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Empirical Models of the Azimuthal Reception Angle—Part II: Adaptation of the Empirical Models in Analytical and Simulation Studies

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Abstract This paper focuses on modeling of the statistical reception angle properties that are the result of the environmental influence on transmitted signals. Numerous measurements have shown that the statistical properties of angle of arrival (AOA) significantly depend on the type of propagation environment. In practice, this means that in the analytical and simulation studies, the use of statistical models of AOA required their adaptation to propagation conditions in research scenario. Despite the large number of papers that are devoted to the AOA modeling, the method of fitting models to the environment is not presented in any of the publications. This paper fills this gap. For the assessment of the propagation properties and environment type classification, the basis is the rms delay spread (DS). Therefore, the presented method of the model adaptation to the environment type consists of determining the relationship between the model parameter and DS of environment. Here, the probability density function (PDF) of models such as the modified Gaussian, modified Laplacian, modified logistic, and von Mises distribution are considered as the statistical models of azimuth AOA. For seven different propagation environments, the measurement results are used as a reference data. A comparative analysis shows that the modified Laplacian PDF provides the smallest error fit to measurement data. However, in theoretical analysis and simulation studies, other empirical models can be used due to relatively low approximation errors. The presented results are the basis for the adaptation error assessment of the empirical model for any of the research scenario.

Keywords Wireless communications · Multipath channel · Scattering · Angle of arrival (AOA) · Probability density function (PDF) · Empirical models of PDF of azimuthal AOA · Laplacian · Gaussian · Logistic · Von Mises distribution

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1 Introduction

For multipath propagation environment, mapping the statistical properties of the angle of arrival (AOA) is important in the analytical and simulation studies of signals in wireless systems. Part I of this paper [1] is focused on a comparative analysis of empirical and geometrical (theoretical) models of the probability density function (PDF) of azimuth AOA (AAOA) for different environmental conditions. Empirical models are based on standard distributions such as Gaussian [2–5], Laplacian [2, 3, 6–12], logistic [13–16], and von Mises [3, 17–21], which are being modified due to limited support of PDF of AAOA. The theoretical models are defined by the geometrical structures that describe the spatial position of scattering elements. In [1], the comparative analysis of PDF models also takes into account the results from [22], which relate to the best geometrical models such as uniform elliptical (receiver (Rx) outside) [23], Gaussian [6, 24, 25], and Rayleigh circular (Rx outside) [26]. A comparison shows that the smallest approximation errors of the measured data are provided by modified Laplacian and uniform elliptical (Rx outside) PDF for empirical and geometrical models, respectively. For all the analyzed scenarios and measures of the approximation accuracy, the average error is smallest for modified Laplacian PDF. The simplicity of the analytical description of empirical models, whose detailed analysis is derived from mathematical statistics, is an additional asset in relation to other models. This feature of the empirical models plays a significant role in theoretical and simulation studies. Therefore, in analytical analysis, e.g. [3, 17], which take into account the impact of AOA on correlational and spectral properties of received signals, the empirical models are primarily used. Numerous measurements, e.g. [7, 27, 28], show that the statistical properties of AOA significantly depend on the type of the propagation environment and spatial parameters of the measurement scenario. In practice, this means that the use of empirical models in analytical studies requires their adaptation to research scenario. In COST 207 [29], the assessment of the propagation conditions and classification of environment type is based on the rms delay spread (DS), σ_τ. This parameter is defined on the basis of the impulse response (IR), power delay profile (PDP), or power delay spectrum (PDS). Therefore, the fit of the model to the type of propagation environment consists of determining the relationship between the model parameter and DS of the environment. It is the fundamental problem of the practical use of empirical models in analytical and simulation studies. Despite the large number of publications devoted to PDF of AOA modeling, the method of fitting the model to any environment is not described in any paper. This paper fills this gap. Here, the method for determining the relationship between DS and the parameter of each PDF model is presented for all environment types. In analytical and simulation studies, this method makes it possible to fit each analyzed model to different propagation environments.

In Part I of this paper [1], the accuracy of AAOA measurement mapping results by empirical and geometrical PDF models is shown. Part II is focused on the adaptation method of the empirical models to the different types of propagation environments. In Sect. 2, the relationship between DS and the rms angle spread (AS) is defined on the basis of the measurement scenarios taken from the open literature. In Sect. 3, the relationships between DS and parameters of PDF models are determined by minimizing the mean square error. Section 4 includes conclusions, which highlights the practical relevance of this method for analytical and simulation studies.

2 Statistical Relationship Between DS and AS

Evaluation of propagation environment properties is based on the signal power distribution over time that is described by such characteristics as IR, PSD, or PDP. These characteristics are the basis to determine σ_τ that defines the environment properties in the time domain. However, the propagation environment is described by parameters that are defined not only in time domain but also in the reception angle domain. In this case, the properties of the propagation environment are described by the standard deviation of reception angle, σ_θ, which is defined based on PDF of AOA. In practice, the evaluation of the statistical properties of reception angle is based on the measurement of the angular power spread. When analyzing the reception angle in the azimuth plane, σ_θ is defined on the basis of the power azimuth spectrum (PAS), $P(\theta)$, as

$$\sigma_\theta = \sqrt{\frac{\int_{-180°}^{180°} \theta^2 P(\theta)\mathrm{d}\theta}{\int_{-180°}^{180°} P(\theta)\mathrm{d}\theta} - \left(\frac{\int_{-180°}^{180°} \theta P(\theta)\mathrm{d}\theta}{\int_{-180°}^{180°} P(\theta)\mathrm{d}\theta}\right)^2} \tag{1}$$

In this case, σ_θ is called AS. For the empirical PAS, σ_θ is marked as σ_E.

The results of the numerous measurements show that σ_τ and σ_E increase with increasing urbanization of environment. In this paper, the relationship between σ_τ and σ_E is determined based on data from the seven measurement scenarios described in [7, 27, 28, 30–32]. In [22], many measurement scenarios are used to assess the approximation errors of geometrical models of PDF of AAOA. For analysis of empirical models, a number of measurement data is limited to these scenarios that include the signal characteristics in both time and angle domain. For all selected measurement scenarios, σ_τ and σ_E are determined on the basis of PDS or PDP and PAS, respectively. The results of numerical calculations are included in Table 1.

The correlation coefficient, ρ, is the basis for assessing the degree of DS and AS correlation [7]

$$\rho = \frac{\sum_{n=1}^{N} (\sigma_E[n] - \bar{\sigma}_E)(\sigma_\tau[n] - \bar{\sigma}_\tau)}{\sqrt{\sum_{n=1}^{N} (\sigma_E[n] - \bar{\sigma}_E)^2 \sum_{n=1}^{N} (\sigma_\tau[n] - \bar{\sigma}_\tau)^2}} \tag{2}$$

where $N = 7$ is the number of scenarios described in Part I [1], $\sigma_E[n]$ and $\sigma_\tau[n]$ are the estimated AS and DS for the nth scenario, while $\bar{\sigma}_E = 7.328°$ and $\bar{\sigma}_\tau = 0.7203\ \mu s$ are the sample means of the estimated ASs and DSs, respectively.

Table 1 DSs and ASs—results of numerical calculation for selected measurement scenarios

Measurement scenario	Matthews et al. [30] (Wong et al. [22, Fig. 4])	Pedersen et al. [27] (Wong et al. [22, Fig. 5])	Takada et al. [31] (Wong et al. [22, Fig. 7])	Fleury et al. [32] (Wong et al. [22, Fig. 8])	Mogensen et al. [28] (Wong et al. [22, Fig. 9])	Pedersen et al. [7]—Aarhus (Wong et al. [22, Fig. 10])	Pedersen et al. [7]—Stockholm (Wong et al. [22, Fig. 11])
	1	2	3	4	5	6	7
DS, σ_τ (µs)	1.4185	0.0932	0.1112	1.2337	1.2337	0.3221	0.6285
AS, σ_E (°)	8.935	2.089	4.448	10.169	9.782	6.428	9.445

For empirical data included in Table 1, the correlation coefficient is computed to be $\rho = 0.8537$. This value indicates that AS and DS are highly correlated. This means that between σ_τ and σ_E is the functional relationship, which makes it possible to assess the reception angle spread on the basis of the spread of signal in the time domain. In [7, Fig. 6], the analysis results of measurement data are a prerequisite to the adaptation of a linear function for mapping the relationship between σ_τ and σ_E. The linear regression line between AS and DS is obtained by using the least square method [33, 34]. For the linear function

$$\tilde{\sigma}_\theta = a \cdot \sigma_\tau + b \tag{3}$$

where a and b are the coefficients described by [33]

$$
\begin{aligned}
a &= \frac{1}{\Delta} \left(N \sum_{n=1}^{N} \sigma_\tau[n]\sigma_\theta[n] - \sum_{n=1}^{N} \sigma_\tau[n] \sum_{n=1}^{N} \sigma_\theta[n] \right) \\
b &= \frac{1}{\Delta} \left(\sum_{n=1}^{N} \sigma_\theta[n] \sum_{n=1}^{N} \sigma_\tau^2[n] - \sum_{n=1}^{N} \sigma_\tau[n] \sum_{n=1}^{N} \sigma_\tau[n]\sigma_\theta[n] \right) \\
\Delta &= N \sum_{n=1}^{N} \sigma_\tau^2[n] - \left(\sum_{n=1}^{N} \sigma_\tau[n] \right)^2
\end{aligned}
\tag{4}
$$

Based on Table 1, the linear relation between σ_E and σ_τ is

$$\tilde{\sigma}_E[°] = 4.65 \cdot \sigma_\tau[\mu s] + 3.98 \tag{5}$$

The empirical data and the regression line for DS and AS are presented in Fig. 1.

For the regression line, the root of the mean square error (RMSE), δ_O is used to evaluate the approximation error
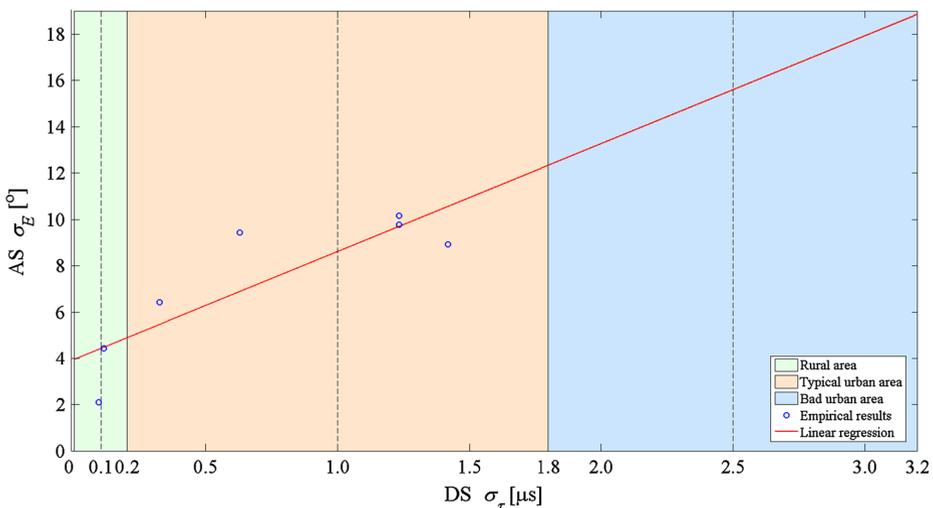

Fig. 1 Empirical data and regression line for σ_τ and σ_E

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\sigma_E[n] - \tilde{\sigma}_\theta[n])^2} \tag{6}$$

where $\sigma_E[n]$ and $\tilde{\sigma}_\theta[n]$ are empirical ASs from Table 1 and from regression line for the nth scenario, respectively. For the empirical data, $\tilde{\sigma}_\theta[n] = \tilde{\sigma}_E[n]$.

On the basis of Eq. (6) and data contained in Table 1, the approximation error is computed to be $RMSE = 1.4953°$. The obtained result indicates that the approximation line yields an error of a value corresponding to the practical accuracy of the measurement angle. Later in this paper, $RMSE$ is used to compare analyzed PDF models of AAOA.

3 Adaptation of the Empirical Models to Propagation Scenarios

The adaptation of the empirical model to the environment type involves selecting such value of model parameter that minimizes the approximation error with respect to measurement data. In Part I of this paper [1], the measure for the parameter selection and evaluation of the approximation error is the least-squares error (LSE). The optimal parameters of the models and corresponding LSEs are shown in [1, Table 4] and [1, Table 5], respectively. For all measurement scenarios and optimal parameters, each empirical model determines specified ASs that are shown in Table 2. To evaluate the accuracy of σ_θ approximation by particular models, RMSEs relative to measurement results are also presented here.

In time domain, each measurement scenario is identified by σ_τ. Taking into account all measurement scenarios and their DSs, the data from Table 2 are the basis for the determination of the relationship between σ_θ and σ_τ for particular empirical models. The measurement results [7, Fig. 6] are a prerequisite to the adaptation of a linear function for the approximation of the relation between AS and DS for all models. Similarly as for (5), the regression lines for each model are obtained by using the method of least squares based on data from Tables 1 and 2. For particular models, the relationships are as follows:

- modified Gaussian

$$\tilde{\sigma}_\theta = 3.72 \cdot \sigma_\tau + 2.26 \tag{7}$$

- modified Laplacian

$$\tilde{\sigma}_\theta = 5.32 \cdot \sigma_\tau + 2.75 \tag{8}$$

- modified logistic

Table 2 AS and RMSE for particular models and measurement scenarios

Empirical PDF model	AS for measurement scenario, σ_θ (°)							RMSE (°)
	1	2	3	4	5	6	7	
Modified Gaussian	7.972	1.173	1.980	6.037	5.598	4.159	7.664	2.693
Modified Laplacian	10.690	1.645	2.477	8.443	8.097	5.617	9.128	1.401
Modified logistic	8.892	1.306	2.140	6.790	6.298	4.549	8.167	2.225
von Mises	7.990	1.173	1.980	6.045	5.604	4.162	7.679	2.687

$$\tilde{\sigma}_\theta = 4.24 \cdot \sigma_\tau + 2.39 \tag{9}$$

- von Mises

$$\tilde{\sigma}_\theta = 3.73 \cdot \sigma_\tau + 2.26 \tag{10}$$

In Fig. 2, the graphical representations of these equations are presented.

The graphs show that the approximation line for the modified Laplacian is the most convergent to the straight line for measurement data. The accuracy of mapping empirical relation of AS versus DS for modified Gaussian and von Mises PDF is practically the same. However, these models introduce a greater approximation error, especially for typical and bad urban environment.

The purpose of adapting the PDF model of AAOA is to provide such AS that corresponds to the propagation environment type. For each model, there is a close relationship between its parameter and σ_θ. These relations for analyzed models are included in Table 3.

The above relationships are the basis for determining the parameters of models as a function of the environment type, that is, depending on σ_τ. For the modified Gaussian and logistic models and taking (7), (9) into account, the relations between σ, s, and σ_τ are, respectively

- modified Gaussian

$$\tilde{\sigma} = 3.73 \cdot \sigma_\tau + 2.23 \tag{11}$$

- modified logistic

$$\tilde{s} = 2.35 \cdot \sigma_\tau + 1.32 \tag{12}$$

The graphs of these equations are presented in Figs. 3 and 4, respectively.

In Table 3 for the modified Laplacian, the relationship between λ and σ_θ is described as a hyperbolic function. Therefore, the relation λ versus σ_τ is
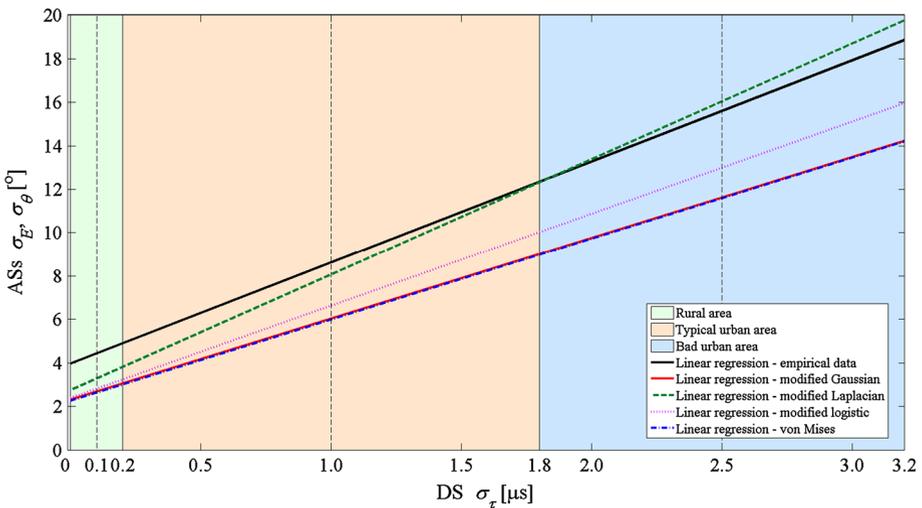

Fig. 2 Linear approximation of the relations between AS and DS for the empirical models

Table 3 AS versus parameters of particular PDF models

Empirical PDF model	Model parameter	Normalized factor	Theoretical AS
Modified Gaussian	σ	C_G	$\sigma_\theta = \sqrt{C_G}\,\sigma$
Modified Laplacian	λ	C_L	$\sigma_\theta = \sqrt{C_L}\,\frac{\sqrt{2}}{\lambda}$
Modified logistic	s	C_S	$\sigma_\theta = \sqrt{C_S}\,s\,\frac{\pi}{\sqrt{3}} = \sqrt{C_S}\,s\,\frac{\pi\sqrt{3}}{3}$
von Mises	κ	–	$\sigma_\theta = \sqrt{1 - \frac{I_1(\kappa)}{I_0(\kappa)}}$

$I_0(\cdot)$, $I_1(\cdot)$ are the zero- and first-order modified Bessel functions, respectively, C_G, C_L, and C_S are the normalized factors of the models defined in [1]

$$\tilde{\lambda} = \frac{1}{9.44 \cdot \sigma_\tau + 0.40} \tag{13}$$

In Fig. 5, the graph of the equation is presented.

For the von Mises PDF, the complexity of relationship between κ and σ_θ makes it difficult to use the least-squares method to determine the approximating function of κ versus σ_τ. Therefore, in this case, the following approximation is adopted:

$$\sigma_\theta = \sqrt{1 - \frac{I_1(\kappa)}{I_0(\kappa)}} \cong \frac{1}{a_{M0} \cdot \kappa + b_{M0}} + c_{M0} \cong \tilde{\sigma}_\theta \tag{14}$$

Taking (10) and (14) into account, the relation κ versus σ_τ takes the form

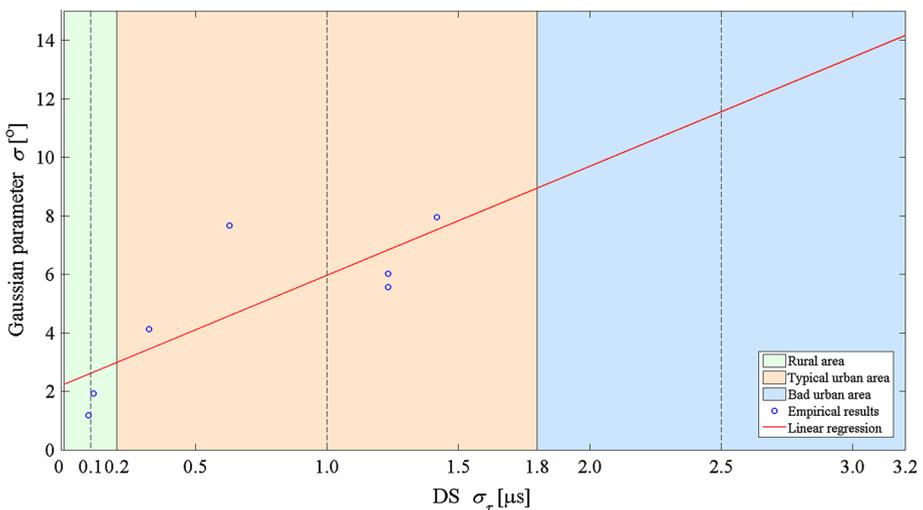$$\tilde{\kappa} = \frac{1}{a_M \cdot \sigma_\tau + b_M} + c_M \tag{15}$$

Fig. 3 σ versus σ_τ for the modified Gaussian
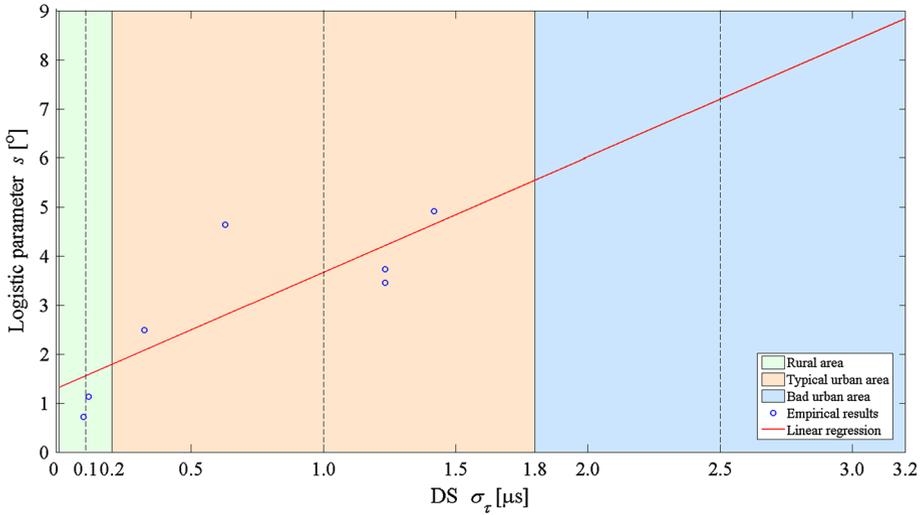
Fig. 4 s versus σ_τ for the modified logistic model
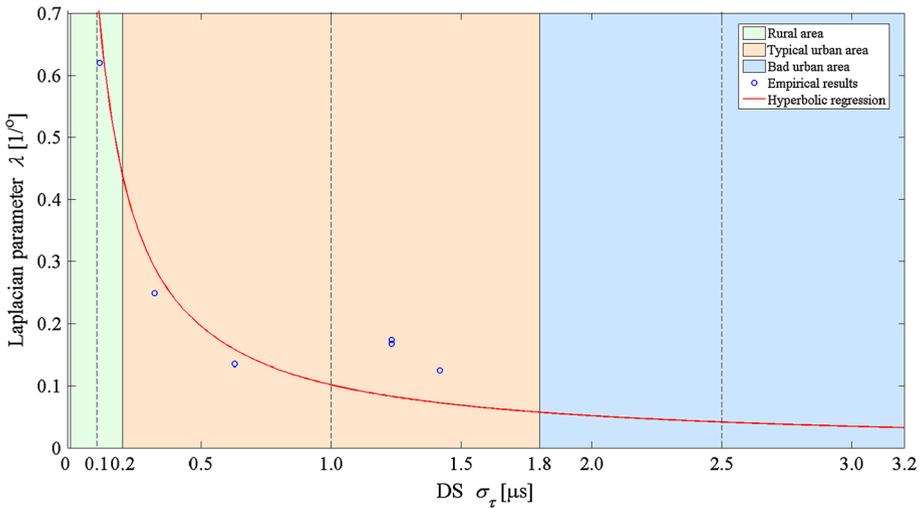

Fig. 5 λ versus σ_τ for the modified Laplacian

In (15), the parameters a_M, b_M, and c_M are determined by the use of the least-squares method relative to optimal κ included in [1, Table 4] and data from Table 2. Obtained results are $a_M = 0.0426$, $b_M = -0.0035$, and $c_M = 59.0287$, so

$$\tilde{\kappa} = \frac{1}{0.0426 \cdot \sigma_\tau - 0.0035} + 59.0287 \tag{16}$$

Graphical representation of (16) is shown in Fig. 6.
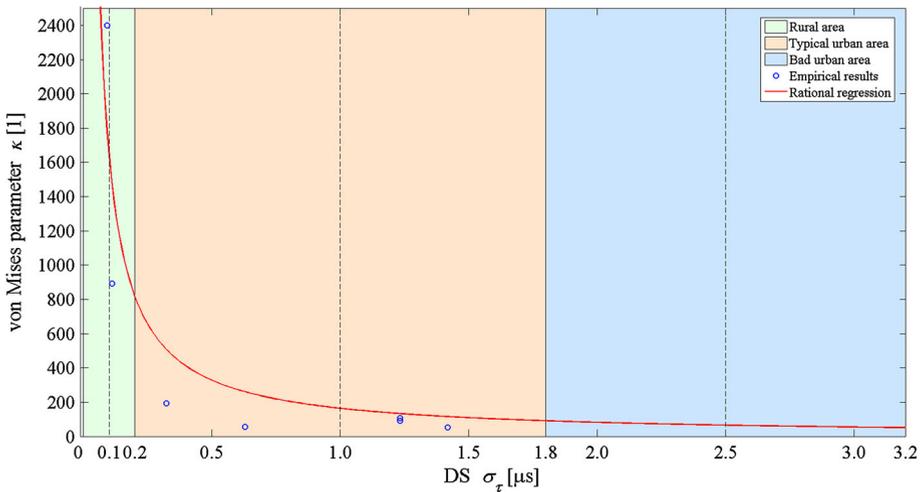
Fig. 6 κ versus σ_τ for the von Mises PDF

Table 4 Average LSEs for the particular models	Empirical PDF model	LSE	LSE_R	$LSE_R - LSE$
	Modified Gaussian	0.522322	3.015436	2.493114
	Modified Laplacian	0.125177	0.203505	0.078328
	Modified logistic	0.358694	2.714384	2.355690
	von Mises	0.521320	0.560357	0.039036

Based on the close relationship contained in Table 3 and for optimal κ from Table 2, RMSE is 2.687°, whereas for (14) is 2.852°. Therefore, the relative error of RMSE is about 1/16. It shows that (14) introduces about 16 times smaller error in relation to (10). This fact justifies the adaptation of (14) to approximate the relationship between σ_θ and κ for the von Mises PDF.

For the particular empirical models, (11), (12), (13), and (16) provide the fit of models to the type environment defined based on σ_τ. However, using these approximation increases the mapping error of AAOA statistical properties when compared with the optimal parameters. As in Part I [1], LSE is used to access the PDF mapping error of AAOA by empirical models. The basis for assessing the mapping error are the average LSE, for the optimal parameters and the average LSE_R, for values from (11), (12), (13), and (16). For the particular models, the results of the numerical calculations of these measures and their difference are contained in Table 4.

The obtained results make it possible to assess the mapping error of PDF of AAOA for each model. Comparison of LSE_R and LSE shows that the adaptation method provides the smallest increment of mapping error for the von Mises PDF. A slightly greater increase in error occurs for the modified Laplacian. However, this model provides the smallest error in the evaluation of PDF of AAOA because it gives the smallest LSE and LSE_R. Relatively worse mapping of real AAOA properties by the modified Gaussian, modified logistic, and von Mises PDFs do not exclude the possibility of their use in theoretical analysis. In many cases, the simplified description of the statistical properties of AAOA is ensured by PDF

model that introduces a greater approximation error. In this paper, the presented adaptation method provides the opportunity to evaluate this error.

4 Conclusion

The paper discusses the adaptation of the PDF models of AAOA to study the propagation scenarios that take into account the diversity of environments. In this publication, the simplicity of the analytical description and a better fit to the measurement results are reasons to focus attention on the empirical models. Especially, these properties are important in analytical and simulation studies of signals in wireless systems. For diverse propagation environments, such PDF models of AAOA as the modified Gaussian, modified Laplacian, modified logistic, and von Mises have been considered in the presented comparative analysis. Similarly as in COST 207 [29], DS is adopted for the classification of the propagation environment type. The adaptation method of empirical model boils down to determining the relationship between DS and the model parameter. The obtained relationships make it possible to fit each PDF model to the real properties of AAOA that correspond to each propagation environment. For seven propagation scenarios, the measurement results are used as a reference data. A comparative analysis shows that the modified Laplacian model provides the smallest fitting error to measurement data. However, in theoretical and simulation studies, other empirical models can also be used due to the relatively small error and simple analytical description in relation to the geometric models. In this case, the presented adaptation method enables the assessment of the approximation error that results from the application of the empirical model.

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