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Stability of a solution set for parametric generalized vector mixed quasivariational inequality problem

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Abstract

In this paper, we study a class of parametric generalized vector mixed quasivariational inequality problems (in short, (MQVIP)) in Hausdorff topological vector spaces. The upper semicontinuity, closedness, the outer-continuity and the outer-openness of the solution set are obtained. Moreover, a key assumption is introduced by virtue of a parametric gap function. Then, by using the key assumption, we establish that the condition $(H_h(\gamma_0, \mu_0))$ is a sufficient and necessary condition for the lower semicontinuity, the Hausdorff lower semicontinuity, the continuity and Hausdorff continuity of solutions for (MQVIP). The results presented in this paper are new and extend some main results in the literature.

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1 Introduction

Let X, Y be two Hausdorff topological vector spaces and let Λ, M be two topological vector spaces. Let $L(X,Y)$ be the space of all linear continuous operators from X to Y. Let $K : X \times \Lambda \to 2^X$, $T : X \times M \to 2^{L(X,Y)}$ be set-valued mappings and let $C : X \to 2^Y$ be a set-valued mapping such that $C(x)$ is a closed pointed convex cone with $\operatorname{int} C(x) \neq \emptyset$. Let $\Omega : X \times X \times \Lambda \to X$, $\Theta : X \times X \times \Lambda \to Y$ be two continuous vector-valued functions satisfying $\Omega(y,y,\gamma) = 0$ and $\Theta(y,y,\gamma) = 0$ for each $y \in X$, $\gamma \in \Lambda$. And let $Q : L(X,Y) \to L(X,Y)$, $\psi : X \to X$ be continuous single-valued mappings. Denoting by $\langle z, x\rangle$ the value of a linear operator $z \in L(X;Y)$ at $x \in X$, we always assume that $\langle \cdot, \cdot \rangle$ is continuous.

For $\gamma \in \Lambda$, $\mu \in M$, we consider the following parametric generalized vector mixed quasivariational inequality problem (in short, (MQVIP)).

(MQVIP) Find $\bar{x} \in K(\bar{x}, \gamma)$ and $\bar{z} \in T(\bar{x}, \mu)$ such that

$$\big\langle Q(\bar{z}), \Omega\big(y, \psi(\bar{x}), \gamma\big)\big\rangle + \Theta\big(y, \psi(\bar{x}), \gamma\big) \notin -\operatorname{int} C(\bar{x}), \quad \forall y \in K(\bar{x}, \gamma).$$

For each $\gamma \in \Lambda$, $\mu \in M$, we let $E(\gamma) := \{x \in X | x \in K(x, \gamma)\}$ and $\Psi : \Lambda \times M \to 2^X$ be a set-valued mapping such that $\Psi(\gamma, \mu)$ is the solution set of (MQVIP). Throughout this paper, we always assume that $\Psi(\gamma, \mu) \neq \emptyset$ for each (γ, μ) in the neighborhood $(\gamma_0, \mu_0) \in \Lambda \times M$.

Special cases of the problem (MQVIP) are as follows:

(a) If we let $K(x,\gamma) = K(x)$, $\Omega(y,\psi(x),\gamma) = \Omega(y,\psi(x))$, $\Theta(y,\psi(x),\gamma) = \Theta(\psi(x),y)$, then the problem (MQVIP) is reduced to the following generalized vector mixed general quasi-variational-like inequality problem:

Find $\bar{x} \in X$ such that $\bar{x} \in K(\bar{x})$ and for each $y \in K(\bar{x})$, there exists $\bar{z} \in T(\bar{x})$ satisfying

$$\bigl\langle Q(\bar{z}), \Omega\bigl(y,\psi(\bar{x})\bigr)\bigr\rangle + \Theta\bigl(\psi(\bar{x}),y\bigr) \notin -\operatorname{int} C(\bar{x}).$$

This problem was studied in [1].

(b) If Q, ψ are identity mappings and $\Omega(y,\psi(x),\gamma) = \Omega(y,x)$, $\Theta(y,\psi(x),\gamma) = \Theta(y,x)$, $K(x,\gamma) = K(\gamma)$, then the problem (MQVIP) is reduced to the following parametric generalized vector quasi-variational-like inequality problem (in short, (PGVQVLIP)):

(PGVQVLIP) Find $\bar{x} \in K(\gamma)$ and $\bar{z} \in T(\bar{x},\mu)$ such that

$$\bigl\langle \bar{z}, \eta(y,\bar{x})\bigr\rangle + \psi(y,\bar{x}) \notin -\operatorname{int} C(\bar{x}), \quad \forall y \in K(\gamma).$$

This problem was studied in [2].

(c) If Q, ψ are identity mappings and $K(x,\gamma) = X$, $T(\bar{x},\mu) = T(\bar{x})$, $\Omega(y,\psi(x),\gamma) = \Omega(y,x)$, $\Theta(y,\psi(x),\gamma) = \Theta(y,x)$ and $C(x) = C$ with $C \subseteq Y$ is a pointed, closed and convex cone in Y with $\operatorname{int} C \neq \emptyset$, then the problem (MQVIP) is reduced to the following generalized vector variational inequality problem:

Find $\bar{x} \in X$ and $\bar{z} \in T(\bar{x})$ such that

$$\bigl\langle \bar{z}, \eta(y,\bar{x})\bigr\rangle + \psi(y,\bar{x}) \notin -\operatorname{int} C, \quad \forall y \in K(\gamma).$$

This problem was studied in [3].

(d) If Q, ψ are identity mappings and $\Omega(y,\eta(x),\gamma) = y - x$, $\Theta(y,\psi(x),\gamma) = 0$, $\Lambda = M$, then the problem (MQVIP) is reduced to the following generalized vector quasivariational inequality problem (in short, (PGVQVI)):

(PGVQVI) Find $\bar{x} \in K(\bar{x},\gamma)$ and $\bar{z} \in T(\bar{x},\gamma)$ such that

$$\langle \bar{z}, y - \bar{x}\rangle \in Y \setminus -\operatorname{int} C(\bar{x}), \quad \forall y \in K(\bar{x},\gamma).$$

This problem was studied in [4].

(e) If Q, ψ are identity mappings and $\Omega(y,\eta(x),\gamma) = y - x$, $\Theta(y,\psi(x),\gamma) = 0$, $K(x,\gamma) = K(\gamma)$, $\Lambda = M$ and $C(x) = C$ with $C \subseteq Y$ is a pointed closed and convex cone in Y with $\operatorname{int} C \neq \emptyset$, then the problem (MQVIP) is reduced to the following parametric set-valued weak vector variational inequality (in short, (PSWVVI)):

(PSWVVI) Find $\bar{x} \in K(\gamma)$ and $\bar{z} \in T(\bar{x},\gamma)$ such that

$$\langle \bar{z}, y - \bar{x}\rangle \notin -\operatorname{int} C, \quad \forall y \in K(\gamma).$$

This problem was studied in [5].

(f) If Q, ψ are identity mappings and $\Omega(y,\eta(x),\gamma) = 0$, $\Theta(y,\psi(x),\gamma) = \Theta(x,y,z)$, $\Lambda = M$, then the problem (MQVIP) is reduced to the following parametric generalized vector quasiequilibrium problem (in short, (PGVQEP)):

(PGVQEP) Find $\bar{x} \in K(\bar{x}, \gamma)$ and $\bar{z} \in T(\bar{x}, \gamma)$ such that

$$\Theta(x, y, z) \in Y \setminus -\operatorname{int} C(\bar{x}), \quad \forall y \in K(\bar{x}, \gamma).$$

This problem was studied in [6].

(g) If Q, ψ are identity mappings and $Y = \mathbb{R}^n$, $C(x) = \mathbb{R}^n_+$, $\Lambda = M$, $K(x, \gamma) = K(\gamma)$, $\Omega(y, \eta(x), \gamma) = y - x$, $\Theta(y, \psi(x), \gamma) = 0$ and $T : X \times \Lambda \to L(X, \mathbb{R}^n)$, then the problem (MQVIP) is reduced to the parametric weak vector variational inequality problem (in short, (PWVVI)):
(PWVVI) Find $\bar{x} \in K(\gamma)$ such that

$$\langle T(\bar{x}, \gamma), y - x \rangle \notin -\operatorname{int} \mathbb{R}^n_+, \quad \forall y \in K(\gamma).$$

This problem was studied in [7].

Stability of solutions for the parametric generalized vector mixed quasivariational inequality problem is an important topic in optimization theory and applications. Recently, the continuity, especially the upper semicontinuity, the lower semicontinuity and the Hausdorff lower semicontinuity of the solution sets for parametric optimization problems, parametric vector variational inequality problems and parametric vector quasiequilibrium problems have been studied in the literature; see [2, 4–17] and the references therein.

The structure of our paper is as follows. In the first part of this article, we introduce the model parametric generalized vector mixed quasivariational inequality problems. In Section 2, we recall definitions for later uses. In Section 3, we establish the upper semicontinuity, closedness, the outer-continuity and the outer-openness, and in Section 4, we establish that the condition $(H_h(\gamma_0, \mu_0))$ is a sufficient and necessary condition for the lower semicontinuity, the Hausdorff lower semicontinuity, the continuity and Hausdorff continuity of the solution set for the parametric generalized vector mixed quasivariational inequality problem in Hausdorff topological vector spaces.

2 Preliminaries

In this section, we recall some basic definitions and some of their properties.

First, we recall two limits in [18, 19]. Let X and Y be two topological vector spaces and $G : X \to 2^Y$ be a multifunction. The superior limit and the superior open limit of G are defined as

$$\limsup_{x \to x_0} G(x) := \{ y \in Y \mid \exists x_\nu \to x_0, \exists y_\nu \in G(x_\nu) : y_\nu \to y, \forall \nu \},$$

$$\operatorname*{limsupo}_{x \to x_0} G(x) := \{ y \in Y \mid \text{ there are an open neighborhood } U \text{ of } y \text{ and a net } \{x_\nu\} \subseteq X, x_\nu \neq x_0 \text{ converging to } x_0 \text{ such that } U \subseteq G(x_\nu), \forall \nu \}.$$

Definition 2.1 ([18, 20, 21]) Let X and Y be topological vector spaces and $G : X \to 2^Y$ be a multifunction.

(i) G is said to be *outer-continuous at* $x_0 \in X$ if $\limsup_{x \to x_0} G(x) \subseteq G(x_0)$. G is said to be outer-continuous in X if it is outer-continuous at each $x_0 \in X$.

(ii) G is said to be *outer-open at* $x_0 \in X$ if $\operatorname{limsupo}_{x \to x_0} G(x) \subseteq G(x_0)$. G is said to be outer-open in X if it is outer-open at each $x_0 \in X$.

(iii) G is said to be *lower semicontinuous (lsc) at* $x_0 \in X$ if $G(x_0) \cap U \neq \emptyset$ for some open set $U \subseteq Y$ implies the existence of a neighborhood N of x_0 such that $G(x) \cap U \neq \emptyset$, $\forall x \in N$. G is said to be lower semicontinuous in X if it is lower semicontinuous at each $x_0 \in X$.

(iv) G is said to be *upper semicontinuous (usc) at* $x_0 \in X$ if for each open set $U \supseteq G(x_0)$, there is a neighborhood N of x_0 such that $U \supseteq G(x)$, $\forall x \in N$. G is said to be upper semicontinuous in X if it is upper semicontinuous at each $x_0 \in X$.

(v) G is said to be *Hausdorff upper semicontinuous (H-usc) at* $x_0 \in X$ if for each neighborhood B of the origin in Z, there exists a neighborhood N of x_0 such that $G(x) \subseteq G(x_0) + B$, $\forall x \in N$. G is said to be Hausdorff upper semicontinuous in X if it is Hausdorff upper semicontinuous at each $x_0 \in X$.

(vi) G is said to be *Hausdorff lower semicontinuous (H-lsc) at* $x_0 \in X$ if for each neighborhood B of the origin in Y, there exists a neighborhood N of x_0 such that $G(x_0) \subseteq G(x) + B$, $\forall x \in N$. G is said to be Hausdorff lower semicontinuous in X if it is Hausdorff lower semicontinuous at each $x_0 \in X$.

(vii) G is said to be *continuous at* $x_0 \in X$ if it is both lsc and usc at x_0 and to be *H-continuous at* $x_0 \in X$ if it is both H-lsc and H-usc at x_0. G is said to be continuous in X if it is both lsc and usc at each $x_0 \in X$ and to be H-continuous in X if it is both H-lsc and H-usc at each $x_0 \in X$.

(viii) G is said to be *closed at* $x_0 \in X$ if and only if $\forall x_n \to x_0$, $\forall y_n \to y_0$ such that $y_n \in G(x_n)$, we have $y_0 \in G(x_0)$. G is said to be closed in X if it is closed at each $x_0 \in X$.

Lemma 2.2 ([20, 21]) *Let X and Y be topological vector spaces and $G : X \to 2^Y$ be a multifunction.*

(i) *If G is usc at x_0, then G is H-usc at x_0. Conversely if G is H-usc at x_0 and if $G(x_0)$ is compact, then G is usc at x_0*;

(ii) *If G is H-lsc at x_0 then G is lsc at x_0. The converse is true if $G(x_0)$ is compact*;

(iii) *If Y is compact and G is closed at x_0, then G is usc at x_0*;

(iv) *If G is usc at x_0 and $G(x_0)$ is closed, then G is closed at x_0*;

(v) *If G has compact values, then G is usc at x_0 if and only if, for each net $\{x_\alpha\} \subseteq X$ which converges to x_0 and for each net $\{y_\alpha\} \subseteq G(x_\alpha)$, there are $y \in G(x_0)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y$.*

Lemma 2.3 ([22, 23]) *Let $e : X \to Y$ be a vector-valued mapping and for any $x \in X$, $e \in \operatorname{int} C(x)$. The nonlinear scalarization function $\xi_e : X \times Y \to \mathbb{R}$ defined by $\xi_e(x,y) := \inf\{r \in \mathbb{R} : y \in re(x) - C(x)\}$ has the following properties*:

(i) $\xi_e(x,y) < r \Leftrightarrow y \in re - \operatorname{int} C(x)$;

(ii) $\xi_e(x,y) \geq r \Leftrightarrow y \notin re - \operatorname{int} C(x)$.

Lemma 2.4 (See [4, 23]) *Let X and Y be two locally convex Hausdorff topological vector spaces, and let $C : X \to 2^Y$ be a set-valued mapping such that, for each $x \in X, C(x)$ is a proper, closed, convex cone in Y with $\operatorname{int} C(x) \neq \emptyset$. Furthermore, let $e : X \to Y$ be the continuous selection of the set-valued map $\operatorname{int} C(\cdot)$. Define a set-valued mapping $V : X \to 2^Y$ by $V(x) = Y \setminus \operatorname{int} C(x)$ for $x \in X$. We have*

(i) *If $V(\cdot)$ is usc in X, then $\xi_e(\cdot,\cdot)$ is upper semicontinuous in $X \times Y$*;

(ii) *If $C(\cdot)$ is usc in X, then $\xi_e(\cdot,\cdot)$ is lower semicontinuous in $X \times Y$.*

From Lemma 2.4, we know that if $V(\cdot)$ and $C(\cdot)$ are both usc in X, then $\xi_e(\cdot,\cdot)$ is continuous in $X \times Y$.

Now we suppose that $K(x,\gamma)$ and $T(x,\mu)$ are compact sets for any $(x,\gamma) \in X \times \Lambda$ and $(x,\mu) \in X \times M$. We define a function $h : X \times \Lambda \times M \to \mathbb{R}$ as follows:

$$h(x,\gamma,\mu) = \min_{z\in T(x,\mu)} \max_{y\in K(x,\gamma)} \bigl\{-\xi_e\bigl(x,\bigl\langle Q(z),\Omega\bigl(y,\psi(x),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x),\gamma\bigr)\bigr)\bigr\}.$$

Since $K(x,\gamma)$ and $T(x,\mu)$ are compact sets, $h(x,\gamma,\mu)$ is well defined.

Lemma 2.5

(i) $h(x,\gamma,\mu) \geq 0$ *for all* $x \in E(\gamma)$;

(ii) $h(x_0,\gamma_0,\mu_0) = 0$ *if and only if* $x_0 \in \Psi(\gamma_0,\mu_0)$.

Proof We define a function $h_1 : X \times L(X,Y) \to \mathbb{R}$ as follows:

$$h_1(x,z) = \max_{y\in K(x,\gamma)} \bigl\{-\xi_e\bigl(x,\bigl\langle Q(z),\Omega\bigl(y,\psi(x),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x),\gamma\bigr)\bigr)\bigr\}.$$

(i) It is easy to see that $h_1(x,z) \geq 0$. Suppose to the contrary that there exists $x_0 \in E(\gamma)$ and $z_0 \in T(x_0,\mu)$ such that $h_1(x_0,z_0) < 0$, then

$$\begin{aligned} 0 > h_1(x_0,z_0) &= \max_{y\in K(x_0,\gamma)} \bigl\{-\xi_e\bigl(x_0,\bigl\langle Q(z_0),\Omega\bigl(y,\psi(x_0),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x_0),\gamma\bigr)\bigr)\bigr\} \\ &\geq -\xi_e\bigl(x_0,\bigl\langle Q(z_0),\Omega\bigl(y,\psi(x_0),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x_0),\gamma\bigr)\bigr). \end{aligned}$$

When $\psi(x_0) = y$, we have

$$\begin{aligned} &\xi_e\bigl(x_0,\bigl\langle Q(z_0),\Omega(y,y,\gamma)\bigr\rangle + \Theta(y,y,\gamma)\bigr) \\ &\quad = \xi_e(x_0,0) \\ &\quad = \inf\bigl\{r \in \mathbb{R} : 0 \in re(x_0) - C(x_0)\bigr\} \\ &\quad = \inf\bigl\{r \in \mathbb{R} : -re(x_0) \in -C(x_0)\bigr\} \\ &\quad = \inf\{r \in \mathbb{R} : r \geq 0\} = 0, \end{aligned}$$

which is a contradiction. Hence,

$$\begin{aligned} h_1(x,z) = \max_{y\in K(x,\gamma)} &\bigl\{-\xi_e\bigl(x,\bigl\langle Q(z),\Omega\bigl(y,\psi(x),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x),\gamma\bigr)\bigr)\bigr\} \geq 0, \\ &x \in E(\gamma), z \in T(x,\gamma). \end{aligned}$$

Thus, since $z \in T(x,\mu)$ is arbitrary, we have

$$h(x,\gamma,\mu) = \min_{z\in T(x,\mu)} \max_{y\in K(x,\gamma)} \bigl\{-\xi_e\bigl(x,\bigl\langle Q(z),\Omega\bigl(y,\psi(x),\gamma\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x),\gamma\bigr)\bigr)\bigr\} \geq 0.$$

(ii) By definition, $h(x_0,\gamma_0,\mu_0) = 0$ if and only if there exists $z_0 \in T(x_0,\mu_0)$ such that $h_1(x_0,z_0) = 0$, *i.e.*,

$$\max_{y\in K(x_0,\gamma_0)} \bigl\{-\xi_e\bigl(x_0,\bigl\langle Q(z_0),\Omega\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr\rangle + \Theta\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr)\bigr\} = 0, \quad x_0 \in E(\gamma_0)$$

if and only if, for any $y \in K(x_0, \gamma_0)$,

$$-\xi_e\big(x_0, \big\langle Q(z_0), \Omega\big(y, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y, \psi(x_0), \gamma_0\big)\big) \le 0,$$

or

$$\xi_e\big(x_0, \big\langle Q(z_0), \Omega\big(y, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y, \psi(x_0), \gamma_0\big)\big) \ge 0.$$

By Lemma 2.3(ii), if and only if

$$\big\langle Q(z_0), \Omega\big(y, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y, \psi(x_0), \gamma_0\big)\big) \notin -\operatorname{int} C(x_0), \quad \forall y \in K(x_0, \gamma_0),$$

that is, $x_0 \in \Psi(\gamma_0, \mu_0)$. □

We may call the function $h(\cdot,\cdot,\cdot)$ a parametric gap function for (MQVIP) if the properties of Lemma 2.5 are satisfied. Many authors have studied the gap functions for vector equilibrium problems and vector variational inequalities; see [3, 24–27] and the references therein.

Example 2.6 Let ψ, Q be identity mappings and $X = Y = \mathbb{R}$, $\Lambda = M = [0,1]$, $C(x) = \mathbb{R}_+$, $K(x,\gamma) = [0,1]$, $T(x,\gamma) = \{2^{\gamma^2+2}\}$, $\Omega(y,\psi(x),\gamma) = \Theta(y,\psi(x),\gamma) = y - x$. Now we consider the problem (MQVIP) of finding $x \in K(x,\gamma)$ and $z \in T(x,\gamma)$ such that

$$\big\langle Q(z), \Omega\big(y, \psi(x), \gamma\big)\big\rangle + \Theta\big(y, \psi(x), \gamma\big) = 2^{\gamma^2+2}(y - x) + y - x \notin -\operatorname{int}\mathbb{R}_+.$$

It follows from a direct computation $\Psi(\gamma,\mu) = \{0\}$ for all $\gamma \in [0,1]$. Now we show that $h(\cdot,\cdot,\cdot)$ is a parametric gap function of (MQVIP). Indeed, taking $e = 1 \in \operatorname{int}\mathbb{R}_+$, we have

$$\begin{aligned} h(x,\gamma,\mu) &= \min_{z\in T(x,\mu)} \max_{y\in K(x,\gamma)} \big\{-\xi_e\big(x\big\langle Q(z), \Omega\big(y, \psi(x), \gamma\big)\big\rangle + \Theta\big(y, \psi(x), \gamma\big)\big)\big\} \\ &= \max_{y\in K(x,\gamma)} \big(\big(1 + 2^{\gamma^2+2}\big)(x - y)\big) \\ &= \begin{cases} 0 & \text{if } x = 0, \\ (1 + 2^{\gamma^2+2})x & \text{if } x \in (0,1]. \end{cases} \end{aligned}$$

Hence, $h(\cdot,\cdot,\cdot)$ is a parametric gap function of (MQVIP).

The following lemma gives a sufficient condition for the parametric gap function $h(\cdot,\cdot,\cdot)$ is continuous in $X \times \Lambda \times M$.

Lemma 2.7 *Consider* (MQVIP). *If the following conditions hold*:

(i) $K(\cdot,\cdot)$ *is continuous with compact values in* Λ;

(ii) $T(\cdot,\cdot)$ *is continuous with compact values in* $X \times \Lambda$;

(iii) $C(\cdot)$ *is upper semicontinuous in* X *and* $e(\cdot) \in \operatorname{int} C(\cdot)$ *is continuous in* X.

Then $h(\cdot,\cdot,\cdot)$ *is continuous in* $X \times \Lambda \times M$.

Proof First we prove that $h(\cdot,\cdot,\cdot)$ is lower semicontinuous in $X \times \Lambda \times M$. Indeed, we let $r \in \mathbb{R}$. Suppose that $\{(x_\alpha, \gamma_\alpha, \mu_\alpha)\} \subseteq X \times \Lambda \times M$ satisfies

$$h(x_\alpha, \gamma_\alpha, \mu_\alpha) \leq r, \quad \forall \alpha$$

and

$$(x_\alpha, \gamma_\alpha, \mu_\alpha) \to (x_0, \gamma_0, \mu_0) \quad \text{as } \alpha \to \infty.$$

It follows that

$$\begin{aligned} &h(x_\alpha, \gamma_\alpha, \mu_\alpha) \\ &\quad = \min_{z \in T(x_\alpha,\mu_\alpha)} \max_{y \in K(x_\alpha,\gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \leq r. \end{aligned}$$

We define the function $g : X \times L(X,Y) \times \Lambda \times M \to \mathbb{R}$ by

$$g(x,z,\gamma,\mu) = \max_{y \in K(x,\gamma)} \bigl\{-\xi_e\bigl(x, \bigl\langle Q(z), \Omega\bigl(y, \psi(x), \gamma\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x), \gamma\bigr)\bigr)\bigr\}, \quad x \in E(\gamma).$$

By the continuity of $\psi(\cdot)$, $\Omega(\cdot,\cdot,\cdot)$, $\Theta(\cdot,\cdot,\cdot)$, $\xi_e(\cdot,\cdot)$ and since $K(\cdot,\cdot)$ is continuous with compact values in $X \times \Lambda$, thus, by Proposition 23 in Section 1 of Chapter 3 [20], we can deduce that $g(x,z,\gamma,\mu)$ is continuous with respect to (x,z,γ,μ). By the compactness of $T(x_\alpha, \mu_\alpha)$, there exists $z_\alpha \in T(x_\alpha, \mu_\alpha)$ such that

$$\begin{aligned} h(x_\alpha, \gamma_\alpha, \mu_\alpha) &= \min_{z \in T(x_\alpha,\mu_\alpha)} \max_{y \in K(x_\alpha,\gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \\ &= g(x_\alpha, z_\alpha, \gamma_\alpha, \mu_\alpha) \\ &= \max_{y \in K(x_\alpha,\gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z_\alpha), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \leq r. \end{aligned}$$

Since $K(\cdot,\cdot)$ is lower semicontinuous in $X \times \Lambda$, for any $y_0 \in K(x_0, \gamma_0)$, there exists $y_\alpha \in K(x_\alpha, \gamma_\alpha)$ such that $y_\alpha \to y_0$. For $y_\alpha \in K(x_\alpha, \gamma_\alpha)$, we have

$$-\xi_e\bigl(x_\alpha, \bigl\langle Q(z_\alpha), \Omega\bigl(y_\alpha, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y_\alpha, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr) \leq r. \tag{2.1}$$

Since $T(\cdot,\cdot)$ is upper semicontinuous with compact values in $X \times M$, there exists $z_0 \in T(x_0, \mu_0)$ such that $z_\alpha \to z_0$ (taking a subnet $\{z_\beta\}$ of $\{z_\alpha\}$ if necessary) as $\alpha \to \infty$. From the continuity of $\xi_e(\cdot, (Q(\cdot, \psi(\cdot), \cdot) + \Theta(\cdot, \psi(\cdot), \cdot)))$, taking the limit in (2.1), we have

$$-\xi_e\bigl(x_0, \bigl\langle Q(z_0), \Omega\bigl(y_0, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y_0, \psi(x_0), \gamma_0\bigr)\bigr) \leq r. \tag{2.2}$$

Since $y_0 \in K(x_0, \gamma_0)$ is arbitrary, it follows from (2.2) that

$$g(x_0, z_0, \gamma_0, \mu_0) = \max_{y \in K(x_0,\gamma_0)} \bigl\{-\xi_e\bigl(x_0, \bigl\langle Q(z_0), \Omega\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr)\bigr\} \leq r.$$

And so, for any $z_0 \in T(x_0, \mu_0)$, we have

$$h(x_0, \gamma_0, \mu_0) = \min_{z \in T(x_0,\mu_0)} \max_{y \in K(x_0,\gamma_0)} \bigl\{-\xi_e\bigl(x_0, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr)\bigr\} \leq r.$$

This proves that, for $r \in \mathbb{R}$, the level set $\{(x, \gamma, \mu) | h(x, \gamma, \mu) \leq r\}$ is closed. Hence, $h(\cdot, \cdot, \cdot)$ is lower semicontinuous in $X \times X \times \Lambda$.

Next, we need to prove that $h(\cdot, \cdot, \cdot)$ is upper semicontinuous in $X \times \Lambda \times M$. Indeed, let $r \in \mathbb{R}$. Suppose that $\{(x_\alpha, \gamma_\alpha, \mu_\alpha)\} \subseteq X \times \Lambda \times M$ satisfies

$$h(x_\alpha, \gamma_\alpha, \mu_\alpha) \geq r, \quad \forall \alpha$$

and

$$(x_\alpha, \gamma_\alpha, \mu_\alpha) \to (x_0, \gamma_0, \mu_0) \quad \text{as } \alpha \to \infty,$$

then

$$\min_{z \in T(x_\alpha, \mu_\alpha)} \max_{y \in K(x_\alpha, \gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \geq r,$$

and so, for any $z \in T(x_\alpha, \mu_\alpha)$, we have

$$\max_{y \in K(x_\alpha, \gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \geq r. \tag{2.3}$$

Since $T(\cdot, \cdot)$ is lower semicontinuous in $X \times M$, for any $z_0 \in T(x_0, \mu_0)$, there exists $z_\alpha \in T(x_\alpha, \mu_\alpha)$ such that $z_\alpha \to z_0$ as $\alpha \to \infty$. Since $z_\alpha \in T(x_\alpha, \mu_\alpha)$, it follows (2.3) that

$$\max_{y \in K(x_\alpha, \gamma_\alpha)} \bigl\{-\xi_e\bigl(x_\alpha, \bigl\langle Q(z_\alpha), \Omega\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr)\bigr\} \geq r. \tag{2.4}$$

By the compactness of $K(\cdot, \cdot)$, there exists $y_\alpha \in K(x_\alpha, \gamma_\alpha)$ such that

$$-\xi_e\bigl(x_\alpha, \bigl\langle Q(z_\alpha), \Omega\bigl(y_\alpha, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr\rangle + \Theta\bigl(y_\alpha, \psi(x_\alpha), \gamma_\alpha\bigr)\bigr) \geq r. \tag{2.5}$$

Since $K(\cdot, \cdot)$ is upper semicontinuous with compact values, there exists $y_0 \in K(x_0, \gamma_0)$ such that $y_\alpha \to y_0$ (taking a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ if necessary) as $\alpha \to \infty$. From the continuity of $\xi_e(\cdot, (Q(\cdot, \psi(\cdot), \cdot) + \Theta(\cdot, \psi(\cdot), \cdot)))$, taking the limit in (2.5), we have

$$-\xi_e\bigl(x_0, \bigl\langle Q(z_0), \Omega\bigl(y_0, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y_0, \psi(x_0), \gamma_0\bigr)\bigr) \geq r.$$

For any $y \in K(x_0, \gamma_0)$, we have

$$\max_{y \in K(x_0, \gamma_0)} \bigl\{-\xi_e\bigl(x_0, \bigl\langle Q(z_0), \Omega\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr)\bigr\} \geq r. \tag{2.6}$$

Since $z \in T(x_0, \gamma_0)$ is arbitrary, it follows from (2.6) that

$$h(x_0, \gamma_0, \mu_0) = \min_{z \in T(x_0, \mu_0)} \max_{y \in K(x_0, \gamma_0)} \bigl\{-\xi_e\bigl(x_0, \bigl\langle Q(z), \Omega\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr)\bigr\} \geq r.$$

This proves that, for $r \in \mathbb{R}$, the level set $\{(x, \gamma, \mu) | h(x, \gamma, \mu) \geq r\}$ is closed. Hence, $h(\cdot, \cdot, \cdot)$ is upper semicontinuous in $X \times \Lambda \times M$. □

Remark 2.8 In special cases as those in Section 1 (d), (e) and (f),

(i) Lemma 2.5 extends Proposition 4.1 of Chen *et al.* in [4] and Lemma 2.6 of Zhong-Huang in [6].

(ii) Lemma 2.7 extends Lemma 4.2 of Chen *et al.* in [4], Lemma 2.7 of Zhong-Huang in [5] and Lemma 2.8 of Zhong-Huang in [6].

3 Upper semicontinuity of a solution set

In this section, we establish the upper semicontinuity, closedness, outer-continuity and outer-openess of the solution set for the parametric generalized vector mixed quasivariational inequality problem (MQVIP).

Theorem 3.1 *Assume for the problem* (MQVIP) *that*

(i) $E(\cdot)$ *is upper semicontinuous with compact values in* Λ *and* $K(\cdot,\cdot)$ *is lower semicontinuous in* $X \times \Lambda$;

(ii) $T(\cdot,\cdot)$ *is upper semicontinuous with compact values in* $X \times M$;

(iii) $W(\cdot) = Y \setminus -\operatorname{int} C(\cdot)$ *is closed in* X.

Then $\Psi(\cdot,\cdot)$ *is upper semicontinuous in* $\Lambda \times M$. *Moreover,* $\Psi(\gamma_0,\mu_0)$ *is a compact set and* $\Psi(\cdot,\cdot)$ *is closed in* $\Lambda \times M$.

Proof First we prove that $\Psi(\cdot,\cdot)$ is upper semicontinuous in $\Lambda \times M$. Indeed, we suppose that $\Psi(\cdot,\cdot)$ is not upper semicontinuous at (γ_0,μ_0), *i.e.*, there is an open subset V of $\Psi(\gamma_0,\mu_0)$ such that for all nets $\{(\gamma_\alpha,\mu_\alpha)\}$ convergent to (γ_0,μ_0), there is $x_\alpha \in \Psi(\gamma_\alpha,\mu_\alpha)$, $x_\alpha \notin V$, $\forall\alpha$. By the upper semicontinuity of $E(\cdot)$ in Λ and the compactness of $E(\gamma)$, one can assume that $x_\alpha \to x_0 \in E(\gamma_0)$ (taking a subnet if necessary). Now we show that $x_0 \in \Psi(\gamma_0,\mu_0)$. If $x_0 \notin \Psi(\gamma_0,\mu_0)$, then $\forall z_0 \in T(x_0,\mu_0)$, $\exists y_0 \in K(x_0,\gamma_0)$ such that

$$\big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big) \in -\operatorname{int} C(x_0). \tag{3.1}$$

By the lower semicontinuity of $K(\cdot,\cdot)$ at (x_0,γ_0), there exists $y_\alpha \in K(x_\alpha,\gamma_\alpha)$ such that $y_\alpha \to y_0$. Since $x_\alpha \in \Psi(\gamma_\alpha,\mu_\alpha)$, there exists $z_\alpha \in T(x_\alpha,\mu_\alpha)$ such that

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big) \notin -\operatorname{int} C(x_\alpha). \tag{3.2}$$

Since $T(\cdot,\cdot)$ is upper semicontinuous and with compact values in $X \times M$, one has $z_0 \in T(x_0,\mu_0)$ such that $z_\alpha \to z_0$ (can take a subnet if necessary) and since $Q(\cdot)$, $\Omega(\cdot,\psi(\cdot),\cdot)$ are continuous, we have

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle \to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle.$$

It follows from the continuity of $\Theta(\cdot,\psi(\cdot),\cdot)$ that

$$\begin{aligned}&\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\\&\quad \to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big).\end{aligned}$$

By the condition (iii), we have

$$\big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big) \notin -\operatorname{int} C(x_0). \tag{3.3}$$

We see a contradiction between (3.1) and (3.3), and so we have $x_0 \in \Psi(\gamma_0, \mu_0) \subseteq V$, which contradicts the fact $x_\alpha \notin V$, $\forall\alpha$. Hence, $\Psi(\cdot,\cdot)$ is upper semicontinuous in $\Lambda \times M$.

Now we prove that $\Psi(\gamma_0, \mu_0)$ is compact. We first show that $\Psi(\gamma_0, \mu_0)$ is a closed set. Indeed, we supposed that $\Psi(\gamma_0, \mu_0)$ is not a closed set, then there exists a net $\{x_\alpha\} \in \Psi(\gamma_0, \mu_0)$ such that $x_\alpha \to x_0$, but $x_0 \notin \Psi(\gamma_0, \mu_0)$. The further argument is the same as above. And so we have $\Psi(\gamma_0, \mu_0)$ is a closed set. Moreover, as $\Psi(\gamma_0, \mu_0) \subseteq E(\gamma_0)$ and $E(\gamma_0)$ is compact, it follows that $\Psi(\gamma_0, \mu_0)$ is compact. Hence, by Lemma 2.2(iv), it follows that $\Psi(\cdot,\cdot)$ is closed in $\Lambda \times M$. □

Remark 3.2 In the special case as that in Section 1 (d), Theorem 3.1 extends Theorem 3.1 of Chen *et al.* in [4].

Theorem 3.3 *Assume for the problem* (MQVIP) *that*

(i) $E(\cdot)$ *is outer-continuous in* Λ *and* $K(\cdot,\cdot)$ *is lower semicontinuous in* $X \times \Lambda$;

(ii) $T(\cdot,\cdot)$ *is upper semicontinuous with compact values in* $X \times M$;

(iii) $W(\cdot) = Y \setminus -\operatorname{int} C(\cdot)$ *is closed in* X.

Then $\Psi(\cdot,\cdot)$ *is outer-continuous in* $\Lambda \times M$.

Proof Let $x_0 \in \limsup_{\gamma\to\gamma_0,\mu\to\mu_0} \Psi(\gamma, \mu)$. There are nets $\{(\gamma_\alpha, \mu_\alpha)\}$ converging to (γ_0, μ_0) and $\{x_\alpha\}$ converging to x_0 with $x_\alpha \in \Psi(\gamma_\alpha, \mu_\alpha)$. By the outer continuity of $E(\cdot)$, we have $x \in E(\gamma_0)$. Now we show that $x_0 \in \Psi(\gamma_0, \mu_0)$. Indeed, by the lower-semicontinuity of $K(\cdot,\cdot)$ in $X \times \Lambda$, for any $y_0 \in K(x_0, \gamma_0)$, there exists $y_\alpha \in K(x_\alpha, \gamma_\alpha)$ such that $y_\alpha \to y_0$. As $x_\alpha \in \Psi(\gamma_\alpha, \mu_\alpha)$, there exists $z_\alpha \in T(x_\alpha, \mu_\alpha)$ such that

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big) \notin -\operatorname{int} C(x_\alpha). \tag{3.4}$$

Since $T(\cdot,\cdot)$ is upper semicontinuous with compact-values in $X \times M$, there exists $z_0 \in T(x_0, \mu_0)$ such that $z_\alpha \to z_0$ (can take a subnet if necessary). Since $Q(\cdot)$, $\Omega(\cdot, \psi(\cdot), \cdot)$ are continuous, we have

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle \to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle.$$

It follows from the continuity of $\Theta(\cdot, \psi(\cdot), \cdot)$ that

$$\begin{aligned}&\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_\alpha), \gamma_\alpha\big)\\ &\quad \to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big).\end{aligned}$$

By the condition (iii) and (3.4), we have

$$\big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big) \notin -\operatorname{int} C(x_0).$$

Hence, $x_0 \in \Psi(\gamma_0, \mu_0)$. Thus, $\Psi(\cdot,\cdot)$ is outer-continuous in $\Lambda \times M$. □

Theorem 3.4 *Assume for the problem* (MQVIP) *that*

(i) $E(\cdot)$ *is outer-open in* Λ *and* $K(x_0,\cdot)$ *is lower semicontinuous in* Λ *for all* $x_0 \in E(\gamma_0)$;

(ii) *for all* $x_0 \in E(\gamma_0)$, $T(x_0,\cdot)$ *is upper semicontinuous with compact values in* M;

(iii) *for all* $x_0 \in E(\gamma_0)$, $W(x_0) = Y \setminus -\operatorname{int} C(x_0)$ *is closed*.
Then $\Psi(\cdot,\cdot)$ *is outer-open in* $\Lambda \times M$.

Proof Let $x_0 \in \operatorname{limsupo}_{\gamma\to\gamma_0,\mu\to\mu_0} \Psi(\gamma,\mu)$. There are a neighborhood V of x_0 and nets $\{\gamma_\alpha\} \subseteq \Lambda$, $\gamma_\alpha \neq \gamma_0$ converging to γ_0 and $\{\mu_\alpha\} \subseteq M$, $\mu_\alpha \neq \mu_0$ converging to μ_0 such that $V \subset \Psi(\gamma_\alpha,\mu_\alpha)$, $\forall\alpha$. By $V \subset E(\gamma_\alpha)$, we have $x \in \operatorname{limsupo}_{\gamma\to\gamma_0} E(\gamma)$. It follows from (i) that $x_0 \in E(\gamma_0)$. Now we show that $x_0 \in \Psi(\gamma_0,\mu_0)$. Indeed, by the lower-semicontinuity of $K(x_0,\cdot)$ in Λ, for any $y_0 \in K(x_0,\gamma_0)$, there exists $y_\alpha \in K(x_0,\gamma_\alpha)$ such that $y_\alpha \to y_0$. As $x_0 \in \Psi(\gamma_\alpha,\mu_\alpha)$, there exists $z_\alpha \in T(x_0,\mu_\alpha)$ such that

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_0), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_0), \gamma_\alpha\big) \notin -\operatorname{int} C(x_0). \tag{3.5}$$

Since $T(x_0,\cdot)$ is upper semicontinuous with compact-values in M, there exists $z_0 \in T(x_0,\mu_0)$ such that $z_\alpha \to z_0$ (can take a subnet if necessary). Since $Q(\cdot)$, $\Omega(\cdot,\psi(\cdot),\cdot)$ are continuous, we have

$$\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_0), \gamma_\alpha\big)\big\rangle \to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle.$$

It follows from the continuity of $\Theta(\cdot,\psi(\cdot),\cdot)$ that

$$\begin{aligned}&\big\langle Q(z_\alpha), \Omega\big(y_\alpha, \psi(x_0), \gamma_\alpha\big)\big\rangle + \Theta\big(y_\alpha, \psi(x_0), \gamma_\alpha\big)\\ &\quad\to \big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big).\end{aligned}$$

By the condition (iii) and (3.5), we have

$$\big\langle Q(z_0), \Omega\big(y_0, \psi(x_0), \gamma_0\big)\big\rangle + \Theta\big(y_0, \psi(x_0), \gamma_0\big) \notin -\operatorname{int} C(x_0).$$

Hence, $x_0 \in \Psi(\gamma_0,\mu_0)$. Thus, $\Psi(\cdot,\cdot)$ is outer-open in $\Lambda \times M$. □

The following example shows that all assumptions of Theorem 3.4 are fulfilled. But the outer-continuity in Theorem 3.3 is not satisfied. Thus, Theorem 3.3 cannot be applied.

Example 3.5 Let $X = Y = \mathbb{R}$, $\Lambda = M = [0,1]$, $\gamma_0 = 0$, $C = [0,+\infty)$, let ψ, Q be identity mappings, and $T(x,\gamma) = [0, 2^{x^2+y^2+2\gamma+2}]$, $K(x,\gamma) = (-1,\gamma)$ and $\Omega(y,x,\gamma) = 0$ and

$$\Theta(y,x,\gamma) = \begin{cases} 0 & \text{if } \gamma = 0,\\ [\frac{1}{2^{\gamma+1}}, 1] & \text{otherwise.}\end{cases}$$

We have $E(\gamma) = (-1,\gamma)$, $\forall\gamma \in [0,1]$. We show that the conditions (i), (ii) and (iii) of Theorem 3.4 are easily seen to be fulfilled. And so $\Psi(\cdot,\cdot)$ is outer-open at $(0,0)$ (in fact, $\Psi(0,0) = (-1,0)$ and $\Psi(\gamma,\mu) = (-1,\gamma)$ for all $\gamma \in (0,1]$), but $E(\cdot)$ is not outer-continuous at 0. Hence $\Psi(\cdot,\cdot)$ is not outer-continuous at $(0,0)$.

The following example shows that all assumptions of Theorem 3.4 and Theorem 3.3 are fulfilled. But Theorem 3.1 cannot be applied.

Example 3.6 Let Q, X, Y, Λ, M, T, C, ψ, Ω, γ_0 as in Example 3.5, and let $K(x,\gamma) = \{(\zeta, \gamma\zeta) : \zeta \in \mathbb{R}\}$ and

$$\Theta(y,x,\gamma) = \begin{cases} 0 & \text{if } \lambda = 0, \\ [\frac{1}{e^{\cos^2\lambda+2}}, 1] & \text{otherwise.} \end{cases}$$

Then, we have $E(\gamma) = \{(\zeta, \gamma\zeta) : \zeta \in \mathbb{R}\}$ for all $\gamma \in [0,1]$. Hence, E is outer-open and outer-continuous at 0. It is not hard to see that (i)-(iii) in Theorem 3.4 and Theorem 3.3 are satisfied. Hence, $\Psi(\cdot,\cdot)$ is outer-open and outer-continuous at $(0,0)$ (in fact, $\Psi(\gamma,\mu) = \{(\zeta,\gamma\zeta) : \zeta \in \mathbb{R}\}$ for all $\gamma \in [0,1]$). We see that $E(\cdot)$ is not upper semicontinuous at 0. Thus, $\Psi(\cdot,\cdot)$ is not upper semicontinuous at $(0,0)$. Hence, we cannot apply Theorem 3.1.

The following example shows that the assumptions in Theorem 3.1, Theorem 3.3 and Theorem 3.4 may be satisfied in every case.

Example 3.7 Let X, Y, Λ, M, ψ, Q, C, γ_0 be as in Example 3.6, and let $T(x,\gamma) = \{\frac{1}{e}\}$, $\Omega(y,x,\gamma) = 2^{\gamma^2+\sin^4 x+1}$, $K(x,\gamma) = [0,1]$ and

$$\Theta(y,x,\gamma) = \begin{cases} 0 & \text{if } \gamma = 0, \\ [\frac{1}{2}, 1] & \text{otherwise.} \end{cases}$$

We see that the conditions (i), (ii) and (iii) in Theorem 3.1, Theorem 3.3 and Theorem 3.4 are satisfied. And so, $\Psi(\cdot,\cdot)$ is outer-open, outer-continuous and upper semicontinuous at $(0,0)$ (in fact, $\Psi(\gamma,\mu) = [0,1]$, $\forall\gamma \in [0,1]$).

4 Lower semicontinuity of a solution set

In this section, we establish that the condition $(H_h(\gamma_0,\mu_0))$ is a sufficient and necessary condition for the lower semicontinuity, the Hausdorff lower semicontinuity, the continuity and Hausdorff continuity of the solution set for the parametric generalized vector mixed quasivariational inequality problem (MQVIP).

Motivated by the hypothesis (H_1) of [15, 17] and the assumption (H_g) in [4, 7], by virtue of the parametric gap function $h(\cdot,\cdot,\cdot)$, now we introduce the following key assumption.

$(H_h(\gamma_0,\mu_0))$ Given $(\gamma_0,\mu_0) \in \Lambda \times M$. For any open neighborhood N of the origin in X, there exist $\alpha > 0$ and a neighborhood $V(\gamma_0,\mu_0)$ of (γ_0,μ_0) such that for all $(\gamma,\mu) \in V(\gamma_0,\mu_0)$ and $x \in E(\gamma) \setminus (\Psi(\gamma,\mu) + N)$, one has $h(x,\gamma,\mu) \geq \alpha$.

As mentioned in Zhao [17] and Kien [15], the above hypothesis $(H_h(\gamma_0,\mu_0))$ is characterized by a common theme used in mathematical analysis. Such a theme interprets a proposition associated with a set in terms of other propositions associated with the complement set. Instead of imposing restrictions on the solution set, the hypothesis $(H_h(\gamma_0,\mu_0))$ lays a condition on the behavior of the parametric gap function on the complement of the solution set.

Geometrically, the hypothesis $(H_h(\gamma_0,\mu_0))$ means that, given a small open neighborhood N of the origin in X, we can find a small positive number $\alpha > 0$ and a neighborhood $V(\gamma_0,\mu_0)$ of (γ_0,μ_0), such that for all (γ,μ) in the neighborhood of (γ_0,μ_0), if a feasible point x is not in the set $\Psi(\gamma,\mu) + N$, then a 'gap' by an amount of at least α will be yielded.

The following Lemma 4.1 is modified from Proposition 3.1 in Kien [15].

Lemma 4.1 *Suppose that all conditions in Lemma* 2.7 *are satisfied. For any open neighborhood N of the origin in X, let*

$$\Phi(\gamma,\mu) := \inf_{x\in E(\gamma)\setminus(\Psi(\gamma,\mu)+N)} h(x,\gamma,\mu).$$

Then $(H_h(\gamma_0,\mu_0))$ holds if and only if for any open neighborhood N of the origin in X, one has

$$\lim_{\gamma\to\gamma_0,\mu\to\mu_0} \inf \Phi(\gamma,\mu) > 0.$$

Proof If $(H_h(\gamma_0,\mu_0))$ holds, then for any open neighborhood N of the origin in X, there exist $\alpha > 0$ and a neighborhood $V(\gamma_0,\mu_0)$ of (γ_0,μ_0) such that for all $(\gamma,\mu) \in V(\gamma_0,\mu_0)$ and $x \in E(\gamma) \setminus (\Psi(\gamma,\mu) + N)$, one has $h(x,\gamma,\mu) \geq \alpha$.

This implies that $\Phi(\gamma,\mu) \geq \alpha$ for every $(\gamma,\mu) \in V(\gamma_0,\mu_0)$, hence

$$\lim_{\gamma\to\gamma_0,\mu\to\mu_0} \inf \Phi(\gamma,\mu) \geq \alpha > 0.$$

Conversely, for any open neighborhood N of the origin in X,

$$\pi = \lim_{\gamma\to\gamma_0,\mu\to\mu_0} \inf \Phi(\gamma,\mu) > 0,$$

then there exists a neighborhood $V(\gamma_0,\mu_0)$ of (γ_0,μ_0) such that

$$\Phi(\gamma,\mu) = \inf_{x\in E(\gamma)\setminus(\Psi(\gamma,\mu)+N)} h(x,\gamma,\mu) \geq \alpha > 0$$

for all $(\gamma,\mu) \in V(\gamma_0,\mu_0)$, where $\alpha := \frac{1}{2}\pi$. Hence, for any $x \in E(\gamma) \setminus (\Psi(\gamma,\mu) + N)$, we have

$$h(x,\gamma,\mu) \geq \alpha > 0,$$

which shows that $(H_h(\gamma_0,\mu_0))$ holds. □

Remark 4.2 ([21])

(i) Let a set $A \subset X$, A is said to be balanced if $\lambda A \subset A$ for every $\lambda \in R$ with $|\lambda| \leq 1$;

(ii) For each neighborhood N of the origin in X, there exists a balanced open neighborhood U of the origin in X such that $U + U + U \subset N$.

Theorem 4.3 *Suppose that the condition $(H_h(\gamma_0,\mu_0))$ holds and*

(i) *$E(\cdot)$ is lower semicontinuous with compact values in Λ;*

(ii) *$K(\cdot,\cdot)$ is continuous with compact values in $X \times \Lambda$;*

(iii) *$T(\cdot,\cdot)$ is continuous with compact values in $X \times M$;*

(iv) *$C(\cdot)$ is upper semicontinuous in X and $e(\cdot) \in \operatorname{int} C(\cdot)$ is continuous in X;*

(v) *$W(\cdot) = Y \setminus -\operatorname{int} C(\cdot)$ is closed in X.*

Then $\Psi(\cdot,\cdot)$ is Hausdorff lower semicontinuous in $\Lambda \times M$.

Proof Suppose to the contrary that $(H_h(\gamma_0,\mu_0))$ holds but $\Psi(\cdot,\cdot)$ is not Hausdorff lower semicontinuous at $(\gamma_0,\mu_0)\in\Lambda\times M$. Then there exist a neighborhood N of the origin in X, a net $\{(\gamma_\alpha,\mu_\alpha)\}\subset\Lambda\times M$ with $(\gamma_\alpha,\mu_\alpha)\to(\gamma_0,\mu_0)$ and a net $\{x_\alpha\}$ such that

$$x_\alpha\in\Psi(\gamma_0,\mu_0)\setminus\bigl(\Psi(\gamma_\alpha,\mu_\alpha)+N\bigr).\tag{4.1}$$

By the compactness of $\Psi(\gamma_0,\mu_0)$, we can assume that $x_\alpha\to x_0\in\Psi(\gamma_0,\mu_0)$. By Lemma 4.2, there exists a balanced open neighborhood U_0 of the origin in X such that $U_0+U_0+U_0\subset N$. Hence, for any given $\varepsilon>0$, $(x_0+\varepsilon U_0)\cap E(\gamma_0)\neq\emptyset$. By $E(\cdot)$ is lower semicontinuous at $\gamma_0\in\Lambda$, there exists some k_1 such that $(x_0+\varepsilon U_0)\cap E(\gamma_k)\neq\emptyset$ for all $k\geq k_1$.

For $\varepsilon\in(0,1]$, suppose that $a_k\in(x_0+\varepsilon U_0)\cap E(\gamma_k)$. We claim that $a_k\notin\Psi(\gamma_k)+U_0$. Otherwise, there exists $t_k\in\Psi(\gamma_k)$ such that $a_k-t_k\in U_0$. Without loss of generality, we may assume that $x_k-x_0\in U_0$ whenever k is sufficiently large. Consequently, we get

$$x_k-t_k=(x_k-x_0)+(x_0-a_k)+(a_k-t_k)\in U_0+(-\varepsilon U_0)+U_0\subset U_0+U_0+U_0\subset N.$$

This implies that $x_k\in\Psi(\gamma_k,\mu_k)+N$, contrary to (4.1). Thus,

$$a_k\notin\Psi(\gamma_k,\mu_k)+U_0.$$

By the assumption $(H_h(\gamma_0,\mu_0))$, there exists $\sigma>0$ such that $h(a_k,\gamma_k,\mu_k)\geq\sigma$. By Lemma 2.7, $h(\cdot,\cdot,\cdot)$ is upper semicontinuous in $X\times\Lambda\times M$. So, for any $\delta>0$ and for k sufficiently large, we have

$$h(a_k,\gamma_k,\mu_k)-\delta\leq h(x_0,\gamma_0,\mu_0).$$

We can take δ such that $\sigma-\delta>0$. Thus,

$$h(x_0,\gamma_0,\mu_0)\geq h(a_k,\gamma_k,\mu_k)-\delta\geq\sigma-\delta>0.$$

Hence

$$h(x_0,\gamma_0,\mu_0)=\min_{z\in T(x_0,\mu_0)}\max_{y\in K(x_0,\gamma_0)}\bigl\{-\xi_e\bigl(x_0,\bigl\langle Q(z),\Omega\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr\rangle+\Theta\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr)\bigr\}>0,$$

and so

$$\max_{y\in K(x_0,\gamma_0)}\bigl\{-\xi_e\bigl(x_0,\bigl\langle Q(z),\Omega\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr\rangle+\Theta\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr)\bigr\}>0,\quad\forall z\in T(x_0,\mu_0).$$

Since $y\in K(x_0,\lambda_0)$ is arbitrary, we have

$$-\xi_e\bigl(x_0,\bigl\langle Q(z),\Omega\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr\rangle+\Theta\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr)>0$$

or

$$\xi_e\bigl(x_0,\bigl\langle Q(z),\Omega\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr\rangle+\Theta\bigl(y,\psi(x_0),\gamma_0\bigr)\bigr)<0.$$

By Lemma 2.3(i), we have

$$\bigl\langle Q(z), \Omega\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr\rangle + \Theta\bigl(y, \psi(x_0), \gamma_0\bigr)\bigr) \in -\operatorname{int} C(x_0),$$

which contradicts $x_0 \in \Psi(\gamma_0, \mu_0)$. Therefore, $\Psi(\cdot,\cdot)$ is Hausdorff lower semicontinuous in $\Lambda \times M$. □

Corollary 4.4 *Suppose that all conditions in Theorem 4.3 are satisfied. Then we have* $\Psi(\cdot,\cdot)$ *is lower semicontinuous in* $\Lambda \times M$.

Theorem 4.5 *Suppose that*

(i) $E(\cdot)$ *is continuous with compact values in* Λ;
(ii) $K(\cdot,\cdot)$ *is continuous with compact values in* $X \times \Lambda$;
(iii) $T(\cdot,\cdot)$ *is continuous with compact values in* $X \times M$;
(iv) $C(\cdot)$ *is upper semicontinuous in* X *and* $e(\cdot) \in \operatorname{int} C(\cdot)$ *is continuous in* X;
(v) $W(\cdot) = Y \setminus -\operatorname{int} C(\cdot)$ *is closed in* X.

Then $\Psi(\cdot,\cdot)$ *is Hausdorff lower semicontinuous in* $\Lambda \times M$ *if and only if* $(H_h(\gamma_0, \mu_0))$ *holds.*

Proof From Theorem 4.3, we only need to prove the necessity. Suppose to the contrary that $\Psi(\cdot,\cdot)$ is Hausdorff lower semicontinuous at $(\gamma_0, \mu_0) \in \Lambda \times M$, but $(H_h(\gamma_0, \mu_0))$ does not hold. By Lemma 4.1, there exists a neighborhood N of the origin in X such that

$$\lim_{\gamma \to \gamma_0, \mu \to \mu_0} \inf \Phi(\gamma, \mu) = 0.$$

Then there exists a net $\{(\gamma_\alpha, \mu_\alpha)\} \subset \Lambda \times M$ with $(\gamma_\alpha, \mu_\alpha) \to (\gamma_0, \mu_0)$ such that

$$\lim_{\alpha \to \infty} \Phi(\gamma_\alpha, \mu_\alpha) = \lim_{\alpha \to \infty} \inf_{x \in E(\gamma_\alpha) \setminus (\Psi(\lambda_\alpha, \mu_\alpha) + N)} h(x, \gamma_\alpha, \mu_\alpha) = 0. \tag{4.2}$$

By $E(\gamma_\alpha) \setminus (\Psi(\gamma_\alpha, \mu_\alpha) + N)$ is a compact set and $h(\cdot,\cdot,\cdot)$ is continuous from Lemma 2.7, there exists $x_\alpha \in E(\gamma_\alpha) \setminus (\Psi(\gamma_\alpha, \mu_\alpha) + N)$ satisfying $\Phi(\gamma_\alpha, \mu_\alpha) = h(x_\alpha, \gamma_\alpha, \mu_\alpha)$. Clearly, (4.2) implies

$$\lim_{\alpha \to \infty} h(x_\alpha, \gamma_\alpha, \mu_\alpha) = 0.$$

Since $E(\cdot)$ is upper semicontinuous with compact values in Λ, we can assume that $x_\alpha \to x_0$ with $x_0 \in E(\gamma_0)$. By the continuity of $h(\cdot,\cdot,\cdot)$, we have $h(x_0, \gamma_0, \mu_0) = 0$ and so $x_0 \in \Psi(\gamma_0, \mu_0)$. For any $t \in \Psi(\gamma_0, \mu_0)$, since $\Psi(\cdot,\cdot)$ is Hausdorff lower semicontinuous at $(\gamma_0, \mu_0) \in \Lambda \times M$, we can find a net $\{t_\alpha\} \subset \Psi(\gamma_\alpha, \mu_\alpha)$ such that $t_\alpha \to t_0$, $\forall \alpha$. By $x_\alpha \in E(\gamma_\alpha) \setminus (\Psi(\gamma_\alpha, \mu_\alpha) + N)$, $t_\alpha - x_\alpha \nsubseteq N$. Letting $\alpha \to \infty$, we have $t_0 - x_0 \nsubseteq N$, $\forall t_0 \in \Psi(\gamma_0, \mu_0)$. Since $x_0 \in \Psi(\gamma_0, \mu_0)$, we have a contradiction. Thus, $(H_h(\gamma_0, \mu_0))$ holds. □

The following example shows that $(H_h(\gamma_0, \mu_0))$ in Theorem 4.5 is essential.

Example 4.6 Let X, Λ, M, γ_0, ψ, Q as in Example 3.5, let $Y = \mathbb{R}^2$, $C = \mathbb{R}^2_+$, $K(x, \gamma) = [-1, 1]$, $T(x, \mu) = [1, \gamma + x^2]$, $\Theta(y, \psi(x), \gamma) = 0$, $\Omega(y, \psi(x), \gamma) = y - x$. Now we consider the problem (MQVIP) of finding $x \in E(\gamma)$ and $z \in T(x, \mu)$ such that

$$\bigl\langle Q(z), \Omega\bigl(y, \eta(x), \gamma\bigr)\bigr\rangle + \Theta\bigl(y, \eta(x), \gamma\bigr) = \bigl((y - x), \bigl(\gamma + x^2\bigr)(y - x)\bigr) \notin -\operatorname{int} \mathbb{R}^2_+.$$

It follows from a direct computation

$$\Psi(\gamma,\mu)=\begin{cases}\{-1,0\} & \text{if } \gamma=0,\\ \{-1\} & \text{otherwise.}\end{cases}$$

Hence $\Psi(\cdot,\cdot)$ is not H-lsc in $\Lambda\times M$. Now we show that condition $(H_h(\gamma_0,\mu_0))$ does not hold at $(0,0)$. Taking $e=(1,1)\in\operatorname{int}\mathbb{R}^2_+$, we have

$$\begin{aligned}h(x,\gamma,\mu)&=\min_{z\in T(x,\gamma)}\max_{y\in K(x,\gamma)}\bigl\{-\xi_e\bigl(x,\bigl\langle Q(z),\Omega\bigl(y,\eta(x),\gamma\bigr)\bigr\rangle+\Theta\bigl(y,\eta(x),\gamma\bigr)\bigr)\bigr\}\\&=\max_{y\in K(x,\gamma)}\bigl(\bigl(\lambda+x^2\bigr)(x-y)\bigr)\\&=\bigl(\gamma+x^2\bigr)(x+1).\end{aligned}$$

We have $h(\cdot,\cdot,\cdot)$ is a parametric gap function of (MQVIP). For given $(\gamma_0,\mu_0)\in\Lambda\times M$, for any open neighborhood $N_\varepsilon(0)=(-\varepsilon,\varepsilon)$, choose ε such that $0<\varepsilon<1$. For any $\alpha>0$, taking $(\gamma_\beta,\mu_\beta)\to(0,0)$ with $0<\gamma_\beta<\alpha$ and $x_\beta=0\in E(\gamma_\beta)\setminus(\Psi(\gamma_\beta,\mu_\beta)+N_\varepsilon(0))$, we have $h(x_\beta,\gamma_\beta,\mu_\beta)=\gamma_\beta<\alpha$. Hence, $(H_h(\gamma_0,\mu_0))$ does not hold at $(0,0)$.

Corollary 4.7

(i) *Suppose that all conditions in Theorem* 4.5 *are satisfied. Then we have* $\Psi(\cdot,\cdot)$ *is lower semicontinuous in* $\Lambda\times M$ *if and only if* $(H_h(\gamma_0,\mu_0))$ *holds.*

(ii) *Suppose that all conditions in Theorem* 4.5 *are satisfied. Then we have* $\Psi(\cdot,\cdot)$ *is both continuous* (*H-continuous*) *and closed in* $\Lambda\times M$ *if and only if* $(H_h(\gamma_0,\mu_0))$ *holds.*

Remark 4.8

(i) In special cases as those in Section 1 (e) and (f), Theorem 4.5 extends Theorem 3.2 in [6] and Theorem 3.1 in [5]. Moreover, our assumption $(H_h(\gamma_0,\mu_0))$ is different from the assumption (H_g) in [5, 6]. Besides, our problem (MQVIP) is considered in Hausdorff topological vector spaces.

(ii) In the special case as that in Section 1 (d), Theorem 4.5 extends Theorem 4.1 in [4], and in the special case as that in Section 1 (b), Corollary 4.7(ii) extends Theorem 3.4 in [2]. Indeed, our assumption $(H_h(\gamma_0,\mu_0))$ is a sufficient and necessary condition for the lower semicontinuity, the Hausdorff lower semicontinuity, the continuity and Hausdorff continuity of the solution set for (MQVIP) while the assumption (H_g) in [2, 4] is only a sufficient condition.

Competing interests

The author declares that they have no competing interests.

Authors' contributions

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