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Solution for counter-current imbibition of 1D immiscible two-phase flow in tight oil reservoir

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Abstract Spontaneous imbibition is an important mechanism for fractured reservoir to enhance oil recovery. Wetting phase enters porous media with the force of capillary pressure and gravity and replaces oil in matrix. To investigate the imbibition of tight reservoirs, on the consideration of tight formation characteristics, this paper derived a one dimension, two phases, counter-current imbibition model, after dimensionless of distance and time, Galerkin method for spatial discretization and time integration, solutions were given, comparisons of conventional sandstone and tight formation were made. The results have indicated that: (1) Imbibition can be divided into gravity assisting, gravity opposing and zero gravity in terms of different gravity conditions. (2) Saturation front of tight formation moves faster than sandstone because of high capillary pressure. (3) Capillary pressure plays the dominant role than gravity in imbibition. Influence of gravity is much greater in high-permeability sandstone than in tight reservoirs. (4) Horizontal well multi-stage fracturing and massive fracturing can increase fracture area and fracture volume, and increase the contact area with wetting phases, this will result in a greater imbibition and a great recovery of oil.

Keywords Two-phase flow · Counter-current imbibition · Capillary pressure · Gravity · Tight oil reservoir · Hydraulic fracturing

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Introduction

Imbibition can be defined as the inflow of wetting phase and the displacement of non-wetting phase in the porous media under the forces of capillary pressure, gravity and buoyancy force. Imbibition can be divided into co-current imbibition and counter-current imbibition according to the flow direction (Mattax and Kyte 1962). In co-current imbibition, wetting phase pushes non-wetting phase out of the matrix in the same direction. While in the counter-current imbibition, wetting phase imbibes into matrix, displacing the non-wetting phase in the opposite direction. Co-current imbibition is much faster and more efficiency than the counter-imbibition. However, as a matter of fact, the counter-current imbibition is the main recovery mechanism because only one face of matrix can contact non-wetting phase in most cases.

Experimental and modeling method for imbibition has been studied by many researchers. Ryzhik (1960) derived a 1D self-similar solution for counter-current model by assuming the linear function of capillary pressure and relative permeability with water saturation. Yortsos et al. (1993) obtained an analytical solution by the assumption of some constrained relationship of capillary pressure, relative

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permeability and water saturation. Reis and Cil (1993) got the approximate solution for 1D counter-current imbibition considering capillary pressure. Kashchiev and Firoozabadi (2003) and Behbahani et al. (2006) solved a counter-current imbibition model of water wetting and fractured reservoirs, and influencing factors of gravity, capillary pressure and viscous resistance . Some scholars also adopted the method of pore scale network (PSN) method (Blunt 2001; Valvatne and Blunt 2004), Lattice Boltzmann (LBM) method (Porter et al. 2009; Galindo-Torres et al. 2013) and computational fluid dynamics (CFD) method (Hammecker et al. 1993; Standnes 2006) to study the mechanism of co-current and counter-current imbibition.

However, some of the above models ignore the influence of gravity (Behbahani et al. 2006; Blunt 2001); some dealing capillary pressure and relative permeability too simple ($J(S) = \ln S$) (Kashchiev and Firoozabadi 2003; Behbahani et al. 2006). Most of all, they are models of fractured reservoir or high-permeability sandstone reservoir, which may make a great difference from tight oil reservoir.

In this paper, we derived a general formula of 1D two immiscible phase flow considering gravity, capillary pressure and buoyancy force considering characteristics of tight oil reservoir. Capillary pressure and relative permeability of tight oil were obtained by matching with experiment data and were substituted into the general formula. After dimensionless, spatial dispersion by Galerkin method and time difference, we solved the high nonlinear partial differential equation. Solutions for three different boundary conditions and gravity conditions were given.

Derivation of the model

Hypothesis

(1) Two-phase flow (wetting and non-wetting); (2) one-dimensional flow; (3) inflow velocity of wetting phase equals to the outflow velocity of non-wetting phase, and in opposite direction; (4) no reaction between different fluids, means immiscible flow; (5) fluid is incompressible; (6) isothermal flow and (7) considering effects of gravity. Three different models are shown as follows.

Derivation

According to Darcy's law, velocity of the wetting phase and non-wetting phase is:

$$v_{\mathrm{w}} = -\frac{k_{\mathrm{rw}}}{\mu_{\mathrm{w}}} K\left(\frac{\partial P_{\mathrm{w}}}{\partial z} + \rho_{\mathrm{w}} g\right) \tag{1}$$

$$v_{\mathrm{nw}} = -\frac{k_{\mathrm{rnw}}}{\mu_{\mathrm{nw}}} K\left(\frac{\partial P_{\mathrm{nw}}}{\partial z} + \rho_{\mathrm{nw}} g\right) \tag{2}$$

where, v_{w} is the seepage velocity of wetting phase, v_{nw} is the seepage velocity of non-wetting phase, K is absolute permeability, μ_{w} is viscosity of wetting phase, μ_{nw} is viscosity of non-wetting phase, k_{rw} is wetting phase relative permeability, k_{rnw} is non-wetting phase relative permeability, ρ_{w} is wetting phase density, and ρ_{nw} is non-wetting phase density.

Capillary pressure can be defined as the subtraction of non-wetting phase pressure and wetting phase pressure; its direction is from wetting phase to non-wetting phase.

$$P_{\mathrm{c}} = P_{\mathrm{nw}} - P_{\mathrm{w}} \tag{3}$$

From Leveratt two-phase flow, we can get that,

$$\frac{\partial P_{\mathrm{c}}}{\partial z} = \frac{\partial P_{\mathrm{nw}}}{\partial z} - \frac{\partial P_{\mathrm{w}}}{\partial z} \tag{4}$$

In counter-current imbibition, the inflow of wetting phase and the outflow of non-wetting phase occurs in the same face, which means the velocity of these two phases can be described as:

$$v_{\mathrm{nw}} + v_{\mathrm{w}} = 0 \tag{5}$$

Seepage velocity of wetting phase can be described as formula (6) by the combination of formula (1), (2), (4) and (5).

$$v_{\mathrm{w}} = \frac{K k_{\mathrm{rw}} k_{\mathrm{rnw}}}{k_{\mathrm{rw}}\mu_{\mathrm{nw}} + k_{\mathrm{rnw}}\mu_{\mathrm{w}}}\left(\frac{\partial P_{\mathrm{c}}}{\partial z} - \Delta\rho g\right) \tag{6}$$

where,

$$\Delta\rho = \rho_{\mathrm{w}} - \rho_{\mathrm{nw}} \tag{7}$$

Continuity equation (material balance equation) of wetting phase can be written as formula (8) when the density variation is not considered.

$$\varphi\frac{\partial S_{\mathrm{w}}}{\partial t} + \frac{\partial v_{\mathrm{w}}}{\partial z} = 0 \tag{8}$$

where, φ is matrix porosity, S_{w} is water saturation.

Take formula (6) into formula (8), we can obtain this:

$$\varphi\frac{\partial S_{\mathrm{w}}}{\partial t} + \frac{\partial}{\partial z}\left(\frac{K k_{\mathrm{rw}} k_{\mathrm{rnw}}}{k_{\mathrm{rw}}\mu_{\mathrm{nw}} + k_{\mathrm{rnw}}\mu_{\mathrm{w}}}\left(\frac{\partial P_{\mathrm{c}}}{\partial z} - \Delta\rho g\right)\right) = 0 \tag{9}$$

This is the general formula of counter-current imbibition of two immiscible phases (considering gravity).

Initial condition, Neumann boundary condition and Dirichlet boundary condition of model (a) (Fig. 1a) and model (c) (Fig. 1c) can be written as:

$$S_{\mathrm{w}} = S_{\mathrm{iw}}, \quad t = 0, \quad 0 \le z \le H \tag{10}$$

$$S_{\mathrm{w}} = 1 - S_{\mathrm{nwr}}, \quad t \ge 0, \quad z = 0 \tag{11}$$

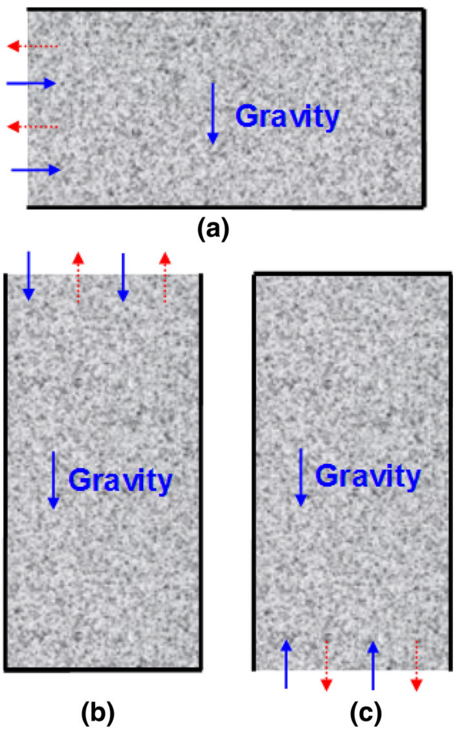

Fig. 1 Counter-current imbibition considering gravity. **a** Non-gravity; **b** gravity assisting; **c** gravity opposing

$$\frac{dS_w}{dz} = 0, \quad t \geq 0, \ z = H \tag{12}$$

Initial condition, Neumann boundary condition and Dirichlet boundary condition of model (b) (Fig. 1b) can be written as:

$$S_w = S_{iw}, \quad t = 0, \ 0 \leq z \leq H \tag{13}$$

$$\frac{dS_w}{dz} = 0, \quad t \geq 0, \ z = 0 \tag{14}$$

$$S_w = 1 - S_{nwr}, \quad t \geq 0, \ z = H \tag{15}$$

where, S_{iw} is irreducible water saturation, S_{nwr} is resistant saturation of non-wetting phase, H is the height of porous media.

Simplification of the formula

We can find that formula (9) is a high nonlinear partial differential equation. Since non-wetting phase relative permeability (k_{ro}), wetting phase relative permeability (k_{rw}) and capillary pressure (P_c) are discontinuous functions of water saturation (S_w), PDE formula (9) can not be solved directly. Dealing with relative permeability and capillary pressure can be done as follows.

Relative permeability

Paul Willhite (1986) describes relative permeability and dimensionless water saturation as formula (16) to formula (18). By matching the relative permeability using these formulas, we obtained the parameters' values as shown in Table 1.

$$k_{rw} = k_{rw}^0 S^a \tag{16}$$

$$k_{rnw} = k_{rnw}^0 (1 - S)^b \tag{17}$$

$$S = \frac{S_w - S_{iw}}{1 - S_{rnw} - S_{iw}} \tag{18}$$

where, k_{rw}^0 is water relative permeability at residual oil saturation, k_{rnw}^0 is oil relative permeability at irreducible water saturation, S_{iw} is irreducible water saturation, S_{rnw} is residual oil saturation, S is normalized water saturation, and m and n is relative permeability index for oil phase and water phase which depends on formation rock's pore scale structure and wettability (Fig. 2).

Capillary pressure

Capillary pressure is related to formation pore structure, fluid property and water saturation. Considering all these factors, all the previous authors (Ryzhik 1960; Yortsos et al. 1993; Kashchiev and Firoozabadi 2003; Pooladi-Darvish and Firoozabadi 2000) used $J(S)$ function to describe two-phase flow.

$$P_c = \sigma \left(\frac{\varphi}{K}\right)^{\frac{1}{2}} J(S) \tag{19}$$

where, σ is oil–water interfacial tension, mN/m. They all simplify $J(S)$ as ln S. This kind of simplification does not correspond to tight oil formation. In this paper, we match the capillary pressure with experimental result by an exponentially fitted method as shown in Fig. 3. This capillary pressure curve was substituted into formula (9). From Fig. 3, we can find that capillary pressure of high-permeability sandstone is much smaller than tight oil. This is a result of the nano-scale pores and throats distributed in tight formation.

Table 1 Parameters used in the model

K ($\times 10^{-3}$ μm^2)	Φ	H (m)	σ (mN/m)	S_{iw}	S_{inw}	m
0.2	0.7	2	20	0.3	0.25	3
k_{rw}^0	k_{rnw}^0	μ_o (mPa s)	μ_w (mPa s)	ρ_o (g/cm^3)	ρ_w (g/cm^3)	n
0.2	1	0.8	1.0	0.8	1	1.5

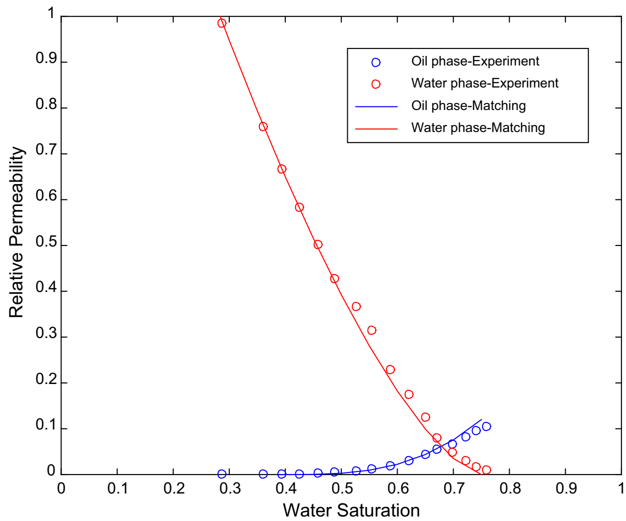

Fig. 2 Relative permeability obtained by matching

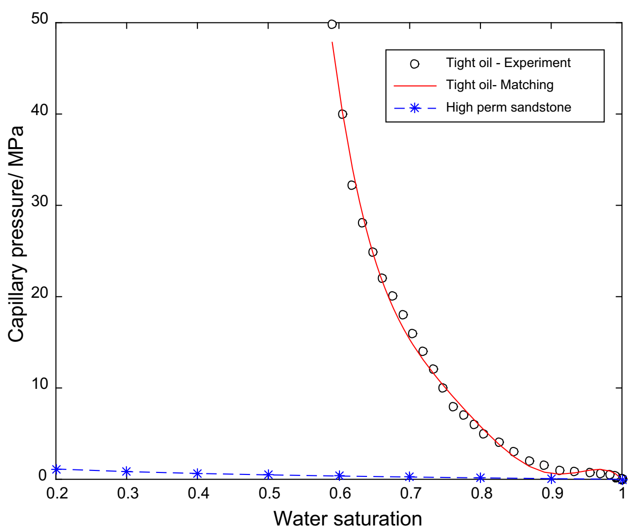

Fig. 3 Capillary pressure of tight formation and high-permeability sandstone

Solution for the model

To solve the partial differential equation, we introduced a dimensionless length Z and dimensionless time T to make this PDE to be dimensionless.

$$Z = \frac{z}{H}, \quad T = \frac{\sigma}{\mu_{\mathrm{w}} H^2}\sqrt{\frac{K}{\varphi}}t \tag{20}$$

where, z is imbibition height, m; H is the total model height, m; Z is the dimensionless height.

There are 3 unknown factors in formula (9), $\frac{\partial S_{\mathrm{w}}}{\partial t}$, $\frac{\partial P_{\mathrm{c}}}{\partial z}$ and $\frac{\partial v_{\mathrm{w}}}{\partial z}$, can be transformed into function of S and T as formula (21–23), respectively.

$$\frac{\partial S_{\mathrm{w}}}{\partial t} = \frac{\partial S_{\mathrm{w}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}t} = (1 - S_{\mathrm{rnw}} - S_{\mathrm{iw}})\frac{\partial S}{\partial T}\frac{\sigma}{\mu_{\mathrm{w}} H^2}\sqrt{\frac{K}{\varphi}} \tag{21}$$

$$\frac{\partial P_{\mathrm{c}}}{\partial z} = \frac{\partial P_{\mathrm{c}}}{\partial Z}\frac{\mathrm{d}Z}{\mathrm{d}z} = \frac{1}{H}\frac{\partial P_{\mathrm{c}}}{\partial Z} = \frac{\sigma}{H}\sqrt{\frac{K}{\varphi}}\frac{\mathrm{d}J(S)}{\mathrm{d}Z} = \frac{\sigma}{H}\sqrt{\frac{K}{\varphi}}\frac{\mathrm{d}J(S)}{\mathrm{d}S}\frac{\partial S}{\partial Z} \tag{22}$$

$$\frac{\partial v_{\mathrm{w}}}{\partial z} = \frac{\partial v_{\mathrm{w}}}{\partial Z}\frac{\mathrm{d}Z}{\mathrm{d}z} = \frac{1}{H}\frac{\partial v_{\mathrm{w}}}{\partial Z} = \frac{1}{H}\frac{\partial}{\partial Z}\left(\frac{Kk_{\mathrm{rw}}^0 S^{\mathrm{a}} k_{\mathrm{rnw}}^0 (1-S)^{\mathrm{b}}}{k_{\mathrm{rw}}^0 \mu_{\mathrm{nw}}^{\mathrm{a}} S^{\mathrm{a}} + k_{\mathrm{rnw}}^0 \mu_w (1-S)^{\mathrm{b}}}\left(\frac{\partial P_{\mathrm{c}}}{\partial z} - \Delta\rho g\right)\right) \tag{23}$$

Substituting formula (21), (22) and (23) into formula (9), we can derive a general function between normalized water saturation (S) and dimensionless time (T), dimensionless height (Z).

$$\frac{\partial S}{\partial T} + \frac{\mu_{\mathrm{w}}}{1 - S_{\mathrm{rnw}} - S_{\mathrm{iw}}}\frac{\partial}{\partial Z} \times \left(\frac{Kk_{\mathrm{rw}}^0 S^{\mathrm{a}} k_{\mathrm{rnw}}^0 (1-S)^{\mathrm{b}}}{k_{\mathrm{rw}}^0 \mu_{\mathrm{nw}} S^{\mathrm{a}} + k_{rnw}^0 \mu_{\mathrm{w}} (1-S)^{b}}\left(\frac{\mathrm{d}J(S)}{\mathrm{d}S}\frac{\partial S}{\partial Z} - N_{\mathrm{B}}\right)\right) = 0 \tag{24}$$

where,

$$N_{\mathrm{B}}^{-1} = \frac{\sigma\sqrt{\frac{\varphi}{K}}}{\Delta\rho g H} \tag{25}$$

NB^{-1} derived in this paper is the same as Schechter et al. (1991) result. It can be described as the ratio of capillary pressure and gravity, a parameter to describe the contribution of capillary pressure and gravity to imbibition. Schechter has a conclusion that capillary pressure is dominated when NB^{-1} is larger than 5, while gravity is dominated when NB^{-1} is less than 0.2.

At the same time, initial condition, Neumann boundary condition and Dirichlet boundary condition of model (b) (Fig. 1b) can be simplified as:

$$S = 0, \quad T = 0, \quad 0 \le Z \le 1 \tag{26}$$

$$S = 1, \quad T \ge 0, \quad Z = 0 \tag{27}$$

$$\frac{\mathrm{d}S}{\mathrm{d}Z} = 0, \quad T \ge 0, \quad Z = 1 \tag{28}$$

While initial condition and boundary condition of model (a) and model (c) (Fig. 1a–c) can be simplified as:

$$S = 0, \quad T = 0, \quad 0 \le Z \le 1 \tag{29}$$

$$\frac{\mathrm{d}S}{\mathrm{d}Z} = 0, \quad T \ge 0, \quad Z = 0 \tag{30}$$

$$S = 1, \quad T \ge 0, \quad Z = 1 \tag{31}$$

Model verification and discussion

Commercial numerical reservoir simulator ECLIPSE (E100) is chosen as a verification. Horizontal flow and vertical flow model were built as shown in Fig. 4. The first grid of this model is wetting phase (water) to act as the initial water saturation at the position of $x = 0$. Grid 2 ~ 100 is non-wetting phase to represent the water saturation at the position of $x > 0$. Relative permeability and capillary pressure are the same as our model. Some other parameters are shown in Table 1.

From Fig. 5, we can see that saturation front solved by the model corresponds to the result solved by ECLIPSE. That is a verification of the accuracy of our model.

Figure 6 is a comparison between high-permeability sandstone imbibition and tight oil imbibition. We can see that because of the high capillary pressure, saturation front of tight formation moves faster than high-permeability sandstone. This means imbibition of tight formation is much greater than sandstone.

Figures 7 and 8 describe the saturation front of gravity assisting imbibition (*model b*) in high-permeability sandstone and tight formation. We can find that: (1) Saturation front of tight oil reservoir moves faster than conventional high-permeability sandstone because of high capillary pressure, which indicates a greater imbibition of tight formation. (2) Gravity shows an obvious influence in conventional high-permeability sandstone, which shows little influence in tight reservoir. (3) Influence of gravity is not obvious at the beginning, which makes a highlight as time goes on (1 h, 20 days, 80 days and 400 days).

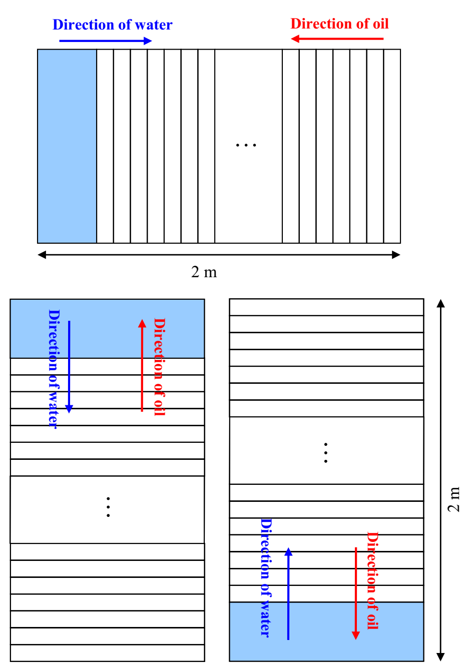

Fig. 4 Numerical model in eclipse

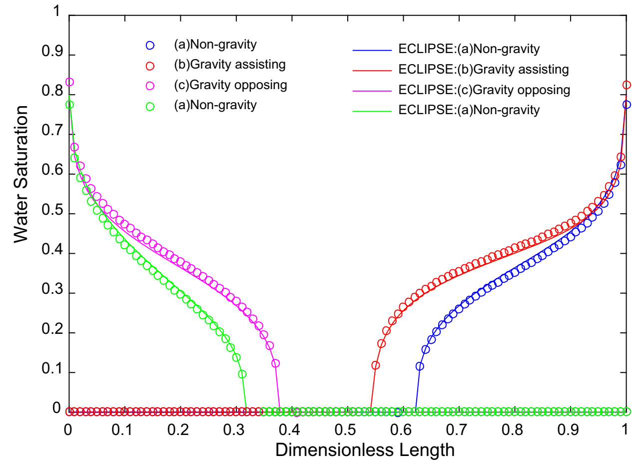

Fig. 5 Matching of the model and eclipse

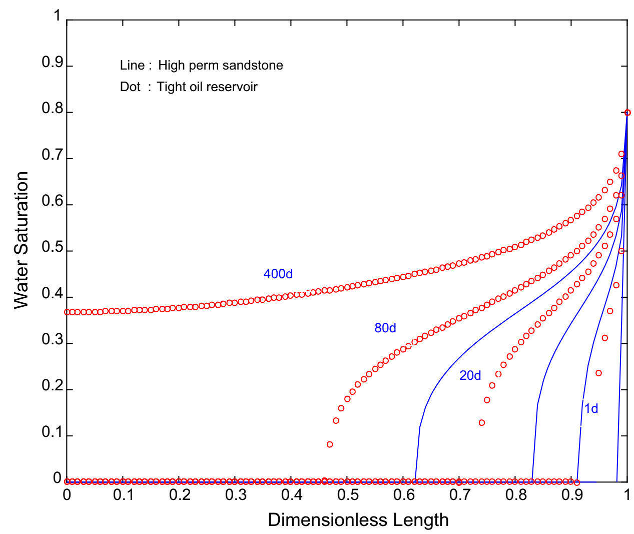

Fig. 6 Saturation front of tight oil and sand formation

Figures 9 and 10 describe the saturation front of gravity opposing imbibition (*model c*) in tight formation and high-permeability sandstone. Except above conclusions, we can also find that: (1) Influence of capillary pressure is much greater than gravity both for gravity assisting imbibition and gravity opposing imbibition. (2) Saturation front moves faster in gravity assisting imbibition than in gravity opposing imbibition in both kinds of rocks.

In 2012, after 6 months' shut-in after hydraulic fracturing, a shale gas well in Marcellus (Cheng 2012) extracted a great amount of gas while a little amount of water. This has inspired researcher to investigate the imbibition mechanism during shut-in periods. We have find that, (1) Imbibition distance of conventional high-permeability sandstone is about 2, 16 and 34 cm for 1, 20 and 80 days. Oil recovery via imbibition in this kind of

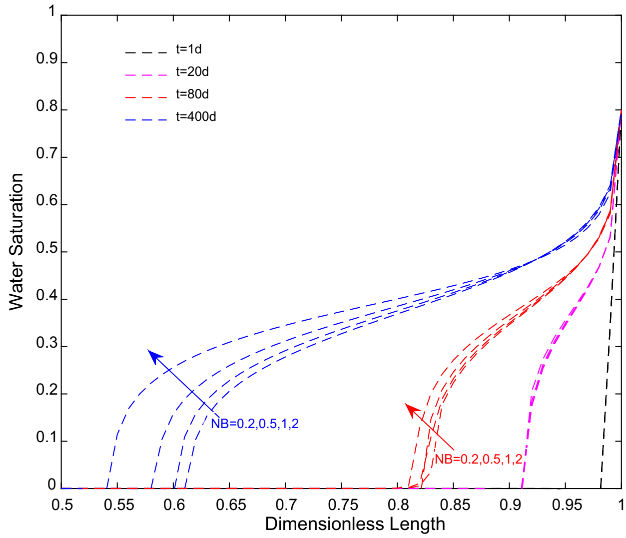

Fig. 7 Saturation front of gravity assisting imbibition (sandstone)

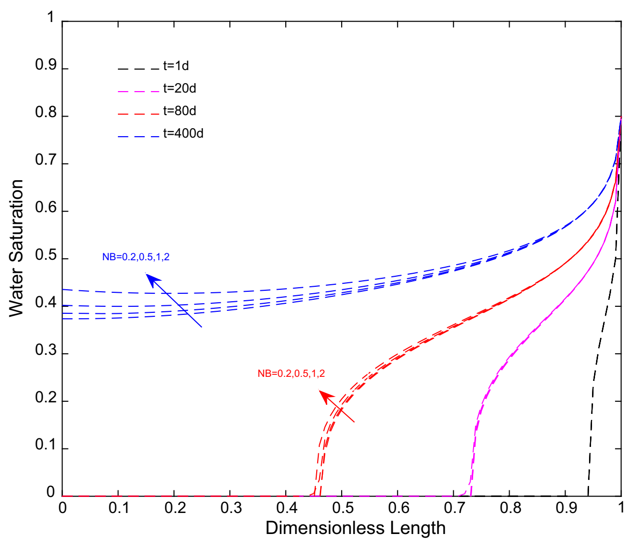

Fig. 8 Saturation front of gravity assisting imbibition (tight oil)

formation is little because of short imbibition distance. (2) While for tight formation, imbibition distance is about 10, 54 and 106 cm in 1, 20 and 80 days. This is a much longer distance and will result in a great oil recovery.

Horizontal well multi-stage fracturing and massive fracturing method not only increase fracture area and fracture volume but also increase the contact area between formation and the wetting fluids injected. These injected chemical fluids can result in a change of interfacial tension and wettability, utilization of imbibition mechanism caused by them will result in a good EOR performance.

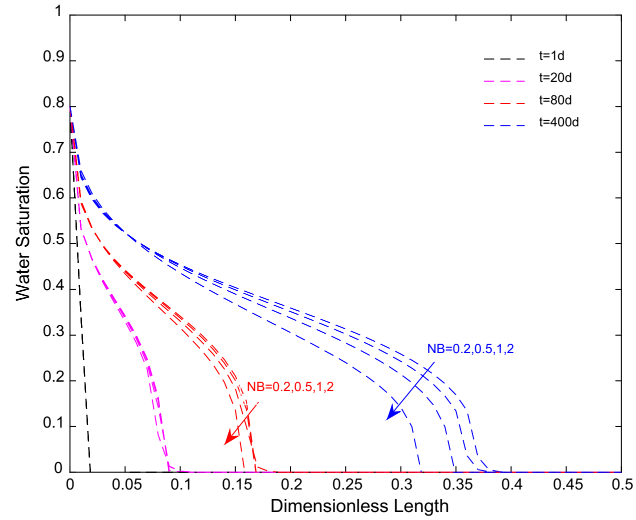

Fig. 9 Saturation front of gravity opposing imbibition (sandstone)

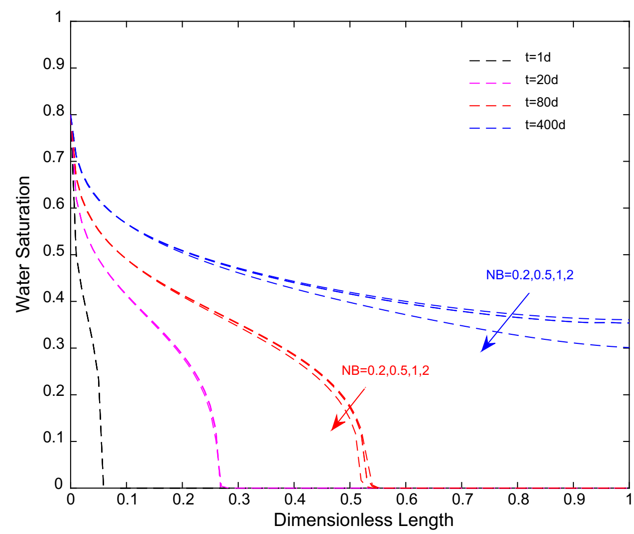

Fig. 10 Saturation front of gravity opposing imbibition (tight oil)

Conclusion

1. Considering characteristics of tight formation, a one-dimensional two-phase counter-current imbibition model for tight formation was derived; after dimensionless, the partial differential equation was solved by Galerkin spatial dispersion and temporal difference. Solution for three different boundary conditions and gravity conditions were given.
2. Imbibition can be divided into gravity assisting, gravity opposing and zero gravity in terms of different gravity conditions. Imbibition of tight formation is much greater than sandstone because of the high capillary pressure.

3. Capillary pressure plays the dominant role in imbibition. Influence of gravity is much greater in high-permeability sandstone than in tight formation.
4. Horizontal well multi-stage fracturing and massive fracturing can increase fracture area and fracture volume and increase the contact area with wetting phase, which may result in a greater oil recovery with the utilization of imbibition mechanism. Imbibition distance may be a reference for engineers to design fracture spacing in horizontal well's hydraulic fracturing.

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