Published for SISSA by Springer

Received: November 12, 2013
Accepted: December 9, 2013
Published: January 2, 2014

$1/N^2$ corrections to the holographic Weyl anomaly

Arash Arabi Ardehali,[a] James T. Liu[a] and Phillip Szepietowski[b]

[a] *Michigan Center for Theoretical Physics, Randall Laboratory of Physics, The University of Michigan, Ann Arbor, MI 48109-1040, U.S.A.*

[b] *Department of Physics, University of Virginia, Box 400714, Charlottesville, VA 22904, U.S.A.*

E-mail: ardehali@umich.edu, jimliu@umich.edu, pgs8b@virginia.edu

Abstract: We compute the $\mathcal{O}(1)$ contribution to holographic $c-a$ for IIB supergravity on $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ and on $\mathrm{AdS}_5 \times T^{1,1}/\mathbb{Z}_n$. In both cases, we find agreement with the dual field theory results, thus providing $1/N^2$ checks of AdS/CFT with reduced supersymmetry. Since the holographic computation involves a sum over shortened multiplets in the KK tower, we provide some details on the S^5 and $T^{1,1}$ spectra in a form that is convenient when considering their $\mathbb{Z}_n$ orbifolds. The computation for the even $\mathbb{Z}_n$ orbifolds of S^5 includes a sum over the multiplets in the twisted sector that is essential for obtaining agreement with the dual field theory.

Keywords: AdS-CFT Correspondence, Anomalies in Field and String Theories, Supergravity Models, Supersymmetric gauge theory

ArXiv ePrint: 1310.2611

doi:10.1007/JHEP01(2014)002

JHEP01(2014)002

Contents

1 Introduction

The AdS/CFT correspondence relates, among other things, strongly coupled superconformal gauge theories to their dual string theories in weakly curved AdS backgrounds. Thus, in principle, one requires some knowledge of the strongly interacting field theory to test the duality. While this is often a challenging situation in general, in some cases it is possible to obtain exact results at strong coupling. These cases include the study of protected operators and BPS states as well as anomalies.

On the CFT side of the duality, the theory may be characterized by two central charges a and c in four dimensions. Moreover, data on these central charges can be obtained at strong coupling based on 't Hooft anomaly matching and supersymmetry arguments [1]. Assuming the CFT admits an AdS dual, it is then possible to reproduce the central charges through the holographic Weyl anomaly [2]. At large N, where the dual string theory can be well approximated by classical supergravity, the result $c = a = \mathcal{O}(N^2)$ exactly matches the CFT result at leading order.

Here we are interested in going beyond the leading order in comparing the central charges on both sides of the duality. In particular, we focus on four-dimensional quiver gauge theories dual to string theory on $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ as well as on $\mathrm{AdS}_5 \times T^{1,1}/\mathbb{Z}_n$. For

JHEP01(2014)002

the $\mathbb{Z}_n$ orbifold of S^5, a gauge theory computation gives

$$\begin{cases} c = a = \frac{N^2-1}{4}, & & n = 1; \\ c = \frac{N^2}{2} - \frac{1}{3}, & a = \frac{N^2}{2} - \frac{5}{12}, & n = 2; \\ c = n\left(\frac{N^2}{4} - \frac{1}{8}\right), & a = n\left(\frac{N^2}{4} - \frac{3}{16}\right), & n \geq 3, \end{cases} \tag{1.1}$$

which matches the leading order holographic Weyl anomaly computation

$$c = a = \frac{N^2}{4}\frac{\pi^3}{\mathrm{vol}(S^5/\mathbb{Z}_n)} = n\frac{N^2}{4}. \tag{1.2}$$

The corresponding expressions for $T^{1,1}/\mathbb{Z}_n$ are

$$c = n\left(\frac{27}{64}N^2 - \frac{1}{4}\right), \qquad a = n\left(\frac{27}{64}N^2 - \frac{3}{8}\right), \tag{1.3}$$

which again matches the holographic computation at leading order.

In order to reproduce the $\mathcal{O}(1)$ terms ($\mathcal{O}(1/N^2)$ relative to the leading $\mathcal{O}(N^2)$ terms) holographically, we must go beyond the tree level and consider loop corrections to the bulk effective action. As argued in [3], these loop effects fall into two categories: *i*) massive string states running in the loop, and *ii*) massless ten-dimensional supergravity states in the loop. From a five-dimensional point of view, the latter includes not just the supergravity states, but also those from the Kaluza-Klein tower obtained from compactifying IIB supergravity on either $S^5/\mathbb{Z}_n$ or $T^{1,1}/\mathbb{Z}_n$.

The case of $\mathcal{N} = 4$ SYM has been investigated in [4–8], where it was shown that the shift $N^2 \to N^2 - 1$ can be accounted for by considering the one-loop contribution from the Kaluza-Klein tower on S^5. In the approach of [6–8], the subleading holographic Weyl anomaly $\delta\mathcal{A}$ may be computed from the expression

$$\delta\mathcal{A} = -\sum(-1)^F \frac{(E_0 - 2)a_2}{32\pi^2}, \tag{1.4}$$

where the sum is over all the bulk KK states that could run in the loop. Here a_2 are four dimensional heat-kernel coefficients for the transverse space in the bulk (which has the same geometry as the regularized boundary), and E_0 is the lowest energy eigenvalue labeling the AdS representation of the field (corresponding to the conformal dimension Δ in the CFT dual). Using the appropriate heat-kernel coefficients, both c and a can be independently extracted from (1.4), provided the full KK spectrum of the five-dimensional bulk theory is available. In particular, when applied to the spectrum of IIB supergravity on $\mathrm{AdS}_5 \times S^5$ [9, 10], this expression successfully reproduces both c and a of $\mathcal{N} = 4$ SYM beyond the leading order [7].

There are, however, several practical issues in applying (1.4). Firstly, since the sum is over an infinite number of states in the KK tower, it needs to be regulated in some manner. Although physical quantities should not depend on choice of regulator, it is not entirely clear how this would work out in general. We will have more to say about this below. Secondly, the full linearized KK spectrum may be difficult to obtain for compactifications with

JHEP01(2014)002

reduced supersymmetry. This is not an issue for orbifolds of S^5 and $T^{1,1}$ considered here, whose complete linearized KK spectrum can be found from the known spectrum on their covers. However it becomes a difficulty when considering other interesting compactification spaces, such as $Y^{p,q}$ or $L^{a,b,c}$ manifolds [11, 12].

For the case of $\mathcal{N} = 2$ supergravity in AdS_5 (dual to $\mathcal{N} = 1$ superconformal field theory), the KK tower organizes itself into a combination of long and shortened multiplets. In ref. [13], we demonstrated that while long multiplets will contribute to the individual central charges c and a obtained from (1.4), their contribution vanishes for the difference $c - a$. In particular, this allows us to compute the difference using only knowledge of the protected data in the bulk. Note that, from the field theory point of view, the combination $c - a$ shows up in both the Weyl anomaly and the R-current anomaly via [1, 14]

$$\begin{aligned} \langle T^\mu_\mu \rangle &= \frac{c-a}{16\pi^2} R^2_{\mu\nu\rho\sigma} + \cdots , \\ \langle \partial_\mu \sqrt{g} R^\mu \rangle &= \frac{c-a}{48\pi^2} \epsilon^{\mu\nu}{}_{\rho\sigma} R_{\mu\nu\alpha\beta} R^{\rho\sigma\alpha\beta} + \cdots . \end{aligned} \tag{1.5}$$

This relation of $c - a$ to the R-current anomaly seems to be connected to the vanishing contribution from long multiplets. Note also that the combination $c - a$ gives the full Weyl anomaly if the field theory lives on a Ricci-flat manifold.

Since $c = a$ at leading order, we can rewrite (1.4) in the useful form

$$c - a = -\frac{1}{2} \sum (-1)^F (E_0 - 2) a_2 \Big|_{R^2_{\mu\nu\rho\sigma} \text{ term}}, \tag{1.6}$$

where the sum is only over states in shortened representations in the KK tower. Here it is worth noting that the coefficient of the Riemann-squared term in a_2 has a particularly simple form [15]. For a four-dimensional field transforming in the irreducible representation (A, B) of the Lorentz group, the expression is

$$180 a_2 \Big|_{R^2_{\mu\nu\rho\sigma} \text{ term}} = d(A, B) \left(1 + f(A) + f(B)\right), \tag{1.7}$$

where $d(A, B) = d(A)d(B) = (2A + 1)(2B + 1)$ is the dimension of the representation and $f(X) = X(X + 1)(6X(X + 1) - 7)$. The crucial observation is that the expression for $c - a$ then splits into a sum of factorized pieces

$$c - a = -\frac{1}{360} \sum (E_0 - 2)(-1)^{2A+2B} \left[d(A)d(B) + (d(A)f(A))d(B) + d(A)(d(B)f(B)) \right]. \tag{1.8}$$

The reason this vanishes for long multiplets is that such multiplets carry an equal number of integer and half-integer helicity states labeled by A and B, so that both $\sum(-1)^{2A} d(A) = 0$ and $\sum(-1)^{2B} d(B) = 0$, along with $\sum E_0 (-1)^{2A} d(A) = 0$ and $\sum E_0 (-1)^{2B} d(B) = 0$. (See e.g. [16] for a summary of unitary representations of $\mathrm{SU}(2, 2|1)$ and shortening conditions.) Note that this cancellation is independent of the explicit form of $f(X)$.

Shortened representations, however, do not have both A and B sums vanishing. Chiral and semi-long II multiplets have $\sum(-1)^{2A} d(A) = 0$ and $\sum E_0 (-1)^{2A} d(A) = 0$, while anti-chiral and semi-long I multiplets have the corresponding sums over B vanishing. Since this

JHEP01(2014)002

is insufficient to ensure the vanishing of all terms in (1.8), $c-a$ will receive a contribution from all short multiplets in the KK spectrum. (In this case, the explicit form of $f(X)$ does enter when computing the anomaly.) This suggests that $c-a$ may be related to some sort of indices on either side of the duality.[1]

Focusing on the difference $c-a$, we see from (1.1) that for $S^5/\mathbb{Z}_n$ orbifolds, the field theory gives

$$c-a=\begin{cases}0, & n=1;\\ \frac{1}{12}, & n=2;\\ \frac{n}{16}, & n\geq 3.\end{cases} \tag{1.9}$$

The $n=1$ case corresponds to the round five-sphere, or equivalently $\mathcal{N}=4$ SYM, and the result $c=a$ was reproduced on the gravity side in [7] by regulating the sum (1.6). We studied the $S^5/\mathbb{Z}_3$ case, corresponding to $\mathcal{N}=1$ $\mathrm{SU}(N)^3$ gauge theory, in [13] and similarly found exact matching with $c-a=3/16$ on both sides of the duality. In this paper, we extend our previous result for $S^5/\mathbb{Z}_3$ to arbitrary $\mathbb{Z}_n$ orbifolds of S^5 and again find exact matching with $c-a=n/16$. For even orbifolds, a contribution from the twisted sector is expected; this may be computed by starting with the low energy effective description of the twisted sector in terms of a $(2,0)$ tensor theory in six dimensions, KK reducing it to five dimensions and then applying eq. (1.6) to the resulting five-dimensional spectrum.

One issue that we have only alluded to so far is the contribution to the bulk effective action from massive string states running in the loop. As argued in [3], such holographic contributions would show up through higher-derivative corrections in the five-dimensional effective action, and they should be added to (1.6) to obtain the complete subleading shift to $c-a$. However, it turns out that these massive string loop contributions vanish for compactifications on S^5 and its $\mathbb{Z}_n$ orbifolds. Therefore the exact matching $c-a=n/16$ (or $c-a=1/12$ for $n=2$) is unaffected by massive string loop considerations.

The issue of massive string loop corrections will arise for other Sasaki-Einstein compactifications of IIB string theory. In particular, the computation in [3] predicts that such string loops would contribute $1/24$ to $c-a$ for the conifold theory. In order to investigate this possible contribution, in this paper we also examine orbifolds of $T^{1,1}$. Curiously, we find that the sum of the KK tower in (1.6) completely reproduces the field theory result, so that massive string loop contributions are in fact *not* necessary (and so would ruin the matching if included). This presents a puzzle for the fate of the massive string loop corrections.

This paper is organized as follows. In the next section, we consider IIB supergravity on $\mathrm{AdS}_5\times S^5/\mathbb{Z}_n$ and find the result $c-a=n/16$ (or $c-a=1/12$ for $n=2$), in agreement with the field theory side of the duality. We also elaborate on the twisted states appearing in the cases with even n and demonstrate that they are necessary for the matching to work. In section 3, we examine IIB supergravity on $\mathrm{AdS}_5\times T^{1,1}/\mathbb{Z}_n$ and find $c-a=n/8$, which matches the field theory result provided there are no further contributions from massive

[1] A similar suspicion was stated in [17] that the anomaly coefficients might be related to the superconformal index on $S^3\times\mathbb{R}$. Since the anomaly coefficients are sensitive to the detailed spectrum only at subleading order, we consider it more likely that if such a relation exists, it would relate the superconformal index to the *subleading* part of the anomaly coefficients and perhaps directly to $c-a$.

JHEP01(2014)002

string loops. Finally, we conclude in section 4 with some open questions. Some details on the twisted sector of the $S^5/\mathbb{Z}_2$ orbifold are presented in appendix A.

2 Orbifolds of S^5

Perhaps the best studied framework for AdS/CFT involves the duality between IIB string theory on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super-Yang-Mills theory. This system preserves 32 real supercharges, and the appropriate supergroup is $\mathrm{SU}(2,2|4)$. Application of (1.4) demonstrates that the leading order Weyl anomaly $c = a = N^2/4$ gets shifted to $c = a = (N^2-1)/4$ [7]. However, the difference $c - a$ continues to vanish because of maximal supersymmetry.

Starting with $\mathrm{AdS}_5 \times S^5$, it is straightforward to consider the family of orbifold models $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ that preserve a reduced amount of supersymmetry. Here the orbifold $S^5/\mathbb{Z}_n$ is obtained by starting with $\mathbb{C}^3$ intersected with the unit sphere and modding out by the $\mathbb{Z}_n$ action generated by

$$\Omega = \begin{pmatrix} \omega & & \\ & \omega & \\ & & \omega^{-2} \end{pmatrix}, \tag{2.1}$$

where $\omega^n = 1$. Since this element is contained in $\mathrm{SU}(3)$, the orbifold generically preserves $\mathcal{N} = 1$ supersymmetry in four dimensions. Note, however, that for $n = 2$ this element is in the center of $\mathrm{SU}(2)$, so the $S^5/\mathbb{Z}_2$ orbifold actually preserves $\mathcal{N} = 2$ supersymmetry in four dimensions. (The $n = 3$ case is also somewhat special, as the element is then in the center of $\mathrm{SU}(3)$, a fact that we found useful in the analysis of [13]).

2.1 The spectrum and shortenings

For orbifolds of S^5, the natural starting point is simply the spectrum of IIB supergravity on the round S^5, originally obtained in [9, 10]. Since here we are interested in $\mathcal{N} = 2$ supergravity in five dimensions, we rewrite the $\mathcal{N} = 8$ spectrum in $\mathcal{N} = 2$ language that will be convenient for further applications. This is shown in table 1, where $\mathcal{D}(E_0, s_1, s_2; r)$ label the irreducible representations of the superalgebra SU(2,2|1).

For the holographic computation of $c - a$, however, only the shortened spectrum of the theory is needed. There are three multiplet-shortening conditions, corresponding to conserved, chiral (anti-chiral) and semi-long I (semi-long II) multiplets. Since these conditions constrain the relation between E_0 and r, for a given KK level p, only terms at the ends of the sums over k in table 1 correspond to shortened states. The shortened spectrum is shown in table 2. In this table, we also present the contribution of each short multiplet to $c - a$ as obtained in [13]. As a check, we have summed over all states shown in table 2, and found a vanishing correction to $c - a$, in agreement with the result of [7] for the round S^5 (dual to $\mathcal{N} = 4$ SYM).

We are, of course, interested in $\mathbb{Z}_n$ orbifolds of S^5 generated by the action of (2.1). Since this element commutes with $\mathrm{SU}(2)$ acting on the first two complex coordinates, it is natural to decompose the original $\mathrm{SU}(4)$ R-symmetry according to

$$\mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_r \supset \mathrm{SU}(2) \times \mathrm{U}(1)_q \times \mathrm{U}(1)_r. \tag{2.2}$$

JHEP01(2014)002

Supermultiplet	Representation	KK level
Graviton	$\sum_{k=0}^{p-2} \mathcal{D}(p+1, \frac{1}{2}, \frac{1}{2}; \frac{1}{3}(2p-4k-4))(k, p-k-2)$	$p \geq 2$
Gravitino I and III	$\sum_{k=0}^{p-1} \mathcal{D}(p+\frac{1}{2}, \frac{1}{2}, 0; \frac{1}{3}(2p-4k+1))(k, p-k-1)$	$p \geq 2$
	$+\sum_{k=0}^{p-1} \mathcal{D}(p+\frac{1}{2}, 0, \frac{1}{2}; \frac{1}{3}(2p-4k-5))(k, p-k-1)$	
Gravitino II and IV	$\sum_{k=0}^{p-3} \mathcal{D}(p+\frac{3}{2}, \frac{1}{2}, 0; \frac{1}{3}(2p-4k-9))(k, p-k-3)$	$p \geq 3$
	$+\sum_{k=0}^{p-3} \mathcal{D}(p+\frac{3}{2}, 0, \frac{1}{2}; \frac{1}{3}(2p-4k-3))(k, p-k-3)$	
Vector I	$\sum_{k=0}^{p} \mathcal{D}(p, 0, 0; \frac{1}{3}(2p-4k))(k, p-k)$	$p \geq 2$
Vector II	$\sum_{k=0}^{p-4} \mathcal{D}(p+2, 0, 0; \frac{1}{3}(2p-4k-8))(k, p-k-4)$	$p \geq 4$
Vector III and IV	$\sum_{k=0}^{p-2} \mathcal{D}(p+1, 0, 0; \frac{1}{3}(2p-4k-10))(k, p-k-2)$	$p \geq 2$
	$+\sum_{k=0}^{p-2} \mathcal{D}(p+1, 0, 0; \frac{1}{3}(2p-4k+2))(k, p-k-2)$	

Table 1. The spectrum of IIB supergravity on S^5 written in terms of $\mathcal{N}=2$ multiplets, and with the decomposition $\mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_r$. The supermultiplets are given in the conventional notation $\mathcal{D}(E_0, s_1, s_2; r)$ with the SU(3) representation given in terms of Dynkin labels (l_1, l_2) appended.

We define the U(1) normalizations by taking

$$\mathbf{4} \to \mathbf{3}_{1/3} + \mathbf{1}_{-1} \to \mathbf{2}_{1,1/3} + \mathbf{1}_{-2,1/3} + \mathbf{1}_{0,-1}. \tag{2.3}$$

Here the R-charge is conventionally normalized, while the $\mathrm{U}(1)_q$ charge is normalized so that the states that survive the $\mathbb{Z}_n$ orbifolding are those that satisfy

$$q = 0 \bmod n. \tag{2.4}$$

It is then simply a matter of group theory to project out the states in the massive KK tower.

Before considering the orbifold, we rewrite the shortened S^5 spectrum in terms of $\mathrm{SU}(2) \times \mathrm{U}(1)_q \times \mathrm{U}(1)_r$ quantum numbers. This is obtained by appropriately branching the representations in table 2, and the result is given in table 3. Of course this contains the same information as table 2. However, it is now in a form that is applicable to the $S^5/\mathbb{Z}_n$ orbifold models.

2.2 Subleading Weyl anomaly computation

We now turn to the computation of $c-a$ for the orbifolds $S^5/\mathbb{Z}_n$. Basically, our goal is to sum the individual contributions given in the last column of table 2 over the shortened representations of table 3 that survive the orbifolding. It is more convenient to rewrite the sums over KK level p and SU(2) representation k in table 3 in terms of sums over the $\mathrm{U}(1)_q$ charge q and KK level p. In this case, we can then restrict the sum over q to those satisfying the projection condition (2.4), namely $q = 0 \bmod n$.

One simplifying step is to note that the contribution to $c-a$ from the conserved multiplets at KK level $p=2$ in fact matches the sum of the corresponding contributions to $c-a$ from the SLI and SLII multiplets, if their contributions were to be extrapolated from $p>2$ to $p=2$. For example, if we took the graviton SLI and SLII contributions from

JHEP01(2014)002

Multiplet	KK level	Shortened representation	Shortening type	$c-a$ for one
Graviton	$p=2$	$\mathcal{D}(3,\frac{1}{2},\frac{1}{2};0)(0,0)$	conserved	$-\frac{5}{8}$
	$p>2$	$\mathcal{D}(p+1,\frac{1}{2},\frac{1}{2};-\frac{2}{3}(p-2))(p-2,0)$	SLI	$-\frac{5}{48}(p+1)$
		$\mathcal{D}(p+1,\frac{1}{2},\frac{1}{2};\frac{2}{3}(p-2))(0,p-2)$	SLII	$-\frac{5}{48}(p+1)$
Gravitino I	$p=2$	$\mathcal{D}(\frac{5}{2},\frac{1}{2},0;\frac{1}{3})(1,0)$	conserved	$\frac{35}{192}$
	$p\geq 2$	$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;\frac{2}{3}(p+\frac{1}{2}))(0,p-1)$	chiral	$-\frac{5}{48}(p-1)$
	$p>2$	$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;-\frac{2}{3}(p-\frac{5}{2}))(p-1,0)$	SLI	$-\frac{1}{96}(p+\frac{1}{2})$
		$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;\frac{2}{3}(p-\frac{3}{2}))(1,p-2)$	SLII	$\frac{5}{48}p$
Gravitino II	$p\geq 3$	$\mathcal{D}(p+\frac{3}{2},\frac{1}{2},0;-\frac{2}{3}(p-\frac{3}{2}))(p-3,0)$	SLI	$-\frac{1}{96}(p+\frac{3}{2})$
Gravitino III	$p=2$	$\mathcal{D}(\frac{5}{2},0,\frac{1}{2};-\frac{1}{3})(0,1)$	conserved	$\frac{35}{192}$
	$p\geq 2$	$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};-\frac{2}{3}(p+\frac{1}{2}))(p-1,0)$	anti-chiral	$-\frac{5}{48}(p-1)$
	$p>2$	$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};\frac{2}{3}(p-\frac{5}{2}))(0,p-1)$	SLII	$-\frac{1}{96}(p+\frac{1}{2})$
		$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};-\frac{2}{3}(p-\frac{3}{2}))(p-2,1)$	SLI	$\frac{5}{48}p$
Gravitino IV	$p\geq 3$	$\mathcal{D}(p+\frac{3}{2},0,\frac{1}{2};\frac{2}{3}(p-\frac{3}{2}))(0,p-3)$	SLII	$-\frac{1}{96}(p+\frac{3}{2})$
Vector I	$p=2$	$\mathcal{D}(2,0,0;0)(1,1)$	conserved	$\frac{1}{32}$
	$p\geq 2$	$\mathcal{D}(p,0,0;\frac{2}{3}p)(0,p)$	chiral	$-\frac{1}{96}(p-\frac{3}{2})$
		$\mathcal{D}(p,0,0;-\frac{2}{3}p)(p,0)$	anti-chiral	$-\frac{1}{96}(p-\frac{3}{2})$
	$p>2$	$\mathcal{D}(p,0,0;-\frac{2}{3}(p-2))(p-1,1)$	SLI	$\frac{1}{96}(p-\frac{1}{2})$
		$\mathcal{D}(p,0,0;\frac{2}{3}(p-2))(1,p-1)$	SLII	$\frac{1}{96}(p-\frac{1}{2})$
Vector II	—	—	—	
Vector III	$p\geq 2$	$\mathcal{D}(p+1,0,0;-\frac{2}{3}(p+1))(p-2,0)$	anti-chiral	$-\frac{1}{96}(p-\frac{1}{2})$
	$p\geq 3$	$\mathcal{D}(p+1,0,0;-\frac{2}{3}(p-1))(p-3,1)$	SLI	$\frac{1}{96}(p+\frac{1}{2})$
Vector IV	$p\geq 2$	$\mathcal{D}(p+1,0,0;\frac{2}{3}(p+1))(0,p-2)$	chiral	$-\frac{1}{96}(p-\frac{1}{2})$
	$p\geq 3$	$\mathcal{D}(p+1,0,0;\frac{2}{3}(p-1))(1,p-3)$	SLII	$\frac{1}{96}(p+\frac{1}{2})$

Table 2. Shortening structure of the S^5 KK tower. Note that Vector Multiplet II is never shortened. The contribution of a single shortened multiplet to $c-a$ is given in the last column. This factor must be multiplied by the dimension of the SU(3) representation to obtain the total contribution to $c-a$.

table 2 and set $p=2$, we would find

$$-\frac{5}{48}(p+1)-\frac{5}{48}(p+1)\Big|_{p=2}=-\frac{5}{8}, \tag{2.5}$$

which agrees with the value for the conserved graviton multiplet. It is easy to see that this holds in general for all of the conserved multiplets.

Continuing with the graviton multiplet, since the SLI and SLII multiplets are conjugates of each other, it is sufficient to consider only one of them, and double the result. We

JHEP01(2014)002

Multiplet	KK level	Shortened representation	Shortening type
Graviton	$p=2$	$\mathcal{D}(3,\frac{1}{2},\frac{1}{2};0)\mathbf{1}_0$	conserved
	$p>2$	$\mathcal{D}(p+1,\frac{1}{2},\frac{1}{2};-\frac{2}{3}(p-2))\sum_{k=0}^{p-2}(\mathbf{k+1})_{-2p+3k+4}$	SLI
		$\mathcal{D}(p+1,\frac{1}{2},\frac{1}{2};\frac{2}{3}(p-2))\sum_{k=0}^{p-2}(\mathbf{k+1})_{2p-3k-4}$	SLII
Gravitino I	$p=2$	$\mathcal{D}(\frac{5}{2},\frac{1}{2},0;\frac{1}{3})\mathbf{1}_{-2}+\mathbf{2}_1$	conserved
	$p\geq 2$	$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;\frac{2}{3}(p+\frac{1}{2}))\sum_{k=0}^{p-1}(\mathbf{k+1})_{2p-3k-2}$	chiral
	$p>2$	$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;-\frac{2}{3}(p-\frac{5}{2}))\sum_{k=0}^{p-1}(\mathbf{k+1})_{-2p+3k+2}$	SLI
		$\mathcal{D}(p+\frac{1}{2},\frac{1}{2},0;\frac{2}{3}(p-\frac{3}{2}))\sum_{k=1}^{p-1}(\mathbf{k+1})_{2p-3k}$ $+\sum_{k=0}^{p-2}(\mathbf{k+1})_{2p-3k-6}$	SLII
Gravitino II	$p\geq 3$	$\mathcal{D}(p+\frac{3}{2},\frac{1}{2},0;-\frac{2}{3}(p-\frac{3}{2}))\sum_{k=0}^{p-3}(\mathbf{k+1})_{-2p+3k+6}$	SLI
Gravitino III	$p=2$	$\mathcal{D}(\frac{5}{2},0,\frac{1}{2};-\frac{1}{3})\mathbf{1}_2+\mathbf{2}_{-1}$	conserved
	$p\geq 2$	$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};-\frac{2}{3}(p+\frac{1}{2}))\sum_{k=0}^{p-1}(\mathbf{k+1})_{-2p+3k+2}$	anti-chiral
	$p>2$	$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};\frac{2}{3}(p-\frac{5}{2}))\sum_{k=0}^{p-1}(\mathbf{k+1})_{2p-3k-2}$	SLII
		$\mathcal{D}(p+\frac{1}{2},0,\frac{1}{2};-\frac{2}{3}(p-\frac{3}{2}))\sum_{k=1}^{p-1}(\mathbf{k+1})_{-2p+3k}$ $+\sum_{k=0}^{p-2}(\mathbf{k+1})_{-2p+3k+6}$	SLI
Gravitino IV	$p\geq 3$	$\mathcal{D}(p+\frac{3}{2},0,\frac{1}{2};\frac{2}{3}(p-\frac{3}{2}))\sum_{k=0}^{p-3}(\mathbf{k+1})_{2p-3k-6}$	SLII
Vector I	$p=2$	$\mathcal{D}(2,0,0;0)\mathbf{1}_0+\mathbf{2}_3+\mathbf{2}_{-3}+\mathbf{3}_0$	conserved
	$p\geq 2$	$\mathcal{D}(p,0,0;\frac{2}{3}p)\sum_{k=0}^{p}(\mathbf{k+1})_{2p-3k}$	chiral
		$\mathcal{D}(p,0,0;-\frac{2}{3}p)\sum_{k=0}^{p}(\mathbf{k+1})_{-2p+3k}$	anti-chiral
	$p>2$	$\mathcal{D}(p,0,0;-\frac{2}{3}(p-2))\sum_{k=0}^{p-1}(\mathbf{k+1})_{-2p+3k+4}$ $+\sum_{k=1}^{p}(\mathbf{k+1})_{-2p+3k-2}$	SLI
		$\mathcal{D}(p,0,0;\frac{2}{3}(p-2))\sum_{k=1}^{p}(\mathbf{k+1})_{2p-3k+2}$ $+\sum_{k=0}^{p-1}(\mathbf{k+1})_{2p-3k-4}$	SLII
Vector II	—	—	—
Vector III	$p\geq 2$	$\mathcal{D}(p+1,0,0;-\frac{2}{3}(p+1))\sum_{k=0}^{p-2}(\mathbf{k+1})_{-2p+3k+4}$	anti-chiral
	$p\geq 3$	$\mathcal{D}(p+1,0,0;-\frac{2}{3}(p-1))\sum_{k=0}^{p-3}(\mathbf{k+1})_{-2p+3k+8}$ $+\sum_{k=1}^{p-2}(\mathbf{k+1})_{-2p+3k+2}$	SLI
Vector IV	$p\geq 2$	$\mathcal{D}(p+1,0,0;\frac{2}{3}(p+1))\sum_{k=0}^{p-2}(\mathbf{k+1})_{2p-3k-4}$	chiral
	$p\geq 3$	$\mathcal{D}(p+1,0,0;\frac{2}{3}(p-1))\sum_{k=0}^{p-3}(\mathbf{k+1})_{2p-3k-8}$ $+\sum_{k=1}^{p-2}(\mathbf{k+1})_{2p-3k-2}$	SLII

Table 3. The shortened multiplets of the S^5 KK tower decomposed in terms of $\mathrm{SU}(2)\times\mathrm{U}(1)_q\times\mathrm{U}(1)_r$ quantum numbers. Note that the SU(2) representations are given in terms of their dimensions.

are thus led to the contribution

$$c-a\Big|_{\text{graviton}}=2\times\sum_{p=2}^{\infty}\sum_{k=0}^{p-2}z^p\left(-\frac{5}{48}\right)(p+1)(k+1), \tag{2.6}$$

JHEP01(2014)002

where z^p is used to regulate the sum over KK modes, and where the $k+1$ factor is the dimension of the SU(2) representation. For the $\mathbb{Z}_n$ orbifold, the sum in (2.6) should be restricted to $q = 0 \bmod n$, where $q = -2p + 3k + 4$.

In order to make the q charge explicit, we write

$$c - a\Big|_{\text{graviton}} = \sum_{p=2}^{\infty}\sum_{k=0}^{p-2} f(-2p+3k+4, p, k), \tag{2.7}$$

where $f(q,p,k)$ is the summand in (2.6). It is a simple exercise to convert this into a set of sums over q

$$\begin{aligned} c - a\Big|_{\text{graviton}} &= \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} f(j, j+3l+2, j+2l) + \sum_{j=1}^{\infty}\sum_{l=0}^{\infty} f(-2j, j+3l+2, 2l) \\ &\quad + \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} f(-2j-1, j+3l+4, 2l+1). \end{aligned} \tag{2.8}$$

In particular, the first sum in (2.8) is over non-negative q, the second sum is over negative even q and the final sum is over negative odd q. Given this decomposition, it is now straightforward to restrict the q charges for the $\mathbb{Z}_n$ orbifold.

Note that for even n, the negative odd q sum in (2.8) drops out, while for odd n all three sums will contribute. Thus we consider even and odd cases separately. For even n, we have

$$c - a\Big|_{\text{graviton}}^{\text{even } \mathbb{Z}_n} = \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} f(nj, nj+3l+2, nj+2l) + \sum_{j=1}^{\infty}\sum_{l=0}^{\infty} f(-nj, nj/2+3l+2, 2l), \tag{2.9}$$

and for odd n we have

$$\begin{aligned} c - a\Big|_{\text{graviton}}^{\text{odd } \mathbb{Z}_n} &= \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} f(nj, nj+3l+2, nj+2l) + \sum_{j=1}^{\infty}\sum_{l=0}^{\infty} f(-2nj, nj+3l+2, 2l) \\ &\quad + \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} f(-n(2j+1), (n(2j+1)-1)/2+3l+4, 2l+1). \end{aligned} \tag{2.10}$$

In both cases, the function $f(q,p,k)$ for the graviton is given in (2.6):

$$f(q,p,k) = -\frac{5}{24} z^p (p+1)(k+1). \tag{2.11}$$

The sums can be evaluated, and the result for the graviton contribution is

$$c - a\Big|_{\text{graviton}} = \begin{cases} -\dfrac{5}{8n(z-1)^4} - \dfrac{5}{4n(z-1)^3} - \dfrac{65}{96n(z-1)^2} + \dfrac{n^4+20n^2+84}{4608n} + \cdots, & n \text{ even;} \\ -\dfrac{5}{8n(z-1)^4} - \dfrac{5}{4n(z-1)^3} - \dfrac{5}{8n(z-1)^2} + \dfrac{n^4+30n^2-31}{4608n} + \cdots, & n \text{ odd.} \end{cases} \tag{2.12}$$

Recall that z is used to regulate the sum over the infinite KK tower; following [7], we expect to ignore the pole terms and keep only the finite contribution to $c - a$.

JHEP01(2014)002

To obtain the full result, we sum over all shortened multiplets in table 3. Since the procedure for the other multiplets parallels that of the graviton multiplet, we omit the details here. However, there is one small detail for some of the other multiplets, which is that the restriction on KK level leads to a few exceptions in the sums for $n = 1$ (ie the round S^5) and $n = 2$ (ie $S^5/\mathbb{Z}_2$). These exceptions are perhaps not surprising, as these cases have additional supersymmetry compared with the generic orbifolds.

2.2.1 Odd orbifolds

For odd n, the $\mathbb{Z}_n$ element (2.1) acts freely on S^5. Hence there is no need to consider any twisted sectors, and the sum over the shortened KK spectrum gives the entire contribution to $c - a$. Curiously, the pole terms vanish identically when summing over all multiplets, and we are left with the simple result

$$c - a\Big|_{S^5/\mathbb{Z}_n} = \begin{cases} 0, & n = 1; \\ \dfrac{n}{16} + \cdots, & n \geq 3 \text{ odd}, \end{cases} \tag{2.13}$$

where the ellipses denote terms vanishing in the limit $z \to 1$. This matches the field theory result (1.9).

2.2.2 Even orbifolds

For even n, there is the added complication that the $\mathbb{Z}_n$ action admits a $\mathbb{Z}_2$ subgroup generated by

$$\Omega^{n/2} = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}. \tag{2.14}$$

This element leaves a fixed plane in $\mathbb{C}^3$, which gives rise to a fixed circle on S^5. Thus, to understand the even orbifolds, we will have to consider the effect of the twisted sector in addition to the KK tower discussed above.

Before discussing the twisted sector, we present the result from the sum over the shortened KK spectrum in the untwisted sector

$$c - a\Big|_{\text{untwisted}} = \begin{cases} -\dfrac{1}{8(z-1)^2} - \dfrac{1}{8(z-1)} + \dfrac{1}{16} + \cdots, & n = 2; \\ -\dfrac{1}{4n(z-1)^2} - \dfrac{1}{4n(z-1)} + \dfrac{5n^2-4}{96n} + \cdots, & n \geq 4 \text{ even}. \end{cases} \tag{2.15}$$

Unlike in the odd case, here the pole terms do not disappear. Note, however, that the leading fourth and third order poles cancel when summed over the complete set of multiplets. The $n = 2$ case is an exception since the dual quiver gauge theory has $\mathcal{N} = 2$ supersymmetry and chiral matter in the adjoint, as indicated in figure 1.

The twisted modes of the even orbifolds are known to arise from the KK reduction of the six-dimensional $(2, 0)$ tensor theory on $\mathrm{AdS}_5 \times S^1$ [18, 19]. Since these states originate from the $\mathbb{Z}_2$ action generated by (2.14), they preserve $\mathcal{N} = 4$ supersymmetry in five dimensions.

JHEP01(2014)002

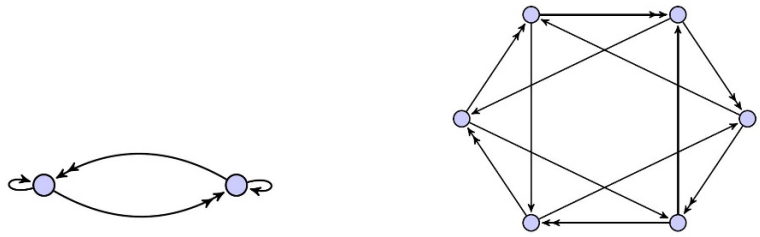

Figure 1. Two of the orbifold quivers. The $\mathbb{Z}_2$ orbifold is special because of the chiral multiplets in the adjoints and is shown on the left. The $\mathbb{Z}_6$ orbifold follows the generic pattern and is shown on the right.

However, they may be further decomposed into $\mathcal{N}=2$ multiplets. The result is presented in table 4. (See appendix A for additional details.) For even $n \geq 4$, an additional projection $q = 0 \bmod n$ must be imposed. In this case, the zero mode ($p = 0$) must be treated separately from the KK tower on S^1. We find

$$c - a\Big|_{p=0 \text{ twisted}} = \begin{cases} 1/48, & n = 2; \\ 1/32, & n \geq 4 \text{ even}, \end{cases} \tag{2.16}$$

and

$$c - a\Big|_{p\geq 1 \text{ twisted}} = \begin{cases} -\dfrac{1}{4(z-1)^2} - \dfrac{1}{4(z-1)} + 0 + \cdots, & n = 2; \\ -\dfrac{1}{2n(z-1)^2} - \dfrac{1}{2n(z-1)} + \dfrac{n^2-3n+4}{96n}, & n \geq 4 \text{ even}. \end{cases} \tag{2.17}$$

Again, a z^p regulator is used, with p the KK level on S^1. Here the double pole is leading, so there is no partial pole cancellation as there was in the untwisted sector. Note that one could introduce a different fugacity for the twisted states, say multiplying each term by y^p instead of z^p, but that would not change the finite part of the final result in (2.18) after one also expands around $y = 1$.

Adding together (2.15), (2.16) and (2.17), we find

$$c - a\Big|_{S^5/\mathbb{Z}_n} = \begin{cases} -\dfrac{3}{8(z-1)^2} - \dfrac{3}{8(z-1)} + \dfrac{1}{12} + \cdots, & n = 2; \\ -\dfrac{3}{4n(z-1)^2} - \dfrac{3}{4n(z-1)} + \dfrac{n}{16} + \cdots, & n \geq 4 \text{ even}. \end{cases} \tag{2.18}$$

Although the second and first order poles survive in this result, if we follow the regulation procedure of [7] and drop the poles, we see that the finite part agrees with the field theory result (1.9).

Thus we have successfully reproduced the field theory result for $c - a$, (1.9), for all $\mathbb{Z}_n$ orbifolds of S^5. For odd n, the regulated sum over the KK tower is finite, and directly gives $c - a = n/16$. For even n, the regulated sum diverges with first and second order poles. However, the finite term correctly gives $c - a = n/16$ (or $c - a = 1/12$ for $n = 2$). This distinction between even and odd orbifolds is presumably related to the presence of a twisted sector in the former case. Furthermore, the holographic contribution to $c - a$ from massive string loops vanishes in this case [3], so the result from the KK tower is complete.

JHEP01(2014)002

KK level	Representation	Shortening type
$p=0$	$\mathcal{D}(2,0,0;0)\mathbf{1}_0$	conserved
	$\mathcal{D}(2,0,0;\frac{4}{3})\mathbf{1}_{-2}$	chiral
	$\mathcal{D}(2,0,0;-\frac{4}{3})\mathbf{1}_2$	anti-chiral
$p\geq 1$	$\mathcal{D}(p+1,0,0;\frac{2}{3}(p+1))\mathbf{1}_{2p+2}$	chiral
	$\mathcal{D}(p+\frac{3}{2},\frac{1}{2},0;\frac{2}{3}(p+\frac{3}{2}))\mathbf{1}_{2p}$	
	$\mathcal{D}(p+2,0,0;\frac{2}{3}(p+2))\mathbf{1}_{2p-2}$	
	$\mathcal{D}(p+1,0,0;-\frac{2}{3}(p+1))\mathbf{1}_{-2p-2}$	anti-chiral
	$\mathcal{D}(p+\frac{3}{2},0,\frac{1}{2};-\frac{2}{3}(p+\frac{3}{2}))\mathbf{1}_{-2p}$	
	$\mathcal{D}(p+2,0,0;-\frac{2}{3}(p+2))\mathbf{1}_{-2p+2}$	

Table 4. The twisted sector states for the orbifold $S^5/\mathbb{Z}_2$ written in an $\mathcal{N}=2$ language. We use the same $\mathrm{SU}(2)\times\mathrm{U}(1)_q\times\mathrm{U}(1)_r$ decomposition as in (2.2).

3 Orbifolds of $T^{1,1}$

Having successfully matched the gravity and field theory results for $c-a$ for the supersymmetric orbifolds $S^5/\mathbb{Z}_n$, we would like to extend this comparison to more quiver gauge theories and their gravitational duals. Because the holographic computation requires knowledge of the shortened KK spectrum, we restrict our consideration to $T^{1,1}$, where the spectrum is known [20, 21].

3.1 The spectrum and shortenings

As demonstrated in [22], the generic KK spectrum for compactification of IIB supergravity on a Sasaki-Einstein manifold consists of nine generic KK multiplets (originally identified for $T^{1,1}$ in [20, 21]), along with possibly additional 'special' KK multiplets and Betti multiplets. An example of the special and Betti multiplets can be seen in the case of $S^5/\mathbb{Z}_2$, were the twisted sector states shown in table 4 can be organized into three chiral and three anti-chiral towers, corresponding to special multiplets, along with the three $q=0$ representations $\mathcal{D}(2,0,0;0)$, $\mathcal{D}(3,0,0,2)$ and $\mathcal{D}(3,0,0,-2)$, corresponding to Betti multiplets.[2] These special and Betti multiplets do not exist for the round S^5 nor for its odd orbifolds.

For a given Sasaki-Einstein manifold, the KK spectrum (excluding special and Betti multiplets) can be obtained in terms of the eigenvalues of the scalar Laplacian [20–22]. For $T^{1,1}$, define the eigenvalues of the scalar Laplacian on $T^{1,1}$ as

$$\Box Y = -H_0 Y, \qquad H_0 = 6[j(j+1)+\ell(\ell+1)-r^2/8], \tag{3.1}$$

where (j,ℓ,r) specify the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ quantum numbers. Here the r-charge is integer quantized, and is bounded by

$$|r|\leq 2\min(j,\ell). \tag{3.2}$$

[2]Recall that the topology of $S^5/\mathbb{Z}_2$ with the fixed circle blown up is the same as $T^{1,1}$ and therefore it admits the same type of Betti multiplets as the latter.

JHEP01(2014)002

Supermultiplet	Representation	e_0 condition
Graviton	$\mathcal{D}(e_0+3,\frac{1}{2},\frac{1}{2};r)$	$e_0 \geq 0$
Gravitino I and III	$\mathcal{D}(e_0+\frac{3}{2},\frac{1}{2},0;r+1)+\mathcal{D}(e_0+\frac{3}{2},0,\frac{1}{2};r-1)$	$e_0 > 0$
Gravitino II and IV	$\mathcal{D}(e_0+\frac{9}{2},\frac{1}{2},0;r-1)+\mathcal{D}(e_0+\frac{9}{2},0,\frac{1}{2};r+1)$	$e_0 \geq 0$
Vector I	$\mathcal{D}(e_0,0,0;r)$	$e_0 > 0$
Vector II	$\mathcal{D}(e_0+6,0,0;r)$	$e_0 \geq 0$
Vector III and IV	$\mathcal{D}(e_0+3,0,0;r-2)+\mathcal{D}(e_0+3,0,0;r+2)$	$e_0 \geq 0$
Betti vector	$\mathcal{D}(2,0,0;0)$	
Betti hyper	$\mathcal{D}(3,0,0;2)+\mathcal{D}(3,0,0;-2)$	

Table 5. The $\mathcal{N}=2$ spectrum of IIB supergravity on $T^{1,1}$. All representations transform as (j,ℓ) under $\mathrm{SU}(2)\times\mathrm{SU}(2)$.

Supermultiplet	Representation	Name given in [24]
Graviton	$\mathcal{D}(3,\frac{1}{2},\frac{1}{2};0)$	supergraviton
Gravitino II and IV	$\mathcal{D}(\frac{9}{2},\frac{1}{2},0;-1)+\mathcal{D}(\frac{9}{2},0,\frac{1}{2},1)$	LH+RH massive gravitino
Vector II	$\mathcal{D}(6,0,0;0)$	massive vector
Vector III and IV	$\mathcal{D}(3,0,0;-2)+\mathcal{D}(3,0,0;2)$	LH+RH chiral

Table 6. The $e_0=0$ multiplets. These are the multiplets that survive the consistent Sasaki-Einstein truncation.

Now let

$$H_0 = e_0(e_0+4), \tag{3.3}$$

or

$$e_0 = \sqrt{H_0+4}-2 \geq 0. \tag{3.4}$$

The Kaluza-Klein supermultiplet spectrum on $T^{1,1}$ is then given in table 5. Note that $e_0=0$ corresponds to the zero mode on $T^{1,1}$, with $j=\ell=r=0$. There are four sets of supermultiplets where this is allowed; these are the ones that may be retained in the massive consistent truncation on Sasaki-Einstein [23–26], and they are shown in table 6.

It is straightforward to work out the multiplet shortening conditions for the $T^{1,1}$ spectrum, and the result is shown in table 7. We have also included the Betti multiplets in this table, as they are part of the shortened spectrum. There are no special multiplets for $T^{1,1}$.

Although the $T^{1,1}$ harmonics do not have an obvious single 'KK level' arrangement (since they involve harmonics on $S^2\times S^2\times S^1$ instead of a single S^5), the shortened multiplets follow the same pattern as those of S^5. Thus in table 7 we have assigned KK levels based on what they would have been for S^5 (or its orbifolds). Note, however, that j can take on both integer and half-integer values. The lowest KK level is $p=3/2$, and it consists of only Vector I (chiral and anti-chiral). In fact, this lowest KK level is unusual, in that the shortened representation $\mathcal{D}(\frac{3}{2},0,0;1)(\frac{1}{2},\frac{1}{2})+\mathcal{D}(\frac{3}{2},0,0;-1)(\frac{1}{2},\frac{1}{2})$ contains a complex

Multiplet	KK level	Shortened representation	Shortening type
Graviton	$p=2$	$\mathcal{D}(3,\frac{1}{2},\frac{1}{2};0)(0,0)$	conserved
	$p=3j+2$	$\mathcal{D}(3j+3,\frac{1}{2},\frac{1}{2};-2j)(j,j)$	SLI ($j>0$)
		$\mathcal{D}(3j+3,\frac{1}{2},\frac{1}{2};2j)(j,j)$	SLII ($j>0$)
Gravitino I	$p=3j+1$	$\mathcal{D}(3j+\frac{3}{2},\frac{1}{2},0;2j+1)(j,j)$	chiral ($j>0$)
		$\mathcal{D}(3j+\frac{3}{2},\frac{1}{2},0;-2j+1)(j,j)$	SLI ($j>0$)
	$p=3j+3$	$\mathcal{D}(3j+\frac{7}{2},\frac{1}{2},0;2j+1)(j+1,j)\oplus(j,j+1)$	SLII
Gravitino II	$p=3j+3$	$\mathcal{D}(3j+\frac{9}{2},\frac{1}{2},0;-2j-1)(j,j)$	SLI
Gravitino III	$p=3j+1$	$\mathcal{D}(3j+\frac{3}{2},0,\frac{1}{2};-2j-1)(j,j)$	anti-chiral ($j>0$)
		$\mathcal{D}(3j+\frac{3}{2},0,\frac{1}{2};2j-1)(j,j)$	SLII ($j>0$)
	$p=3j+3$	$\mathcal{D}(3j+\frac{7}{2},0,\frac{1}{2};-2j-1)(j+1,j)\oplus(j,j+1)$	SLI
Gravitino IV	$p=3j+3$	$\mathcal{D}(3j+\frac{9}{2},0,\frac{1}{2};2j+1)(j,j)$	SLII
Vector I	$p=2$	$\mathcal{D}(2,0,0;0)(1,0)\oplus(0,1)$	conserved
	$p=3j$	$\mathcal{D}(3j,0,0;2j)(j,j)$	chiral ($j>0$)
		$\mathcal{D}(3j,0,0;-2j)(j,j)$	anti-chiral ($j>0$)
	$p=3j+2$	$\mathcal{D}(3j+2,0,0;-2j)(j+1,j)\oplus(j,j+1)$	SLI ($j>0$)
		$\mathcal{D}(3j+2,0,0;2j)(j+1,j)\oplus(j,j+1)$	SLII ($j>0$)
Vector II	—	—	—
Vector III	$p=3j+2$	$\mathcal{D}(3j+3,0,0;-2j-2)(j,j)$	anti-chiral
	$p=3j+4$	$\mathcal{D}(3j+5,0,0;-2j-2)(j+1,j)\oplus(j,j+1)$	SLI
Vector IV	$p=3j+2$	$\mathcal{D}(3j+3,0,0;2j+2)(j,j)$	chiral
	$p=3j+4$	$\mathcal{D}(3j+5,0,0;2j+2)(j+1,j)\oplus(j,j+1)$	SLII
Betti vector	–	$\mathcal{D}(2,0,0;0)(0,0)$	conserved
Betti hyper	–	$\mathcal{D}(3,0,0;2)(0,0)$	chiral
		$\mathcal{D}(3,0,0;-2)(0,0)$	anti-chiral

Table 7. Shortening structure of the $T^{1,1}$ KK tower. The supermultiplets are given in the conventional notation $\mathcal{D}(E_0,s_1,s_2;r)$ with the SU(2) × SU(2) representation (j,ℓ) appended. Here $j=0,\frac{1}{2},1,\frac{3}{2},\ldots$, unless otherwise indicated. Note that Vector Multiplet II is never shortened. The 'KK level' is suggested by analogy with the S^5 spectrum.

scalar with $E_0=3/2$. This is in the range where both modes are normalizable. Thus the scalar needs to be quantized with Neumann (as opposed to the usual Dirichlet) boundary conditions in order to select out the $E_0=3/2$ mode. We will have more to say more about this below.

3.2 Subleading Weyl anomaly computation

The holographic computation of $c-a$ proceeds along the same lines as that for S^5. We essentially take the contributions to $c-a$ from table 2 and sum over the shortened $T^{1,1}$ spectrum of table 7. As in the S^5 case, we can simplify the sum over the spectrum by ignoring the conserved Graviton and Vector I multiplets and instead extend the sums for the corresponding SLI and SLII towers to include $j=0$.

JHEP01(2014)002

As an example, we present the computation of $c-a$ for the graviton tower. Using the same regularization procedure of multiplying by z^p (where p is the assigned KK level), we have

$$\begin{aligned} c-a\Big|_{\text{graviton}} &= 2\times\sum z^p\left(-\frac{5}{48}\right)(p+1)(2j+1)(2l+1) \\ &= 2\times\sum_j z^{3j+2}\left(-\frac{5}{48}\right)(3j+3)(2j+1)^2, \end{aligned} \tag{3.5}$$

where $j=0,\frac{1}{2},1,\ldots$, and the overall factor of two takes care of the conjugate multiplets. In the first line, the factor $(2j+1)(2l+1)$ corresponds to the dimension of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ representation, and in the second line we have substituted in the relation between p, j and l as shown in table 7. The sum can be easily evaluated, and the result for the graviton contribution is

$$c-a\Big|_{\text{graviton}} = -\frac{10}{27(z-1)^4}-\frac{20}{27(z-1)^3}-\frac{125}{324(z-1)^2}+\frac{385}{31104}+\cdots. \tag{3.6}$$

The contributions from the other towers can be worked out in a similar manner. In addition, the contribution from the Betti vector $(1/32)$ cancels against that from the Betti hyper $(-1/32)$. There is one subtlety, however, and that is related to the quantization of the $E_0=3/2$ scalar in the $p=3/2$ KK level, as mentioned above. The alternate boundary conditions used to quantize this scalar may modify its contribution to $c-a$ [27]. Thus we add a term $\delta_{\text{alt. quant.}}$ that accounts for this contribution. This, however, is at most a finite shift, and will not affect the convergence of the sum over the KK tower. Putting everything together, one arrives at

$$c-a\Big|_{T^{1,1}} = -\frac{2}{9(z-1)^2}-\frac{2}{9(z-1)}+\frac{1}{8}+\delta_{\text{alt. quant.}}+\cdots. \tag{3.7}$$

While the fourth and third order poles cancel at $z=1$, the second and first order poles do not. Hence the sum over the KK tower is divergent. Following the prescription of [7], we drop the pole terms, so we are left with the finite result $c-a=1/8+\delta_{\text{alt. quant.}}$ for $T^{1,1}$.

The shift $\delta_{\text{alt. quant.}}$ due to imposing Neumann boundary conditions for the $E_0=3/2$ scalar is not well understood. Ref. [27] reports an answer for this shift corresponding to $\delta=-1/180$ for each real scalar with $E_0<2$, but finds disagreement with the established results [28–30] on the shift in the a central charge due to the alternative boundary condition. We are not able to resolve this contradiction. However, it is interesting to note that the conifold gauge theory has $c-a=1/8$, suggesting that $\delta_{\text{alt. quant.}}$ should in fact vanish.

3.2.1 The $T^{1,1}/\mathbb{Z}_2$ orbifold

One way to avoid the issue of working with alternate boundary conditions is to consider orbifolds of $T^{1,1}$ where the $p=3/2$ KK level is projected out. We first consider the orbifold $T^{1,1}/\mathbb{Z}_2$ defined by taking the period along the U(1) fiber to be 2π instead of the normal 4π. This orbifold maintains the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ isometry of $T^{1,1}$, but projects to integer SU(2) charges only. This corresponds to taking integer j in table 7, so the KK level

JHEP01(2014)002

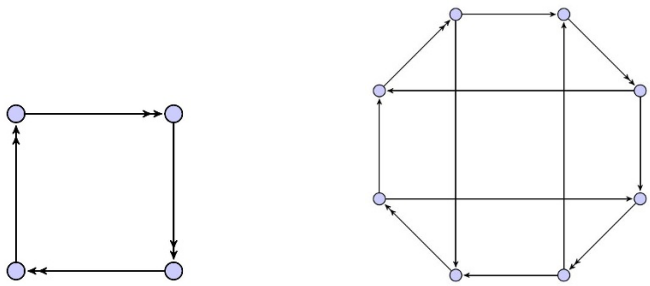

Figure 2. The quivers corresponding to $T^{1,1}/\mathbb{Z}_2$ (on the left) and $T^{1,1}/\mathbb{Z}_4$ (on the right). The latter is an example of typical quivers corresponding to orbifolds of $T^{1,1}$.

p is now an integer; in particular this removes the $p = 3/2$ multiplets from the spectrum. The dual quiver is shown in figure 2.

Computing the holographic $c - a$ is quite similar to the case of $T^{1,1}$, but the sums are now over integer j. We find

$$c - a\Big|_{T^{1,1}/\mathbb{Z}_2} = -\frac{1}{9(z-1)^2} - \frac{1}{9(z-1)} + \frac{1}{4} + \cdots . \tag{3.8}$$

Keeping the finite part gives $c - a = 1/4$, in perfect agreement with the field theory result corresponding to the four-node dual quiver.

3.2.2 The $T^{1,1}/\mathbb{Z}_n$ orbifolds

We now consider the $Y^{n,0} = T^{1,1}/\mathbb{Z}_n$ orbifolds obtained by taking a $\mathbb{Z}_n$ quotient of the conifold. In particular, we take the conifold to be defined by

$$xy - zw = 0. \tag{3.9}$$

Then the $\mathbb{Z}_n$ action is defined by [31]

$$x \to e^{2\pi i/n} x, \quad y \to e^{-2\pi i/n} y, \quad z \to e^{2\pi i/n} z, \quad w \to e^{-2\pi i/n} w. \tag{3.10}$$

This corresponds to a $\mathbb{Z}_n$ quotient of the $\mathrm{SU}(2)_j$ subgroup of the isometry group $\mathrm{SU}(2)_j \times \mathrm{SU}(2)_l \times \mathrm{U}(1)_r$ of $T^{1,1}$. Note that these orbifolds are all fixed-point-free, and that the $n = 2$ case corresponds to taking integer j, and hence reduces to the $\mathbb{Z}_2$ orbifold considered above. For $n > 2$, the isometry group of the orbifold is reduced to $\mathrm{SU}(2)_l \times \mathrm{U}(1)_j \times \mathrm{U}(1)_r$.

To find the $\mathbb{Z}_n$-singlet states, one simply decomposes $\mathrm{SU}(2)_j \supset \mathrm{U}(1)_j$, where $\mathrm{U}(1)_j$ is just the third component of isospin, and then keeps $j_z = 0 \bmod n/2$. For example, the conserved graviton multiplet with $(j, l) = (0, 0)$ along with the conserved vector multiplet with $(j, l) = (0, 1)$ survives the orbifolding. However, the conserved vector with $(j, l) = (1, 0)$ will be branched to $(0)_{-1} + (0)_0 + (0)_1$ (where the l quantum number is shown inside the parentheses and the $\mathrm{U}(1)_j$ charge is subscripted). Only the $(0)_0$ state will survive the $\mathbb{Z}_n$ projection for $n > 2$.

As above, we highlight the computation of $c - a$ for the graviton tower. We have

$$c - a\Big|_{\text{graviton}} = 2 \times \sum_j z^p \left(-\frac{5}{48}\right) (p+1)\gamma_j^{(n)}(2l+1). \tag{3.11}$$

This expression is identical to the first line of (3.5), except that the dimension of the complete $\mathrm{SU}(2)_j$ representation, $2j+1$, is replaced by $\gamma_j^{(n)}$, which counts the number of states surviving the orbifolding by $\mathbb{Z}_n$. For example, take $n=3$ and consider the $\mathrm{SU}(2)_j$ representation given by $j=4$. The j_z charges are then all integers from -4 to 4, and only three of the states, with $j_z=-3,0,3$ survive the projection. Hence we find $\gamma_4^{(3)}=3$. For the general case, we may write $j=(n\alpha+\beta)/2$, with α, β nonnegative integers where $\beta<n$. When n is even, it turns out that $\gamma_j^{(n)}=2\alpha+1$. When n is odd, $\gamma_j^{(n)}=\alpha$ for odd β, and $\gamma_j^{(n)}=\alpha+1$ for even β.

We skip the rest of the details and report the final answer

$$c-a\Big|_{T^{1,1}/\mathbb{Z}_n}=-\frac{2}{9n(z-1)^2}-\frac{2}{9n(z-1)}+\frac{n}{8}+\cdots\,. \tag{3.12}$$

Interestingly, although the computation bifurcates depending on even or odd n, this final result takes the same form in both cases. Setting $n=1$ reproduces the $T^{1,1}$ answer (3.7), but without $\delta_{\text{alt. quant.}}$. Keeping only the finite part, we find $c-a=n/8$, again in agreement with the field theory result for the $2n$-node quiver corresponding to $Y^{n,0}$.

It appears that we have been successful in reproducing the quiver field theory result $c-a=n/8$ for the entire family of $\mathbb{Z}_n$ orbifolds of $T^{1,1}$. However, this does raise a puzzle in that it was argued in [3] that $c-a$ for $T^{1,1}$ would receive an additional contribution of $1/24$ from massive string loop corrections, and it can be shown that this corresponds to a contribution of $n/24$ for $T^{1,1}/\mathbb{Z}_n$. For $T^{1,1}$ itself, this would suggest that $\delta_{\text{alt. quant.}}$ in (3.7) should take the value $-1/24$, so as to cancel the massive string loop correction. However, there is no added room for removing the $n/24$ contribution for the orbifolds with $n>1$. This suggests that the conjecture in [3] that one simply adds the massive string loop to the supergravity KK loop contributions in order to obtain $c-a$ needs refinement.

4 Discussion

Our main result is the exact matching of the holographic $c-a$ with the gauge theory result for the families of theories dual to IIB string theory on $S^5/\mathbb{Z}_n$ and $T^{1,1}/\mathbb{Z}_n$. This exact matching is achieved by considering in the bulk effective supergravity theory the one-loop contribution[3] on AdS_5 with all possible states in the shortened KK spectrum running in the loop. (The long multiplets have a vanishing contribution, and hence can be discarded from the computation.) Since the KK tower is unbounded, the sum (1.6) over the tower does not converge, and needs to be regulated. We have followed the regularization method of [8], which is to multiply the contribution at each KK level by z^p, where p is the level. This sum converges for $|z|<1$, and the value of $c-a$ is obtained by dropping the pole terms and keeping only the finite term when $z\to 1$.

One difficulty with this regulator is how to extend the notion of a KK level p to the case of a generic Sasaki-Einstein compactification. For S^5 and its orbifolds, one can take p to be the usual KK level on the round S^5 before projection. However, for a space like $T^{1,1}$,

[3] We need α' and g_s to be small enough (or λ and N large enough) that the ten dimensional supergravity gives a good approximation to the bulk effective action, but we must not take strict limits, as it would hide the subleading effects due to respectively massive and massless loops. The fact that $c-a$ of the quivers are independent of both λ and N then seems to guarantee the one-loop exactness of the gravitational results.

JHEP01(2014)002

there is no unambiguous notion of a KK level. Nevertheless, we have proposed a working definition of 'KK level' based on associating the E_0 values of the shortened spectrum with p values corresponding to what they would have been had they come from compactification on S^5. Although this yields non-integer levels p for $T^{1,1}$ and its odd orbifolds, the agreement we have found in the $c-a$ values suggests that this is a valid regulator. In fact, the number p has a clear AdS (or CFT) interpretation for the multiplets of S^5 compactification: it is the number of oscillator pairs that make up the representations of the isometry group $\mathrm{SO}(4,2) \sim \mathrm{SU}(2,2)$ [9, 32] (see also [33]). It seems likely that the 'KK level' we have assigned to the multiplets on $T^{1,1}$ has a similar purely AdS interpretation. It would be interesting to establish this explicitly.

Curiously, we have found that the regulated sum contributing to $c-a$ is finite at $z=1$ for the odd orbifolds of S^5. As we have shown in ref. [13], a zeta-function regularization yields the same result as the z^p regulator for $S^5/\mathbb{Z}_3$; we have also checked that this is the case for $S^5/\mathbb{Z}_5$, and expect it to hold for all the odd orbifolds. However, the regulated sum does have double and single poles at $z=1$ for the case of even orbifolds of S^5 and for all orbifolds of $T^{1,1}$. In these cases, it appears that a zeta function regularization will produce a different result. This is something we do not fully understand.

In fact, any time pole terms are present in the regulated $c-a$, it is possible to shift the finite part simply by transforming z. For example, if we took the result (3.7) for $T^{1,1}$ and let $z \to z^2$, we would end up with $5/36 + \delta_{\text{alt. quant.}}$ instead. Of course, such a transformation corresponds to a redefinition of the effective KK level. Hence this ambiguity in the finite term is closely related to how we define the KK level.

Curiously, whenever we have found a divergent expression for $c-a$, it has taken the form

$$c-a = \frac{\alpha}{(z-1)^2} + \frac{\alpha}{z-1} + \text{finite} = \alpha \frac{z}{(z-1)^2} + \text{finite}. \tag{4.1}$$

(The third and fourth order poles that could be present always seem to vanish when the contributions from the different multiplets are combined.) This suggests that the pole terms may be attributed to the sum over the KK tower as follows

$$\alpha \frac{z}{(z-1)^2} = \sum_{p=1}^{\infty} z^p (\alpha p). \tag{4.2}$$

It would be curious to see if there is any physical interpretation of this sum and in particular of the value of α. Note that $\alpha = 0$ for odd orbifolds of S^5, $\alpha = -3/4n$ for even orbifolds of S^5 and $\alpha = -2/9n$ for orbifolds of $T^{1,1}$.

One possible way around the possible ambiguities in choosing a regulator would be to work directly in ten-dimensional IIB supergravity. Then instead of summing over the KK tower and regulating this sum by multiplying by z^p, we could simply use a ten-dimensional heat kernel regularization (or nine-dimensional heat kernel for the directions transverse to the radial direction). This can be facilitated by taking advantage of the factorizability of the heat kernel on product spaces. Of course, this would just replace the spectral analysis on the internal manifold with an essentially equivalent heat kernel computation. However, it would naturally provide a uniform regularization instead of having separate ones involving the four-dimensional heat kernel coefficient along with the z^p regulator.

JHEP01(2014)002

Up to a possible ambiguity due to the unknown factor $\delta_{\text{alt. quant.}}$ for the case of $T^{1,1}$, we have shown that the holographic computation of $c-a$ in the IIB supergravity theory reproduces the corresponding field theory result. In particular, this leaves no room for contributions from massive string states running in the loop. This does not present a difficulty for the orbifolds of S^5, as the string loop contribution to $c-a$ vanishes in this case [3]. However, the contribution does not appear to vanish for orbifolds of $T^{1,1}$, and adding this contribution to the supergravity result would then destroy the perfect agreement with the dual gauge theory. One possible explanation for this disparity is that the string loop computation may not be completely independent of the supergravity computation. Although the supergravity computation necessarily excludes massive string states, there may be overlap in the massless sector.[4] In this case, adding the string loop result to the supergravity result would then end up double counting some of the contributions to $c-a$.

In the course of the present work, we have received insightful suggestions[5] as to how the z^p regulator may be generalized to cases where the KK level p may be ill-defined. A potentially fruitful idea is to introduce separate chemical potentials for the individual quantum numbers (or charges) associated to the isometry group of the internal manifold. This idea is particularly appealing when recalling the index-like nature of the holographic $c-a$. For example, the KK multiplets on $T^{1,1}$ are labeled by three quantum numbers j, l, r, corresponding to $\mathrm{SU}(2)_j \times \mathrm{SU}(2)_l \times \mathrm{U}(1)_r$. In this case, one would regulate the $T^{1,1}$ tower by multiplying by $z_1^j z_2^l z_3^r$. However, j, l and r are all related to each other in the shortened towers, so it is not clear if anything is gained by introducing separate chemical potentials for all three quantum numbers.

Another possibility is to regulate the sum by z^L, associating a chemical potential to the length L of the superfield dual to a given bulk multiplet. For S^5 and its orbifolds, this matches the z^p regularization in the untwisted sector. However, the length of the dual superfield and the assigned 'KK level' no longer coincide for $T^{1,1}$ and its orbifolds. Using the z^L regulator in these cases would yield results in disagreement with the field theory expectation (whether the massive string loop corrections suggested in [3] are included or not).

Some of these regulator issues, as well as puzzles about the possible contribution from massive string loops could potentially be resolved by studying additional pairs of AdS/CFT duals. A natural extension would be to consider the Sasaki-Einstein manifolds $Y^{p,q}$. Although knowledge of the full spectrum appears to be out of reach, we would only need information about the shortened spectrum in order to investigate $c-a$. A partial analysis for $Y^{p,q}$ was performed in [35], and we anticipate that this can be extended to provide information on the complete shortened spectrum. This is currently under investigation.

Acknowledgments

We are thankful to D. Minic and L. A. Pando Zayas for insightful comments on the regularization of the sum over the KK towers. A. A. A. is also grateful to G. Dall'Agata and

[4] One may suspect that the massive string states would fall into long representations of $\mathrm{SU}(2,2|1)$, much like the $\mathrm{SU}(2,2|4)$ case as discussed in [34], and hence would not contribute to $c-a$. If this were the case, then the computation of [3] would indeed represent a contribution from the massless sector.

[5] We are particularly thankful to D. Minic and L. A. Pando Zayas for stimulating discussions on this point.

JHEP01(2014)002

A. Hanany for helpful correspondence on the KK spectroscopy of $T^{1,1}$ and $T^{1,1}/\mathbb{Z}_2$. We would also like to thank M. Duff, R. Eager, Z. Komargodski, J. Schmude, Y. Tachikawa and B. Wecht for useful correspondence. This work was supported in part by the US Department of Energy under grants DE-SC0007859 and DE-SC0007984.

A The twisted sector states for the $S^5/\mathbb{Z}_2$ orbifold

The $S^5/\mathbb{Z}_2$ orbifold preserves 16 real supercharges, and the twisted sector may be described by a six-dimensional (2,0) theory with a single tensor multiplet [36]. The field content is $(B^-_{\mu\nu}, 5\phi, 4\chi)$ transforming as the $\mathbf{1}+\mathbf{5}+\mathbf{4}$ of USp(4). This may be reduced on $\mathrm{AdS}_5 \times S^1$ to give an effective five-dimensional $\mathcal{N}=4$ spectrum classified by SU(2,2|2) [18, 19]. Making use of the decompositions $\mathbf{5} \to \mathbf{3}_0 + \mathbf{1}_2 + \mathbf{1}_{-2}$ and $\mathbf{4} \to \mathbf{2}_1 + \mathbf{2}_{-1}$ under $\mathrm{USp}(4) \supset \mathrm{SU}(2)_R \times \mathrm{U}(1)_R$, the zero mode on the circle gives rise to the shortened $\mathcal{N}=4$ multiplet [19]

$$\begin{aligned}\mathcal{D}(2,0,0;\mathbf{3}_0) = {} & D(2,0,0)\mathbf{3}_0 + D\left(\frac{5}{2},\frac{1}{2},0\right)\mathbf{2}_{-1} + D\left(\frac{5}{2},0,\frac{1}{2}\right)\mathbf{2}_1 + D\left(3,\frac{1}{2},\frac{1}{2}\right)\mathbf{1}_0 \\ & + D(3,0,0)\mathbf{1}_{-2} + D(3,0,0)\mathbf{1}_2, \end{aligned} \tag{A.1}$$

where the AdS_5 representations are labeled by $D(E_0, s_1, s_2)$, and the $\mathrm{SU}(2)_R \times \mathrm{U}(1)_R$ quantum numbers are appended. The non-zero-modes are also shortened. For positive KK level $p \geq 1$, we have [18, 19]

$$\begin{aligned}\mathcal{D}(p{+}1,0,0;\mathbf{1}_{2p+2}) = {} & D(p{+}1,0,0)\mathbf{1}_{2p+2} + D\left(p{+}\frac{3}{2},\frac{1}{2},0\right)\mathbf{2}_{2p+1} + D(p{+}2,1,0)\mathbf{1}_{2p} \\ & + D(p{+}2,0,0)\mathbf{3}_{2p} + D\left(p{+}\frac{5}{2},\frac{1}{2},0\right)\mathbf{2}_{2p-1} + D(p{+}3,0,0)\mathbf{1}_{2p-2}. \end{aligned} \tag{A.2}$$

The negative KK modes are just the conjugates of the positive ones.

The reduction of the $\mathcal{N}=4$ representations to $\mathcal{N}=2$ follows from the decomposition $\mathrm{SU}(2)_R \times \mathrm{U}(1)_R \supset \mathrm{U}(1)_q \times \mathrm{U}(1)_r$, where

$$q = R - 2T^3, \qquad r = \frac{1}{3}(R + 4T^3). \tag{A.3}$$

Here T^3 is the Cartan generator of $\mathrm{SU}(2)_R$. The $\mathrm{U}(1)_q$ normalization is chosen to match that of the untwisted sector, while $\mathrm{U}(1)_r$ takes the conventional normalization for the $\mathcal{N}=2$ R-charge. The zero mode then breaks up into three $\mathcal{N}=2$ multiplets

$$\mathcal{D}(2,0,0;\mathbf{3}_0) = \mathcal{D}\left(2,0,0;-\frac{4}{3}\right)_2 + \mathcal{D}(2,0,0;0)_0 + \mathcal{D}\left(2,0,0;\frac{4}{3}\right)_{-2}, \tag{A.4}$$

where the q-charge is subscripted. The positive KK tower breaks up according to

$$\begin{aligned}\mathcal{D}(p+1,0,0;\mathbf{1}_{2p+2}) = {} & \mathcal{D}\left(p+1,0,0;\frac{2}{3}(p+1)\right)_{2p+2} + \mathcal{D}\left(p+\frac{3}{2},\frac{1}{2},0;\frac{2}{3}\left(p+\frac{3}{2}\right)\right)_{2p} \\ & + \mathcal{D}\left(p+2,0,0;\frac{2}{3}(p+2)\right)_{2p-2}. \end{aligned} \tag{A.5}$$

These are all shortened $\mathcal{N}=2$ states. This information is presented in table 4, where it is noted that they all transform as singlets under the SU(2) corresponding to rotations in the first two complex planes acted upon by the $\mathbb{Z}_2$ generator (2.14).

JHEP01(2014)002

References

[1] D. Anselmi, D. Freedman, M.T. Grisaru and A. Johansen, *Nonperturbative formulas for central functions of supersymmetric gauge theories*, *Nucl. Phys.* **B 526** (1998) 543 [hep-th/9708042] [INSPIRE].

[2] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *JHEP* **07** (1998) 023 [hep-th/9806087] [INSPIRE].

[3] J.T. Liu and R. Minasian, *Computing $1/N^2$ corrections in AdS/CFT*, arXiv:1010.6074 [INSPIRE].

[4] A. Bilal and C.-S. Chu, *A note on the chiral anomaly in the AdS/CFT correspondence and $1/N^2$ correction*, *Nucl. Phys.* **B 562** (1999) 181 [hep-th/9907106] [INSPIRE].

[5] A. Bilal and C.-S. Chu, *Testing the AdS/CFT correspondence beyond large-N*, hep-th/0003129 [INSPIRE].

[6] P. Mansfield and D. Nolland, *Order $1/N^2$ test of the Maldacena conjecture: cancellation of the one loop Weyl anomaly*, *Phys. Lett.* **B 495** (2000) 435 [hep-th/0005224] [INSPIRE].

[7] P. Mansfield, D. Nolland and T. Ueno, *Order $1/N^2$ test of the Maldacena conjecture. 2. The full bulk one loop contribution to the boundary Weyl anomaly*, *Phys. Lett.* **B 565** (2003) 207 [hep-th/0208135] [INSPIRE].

[8] P. Mansfield, D. Nolland and T. Ueno, *The boundary Weyl anomaly in the $\mathcal{N} = 4$ SYM/type IIB supergravity correspondence*, *JHEP* **01** (2004) 013 [hep-th/0311021] [INSPIRE].

[9] M. Günaydin and N. Marcus, *The spectrum of the S^5 compactification of the chiral $N = 2$, $D = 10$ supergravity and the unitary supermultiplets of* $\mathrm{U}(2, 2/4)$, *Class. Quant. Grav.* **2** (1985) L11 [INSPIRE].

[10] H. Kim, L. Romans and P. van Nieuwenhuizen, *The mass spectrum of chiral $N = 2$ $D = 10$ supergravity on S^5*, *Phys. Rev.* **D 32** (1985) 389 [INSPIRE].

[11] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Sasaki-Einstein metrics on $S^2 \times S^3$*, *Adv. Theor. Math. Phys.* **8** (2004) 711 [hep-th/0403002] [INSPIRE].

[12] M. Cvetič, H. Lü, D.N. Page and C. Pope, *New Einstein-Sasaki spaces in five and higher dimensions*, *Phys. Rev. Lett.* **95** (2005) 071101 [hep-th/0504225] [INSPIRE].

[13] A. Arabi Ardehali, J.T. Liu and P. Szepietowski, *The spectrum of IIB supergravity on $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_3$ and a $1/N^2$ test of AdS/CFT*, *JHEP* **06** (2013) 024 [arXiv:1304.1540] [INSPIRE].

[14] D. Anselmi and A. Kehagias, *Subleading corrections and central charges in the AdS/CFT correspondence*, *Phys. Lett.* **B 455** (1999) 155 [hep-th/9812092] [INSPIRE].

[15] S. Christensen and M. Duff, *Axial and conformal anomalies for arbitrary spin in gravity and supergravity*, *Phys. Lett.* **B 76** (1978) 571 [INSPIRE].

[16] D. Freedman, S. Gubser, K. Pilch and N. Warner, *Renormalization group flows from holography supersymmetry and a c theorem*, *Adv. Theor. Math. Phys.* **3** (1999) 363 [hep-th/9904017] [INSPIRE].

JHEP01(2014)002

[17] A. Gadde, L. Rastelli, S.S. Razamat and W. Yan, *On the superconformal index of $\mathcal{N}=1$ IR fixed points: a holographic check*, *JHEP* **03** (2011) 041 [arXiv:1011.5278] [INSPIRE].

[18] S. Gukov, *Comments on $\mathcal{N}=2$ AdS orbifolds*, *Phys. Lett.* **B 439** (1998) 23 [hep-th/9806180] [INSPIRE].

[19] A. Gadde, E. Pomoni and L. Rastelli, *The Veneziano limit of $\mathcal{N}=2$ superconformal QCD: towards the string dual of $N=2$ SU(N_c) SYM with $N_f=2N_c$*, arXiv:0912.4918 [INSPIRE].

[20] A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, *Spectrum of type IIB supergravity on $\mathrm{AdS}_5\times T^{11}$: predictions on $\mathcal{N}=1$ SCFT's*, *Phys. Rev.* **D 61** (2000) 066001 [hep-th/9905226] [INSPIRE].

[21] A. Ceresole, G. Dall'Agata and R. D'Auria, *KK spectroscopy of type IIB supergravity on $\mathrm{AdS}_5\times T^{11}$*, *JHEP* **11** (1999) 009 [hep-th/9907216] [INSPIRE].

[22] R. Eager, J. Schmude and Y. Tachikawa, *Superconformal indices, Sasaki-Einstein manifolds and cyclic homologies*, arXiv:1207.0573 [INSPIRE].

[23] D. Cassani, G. Dall'Agata and A.F. Faedo, *Type IIB supergravity on squashed Sasaki-Einstein manifolds*, *JHEP* **05** (2010) 094 [arXiv:1003.4283] [INSPIRE].

[24] J.T. Liu, P. Szepietowski and Z. Zhao, *Consistent massive truncations of IIB supergravity on Sasaki-Einstein manifolds*, *Phys. Rev.* **D 81** (2010) 124028 [arXiv:1003.5374] [INSPIRE].

[25] J.P. Gauntlett and O. Varela, *Universal Kaluza-Klein reductions of type IIB to $\mathcal{N}=4$ supergravity in five dimensions*, *JHEP* **06** (2010) 081 [arXiv:1003.5642] [INSPIRE].

[26] K. Skenderis, M. Taylor and D. Tsimpis, *A consistent truncation of IIB supergravity on manifolds admitting a Sasaki-Einstein structure*, *JHEP* **06** (2010) 025 [arXiv:1003.5657] [INSPIRE].

[27] D. Nolland, *AdS/CFT boundary conditions, multitrace perturbations and the c theorem*, *Phys. Lett.* **B 584** (2004) 192 [hep-th/0310169] [INSPIRE].

[28] S.S. Gubser and I. Mitra, *Double trace operators and one loop vacuum energy in AdS/CFT*, *Phys. Rev.* **D 67** (2003) 064018 [hep-th/0210093] [INSPIRE].

[29] S.S. Gubser and I.R. Klebanov, *A universal result on central charges in the presence of double trace deformations*, *Nucl. Phys.* **B 656** (2003) 23 [hep-th/0212138] [INSPIRE].

[30] T. Hartman and L. Rastelli, *Double-trace deformations, mixed boundary conditions and functional determinants in AdS/CFT*, *JHEP* **01** (2008) 019 [hep-th/0602106] [INSPIRE].

[31] S. Franco, A. Hanany and A.M. Uranga, *Multi-flux warped throats and cascading gauge theories*, *JHEP* **09** (2005) 028 [hep-th/0502113] [INSPIRE].

[32] M. Günaydin, D. Minic and M. Zagermann, *4D doubleton conformal theories, CPT and IIB string on $\mathrm{AdS}_5\times S^5$*, *Nucl. Phys.* **B 534** (1998) 96 [*Erratum ibid.* **B 538** (1999) 531] [hep-th/9806042] [INSPIRE].

[33] M. Günaydin and N. Warner, *Unitary supermultiplets of $\mathrm{Osp}(8/4,\mathbb{R})$ and the spectrum of the S^7 compactification of eleven-dimensional supergravity*, *Nucl. Phys.* **B 272** (1986) 99 [INSPIRE].

[34] M. Bianchi, J.F. Morales and H. Samtleben, *On stringy $\mathrm{AdS}_5\times S^5$ and higher spin holography*, *JHEP* **07** (2003) 062 [hep-th/0305052] [INSPIRE].

[35] H. Kihara, M. Sakaguchi and Y. Yasui, *Scalar Laplacian on Sasaki-Einstein manifolds $Y^{p,q}$*, *Phys. Lett.* **B 621** (2005) 288 [hep-th/0505259] [INSPIRE].

[36] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167 [INSPIRE].

JHEP01(2014)002