Zhao et al. *Journal of Inequalities and Applications* 2013, **2013**:228
http://www.journalofinequalitiesandapplications.com/content/2013/1/228

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

On the Hermite-Hadamard type inequalities

Chang-Jian Zhao[1*], Wing-Sum Cheung[2] and Xiao-Yan Li[3]

*Correspondence:
chjzhao@163.com;
chjzhao@yahoo.com.cn
[1]Department of Mathematics, China Jiliang University, Hangzhou, 310018, P.R. China
Full list of author information is available at the end of the article

Abstract
In the present paper, we establish some new Hermite-Hadamard type inequalities involving two functions. Our results in a special case yield recent results on Hermite-Hadamard type inequalities.
MSC: 26D15

Keywords: Hermite-Hadamard inequality; Barnes-Godunova-Levin inequality; Minkowski integral inequality; Hölder inequality

1 Introduction

The following inequality is well known in the literature as Hermite-Hadamard's inequality [1].

Theorem 1.1 *Let $f : [a,b] \subset \mathbb{R} \to \mathbb{R}$ be a convex function on an interval of real numbers. Then the following Hermite-Hadamard inequality for convex functions holds*:

$$f\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{2}. \tag{1.1}$$

If the function f is concave, the inequality (1.1) *can be written as follows*:

$$f\left(\frac{a+b}{2}\right) \ge \frac{1}{b-a}\int_a^b f(x)\,dx \ge \frac{f(a)+f(b)}{2}. \tag{1.2}$$

Recently, many generalizations, extensions and variants of this inequality have appeared in the literature (see, *e.g.*, [2–10]) and the references given therein. In particular, in 2010, Özdemir and Dragomir [11] established some new Hermite-Hadamard inequalities and other integral inequalities involving two functions in $\mathbb{R}$. Following this work, the main purpose of the present paper is to establish some dual Hermite-Hadamard type inequalities involving two functions in $\mathbb{R}^2$. Our results provide some new estimates on such type of inequalities.

2 Preliminaries

A region $D \subset \mathbb{R}^2$ is called convex if it contains the close line segment joining any two of its points, or equivalently, if $\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2 \in D$ whenever $x(x_1,y_1), y(x_2,y_2) \in D$ and $0 \le \lambda \le 1$.

Let $z = f(x,y)$ be a duality function on the convex region $D \subset \mathbb{R}^2$. $z = f(x,y)$ is called a duality convex function on the convex region D if

$$f\bigl[\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2\bigr] \le \lambda f(x_1,y_1) + (1-\lambda)f(x_2,y_2), \tag{2.1}$$

whenever $(x_1,y_1), (x_2,y_2) \in D$ and $0 \le \lambda \le 1$.

If the function $f(x,y)$ is concave, the inequality (2.1) can be written as follows:

$$f\bigl[\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2\bigr] \ge \lambda f(x_1,y_1) + (1-\lambda)f(x_2,y_2). \tag{2.2}$$

Let $x = (x_{11},\ldots,x_{1n},\ldots,x_{m1},\ldots,x_{mn})$ and $p = (p_{11},\ldots,p_{1n},\ldots,p_{m1},\ldots,p_{mn})$ be two positive nm-tuples, and let $r \in \mathbb{R} \cup \{+\infty,-\infty\}$. Then, on putting $P_{mn} = \sum_{k_2=1}^{n}\sum_{k_1=1}^{m} p_{k_1k_2}$, it easy follows that if $-\infty \le r < s \le +\infty$, then

$$M_{mn}^{[r]} \le M_{mn}^{[s]} \tag{2.3}$$

(also see, *e.g.*, [1, p.15]). Here, the rth power mean of x with weights p is the following: $M_{mn}^{[r]} = (\frac{1}{P_{mn}}\sum_{k_2=1}^{n}\sum_{k_1=1}^{m} p_{k_1k_2}x_{k_1k_2}^{r})^{1/r}$ if $r \ne +\infty, 0, -\infty$; $M_{mn}^{[r]} = (\prod_{k_2=1}^{n}\prod_{k_1=1}^{m} x_{k_1k_2}^{p_{k_1k_2}})^{P_{mn}}$ if $r = 0$; $M_{mn}^{[r]} = \min(x_{11},\ldots,x_{1n},\ldots,x_{n1},\ldots,x_{mn})$ if $r = -\infty$ and $M_{mn}^{[r]} = \max(x_{11},\ldots,x_{1n},\ldots,x_{n1},\ldots,x_{mn})$ if $r = +\infty$.

Let $f(x,y) : [a,b] \times [c,d] \to \mathbb{R}$, and $p \ge 1$. Now, we define the p-norm of the function $f(x,y)$ on $[a,b] \times [c,d]$ as follows:

$$\bigl\|f(x,y)\bigr\|_p = \left(\int_a^b \int_c^d \bigl|f(x,y)\bigr|^p \,dx\,dy\right)^{1/p}, \quad 1 \le p < \infty,$$

and

$$\bigl\|f(x,y)\bigr\|_p = \sup\bigl|f(x,y)\bigr|, \quad p = \infty,$$

and $L^p([a,b] \times [c,d])$ is the set of all functions $f(x,y) : [a,b] \times [c,d] \to \mathbb{R}$ such that $\|f(x,y)\|_p < \infty$.

Lemma 2.1 (see [12]) (Barnes-Godunova-Levin inequality) *Let $f(x,y)$, $g(x,y)$ be nonnegative concave functions on $[a,b] \times [c,d]$, then for $p,q > 1$ we have*

$$\bigl\|f(x,y)\bigr\|_p\bigl\|g(x,y)\bigr\|_q \le B(p,q)\int_a^b \int_c^d f(x,y)g(x,y)\,dx\,dy, \tag{2.4}$$

where

$$B(p,q) = \frac{6[(b-a)(d-c)]^{1/p+1/q-1}}{(p+1)^{1/p}(q+1)^{1/q}}.$$

Lemma 2.2 (see [1]) (Hermite-Hadamard inequality) *Let $f(x,y) : [a,b] \times [c,d] \subset \mathbb{R}^2 \to \mathbb{R}$ be a convex function. Then the following dual Hermite-Hadamard inequality for convex functions holds*:

$$f\left(\frac{a+c}{2}, \frac{b+d}{2}\right) \le \frac{1}{(b-a)(d-c)}\int_a^b \int_c^d f(x,y)\,dx\,dy \le \frac{f(a,b)+f(c,d)}{2}. \tag{2.5}$$

The inequality is reversed if the function $f(x,y)$ is concave.

Lemma 2.3 (see [13]) (A reversed Minkowski integral inequality) *Let $f(x,y)$ and $g(x,y)$ be positive functions satisfying*

$$0 < m \leq \frac{f(x,y)}{g(x,y)} \leq M, \quad (x,y) \in [a,b] \times [c,d]. \tag{2.6}$$

Then

$$\|f(x,y)\|_p + \|g(x,y)\|_p \leq c\|f(x,y) + g(x,y)\|_p, \tag{2.7}$$

where $c = [M(m+1) + (M+1)]/[(m+1)(M+1)]$.

3 Main results

Our main results are established in the following theorems.

Theorem 3.1 *Let $p, q > 1$ and let $f(x,y), g(x,y) : [a,b] \times [c,d] \to \mathbb{R}$ be nonnegative functions such that $f(x,y)^p$ and $g(x,y)^q$ are concave on $[a,b] \times [c,d]$. Then*

$$\begin{aligned} &\frac{f(a,b) + f(c,d)}{2} \times \frac{g(a,b) + g(c,d)}{2} \\ &\quad \leq \frac{1}{[(b-a)(d-c)]^{1/p+1/q}} B(p,q) \int_a^b \int_c^d f(x,y) g(x,y)\, dx\, dy, \end{aligned} \tag{3.1}$$

where $B(p,q)$ is the Barnes-Godunova-Levin constant given by (2.4).

Proof Observe that whenever $f^p(x,y)$ is concave on $[a,b] \times [c,d]$, the nonnegative function $f(x,y)$ is also concave on $[a,b] \times [c,d]$. Namely,

$$f\big[\lambda a + (1-\lambda)c, \lambda b + (1-\lambda)d\big]^p \geq \lambda f(a,b)^p + (1-\lambda) f(c,d)^p,$$

that is,

$$f\big[\lambda a + (1-\lambda)c, \lambda b + (1-\lambda)d\big] \geq \big(\big(\lambda f(a,b)^p + (1-\lambda) f(c,d)^p\big)\big)^{1/p},$$

and $p > 1$, using the power-mean inequality (2.3), we obtain

$$f\big[\lambda a + (1-\lambda)c, \lambda b + (1-\lambda)d\big] \geq \lambda f(a,b) + (1-\lambda) f(c,d).$$

For $q > 1$, similarly, if $g^q(x,y)$ is concave on $[a,b] \times [c,d]$, the nonnegative function $g(x,y)$ is concave on $[a,b] \times [c,d]$.

In view that $f^p(x,y)$ and $g^q(x,y)$ are concave functions on $[a,b] \times [c,d]$, from Lemma 2.2, we get

$$\begin{aligned} \left(\frac{f(a,b)^p + f(c,d)^p}{2}\right)^{1/p} &\leq \frac{1}{[(b-a)(d-c)]^{1/p}} \left(\int_a^b \int_c^d f(x,y)^p\, dx\, dy\right)^{1/p} \\ &\leq f\left(\frac{a+c}{2}, \frac{b+d}{2}\right), \end{aligned} \tag{3.2}$$

and

$$\left(\frac{g(a,b)^p + g(c,d)^q}{2}\right)^{1/q} \le \frac{1}{[(b-a)(d-c)]^{1/q}}\left(\int_a^b\int_c^d g(x,y)^q\,dx\,dy\right)^{1/q} \le g\left(\frac{a+c}{2},\frac{b+d}{2}\right). \tag{3.3}$$

By multiplying the above inequalities, we obtain

$$\left(\frac{f(a,b)^p + f(c,d)^p}{2}\right)^{1/p}\left(\frac{g(a,b)^p + g(c,d)^q}{2}\right)^{1/q} \le \frac{1}{[(b-a)(d-c)]^{1/p+1/q}}\left(\int_a^b\int_c^d f(x,y)^p\,dx\,dy\right)^{1/p}\left(\int_a^b\int_c^d g(x,y)^q\,dx\,dy\right)^{1/q}. \tag{3.4}$$

If $p, q > 1$, then it is easy to show that

$$\left(\frac{f(a,b)^p + f(c,d)^p}{2}\right)^{1/p} \ge \frac{f(a,b) + f(c,d)}{2}, \tag{3.5}$$

and

$$\left(\frac{g(a,b)^q + g(c,d)^q}{2}\right)^{1/q} \ge \frac{g(a,b) + g(c,d)}{2}. \tag{3.6}$$

Thus, by applying Barnes-Godunova-Levin inequality to the right-hand side of (3.4) with (3.5), (3.6), we get (3.1).

The proof is complete. □

Remark 3.1 By multiplying inequalities (3.2), (3.3), we obtain

$$\frac{1}{[(b-a)(d-c)]^{1/p+1/q}}\left(\int_a^b\int_c^d f(x,y)^p\,dx\,dy\right)^{1/p}\left(\int_a^b\int_c^d g(x,y)^q\,dx\,dy\right)^{1/q} \le f\left(\frac{a+c}{2},\frac{b+d}{2}\right)g\left(\frac{a+c}{2},\frac{b+d}{2}\right). \tag{3.7}$$

By applying the Hölder inequality to the left-hand side of (3.7) with $(1/p) + (1/q) = 1$, we get

$$\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)g(x,y)\,dx\,dy \le f\left(\frac{a+c}{2},\frac{b+d}{2}\right)g\left(\frac{a+c}{2},\frac{b+d}{2}\right). \tag{3.8}$$

Remark 3.2 Let $f(x,y)$ and $g(x,y)$ change to $f(x)$ and $g(x)$, respectively, and with suitable changes in Theorem 3.1 and Remark 3.1, we have the following.

Corollary 3.1 *Let $p, q > 1$ and let $f(x), g(x) : [a,b] \to \mathbb{R}$, $a < b$, be nonnegative functions such that $f(x)^p$ and $g(x)^q$ are concave on $[a,b]$. Then*

$$\frac{f(a)+f(b)}{2}\cdot\frac{g(a)+g(b)}{2} \le \frac{1}{(b-a)^{1/p+1/q}}B(p,q)\int_a^b f(x)g(x)\,dx,$$

and if $(1/p) + (1/q) = 1$, *then one has*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le f\left(\frac{a+b}{2}\right)g\left(\frac{a+b}{2}\right).$$

This is just Theorem 2.1 established by Özdemir and Dragomir [11].

Theorem 3.2 *Let* $p \ge 1$ *and let* $\int_a^b \int_c^d f(x,y)^p\,dx\,dy < \infty$ *and* $\int_a^b \int_c^d g(x,y)^p\,dx\,dy < \infty$, *and let* $f(x,y), g(x,y) : [a,b] \times [c,d] \to \mathbb{R}$ *be positive functions with*

$$0 < m \le \frac{f(x,y)}{g(x,y)} \le M, \quad \forall (x,y) \in [a,b] \times [c,d].$$

Then

$$\|f(x,y)\|_p^2 + \|g(x,y)\|_p^2 \ge \left(\frac{(M+1)(m+1)}{M} - 2\right)\|f(x,y)\|_p \|g(x,y)\|_p. \tag{3.9}$$

Proof Since $f(x,y)$, $g(x,y)$ are positive, as in the proof of Lemma 2.3 (see [13, p.2]), we have

$$\left(\int_a^b \int_c^d f(x,y)^p\,dx\,dy\right)^{1/p} \le \frac{M}{M+1}\left(\int_a^b \int_c^d \bigl(f(x,y)+g(x,y)\bigr)^p\,dx\,dy\right)^{1/p}$$

and

$$\left(\int_a^b \int_c^d g(x,y)^p\,dx\,dy\right)^{1/p} \le \frac{1}{m+1}\left(\int_a^b \int_c^d \bigl(f(x,y)+g(x,y)\bigr)^p\,dx\,dy\right)^{1/p}.$$

By multiplying the above inequalities and in view of the Minkowski inequality, we get

$$\begin{aligned}
&\left(\int_a^b \int_c^d f(x,y)^p\,dx\,dy\right)^{1/p}\left(\int_a^b \int_c^d g(x,y)^p\,dx\,dy\right)^{1/p} \\
&\quad \le \frac{M}{(M+1)(m+1)}\left(\int_a^b \int_c^d \bigl(f(x,y)+g(x,y)\bigr)^p\,dx\,dy\right)^{2/p} \\
&\quad \le \frac{M}{(M+1)(m+1)}\left(\left(\int_a^b \int_c^d f(x,y)^p\,dx\,dy\right)^{1/p} \right. \\
&\qquad \left. + \left(\int_a^b \int_c^d g(x,y)^p\,dx\,dy\right)^{1/p}\right)^2.
\end{aligned} \tag{3.10}$$

Hence

$$\begin{aligned}
&\left(\int_a^b \int_c^d f(x,y)^p\,dx\,dy\right)^{2/p} + \left(\int_a^b \int_c^d g(x,y)^p\,dx\,dy\right)^{2/p} \\
&\quad \ge \left(\frac{(M+1)(m+1)}{M} - 2\right)\left(\int_a^b \int_c^d f(x,y)^p\,dx\,dy\right)^{1/p}\left(\int_a^b \int_c^d g(x,y)^p\,dx\,dy\right)^{1/p}.
\end{aligned}$$

This proof is complete. □

Remark 3.3 Let $f(x,y)$ and $g(x,y)$ change to $f(x)$ and $g(x)$, respectively, and with suitable changes in (3.9), (3.9) reduces to an inequality established by Özdemir and Dragomir [11].

Theorem 3.3 *If $f^p(x,y)$ and $g^q(x,y)$ are as in Theorem* 3.1, *then the following inequality holds*:

$$\frac{1}{(b-a)(d-c)}\|f(x,y)\|_p^p \cdot \|g(x,y)\|_q^q \ge \frac{(f(a,b)+f(c,d))^p(g(a,b)+g(c,d))^q}{2^{p+q}}. \tag{3.11}$$

Proof If $f^p(x,y)$ and $g^q(x,y)$ are concave on $[a,b]\times[c,d]$, then from Lemma 2.2, we get

$$\frac{f(a,b)^p+f(c,d)^p}{2} \le \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)^p\,dx\,dy$$

and

$$\frac{g(a,b)^q+g(c,d)^q}{2} \le \frac{1}{(b-a)(d-c)}\int_a^b\int_c^d g(x,y)^q\,dx\,dy,$$

which imply that

$$\begin{aligned}&\frac{[f(a,b)^p+f(c,d)^p][g(a,b)^q+g(c,d)^q]}{4}\\&\quad\le \frac{1}{[(b-a)(d-c)]^2}\int_a^b\int_c^d f(x,y)^p\,dx\,dy\int_a^b\int_c^d g(x,y)^q\,dx\,dy.\end{aligned} \tag{3.12}$$

On the other hand, if $p,q\ge 1$, from (2.3) we get

$$\frac{f(a,b)^p+f(c,d)^p}{2} \le 2^{-p}\big[f(a,b)+f(c,d)\big]^p$$

and

$$\frac{g(a,b)^q+g(c,d)^q}{2} \le 2^{-q}\big[g(a,b)+g(c,d)\big]^q,$$

which imply that

$$\begin{aligned}&\frac{[f(a,b)^p+f(c,d)^p][g(a,b)^p+g(c,d)^q]}{4}\\&\quad\ge 2^{-p-q}\big[f(a,b)+f(c,d)\big]^p\big[g(a,b)+g(c,d)\big]^q.\end{aligned} \tag{3.13}$$

Combining (3.12) and (3.13), we obtain the desired inequality as

$$\begin{aligned}&2^{-p-q}\big[f(a,b)+f(c,d)\big]^p\big[g(a,b)+g(c,d)\big]^q\\&\quad\le \frac{1}{[(b-a)(d-c)]^2}\|f(x,y)\|_p^p \cdot \|g(x,y)\|_q^q.\end{aligned}$$

This proof is complete. □

Remark 3.4 Let $f(x,y)$ and $g(x,y)$ change to $f(x)$ and $g(x)$, respectively, and with suitable changes in (3.11), (3.11) reduces to an inequality established by Özdemir and Dragomir [11].

Theorem 3.4 *Let $f(x,y), g(x,y) : [a,b] \times [c,d] \to \mathbb{R}^+$ be functions such that $f(x,y)^p$, $g(x,y)^q$ and $f(x,y)g(x,y)$ are in $L_1([a,b] \times [c,d])$, and*

$$0 < m \le \frac{f(x,y)}{g(x,y)} \le M, \quad \forall (x,y) \in [a,b] \times [c,d], a,b,c,d \in [0,\infty).$$

Then

$$\int_a^b \int_c^d f(x,y)g(x,y)\,dx\,dy \le c_1\left(\frac{\|f(x,y)\|_p^p + \|g(x,y)\|_p^p}{2}\right) + c_2\left(\frac{\|f(x,y)\|_q^q + \|g(x,y)\|_q^q}{2}\right), \tag{3.14}$$

where

$$c_1 = \frac{2^p}{p}\left(\frac{M}{M+1}\right)^p, \qquad c_2 = \frac{2^q}{q}\left(\frac{1}{m+1}\right)^q,$$

and $(1/p) + (1/q) = 1$ *with* $p > 1$.

Proof Since $0 < m \le \frac{f(x,y)}{g(x,y)} \le M$, $\forall (x,y) \in [a,b] \times [c,d]$, we have

$$f(x,y) \le \frac{M}{M+1}\bigl(f(x,y) + g(x,y)\bigr)$$

and

$$g(x,y) \le \frac{1}{m+1}\bigl(f(x,y) + g(x,y)\bigr).$$

In view of the Young-type inequality and using the elementary inequality

$$(a+b)^p \le 2^{p-1}\bigl(a^p + b^p\bigr), \quad p > 1, a, b \in \mathbb{R}^+,$$

we have

$$\begin{aligned} &\int_a^b \int_c^d f(x,y)g(x,y)\,dx\,dy \\ &\quad\le \frac{1}{p}\left(\frac{M}{M+1}\right)^p \int_a^b \int_c^d \bigl(f(x,y) + g(x,y)\bigr)^p\,dx\,dy \\ &\qquad + \frac{1}{q}\left(\frac{1}{m+1}\right)^q \int_a^b \int_c^d \bigl(f(x,y) + g(x,y)\bigr)^q\,dx\,dy \\ &\quad\le \frac{1}{p}\left(\frac{M}{M+1}\right)^p 2^{p-1} \int_a^b \int_c^d \bigl[f(x,y)^p + g(x,y)^p\bigr]\,dx\,dy \\ &\qquad + \frac{1}{q}\left(\frac{1}{m+1}\right)^q 2^{q-1} \int_a^b \int_c^d \bigl[f(x,y)^q + g(x,y)^q\bigr]\,dx\,dy. \end{aligned}$$

This completes the proof. □

Remark 3.5 Let $f(x,y)$ and $g(x,y)$ change to $f(x)$ and $g(x)$, respectively, and with suitable changes in (3.14), (3.14) reduces to an inequality established by Özdemir and Dragomir [11].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
C-JZ, W-SC and X-YL jointly contributed to the main results Theorems 3.1-3.4. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, China Jiliang University, Hangzhou, 310018, P.R. China. [2]Department of Mathematics, The University of Hong Kong, Pokfulam Road, Hong Kong, P.R. China. [3]Department of Mathematics, Hunan Normal University, Changsha, 410000, P.R. China.

Acknowledgements
The first author's research is supported by Natural Science Foundation of China (10971205). The second author's research is partially supported by a HKU Seed Grant for Basic Research.

Received: 20 October 2012 Accepted: 18 April 2013 Published: 7 May 2013

References
1. Mitrinović, DS, Pecarič, JE, Fink, AM: Classical and New Inequalities in Analysis. Mathematics and Its Applications (East European Series), vol. 61. Kluwer Academic, Dordrecht (1993)
2. Kavurmaci, H, Avci, M, Özdemir, ME: New inequalities of Hermite-Hadamard type for convex functions with applications. J. Inequal. Appl. **2011**, 86 (2011)
3. Özdemir, ME, Kavurmaci, H, Ocak Akdemir, A, Avci, M: Inequalities for convex and *s*-convex functions on $\Delta = [a,b] \times [c,d]$. J. Inequal. Appl. **2012**, 20 (2012)
4. Dragomir, SS: A sequence of mappings associated with the Hermite-Hadamard inequalities and applications. Appl. Math. **49**, 123-140 (2004)
5. Kotrys, D: Hermite-Hadamard inequality for convex stochastic processes. Aequ. Math. **83**, 143-151 (2012)
6. Makó, J, Páles, Z: Hermite-Hadamard inequalities for generalized convex functions. Aequ. Math. **69**, 32-40 (2005)
7. Ger, R, Pečarić, J: On vector Hermite-Hadamard differences controlled by their scalar counterparts. Int. Ser. Numer. Math. **161**, 165-173 (2010)
8. Xi, B-Y, Bai, R-F, Qi, F: Hermite-Hadamard type inequalities for the m- and (α, m)-geometrically convex functions. Filomat **27**, 1-7 (2013)
9. Dragomir, SS, Hunt, E, Pearce, CEM: Interpolating maps, the modulus map and Hadamard's inequality. In: Optimization, Part 1. Springer Optimization and Its Applications, vol. 32, pp. 207-223 (2009)
10. Niculescu, CP: The Hermite-Hadamard inequality for convex functions of a vector variable. Math. Inequal. Appl. **5**(4), 619-623 (2002)
11. Set, E, Özdemir, ME, Dragomir, SS: On the Hermite-Hadamard inequality and other integral inequalities involving two functions. J. Inequal. Appl. **2010**, Article ID 148102 (2010)
12. Pachpatte, BG: Inequalities for Differentiable and Integral Equations. Academic Press, Boston (1997)
13. Bougoffa, L: On Minkowski and Hardy integral inequalities. J. Inequal. Pure Appl. Math. **7**(2), Article ID 60 (2006)

doi:10.1186/1029-242X-2013-228
Cite this article as: Zhao et al.: **On the Hermite-Hadamard type inequalities.** *Journal of Inequalities and Applications* 2013 **2013**:228.