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NHL expansion and fan allegiance: a mathematical modelling study

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Abstract
Expansion of the NHL hockey league is of current interest. Success of an expansion team will rely on approved geographic locations, and most importantly fan support. Fan allegiance can be likened to infectious disease transmission, whereby individuals are susceptible, infected and recovered from a current team's hockey season. We employ an infectious disease model to assess the viability of certain Canadian and American cities that have been discussed as potential locations for NHL expansion in the past decade. We find that Quebec City presents an ideal location for expansion, where as, Las Vegas is suboptimal.

Introduction

Expansion of the National Hockey League (NHL) has been a major conversation topic of team owners, players and fans over the past decade [3, 26, 31, 34]. Many cities in Canada and the USA have been featured in local and national media outlets as potential locations. Expansion to add another team to the Greater Toronto Area has been discussed widely [3, 16], as well as expansions to Las Vegas, Seattle and Kansas City (among others) [3]. The 2011 relocation of the Atlanta Thrashers to Winnipeg (the team was renamed the Winnipeg Jets, a team that had been moved out of Winnipeg in the year 1996 to become the Phoenix Coyotes) has also led to speculation that Quebec City, a city that lost its NHL team in 1995 (the Quebec Nordiques were moved to form the Colorado Avalanche), could provide an ideal location for relocation or expansion [26].

Success of a new NHL team will depend on the locational quality of a city or geographic region that will be able to support a team through thick and thin. Currently, several existing NHL teams are struggling financially, which may be attributed to poor performance. However, it is possible for a team to struggle financially, despite performing well on the ice. For example, while NHL teams in Canada and the northeastern United States have fairly steady revenues, teams in the southern US regularly rank in the bottom of league revenue and attendance lists [7–11, 21–24, 29] - hockey may not be of interest to individuals living in cities in the sun-belt. Poor performance and poor locational quality simply lead to a lack of fans. NHL teams rely heavily on fan support to bring in revenue from ticket sales and merchandise purchases (note that the NHL is much more heavily reliant on its gate revenue than the NFL because of significantly less lucrative national television deals [2, 5, 19]). Thus fan allegiance is needed for an NHL team to ultimately survive.

We propose that the spread of fan allegiance can be likened to transmission of an infectious disease. An individual can be 'infected' through contact with fans in the population - typically due to the influence of family or friends, but individuals may also lose interest and recover or become immune to all future exposures. Also, in situations where there are multiple teams close by, a multi-strain model of infectious disease spread can be considered. We thus present a mathematical modelling study of fan allegiance using an infectious disease dynamical system. The model is then used to assess locations in Canada and the USA where NHL expansion may be successful.

In the sections that follow, we apply a Susceptible-Infected-Recovered modelling framework of fan allegiance so as to evaluate the locational quality of cities interested in housing new (or relocated) NHL teams. Model parameters are defined by the location attributes that have been shown to affect team financial success [17, 18]. The Basic Reproductive Ratio ($\mathcal{R}_0$), the number of new fans produced by one fan that is introduced into a totally susceptible population, is determined. Locations associated with $\mathcal{R}_0$ values greater than unity are deemed acceptable for NHL expansion.

Note that in the course of this work, the NHL announced a bidding process for expansion [32]. Two cities filed applications: Quebec City and Las Vegas [30]. These cities were previously included in our analysis, so no modifications to the modelling study were needed.

Methods

Model

We propose that fan allegiance to a sports team spreads mainly through contact with other fans. As a result, it is appropriate to use an epidemiological model to explain the transmission of fan allegiance in the NHL.

Standard SIR model

The basic Susceptible-Infected-Recovered model of infectious disease dynamics is given by:

$$\dot{S} = -\beta SI \tag{1}$$

$$\dot{I} = \beta SI - \gamma I \tag{2}$$

$$\dot{R} = \gamma I \tag{3}$$

$$N = S + I + R \tag{4}$$

with the basic reproductive ratio:

$$R_0 = \frac{\beta N}{\gamma}\,. \tag{5}$$

Here, $\beta > 0$ represents the mean transmission rate of the disease, and $1/\gamma, \gamma > 0$ represents the mean infectious period. It should be noted that for S, I, and R to represent proportions of the population, we set $N = 1$ in the last equation.

Expanding the model

In a 2014 survey, The Forum Poll asked respondents in Canada to rate their dedication to NHL hockey as 'part-time,' 'regular,' 'enthusiastic,' or 'extreme' [13]. Combining the part-time and regular fans in one group, and the enthusiastic and extreme fans in another, we expanded the classic SIR model to include a chronically infected compartment (C).

Essentially, once a person is infected by the 'disease' of fan allegiance, it is possible for this infected person to become a chronic fan of one team, beyond recovery. This compartment represents enthusiastic and extreme fans who are very devoted to their respective teams of choice and are unlikely to recover from this 'disease'.

The National Hockey League includes thirty teams, each of which draws fans at varying rates. For most NHL teams there exists little to no competition for fan allegiance among locals, as most teams are located far from each other geographically, in different and distinct cites. In New York City and Los Angeles the situation is different, however, with the LA Kings and Anaheim Ducks less than 100 km from each other, and the NY Rangers, NY Islanders and NJ Devils residing within a 50 km radius. We thus expand the model to consider one, two or three teams so that competition for fans can be realized. In the case of two teams the model is as follows:

$$\begin{aligned}
\dot{S} &= -S[\beta_1(I_1 + C_1) + \beta_2(I_2 + C_2)] \\
\dot{I_1} &= \beta_1 S(I_1 + C_1) - \gamma I_1 - \alpha_1 I_1 + \delta_{21}[(I_1 + C_1)I_2] - \delta_{12}[I_1(I_2 + C_2)] \\
\dot{I_2} &= \beta_2 S(I_2 + C_2) - \gamma I_2 - \alpha_2 I_2 + \delta_{12}[I_1(I_2 + C_2)] - \delta_{21}[(I_1 + C_1)I_2] \\
\dot{C_1} &= \alpha_1 C_1 \\
\dot{C_2} &= \alpha_2 C_2 \\
\dot{R} &= \gamma(I_1 + I_2) \\
N &= S + I_1 + I_2 + C_1 + C_2 + R
\end{aligned} \tag{6}$$

where N is the total number of susceptible individuals at the time an NHL expansion team is located in the geographic region of the city. A schematic of the model is provided in Fig. 1. Briefly, β and γ are as above; α represents the mean rate at which infection becomes chronic (i.e. the rate of movement from compartment I to compartment C); and δ represents the mean rate at which an individual infected with strain i becomes infected with strain j instead (i.e. 'switches' teams).

At the time of expansion it is expected that there will be no chronically infected individuals in the population, and that switching between teams will be negligible. Therefore, we can determine that the basic reproductive ratio of a new team in a geographic region will be given by

$$R_0 = \frac{\beta N}{\gamma}. \tag{7}$$

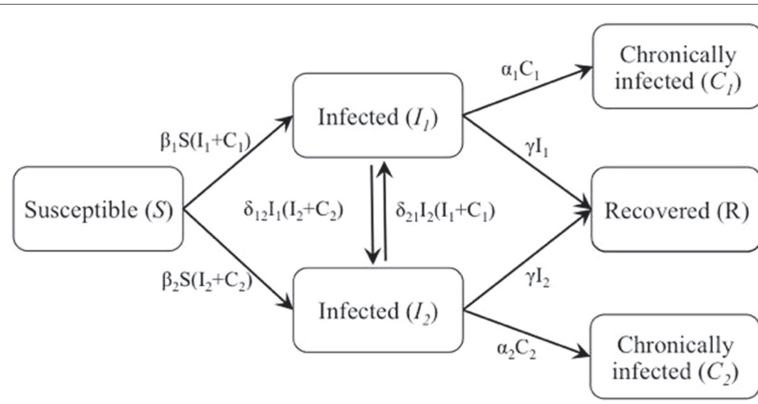

Fig. 1 Flow diagram

Note that demographics (birth and death) are ignored here. This permits the study of $\mathcal{R}_0$ for one NHL season.

Parameters

The mean rate of transmission of infection for team t, β_t, has several influencing factors. Firstly, team success is an important factor in fan allegiance since successful teams attract more fans, as seen during the 2015 playoff run of the Toronto Blue Jays [4]. Secondly, a team's physical location may affect its ability to attract fans for several reasons: a city with a larger population means there are more potential fans nearby; the average income of the population has an effect; and because hockey is a winter sport, cities with colder climates have more inhabitants who are hockey fans. We propose that the last reason holds because in colder climates, inhabitants of the city are more likely to play the sport themselves, whether casually or in an organized league. It is worth mentioning that many children in Canada and the northeastern United States grow up playing shinny hockey in winter and road hockey in summer. We suggest that this early interest in the sport often leads to following an NHL team in adulthood.

Finally, following professional sports leagues tends to be a social phenomenon [15]: most hockey fans have either played the sport themselves or have been influenced by family and friends to become a fan. Thus, a susceptible individual's social environment - their friends, family, and coworkers - is an important factor in determining the rate at which individuals become a fan of an NHL team. An individual whose social environment includes few hockey fans will be much less likely to become 'infected' than an individual whose close family and friends are 'chronic' hockey fans.

In order to take team success into account in determination of β, we focused on success in the NHL's post-season. Called the Stanley Cup playoffs, the post-season consists of four rounds, each a best-of-seven-games series. Many NHL players readily admit that winning the Stanley Cup is the only important type of success (e.g., "I can't hear what Jeremy says, because I've got my two Stanley Cup rings plugging my ears." - Patrick Roy, four-time Stanley Cup winning NHL goaltender from 1985 to 2003, in response to a comment made by Jeremy Roenick, NHL forward from 1988 to 2009, during the 1996 NHL playoffs. (Roenick did not win a Cup during his career)). We considered only the seasons from 2000-01[1] to 2014-15, the most recent completed season.[2]

However, since only one Stanley Cup is awarded for each season, there is simply not enough data to draw meaningful conclusions. We considered separately either playoff series wins or playoff series appearances (which includes series won and lost), but neither of these metrics gives a complete picture on its own. Any useful measurement of team success rates must take into account Stanley Cup wins, playoff series wins, and also playoff series appearances (see Table 1 and Fig. 2). Simply adding the number of Cup wins, series wins, and series appearances, however, will also not suffice, since appearances should not influence fans as much as wins, and playoff wins will not influence fans as much as Stanley Cup championships. Also, such simple addition may rank teams with more Stanley Cups lower than a team with less Cups, but more playoff appearances (e.g., Chicago and Detroit, Fig. 2).

Table 1 Revenue and playoff success for teams in the NHL

Financial and on-ice success of NHL teams				
Team	Revenue[a]	Playoff appearances[b]	Playoff wins[c]	Stanley cup wins[d]
Anaheim ducks	107	21	13	1
Arizona coyotes	80	6	2	0
Boston bruins	164	19	10	1
Buffalo sabres	103	10	5	0
Calgary flames	122	10	4	0
Carolina hurricanes	91	12	9	1
Chicago blackhawks	172	22	16	3
Colorado avalanche	104	16	9	1
Columbus blue Jackets	86	2	0	0
Dallas stars	113	11	4	0
Detroit red wings	134	29	17	2
Edmonton oilers	119	6	3	0
Florida panthers	83	2	0	0
Los Angeles kings	146	16	11	2
Minnesota wild	111	10	4	0
Montreal canadiens	187	18	8	0
Nashville predators	98	10	2	0
New Jersey devils	111	21	12	1
New York islanders	83	6	0	0
New York rangers	217	19	10	0
Ottawa senators	117	18	8	0
Philadelphia flyers	136	21	10	0
Pittsburgh penguins	141	22	13	1
St. Louis blues	98	13	4	0
San Jose sharks	117	23	11	0
Tampa Bay lightning	97	16	10	1
Toronto maple leafs	190	9	4	0
Vancouver canucks	154	18	7	0
Washington capitals	117	13	4	0
Winnipeg jets[e]	102	1	0	0
Mean	123.00	14.03	7.00	0.47

Notes: [a]in millions of USD, for the 2013–2014 season [24]; [b]Playoff series appeared in, 2001–2015 [14]; [c]Playoff series won, 2001–2015 [14]; [d]Stanley Cups won, 2001–2015 [14]; [c]Success values given for the Winnipeg Jets are calculated only from results obtained since the franchise moved to Winnipeg for the 2011-12 season

We therefore propose S_t as the weighted per-season success rating of team t for the seasons 2000-01 to 2014-15, given by

$$S_t = \frac{a \cdot PSA + b \cdot PSW + c \cdot SC}{N}, \tag{8}$$

where $N = 14$ is the number of seasons in our analysis, SC represents Stanley Cup wins, SW is playoff series wins in any round, SA is playoff series appearances; and the coefficients a, b, c give the relative significance or 'weight' of each type of success, as follows:

- a: Of the thirty teams in the NHL, sixteen play in the first round of the playoffs each season, so that each of these teams gains one playoff series appearance. Thus, $a = \frac{30}{16} = 1.875$.

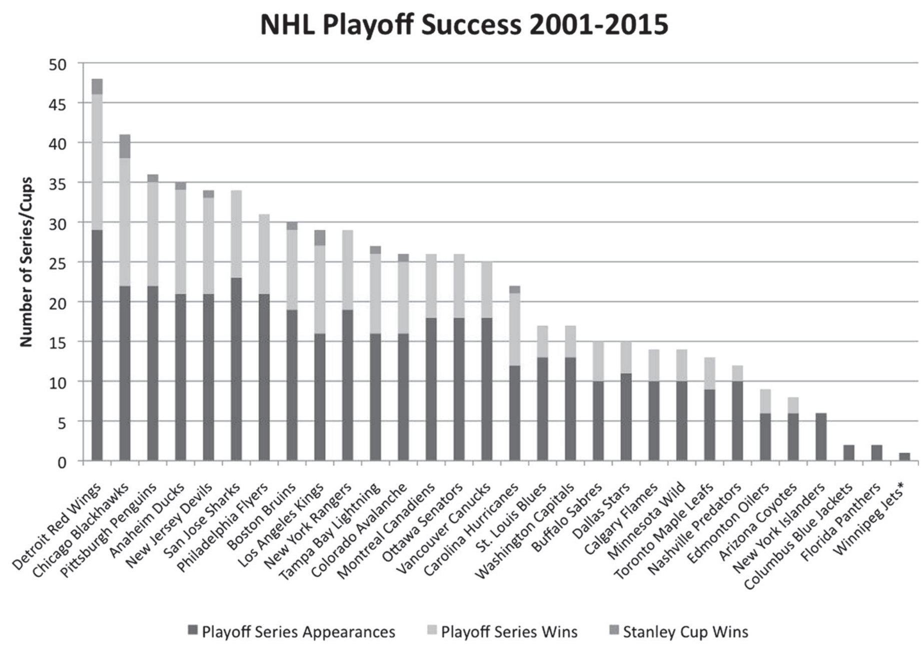

Fig. 2 A graph of playoff series appearances, playoff series wins, and Stanley Cup wins from 2001-15 for all thirty NHL teams (see Table 1). * Note: success values given for the Winnipeg Jets were calculated only from results obtained since the franchise moved to Winnipeg for the 2011-12 season

- b: Half of the teams in the first round of the playoffs win their respective series. Thus $b = 2 \cdot \frac{30}{16} = 3.75$, which is twice the value of a.
- c: Only one NHL team wins the Stanley Cup each season, so we take $c = \frac{30}{1} = 30$.

For comparison between teams the S_t values are divided by the mean team success rate μ_{S_t} to obtain

$$s_t = \frac{S_t}{\mu_{S_t}}. \tag{9}$$

Table 2 lists the success ratings for all thirty NHL teams using the Stanley Cup playoff data provided in Appendix: Table 1. Chicago is ranked first in the league with three Stanley Cup wins in the last 14 years, followed by Detroit and then Los Angeles, each with two championships.

To quantify the second influencing factor, team location, we slightly modified part of a statistical model given in Jones and Ferguson [18] that defined H_t, the locational quality of team t's home city, as a function of the city population size, mean income, and country. Jones and Ferguson defined H_t as:

$$H_t = \frac{1}{4}\frac{a_t^2}{b_t^2} \tag{10}$$

and

$$\log a_t = \alpha_0 + \alpha_1 CAN + \alpha_2 \log POP + \alpha_3 \log INC \tag{11}$$

$$\log b_t = \beta_0 + \beta_1 CAN + \beta_2 \log POP + \beta_3 \log INC \tag{12}$$

where CAN is a dummy variable with a value of 1 if the team is located in Canada and zero otherwise, POP is the city's population, and INC is the mean income in the city. Before

Table 2 Success ratings for teams in the NHL

	Success ratings of NHL teams	
Team	S_t[a]	s_t[b]
Anaheim ducks	8.4375	1.7722
Arizona coyotes	1.3393	0.2813
Boston bruins	7.3661	1.5471
Buffalo sabres	2.6786	0.5626
Calgary flames	2.4107	0.5063
Carolina hurricanes	6.1607	1.2940
Chicago blackhawks	13.6607	2.8692
Colorado avalanche	6.6964	1.4065
Columbus blue jackets	0.2679	0.0563
Dallas stars	2.5446	0.5345
Detroit red wings	12.7232	2.6723
Edmonton oilers	1.6071	0.3376
Florida panthers	0.2679	0.0563
Los Angeles kings	9.3750	1.9691
Minnesota wild	2.4107	0.5063
Montreal canadiens	4.5536	0.9564
Nashville predators	1.8750	0.3938
New Jersey devils	8.1696	1.7159
New York islanders	0.8036	0.1688
New York rangers	5.2232	1.0970
Ottawa senators	4.5536	0.9564
Philadelphia flyers	5.4911	1.1533
Pittsburgh penguins	8.5714	1.8003
St. Louis blues	2.8125	0.5907
San Jose sharks	6.0268	1.2658
Tampa bay lightning	6.9643	1.4627
Toronto maple leafs	2.2768	0.4782
Vancouver canucks	4.2857	0.9001
Washington capitals	2.8125	0.5907
Winnipeg jets[c]	0.4688	0.0985
Mean	4.7612	1.0000

Notes: [a]See Table 1 and Eq. (8); [b]See Eq. (9); [c]Success values given for the Winnipeg Jets are calculated only from results obtained since the franchise moved to Winnipeg for the 2011-12 season

applying the model in Jones and Ferguson to the thirty current NHL teams using current data, we first made one modification to the *CAN* variable. After looking at climate data for all current NHL cities [6], we chose to give a value of 1 for this variable to cities with climates similar to those in Canadian cities. We considered the average high temperature for each city's coldest month; in Vancouver, the mildest of the Canadian cities with an NHL team, this temperature is 6 °C. We took this value as our benchmark so that any team whose city had a coldest month average high temperature of 6 °C or less was given a *CAN* value of 1. We therefore changed the variable name from *CAN* to *WINT*. Parameter values a_t and b_t are listed in Table 3. Note that we used gross domestic product per capita as an approximation of income per capita, all in USD. The final modification made to the Jones and Ferguson model was to again normalize the locational qualities to obtain a mean value of 1, defining:

$$h_t = \frac{H_t}{\mu_{H_t}}, \tag{13}$$

Table 3 Locational qualities for teams in the NHL and selected other North American cities

	Locational qualities of NHL teams and selected other North American cities							
Team or City	Population (millions)[a]	GDP per capita[a]	Mean cold temp[b]	WINT[c]	a_t[d]	b_t[e]	H_t[f]	h_t[g]
Anaheim ducks	5.552807	$65,052.00	21	0.00	23872.2091	1256.5751	90.2294	0.7639
Arizona coyotes	4.459692	$46,430.00	20	0.00	26569.6757	1330.5856	99.6843	0.8439
Boston bruins	4.725601	$76,204.00	2	1.00	27631.2531	1251.2492	121.9140	1.0321
Buffalo sabres	1.133666	$64,090.00	0	1.00	25899.9405	1241.9735	108.7209	0.9204
Calgary flames	1.401800	$69,826.00	-1	1.00	25566.3899	1229.4164	108.1137	0.9153
Carolina hurricanes	1.244021	$51,729.00	11	0.00	22711.8760	1258.0512	81.4797	0.6898
Chicago blackhawks	9.568133	$58,861.00	-1	1.00	32472.5174	1339.6617	146.8864	1.2435
Colorado avalanche	2.746768	$61,795.00	6	1.00	28460.1760	1281.8863	123.2299	1.0432
Columbus blue jackets	1.988002	$54,780.00	3	1.00	28918.0283	1299.4250	123.8155	1.0482
Dallas stars	6.937652	$59,483.00	14	0.00	25195.9012	1285.8766	95.9845	0.8126
Detroit red wings	4.286134	$48,421.00	0	1.00	32487.1820	1358.8221	142.9022	1.2098
Edmonton oilers	1.320300	$62,832.00	-6	1.00	26458.4329	1251.9393	111.6611	0.9453
Florida panthers	5.905918	$44,480.00	25	0.00	27701.5572	1351.9469	104.9613	0.8886
Los Angeles kings	7.668163	$65,052.00	20	0.00	24583.7917	1267.9831	93.9747	0.7956
Minnesota wild	3.491620	$60,544.00	-5	1.00	29313.3379	1295.4964	127.9965	1.0836
Montreal canadiens	4.011200	$38,867.00	-5	1.00	35078.3776	1413.4876	153.9696	1.3035
Nashville predators	1.785349	$53,193.00	8	0.00	23225.5464	1264.1947	84.3809	0.7143
New Jersey devils	5.419961	$69,915.00	4	1.00	28900.5599	1276.5556	128.1366	1.0848
New York islanders	4.014786	$69,915.00	4	1.00	28121.9798	1265.8737	123.3818	1.0445
New York rangers	10.639183	$69,915.00	4	1.00	30729.9067	1300.8921	139.5018	1.1810
Ottawa senators	1.318400	$44,149.00	-6	1.00	30215.1591	1337.7141	127.5449	1.0798
Philadelphia flyers	6.060560	$57,166.00	5	1.00	31495.4880	1329.9238	140.2113	1.1870
Pittsburgh penguins	2.361681	$58,615.00	2	1.00	28635.7681	1289.2109	123.3417	1.0442
San Jose sharks	1.945539	$82,414.00	14	0.00	19849.6403	1167.1598	72.3077	0.6121
St. Louis blues	2.807760	$50,070.00	4	1.00	30868.6490	1334.4001	133.7837	1.1326
Tampa Bay lightning	2.895946	$44,999.00	21	0.00	25848.9340	1322.3520	95.5281	0.8087

Table 3 Locational qualities for teams in the NHL and selected other North American cities (*Continued*)

Toronto maple leafs	6.036800	$45,771.00	-1	1.00	34233.6201	1386.5012	152.4071	1.2902
Vancouver canucks	2.476600	$44,337.00	6	1.00	31948.2055	1360.4518	137.8690	1.1672
Washington capitals	6.056296	$73,017.00	6	1.00	28720.5782	1270.1267	127.8298	1.0822
Winnipeg jets	0.778500	$41,719.00	-11	1.00	29421.4851	1332.2235	121.9312	1.0322
Hamilton	0.765228	$35,315.00	-1	1.00	31278.1681	1373.9382	129.5652	1.0969
Kansas City	2.070981	$51,157.00	4	1.00	29783.5302	1317.7477	127.7109	1.0812
Las Vegas	2.051796	$45,744.00	14	0.00	24896.4995	1305.6199	90.9040	0.7696
Québec city	0.797500	$41,907.00	-7	1.00	29436.2287	1331.9975	122.0949	1.0336
Seattle	3.663399	$73,012.00	8	0.00	22007.7246	1215.3763	81.9726	0.6940
MEAN (NHL teams)	4.034628	$57,788.03	5.47	0.700	28171.2056	1298.3985	118.1226	1.0000

Notes: [a][25]; [b]Average coldest month high temperature [6]; [c]Teams with average coldest month high temperatures of 6 °C or less given value of 1, and 0 otherwise; [d]See Eq. (11); [e]See Eq. (12); [f]See Eq. (10); [g]See Eq. (13)

where μ_{H_t} is the mean locational quality for all thirty NHL teams. The locational quality for all thirty NHL teams is shown in Table 3 and Fig. 3. It is also shown for the five expansion cities being considered. It is interesting to note that little correlation exists between the city population size and the locational quality.

We now need to quantify the final influencing factor, namely the effect of an individual's social environment on their susceptibility to becoming a fan of an NHL team. Defining p as the 'rating' of an individual's social environment, we chose three representative values in order to simplify our model, as follows:

1. $p = 0.05$ represents a scenario where a susceptible person does not have any close friends or family who are hockey fans and does not play hockey at any level, on any surface.
2. $p = 0.25$ represents a scenario where a susceptible person has some friends or family who are regular fans (i.e. they belong to class I_t for team t)
3. $p = 0.75$ represents a scenario where a susceptible person's close friends or family are devoted fans (i.e. they belong to class C_t for team t)

Bringing the three influencing factors together, we now define

$$\beta_t = p\, s_t\, h_t. \tag{14}$$

Other model parameters are easier to determine. We set the mean infectious period, $1/\gamma$, at 9 months, the length of a typical NHL season including playoffs (approximately October through June). According to The Forum Poll [13], approximately 30 % of hockey fans are 'enthusiastic' or 'extreme' fans, combined. Thus, α could be calibrated so that 30 % of NHL teams are chronically infected. However, a casual fan is more likely to become chronically infected by fan allegiance to a well-located, winning team, and therefore α must also depend on locational quality and success rates. We thus assume that α is proportional to β, which is very small when β is small, properly reflecting that fact that the rate of becoming a chronically infected fan of a team will be very small (and may be almost

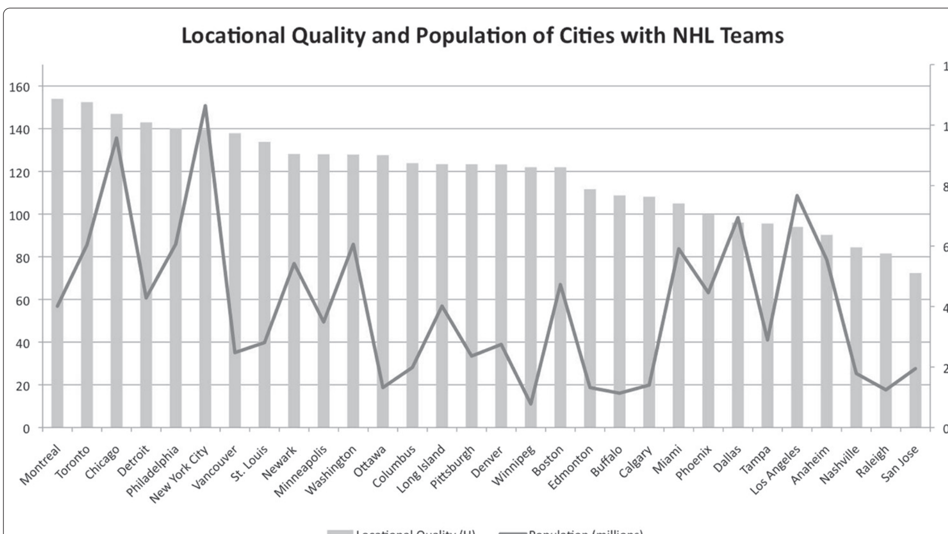

Fig. 3 Locational quality. The calculated locational quality for the thirty NHL team host cities is shown

zero) for expansion teams, especially in their first season. For the purposes of this study, we set $\alpha = 0.3\beta$.

Though not relevant to the results below, for the sake of thoroughness we will define $\delta_{t_1 t_2}$, the mean rate of 'switching' allegiance from team t_1 to team t_2. Switching team allegiance is not a common phenomenon. We suggest that it happens mainly among casual fans (class I), and that it depends on an individual's social environment as well as on the success ratings of both the 'old' and 'new' teams. We further propose that this 'switching' only occurs when the new team's success rating is higher than that of the old team. We therefore define $\delta_{t_1 t_2}$

$$\delta_{t_1 t_2} = \begin{cases} p \cdot (s_{t_2} - s_{t_1})/\mu_{s_t} & \text{if } s_{t_2} > s_{t_1}, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Results

The equilibria of Eq. (6) correspond to an uninfected equilibrium $E_0 = \{S, 0, 0, 0, 0, N - S\}$ and an infected equilibrium $E_1 = \{0, 0, 0, C_1, C_2, N - C1 - C2\}$. However, over one NHL season E_1 cannot be achieved, and thus every season ends with a population distribution across all compartments S, I_1, I_2, C_1, C_2, R; it is assumed that over the course of the summer the casually infected individuals may wane back to the susceptible class S. It is now possible to determine the circumstances under which a given team's fanbase is likely to grow; in other words, when the 'infection' will spread. This corresponds to determining the point at which $\mathcal{R}_0 > 1$ in an epidemiological model. Here,

$$\mathcal{R}_{0,t} = \frac{\beta_t S}{\gamma} = \frac{p h_t s_t S}{\gamma} \tag{16}$$

$\mathcal{R}_0$ has a useful predictive property whereby we can expect that an infection will spread in a population only when $\mathcal{R}_0 > 1$. We will utilize this property of $\mathcal{R}_0$ in order to investigate the ability of different NHL teams to grow their respective fanbases.

It is important to note that once the chronically infected compartment is populated, E_1 is always stable, since chronically infected fans will always infect others in the population. This, however, does not imply that a team will be financially viable, since a small number of chronically infected fans will not be able to support a team financially (through merchandise purchases and ticket sales). The chronically infected compartment must reach a threshold level for this to be true. Determination of this threshold is beyond the scope of this study. We are providing a first step towards this calculation, determining geographical locations where 'infection' will spread.

Current NHL teams

First, we look at the current NHL teams to determine $\mathcal{R}_0$ with different success rates, and different locational qualities. Table 4 lists the $\mathcal{R}_0$ values for all NHL teams when $p = 0.05$, 0.25, and 0.75. The $\mathcal{R}_0$ values are also shown in Fig. 4.

Table 4 Basic reproduction ratios for teams in the NHL, calculated for $p = 0.05$, 0.25, and 0.75

	$\mathcal{R}_0$ for $p = 0.05, p = 0.25, p = 0.75$		
Team	$\mathcal{R}_0, p = 0.05$	$\mathcal{R}_0, p = 0.25$	$\mathcal{R}_0, p = 0.75$
Anaheim ducks	0.6092	3.0458	9.1373
Arizona coyotes	0.1068	0.5341	1.6024
Boston bruins	0.7185	3.5927	10.7782
Buffalo sabres	0.2330	1.1651	3.4952
Calgary flames	0.2085	1.0427	3.1281
Carolina hurricanes	0.4016	2.0082	6.0247
Chicago blackhawks	1.6055	8.0277	24.0831
Colorado avalanche	0.6603	3.3014	9.9041
Columbus blue jackets	0.0265	0.1327	0.3980
Dallas stars	0.1954	0.9772	2.9315
Detroit red wings	1.4548	7.2740	21.8219
Edmonton oilers	0.1436	0.7179	2.1538
Florida panthers	0.0225	0.1125	0.3374
Los Angeles kings	0.7049	3.5247	10.5740
Minnesota wild	0.2469	1.2345	3.7034
Montreal canadiens	0.5610	2.8049	8.4148
Nashville predators	0.1266	0.6330	1.8989
New Jersey devils	0.8376	4.1881	12.5642
New York islanders	0.0793	0.3967	1.1900
New York rangers	0.5830	2.9151	8.7453
Ottawa senators	0.4647	2.3235	6.9706
Philadelphia flyers	0.6160	3.0802	9.2406
Pittsburgh penguins	0.8459	4.2296	12.6888
St. Louis blues	0.3011	1.5053	4.5160
San Jose sharks	0.3487	1.7434	5.2303
Tampa Bay lightning	0.5323	2.6616	7.9848
Toronto maple leafs	0.2776	1.3882	4.1647
Vancouver canucks	0.4728	2.3639	7.0917
Washington capitals	0.2877	1.4383	4.3150
Winnipeg jets	0.0457	0.2287	0.6860

As an example, consider the Chicago Blackhawks, 2015 Stanley Cup champions. The basic reproductive ratio in Chicago when $p = 0.05$ is

$$\begin{aligned}\mathcal{R}_0 &= 9 \cdot 0.05\, h_{Chi}\, s_{Chi}\\ &= 0.45 \cdot 1.2435 \cdot 2.8692\\ &= 1.6055\end{aligned}$$

The Blackhawks have $\mathcal{R}_0 > 1$ even if p is small. Winners of three Stanley Cups since the 2004-05 lockout, the Blackhawks have the highest $\mathcal{R}_0$ values in the league. They are followed closely by the Detroit Red Wings, winners of two Cups since 2001. For all three p-values, both Chicago and Detroit have $\mathcal{R}_0 > 1$.

Three teams in the league, the Florida Panthers, Columbus Blue Jackets, and Winnipeg Jets, have $\mathcal{R}_0 < 1$ for all three values of p. This may be related to a warm geographic location (Florida), poor playoff success (Columbus), and the fact that a team is fairly new to the NHL (Winnipeg). A further five teams have $\mathcal{R}_0 < 1$ for $p \leq 0.25$: the New York

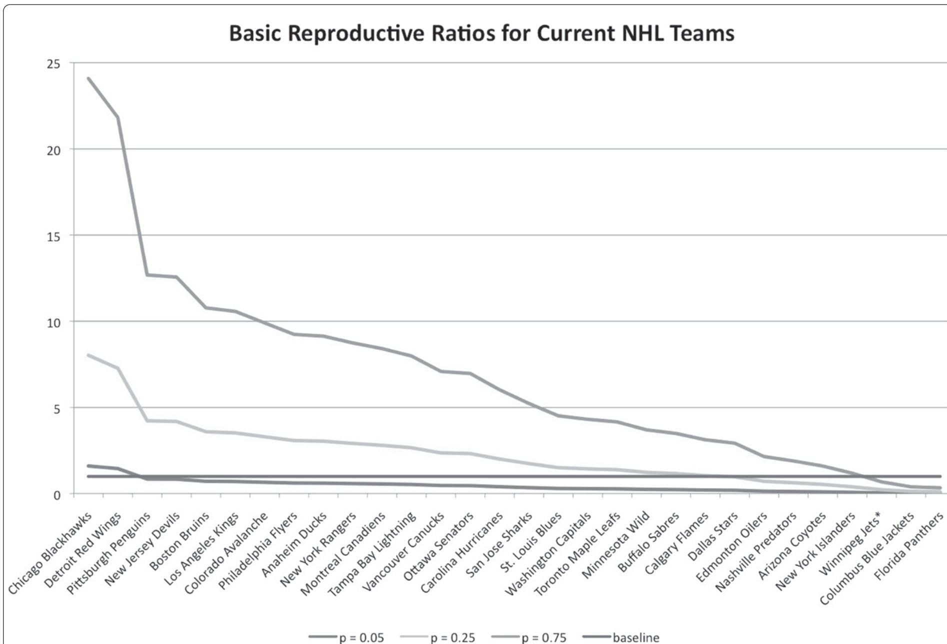

Fig. 4 Basic reproductive ratio $\mathcal{R}_0$. $\mathcal{R}_0$ is shown for all current NHL teams when $p = 0.05, 0.25, 0.75$

Islanders, Arizona Coyotes, Nashville Predators, Edmonton Oilers, and Dallas Stars - all teams with few playoff appearances in the timeframe of reference. It is interesting to note that the Toronto Maple Leafs, despite having similar playoff appearances to these teams, have $\mathcal{R}_0 > 1$ when $p = 0.25$.

Appendix: Figs. 5 and 6, show that a team's success rate has a much stronger effect on $\mathcal{R}_0$ than its locational quality. The outliers whose success rates are greater than their $\mathcal{R}_0$ values (Los Angeles, Anaheim, Tampa Bay, Carolina, San Jose, Dallas, and Nashville) all have locational qualities well below the league average, and the outliers whose success rates are lower than their $\mathcal{R}_0$ values all have high locational qualities. Thus locational quality does have an effect on $\mathcal{R}_0$, but this is small compared to team success. It is interesting to note that all the teams that have a lower success rate than their corresponding $\mathcal{R}_0$ are either Original Six teams,[3] located in Canada, or have been in the NHL since the 1967-68 expansion.

Although the success rate is the most significant factor in determining a team's $\mathcal{R}_0$, a low locational quality is certainly a disadvantage for a team trying to grow its fanbase. Such a team requires a higher success rate to attract the same number of new fans compared to a team with a higher locational quality. According to this rationale, of the three teams that have $\mathcal{R}_0$ values below 1 for all values of p, the Florida Panthers need a higher success rate than the Columbus Blue Jackets and Winnipeg Jets in order to be viable. In order to attract fans in the $p = 0.75$ category, Florida must increase its success rate to approximately 6 playoff series appearances in 14 years, meaning that the team needs to reach the playoffs nearly every second year. Columbus and Winnipeg each need to increase their respective success rates to approximately 5 playoff series appearances in 14 years, which means reaching the playoffs at least every third year. In order to attract fans in the $p = 0.25$ category, Florida must increase its success rate to one of the following: 10 series appearances and 4 series wins in 14 years (reaching the playoffs six times); 12 series appearances and 3 series wins in 14 years (reaching the playoffs 9 times); 14 series appearances and 2 series wins in 14 years (reaching the playoffs 12 times); or 16 series appearances and one series win in 14 years (reaching the playoffs every year). Columbus and Winnipeg, however, need increase their respective success rates to, for example, 9 series appearances and 3 series wins in 14 years (reaching the playoffs six times), or 11 series appearances and 2 series wins in 14 years (reaching the playoffs 9 times).

Given that success rates have a greater impact than location on a team's ability to attract fans, it is likely that many cities in North America are capable of financially sustaining a successful NHL team. However, as evidenced by the outliers in Fig. 5, a high locational quality is advantageous, as it lowers the success rate needed to ensure a team's financial viability. It is not a given that a city with a low locational quality will be unable to support an NHL team; however, a high locational quality is an advantage to a new team as it works to attract fans in the team's first years. Since many expansion or relocation teams struggle on-ice for the first several years, this advantage may be quite significant in the long run.[4]

Potential expansion sites

Current NHL expansion considerations - Québec City and Las Vegas

Las Vegas and Québec City are involved in the current NHL expansion considerations [20]. In terms of locational quality, Québec City, at 122.1, is just higher than the league average of 118.1, and on par with Boston and Winnipeg, both at approximately 121.9. Las Vegas' locational quality is much lower; at only 90.9, it is closest to Anaheim at 90.2, and is higher than the cities of only three other current teams: San Jose at 72.3, Raleigh at 81.5, and Nashville at 84.4 (See Table 3).

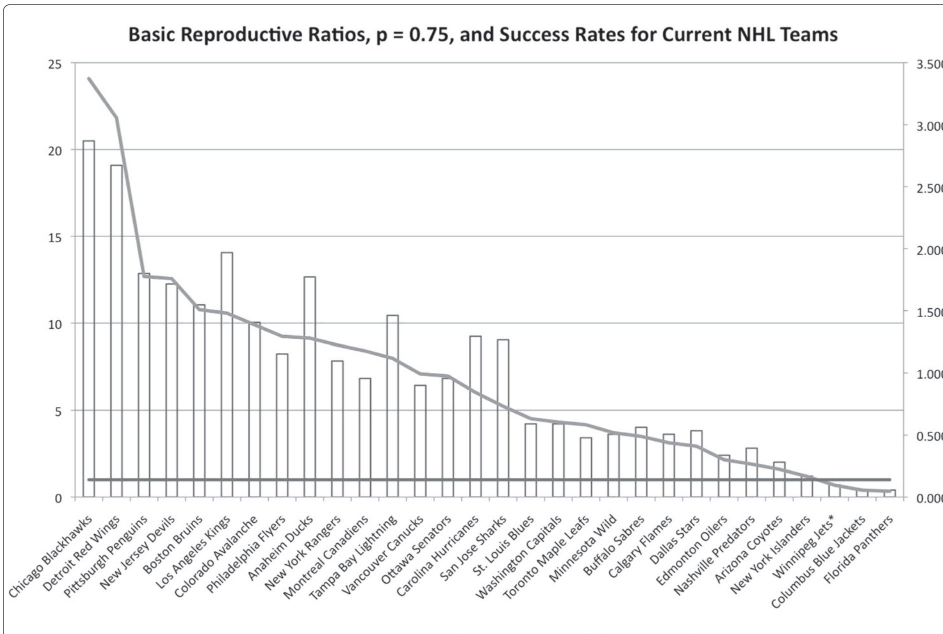

Fig. 5 Yearly success rates and $\mathcal{R}_0$. The normalized yearly success rate and basic reproduction ratio $\mathcal{R}_0$ are shown for all thirty NHL teams. It is assumed that $p = 0.75$

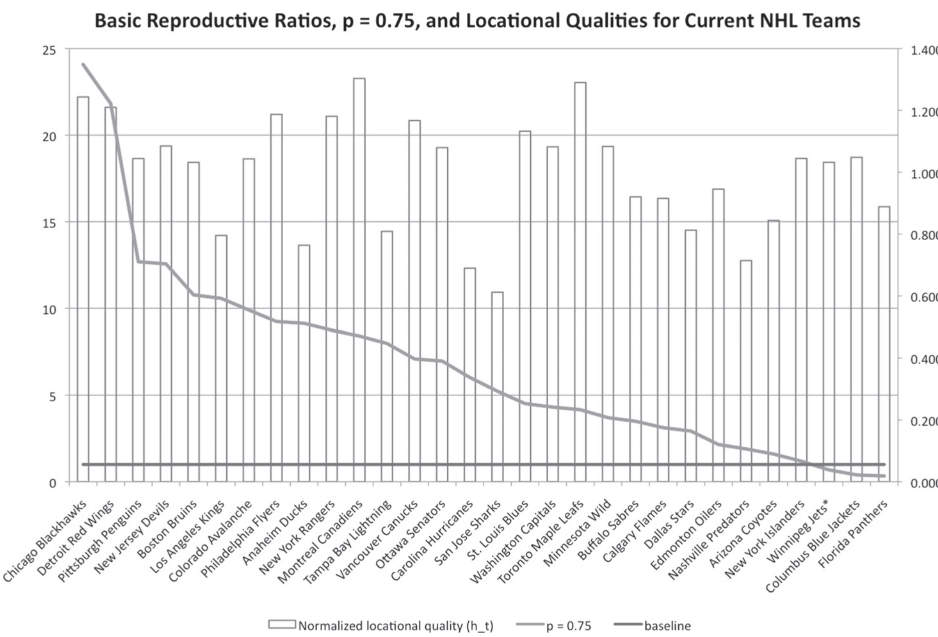

Fig. 6 Locational quality and $\mathcal{R}_0$. The normalized locational quality and basic reproduction ratio $\mathcal{R}_0$ are shown for all thirty NHL teams. It is assumed that $p = 0.75$

Table 5 lists the minimum success rates required to attain an $R_0 > 1$ for potential expansion sites.

For a team in Québec City to be viable, it would need to have similar success rates to Boston ($\mathcal{R}_0 > 1$ when $p = 0.25, 0.75$), Colorado ($\mathcal{R}_0 > 1$ when $p = 0.25, 0.75$) or the New York Islanders ($\mathcal{R}_0 > 1$ when $p = 0.75$). We note that expansion teams, in general, do not have very good success rates [14] (see Footnote 4 above), so a success rate similar to the New York Islanders could be expected, but a large p is needed for viability. Québec City previously hosted the Nordiques in the NHL from 1979 until the team relocated to Denver, Colorado in 1995. Many inhabitants of the city have not forgotten their old team and would be eager to support a new Nordiques team if the league allows it [1]. It is also expected that season tickets would sell out quickly in Québec City, similar to what occurred in Winnipeg when the Jets returned to that city in 2011 [33]. Thus, it is not unreasonable to believe that p could be high, and thus, a team in Québec City could thrive. Drawbacks that must be considered include the low Canadian dollar [27] and the relatively small size of the city [25].

Table 5 Minimum success rates required to attain a basic reproduction ratio greater than one for a theoretical team located in one of the listed cities

	Minimum s_t to attain $\mathcal{R}_0 > 1$ for $p = 0.05, p = 0.25, 0.75$			
Team location	H_t	$s_t, p = 0.05$	$s_t, p = 0.25$	$s_t, p = 0.75$
Hamilton	129.5652	2.0260	0.4052	0.1351
Kansas city	127.7109	2.0554	0.4111	0.1370
Las Vegas	90.9040	2.8876	0.5775	0.1925
Québec city	122.0949	2.1499	0.4300	0.1433
Seattle	81.9726	3.2022	0.6404	0.2135

In Las Vegas, because of the city's much lower locational quality, a team would need to make 7 series appearances in 14 years for a $p = 0.75$. This means that the team must reach the postseason every second year. In the $p = 0.25$ category, a team in Las Vegas would need 11 series appearances and 5 series wins in order to ensure that $\mathcal{R}_0$ is greater than 1. Again, citing the low expected success rates of expansion teams [14], these success ratings are very unlikely.

Bill Foley, who leads the group in Las Vegas, successfully ran a season ticket drive that secured more than 13 200 deposits for season tickets [30]. Interestingly, Las Vegas does not currently host a professional sports team from any of the major sports leagues [30],[5] which may help a potential NHL team gain fans in the city. Its status as a popular tourist destination means that the many visitors from colder climates can be expected to purchase single-game tickets, as is already the case among 'snowbirds'[6] who visit Miami and Tampa in Florida [12]. The main drawback to putting a team in Las Vegas is the hot, dry climate, which is not conducive to ice hockey. As a result, there is little local interest in the sport, as evidenced by the low participation levels in amateur hockey in the state of Nevada [29].

Other potential expansion cities - Hamilton, Kansas City and Seattle

Although applications for teams in Kansas City, Seattle, and Hamilton - or the Greater Toronto Area - were not submitted for consideration in the current expansion process, these cities remain interesting as possible future sites for an expansion or relocation team. Hamilton, approximately 60 kilometres outside of Toronto, has the highest locational quality of the three; at 129.6, its locational quality is higher than those of all current Canadian teams except Montreal, Toronto, and Vancouver. Kansas City is not far behind at 127.7, which is on a par with Newark, NJ at 128.1, Minneapolis-St. Paul, MN at 128.0, Washington, D.C. at 127.8 and Ottawa at 127.5. Seattle's locational quality is low at 82.0, higher than only two current teams: Raleigh, NC and San Jose.

Hamilton and Kansas City have higher locational qualities than Québec City; thus, a team in each of these cities would need a slightly lower success rate than Boston, Colorado, and the New York Islanders: for potential fans in the $p = 0.25$ category, teams in these cities would only need 8 series appearances and 3 series wins, or 10 series appearances and 2 series wins, to reach $\mathcal{R}_0 > 1$.

Based on its high locational quality, Kansas City could be an excellent place in which to put an expansion team. No potential ownership group completed an application in the current NHL expansion application process, but this does not rule it out as a potential location for an expansion or relocation team in the future.

Based on its locational quality, Hamilton seems to be an excellent place to put a new NHL team; however, its situation is complicated. Due to its proximity to both Toronto and Buffalo, a team in Hamilton could potentially 'steal' fans and revenue from either of those two cities, which might affect the viability of those teams. This proximity would not be expected to harm the Toronto Maple Leafs' viability since the Leafs remain the most financially successful team in the league despite reaching the playoffs only once in the last ten years [14, 16, 24]; however, the less successful Buffalo Sabres could potentially experience significant damage.

In order to gain more insight into Hamilton's situation, we looked at the situation in New York City, where three teams - the New York Rangers in Manhattan, the New

York Islanders in Long Island, and the New Jersey Devils in Newark - are located within metropolitan New York City. In order to divide up the population of metro New York for our calculations, we looked at how the city's 2014 hockey revenue was split between the three teams. The revenue split was as follows: the Rangers had 53 %, the Islanders, 20 %, and the Devils, 27 %, of the city's hockey revenue. While the situation in Toronto-Hamilton-Buffalo is not identical due to a greater distance between the teams, the revenue split in New York City provides insight into whether a team in Hamilton would be financially viable.

It should be noted that this comparison rests on two assumptions: first, that Buffalo's revenue would not decrease; and second, that a team in Hamilton would bring in new revenue. If a team in Hamilton has a 20 % share of the revenue in the region, similar to the Islanders, then such a team can expect a revenue of $73.25 million, which is significantly lower than any current team. Since several current teams struggle even with higher revenues, such a low amount does not point to financial viability. However, if a team in Hamilton has a 27 % share of the region's revenue, it can expect a much more viable revenue of $108.37 million. If we relax the second assumption and allow the team in Hamilton to rely on revenue 'stolen' from Toronto and Buffalo, it will not be viable based on the above revenue split, and, furthermore, will greatly damage the Buffalo Sabres' financial viability.

The NHL may be hesitant to approve a team in Hamilton because of the potential danger to the viability of the Buffalo Sabres, who have not reached the postseason since 2011. Unless the Sabres start to reach the playoffs more regularly, thereby stabilizing their fanbase and securing the team's financial health, the NHL is unlikely to approve a team in Hamilton in the near future.

An expansion team in Seattle would need at least 8 series appearances in 14 years for $p = 0.75$ and at least 13 appearances and 5 wins for $p = 0.25$ for the team to be viable, even more than a team in Las Vegas. Thus, Seattle might not be a good place in which to put an expansion team. We note, however, that Seattle previously hosted a professional ice hockey team, the Seattle Metropolitans, which played in the Pacific Coast Hockey League from 1915-24 [14]. This history, along with its proximity to the Canadian border, may have a positive effect on the success rate needed in this city.

Conclusions

The applications for expansion teams Québec City, Québec, and Las Vegas, Nevada by potential ownership groups have passed the first round of the application process (started by the NHL in June 2015), and are currently being evaluated by the league. Our study suggests that NHL expansion in Québec City would be viable, but that expansion to Las Vegas would be very risky. Our study also found that other North American cities that have shown interest in hosting NHL teams could be worthy of consideration, in particular Hamilton and Kansas City. We conclude that Québec City represents a viable option for NHL expansion, with Hamilton and Kansas City deserving consideration as well.

Our results are supported by a locational quality and success rating evaluation of all current NHL teams. The locational quality was determined using the population, GDP per capita, and mean winter temperature of all NHL cities and potential expansion cities. The success ratings of all current NHL teams were determined from Stanley Cup playoff and championship data for the years 2001-15. Given the locational quality of the potential

NHL expansion cities, we then determined the success rating that an expansion team in each city would need to achieve in order to attract new fans.

Model results show that most current NHL teams can expect to have trouble gaining fans with little connection to hockey (see Table 4 and Fig. 4). Among those susceptible individuals in the category $p = 0.25$, some fanbases could grow but many others would not, including some Canadian teams. Stability analysis of the model equilibria shows that the existence of chronically infected fans will enable growth in fan allegiance; however, for this growth to occur, susceptible individuals must have exposure to devoted fans. Calculation of a threshold of devoted fans needed to guarantee team financial stability is a course for future work.

In our analysis, we included a parameter that involved average winter temperature as a variable in our evaluation of locational quality to reflect 'fan interest' in the sport. We note here that involvement in amateur hockey and local leagues may be a better indication of hockey interest. Data compilation on participation rates in ice hockey, roller hockey, and ball hockey leagues is a course for future work.

Along a similar argument to that above, fans are more likely to be found in places where a particular sport is of interest. We suggest that expansion of the NHL to a particular geographic location could be preceded by NHL sponsorship or investment in hockey activities at the local level. This could involve sponsorship of house leagues, encouraging NHL player involvement in community activities, or even financial bursaries to renovate or build hockey facilities.

Finally, our analysis did not consider future expansion in the Greater Toronto Area (GTA). Considering that Los Angeles and New York City each support more than one NHL team, perhaps metropolitan Toronto is capable of supporting a second team as well. Toronto has a high locational quality that is second in the league, behind only Montreal, and at 6 million, it has the largest metropolitan population in Canada. Los Angeles hosts two teams, thus a comparison of Toronto to Los Angeles could be warranted. Revenue in Los Angeles for 2014 was split 58–42 % between the Kings and Ducks. If putting a new team in the GTA does not bring in new revenue but rather splits the Toronto Maple Leafs' current revenue of $190 million - with the Leafs retaining the larger share - the new team would have a revenue of only $79.80 million. Given that the Arizona Coyotes had the lowest 2014 revenue in the league at $80 million and are known to be struggling financially [24, 28], $79.8 million may not be a viable revenue for a new team. This is a very simple analysis, however. Consideration must also be given to the cost of seasons tickets, individual tickets sales, and the seating capacity of the Maple Leaf arena, all factors that limit the spending habits of Maple Leaf fans. Thus, it could be expected that a new NHL team in the GTA could bring in new revenue. A game theoretic analysis of fan allegiance and merchandise sales would be informative here, and is a course for future work.

Endnotes

[1] Two teams, the Minnesota Wild and the Columbus Blue Jackets, joined the league for the 2000-01 season, bringing the total number of teams in the league to 30, the current number.

[2] Since the entire 2004-05 season was lost to a labour dispute, or 'lockout', this timeframe includes only 14 seasons.

[3] A term commonly used to refer to the six teams in the NHL from 1943-1967: Boston, Chicago, Detroit, Montreal, New York Rangers, Toronto.

[4] None of the nine expansion teams added to the NHL since 1991 reached the playoffs in its first year in the league; only one team, the Carolina Hurricanes, reached the playoffs in its second year; and two other teams, the San Jose Sharks and the Minnesota Wild, reached the playoffs in their third year [14].

[5] The major sports leagues in North America, often referred to as the 'Big Four', include the NHL, the National Football League, the National Basketball Association, and Major League Baseball.

[6] Canadians who spend the winter months in Florida are often referred to as 'snowbirds.'

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Authors' contributions
JL, AC and JMH particpated in the development of the model, analysis and interpretation of results, and in the writing of the manuscript. All authors read and approved the final manuscript.

Competing interests
The author declares that that they have no competing interests.

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