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Upper bounds on ε'/ε parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from large N QCD and other news

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ABSTRACT: We demonstrate that in the large N approach developed by the authors in collaboration with Bardeen, the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ parametrizing the $K \to \pi\pi$ matrix elements $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ of the dominant QCD and electroweak operators receive both *negative* $\mathcal{O}(1/N)$ corrections such that $B_6^{(1/2)} \le B_8^{(3/2)} < 1$ in agreement with the recent lattice results of the RBC-UKQCD collaboration. We also point out that the pattern of the size of the hadronic matrix elements of all QCD and electroweak penguin operators Q_i contributing to the $K \to \pi\pi$ amplitudes A_0 and A_2, obtained by this lattice collaboration, provides further support to our large N approach. In particular, the lattice result for the matrix element $\langle Q_8 \rangle_0$ implies for the corresponding parameter $B_8^{(1/2)} = 1.0 \pm 0.2$ to be compared with large N value $B_8^{(1/2)} = 1.1 \pm 0.1$. We discuss briefly the implications of these findings for the ratio ε'/ε. In fact, with the precise value for $B_8^{(3/2)}$ from RBC-UKQCD collaboration, our upper bound on $B_6^{(1/2)}$ implies ε'/ε in the SM roughly by a factor of two below its experimental value $(16.6 \pm 2.3) \times 10^{-4}$. We also briefly comment on the parameter $\hat{B}_K$ and the $\Delta I = 1/2$ rule.

KEYWORDS: CP violation, Kaon Physics

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1 Introduction

The decays $K \to \pi\pi$ have played a very important role since their discovery in the 1950s, both in the construction of the Standard Model (SM) and more recently in the tests of its possible extensions. Most of the discussions in the literature centred on the following quantities:

- The ratio
$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 22.4\,, \tag{1.1}$$
which expresses the so-called $\Delta I = 1/2$ rule [1, 2].

- The parameter ε_K, a measure of indirect CP-violation in $K_L \to \pi\pi$ decays, found to be
$$\varepsilon_K = 2.228(11) \times 10^{-3} e^{i\phi_\varepsilon}, \tag{1.2}$$
where $\phi_\varepsilon = 43.51(5)^\circ$.

- The ratio of the direct CP-violation and indirect CP-violation in $K_L \to \pi\pi$ decays measured to be [3–6]
$$\mathrm{Re}(\varepsilon'/\varepsilon) = (16.6 \pm 2.3) \times 10^{-4}. \tag{1.3}$$

Unfortunately, due to non-perturbative uncertainties originating in the hadronic matrix elements of contributing four-quark operators, it took a long time to obtain meaningful results for all these observables in QCD. But already in the second half of the 1980s, we have developed an approach to $K^0 - \bar{K}^0$ mixing and non-leptonic K-meson decays [7–11] based on the dual representation of QCD as a theory of weakly interacting mesons for large N, where N is the number of colours [12–15]. The most recent results from our approach can be found in [16, 17].

This approach provided, in particular, first results within QCD for the amplitudes ReA_0 and ReA_2 in the ballpark of experimental values [10]. In this manner, for the first time, the SM dynamics behind the $\Delta I = 1/2$ rule has been identified. In particular, it has been emphasized that at scales $\mathcal{O}(1\,\mathrm{GeV})$ long distance dynamics in hadronic matrix elements of current-current operators and not QCD-penguin operators, as originally proposed in [18], are dominantly responsible for this rule. Moreover, it has been demonstrated analytically why ReA_0 is enhanced and why ReA_2 is suppressed relative to the vacuum insertion approximation (VIA) estimates. In this context, we have emphasized that the so-called Fierz terms in the latter approach totally misrepresent $1/N$ corrections to the strict large N limit for these amplitudes [10].

Our approach, among other applications, allowed us to consistently calculate, for the first time within QCD, the non-perturbative parameters $\hat{B}_K$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$ governing the corresponding matrix elements of $\Delta S = 2$ SM current-current operator and $K \to \pi\pi$ matrix elements of the dominant QCD-penguin (Q_6) and electroweak penguin (Q_8) operators. These parameters are crucial for the evaluation of ε_K and ε'/ε within the SM and its various extensions. Other applications of large N ideas to $K \to \pi\pi$ and $\hat{B}_K$, but in a different spirit than our original approach, are reviewed in [19]. We will comment in section 6 on those which reached very different conclusions from ours.

It is interesting and encouraging that most of our results have been confirmed by several recent lattice QCD calculations that we will specify below. While the lattice QCD approach has a better control over the errors than our approach, it does not provide the physical picture of the dynamics behind the obtained numerical results. This is in particular seen in the case of the $\Delta I = 1/2$ rule where our analytic approach offers a very simple picture of the dynamics behind this rule, as summarized again in [16, 17].

In the present paper, we briefly compare in section 2 the status of lattice results for $\hat{B}_K$ and the $\Delta I = 1/2$ rule with the ones obtained in our approach. Subsequently, in section 3 we demonstrate that the pattern of the size of the matrix elements for penguin operators presented recently by the RBC-UKQCD collaboration for A_0 [20] and A_2 amplitudes [21] gives another support to our approach. In section 4, we derive upper bounds on the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ and discuss briefly in section 5 their phenomenological implications for ε'/ε. In section 6 we describe briefly the results obtained in other large N QCD approaches. An outlook is presented in section 7.

2 $\hat{B}_K$ and the $\Delta I = 1/2$ rule

2.1 $\hat{B}_K$

The scale and renormalization scheme dependent parameter $B_K(\mu)$ is related to the relevant hadronic matrix element of the $\Delta S = 2$ operator

$$Q = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \tag{2.1}$$

as follows[1]

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = B_K(\mu) \frac{8}{3} F_K^2 m_K^2 . \tag{2.2}$$

[1] In this paper, we use the normalization of weak decay constants given in (3.17).

More useful is the renormalization group invariant parameter $\hat{B}_K$ that is given by [22]

$$\hat{B}_K = B_K(\mu)\left[\alpha_s^{(3)}(\mu)\right]^{-d}\left[1+\frac{\alpha_s^{(3)}(\mu)}{4\pi}J_3\right], \qquad d=\frac{9(N-1)}{N(11N-6)}\,. \tag{2.3}$$

We have shown the N-dependence of the exponent d in the leading term to signal that d vanishes in the large N limit. The coefficient J_3 is renormalization scheme dependent. This dependence cancels the one of $B_K(\mu)$.

As in the strict large N limit the exponent in (2.3) and the NLO term involving J_3 vanish, one finds [7] that independently of any renormalization scale or renormalization scheme for the operator Q

$$\hat{B}_K \to 0.75, \qquad (\text{in large N limit, } 1986). \tag{2.4}$$

It can be shown that including $1/N$ corrections suppresses $\hat{B}_K$ so that [23]

$$\hat{B}_K \leq 0.75\,, \qquad (\text{in 1/N expansion}). \tag{2.5}$$

Our latest analysis in our approach gave [16]

$$\hat{B}_K = 0.73 \pm 0.02, \qquad (\text{in dual QCD}), \tag{2.6}$$

where the error should not be considered as a standard deviation. Rather, this result represents the range for $\hat{B}_K$ we expect in our approach after the inclusion of NLO QCD corrections and the contributions of pseudoscalar and vector mesons as discussed in detail in [16].

On the other hand, the world lattice average for $\hat{B}_K$ based on the calculations of various groups [24–29] reads for $N_f = 2+1$ calculations (recent FLAG update of [27])

$$\hat{B}_K = 0.766 \pm 0.010, \qquad (\text{in lattice QCD, } 2014). \tag{2.7}$$

See also the recent analyses in [30, 31]. While this result violates the bound in (2.5), it should be noted that a number of lattice groups among [24–29] published results with central values satisfying the bound in (2.5) but the errors did not allow for a clear cut conclusion. In fact, the most recent update from staggered quarks [31] quotes precisely $\hat{B}_K = 0.738 \pm 0.005$ but additional systematic error of 0.037 does not allow for definite conclusions. Similarly, the Rome group [32] finds basically the result in (2.6). We expect therefore that improved lattice calculations will satisfy our bound one day and in a few years from now lattice average for $\hat{B}_K$ will read $\hat{B}_K \approx 0.74$.

Finally, let us remark that while the lattice approach did not provide the explanation why $\hat{B}_K$ is so close to its large N limit 0.75, in our approach the smallness of $1/N$ corrections follows from the approximate cancellation of negative pseudoscalar meson contributions by the positive vector meson contributions.

2.2 $\Delta I = 1/2$ rule

A very detailed comparison of the calculations of $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ in our approach and the lattice QCD has been presented in [16] and our present discussion is meant to be an update due to new results of the RBC-UKQCD collaboration on $\mathrm{Re}A_0$ [20].

First, let us mention that both the dual approach to QCD and lattice approach obtain satisfactory results for the amplitude $\mathrm{Re}A_2$. On the other hand, whereas we find [16]

$$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)_{\text{dual QCD}} = 16.0 \pm 1.5\,, \tag{2.8}$$

the most recent result from the RBC-UKQCD collaboration reads [20]

$$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)_{\text{lattice QCD}} = 31.0 \pm 6.6\,. \tag{2.9}$$

Due to large error in the lattice result, both results are compatible with each other and both signal that this rule follows dominantly from the QCD dynamics related to current-current operators. But our approach, being analytic, allows to connect the $\Delta I = 1/2$ rule to the main properties of QCD: asymptotic freedom and the related evolutions of weak matrix elements which at long distance scales can be performed in the dual representation of QCD as a theory of weakly interacting mesons for large N. As lattice QCD calculations are performed basically at a single energy scale, no such physical explanation of this rule is expected from that framework. To this end, lattice calculations would have to be performed at scales below 1 GeV which is straightforward in our approach but appears impossible by lattice methods at present.

On the other hand, from the present perspective only lattice simulations can provide precise value of $\mathrm{Re}A_0$ one day, so that we will know whether some part of this rule at the level of $(20-30)\%$, as signalled by the result in (2.8), originates in new physics (NP) contributions. Indeed, as demonstrated in [33], a heavy Z' and in particular a heavy G' in the reach of the LHC could be responsible for the missing piece in $\mathrm{Re}A_0$ in (2.8). On the basis of the analysis in [33] it is much harder to bring this ratio with the help of NP from 31 down to 22 without violating ΔM_K constraint, but this requires a separate study.

Of some interest is the ratio of the matrix elements $\langle Q_2\rangle_0$ and $\langle Q_1\rangle_0$. It equals -2 in the large N limit, corresponding to $\mu = 0$ [16]. Evolving these matrix elements to $\mu = 1\,\mathrm{GeV}$ in the meson theory and subsequently to $\mu = 1.53\,\mathrm{GeV}$ in the quark theory, we find in the NDR-$\overline{\mathrm{MS}}$ scheme[2]

$$\frac{\langle Q_2\rangle_0}{\langle Q_1\rangle_0} = -1.50 \pm 0.10, \qquad \mu = 1.53\,\mathrm{GeV}, \qquad \text{(dual QCD)}. \tag{2.10}$$

The corresponding result in [20] reads

$$\frac{\langle Q_2\rangle_0}{\langle Q_1\rangle_0} = -1.12 \pm 0.49, \qquad \mu = 1.53\,\mathrm{GeV}, \qquad \text{(lattice QCD)}. \tag{2.11}$$

In view of large uncertainty in the lattice result, these two ratios are compatible with each other. We expect on the basis of the results in (2.8) and (2.9) that this ratio will be eventually found in the ballpark of -1.4.

[2]We thank Martin Gorbahn for checking this result.

3 Matrix elements of penguin operators

3.1 Preliminaries

We will consider the usual basis of operators contributing to $K \to \pi\pi$ amplitudes [34], namely

Current-Current:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A}\,, \qquad Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A} \tag{3.1}$$

QCD-penguins:

$$Q_3 = (\bar{s}d)_{V-A}\sum_{q=u,d,s}(\bar{q}q)_{V-A}, \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\alpha)_{V-A} \tag{3.2}$$

$$Q_5 = (\bar{s}d)_{V-A}\sum_{q=u,d,s}(\bar{q}q)_{V+A}, \qquad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\alpha)_{V+A} \tag{3.3}$$

Electroweak penguins:

$$Q_7 = \frac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s}e_q\,(\bar{q}q)_{V+A}, \qquad Q_8 = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}e_q(\bar{q}_\beta q_\alpha)_{V+A} \tag{3.4}$$

$$Q_9 = \frac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s}e_q(\bar{q}q)_{V-A}, \qquad Q_{10} = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}e_q\,(\bar{q}_\beta q_\alpha)_{V-A} \tag{3.5}$$

Here, α, β denote colour indices and e_q denotes the electric quark charges reflecting the electroweak origin of $Q_7, \ldots, Q_{10}$. Finally, $(\bar{s}d)_{V-A} \equiv \bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha$ as in (2.1).

Recently, the RBC-UKQCD collaboration published their results for the matrix elements $\langle Q_i\rangle_0$ [20]. Their matrix elements are given for *three* dynamical quarks at $\mu = 1.53\,\text{GeV}$, which is too high for the direct comparison with our approach in the case of current-current operators. On the other hand, the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ of the QCD penguin operator Q_6 and the electroweak penguin operator Q_8 are known [34] to be practically scale independent for $1.0\,\text{GeV} \le \mu \le 3.0\,\text{GeV}$. Therefore these results constitute a useful test of our approach. Another issue is the colour suppression of some matrix elements of other penguin operators which is predicted within our approach. We would like to check whether the pattern of this suppression is also seen in the lattice data.

3.2 Hadronic matrix elements

The hadronic matrix elements of operators Q_i that are most useful for our discussions are

$$\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I\,|Q_i|\,K\rangle\,, \tag{3.6}$$

with $I = 0, 2$ being strong isospin.

It should be recalled that for $\mu \le m_c$, when charm quark has been integrated out, only seven of the operators listed above are independent of each other. Eliminating then Q_4, Q_9

and Q_{10} in terms of the remaining seven operators allows, in the isospin symmetry limit, to find the following important relations [34][3]

$$\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + \langle Q_2\rangle_0 - \langle Q_1\rangle_0\,, \tag{3.7}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0\,, \tag{3.8}$$

$$\langle Q_{10}\rangle_0 = \langle Q_2\rangle_0 + \frac{1}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0\,, \tag{3.9}$$

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_1\rangle_2\,, \tag{3.10}$$

where we have used

$$\langle Q_1\rangle_2 = \langle Q_2\rangle_2\,. \tag{3.11}$$

We have checked that these relations have been used in [20].

Of particular importance for our discussion are the matrix elements

$$\langle Q_6(\mu)\rangle_0 = -\,h\left[\frac{2m_{\rm K}^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}\,, \tag{3.12}$$

$$\langle Q_8(\mu)\rangle_2 = \frac{h}{2\sqrt{2}}\left[\frac{2m_{\rm K}^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(3/2)}\,, \tag{3.13}$$

$$\langle Q_8(\mu)\rangle_0 = \frac{h}{2}\left[\frac{2m_{\rm K}^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(1/2)}\,, \tag{3.14}$$

with [7, 8, 35]

$$B_6^{(1/2)} = B_8^{(3/2)} = B_8^{(1/2)} = 1, \qquad \text{(large N Limit)}\,. \tag{3.15}$$

Note, that using the definition of B_i parameters consistent with the large N limit of QCD, as given above, implies that their values in the VIA [34] read

$$B_6^{(1/2)} = 1, \qquad B_8^{(3/2)} \approx 0.99, \qquad B_8^{(1/2)} \approx 1.2 \qquad \text{(VIA)}\,. \tag{3.16}$$

We will return to this point in the next section.

The input values of parameters entering these expressions are given by [36, 37]

$$F_\pi = 130.41(20)\,\text{MeV}, \qquad \frac{F_K}{F_\pi} = 1.194(5) \tag{3.17}$$

$$m_s(m_c) = 109.1(2.8)\,\text{MeV}, \qquad m_d(m_c) = 5.44(19)\,\text{MeV}\,. \tag{3.18}$$

It should be emphasized that the overall factor h in these expressions depends on the normalisation of the amplitudes $A_{0,2}$. In [34] and recent papers of the RBC-UKQCD collaboration [21, 38] $h = \sqrt{3/2}$ is used whereas in most recent phenomenological papers [16, 19, 33, 39], $h = 1$. In the present paper we will keep general h so that, e.g., the decay amplitude $K^+ \to \pi^+\pi^0$ reads $(3/2h)A_2$.

[3] In writing (3.7) we neglect a small $\mathcal{O}(\alpha_s)$ correction in the NDR scheme which is explicitly given in (4.44) of [34].

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Comparing the expressions (3.12) and (3.14) with the lattice results in [20], we find (see also [40])[4]

$$B_6^{(1/2)} = 0.57 \pm 0.19, \qquad B_8^{(1/2)} = 1.0 \pm 0.2\,, \qquad \text{(lattice QCD)}. \tag{3.19}$$

On the other hand, comparing (3.13) with the value for this matrix element obtained by RBC-UKQCD collaboration in [21] one extracts [39]

$$B_8^{(3/2)} = 0.76 \pm 0.05\,, \qquad \text{(lattice QCD)}. \tag{3.20}$$

All these results are very weakly dependent on the renormalization scale. The quoted values correspond to $\mu = 1.53\,\text{GeV}$. Basically, identical results are obtained for $\mu = m_c$ used in [40]. However, as stated before (3.19), in extracting these parameters from [20] it is important to use the quark masses at that scale.

As we will demonstrate in the next section, these lattice results are consistent with the large N approach. Indeed, we will show that the following pattern emerges at next-to-leading order in our dual approach:

$$B_6^{(1/2)} = 1 - \left[\frac{F_\pi}{F_K - F_\pi}\right] \mathcal{O}\left(\frac{1}{N}\right) < 1\,, \tag{3.21}$$

$$B_8^{(3/2)} = 1 - \mathcal{O}\left(\frac{1}{N}\right) < 1\,, \tag{3.22}$$

$$B_8^{(1/2)} = 1 + \mathcal{O}\left(\frac{1}{N}\right) > 1\,. \tag{3.23}$$

We would like to recall that strong indication for the suppression of $B_8^{(3/2)}$ below unity in our approach have been found already in 1998 in [41], while in the case of $B_6^{(1/2)}$ no clear cut conclusions could be reached. Our present analysis of both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ clearly indicates the negative signs of $1/N$ corrections to the leading result in (3.15).

Finally, the lattice results in [20] and [21] exhibit colour suppression of the matrix elements of Q_3, Q_5 and Q_7 operators relative to the ones of Q_4, Q_6 and Q_8, respectively:

$$\frac{\langle Q_3\rangle_0}{\langle Q_4\rangle_0} = -0.18 \pm 0.25, \qquad \left(\frac{F_K}{F_\pi} - 1\right)\frac{\langle Q_5\rangle_0}{\langle Q_6\rangle_0} = 0.10 \pm 0.05\,, \tag{3.24}$$

$$\frac{\langle Q_7\rangle_0}{\langle Q_8\rangle_0} = 0.13 \pm 0.04\,, \qquad \frac{\langle Q_7\rangle_2}{\langle Q_8\rangle_2} = 0.22 \pm 0.01\,. \tag{3.25}$$

These results are consistent with the large N approach. Indeed, as we will demonstrate soon, the ratios in (3.24) are $\mathcal{O}(1/N^2)$ while the ratios in (3.25) are $\mathcal{O}(1/N)$.

These results allow to simplify some of the relations between the matrix elements so that it is justified to use the relations

$$\langle Q_4\rangle_0 = \langle Q_2\rangle_0 - \langle Q_1\rangle_0\,, \tag{3.26}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\langle Q_1\rangle_0\,, \tag{3.27}$$

$$\langle Q_{10}\rangle_0 = \langle Q_2\rangle_0 + \frac{1}{2}\langle Q_1\rangle_0, \tag{3.28}$$

which simplify the phenomenological analysis of ε'/ε in [40].

[4]To this end, the values $m_s = 102.27\,\text{MeV}$ and $m_d = 5.10\,\text{MeV}$ at $\mu = 1.53\,\text{GeV}$ have to be used.

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4 Derivations

The large N numerical values of the $|\Delta S| = 1$ matrix elements already displayed in section 3.2 are most easily derived from the effective theory for the the pseudo-Goldstone field

$$\mathrm{U}(\pi) \equiv \exp\left(i\sqrt{2}\frac{\pi}{f}\right) \tag{4.1}$$

with $\pi = \lambda_a \pi^a$, the meson nonet lying below the one GeV and f, the associated weak decay constant scaling like $\sqrt{N}$. In particular, the electroweak penguin operator introduced in (3.4) and "Fierzed" into a product of two colour-singlet quark densities, namely,

$$Q_8 = -12 \sum_{q=u,d,s} (\bar{s}_L q_R) e_q (\bar{q}_R d_L) \tag{4.2}$$

can be hadronized by considering the leading chiral effective Lagrangian in the large N limit:

$$L_{\text{eff}}(p^2, N) = \frac{f^2}{8} Tr\left[\partial_\mu U \partial^\mu U^+ + r(mU^\dagger + Um^\dagger)\right]. \tag{4.3}$$

Indeed, a straightforward identification of the second term in this equation with the standard Dirac mass term in QCD

$$L_{\text{QCD}}(\text{mass}) = -(\bar{q}_L m q_R + \bar{q}_R m^\dagger q_L) \tag{4.4}$$

allows us to hadronize all colour-singlet quark densities

$$\bar{q}^a_R q^b_L = -\frac{f^2}{8} r U^{ba} \tag{4.5}$$

$$\bar{q}^a_L q^b_R = -\frac{f^2}{8} r U^{\dagger ba} \tag{4.6}$$

such that

$$Q_8 = -\frac{3}{16} f^4 r^2 \sum_{q=u,d,s} U^{dq} e_q U^{\dagger qs}. \tag{4.7}$$

Consequently, the factorized matrix elements of the $\Delta I = 1/2$ and $\Delta I = 3/2$ components of Q_8 in the large N limit are

$$\langle Q_8 \rangle_0 = \frac{h}{2} f r^2 \tag{4.8}$$

$$\langle Q_8 \rangle_2 = \frac{h}{2\sqrt{2}} f r^2. \tag{4.9}$$

Similarly, the QCD penguin operator Q_6 introduced in (3.3) and "Fierzed" into a product of two colour-singlet densities reads

$$Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L) = -\frac{1}{8} f^4 r^2 \sum_q U^{dq} U^{\dagger qs} = 0. \tag{4.10}$$

As a matter of fact, one has the relation

$$r(\mu) = \frac{2m_K^2}{m_s(\mu) + m_d(\mu)} \tag{4.11}$$

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at the level of $L_{\rm eff}(p^2,N)$. Yet, at this level, the absence of SU(3) splitting among the weak decay constants implies ill-defined $\langle Q_8\rangle_{0,2}$ matrix elements in (4.8) and (4.9) as well as a vanishing Q_6 operator in (4.10).

It is well known [8, 42] that the next-to-leading term in the chiral effective Lagrangian

$$L_{\rm eff}(p^4,N) = -\frac{f^2}{8}\frac{r}{\Lambda_\chi^2} Tr\left[m\partial^2 U^\dagger + \partial^2 U m^\dagger\right] \tag{4.12}$$

solves both problems since it leads to realistic weak decay constants

$$F_\pi = \left(1+\frac{m_\pi^2}{\Lambda_\chi^2}\right) f \tag{4.13}$$

$$\frac{F_K}{F_\pi} = 1+\frac{m_K^2-m_\pi^2}{\Lambda_\chi^2} \tag{4.14}$$

thanks to its derivative dependence and, simultaneously, it implies

$$Q_6 = -\frac{f^4}{4}\left(\frac{r}{\Lambda_\chi}\right)^2 (\partial_\mu U\partial^\mu U^\dagger)^{ds} + \mathcal{O}\left(\frac{1}{\Lambda_\chi^4}\right) \approx -\left(\frac{r}{\Lambda_\chi}\right)^2 Q_4 \tag{4.15}$$

from the shift induced in the hadronized quark densities

$$U \to U - \frac{1}{\Lambda_\chi^2}\partial^2 U \tag{4.16}$$

through its mass dependence. Taking these corrections into account, we now reproduce the large N matrix elements given in (3.12)–(3.14), with the normalization (3.15) for the $B_{6,8}$ coefficients if contributions $\mathcal{O}(m_\pi^2/\Lambda_\chi^2)$ to Q_8 are neglected.

At this point, it is worth emphasizing that, here, we consistently normalize the $|\Delta S| = 1$ $B_{6,8}$ to unity in the large N limit. Such is unfortunately not the case for the $|\Delta S| = 2$ B_K parameter conventionally normalized to one with respect to VIA in (2.2). Had the $\Delta S = 2$ matrix element in (2.2) been normalized relative to its large N value, the most precise $\hat{B}_K$ parameter extracted from lattice QCD in (2.7) would read $\hat{B}_K = 1.021 \pm 0.013$ nowadays and our result in (2.6) $\hat{B}_K = 0.97 \pm 0.03$. In [34, 41], B_6 and B_8 were also normalized with respect to the VIA as in (3.16).

We are now in an ideal position to estimate $1/N$ corrections encoded in the $B_{6,8}$ parameters. The factorizable $1/N$ corrections to $|\Delta S| = 1$ density-density operators are fully included [35] in the running of quark masses in (4.11). Let us thus focus on non-factorizable one loop corrections induced by $L_{\rm eff}(p^2,N)$. Applying the background field method of [43], we find

$$U^{dq}U^{\dagger q's}(\Lambda) = U^{dq}U^{\dagger q's}(M) - \frac{16}{f^4}\frac{\ln(\Lambda^2/M^2)}{(4\pi f)^2}\left[2J_L^{ds}J_R^{q'q} + (J_LJ_L)^{ds}\delta^{q'q}\right](M) \tag{4.17}$$

with $\Lambda = \mathcal{O}(1\,\text{GeV})$ the euclidean ultraviolet cut-off of the effective theory (4.3) to be matched with the non-factorizable short distance evolution, $M = \mathcal{O}(m_K)$ and

$$J_L^{ab} = \bar{q}_L^b\gamma_\mu q_L^a = i\frac{f^2}{4}(\partial_\mu U U^\dagger)^{ab} \tag{4.18}$$

$$J_R^{ab} = \bar{q}_R^b\gamma_\mu q_R^a = i\frac{f^2}{4}(\partial_\mu U^\dagger U)^{ab} \tag{4.19}$$

the colour-singlet left-handed and right-handed hadronic currents derived from $L_{\rm eff}(p^2,N)$, respectively.

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Applied to the specific $K \to \pi\pi$ decay processes,

- the first (L-R) current-current operator in (4.17), also present with the right relative sign in the VIA through a Fierz transformation, does not contribute to the matrix element $\langle Q_6 \rangle_0$ since

$$\mathrm{Tr}(J_R) = \frac{\sqrt{3}}{2} f \partial_\mu \eta^0 \tag{4.20}$$

- the second (L-L) current-current operator in (4.17), absent in the VIA, does not contribute to the matrix elements $\langle Q_8 \rangle_{0,2}$ since

$$\mathrm{Tr}(e_q) = 0\,. \tag{4.21}$$

An explicit calculation of the surviving Q_4 and Q_7 matrix elements (in the large N limit) gives then, respectively,

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_6^2}\right) = 1 - 0.66\,\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_6^2}\right) \tag{4.22}$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 + 0.08\,\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) \tag{4.23}$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 - 0.17\,\ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) \tag{4.24}$$

with pseudoscalar mass scale parameters bounded necessarily by the effective cut-off around 1 GeV:

$$\tilde{m}_{6,8} \le \Lambda\,. \tag{4.25}$$

First, we emphasize most important properties of these results:

- For $\Lambda = 0$, corresponding to strict large N limit and matrix elements evaluated at zero momentum, $B_6^{(1/2)} = B_8^{(3/2)} = B_8^{(1/2)} = 1$ in accordance with (3.15).

- With increasing Λ, the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ decrease below unity and $B_6^{(1/2)}$ decreases faster than $B_8^{(3/2)}$. Consequently, at scales $\mathcal{O}(1\,\mathrm{GeV})$ relevant for the phenomenology both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are predicted to be below unity and there is strong indication that $B_6^{(1/2)} < B_8^{(3/2)}$.

- While the dependence of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ on $\Lambda < 1\,\mathrm{GeV}$ is stronger than their dependence on μ in the perturbative regime, these two properties of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are at the qualitative level consistent with the numerical analysis performed for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ by means of the standard renormalization group running in [34]. Indeed as seen in figures 11 and 12 of that paper $B_6^{(1/2)}$ decreases with increasing μ, faster than $B_8^{(3/2)}$, albeit in this perturbative range the dependence of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ on μ is very weak. While the analysis in [34] includes NLO QCD and QED corrections, the inspection of the one-loop anomalous dimension matrix allows to see these properties

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explicitly. In particular Q_6 mixes with the linear combination $(Q_4 + Q_6)$ and we find for $\mu_1 \leq \mu_2 \leq m_c$

$$B_6^{(1/2)}(\mu_2) = B_6^{(1/2)}(\mu_1)\left[1 - \frac{\alpha_s(\mu_1)}{2\pi}\ln\left(\frac{\mu_2}{\mu_1}\right)\left(1 + \frac{\langle Q_4(\mu_1)\rangle_0}{\langle Q_6(\mu_1)\rangle_0}\right)\right]. \tag{4.26}$$

From (4.15) $|\langle Q_6(\mu_1)\rangle_0| > |\langle Q_4(\mu_1)\rangle_0|$ such that $B_6^{(1/2)}$ decreases with increasing μ. On the other hand, in the LO Q_8 runs only by itself and the one-loop anomalous dimension matrix implies

$$B_8^{(1/2,3/2)}(\mu_2) = B_8^{(1/2,3/2)}(\mu_1)\,, \tag{4.27}$$

which follows from exact SU(3) symmetry imposed in SD calculations. The breakdown of SU(3) is only felt in the matrix elements of Q_8 making in the LD range $B_8^{(3/2)}$ dependent weakly on the scales involved. In view of this, the suppression of both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ below the unity can be considered as a solid result and our explicit calculation as well as different behaviour of Q_6 and Q_8 under flavour SU(3) provide a strong support for $B_6^{(1/2)} < B_8^{(3/2)}$. On the other hand,

$$B_8^{(1/2)} \approx [B_8^{(3/2)}]^{-1/2} \tag{4.28}$$

slightly increases with Λ which is also consistent with the standard renormalization group running [40].

Next, we observe that:

- The numerical value of the parameter $B_6^{(1/2)}$ suffers from rather large uncertainties. This feature is related to the fact that Q_6 vanishes at leading order in chiral perturbation theory (see (4.10)). The $1/N$ logarithmic correction in (4.22) is therefore *artificially enhanced* by the factor $F_\pi/(F_K - F_\pi) \approx 5$ such that

$$B_6^{(1/2)} < 0.6\,. \tag{4.29}$$

- The parameter $B_8^{(1/2)}$ has a very small $1/N$ correction. At $\mathcal{O}(1/N^2)$, one larger contribution might arise from the anomalous effective Lagrangian

$$L_{\text{eff}}(p^0, 1/N) = \frac{f^2}{32}\left(\frac{m_0^2}{N}\right)\left[Tr(\ln U - \ln U^\dagger)\right]^2 \tag{4.30}$$

that solves the so-called $\mathrm{U}(1)_A$ problem [44] by providing the η' pseudoscalar with a physical mass in the large N limit [45]:

$$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \approx m_0^2 \approx 0.7\,\mathrm{GeV}^2\,. \tag{4.31}$$

Applying again the background field method, we obtain

$$U^{dq}U^{\dagger q's}(\Lambda) = \left[1 - \frac{4}{N}\frac{m_0^2}{(4\pi f)^2}\ln\left(\frac{\Lambda^2}{M^2}\right)\right]U^{dq}U^{\dagger q's}(M). \tag{4.32}$$

Such a negative contribution to Q_8 has been included in [41]. However, any consistent estimate beyond

$$B_8^{(1/2)} \approx 1 \tag{4.33}$$

would require a full calculation at $\mathcal{O}(1/N^2)$.

- The parameter $B_8^{(3/2)}$, for which the $1/N$ expansion is more reliable, is found in the range

$$0.7 \leq B_8^{(3/2)} \leq 0.9 \tag{4.34}$$

if $\tilde{m}_8 \geq m_K$.

Lattice result for $B_6^{(1/2)}$ in (3.19) turns out to almost saturate our bound. But one should realize that although we are confident about the suppression of $B_6^{(1/2)}$ below unity, its actual size is rather uncertain. For instance the inclusion of dynamical scalars presently frozen in Λ_χ could reduce the coefficient in front of the logarithm in (4.22) making $B_6^{(1/2)}$ larger. This uncertainty in the value of $B_6^{(1/2)}$ explains also why it took so long to calculate $B_6^{(1/2)}$ in lattice QCD even with a large uncertainty as seen in (3.19). On the other hand, the range for $B_8^{(3/2)}$ in (4.34) is consistent with the one in (3.20). These results indicate that indeed $B_6^{(1/2)}$ could be smaller than $B_8^{(3/2)}$. Yet, in view of the large numerical uncertainties in the case of $B_6^{(1/2)}$, we cannot exclude that $B_6^{(1/2)}$ is as large as $B_8^{(3/2)}$. We therefore believe that the best way of summarizing our results for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ is given in (3.21) and (3.22) together with

$$B_6^{(1/2)} \leq B_8^{(3/2)} < 1\,. \tag{4.35}$$

Below 1 GeV we have seen in (4.17) that density-density operators transmute into current-current ones at $\mathcal{O}(1/N)$. But power counting in our effective theory does not allow the other way around, namely current-current operators evolving into density-density ones. This is fully consistent with the evolution of hadronic matrix elements above $\mu =$ 1 GeV studied already in [34] and is opposite to the evolution of the corresponding Wilson coefficients. Now, in the large N approach, it has already been shown [16, 43] that any (L-L) current-current operator evolves as

$$J_L^{ab}J_L^{cd}(\Lambda) = J_L^{ab}J_L^{cd}(0) - \mathcal{O}\left(\frac{1}{N}\right)\left[2J_L^{ad}J_L^{cb} - \delta^{ad}(J_LJ_L)^{cb} - \delta^{cb}(J_LJ_L)^{ad}\right](0) \tag{4.36}$$

to stand in contrast with the wrong relative sign in the VIA analogue

$$J_L^{ab}J_L^{cd}(\Lambda) = J_L^{ab}J_L^{cd}(0) + \frac{1}{N}J_L^{ad}J_L^{cb}(0)\,. \tag{4.37}$$

As a consequence, summing over $c = d = u, d, s$ we conclude that the matrix element $\langle Q_3\rangle_0$ which vanishes in the large N limit is formally $\mathcal{O}(1/N^2)$ relative to $\langle Q_4\rangle_0$ in our effective theory. Such a strong suppression could have been anticipated from the LO short-distance evolution of the four-quark operator Q_3 into another linear combination of Q_4 and Q_6:

$$Q_3(\mu_2) = Q_3(\mu_1) - \mathcal{O}\left(\frac{1}{N}\right)\left[\frac{11}{2}Q_4 + Q_6\right](\mu_1). \tag{4.38}$$

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At long distance, the further Q_3 evolution undergoes an important numerical cancellation since (4.15) tells us that the Q_4 and Q_6 operators are not independent anymore. Following (4.36), this numerical cancellation is not a mere coincidence but the result of a consistent $1/N$ expansion.

In the same manner, it has been proved [43] that any (L-R) current-current operator evolves as

$$J_L^{ab} J_R^{cd}(\Lambda) = J_L^{ab} J_R^{cd}(0) + \mathcal{O}\left(\frac{1}{N}\right) \left[U^{ad}(\delta U^\dagger)^{cb} + \delta U^{ad}(U^\dagger)^{cb}\right](0) \tag{4.39}$$

with δU proportional to $(\Box U - U \Box U^\dagger U)$.

Consequently, the matrix element $\langle Q_5 \rangle_0$ which vanishes in the large N limit is $\mathcal{O}(1/N^2)$ relative to $F_\pi/(F_K - F_\pi)\langle Q_6\rangle_0$ in our effective theory. As already mentioned, an enhancement factor has to be introduced to compensate for the "accidental" chiral suppression of $\langle Q_6\rangle_0$ in (4.10). On the other hand, the matrix elements

$$\langle Q_7\rangle_0 = \frac{h}{2} F_\pi (m_K^2 - m_\pi^2) \tag{4.40}$$

$$\langle Q_7\rangle_2 = -\frac{h}{\sqrt{2}} F_\pi (m_K^2 - m_\pi^2) \tag{4.41}$$

are $\mathcal{O}(p^2)$ but non zero in the large N limit. With the matrix elements $\langle Q_8\rangle_{0,2}$ given in (4.8), (4.9) and being $\mathcal{O}(p^0)$, the resulting ratios $\langle Q_7\rangle_{0,2}/\langle Q_8\rangle_{0,2}$ are at the level of a few percent and can thus be neglected. The long-distance evolution of Q_7 in (4.39) leads then to matrix elements proportional to $\langle Q_8\rangle_{0,2}$, though $\mathcal{O}(p^2)$. This is clearly at variance with its LO short-distance evolution, namely

$$Q_7(\mu_2) = Q_7(\mu_1) + \mathcal{O}\left(\frac{1}{N}\right) Q_8(\mu_1)\,. \tag{4.42}$$

As already explicitly stated in [43], this suggests the necessity to introduce higher resonances beyond our effective theory truncated to the low-lying pseudoscalars. In a dual representation of QCD the matrix elements $\langle Q_7\rangle_{0,2}$ should then be dominantly $\mathcal{O}(p^0)$, but $1/N$-suppressed, with the bound

$$\frac{\langle Q_7\rangle_0}{\langle Q_7\rangle_2} < \sqrt{2} \tag{4.43}$$

resulting from the isospin decompositions in (4.40)–(4.41) and (4.8)–(4.9).

5 Implications for ε'/ε

We will now briefly discuss the implications of our results for ε'/ε. To this end we will use the analytic formula for ε'/ε in the SM derived recently in [40]. In obtaining this formula it has been assumed that the SM describes exactly the data on CP-conserving $K \to \pi\pi$ amplitudes: $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$. This allowed to determine the contributions of the $(V-A)\otimes(V-A)$ QCD penguin operator Q_4 and of the electroweak penguin operators Q_9 and Q_{10} to ε'/ε much more precisely than it is presently possible by lattice QCD and large N approach. This determination was facilitated by our results on the suppression of the matrix element $\langle Q_3\rangle_0$ implying the relations (3.26)–(3.28).[5]

[5] The final numerical analysis in [40] leading to (5.1) included also small corrections from Q_3 and other corrections from subleading operators.

The formula in question reads [40]

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[a\big(1-\hat{\Omega}_{\rm eff}\big)\big(-4.1+24.7\,B_6^{(1/2)}\big)+1.2-10.4\,B_8^{(3/2)}\right], \tag{5.1}$$

where

$$\mathrm{Im}\lambda_{\rm t} = \mathrm{Im}(V_{td}V_{ts}^*) = |V_{ub}||V_{cb}|\sin\gamma \tag{5.2}$$

and [40, 46–48]

$$a = 1.017, \qquad \hat{\Omega}_{\rm eff} = (14.8\pm 8.0)\times 10^{-2}\,. \tag{5.3}$$

$|V_{ub}|$ and $|V_{cb}|$ are the elements of the CKM matrix and γ is an angle in the unitarity triangle. The parameters a and $\hat{\Omega}_{\rm eff}$ represent isospin breaking corrections [46–48]. See these papers and [40] for details.

Setting all parameters, except for $B_6^{(1/2)}$ and $B_8^{(3/2)}$, in (5.1) to their central values we find

$$\mathrm{Re}(\varepsilon'/\varepsilon) = 8.6\times 10^{-4}, \qquad (B_6^{(1/2)}=1.0,\ B_8^{(3/2)}=1.0) \tag{5.4}$$

$$\mathrm{Re}(\varepsilon'/\varepsilon) = 6.4\times 10^{-4}, \qquad (B_6^{(1/2)}=0.8,\ B_8^{(3/2)}=0.8) \tag{5.5}$$

$$\mathrm{Re}(\varepsilon'/\varepsilon) = 2.2\times 10^{-4}, \qquad (B_6^{(1/2)}=0.6,\ B_8^{(3/2)}=0.8)\,. \tag{5.6}$$

A detailed anatomy of ε'/ε in the SM is presented in [40], where various uncertainties related to NNLO QCD corrections and other uncertainties are discussed. But these three examples indicate that taking our bounds into account and guided by the results on $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from lattice QCD and our dual approach, our SM prediction for ε'/ε appears to be significantly below the data given in (1.3).

In each of these predictions, there are uncertainties from the value of $\mathrm{Im}\lambda_t$, the unknown complete NNLO corrections to Wilson coefficients of contributing operators, α_s, m_t and other input parameters. But they appear not to change the conclusion that, presently, the SM prediction for ε'/ε is significantly below the data. Our upper bound on $B_6^{(1/2)}$ plays an important role in this result as otherwise increasing $B_6^{(1/2)}$ above unity would allow to fit easily the data.

6 Comments on other large N QCD approaches

In [49] the authors analyse $\Delta I = 1/2$ rule and ε'/ε in the chiral limit including $1/N$ corrections. Their results differ drastically from our results. In particular, their low energy "Extended Nambu-Jona-Lasinio" model (ENJL) gives

$$B_6^{(1/2)} \approx 3, \qquad B_8^{(3/2)} \approx 1.3, \qquad \text{(ENJL)} \tag{6.1}$$

namely $B_6^{(1/2)}$ roughly by a factor of five larger than lattice calculations and our results. With such high values of $B_6^{(1/2)}$, QCD penguins play an important role in the explanation of the $\Delta I = 1/2$ rule and the experimental value of ε'/ε can easily be reproduced, again in contrast with lattice QCD and our dual QCD approach.

In [50] the authors consider a low energy model including the light pseudo-scalar, vector and scalar poles only in the chiral limit. Within this so-called "Minimal Hadronic Approximation" (MHA), they also obtain much larger values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ than in our approach

$$B_6^{(1/2)} \approx 3, \qquad B_8^{(3/2)} \approx 3.5, \qquad \text{(MHA)} \tag{6.2}$$

and find then good agreement with data for ε'/ε.

In [51–53] the authors rely on dispersion relations and "Finite Energy Sum Rules" (FESR) in the chiral limit to extract the electroweak penguin matrix elements from ALEPH and OPAL data. Doing so, they obtain central values for $B_8^{(3/2)}$ by a factor of two to three larger than in lattice QCD and our dual QCD approach

$$1.3 \leq B_8^{(3/2)} \leq 2.5, \qquad \text{(FESR)}\,. \tag{6.3}$$

We conclude that the recent lattice QCD results tend to demonstrate that the various models considered in [49–53] do not represent properly the low energy QCD dynamics at work for penguin matrix elements, but confirm the structure of our dual QCD approach.

7 Summary and outlook

In the present paper, we have compared the structure of the hadronic matrix elements in $K \to \pi\pi$ decays obtained within the dual approach to QCD with the one obtained recently by the RBC-UKQCD lattice approach to QCD and commented briefly on the status of the parameter $\hat{B}_K$ and the $\Delta I = 1/2$ rule. Our main results are as follows:

- The status of $\hat{B}_K$ is very good as both our approach and the lattice QCD calculations give this parameter very close to 0.75. But we expect that $\hat{B}_K$ from the lattice approach will decrease by a few % in the coming years.

- While the results for $\mathrm{Re}A_2$ obtained in both approaches agree well with the data, the central value of $\mathrm{Re}A_0$ from RBC-UKQCD collaboration is by a factor ot two larger than in our approach and 40% above the data. While our result in (2.8) appears from present perspective to be final in our approach, significant improvement on the lattice result is expected in the coming years. This will allow to find out whether at some level of 20% new physics could still be responsible for the $\Delta I = 1/2$ rule. An analysis anticipating such possibility has been presented in [33].

- As the upper bound on $B_8^{(3/2)}$ in (3.22) has been already indicated in [41], one of the most important results of our paper is the upper bound on $B_6^{(1/2)}$. Our estimate suggests that $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$, but the precise values can only be obtained by lattice methods.

- Among other results of our approach supported by recent results from RBC-UKQCD is the strong suppression of $\langle Q_{3,5}(\mu)\rangle$ and $B_8^{(1/2)} \approx 1$.

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If indeed the emerging pattern $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$ with $B_8^{(3/2)} = 0.8 \pm 0.1$ will be confirmed by more precise calculations one day, the very recent analysis in [40] and our paper show that ε'/ε within the SM will be found roughly by a factor of two below the data. For a detailed phenomenological discussion of the state of ε'/ε within the SM including all errors and future theoretical and experimental prospects we refer to [40]. On the other hand, first phenomenological implications of our results on new physics models have been presented in [54, 55].

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