tr} [\bar{t}_R \Omega_1 \Psi_{\widetilde{T}}] + \text{h.c.} \\
 &+ \text{Tr} [\bar{\Psi}_T (i\not{D}^p - m_T) \Psi_T] + \text{Tr} [\bar{\Psi}_{\widetilde{T}} (i\not{D}^p - m_{\widetilde{T}}) \Psi_{\widetilde{T}}] \\
 &- Y_T \Phi_2 \bar{\Psi}_{T,L} \Psi_{\widetilde{T},R} \Phi_2^T - m_{Y_T} \text{Tr} [\bar{\Psi}_{T,L} \Psi_{\widetilde{T},R}] + \text{h.c.}
 \end{aligned}
 \tag{A.14}$$

The effective lagrangian for the chiral sources  $\Psi_{q_L}$  and  $\Psi_{u_R}$  in the **10** rep, in the background of a constant GB field, is

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{CHM}_{10}} = & \sum_{r=q_L, u_R} \left[ \text{Tr}(\bar{\Psi}_r \not{p} \hat{\Pi}_0^r(p^2) \Psi_r) + \Phi \bar{\Psi}_r \not{p} \hat{\Pi}_1^r(p^2) \Psi_r \Phi^T \right] \\ & + \left[ \text{Tr}(\bar{\Psi}_{q_L} \widehat{M}_0^u(p^2) \Psi_{u_R}) + \Phi \bar{\Psi}_{q_L} \widehat{M}_1^u(p^2) \Psi_{u_R} \Phi^T \right] + \text{h.c.} . \end{aligned} \quad (\text{A.15})$$

Also in this case we can write the form factors in terms of (A.9),

$$\begin{aligned} \hat{\Pi}_0^{qL} &= \hat{\Pi}[m_T, m_{\tilde{T}}, m_{Y_T}], & \hat{\Pi}_1^{qL} &= 2\hat{\Pi}[m_T, m_{\tilde{T}}, m_{Y_T} + Y_T/2] - 2\hat{\Pi}[m_T, m_{\tilde{T}}, m_{Y_T}], \\ \hat{\Pi}_0^{uR} &= \hat{\Pi}[m_{\tilde{T}}, m_T, m_{Y_T}], & \hat{\Pi}_1^{uR} &= 2\hat{\Pi}[m_{\tilde{T}}, m_T, m_{Y_T} + Y_T/2] - 2\hat{\Pi}[m_{\tilde{T}}, m_T, m_{Y_T}], \\ \widehat{M}_0^u &= \widehat{M}[m_T, m_{\tilde{T}}, m_{Y_T}], & \widehat{M}_1^u &= 2\hat{\Pi}[m_T, m_{\tilde{T}}, m_{Y_T} + Y_T/2] - 2\hat{\Pi}[m_T, m_{\tilde{T}}, m_{Y_T}]. \end{aligned} \quad (\text{A.16})$$

To match with eq. (A.8) we define:

$$\begin{aligned} \Pi_0^q &= \frac{1}{y_{t_L}^2} + \hat{\Pi}_0^{qL} + \frac{1}{2} \hat{\Pi}_1^{qL}, & \Pi_1^{q1} &= -\frac{1}{2} \hat{\Pi}_1^{qL}, & \Pi_1^{q2} &= -\hat{\Pi}_1^{qL}, & M_1^u &= \frac{1}{2\sqrt{2}} \widehat{M}_1^u, \\ \Pi_0^u &= \frac{1}{y_{t_R}^2} + \hat{\Pi}_0^{uR}, & \Pi_1^u &= \frac{1}{2} \hat{\Pi}_1^{uR}, \end{aligned} \quad (\text{A.17})$$

which include kinetic terms of elementary fields. Differently from CHM<sub>5</sub>, now  $\Pi_1^{q2}$  is not negligible.

## B Effective potential

In this appendix we provide the details of the computation of the effective potential. This can be expressed in terms of integrals of the two-point functions of the strong sector listed in appendix A. In particular from the asymptotic behavior of the form factors one can easily check the convergence of the potential.

### B.1 Gauge contribution

We start considering the contribution to the Higgs potential induced by  $SU(2)_L$  gauge fields. Integrating over the gauge fields in (A.6) one has,

$$V(h)_{\text{gauge}} = \frac{9}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[ 1 + \frac{1}{4} \frac{\Pi_1(p^2)}{\Pi_0(p^2)} \sin^2 \frac{h}{f} \right], \quad (\text{B.1})$$

where  $\Pi_0(p^2)$  and  $\Pi_1(p^2)$  are defined in eq. (A.5). The integral is UV convergent because  $\Pi_1/\Pi_0$  scales as

$$\frac{\Pi_1(p^2)}{\Pi_0(p^2)} = -\frac{2g_0^2 m_\rho^4 (m_{a_1}^2 - m_\rho^2)}{g_\rho^2 p^2 [p^2 - m_\rho^2 (1 + g_0^2/g_\rho^2)] (p^2 - m_{a_1}^2)} \xrightarrow{p^2 \rightarrow \infty} \frac{1}{p^6}. \quad (\text{B.2})$$



To leading order

$$\begin{aligned}
 V(h)_{\text{gauge}} &\approx \int \frac{d^4p}{(2\pi)^4} \frac{9}{8} \frac{\Pi_1}{\Pi_0} \sin^2 \frac{h}{f} \\
 &= \frac{9}{4} \frac{1}{16\pi^2} \frac{g_0^2}{g_\rho^2} \frac{m_\rho^4 (m_{a_1}^2 - m_\rho^2)}{m_{a_1}^2 - m_\rho^2 (1 + g_0^2/g_\rho^2)} \ln \left[ \frac{m_{a_1}^2}{m_\rho^2 (1 + g_0^2/g_\rho^2)} \right] \sin^2 \frac{h}{f}. \quad (\text{B.3})
 \end{aligned}$$

The curvature of the potential at the origin is positive, a general feature of gauge interactions which tend to preserve the symmetry.

For  $m_{a_1} \rightarrow \infty$  (corresponding to  $f_2 \rightarrow \infty$  in our setup) the potential becomes logarithmically divergent, showing that the coset resonances are crucial for the finiteness of the potential. The finiteness of the potential is spoiled once non-local terms are added. In the language of the self-energies this corresponds to adding a constant to  $\Pi_1$ . One easily sees that in this case the potential is quadratically divergent.

## B.2 Fermionic contribution

The fermionic contribution to the Higgs effective potential is [5]:

$$V(h)_{\text{fermions}} = -2N_c \int \frac{d^4p}{(2\pi)^4} \left[ \ln \Pi_{b_L} + \ln (p^2 \Pi_{t_L} \Pi_{t_R} - \Pi_{t_L t_R}^2) \right]. \quad (\text{B.4})$$

From (A.8) we find,

$$\begin{aligned}
 \Pi_{t_L} &= \Pi_0^q + \frac{\sin^2(h/f)}{2} \Pi_1^{q_1} \\
 \Pi_{b_L} &= \Pi_0^q + \frac{\sin^2(h/f)}{2} \Pi_1^{q_2} \\
 \Pi_{t_R} &= \Pi_0^u + \frac{\sin^2(h/f)}{2} \Pi_1^u \\
 \Pi_{t_L t_R} &= \frac{\sin(h/f) \cos(h/f)}{\sqrt{2}} M_1^u.
 \end{aligned} \quad (\text{B.5})$$

Let us focus on the CHM<sub>5</sub>. Due to the choice of representations in this model, for  $\Delta_{b_L} \ll \Delta_{t_L}$  the contribution to the potential of  $\Pi_{b_L}$  is negligible. Expanding (B.4), we can identify the coefficients  $\alpha$  and  $\beta$  of eq. (4.1),

$$\alpha = -2N_c \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{2} \frac{\Pi_1^{q_1}}{\Pi_0^q} + \frac{1}{2} \frac{\Pi_1^u}{\Pi_0^u} \right], \quad (\text{B.6})$$

$$\beta = -N_c \int \frac{d^4p}{(2\pi)^4} \left[ \frac{(M_1^u)^2}{p^2 \Pi_0^q \Pi_0^u} \right]. \quad (\text{B.7})$$

Using the expressions listed in (A.11), (A.12) it is easy to check that the integrals are finite as each term goes at most as  $1/p^6$  at large momenta. For example,

$$\frac{\Pi_1^q}{\Pi_0^q} = -\frac{Y_T(2m_{Y_T} + Y_T)m_T^2 \Delta_{t_L}^2}{p^6} + \mathcal{O}\left(\frac{1}{p^8}\right). \quad (\text{B.8})$$

For  $Y_T \rightarrow \infty$  the integral becomes logarithmically divergent. This is similar to the potential generated by gauge loops when  $f_2 \rightarrow \infty$  since we do not have complete SO(5) multiplets

in this limit. This shows that a complete  $G$  multiplet of resonances is the minimal number of degrees of freedom which allows the leading contribution to the potential to be finite. The finiteness of the potential is also spoiled if the most general lagrangian without  $LR$  structure is written on the last site in eq. (3.14), again with a logarithmic divergence.

The most important feature of this scenario is that the potential is tunable because we have contributions with different functional dependence, on  $\sin^2(h/f)$  and  $\sin^2(h/f)\cos^2(h/f)$ . Successful EWSB can be obtained for  $\Delta_{t_L} \sim \Delta_{t_R}$  as the two contributions to  $\alpha$  in (B.6) tend to cancel each other leading to  $\alpha$  smaller than its natural size. One can also see that the leading contributions to  $\alpha$  vanish for  $Y_T \rightarrow -2m_{Y_T}$ .

CHM<sub>10</sub> works in a similar way, in particular also in this case the potential is finite.

### C Interaction vertices $\rho\pi\pi$

Here we present the general formulas for trilinear vertices among composite spin-1 resonances and GBs. We focus on the SO(4) resonances as the coset resonances have suppressed trilinear couplings to GBs (in absence of mixing to the elementary fields they are zero by SO(4) invariance, and only a small coupling is induced after EWSB). We will include non-minimal interactions (5.1) which, if sizable, importantly modifies the decay widths. In the general case the GBs are conveniently parametrized as,

$$\Omega_1 = e^{\frac{i}{f} \frac{f_2^2}{f_1^2 + f_2^2} \Pi}, \quad \Phi_2 = \phi_0 e^{-\frac{i}{f} \frac{f_1^2}{f_1^2 + f_2^2} \Pi}, \quad (C.1)$$

where  $f$  is the low energy decay constant (2.14). Expanding the lagrangian (3.1) + (5.1) around  $\Pi = 0$  we obtain,

$$\begin{aligned} \mathcal{L}_{\text{cubic}} = & -\frac{f_0^2}{f^2} g_0 \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} A_\mu^{c'} f^{\hat{a}\hat{b}c'} - \frac{(f^2 - f_0^2)^2}{2f^2 f_1^2} g_0 \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} A_\mu^{c'} f^{\hat{a}\hat{b}c'} \\ & - \frac{f^2 - f_0^2}{2f^2} g_\rho \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} \rho_\mu^c f^{\hat{a}\hat{b}c} - \frac{(f^2 - f_0^2)^2}{2f^2 f_2^2} g_\rho \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} \rho_\mu^c f^{\hat{a}\hat{b}c}. \end{aligned} \quad (C.2)$$

Here the elementary fields are  $A_\mu = A_\mu^{c'} T^{c'}$ , where  $T^{c'} = \{T_L^a, Y\}$  and composite ones  $\rho_\mu = \rho_\mu^c T^c$  with  $T^c = \{T_L^a, T_R^a, X\}$ . We have also traded  $h^{\hat{a}}$  for  $\pi^{\hat{a}}$ . We derive that, in absence of couplings to the elementary fields, the  $\rho\pi\pi$  coupling is ,

$$g_{\rho\pi\pi} = \frac{f^2 - f_0^2}{2f^2} \left( 1 + \frac{f^2 - f_0^2}{f_2^2} \right) g_\rho. \quad (C.3)$$

Eq. (C.3) shows that in the limit  $f_2 = 0$  the heavy resonances and GBs are decoupled, while for  $f_2 \rightarrow \infty$  we recover (5.2). From this we also obtain

$$m_\rho^2 = \frac{f_2^6}{(f_2^4 - (f^2 - f_0^2)^2)(f_2^2 + f^2 - f_0^2)} \times \frac{2f^2}{f^2 - f_0^2} g_{\rho\pi\pi}^2 f^2 \quad (C.4)$$

which generalizes (5.3) to finite  $f_2$ .

To evaluate effects due to the mixing with elementary states, let us consider the  $SU(2)_L$  fields. In the mass basis (before EWSB), we have:<sup>9</sup>

$$\left\{ g \cot \theta_L \left[ \frac{f^2 - f_0^2}{2f^2} \left( 1 + \frac{f^2 - f_0^2}{f_2^2} \right) \right] - g \tan \theta_L \left[ \frac{f_0^2}{f^2} + \frac{(f^2 - f_0^2)^2}{2f_1^2 f^2} \right] \right\} \partial^\mu \pi^{\hat{a}} \pi^{\hat{b}} \rho_{cL}^\mu f^{\hat{a}\hat{b}cL}, \quad (\text{C.5})$$

where  $g$  is defined in (3.10) and  $\theta_L$  in (3.9). To compare with the results of [10] one should consider the limit  $f_2 \rightarrow \infty$  and  $f_0 \rightarrow 0$  in (C.5). The trilinear coupling of  $SU(2)_L$  heavy vectors to GBs simplifies in this limit as

$$\frac{g}{2} (\cot \theta_L - \tan \theta_L) \rho_{cL}^\mu \partial_\mu \pi^{\hat{a}} \pi^{\hat{b}} f^{\hat{a}\hat{b}cL}. \quad (\text{C.6})$$

Analogous formulas hold for the other resonances. In our setup, contrary to [10], in the elementary composite basis the Higgs couples to the elementary fields through nearest neighbor interactions producing the additional (subleading) contribution proportional to  $g \tan \theta_L$ .

The effects of EWSB can be easily taken into account. Eq. (C.2) still holds with the replacement

$$\begin{aligned} f^{\hat{a}\hat{b}c'} &= -i \text{Tr} \left[ [T^{\hat{a}}, T^{\hat{b}}] \bar{T}^{c'} \right] \\ \bar{T}^{c'} &= e^{-i \frac{\bar{\Pi}}{f}} T^{c'} e^{i \frac{\bar{\Pi}}{f}}, \end{aligned} \quad (\text{C.7})$$

where the  $\bar{\Pi} = \sqrt{2} \langle h \rangle T^{\hat{4}}$  is the Higgs VEV. To see this we just have to notice that EWSB is equivalent to a rotation of the elementary fields. As expected we find in that the EWSB is negligible for practical purposes.

### C.1 $\rho \rightarrow W_L^+ W_L^-$ , $Z_L h$ decay widths

Composite resonances decay mostly into longitudinal SM gauge bosons and third generation quarks which are the SM states coupled more strongly to the composite sector. By using the equivalence theorem the decay into longitudinal SM gauge bosons are equal to leading order to the decay into the GBs. This allows to use (C.6) to compute the partial decay widths of the heavy neutral resonances in  $W_L^+ W_L^-$  and  $Z_L h$ . Two examples of neutral gauge bosons decay (with masses around 2 TeV), are given in the tables below. In absence of mixing with the elementary fields the resonance associated to  $U(1)_X$  does not decay into longitudinal SM gauge bosons. A very small decay width is induced by the mixing, the coupling of  $\rho_X$  to  $W/Z$  is of the order of  $g_{0Y}(g_{0Y}/g_\rho)$ . Before EWSB the decay of  $\rho_{3L}$  and  $\rho_{3R}$  is as expected identical in the two channels. After EWSB the neutral states mix and the exact mass eigenstates decay unequally in  $W_L^+ W_L^-$  and  $Z_L h$ . However since the mass difference is very small it is safe to neglect this effect for practical purposes. While  $\rho_{3L}$  and  $\rho_{3R}$  are almost degenerate in our general setup we do not expect  $\rho_X$  to be closely degenerate. As a consequence it will not have a large mixing with the others even after EWSB and its decay remains very small as found in the numerical examples above. This state can then only significantly decay into third generation quarks. For the same reason

<sup>9</sup>Some commutation rules are  $[T^{\hat{a}}, T^{\hat{b}}] = \frac{i}{2} \epsilon^{abc} (T_L^c + T_R^c)$  and  $[T^{\hat{a}}, T^{\hat{4}}] = \frac{i}{2} (T_L^a - T_R^a)$ .

	$\rho_{3L}$	$\rho_{3R}$	$\rho_X$
$m(\text{TeV})$	2.5	2.5	2.1
$\Gamma_{W_L^+W_L^-} = \Gamma_{Z_L h}$ (GeV)	50	50	$4 \cdot 10^{-3}$

**Table 1.** Decay widths of neutral resonances into  $W_L^+W_L^-$  and  $Z_L h$ .  $g_\rho = 7$ ,  $g_{\rho_X} = 6$  and  $f = 0.5$  TeV.

	$\rho_{3L}$	$\rho_{3R}$	$\rho_X$
$m(\text{TeV})$	2.9	2.8	1.4
$\Gamma_{W_L^+W_L^-} = \Gamma_{Z_L h}$ (GeV)	18	18	$6 \cdot 10^{-3}$

**Table 2.** Decay widths of neutral resonances into  $W_L^+W_L^-$  and  $Z_L h$ .  $g_\rho = 4$ ,  $g_{\rho_X} = 2$  and  $f = 1$  TeV.

the production through vector boson fusion is suppressed and proceeds through Drell-Yan process. The latter may be important even for  $\rho_{3L}$  and  $\rho_{3R}$  whose coupling to  $W$ ,  $Z$  is not suppressed. This result is in general different from the one of 5D models where the resonances are approximately degenerate, see [37].

**Open Access.** This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

## References

- [1] D.B. Kaplan and H. Georgi,  $SU(2) \times U(1)$  breaking by vacuum misalignment, *Phys. Lett.* **B 136** (1984) 183 [INSPIRE].
- [2] H. Georgi, D.B. Kaplan and P. Galison, Calculation of the composite Higgs mass, *Phys. Lett.* **B 143** (1984) 152 [INSPIRE].
- [3] H. Georgi and D.B. Kaplan, Composite Higgs and custodial  $SU(2)$ , *Phys. Lett.* **B 145** (1984) 216 [INSPIRE].
- [4] M.J. Dugan, H. Georgi and D.B. Kaplan, Anatomy of a composite Higgs model, *Nucl. Phys.* **B 254** (1985) 299 [INSPIRE].
- [5] K. Agashe, R. Contino and A. Pomarol, The minimal composite Higgs model, *Nucl. Phys.* **B 719** (2005) 165 [hep-ph/0412089] [INSPIRE].
- [6] N. Arkani-Hamed, M. Porrati and L. Randall, Holography and phenomenology, *JHEP* **08** (2001) 017 [hep-th/0012148] [INSPIRE].
- [7] R. Rattazzi and A. Zaffaroni, Comments on the holographic picture of the Randall-Sundrum model, *JHEP* **04** (2001) 021 [hep-th/0012248] [INSPIRE].
- [8] R. Contino, The Higgs as a composite Nambu-Goldstone boson, [arXiv:1005.4269](https://arxiv.org/abs/1005.4269) [INSPIRE].

- [9] D. Becciolini, M. Redi and A. Wulzer, *AdS/QCD: the relevance of the geometry*, *JHEP* **01** (2010) 074 [[arXiv:0906.4562](#)] [[INSPIRE](#)].
- [10] R. Contino, T. Kramer, M. Son and R. Sundrum, *Warped/composite phenomenology simplified*, *JHEP* **05** (2007) 074 [[hep-ph/0612180](#)] [[INSPIRE](#)].
- [11] G. Panico and A. Wulzer, *The discrete composite Higgs model*, *JHEP* **09** (2011) 135 [[arXiv:1106.2719](#)] [[INSPIRE](#)].
- [12] S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 1.*, *Phys. Rev.* **177** (1969) 2239 [[INSPIRE](#)].
- [13] C.G. Callan Jr., S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 2.*, *Phys. Rev.* **177** (1969) 2247 [[INSPIRE](#)].
- [14] B. Gripaios, A. Pomarol, F. Riva and J. Serra, *Beyond the minimal composite Higgs model*, *JHEP* **04** (2009) 070 [[arXiv:0902.1483](#)] [[INSPIRE](#)].
- [15] J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra, et al., *The other natural two Higgs doublet model*, *Nucl. Phys. B* **853** (2011) 1 [[arXiv:1105.5403](#)] [[INSPIRE](#)].
- [16] G. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, *The strongly-interacting light Higgs*, *JHEP* **06** (2007) 045 [[hep-ph/0703164](#)] [[INSPIRE](#)].
- [17] D. Son and M. Stephanov, *QCD and dimensional deconstruction*, *Phys. Rev. D* **69** (2004) 065020 [[hep-ph/0304182](#)] [[INSPIRE](#)].
- [18] H.-C. Cheng, J. Thaler and L.-T. Wang, *Little M-theory*, *JHEP* **09** (2006) 003 [[hep-ph/0607205](#)] [[INSPIRE](#)].
- [19] J. Thaler, *Little technicolor*, *JHEP* **07** (2005) 024 [[hep-ph/0502175](#)] [[INSPIRE](#)].
- [20] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, *Is  $\rho$  meson a dynamical gauge boson of hidden local symmetry?*, *Phys. Rev. Lett.* **54** (1985) 1215 [[INSPIRE](#)].
- [21] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *(De)constructing dimensions*, *Phys. Rev. Lett.* **86** (2001) 4757 [[hep-th/0104005](#)] [[INSPIRE](#)].
- [22] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett. B* **513** (2001) 232 [[hep-ph/0105239](#)] [[INSPIRE](#)].
- [23] C.T. Hill and A.K. Leibovich, *Deconstructing 5D QED*, *Phys. Rev. D* **66** (2002) 016006 [[hep-ph/0205057](#)] [[INSPIRE](#)].
- [24] R. Contino, D. Marzocca, D. Pappadopulo and R. Rattazzi, *On the effect of resonances in composite Higgs phenomenology*, *JHEP* **10** (2011) 081 [[arXiv:1109.1570](#)] [[INSPIRE](#)].
- [25] R. Barbieri, B. Bellazzini, V.S. Rychkov and A. Varagnolo, *The Higgs boson from an extended symmetry*, *Phys. Rev. D* **76** (2007) 115008 [[arXiv:0706.0432](#)] [[INSPIRE](#)].
- [26] R. Foadi, J.T. Lavery, C.R. Schmidt and J.-H. Yu, *Radiative electroweak symmetry breaking in a little Higgs model*, *JHEP* **06** (2010) 026 [[arXiv:1001.0584](#)] [[INSPIRE](#)].
- [27] R. Contino, L. Da Rold and A. Pomarol, *Light custodians in natural composite Higgs models*, *Phys. Rev. D* **75** (2007) 055014 [[hep-ph/0612048](#)] [[INSPIRE](#)].
- [28] K. Agashe, A. Azatov, T. Han, Y. Li, Z.-G. Si, et al., *LHC signals for coset electroweak gauge bosons in warped/composite PGB Higgs models*, *Phys. Rev. D* **81** (2010) 096002 [[arXiv:0911.0059](#)] [[INSPIRE](#)].

- [29] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *A custodial symmetry for  $Zb\bar{b}$* , *Phys. Lett. B* **641** (2006) 62 [[hep-ph/0605341](#)] [[INSPIRE](#)].
- [30] G. Panico, M. Safari and M. Serone, *Simple and realistic composite Higgs models in flat extra dimensions*, *JHEP* **02** (2011) 103 [[arXiv:1012.2875](#)] [[INSPIRE](#)].
- [31] D0 collaboration, V.M. Abazov et al., *Search for single vector-like quarks in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.96$  TeV*, *Phys. Rev. Lett.* **106** (2011) 081801 [[arXiv:1010.1466](#)] [[INSPIRE](#)].
- [32] CMS collaboration, S. Chatrchyan et al., *Search for a vector-like quark with charge 2/3 in  $t + Z$  events from  $pp$  collisions at  $\sqrt{s} = 7$  TeV*, *Phys. Rev. Lett.* **107** (2011) 271802 [[arXiv:1109.4985](#)] [[INSPIRE](#)].
- [33] C. Csáki, C. Grojean, H. Murayama, L. Pilo and J. Terning, *Gauge theories on an interval: unitarity without a Higgs*, *Phys. Rev. D* **69** (2004) 055006 [[hep-ph/0305237](#)] [[INSPIRE](#)].
- [34] C. Csáki, C. Grojean, L. Pilo and J. Terning, *Towards a realistic model of Higgsless electroweak symmetry breaking*, *Phys. Rev. Lett.* **92** (2004) 101802 [[hep-ph/0308038](#)] [[INSPIRE](#)].
- [35] A. Falkowski, C. Grojean, A. Kaminska, S. Pokorski and A. Weiler, *If no Higgs then what?*, *JHEP* **11** (2011) 028 [[arXiv:1108.1183](#)] [[INSPIRE](#)].
- [36] R. Casalbuoni, S. De Curtis and D. Dominici, *Moose models with vanishing  $S$  parameter*, *Phys. Rev. D* **70** (2004) 055010 [[hep-ph/0405188](#)] [[INSPIRE](#)].
- [37] K. Agashe, H. Davoudiasl, S. Gopalakrishna, T. Han, G.-Y. Huang, et al., *LHC signals for warped electroweak neutral gauge bosons*, *Phys. Rev. D* **76** (2007) 115015 [[arXiv:0709.0007](#)] [[INSPIRE](#)].