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# Direct probes of flavor-changing neutral currents in $e^+e^-$-collisions

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Abstract: We propose a novel method to study flavor-changing neutral currents in the $e^+e^- \to D^{*0}$ and $e^+e^- \to B_s^*$ transitions, tuning the energy of $e^+e^-$- collisions to the mass of the narrow vector resonance $D^{*0}$ or $B_s^*$. We present a thorough study of both short-distance and long-distance contributions to $e^+e^- \to D^{*0}$ in the Standard Model and investigate possible contributions of new physics in the charm sector. This process, albeit very rare, has clear advantages with respect to the $D^0 \to e^+e^-$ decay: the helicity suppression is absent, and a richer set of effective operators can be probed. Implications of the same proposal for $B_s^*$ are also discussed.

Keywords: Rare Decays, Heavy Quark Physics, Beyond Standard Model

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## Contents



## 1 Introduction

Experimental studies of the flavor-changing neutral currents (FCNC) are among the most promising ways to reveal virtual effects of possible new physics (NP) in heavy meson decays. The FCNC transitions have been thoroughly studied in the $b$-quark sector, where the Standard Model (SM) is seen to dominate the decay amplitudes [1, 2]. Nonetheless, recent hints at anomalies in the exclusive $B \to K^{(*)}\ell^+\ell^-$ decays [3–5] call for additional studies in the $b$-flavor FCNCs, preferably, with observables having less hadronic uncertainties than in the rare semileptonic decays. The leptonic decays $B_{s,d} \to \ell^+\ell^-$ remain the "cleanest" probes, however, only for the axial-vector/pseudoscalar effective operators. Moreover, due to helicity suppression, only the $\ell = \mu, \tau$ modes are accessible in leptonic decays, leaving the detection of the electron modes extremely challenging.

Still, even if the $b \to s(d)\ell^+\ell^-$ transitions conform with the SM, the situation might be different in the charm-quark FCNCs, where ample room for NP effects in the $c \to u\ell^+\ell^-$ transitions is available (see, e.g., [6–9]). Rare decays of the type $D^0 \to \ell^+\ell^-$ for $\ell = \mu, e$ have a potential to probe a variety of NP scenarios. The studies of these decays are seemingly appealing because for the local operators mediating these transitions, both in SM and beyond, all nonperturbative effects are accumulated in a single parameter, the $D^0$ decay constant. However, in SM the long-distance effects dominate the rare decays of charmed mesons. It is extremely difficult to estimate these contributions model-independently. Moreover, since the initial state in the $D^0 \to e^+e^-$ decay is a pseudoscalar meson, the helicity suppression again makes the observation of this process very difficult. This feature persists in many NP models as well. In addition, the rare radiative decay $D^0 \to \gamma e^+e^-$ has a branching ratio that is enhanced by a factor of $\mathcal{O}(\alpha m_D^2/m_e^2)$ compared to $D^0 \to e^+e^-$, since the additional



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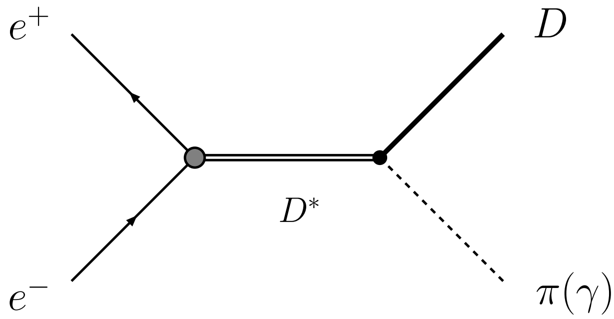


**Figure 1**. Probing the $c\bar{u} \to e^+e^-$ vertex with the $D^*(2007)^0$ resonance production in $e^+e^-$ collisions.

photon lifts the helicity suppression [10, 11]. If the emitted photon is soft, this transition can be easily misidentified as $D^0 \to e^+e^-$, which further complicates its experimental observation.

An interesting alternative to $D^0 \to e^+e^-$ process that is *not* helicity-suppressed is a related decay $D^*(2007)^0 \to e^+e^-$. While also probing the FCNC $c\bar{u} \to \ell^+\ell^-$ transition, this decay is sensitive to the contributions of operators that $D^0 \to \ell^+\ell^-$ cannot be sensitive to. Unfortunately, a direct study of the $D^* \to e^+e^-$ decay is practically impossible, since the $D^*$ decays strongly or electromagnetically.

Nevertheless, as we shall argue in this paper, it might be possible to probe the $D^* \to e^+e^-$ transition experimentally. Assuming time-reversal invariance, it would be equivalent to measure the corresponding production process $e^+e^- \to D^*$, as shown in figure 1. In order to do so, we propose to run an $e^+e^-$ collider, such as BEPCII [12], at the center-of-mass energy corresponding to the mass of the $D^*$ meson. Note that BEPCII already scanned this region of energies, achieving the luminosity of about $5 \times 10^{31}$ cm$^{-2}$ s$^{-1}$ around $\sqrt{s} = 2$ GeV [13]. If produced, the $D^{*0}$ resonance will decay via strong ($D^{*0} \to D^0\pi^0$) or electromagnetic ($D^{*0} \to D^0\gamma$) interactions with branching fractions of $(61.9 \pm 2.9)\%$ and $(38.1 \pm 2.9)\%$ respectively.[1]

In the setup discussed in this paper, the $D^*$ production process is very rare. However, the identification of even a single charmed-meson final state from the $e^+e^- \to D^{*0} \to D^0\pi^0$ decay would provide an unambiguous tag for this flavor-changing production process. Naturally, one also needs an adequate quality of the $\pi^0 \to 2\gamma$ identification and pion-kaon separation in the $D^0$ decays in order to reject background processes.

Our proposal may also be realized in the $b$-quark sector by scanning the region of the $B^*_{s,d}$ resonances at an $e^+e^-$ collider. This will probe the processes $e^+e^- \to B^*_{s,d}$ originating from the $b \to s(d)\ell^+\ell^-$ quark currents. In fact, studying the transitions involving electrons could also shed some light on recent hints at lepton non-universality in $b \to se^+e^-$ versus $b \to s\mu^+\mu^-$ [5].

Note also that tuning an $e^+e^-$ accelerator to the masses of resonances is not the only possibility to access their production. Some sensitivity to these processes could be also

[1]Note that the charged mode $D^{*0} \to D^+\pi^-$ is forbidden by the lack of the available phase space.

achieved by studying radiative return events at currently running $e^+e^-$ machines operating at their nominal energies.

The rest of this paper is devoted to a more detailed discussion of this proposal and to the relevant theoretical estimates.

## 2 $e^+e^- \to D^*$ resonant production

Let us consider a generic scattering amplitude of $e^+e^- \to D\pi$, and assess the contribution of the narrow resonance $D^*$ to this process depicted in figure 1. Writing this amplitude as a matrix element of a generic lepton-quark interaction

$$\mathcal{H} = \frac{\lambda'}{M^2}(\bar{c}\gamma_\mu u)(\bar{e}\gamma^\mu e), \tag{2.1}$$

where only the vector currents are kept for simplicity and an effective scale $M$ and dimensionless coupling $\lambda'$ are introduced, we obtain

$$\begin{aligned}\mathcal{M}(e^+e^- \to D^0\pi^0) &= \langle D^0(p_D)\pi^0(p_\pi)|\mathcal{H}|e^+(p_+)e^-(p_-)\rangle \\ &= \frac{\lambda'}{M^2}\langle D^0(p_D)\pi^0(p_\pi)|\bar{c}\gamma_\mu u|0\rangle\langle 0|\bar{e}\gamma^\mu e|e^+(p_+)e^-(p_-)\rangle \\ &= \frac{\lambda'}{M^2}\left(2f^+_{D^0\pi^0}(s)p_{\pi_\mu}\right)\,\overline{v}(p_+)\gamma^\mu u(p_-)\,,\end{aligned} \tag{2.2}$$

where the lepton current is factorized out and the hadronic matrix element is expressed via the $D^0 \to \pi^0$ vector form factor at $s = (p_D + p_\pi)^2 \geq (m_D + m_\pi)^2$. Note that $\sqrt{s}$ is the center-of-mass energy of the $e^+e^-$-collision. Up to an isospin factor, the same vector form factor appears in the semileptonic $D^0 \to \pi^-\ell^+\nu_\ell$ decay, where $s \leq (m_D - m_\pi)^2$.

At first place, it is the effective coupling in eq. (2.1) that determines the value of the cross section calculated from (2.3). Yet, the presence of a narrow resonance in the form factor is also crucial. To see that, we isolate the two lowest resonance contributions to the form factor, that is, $D^*$ and $D^{*'} = D(2600)$, where the latter, with the mass $m_{D^{*'}} = 2612 \pm 6\,\text{MeV}$ and total width $\Gamma_{D^{*'}} = 93 \pm 14\,\text{MeV}$ [14], is the most suitable candidate for the first radial excitation of $D^*$-meson. In the resulting decomposition

$$f^+_{D^0\pi^0}(s) = \frac{f_{D^{*0}}g_{D^{*0}D^0\pi^0}m_{D^{*0}}}{2(m^2_{D^{*0}} - s - im_{D^{*0}}\Gamma_0)} + \frac{f_{D^{*0\prime}}g_{D^{*0\prime}D^0\pi^0}m_{D^{*0\prime}}}{2(m^2_{D^{*0\prime}} - s - im_{D^{*0\prime}}\Gamma_{D^{*0\prime}})} + \left[f^+_{D^0\pi^0}(s)\right]_{\text{bgr}}\,, \tag{2.3}$$

we expressed the residues of both poles via decay constants of vector resonances and their strong couplings to $D\pi$, defined, respectively, as

$$\langle 0|\overline{u}\gamma^\mu c|D^*(p)\rangle = f_{D^*}m_{D^*}\epsilon^\mu(p)\,, \tag{2.4}$$

$$\langle D^0(p_D)\pi^0(p_\pi)|D^{*0}(p)\rangle = -g_{D^{*0}D^0\pi^0}(\epsilon(p)\cdot p_\pi)\,, \tag{2.5}$$

where $\epsilon^\mu(p)$ is the $D^*$ polarization vector, $p = p_D + p_\pi$, and we isolated the background contribution $\left[f^+_{D^0\pi^0}(s)\right]_{\text{bgr}}$ with respect to the two resonances.

While experimentally only an upper bound for the total width of $D^{0*}$, $\Gamma_0 < 2.1\,\text{MeV}$, is available [14], we can compute the actual value of this width from the measured total width of the charged $D^{+*}$ meson, $\Gamma_+ = 83.4 \pm 1.8$ keV [14]. Using the isospin symmetry to relate the strong $D^*D\pi$ couplings and taking into account the phase space correction (see, e.g., [15]) we obtain:

$$\begin{aligned}\Gamma_0 &= \Gamma(D^{*0} \to D^0\pi^0) + \Gamma(D^{*0} \to D^0\gamma) \\ &\simeq \frac{\Gamma_+ \mathcal{B}_{D^{*+}\to D^0\pi^+}}{2} \left( \frac{\lambda(m^2_{D^{*0}}, m^2_{D^0}, m^2_{\pi^0})}{\lambda(m^2_{D^{*+}}, m^2_{D^0}, m^2_{\pi^+})} \right)^{3/2} \left( 1 + \frac{\mathcal{B}_{D^{*0}\to D^0\gamma}}{\mathcal{B}_{D^{*0}\to D^0\pi^0}} \right) \simeq 60 \text{ keV}\,, \end{aligned} \tag{2.6}$$

where $\mathcal{B}_{D^*\to f} = \Gamma(D^* \to f)/\Gamma_0$ is the branching fraction of $D^* \to f$, $\lambda(x, y, z)$ is the kinematic Källen function and we employed the data on branching fractions from [14], neglecting small experimental errors.

The Breit-Wigner form used in eq. (2.3) is certainly applicable for such a narrow resonance as $D^*$. We adopt the same approximation for the broad resonance $D^{*'}$, which is sufficient for an order-of-magnitude estimate. At $s = m^2_{D^*}$ the magnitude of the excited state contribution to the form factor is suppressed with respect to the $D^{*0}$-pole term by the factor, approximately,

$$\left| \frac{f_{D^{0*\prime}}\, g_{D^{*\prime 0}D^0\pi^0} m_{D^{*0\prime}}}{f_{D^{0*}}\, g_{D^{*0}D^0\pi^0} m_{D^{*0}}} \right| \times \left| \frac{i\Gamma_0}{2\Delta - i\Gamma_{D^{*\prime}}} \right| \sim 5.0 \cdot 10^{-5}\,, \tag{2.7}$$

where $\Delta = m_{D^{*0\prime}} - m_{D^*} \simeq 600\,\text{MeV}$ and the $O(\Delta/m_{D^*})$ terms are neglected. In the above, we assume that the strong couplings and decay constants of the radially excited and ground $D^*$ states are in the same ballpark, so that the first factor in this ratio is of $O(1)$. In fact, the ratio of decay constants in eq. (2.7) is less than one, as the QCD sum rule predictions indicate [16] . Our estimate also implies that if one produces a resonance with a width of $O(100 - 150)$ MeV, typical for the light vector mesons such as $\rho$ or $K^*$, there is no relative gain in the resonance cross section. We emphasize that the very small width of $D^*$, driving up the cross section, essentially originates from the miniscule phase space of its single strong decay mode. In this situation, the radiative mode of $D^*$ becomes equally important and the FCNC transition acquires a tiny but non-negligible branching fraction.

Having in mind a strong suppression of all other than $D^*$ contributions to eq. (2.3) at $\sqrt{s} \simeq m_{D^*}$, the cross section of $e^+e^- \to D\pi$ can be written in a standard resonance form:

$$\sigma(e^+e^- \to D\pi)_{\sqrt{s}\simeq m_{D^*}} \equiv \sigma_{D^*}(s) = \frac{12\pi}{m^2_{D^*}}\, \mathcal{B}_{D^*\to e^+e^-}\mathcal{B}_{D^*\to D\pi}\, \frac{m^2_{D^*}\Gamma_0^2}{(s - m^2_{D^*})^2 + m^2_{D^*}\Gamma_0^2}, \tag{2.8}$$

To exploit the $D^*$-resonance enhancement around $\sqrt{s} = m_{D^*}$, described by eq. (2.8), we tacitly assume that an appropriate tuning of the electron-positron accelerator beams can be performed, so that their energy resolution is smaller than the spread of the resonance cross section. Then we can simply use eq. (2.8) at $s = m^2_{D^*}$, yielding

$$\sigma_{D^*}(m^2_{D^*}) = \mathcal{B}_{D^*\to e^+e^-}\mathcal{B}_{D^*\to D\pi}\frac{12\pi}{m^2_{D^*}} \simeq \mathcal{B}_{D^*\to e^+e^-}(2.26 \times 10^6)\,\text{nb}\,. \tag{2.9}$$

Let us recall that the total cross section $\sigma(e^+e^- \to \mathit{hadrons})$ is about 50 nb at $\sqrt{s} = 2.0\,\text{GeV}$ [14].



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The expected number of $e^+e^- \to D\pi$ events at $\sqrt{s} = m_{D^*}$ is given by the product

$$N_{D^*} = \sigma_{D^*}(m_{D^*}^2)\, \epsilon \int L dt \tag{2.10}$$

where $\epsilon$ and $\int L dt$ are the detection efficiency and time-integrated luminosity, respectively. The condition $N_{D^*} \geq 1$ leads to a lower bound on the $D^* \to e^+e^-$ branching fraction that still allows one to detect the process $e^+e^- \to D^* \to D\pi$:

$$\mathcal{B}_{D^*\to e^+e^-} \geq \left(\frac{1}{\epsilon \int L dt}\right) \times \frac{m_{D^*}^2}{12\pi\ \mathcal{B}_{D^*\to D\pi}}. \tag{2.11}$$

For example, an average $e^+e^-$ luminosity at the level of $L \approx 1.0 \times 10^{32}\ cm^{-2}s^{-1}$, with a "Snowmass year" ($\sim 10^7 s$) of running at the $D^*$ resonance yields $\int L dt = 1.0\,\text{fb}^{-1}$.

Under these conditions, the single-event sensitivity implied by the bound (2.11) means that branching fractions

$$\mathcal{B}_{D^*\to e^+e^-} > 4 \times 10^{-13} \tag{2.12}$$

could in principle be probed. While the above bound is a very crude estimate of the possible sensitivity, as we did not take into account detection efficiency, assuming $\epsilon = 1$, it can serve as a useful criterion. In order to see if and when this bound can be approached, let us first calculate the branching fraction $\mathcal{B}_{D^*\to e^+e^-}$ in the SM. We need this quantity as it is the key parameter determining the $e^+e^- \to D^*$ cross section.

## 3 $D^* \to e^+e^-$ transition

The most general expression for the $D^* \to e^+e^-$ decay amplitude can be written as

$$\begin{aligned} A(D^* \to e^+e^-) = \overline{u}(p_-, s_-) \Bigg[ & A\gamma_\mu + B\gamma_\mu\gamma_5 + \frac{C}{m_{D^*}}(p_+ - p_-)_\mu \\ & + \frac{D}{m_{D^*}}(p_+ - p_-)_\mu i\gamma_5 \Bigg] v(p_+, s_+)\ \epsilon^\mu(p), \end{aligned} \tag{3.1}$$

where $A$, $B$, $C$, and $D$ are dimensionless constants which absorb the underlying effective quark-lepton interaction and the vacuum $\to D^*$ hadronic matrix elements (e.g., the $D^*$ decay constant). Note that the simplified effective interaction (2.3) used in the previous section corresponds to $A = (\lambda'/M^2) f_{D^*} m_{D^*}$ and all other constants put to zero. The amplitude (3.1) leads to the branching fraction

$$\mathcal{B}_{D^*\to e^+e^-} = \frac{m_{D^*}}{12\pi\Gamma_0}\left[\left(|A|^2 + |B|^2\right) + \frac{1}{2}\left(|C|^2 + |D|^2\right)\right], \tag{3.2}$$

where we neglected the mass of the electron.

It is straightforward to compute the SD part of the $D^* \to e^+e^-$ amplitude triggered by a generic effective Hamiltonian, not necessary containing only the SM operators. The decay amplitude is

$$\langle e^+e^-|\mathcal{H}_{\text{eff}}|D^*\rangle = G\sum_i c_i(\mu)\langle e^+e^-|\widetilde{Q}_i|D^*\rangle|_\mu\,, \tag{3.3}$$

where $G$ is a constant with the dimension of inverse squared mass that sets the scale for the operators. For example, $G = 4G_F/\sqrt{2}$ in the SM or $G = 1/\Lambda^2$ for new physics. In the above, $\widetilde{Q}_i$ are the effective operators of dimension six, and $c_i$ are the corresponding (dimensionless) Wilson coefficients. The most general set of ten local operators producing the $c\bar{u} \to \ell^+\ell^-$ transitions reads [17]

$$\begin{aligned} \widetilde{Q}_1 &= (\bar{\ell}_L\gamma_\mu\ell_L)\;(\overline{u}_L\gamma^\mu c_L), \quad \widetilde{Q}_4 = (\bar{\ell}_R\ell_L)\;(\overline{u}_R c_L), \\ \widetilde{Q}_2 &= (\bar{\ell}_L\gamma_\mu\ell_L)\;(\overline{u}_R\gamma^\mu c_R), \quad \widetilde{Q}_5 = (\bar{\ell}_R\sigma_{\mu\nu}\ell_L)\;(\overline{u}_R\sigma^{\mu\nu} c_L), \\ \widetilde{Q}_3 &= (\bar{\ell}_L\ell_R)\;(\overline{u}_R c_L), \end{aligned} \tag{3.4}$$

with five additional operators $\widetilde{Q}_6, \ldots, \widetilde{Q}_{10}$ obtained respectively from those in eq. (3.4) by the substitutions $L \to R$ and $R \to L$. We will be using the set of operators $\widetilde{Q}_{i=1,\ldots,10}$ to assess possible contributions of any generic new physics model to $D^* \to e^+e^-$.

The operators with (pseudo)scalar quark currents $\widetilde{Q}_{3-4}$ and $\widetilde{Q}_{8-9}$ do not contribute to this process. On the other hand, contrary to the $D^0 \to \ell^+\ell^-$ decay, not only the vector operators $\widetilde{Q}_{1-2}$ and $\widetilde{Q}_{6-7}$ but also the tensor operators $\widetilde{Q}_5$ and $\widetilde{Q}_{10}$ could, in principle, contribute to $D^* \to e^+e^-$. However, in most of NP models, these effective operators are absent, motivating us to neglect their contributions. This is equivalent to setting $C = D = 0$ in eq. (3.1). In terms of the Wilson coefficients $c_i$ the constants $A$ and $B$ are

$$\begin{aligned} A &= \frac{G}{4} f_{D^*} m_{D^*} \left(c_1 + c_2 + c_6 + c_7\right), \\ B &= -\frac{G}{4} f_{D^*} m_{D^*} \left(c_1 + c_2 - c_6 - c_7\right). \end{aligned} \tag{3.5}$$

The effective operators in SM (see [18] for the definition) mediating the $D^* \to e^+e^-$ transition in SM are easily matched to the operator set in eq. (3.4),

$$O_9 = \frac{e^2}{16\pi^2}\left(\widetilde{Q}_1 + \widetilde{Q}_7\right), \quad O_{10} = \frac{e^2}{16\pi^2}\left(\widetilde{Q}_7 - \widetilde{Q}_1\right). \tag{3.6}$$

The expressions for the corresponding Wilson coefficients $C_9^c$ and $C_{10}^c$, where a superscript $c$ indicates that CKM matrix elements are included in their definitions, can also be found in [18].

In addition to $O_9$ and $O_{10}$, in SM the $D^* \to e^+e^-$ amplitude receives a contribution from the magnetic dipole operator $O_7$ coupled to the leptons via virtual photon. The corresponding part of the effective Hamiltonian reads

$$H_{\rm eff}^{(7\gamma)} = \frac{4G_F}{\sqrt{2}} C_7^{c,\rm eff}\left(\frac{e}{16\pi^2} m_c\, \overline{u}_L\sigma^{\mu\nu} c_R F_{\mu\nu}\right). \tag{3.7}$$

A calculation of this contribution requires the knowledge of the tensor (transverse) decay constant of the $D^*$:

$$\langle 0|\overline{u}\sigma^{\mu\nu}c|D^*(p)\rangle = i f_{D^*}^T\left(\epsilon^\mu p^\nu - p^\mu\epsilon^\nu\right). \tag{3.8}$$

In the absence of the estimate of $f_{D^*}^T$, we rely on the properties of light vector mesons for which the vector (longitudinal) and tensor decay constants are in the same ballpark and



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simply assume that $f^T_{D^*} = f_{D^*}$. One can now calculate the SD contribution to $D^* \to e^+e^-$ in the SM in terms of effective constants:

$$
\begin{aligned}
A^{(SD)} &= \frac{\alpha}{2\pi}\frac{G_F}{\sqrt{2}} f_{D^*} m_{D^*} \left[ C_9^{\mathrm{c,eff}} + 2\frac{m_c}{m_{D^*}}\frac{f^T_{D^*}}{f_{D^*}} C_7^{\mathrm{c,eff}} \right], \\
B^{(SD)} &= \frac{\alpha}{2\pi}\frac{G_F}{\sqrt{2}} f_{D^*} m_{D^*} C_{10}^{c}\,,
\end{aligned} \tag{3.9}
$$

which results in the SD contribution to the branching fraction:

$$
\mathcal{B}_{D^*\to e^+e^-} = \frac{\alpha^2 G_F^2}{96\pi^3\Gamma_0} m^3_{D^*} f^2_{D^*} \left( \left| C_9^{\mathrm{c,eff}} + 2\frac{m_c}{m_{D^*}}\frac{f^T_{D^*}}{f_{D^*}} C_7^{\mathrm{c,eff}} \right|^2 + |C_{10}^{c}|^2 \right). \tag{3.10}
$$

Adopting $m_c = 1.3\,\mathrm{GeV}$, we use for the Wilson coefficient,[2] $C_9^{\mathrm{c}}(\mu = m_c) = 0.198|V^*_{ub}V_{cb}|$, neglect $C_{10}^{\mathrm{c}}(\mu = m_c)$ and employ the results of the two-loop calculation [19] for the remaining effective coefficient, $C_7^{\mathrm{c,eff}}(\mu = m_c) = -0.0025$. We also use the central value of the QCD sum rule estimate $f_{D^*} \approx 242\,\mathrm{MeV}$ [20]. Substituting all input paramaters in eq. (3.10), we find

$$
\mathcal{B}^{SD}_{D^*\to e^+e^-} \approx 2.0 \times 10^{-19}\,. \tag{3.11}
$$

As expected, this number is extremely small, several orders of magnitude below the lowest accessible branching fraction eq. (2.12) and thus beyond any realistic experimental setup. Still, it is instructive to remind the reader that the short-distance width of the similar decay of the pseudoscalar $D^0$ is many orders of magnitude smaller: $\mathcal{B}^{SD}_{D^0\to e^+e^+} \sim 10^{-23}$, whereas $\mathcal{B}^{SD}_{D^0\to \mu^+\mu^-} \sim 10^{-18}$ (see, e.g., [21]).

## 4 Long-distance contributions

Generally, in rare charm decays, a significant enhancement of the decay rate is expected in SM due to LD contributions, generated by the four-quark weak interaction combined with the emission of the $e^+e^-$-pair via virtual photon. It is very difficult to reliably estimate these contributions in $D \to \ell^+\ell^-$ decay because the two-photon intermediate state overlaps with long-distance hadronic interactions.

To investigate the case of $D^* \to e^+e^-$ decay, we isolate the relevant $\Delta C = 1$, single Cabibbo-suppressed transitions in the effective Hamiltonian $\mathcal{H}_w$ of the SM, representing it in a form of the two four-quark operators:

$$
\begin{aligned}
\mathcal{H}_w = \frac{4G_F}{\sqrt{2}} \sum_{q=d,s} \Bigg[ & \left( C_1^{c(q)} + \frac{C_2^{c(q)}}{N_c} \right) (\overline{q}_L \gamma_\nu q_L)(\overline{u}_L \gamma_\nu c_L) \\
& + 2C_2^{c(q)} (\overline{q}_L \gamma_\nu T^a q_L)(\overline{u}_L \gamma_\nu T^a c_L) \Bigg],
\end{aligned} \tag{4.1}
$$

where $T^a$ are the color-octet matrices. Note that we again included relevant CKM matrix elements into the definition of $C_1^{c(q)}$ and $C_2^{c(q)}$.

[2]We thank Dirk Seidel for providing us this coefficient calculated at NLL order.

The LD contribution to the $D^* \to e^+e^-$ decay amplitude is given by the matrix element

$$\langle e^+e^-|\mathcal{H}_w|D^*(p)\rangle = -e^2\bar{u}(p_-,s_-)\gamma^\mu v(p_+,s_+)\left(\frac{\Sigma_\mu(p^2)}{p^2}\right)\Bigg|_{p^2=m_{D^*}^2}, \tag{4.2}$$

where we factorized out the lepton current, the photon propagator and defined the hadronic matrix element in a form of the correlation function

$$\Sigma_\mu(p^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\left\{j_\mu^{em}(x)\mathcal{H}_w(0)\right\}|D^*(p)\rangle\,, \tag{4.3}$$

where $j_\mu^{em} = \sum_{q=u,d,s,c} Q_q\bar{q}\gamma_\mu q$ is the electromagnetic (e.m.) quark current. The quark-level diagrams corresponding to this amplitude are depicted in figure 2, where we distinguish two main topologies corresponding to the virtual photon interacting with (a) the $q = d, s$ quarks (upper panel) and (b) with the $u$ quark (lower panel). Both contributions develop imaginary parts due to the intermediate light-hadron states. We neglect the e.m. interaction with the heavy $c$-quark.

To estimate the LD contribution corresponding to the diagram figure 2a we adopt factorization approximation, that is, neglect the gluon exchanges between $\bar{q}q$ and $\bar{u}c$ fields in $\mathcal{H}_w$. Hence, we retain in eq. (4.3) only the product of color-neutral vector currents and obtain:

$$\begin{aligned}\Sigma_\mu^{(a)}(p^2) = \frac{G_F}{\sqrt{2}}\sum_{q=d,s} Q_q\left(C_1^{c(q)} + \frac{C_2^{c(q)}}{N_c}\right)\left\{i\int d^4x e^{ip\cdot x}\langle 0|T\left\{\bar{q}\gamma_\mu q(x)\;\bar{q}\gamma_\nu q(0)\right\}|0\rangle\right\}\\ \times\langle 0|\bar{u}\gamma^\nu c|D^*(p)\rangle\,,\end{aligned} \tag{4.4}$$

where we recognize the quantity in the curly braces as the polarization tensor $\Pi_{\mu\nu}^{(q)}$ for the $q$-flavored vector current. Substituting its decomposition

$$\Pi_{\mu\nu}^{(q)}(p) = \left(-g_{\mu\nu}p^2 + p_\mu p_\nu\right)\Pi^{(q)}(p^2) \tag{4.5}$$

in eq. (4.4) and parametrizing the matrix element of the $\bar{u}c$ current according to eq. (2.4), we finally obtain the LD transition amplitude in the form (3.1) with

$$A^{(LD,a)} = 4\pi\alpha Q_q\frac{G_F}{\sqrt{2}}f_{D^*}m_{D^*}\left(C_1^c + \frac{C_2^c}{N_c}\right)\left(\Pi^{(d)}(p^2) - \Pi^{(s)}(p^2)\right)\Bigg|_{p^2=m_{D^*}^2}, \tag{4.6}$$

and $B^{(LD,a)} = 0$. In the above expression, we slightly modified the Wilson coefficients, so that $C_{1,2}^c \equiv C_{1,2}^{c(s)} \simeq -C_{1,2}^{c(d)}$, taking into account that the CKM factors for the $s$- and $d$-quark parts of $\mathcal{H}_w$ are approximately equal and have opposite signs. As expected, in the flavor SU(3) limit the whole LD contribution vanishes, reflecting the GIM cancellation.

Note that the factorizable quark-gluon effects are implicitly retained in the two separate hadronic quantities, the decay constant and the polarization operator. Nonfactorizable QCD corrections will involve contributions of the color-octet and axial-vector quark currents. We expect that these corrections are suppressed either by $\alpha_s(m_c)$ or by the powers

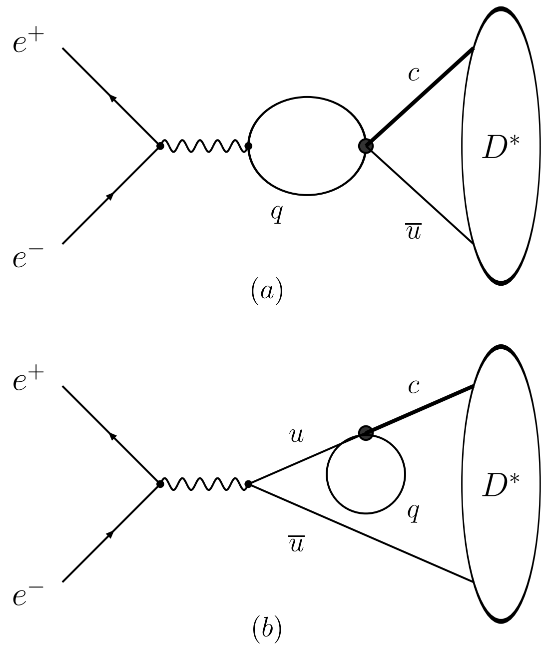


**Figure 2**. Long-distance contributions to $e^+e^- \to D^{*0}$ caused by the virtual photon interaction (a) with $q = d, s$ quark in factorizable approximation, forming the polarization operator; (b) with $u$ quark.

of $\Lambda_{QCD}/m_c$, since the characteristic momenta flowing through the quark loop in the diagram of figure 2a are of order of $m_D \sim m_c$. For the same reason, one may also argue that the mixing between $\bar{s}s$ and $\bar{d}d$ loops in the polarization operator is suppressed, which is also in line with the OZI suppression valid in the vector-meson channel. To substantiate the expected suppression of higher-order effects, one has to calculate the nonfactorizable multi-loop diagrams explicitly, a technically difficult task we postpone to future studies.

Returning to the LD contribution (4.6) we estimate first the polarization operators. Note that $\Pi^{(q)}(p^2)$ satisfies a (once-subtracted) dispersion relation

$$\Pi^{(q)}(p^2) = \frac{p^2}{12\pi^2 Q_q^2} \int_0^\infty ds \frac{R^{(q)}(s)}{s(s - p^2 - i\epsilon)}\,, \tag{4.7}$$

where $R^{(q)}(s)$ is the normalized $e^+e^-$ cross section to hadrons initiated by the quark flavor $q = s, d$, so that below the charm-anticharm threshold

$$R(s) \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{q=u,d,s} R^{(q)}(s)\,. \tag{4.8}$$

In principle, $R_q(s)$ could be extracted from the experimental data. In practice, however, it is hard to disentangle hadronic states generated by the $d$ and $s$-quark currents. Therefore, for the sake of estimate, we employ the parametrization of $R_q(s)$ stemming from the QCD sum rule analysis [22, 23] of the $u, d$- and $s$-quark correlation functions and based on the quark-hadron duality:

$$R^{(d)}(s) = 12\pi^2 Q_d^2 \left[ \sum_{V=\rho^0,\omega} \frac{f_V^2}{2} \delta(s - m_V^2) + \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \theta(s - s_0^d) \right], \tag{4.9}$$

$$R^{(s)}(s) = 12\pi^2 Q_s^2 \left[ f_\phi^2 \delta(s - m_\phi^2) + \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \theta(s - s_0^s) \right], \tag{4.10}$$

where the decay constants of the light vector mesons are defined via matrix elements of the electromagnetic current:

$$\langle 0 | j_\mu^{em} | V \rangle = \kappa_V m_V f_V \epsilon_{(V)}^\mu \, . \tag{4.11}$$

Here the coefficients $\kappa_\rho = 1/\sqrt{2}$, $\kappa_\omega = 1/(3\sqrt{2})$ and $\kappa_\phi = -1/3$ reflect the valence quark content of these mesons. Furthermore, we take into account the total widths of the vector mesons, replacing delta-functions in eqs. (4.9)–(4.10) with the Breit-Wigner approximations.

To calculate the difference of the polarization operators entering eq. (4.6), we use $f_\rho = 220\,\text{MeV}$, $f_\omega = 197\,\text{MeV}$, and $f_\phi = 228\,\text{MeV}$, obtained from the experimental values of the $\rho, \omega$ and $\phi$ leptonic widths [14], We also choose the effective thresholds [22, 23]: $s_0^d = 1.5\,\text{GeV}^2$ and $s_0^s = 1.95\,\text{GeV}^2$. To finalize our estimate of the LD amplitude, we take the relevant combinations of CKM parameters $\eta_{CKM} \equiv V_{us}^* V_{cs} \simeq -V_{ud}^* V_{cd} = \lambda(1 - \lambda^2/2)$, where $\lambda = 0.22537$ [14] is the Wolfenstein parameter, and use the Wilson coefficients at $\mu = m_c = 1.3\,\text{GeV}$ at LO(NLO): $C_1^c/\eta_{CKM} = -0.53(-0.41)$ and $C_2^c/\eta_{CKM} = 1.28(1.21)$.

If we assume that the LD contribution of the annihilation type calculated above saturates alone the decay amplitude, the resulting branching fraction of the $D^* \to e^+e^-$ decay, turns out quite sensitive to the mutual cancellation of the Wilson coefficients $C_1$ and $C_2/3$:

$$\mathcal{B}^{LD,A}_{D^* \to e^+e^-} \simeq \begin{cases} 4.7 \times 10^{-20} & \text{(NLO)} \\ 5.7 \times 10^{-18} & \text{(LO)} \end{cases} . \tag{4.12}$$

The cancellation is numerically less pronounced in LO, in which case the above branching fraction grows by almost two orders of magnitude with respect to the NLO case, becoming substantially larger than the SD width (3.11).

One could also try to estimate the difference of polarization operators in eq. (4.6) by subtracting the $d$- and $s$-quark loop diagrams from each other. This is equivalent to replacing $R^{(q)}(s)$ by the parton spectral density (the second term in brackets in eqs. (4.9)–(4.10)) and integrating from $4m_q^2$ to infinity. In this case, to account for a correct normalization of the effective operators in $H_{\text{eff}}$ and to avoid infrared unstable terms of $O(\ln(m_d/m_s))$ in the difference of the polarization operators, one has to add a (scheme dependent) constant term to the dispersion integral, which contains $\ln m_q^2/\mu^2$. The resulting difference of the loop functions is then proportional to $(m_s^2 - m_d^2)/\mu^2$ as it should be in the GIM cancellation.[3]

[3]In the hadronic representation of the polarization operator used above, the same cancellation can be traced in the difference of QCD sum rules for $f_{\rho,\omega}^2$ and $f_\phi^2$ and stems from the difference of loop diagrams in the perturbative part and, in addition, from the $O(m_s\langle \bar{s}s\rangle - m_d\langle \bar{d}d\rangle)$ terms originating from the vacuum condensate contributions.

The simple loop estimate yields, instead of eq. (4.12),

$$\mathcal{B}^{LD,A}_{D^*\to e^+e^-} \simeq \begin{cases} 2.0\times 10^{-20} & \text{(NLO)} \\ 2.4\times 10^{-18} & \text{(LO)} \end{cases} , \tag{4.13}$$

in the same ballpark as the estimate (4.12).

Turning to the LD contribution represented by the diagram in figure 2 b, we present a rough estimate of its amplitude, taking the imaginary part and accounting only for the lowest $\pi^+\pi^-$ and $K^+K^-$ intermediate states. Other contributions, including multiparticle ones, are possible. Those can in principle give even larger amplitudes due to phase space suppression of the SU(3)-related intermediate states [28].

According to the unitarity condition, the imaginary part of the $D^* \to e^+e^-$ amplitude generated by these particular intermediate states is expressed via the $e^+e^- \to P^+P^-$ e.m. amplitude which contains the $\langle 0|Q_u\overline{u}\gamma_\mu u|P^+P^-\rangle$ form factor taken at the timelike momentum transfer $p^2 = m^2_{D^*}$. The e.m. amplitude is multiplied by the $D^* \to P^+P^-$ nonleptonic amplitude ($P = \pi, K$) and integrated over the $P^+P^-$ phase space.

Applying the naive factorization approximation for the nonleptonic amplitude, we introduce the relevant hadronic matrix element

$$p^+_\alpha\langle P^-(p_-)|\overline{q}\gamma^\alpha\gamma_5 c|D^*(p)\rangle = 2i(\epsilon(p)\cdot p_+)A_0^{D^*\to P}(p^2 = m^2_{D^*})m_{D^*} , \tag{4.14}$$

which depends on the one particular form factor of the $D^* \to P$ transition, (we define these form factors analogous to the well familiar $D \to V$ form factors). Using also the decay constant of the light pseudoscalar meson:

$$\langle P^+(p_+)|\overline{u}\gamma^\rho\gamma_5 q|0\rangle = -ip^\rho_+ f_P , \tag{4.15}$$

we obtain the LD contribution in the following form:

$$\begin{aligned}\mathrm{Im}A^{(LD,b)} = \frac{\alpha}{12}\frac{G_F}{\sqrt{2}}\left(C_2^c + \frac{C_1^c}{N_c}\right) f_\pi A_0^{D^*\pi}(m_\pi^2)Q_u F_\pi^{em}(m^2_{D^*})\beta_\pi^3 m_{D^*} \\ \times\left(1 - \frac{f_K\beta_K^3}{f_\pi\beta_\pi^3}\frac{A_0^{D^*K}(m_K^2)F_K^{em}(m^2_{D^*})}{A_0^{D^*\pi}(m_\pi^2)F_\pi^{em}(m^2_{D^*})}\right) ,\end{aligned} \tag{4.16}$$

where $\beta_P = \sqrt{1 - 4m_P^2/m^2_{D^*}}$. For an accurate numerical estimate we need to calculate the form factors of $D^* \to P$ transition, which is beyond our scope, demanding, e.g., a dedicated application of QCD light-cone sum rules. For an order-of-magnitude estimate we assume that these form factors are in the same ballpark as the (correspondingly normalized) $D \to \pi, K$ form factors and take $A_0^{D^*\pi}(m_\pi^2) \sim 1$. The pion e.m. form factor $F_\pi^{em}(m^2_{D^*}) \simeq 0.28$ is known from a measurement [24] in the time-like momentum region. For the SU(3)-violating ratio of the e.m. and heavy-light form factors we conservatively assume that they can vary within $\pm 30\%$

$$\frac{A_0^{D^*K}(m_K^2)F_K^{em}(m^2_{D^*})}{A_0^{D^*\pi}(m_\pi^2)F_\pi^{em}(m^2_{D^*})} = 1 \pm 0.3 . \tag{4.17}$$

After that we obtain the lower limit on the branching fraction assuming that the LD contribution (4.16) is dominant:

$$\mathcal{B}^{(LD,b)}_{D^*\to e^+e^-} \geq (0.1-5.0)\times 10^{-19}\,, \tag{4.18}$$

where the broad interval mainly reflects the variation within the limits assumed in 4.17, whereas switching from NLO to LO Wilson coefficients is a minor effect in this case.

A more complete analysis of LD effects including possible interference between contributions of the two types considered above as well as a more complete set of intermediate states is an interesting task we postpone for the future. The estimates presented above demonstrate possible approaches which could be developed further.

The branching fractions presented in eqs. (4.12), (4.13) and (4.18), can achieve the level of $10^{-18}$, that is, not very much larger than the typical values (3.11) in the presence of only SD effects. Altogether, the probability of $e^+e^-\to D^{*0}$ in SM remains considerably smaller than the estimated minimal level (2.12) reachable (optimistically) by experiment. Therefore, if observed or at least constrained at the level of eq. (2.12), the $e^+e^-\to D^*$ events will definitely have no SM background. While this is basically true for a majority of FCNC transitions in the charm sector, such as $D\to\ell^+\ell^-$ or semileptonic decays, the $e^+e^-\to D^*$ transition has a clear advantage of having a relatively moderate LD background.

## 5 $D^*\to e^+e^-$ transitions and new physics

It is interesting to estimate what possible NP scale could be probed by the processes discussed in this paper. As follows from eq. (3.4), there are ten possible operators that parameterize any NP contribution to any $c\bar{u}\to e^+e^-$ process. Some of those operators can be probed both in $D^0\to e^+e^-$ and $D^{0*}\to e^+e^-$ transitions. Some can only be reached in $D^{0*}\to e^+e^-$ or (more challenging from hadronic point of view) rare semileptonic charm decays $D\to Me^+e^-$. It would be interesting to compare the possible reach of $D^0$ and $D^{0*}$ decays.

Assuming that a NP contribution is dominated by a single operator, it is easy to see from eqs. (3.2) and (3.5) that

$$\Lambda \sim \left(\frac{1}{3\pi}\frac{m^3_{D^*}f^2_{D^*}}{32\Gamma_0}\frac{C^2}{\mathcal{B}_{D^*\to e^+e^-}}\right)^{1/4}, \tag{5.1}$$

where $C=|c_i|$ for $i=1,2,6$, or 7. Employing the upper bound (2.12) we find that observation of a single event in a "Snowmass year" of running would probe NP scales of the order of $\Lambda\sim 2.7\,\mathrm{TeV}$ provided that $C\sim 1$. Let us compare this to the NP reach of $D^0\to e^+e^-$ decay for the same operators. Using [17],

$$\Lambda \sim \left(\frac{m_D m_e^2 f_D^2}{32\pi\Gamma_D}\frac{C^2}{\mathcal{B}_{D\to e^+e^-}}\right)^{1/4}, \tag{5.2}$$

where $\Gamma_D$ is the total width of the $D^0$ meson, and the current experimental bound, $\mathcal{B}_{D\to e^+e^-}=7.9\times10^{-8}$, we find that only scales $\Lambda\sim 200\,\mathrm{GeV}$ are currently probed by



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$D \to e^+e^-$ decay. It is the presence of the lepton mass factor that severely limits the NP scale sensitivity in this process.

To exemplify this discussion, let us consider two particular models of NP to see how well they can be probed in $D^* \to e^+e^-$ transition. The selected models are by no means unique. The chosen examples simply illustrate the differences in sensitivities between $D^0 \to \ell^+\ell^-$ and $D^* \to e^+e^-$.

**R-parity violating (RPV) SUSY.** R-parity violating SUSY models can be probed in FCNC charm decays [17, 18]. The relevant part of the superpotential can be written as

$$\begin{aligned} W_{\lambda'} = \tilde{\lambda}'_{ijk} \Big\{ & V_{jl} \left[ \tilde{\nu}^i_L \bar{d}^k_R d^l_L + \tilde{d}^l_L \bar{d}^k_R \nu^i_L + (\tilde{d}^k_R)^* (\bar{\nu}^i_L)^c d^l_L \right] \\ & - \tilde{e}^i_L \bar{d}^k_R u^j_L - \tilde{u}^j_L \bar{d}^k_R u^j_L - (\tilde{d}^k_R)^* (\bar{e}^i_L)^c u^j_L \Big\} , \end{aligned} \tag{5.3}$$

where the coupling parameters $\tilde{\lambda}'_{ijk}$ are defined such that $i$ denotes a generation number for leptons or sleptons, $j$ — for up-type quarks, and $k$ — for down-type quarks or squarks. As can be seen from eq. (5.3), the relevant FCNC quark transition $c+\bar{u} \to e^+e^-$ is mediated by a tree-level $d$-squark exchange. Since its mass is much larger than the energy scale at which $D^*$ decay takes place, it can be integrated out, resulting in the effective Lagrangian [17, 18],

$$\mathcal{L}^{\not{R}_p}_{\text{eff}} = \frac{\tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k}}{2 m^2_{\tilde{d}^k_R}} \, \widetilde{Q}_1 , \tag{5.4}$$

which implies that $G c_1 = \tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k} / (2 m^2_{\tilde{d}^k_R})$ in eq. (3.3). Extracting the coefficients $A$ and $B$ and using eq. (3.5) leads to the branching fraction

$$\mathcal{B}^{\not{R}_p}_{D^* \to e^+e^-} = \frac{1}{384\pi} \frac{m^3_{D^*} f^2_{D^*}}{m^4_{\tilde{d}^k_R} \Gamma_0} \left| \tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k} \right|^2 . \tag{5.5}$$

Contrary to the case of $D^0 \to \ell^+\ell^-$, no helicity suppression (factors of $m^2_\ell / m^2_D$) is seen in eq. (5.5), which in principle makes this process more sensitive to the NP parameters. This feature exists for any NP model that is represented by $V \pm A$ interactions. Numerically, taking updated bounds on $\tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k}$ from [25], conservatively (see also [26]),

$$\left| \tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k} \right| \leq 3.83 \times 10^{-3} \left[ \frac{m_{\tilde{d}^k_R}}{300 \text{ GeV}} \right] , \tag{5.6}$$

we estimate that $\mathcal{B}^{\not{R}_p}_{D^* \to e^+e^-} < 1.7 \times 10^{-14}$, implying that some prospects exist for improvement on this bound using the described process.

**Models with $Z'$-mediated gauge interactions.** Another interesting and representative model that we would like to consider here is a model with flavor-changing $Z'$-mediated interactions. In general,

$$\begin{aligned} \mathcal{L}_{Z'} = & -g'_{Z'1} \bar{\ell}_L \gamma_\mu \ell_L Z'^\mu - g'_{Z'2} \bar{\ell}_R \gamma_\mu \ell_R Z'^\mu \\ & - g^{cu}_{Z'1} \bar{u}_L \gamma_\mu c_L Z'^\mu - g^{cu}_{Z'2} \bar{u}_L \gamma_\mu c_L Z'^\mu . \end{aligned} \tag{5.7}$$

For $m_{Z'} \gg m_D$ the Lagrangian in eq. (5.7) leads to

$$\mathcal{L}_{\text{eff}}^{Z'} = -\frac{1}{M_{Z'}^2}\left[g'_{Z'1} g_{Z'1}^{cu} \widetilde{Q}_1 + g'_{Z'1} g_{Z'2}^{cu} \widetilde{Q}_2 + g'_{Z'2} g_{Z'2}^{cu} \widetilde{Q}_6 + g'_{Z'2} g_{Z'1}^{cu} \widetilde{Q}_7\right]. \tag{5.8}$$

Again, identifying the Wilson coefficients $c_i$ from eq. (5.8) and computing $A$ and $B$ leads to the following branching fraction,

$$\mathcal{B}_{D^*\to e^+e^-}^{Z'} = \frac{1}{12\pi}\frac{m_{D^*}^3 f_{D^*}^2}{M_{Z'}^4 \Gamma_0}\left|g_{Z'1}^{cu} + g_{Z'2}^{cu}\right|^2 \left(\left|g'_{Z'1}\right|^2 + \left|g'_{Z'2}\right|^2\right). \tag{5.9}$$

As with our previous example, eq. (5.9) does not exhibit helicity suppression of the rate. Most importantly, $\mathcal{B}_{D^*\to e^+e^-}^{Z'}$ is non-zero for purely vectorial interactions of the $Z'$, which will be realized if, for example, $g_{Z'1}^{cu} = g_{Z'2}^{cu}$. This is contrary to $D^0 \to \ell^+\ell^-$ decay rate, where such vectorial couplings are forbidden by vector current conservation [17]. There are five parameters that describe generic $Z'$ interactions with quarks and leptons, $g_{Z'1}^{cu}$, $g_{Z'2}^{cu}$, $g'_{Z'1}$, $g'_{Z'2}$, and $M_{Z'}$. To assess the sensitivity of the $e^+e^- \to D^*$ production mechanism to $Z'$ models numerically, let us make two simplifying assumptions. First, let us assume that $Z'$ only couples to left-handed quarks,[4] which would mean that $g_{Z'2}^{cu} = 0$. Second, let us assume that the $Z'$ has SM-like diagonal couplings to leptons,

$$g'_{Z'1} = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right), \qquad g'_{Z'2} = \frac{g\sin^2\theta_W}{\cos\theta_W}, \tag{5.10}$$

where $g$ is the SM SU(2) gauge coupling. The branching fraction would then only depend on the combination $g_{Z'1}^{cu}/M_{Z'}^2$,

$$\mathcal{B}_{D^*\to e^+e^-}^{Z'} = \frac{\sqrt{2}G_F}{3\pi\Gamma_0} m_{D^*}^3 f_{D^*}^2 \frac{\left|g_{Z'1}^{cu}\right|^2}{M_{Z'}^2}\frac{M_Z^2}{M_{Z'}^2}\left(\frac{1}{4} - \sin^2\theta_W + 2\sin^4\theta_W\right). \tag{5.11}$$

Taking the constraint $M_{Z'}/\sqrt{g_{Z'1}^{cu}} > 8.7 \times 10^2\,\text{GeV}$ from $D^0 \to \mu^+\mu^-$ [17] yields

$$\mathcal{B}_{D^*\to e^+e^-}^{Z'} < 2.5 \times 10^{-11}, \tag{5.12}$$

which is far above the SM predictions for this rate.

## 6 Implications for $B_s^*$ FCNC decays

Similarly to $D^0 \to e^+e^-$, the $B_{s(d)} \to e^+e^-$ decay is helicity-suppressed. Hence, it might be interesting to see if a $e^+e^-$ production process can be used to probe the $B_{s(d)}^* \to e^+e^-$ transitions. Hereafter we will concentrate on the $B_s^*$ resonant production, the corresponding process with $B_d^*$ is very similar, but CKM suppressed and has therefore less chances to be detected. There are two important differences between the beauty and the charm case. First of all, the strong decays of the $B_s^*$ are kinematically forbidden, and the dominant channel is the radiative one, $B_s^{0*} \to B_s^0\gamma$. This feature is welcome because it leads to a

[4] Equivalently, we could have assumed that $Z'$ only couples to the right-handed currents. Then $g_{Z'1}^{cu} = 0$ and constraints would be obtained for $g_{Z'2}^{cu}$.

smaller total width than for $D^*$. However, additional challenges might emerge for triggering the final state of the produced $B^*_{(s)}$. Second, in the SM the $B_s^* \to e^+e^-$ decay is dominated by the SD contributions, stemming from the well defined effective Hamiltonian. Hence one may expect a reasonable accuracy in predicting the decay rate, also because the only hadronic parameter involved is the $B_s^*$ decay constant. On the other hand, one also has to assess the possible LD contributions, e.g., the effect similar to the one shown in figure 2a, but with the $c$-quark polarization operator.

For $e^+e^- \to B_s^*$, we use the resonant cross section

$$\sigma_{B_s^*}(m^2_{B_s^*}) = \mathcal{B}_{B_s^*\to e^+e^-}\frac{12\pi}{m^2_{B_s^*}} \tag{6.1}$$

and find that with the branching ratios

$$\mathcal{B}(B_s^* \to e^+e^-) > 2.0 \times 10^{-12}\,, \tag{6.2}$$

the resonance production can be observed with at least one event. Here we again assume one year running at the $B_s^*$ resonance energy, with the luminosity of $\sim 10^{32}$ cm$^{-2}$ s$^{-1}$ and $\epsilon \sim 1$ detection efficiency. While no $e^+e^-$ collider is currently operating at this energy, future upgrades of the existing $e^+e^-$ machines, including also the radiative return setup, might make this energy region available for experimental studies.

Before calculating the branching fraction of $B_s^* \to e^+e^-$, it is important to fix the total width of $B_s^*$ which practically coincides with the width of the single flavor-conserving radiative decay:

$$\Gamma^{\rm tot}_{B_s^*} \simeq \Gamma(B_s^* \to B_s\gamma) = \frac{\alpha}{24}|g_{B_s^*B_s\gamma}|^2\left(\frac{m^2_{B_s^*}-m^2_{B_s}}{m_{B_s^*}}\right)^3. \tag{6.3}$$

Here we use the definition of the $B_s^*B_s\gamma$ coupling:

$$\langle B_s(p)\gamma(q)|B_s^*(p+q)\rangle = \sqrt{4\pi\alpha}\,g_{B_s^*B_s\gamma}\,\varepsilon^{\mu\nu\rho\lambda}\epsilon_\mu^{*(\gamma)}q_\nu\epsilon_\rho^{(B_s^*)}p_\lambda\,, \tag{6.4}$$

The dominant contribution to the $H^* \to H\gamma$ $(H = D, B)$ transitions stems from the long-distance photon emission off the light quark in the heavy meson. The analyses of these couplings in terms of heavy-hadron ChPT and related approaches [29–32] allow for a simplified parametrization of the coupling:[5]

$$g_{H^*H\gamma} \simeq \frac{Q_Q}{m_{H^*}} + \frac{Q_q}{\mu_q}\,, \tag{6.5}$$

where $Q_{Q(q)}$ is the charge factor of the heavy(light) quark $Q = c, b$ $(q = u, d, s)$ in $H^{(*)}$, and $\mu_q$ is a nonperturbative parameter, which does not scale with the heavy mass. Numerically, this relation describes well the two experimentally measured $D^{*0,+} \to D\gamma$ widths,

[5]The $H^* \to H\gamma$ couplings are also estimated [33–36] from QCD sum rules, relating them to the electromagnetic susceptibility of QCD vacuum, the latter parameter is however known with a rather large uncertainty.

if $\mu_{u,d} \simeq 420 - 430$ MeV is taken. Using the same value of $\mu_{u,d}$ for the $g_{B^{*0}B^0\gamma}$ coupling, we obtain $\Gamma(B^{*0} \to B^0\gamma) \sim 0.2$ keV, in the ballpark of the estimates obtained in [29–32]. To account for the SU(3)-flavor symmetry violation, we adopt the model of [32] where the photon emission from the light-quark is described via vector-meson dominance, so that $\mu_s \simeq \mu_{u,d}(m_\rho^2/m_\phi^2)$. We obtain then from eq. (6.3)

$$\Gamma_{B_s^*}^{\rm tot} \simeq 0.07\ {\rm keV}\,. \tag{6.6}$$

Our conclusion is that the $B_s^*$-resonance is considerably narrower than $D^*$.

To estimate the probability of $B_s^* \to e^+e^-$ in SM, we employ the relevant SD part of the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transitions:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=7,9,10} C_i O_i + h.c.\,, \tag{6.7}$$

involving the operators:

$$\begin{aligned} O_7 &= -\frac{em_b}{16\pi^2}\, \overline{q}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \\ O_9 &= \frac{e^2}{16\pi^2} \left(\overline{q}_L \gamma^\mu b_L\right) \left(\overline{\ell}\gamma_\mu \ell\right), \\ O_{10} &= \frac{e^2}{16\pi^2} \left(\overline{q}_L \gamma^\mu b_L\right) \left(\overline{\ell}\gamma_\mu\gamma_5 \ell\right). \end{aligned} \tag{6.8}$$

Using eq. (3.10) with the obvious substitutions of charm meson parameters by the beauty ones, we obtain for the branching fraction:

$$\mathcal{B}(B_s^* \to e^+e^-) = \frac{\alpha^2 G_F^2}{96\pi^3 \Gamma_{B_s^*}^{\rm tot}} m_{B_s^*}^3 f_{B_s^*}^2 \, |V_{tb}V_{ts}^*|^2 \left( \left| C_9 + 2\frac{m_b}{m_{B_s^*}} \frac{f_{B_s^*}^T}{f_{B_s^*}} C_7^{\rm eff} \right|^2 + |C_{10}|^2 \right). \tag{6.9}$$

To refine our estimate, we also should add the nonlocal contribution generated by the combination of the current-current weak operator $O_{1,2}$ and the quark e.m. current. One of the important effects is the lepton pair emitted from the intermediate $\bar{c}c$ pair, described by the diagram similar to figure 2a. In the factorizable approximation, this contribution is estimated in a full analogy with the LD effect for the charmed vector-meson leptonic decay, i.e., we can use the same expression but replace the difference of $d$ and $s$ polarization operators with the single $c$-quark one:

$$A^{(LD)} = 4\pi\alpha Q_c \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^* f_{B_s^*} m_{B_s^*} \left(C_1 + \frac{C_2}{N_c}\right) \left[ -\frac{1}{4\pi^2}\left(\ln\frac{m_c^2}{\mu^2} + 1\right) + \Pi^{(c)}(p^2 = m_{B_s^*}^2) \right], \tag{6.10}$$

where the constant term reflects the correct renormalization of the effective operators. The corresponding $u$-quark loop effect is CKM suppressed and therefore neglected. The $c$-quark polarization operator in eq. (6.10) at $p^2 = m_{B_s^*}^2$, far above the charm-anticharm threshold can be estimated using the dispersion representation for the simple $c$-quark loop diagram ("global duality" approximation)

$$\Pi^{(c)}(p^2) = \frac{p^2}{4\pi^2} \int\limits_{4m_c^2}^{\infty} \frac{ds}{s(s-p^2-i\epsilon)} \sqrt{1 - \frac{4m_c^2}{s}} \left(1 + \frac{2m_c^2}{s}\right). \tag{6.11}$$

Note that the amplitude (6.10) can be cast in terms of an effective process-dependent addition $\Delta C_9$ to the coefficient $C_9$ in the branching fraction (6.9):

$$\Delta C_9^{B_s^* \to e^+e^-} = 8\pi^2 Q_c \left(C_1 + \frac{C_2}{3}\right) \left[ -\frac{1}{4\pi^2}\left(\ln\frac{m_c^2}{\mu^2} + 1\right) + \Pi^{(c)}(m_{B_s^*}^2) \right], \tag{6.12}$$

in full analogy with the analysis of the nonlocal charm-loop effect in the semileptonic decay, such as $B \to K\ell\ell$ (see, e.g., [37]). At $m_c = 1.3\,\mathrm{GeV}$, $\mu = 4.5\,\mathrm{GeV}$ and at the Wilson coefficients taken at the same scale: $C_1(\mu) = -0.255$, $C_2(\mu) = 1.11$, the numerical calculation yields:

$$\Delta C_9^{B_s^* \to e^+e^-} = 0.11 - 0.47i \tag{6.13}$$

revealing a very small correction to the short-distance coefficient $C_9$. We therefore skip the other LD effects, e.g., the one with the e.m. interaction of the $s$-quark in $B_s^*$.

Finally, to estimate the branching fraction (6.9) numerically, we replace $C_9 \to C_9 + \Delta C_9$, use the numerical values for the relevant Wilson coefficients at the scale $\mu = 4.5\,\mathrm{GeV}$: $C_7^{\rm eff}(\mu) = -0.316$, $C_9(\mu) = 4.293$, $C_{10}(\mu) = -4.493$, and employ the QCD sum rule estimate [20] for the $B_s^*$ decay constant $f_{B_s^*} \simeq 250\,\mathrm{MeV}$. We arrive at the following prediction for the branching fraction:

$$\mathcal{B}_{B_s^* \to e^+e^-} = 0.98 \times 10^{-11}\,. \tag{6.14}$$

This estimate implies that already within the SM, one could expect several events of the type $e^+e^- \to B_{(s)}^* \to B_s\gamma$ to be observed. The signature of the final state is a combination of monochromatic low-energy photon and a flavor-violating $B_s$ meson.

## 7 Conclusion

We argued that the rare leptonic decays of heavy mesons $H^* = D^*, B^*$ can be probed in the reverse process of the $e^+e^- \to H^*$ production, provided that the beams of $e^+e^-$ collider are tuned in resonance with $m_{H^*}$.

We calculated relevant transition rates for charm and beauty modes. In the case of charmed mesons we paid particular attention to the LD effects, which are calculable and found them to exceed the typical SD rate by at most one order of magnitude. We also studied several examples of NP scenarios and considered similar production effects for the beauty mesons. More efforts can be invested in improving the accuracy of the estimates presented in this paper, by calculating the QCD corrections to the LD effects and by extending the set of NP scenarios sensitive to the processes we considered here.

It would be interesting to note that similar single-charm (single-$b$) final states can be produced in non-leptonic weak decays of heavy quarkonium states, such as $J/\psi \to D\pi$. Since heavy charmonium states lie reasonably far away from the energy region discussed in this paper, these transitions will not be producing any backgrounds for $e^+e^- \to D^* \to D\pi$, but can be used to study experimental systematics associated with such final state. In addition, these transitions are interesting on their own and will be discussed elsewhere.

Although the experimental setup suggested here might look futuristic, we are convinced that the continuous progress of collider and detector technique will make the tasks suggested



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in this paper real. We hope that our first exploratory estimates will simulate dedicated experimental studies of the heavy vector-meson production in electron-positron collisions.

**Note added.** While we were finishing this paper, the work [38] appeared where similar considerations for $B^*$-meson were presented.

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