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Letter

Observational constraint on generalized Chaplygin gas model

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Abstract We investigate observational constraints on the generalized Chaplygin gas (GCG) model as the unification of dark matter and dark energy from the latest observational data: the Union SNe Ia data, the observational Hubble data, the SDSS baryon acoustic peak and the five-year WMAP shift parameter. The result is obtained that the best-fit values of the GCG model parameters with their confidence level are $A_s = 0.73^{+0.06}_{-0.06}$ (1σ) $^{+0.09}_{-0.09}$ (2σ), $\alpha = -0.09^{+0.15}_{-0.12}$ (1σ) $^{+0.26}_{-0.19}$ (2σ). Furthermore, in this model, we can see that the evolution of equation of state (EOS) for dark energy is similar to quiessence, and its current best-fit value is $w_{0de} = -0.96$ with the 1σ confidence level $-0.91 \geq w_{0de} \geq -1.00$.

1 Introduction

The recent cosmic observations from type Ia supernovae (SNe Ia) [1, 2], cosmic microwave background (CMB) [3], clusters of galaxies [4] etc., all suggest that the expansion of the present universe is speeding up rather than slowing down. And it indicates that the baryon matter component is about 5% of the total energy density, and about 95% of the energy density in the universe is invisible. Considering four-dimensional standard cosmology, the accelerated expansion of the present universe is usually attributed to the fact that dark energy (DE) is an exotic component with negative pressure. It is shown that DE takes up about two-thirds of the total energy density from cosmic observations. Many kinds of DE models have already been constructed such as ΛCDM [5], quintessence [6], phantom [7, 8], quintom [9], generalized Chaplygin gas (GCG) [10, 11], modified Chaplygin gas [12–14], holographic dark energy [15–17], agegraphic dark energy [18], and so forth. Furthermore, a

model-independent method[1] and modified gravity theories (such as scalar-tensor cosmology [26], braneworld models [27, 28]) to interpret accelerating universe have also been discussed.

It is well known that the GCG model has been widely studied for interpreting the accelerating universe [29–35]. The most interesting property for this scenario is that two unknown dark sections in universe—dark energy and dark matter—can be unified by using an exotic equation of state (EOS). In this paper, we use the latest observational data: the Union SNe Ia data [36], the observational Hubble data (OHD) [37], the baryon acoustic oscillation (BAO) peak from Sloan Digital Sky Survey (SDSS) [38] and the five-year WMAP CMB shift parameter [39], to constrain the GCG model. And we discuss whether the parameter degeneration [40, 41] for the GCG model can be broken by the latest observed data, since it is always expected that the model degeneration problem can be solved by more accurate observational data.

The paper is organized as follows. In Sect. 2, the GCG model as the unification of dark matter and dark energy is introduced briefly. Based on the observational data, we constrain the GCG model parameter in Sect. 3. The evolution of EOS of DE and the deceleration parameter for the GCG model are presented in Sect. 4. Section 5 is for the conclusions.

2 Generalized Chaplygin gas model

The GCG background fluid with its energy density ρ_{GCG} and pressure p_{GCG} are related by the EOS [10, 11]

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[1]Using mathematical fundament, one expands equation of state of DE w_{de} or deceleration parameter q with respect to scale factor a or redshit z. For example, $w_{de}(z) = w_0 = \text{const}$ [19], $w_{de}(z) = w_0 + w_1 z$ [20], $w_{de}(z) = w_0 + w_1 \ln(1 + z)$ [21], $w_{de}(z) = w_0 + \frac{w_1 z}{1+z}$ [22–24], $q(z) = q_0 + q_1 z$ [19], $q(z) = q_0 + \frac{q_1 z}{1+z}$ [25], where $w_0, w_1,$ or q_0, q_1 are model parameters.

$$p = -\frac{A}{\rho^\alpha}, \tag{1}$$

where A and α are parameters in the model. When $\alpha = 1$, it is reduced to the CG scenario.

Considering the Friedmann–Robertson–Walker (FRW) cosmology, by using the energy conservation equation, $\mathrm{d}(\rho a^3) = -p\,\mathrm{d}(a^3)$, the energy density of GCG can be derived as

$$\rho_{\mathrm{GCG}} = \rho_{0\mathrm{GCG}}\big[A_s + (1 - A_s)(1+z)^{3(1+\alpha)}\big]^{\frac{1}{1+\alpha}}, \tag{2}$$

where a is the scale factor, $A_s = \frac{A}{\rho_0^{1+\alpha}}$. For the GCG model, as a scenario of the unification of dark matter and dark energy, the GCG fluid is decomposed into two components: the dark-energy component and the dark-matter component, i.e., $\rho_{\mathrm{GCG}} = \rho_{\mathrm{de}} + \rho_{\mathrm{dm}}$, $p_{\mathrm{GCG}} = p_{\mathrm{de}}$. Then according to the common assumption about dark matter,

$$\rho_{\mathrm{dm}} = \rho_{0\mathrm{dm}}(1+z)^3, \tag{3}$$

the energy density of the DE in the GCG model is given by

$$\begin{aligned}
\rho_{\mathrm{de}} &= \rho_{\mathrm{GCG}} - \rho_{\mathrm{dm}} \\
&= \rho_{0\mathrm{GCG}}\big[A_s + (1 - A_s)(1+z)^{3(1+\alpha)}\big]^{\frac{1}{1+\alpha}} \\
&\quad - \rho_{0\mathrm{dm}}(1+z)^3.
\end{aligned} \tag{4}$$

Next, we assume that the universe is filled with two components; one is the GCG component, and the other is the baryon matter component, i.e., $\rho_t = \rho_{\mathrm{GCG}} + \rho_b$. In a flat universe, making use of the Friedmann equation, the Hubble parameter H is expressed as

$$\begin{aligned}
H^2 &= \frac{8\pi G \rho_t}{3} = H_0^2 E^2 \\
&= H_0^2\big\{(1 - \Omega_{0b})\big[A_s + (1 - A_s)(1+z)^{3(1+\alpha)}\big]^{\frac{1}{1+\alpha}} \\
&\quad + \Omega_{0b}(1+z)^3\big\}.
\end{aligned} \tag{5}$$

Here $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the present Hubble constant, $h = 0.72 \pm 0.08$ is given by the Hubble Space Telescope key projects [42]. Ω_{0b} is the present value of the dimensionless baryon matter density, and a joint analysis of five-year WMPA, SNe Ia and BAO data gives $\Omega_{0b}h^2 = 0.02265 \pm 0.00059$ [43]. In the following section, we will use the cosmic observations to constrain the GCG model parameter (A_s, α).

3 Constraint on GCG model parameter

It is necessary for the investigation of type Ia supernovae to explore dark energy and constrain the models. Since SNe Ia behave as excellent standard candles, they can be used to directly measure the expansion rate of the universe up to high redshift, comparing with the present rate. Theoretical dark-energy model parameters are determined by minimizing the quantity [44–47]

$$\chi^2_{\mathrm{SNe}}(\theta) = \sum_{i=1}^{N} \frac{(\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(\theta; z_i))^2}{\sigma^2_{\mathrm{obs};i}}, \tag{6}$$

where $N = 307$ for the Union SNe Ia data [36], which include the SNe samples from the Supernova Legacy Survey [48], ESSENCE Surveys [49], distant SNe discovered by the Hubble Space Telescope [50], nearby SNe [51, 52] and several other, small data sets. The 1σ errors $\sigma_{\mathrm{obs};i}$ are from the flux uncertainty, intrinsic dispersion of SNe Ia absolute magnitude and peculiar velocity dispersion, which are assumed to be Gaussian and uncorrelated. θ denotes the model parameters. μ_{obs} is the observed value of the distance modulus and can be given by the SNe data set. The theoretical distance modulus μ_{th} is defined as

$$\mu_{\mathrm{th}}(z_i) \equiv m_{\mathrm{th}}(z_i) - M. \tag{7}$$

Here $m_{\mathrm{th}}(z)$ is the apparent magnitude of the SNe at peak brightness

$$m_{\mathrm{th}}(z) = \overline{M} + 5\log_{10}\big(D_{\mathrm{L}}(z)\big), \tag{8}$$

and the absolute magnitude M can be given by relating to the magnitude zero point offset $\overline{M}$,

$$\overline{M} = M + \mu_0 \tag{9}$$

with $\mu_0 = 5\log_{10}(\frac{H_0^{-1}}{\mathrm{Mpc}}) + 25 = 42.38 - 5\log_{10}h$. Thus according to (7), (8) and (9), the theoretical distance modulus can be written as

$$\mu_{\mathrm{th}}(z) = 5\log_{10}\big(D_{\mathrm{L}}(z)\big) + \mu_0, \tag{10}$$

where $D_{\mathrm{L}}(z)$ is the Hubble free luminosity distance

$$D_{\mathrm{L}}(z) = H_0 d_{\mathrm{L}}(z) = (1+z)\int_0^z \frac{\mathrm{d}z'}{E(\theta; z')}. \tag{11}$$

Since the nuisance parameter μ_0 is independent of the data and the data set, from the above equations one can see that the distance modulus of different SNe (i.e. at different redshift z), $\mu(z_i)$ and $\mu(z_j)$ are uncorrelated. So, the covariance matrix included in the χ^2_{SNe} (see (6)) is diagonal with entries σ_i.

Furthermore, by expanding the χ^2_{SNe} of expression (6) relative to μ_0, the minimization with respect to μ_0 can be made trivially [44–47, 53–55]:

$$\chi^2_{\mathrm{SNe}}(\theta) = A(\theta) - 2\mu_0 B(\theta) + \mu_0^2 C, \tag{12}$$

where

$$A(\theta) = \sum_{i=1}^{N} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \theta)]^2}{\sigma_i^2}, \quad (13)$$

$$B(\theta) = \sum_{i=1}^{N} \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \theta)}{\sigma_i^2}, \quad (14)$$

$$C = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}. \quad (15)$$

Evidently, (6) has a minimum for $\mu_0 = B/C$ at

$$\widetilde{\chi}^2_{\text{SNe}}(\theta) = A(\theta) - B(\theta)^2/C. \quad (16)$$

Since $\chi^2_{\text{SNe,min}} = \widetilde{\chi}^2_{\text{SNe,min}}$ and $\widetilde{\chi}^2_{\text{SNe}}$ is independent of the nuisance parameter μ_0 [55], here we utilize the expression (16) to displace (6) for the SNe constraint.

Since the Hubble parameter $H(z)$ depends on the differential age of the universe,

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}, \quad (17)$$

the value of $H(z)$ can directly be measured through a determination of dz/dt. By using the differential ages of passively evolving galaxies from the GDDS [56] and archival data [57, 58], Ref. [37] got nine values of $H(z)$ in the range of $0 < z < 1.8$ (see Table 1). Here the observed Hubble data $H(z_i)$ and $H(z_j)$ are uncorrelated, for they are obtained by the observations of galaxies at different redshift. Using these nine observational Hubble data one can constrain DE models by minimizing [59–64]

$$\chi^2_{\text{Hub}}(H_0, \theta) = \sum_{i=1}^{N} \frac{[H_{\text{th}}(H_0, \theta, z_i) - H_{\text{obs}}(z_i)]^2}{\sigma^2_{\text{obs};i}}, \quad (18)$$

where H_{th} is the predicted value of the Hubble parameter, H_{obs} is the observed value, $\sigma_{\text{obs};i}$ is the 1σ uncertainty of the measurement of standard deviation. Here H_0 contained in the $\chi^2_{\text{Hub}}(H_0, \theta)$ as a nuisance parameter is marginalized by integrating the likelihood $L(\theta) = \int dH_0 P(H_0) \exp(-\chi^2(H_0, \theta)/2)$. $P(H_0)$ is the prior distribution function of the present Hubble constant, and a Gaussian prior $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} [42] is adopted in this paper.

Using a joint analysis of Union SNe Ia data and OHD (i.e., $\chi^2_{\text{total}} = \chi^2_{\text{SNe}} + \chi^2_{\text{Hub}}$), Fig. 1 shows the constraint on

the GCG parameter space A_s–α at the 1σ (68.3%) and 2σ (95.4%) confidence levels. For this analysis the best-fit parameters are $A_s = 0.80$ and $\alpha = 0.42$. It is obvious that two model parameters, A_s and α, are degenerate. And it can be seen that the model parameter α has the larger variable range. Then in order to get the stringent constraint and diminish systematic uncertainties, in what follows we combine the standard ruler data (the BAO peak from SDSS and the five-year WMAP CMB shift parameter R) with the Union SNe Ia data and the OHD to constrain the GCG model.

Because the universe has a fraction of baryons, the acoustic oscillations in the relativistic plasma would be imprinted onto the late-time power spectrum of the nonrelativistic matter [65]. Then the observations of acoustic signatures in the large-scale clustering of galaxies are very important for constraining cosmological models. From the BAO constraint, the best-fit values of the parameters in the DE models can be determined by constructing [66–69]

$$\chi^2_{\text{BAO}}(\theta) = \frac{[A(\theta) - A_{\text{obs}}]^2}{\sigma_A^2}. \quad (19)$$

Here

$$A(\theta) = \sqrt{\Omega_{0\text{m}}} E(z_{\text{BAO}})^{-1/3} \left[\frac{1}{z_{\text{BAO}}} \int_0^z \frac{dz'}{E(z'; \theta)}\right]^{2/3}, \quad (20)$$

$\Omega_{0\text{m}}$ is the effective matter density parameter given by $\Omega_{0\text{m}} = \Omega_{0\text{b}} + (1 - \Omega_{0\text{b}})(1 - A_s)^{\frac{1}{1+\alpha}}$ [40, 41, 70, 71]. The observed value A_{obs} with its 1σ error σ_A is $A_{\text{obs}} = 0.469 \times (n_s/0.98)^{-0.35} \pm 0.017$ measured from the SDSS at $z_{\text{BAO}} = 0.35$, where n_s is the scalar spectral index [72] and its value is taken to be 0.96 as shown in Ref. [43].

The structure of the anisotropies of the cosmic microwave background radiation depends on two eras in cosmology, i.e., the last scattering era and today. They can also be applied to limit the DE models by minimizing [73–75]

$$\chi^2_{\text{CMB}}(\theta) = \frac{(R(\theta) - R_{\text{obs}})^2}{\sigma_R^2}. \quad (21)$$
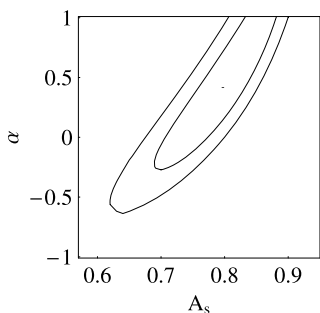
Here the shift parameter [76]

$$R(\theta) = \sqrt{\Omega_{0\text{m}}} \int_0^{z_{\text{rec}}} \frac{dz'}{E(z'; \theta)}, \quad (22)$$

$z_{\text{rec}} = 1089$ is the redshift of recombination. The observed value $R_{\text{obs}} = 1.710$, and its corresponding 1σ error is $\sigma_R = 0.019$ according to the five-year WMAP result [39].

Table 1 The observational $H(z)$ data [37, 59, 60]

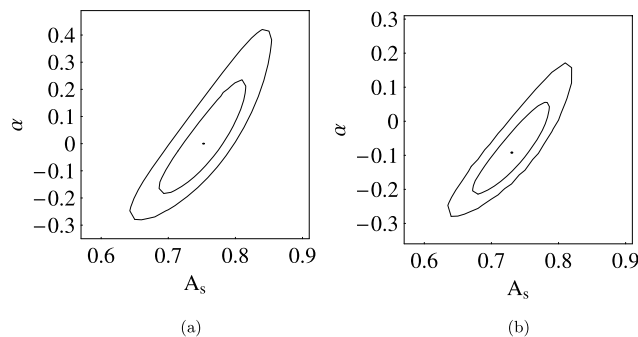| z	0.09	0.17	0.27	0.40	0.88	1.30	1.43	1.53	1.75
$H(z)$ (km s^{-1} Mpc^{-1})	69	83	70	87	117	168	177	140	202
1σ uncertainty	±12	±8.3	±14	±17.4	±23.4	±13.4	±14.2	±14	±40.4

Fig. 1 The 68.3% and 95.4% confidence level contours for A_s versus α from the Union SNe data plus the OHD

Fig. 2 The 68.3% and 95.4% confidence level contours for A_s versus α from the Union SNe data plus the BAO data (**a**), and a combined analysis of the Union SNe, OHD, BAO and CMB data (**b**)

The above four observational data are uncorrelated for each other, since they are given by different experiments and methods. Then the total likelihood χ^2_{total} can be constructed as

$$\chi^2_{\text{total}} = \chi^2_{\text{SNe}} + \chi^2_{\text{Hub}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}}. \tag{23}$$

Using (23) we get the best-fit values of the GCG model parameters (A_s, α); they are $(0.73, -0.09)$ with $\chi^2_{\text{min}} = 322.87$, and the reduced χ^2 value is[2] $\chi^2_{\text{min}}/\text{dof} = 1.03$. The 1σ and 2σ confidence level contours of GCG model parameters are plotted in Fig. 2(b). From this figure, we obtain the values of the model parameters with the confidence levels, $A_s = 0.73^{+0.06}_{-0.06}(1\sigma)^{+0.09}_{-0.09}(2\sigma)$ and $\alpha = -0.09^{+0.15}_{-0.12}(1\sigma)^{+0.26}_{-0.19}(2\sigma)$. It can be seen that parameters A_s and α are also degenerate, and at the 1σ confidence level these results are consistent with the standard dark energy plus dark matter scenario (i.e., the case of $\alpha = 0$). Furthermore, one can see that this constraint on the parameter α is more stringent than the results in Refs. [40, 41], where the constraint results for the GCG model parameters are $A_s = 0.70^{+0.16}_{-0.17}$ and $\alpha = -0.09^{+0.54}_{-0.33}$ at 2σ confidence level by using the X-ray gas mass fractions of galaxy clusters and the dimensionless coordinate distance of SNe Ia and FRIIb radio galaxies [40], and $A_s = 0.75^{+0.08}_{-0.08}$, $\alpha = 0.05^{+0.37}_{-0.26}$ at 2σ confidence level by means of the observational Hubble data, the 115 SNLS SNe Ia data and the SDSS baryonic acoustic oscillations peak [41].

Finally, we also consider the constraint on the GCG model parameter from a combination of the Union SNe Ia and BAO data; the best fit happens at $A_s = 0.75$ and $\alpha = 0$, which can be reduced to the standard dark energy plus dark matter scenario. But at their confidence levels, the two parameters are also highly degenerate. In Fig. 2(a), we display the constraint result for this analysis.

4 Constraint on EOS of dark energy and deceleration parameter

According to (5), the deceleration parameter q in the GCG model can be obtained by

$$q = (1+z)\frac{1}{H}\frac{dH}{dz} - 1. \tag{24}$$

The equation of state of the dark energy is derived as

$$w_{\text{de}} = \frac{p_{\text{de}}}{\rho_{\text{de}}}$$
$$= \frac{-(1-\Omega_{0b})A_s[A_s + (1-A_s)(1+z)^{3(1+\alpha)}]^{-\frac{\alpha}{1+\alpha}}}{(1-\Omega_{0b})[A_s + (1-A_s)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}} - \Omega_{0\text{dm}}(1+z)^3}, \tag{25}$$

where $\Omega_{0\text{dm}}$ is the present value of dimensionless dark matter density. Based on (24) and (25), the confidence levels of the best fit $w_{\text{de}}(z)$ and $q(z)$ calculated by using the covariance matrix are plotted in Fig. 3. From Fig. 3(a), it is easy to see that the best-fit value $w_{0\text{de}} \equiv w_{\text{de}}(z=0) = -0.96 > -1$, and the 1σ confidence level of $w_{0\text{de}}$ is $-0.91 \geq w_{0\text{de}} \geq -1.00$. In addition, it can be found that the best-fit evolution of $w_{\text{de}}(z)$ for GCG is similar to the quiessence model ($w_{\text{de}}(z) = \text{const} \neq -1$). From Fig. 3(b), we can see that the best-fit values of the transition redshift and current deceleration parameter with confidence levels are $z_T = 0.74^{+0.04}_{-0.05}$ (1σ), $q_0 = -0.55^{+0.05}_{-0.06}$ (1σ). One knows that z_T describes the expansion of universe from deceleration to acceleration, and q_0 indicates the expansion rhythm of the current universe. Comparing our results with Ref. [77], where $z_T = 0.49^{+0.14}_{-0.07}$ (1σ) and $q_0 = -0.73^{+0.21}_{-0.20}$ (1σ) are obtained from Union SNe Ia data by using a linear two-parameter expansion for the decelerating parameter, $q(z) = q_0 + q_1 z$, it is clear for our constraint that the universe tends to an earlier time to acceleration and a milder expansion rhythm at present.

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[2]The value of dof (degrees of freedom) for the model equals the number of observational data points minus the number of parameters.
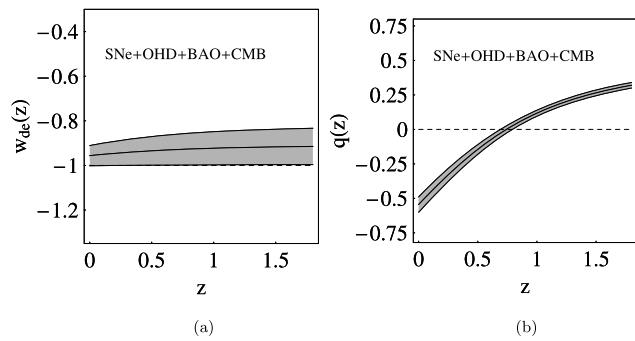
Fig. 3 The best fits of $w_{de}(z)$ and $q(z)$ with 1σ confidence level for GCG model

5 Conclusion

The constraints on the GCG model as the unification of dark matter and dark energy are studied in this paper by using the latest observational data: the Union SNe Ia data, the observational Hubble data, the SDSS baryon acoustic peak and the five-year WMAP shift parameter. We find that the model parameters A_s and α are degenerate, and their values are constrained to $A_s = 0.73^{+0.06}_{-0.06}$ (1σ) $^{+0.09}_{-0.09}$ (2σ) and $\alpha = -0.09^{+0.15}_{-0.12}$ (1σ) $^{+0.26}_{-0.19}$ (2σ). This constraint on the parameter α is more stringent than the results in Refs. [40, 41]. Furthermore, it is shown that the evolution of EOS of dark energy for the GCG model is similar to quiessence, and the best-fit value of current EOS of DE $w_{0de} = -0.96 > -1$. And it indicates that the values of transition redshift and current deceleration parameter are $z_T = 0.74^{+0.05}_{-0.05}$ (1σ), $q_0 = -0.55^{+0.06}_{-0.05}$ (1σ).

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