

# Application of the fractal theory for evaluating effects of coal comminution by waterjet

Yaqing Li · Greg Galecki · Gul Akar · Sezai Sen · Yutao Zhang

Received: 17 July 2014/Revised: 8 November 2014/Accepted: 11 November 2014/Published online: 25 December 2014  
© The Author(s) 2014. This article is published with open access at Springerlink.com

**Abstract** Comminution of coal to ultrafine sizes by high-pressure waterjet provides a novel method for preparation of coal-water fuels for next generation, near-zero emission electric power generation. The particle size distribution (PSD) of ground coal is a key parameter in the preparation of slurries as it determines the settling behavior of the particles and viscosity of the coal-water mixture. There are several methods available for representation and evaluation of particle size analysis data. However, fractal theory provides a means by which the entire PSD of comminuted materials can be quantified by using of a specific and exact value. In this paper, a volume-based fractal model was deduced to characterize the PSD of the coal which is ground in a specially designed comminution cell. During the size reduction process, the inlet pressures up to 276 MPa were used.

**Keywords** Coal comminution · Fractal theory · Particle size distribution · Waterjet

## 1 Introduction

Crushing and grinding coal to ultrafine sizes is the main requirement for successful substitution of oil with coal. Additionally, it also plays an important role in the reduction of impurities. As a result of the comminution mechanism employed, conventional comminution processes are not energy efficient. It has been shown that more than 96 %

of energy consumed in current comminution processes is wasted (Cui et al. 2006). As a result, the sustainable development and utilization of coal requires more efficient comminution methods.

Since most brittle materials have higher compressive strength than tensile strength, fragmentation of brittle minerals through the development of tensile stresses within the minerals is more economical. This is particularly true for coal, which has an internal structure pervaded by small micro-cracks and fissures. However, this fact has not been utilized until high-pressure waterjets were applied for the purpose of coal comminution (Galecki and Mazurkiewicz 1998).

Since its introduction by Galecki and Mazurkiewicz (1987, 1998), many valuable views about high-pressure waterjet comminution have been presented. Fu et al. (2001) adopted the idea of comminution with waterjets and provided comprehensive analysis of the comminution mechanism. Based on findings from their research, they characterized high-pressure waterjet comminution as “high-efficiency, clean, low energy-consumption, and a promising new comminution engineering”. Hou and Sun (2003) characterized high-pressure waterjet comminution as a selective and efficient grinding process. Cui et al. (2006) shared the same view. Liu and Sun

---

Y. Li · Y. Zhang  
Key Lab of Mining and Disaster Prevention and Control in  
Western Mine, Ministry of Education, Xi'an, 710054, China

Y. Li (✉) · Y. Zhang  
School of Energy Science and Engineering, Xi'an University of  
Science and Technology, Xi'an 710054, China  
e-mail: lyqcumt@126.com

G. Galecki  
Department of Mining and Nuclear Engineering, Missouri  
University of Science and Technology, Rolla, MO 65401, USA

G. Akar · S. Sen  
Department of Mining Engineering, Dokuz Eylul University,  
Izmir, Turkey

(2005) pointed out the potential of a high-pressure fluid jet as a tool for comminuting thermally sensitive, inflammable, and explosive materials.

Cui et al. (2007) used a high pressure waterjet for ultra-clean micronized coal slurry preparation. Their studies included a comparison of combustible matter recovery, ash content of the clean coal, separation efficiency, and energy consumption between the new and traditional coal preparation process. They had encouraging results which led them to propose a process for preparation of ultra-clean superfine coal–oil slurry (Cui et al. 2008). Results obtained from their studies validated the unique advantages of high-pressure waterjet comminution for developing coal slurry as a viable alternative energy resource.

To increase the particle size reduction during comminution with waterjets, the slurry was injected into a cavitation chamber in our study. Size reduction in a cavitation cell then is the result of the combined effects of rapid dynamic shear stress, cavitation bubble growth and collapse, and direct impact of particles against a rigid anvil. Some results of recent studies are overviewed by Galecki et al. (2011).

Usually, the particle size analysis results are graphically presented using a particle size distribution (PSD) plot to characterize the process of particle segregation into pre-defined categories (Hyslip and Vallejo 1997). This is a very useful and widely accepted method for presenting the particle size distribution. However, quantification of PSD by only considering a few points such as  $P_{80}$  or  $P_{50}$ , or a limited portion of the PSD curve has inherent limitations. Therefore the search for a more versatile method of presenting comminution results is of interest. A solution to this problem was fractal theory which provides a means by which the entire PSD of comminuted materials may be quantified through a specific and exact value.

Fractal theory was proposed by Mandelbrot (1977) to characterize some irregular, unsmooth, and non-differentiable objects or shapes in nature. The concept of fractal theory and its applications were systematically described by Mandelbrot (1982) in the book of *The fractal geometry of nature*. Fractal theory differs from Euclidean geometry as fractal theory states that the dimension ( $D$ ) of an object is not necessarily an integer but also can be a specific fraction varying from 0 to 3 depending on true fractal sets (Mandelbrot 1982).

One of the first applications of this theory was in animal and plant morphology studies to characterize the complexity of neurons and the shapes of glia cells (Smith et al. 1989; Neale et al. 1993; Corbit and Garbary 1995). On a larger scale, fractal theory was used to characterize the complexity of the habitats (Morse et al. 1985; Gunnarsson 1992; Gee and Warwick 1994). Fractal theory was also successfully used to characterize particle, pore, and

aggregate size distribution in soils (Bartoli et al. 1991; Rieu and Sposito 1991; Perfect et al. 1992; Crawford and Young 1993; Wu et al. 1993; Kozak et al. 1996). More recent successful applications of fractal theory were in roughness, pore size distribution, and adsorption behavior characterization of porous media such as coal (Friesen and Ogunsola 1995; Zhang and Li 1995; Huang et al. 2003). As defined by Mandelbrot (1982), fractals are hierarchical and very often highly irregular, geometric systems and as a novel tool for comminution products analysis, fractal theory was applied for homogeneous and heterogeneous materials (Carpinteri and Pugno 2002a, b). A different aspect of fractal theory use was its application for quantitative measurement of fractures and faults (Boadu and Long 1994).

Basically, there are two types of fractal theories: self-similar fractal and self-affine fractal. Since the self-similarity of coal comminution using fractal PSD analysis was demonstrated by Zeng et al. (1999), self-similar fractal is widely used in comminution engineering. Cui et al. (2006) employed fractal PSD to analyze the fineness of comminution products by waterjet. They demonstrated the use of a single parameter applied for the characterization of comminution products. Tasdemir (2009) proved that the fragmentation processes of chromite ores can be quantified by using the fractal dimension of the PSDs. At the moment, most applications of the fractal concept for PSD analysis are based on the model developed by Turcotte (1986). In this model, the fractal dimension appears as an exponent in the relationship for expressing the cumulative number or mass of particles as a function of particle sizes (Cui et al. 2006; Tasdemir 2009).

The use of laser particle size analyzers is a very convenient method for PSD analysis of fine and ultrafine coal particles generated by the waterjet mill. It provides volume based size distribution of particles, which can be used for calculation of the fractal dimension. However, no volume-based model has been constructed to assess the size and size distribution of the product of disintegration. Therefore, a volume-based fractal model is proposed to characterize PSD of coal ground in a high-pressure waterjet mill coupled with a cavitation cell in this research. Grinding experiments for the range of inlet pressures up to 276 MPa was carried out to validate the model. Additionally, the proposed fractal model was applied to investigate the effect of the inlet pressure on the size distribution.

## 2 Fractal model for particle size distribution

According to self-similar fractal theory, the size distribution of elements in a fractal system is given by the following equation (Chen 2005):

$$n_{(R \geq r)} = cr^{-D} \quad (1)$$

where,  $r$  is a defined equivalent radius of particles;  $n_{(R \geq r)}$  is the number of particles greater than or equal to  $r$  in radius;  $c$  is a constant and  $D$  is the fractal dimension.

When  $r$  is equal to the minimum radius  $r_0$ , the total number of fragments  $n_0$  with the minimum radius is:

$$n_0 = cr_0^{-D} \quad (2)$$

Combining Eqs. (1) and (2) gives the following relationship.

$$\frac{n_{(R \geq r)}}{n_0} = \left(\frac{r}{r_0}\right)^{-D} \quad (3a)$$

or

$$n_{(R \geq r)} = n_0 \left(\frac{r}{r_0}\right)^{-D} \quad (3b)$$

After derivation, Eq. (3) becomes,

$$dn_{(R \geq r)} = -n_0 r_0^D D r^{-(D+1)} dr \quad (4)$$

The cumulative volume of particles whose radius greater than  $r$  is:

$$dV_{(R \geq r)} = kr^3 dn_{(R \geq r)} \quad (5)$$

Substituting Eq. (4) into Eq. (5) yields,

$$dV_{(R \geq r)} = -kn_0 r_0^D D r^{2-D} dr \quad (6)$$

where,  $k$  is the coefficient of volume. Then, Eq. (7) can be formulated by taking integration of Eq. (6) as following:

$$\int_{V_0}^{V_{(R \geq r)}} dV_{(R \geq r)} = \int_{r_0}^r -kn_0 r_0^D D r^{2-D} dr \quad (7)$$

Mathematically, the relationship between accumulated volume of particles whose radius is greater than defined radius  $r$  and the total volume of all particles yields the following equation,

$$V_0 - V_{(R \geq r)} = \frac{kn_0 r_0^D D}{3-D} r^{3-D} \quad (8)$$

where,  $V_0$  is the accumulated total volume of all fragments;  $V_{(R \geq r)}$  is the volume of particles whose radius is greater than  $r$ ; The term  $\frac{kn_0 r_0^D D}{3-D}$  is the coefficient depending on the properties of materials, and will be referred as  $K_v$  later. Since  $K_v$  is a constant for a given coal sample, then Eq. (8) simplifies to

$$V_0 P \% = K_v r^{3-D} \quad (9)$$

where,  $P$  % is the accumulative volume percentage of particles with radius smaller than  $r$ . Then Eq. (9) can be transformed into a linear relationship as,

$$\text{Log } P = (3 - D) \log r + \log K_v - \log V_0 + 2 \quad (10a)$$

Further this equation can be simplified to the form,

$$\text{Log } P = a \log r + \log K \quad (10b)$$

where,  $a$  is the slope coefficient of the linear regression lines. If the size distribution of the fragments is fractal,  $\log P$  should correlate to  $\log r$  linearly. Each fractal dimension  $D$  that represents a particular PSD could be calculated from the slope of the best-fit linear regression line using Eq. (11),

$$D = 3 - a \quad (11)$$

### 3 Materials and methods

#### 3.1 Sample selection and preparation

In these experiments a low-ash bituminous coal with 2.83 % ash content was used. Samples were prepared using two-step crushing. First, run-of-mine coal was crushed to minus 5 mm by a jaw crusher and a 2,000 g sample was collected. Then this amount of coal was processed in a second run through another crusher to collect 500 g of a representative sample in size minus 850 microns.

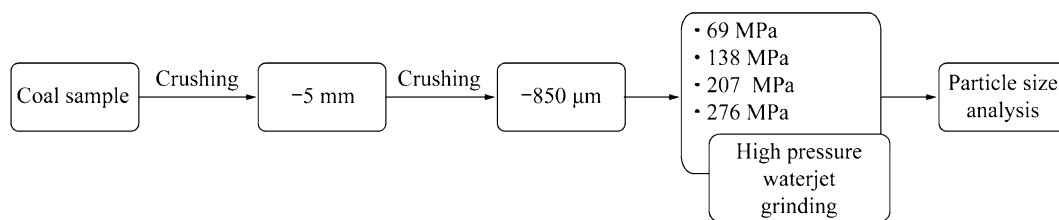
#### 3.2 Experimental procedure and apparatus

The experiments were carried out in a specially designed cavitation cell. Inlet pressure and standoff distance were the parameters. The standoff distance, which is defined as the distance from the mixing nozzle to the anvil, was kept constant at 19 mm in this study. For measuring the product particle size, a laser diffraction Microtrac S3500 series particle size analyzer was employed. The flow-sheet of experiments and the schematic of the high-pressure waterjet mill system are illustrated in Figs. 1 and 2.

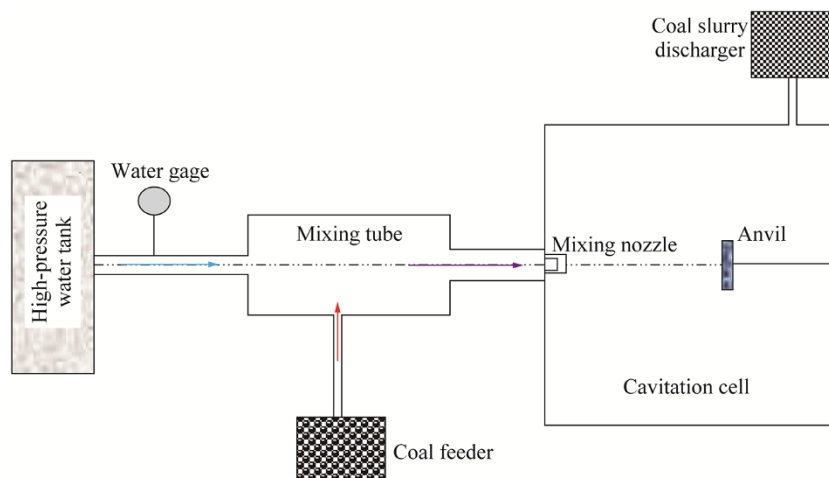
### 4 Results and discussion

In order to use the fractal model for particle size distribution, data obtained from the Microtrac S3500 size analyzer was presented on  $\log P$ – $\log r$  scale, Fig. 3. Curves in Fig. 3 represent PSD as a function of inlet pressure varied in this series of experiments.

When the PSD curve is presented on  $\log$ – $\log$  scale, it consists of two linear segments, as introduced by Tasdemir (2009). Following his interpretation, these segments represent fine and coarse particle fractal domains. This method was adopted for data analysis of coal comminution by waterjets with the exception that we introduced a volume-based fractal model instead of a mass-based model used by Tasdemir.



**Fig. 1** The flow-sheet of experiment



**Fig. 2** Schematic of the high-pressure waterjet mill

Since the data presented on log–log scale consisted of two linear segments, each slope represented a different fractal domain. The first segment represents the fine particles. This segment was determined by solving the highest linear regression coefficient ( $R^2$ ) for contiguous data points starting from the first data point. Then the remaining data were fit into a line representing the second domain. The slopes of these two segments were used in calculating the fractal dimensions of each domain,  $D_1$  and  $D_2$  as are given as in Table 1.

According to previous studies by Lu et al. (2003) and Cui et al. (2006), each fractal dimension indicates a particular PSD with the understanding that the higher fractal dimension indicates higher amount of fine particles. As suggested by Turcotte (1986) and Cui et al. (2006), the fractal dimensions in material fragmentation should be within the range of 1.44–3. The experimental results presented in Table 1 support this statement.

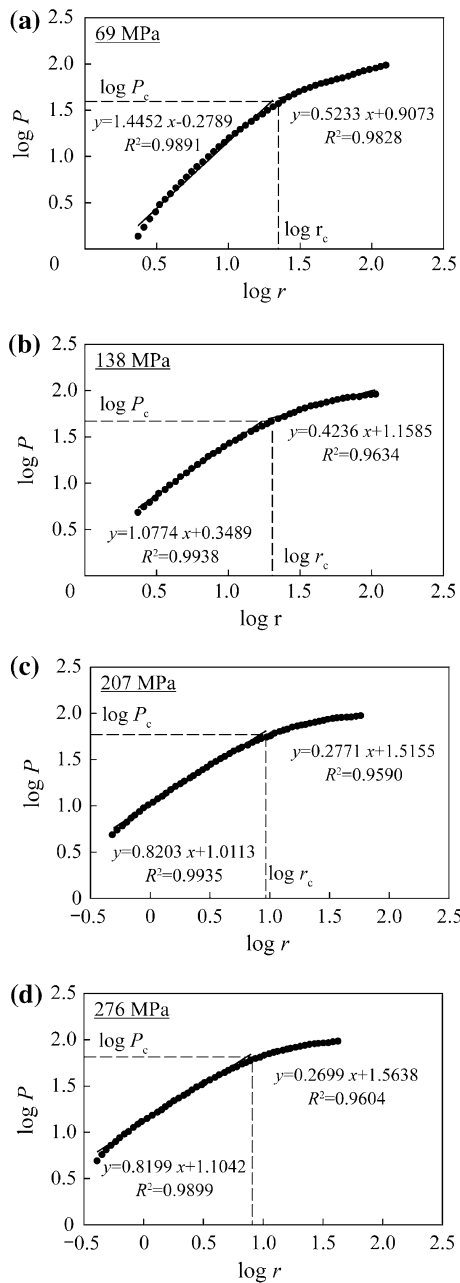
Data presented in Table 1, is the summary of fractal dimensions of  $D_1$  and  $D_2$ , regression coefficient  $R^2$ , cut-off point  $\log r_c$ , and corresponding  $\log P_c$  that will be used in analysis of particle size distribution. Domains  $D_1$  and  $D_2$  show that the fractal dimensions of the product increase with the increasing inlet pressure. This is important for applications that require a finer product.

Data listed in Table 1 show that fractal dimensions in the first domain ( $D_1$ ) range from 1.6101 to 2.1801. For the same experimental condition, fractal dimensions in the

second domain ( $D_2$ ) are higher, ranging from 2.4767 to 2.7301. As introduced and experimentally confirmed by Carpinteri and Pugno (2002a, b) and Tasdemir (2009), the first domain represents fine particles generated by the surface-dominated mechanism while the particles in the second domain are coarser and resulted from volume-dominated size reduction mechanism. Since the existence of two fractal domains was the indication of two breakage mechanisms, this two—domain model established is applicable for all comminution products generated from two breakage mechanisms. According to Palaniandy et al. (2008), the breakage mode of minerals is either destructive (volume-dominated) or abrasion (surface-dominated). Combining his explanation with the analysis of data listed in Table 1, it can be stated that the fractal domains of the PSD should not exceed two.

Further analysis of the results presented in Table 1 show that the increased inlet pressure is associated with an increase in the volume percentage of particles ( $\log P_c$ ) smaller than the cut-off point size ( $\log r_c$ ). The relationship between the fractal dimension of the products and inlet pressure is also depicted in Fig. 4.

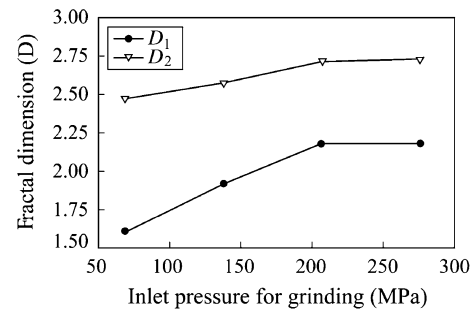
According to Fig. 4, fractal dimensions of products increase with increasing inlet pressure. However, in the same range of pressures, the upward trend is more obvious for the fractal dimension in the first domain ( $D_1$ ). This finding suggests that the surface-dominated phenomenon is



**Fig. 3** Regressive curves of coal particle size distribution with different pressure

**Table 1** Fractal dimension of products ground by cavitation cell under different inlet pressure

Inlet pressure (MPa)	First domain		Second domain		Log $r_c$	Log $P_c$
	$D_1$	$R^2$	$D_2$	$R^2$		
69	1.6101	0.9891	2.4767	0.9828	1.3424	1.5684
138	1.9226	0.9938	2.5764	0.9634	1.2671	1.6569
207	2.1797	0.9935	2.7229	0.9590	1.0036	1.7678
276	2.1801	0.9604	2.7301	0.9604	0.9284	1.7921



**Fig. 4** Fractal dimensions of particle size distribution versus inlet pressure

the prevailing mechanism for the generation of fine particles.

The tendency of fractal dimensions to increase with increasing pressure was observed to the pressure level of 207 MPa. As previously stated by researchers (Cui et al. 2006), the relationship between the inlet pressure and the fractal dimension was not linear mostly due to the losses in the momentum transfer efficiency. Momentum transfer efficiency is a function of the inlet pressure to the mill and is associated with the level of turbulence induced, which also strongly depends on the inlet pressure. As an effect of these various factors, the relationship between the pressure and fractal dimension exhibits a diminishing trend, representing energy loss occurring. This phenomenon would be more visible when presented as process specific energy. The focus of this research was on analyzing particle size reduction. Efficiency of comminution with waterjet will be emphasized in future publications. It is a well-known fact that the PSD of coal comminuted by waterjet is also affected by its inherent characteristics, e.g., the mechanical characteristics, the pore structure, the sturdiness coefficient, etc. As it is emphasized in introduction that the focus of this paper is to propose and validate a fractal model representing the PSD of coal comminution by waterjet, the effects of inherent factors and comminution conditions will be comprehensively investigated using this established mode in further studies.

### 5 Conclusions

Experimental results analysis of this study was the basis for drawing the following conclusions:

- (1) The application of the proposed volume-based fractal model for characterizing the PSD using a single parameter was found to be very useful.
- (2) The proposed fractal model's use in characterizing the PSD of coal comminuted in a high-pressure waterjet mill was well validated by the experimental results.

- (3) Based on the proposed fractal model, it was found that the particle size distributions of the coal products comminuted by the high-pressure waterjet mill exhibited a bi-fractal performance.
- (4) The bi-fractal performance suggested that there were two mechanisms involved in size reduction in the high-pressure waterjet mill. One is the surface-dominated size reduction mechanism, the other is the volume-based size reduction mechanism. The surface-dominated mechanism predominantly produces smaller particles while the generation of relatively coarse particles is more dependent on the volume-dominated mechanism.
- (5) Experimental data suggested that, for the same domain, a higher fractal dimension indicates a higher amount of fine particles. It was also indicated that fractal dimensions of the products increase with inlet pressure rising.

**Acknowledgments** This research was supported by the Missouri University of Science and Technology/Waterjet Laboratory and funded by the China Postdoctoral Science Foundation (Grant No. 2014M552555XB) and Doctoral Program in Xi'an University of Science and Technology (Grant No. 2013QDJ039).

**Open Access** This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

## References

- Bartoli F, Philipp R, Doirisse M, Niquet S, Dubuit M (1991) Silty and sandy soil structure and self-similarity: the fractal approach. *J Soil Sci* 42:167–185
- Boadu FK, Long LT (1994) The fractal character of fracture spacing and RQD. *Int J Rock Mech Min Sci Geomech* 31(2):127–134
- Carpiniteri A, Pugno N (2002a) A fractal comminution approach to evaluate the drilling energy dissipation. *Int J Numer Anal Meth Geomech* 26:499–513
- Carpiniteri A, Pugno N (2002b) A multifractal comminution approach for drilling scaling laws. *Powder Technol* 131:93–98
- Chen Y (2005) *Fractal geometry*, 2nd edn. The Earthquake Press, Beijing, p 289 (In Chinese)
- Corbit JD, Garbary DJ (1995) Fractal dimension as a quantitative measure of complexity in plant development. *Biol Sci* 262(1363):1–6
- Crawford LW, Young IM (1993) Quantification of fungal morphology, gaseous transport and microbial dynamics in soil: an integrated framework utilizing fractal geometry. *Geoderma* 56:157–172
- Cui L, An L, Gong W (2006) Effects of process parameters on the comminution capacity of high pressure water jet mill. *Int J Miner Process* 81:113–121
- Cui L, An LQ, Gong WL, Jiang H (2007) A novel process for preparation of ultra-clean micronized coal by high pressure water-jet comminution technique. *Fuel* 86:750–757
- Cui L, An L, Jiang H (2008) A novel process for preparation of an ultra-clean superfine coal-oil slurry. *Fuel* 87:2296–2303
- Friesen WI, Ogunsola OI (1995) Mercury porosimetry of upgraded western canadian coals. *Fuel* 74(4):604–609
- Fu S, Duan X, Gao Y (2001) Development of high-pressure water jet comminution. *J Coal Sci Eng* 1 29(1):1–4 (In Chinese with English abstract)
- Galecki G, Mazurkiewicz M (1987) Effectiveness of coal comminution by high pressure waterjet. In: *Proceedings of the 8th international conference on alternative energy sources*. Miami Beach, December 1987
- Galecki G, Mazurkiewicz M (1998) *Comminution by waterjets*. In: Momber A (ed) *Book of water jet applications in construction engineering*. A. Balkema Publishers Press, Rotterdam, p 424
- Galecki G, Akar G, Sen S, Li YQ (2011) Enhanced cleaning of the coal feedstock for power generation, In: *Proceedings of the mining engineering conference on innovations in mining engineering*. Rolla, 30 August 1 September 2011
- Gee JM, Warwick RM (1994) Metazoan community structure in relation to the fractal dimensions of marine macroalgae. *Mar Ecol Prog Ser* 103:141–150
- Gunnarsson BB (1992) Fractal dimension of plants and body size distribution in spiders. *Funct Ecol* 6:636–641
- Hou S, Sun Z (2003) High-pressure water jet technology. *J Coal Sci Technol* 29:1–4 (In Chinese with English abstract)
- Huang G, Xu S, Li X (2003) Characterizations of PSD fractal of porous medium. *Trans Tianjin Univ* 9:170–173
- Hyslip PJ, Vallejo EL (1997) Fractal analysis of the roughness and size distribution of granular materials. *Eng Geol* 48:231–244
- Kozak E, Ya A, Sokolowski S, Sokolowska Z, Stepniewski W (1996) A modified number-based method for estimating fragmentation fractal dimensions of soils. *Soil Sci Soc Am J* 60:1291–1297
- Liu Z, Sun Z (2005) Wet comminution of raw salt using high-pressure fluid jet technology. *Powder Technol* 160:194–197
- Lu P, Jefferson IF, Rosenbaum MS, Smalley IJ (2003) Fractal characteristics of loess formation: evidence from laboratory experiments. *Eng Geol* 69:287–293
- Mandelbrot, BB (1977) *Fractals: form, chance and dimension*, W H Freeman and Co, San Francisco, ISBN 0716704730
- Mandelbrot BB (1982) *The fractal geometry of nature*. W.H. Freeman and Company Press, New York, p 497
- Morse DR, Lawton JH, Dodson M, Williamson MH (1985) Fractal dimension of vegetation and the distribution of arthropod body lengths. *Nature* 314:731–733
- Neale EA, Bowers LM, Smith TG (1993) Early dendrite development in spinal cord cell cultures: a quantitative study. *J Neurosci Res* 34:54–66
- Palaniandy A, Azizli KAM, Hussin H, Hasim SY, Azizli K (2008) Effect of operational parameters on the breakage mechanism of silica in a jet mill. *Miner Eng* 21:380–388
- Perfect E, Rasiah V, Kay BD (1992) Fractal dimensions of soil aggregate-size distributions calculated by number and mass. *Soil Sci Soc Am J* 56:1407–1409
- Rieu M, Sposito G (1991) Fractal fragmentation, soil porosity, and soil water properties: II application. *Soil Sci Soc Am J* 55:1239–1244
- Smith TG, Marks WB, Lange GD, Sheriff WH, Neale EA (1989) A fractal analysis of cell images. *J Neurosci Methods* 27(2):173–180
- Tasdemir A (2009) Fractal evaluation of particle size distributions of chromites in different comminution environments. *Miner Eng* 22:156–167
- Turcotte DL (1986) Fractals and fragmentation. *J Geophys Res* 91:1921–1926
- Wu Q, Borkovec M, Sticher H (1993) On particle-size distributions in soils. *Soil Sci Soc Am J* 57:883–890
- Zeng F, Wang Z (1999) The fractal characteristics of particle size distribution in coal grinding process. *Coal Conversion* 22(1):27–29 (In Chinese)
- Zhang B, Li S (1995) Determination of the surface fractal dimension for porous media by mercury porosimetry. *Ind Eng Chem Res* 34:1383–1386