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$\mathcal{N} = 1$ Chern-Simons theories, orientifolds and Spin(7) cones

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Abstract: We construct three dimensional $\mathcal{N} = 1$ Chern-Simons theories living on M2 branes probing Spin(7) cones. We consider Spin(7) manifolds obtained as quotients of Calabi-Yau four-folds by an anti-holomorphic involution, following a construction by Joyce. The corresponding Chern-Simons theories can be obtained from $\mathcal{N} = 2$ theories by an orientifolding procedure. These theories are holographically dual to M theory solutions $AdS_4 \times H$, where the weak G_2 manifold H is the base of the Spin(7) cone.

Keywords: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, M-Theory

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Contents

1 Introduction

There has been some progress in understanding the conformal field theory living on a stack of N M2 branes at the tip of a non compact eight-dimensional cone. This theory is the holographic dual of a Freund-Rubin solution in M theory of the form $AdS_4 \times H$, where H is the seven-dimensional compact base of the cone. In the case of large supersymmetry $\mathcal{N} \geq 3$ the conformal field theory has been identified with a Chern-Simons theory [1–5]. The case where the cone is a Calabi-Yau four-fold corresponds to $\mathcal{N} = 2$ supersymmetry. A general construction of $\mathcal{N} = 2$ Chern-Simons theories dual to Calabi-Yau cones has been discussed in [6, 7] and a large number of models have been subsequently constructed [8–15]. Much less is known about $\mathcal{N} = 1$ theories and some recent studies can be found in [17–20]. Actually three dimensional $\mathcal{N} = 1$ superconformal field theories are particularly interesting because they are still supersymmetric but they do not have any holomorphic property. This is a peculiarity of the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence with respect to the usual $\mathrm{AdS}_5/\mathrm{CFT}_4$, and it is worthwhile to study it.

In this paper we are interested in the three dimensional conformal field theory living on N M2 branes at the tip of Spin(7) cones. This case corresponds to $\mathcal{N}=1$ supersymmetry. We will use a general construction due to Joyce [21] and we will realize Spin(7) cones X as quotients of four dimensional Calabi Yau cones Y by an anti-holomorphic involution Θ: $X = Y/\Theta$. We will obtain theories corresponding to Spin(7) cones $X = Y/\Theta$ by performing an orientifold quotient of the $\mathcal{N}=2$ Chern-Simons theory corresponding to Y. The resulting theory will have a residual $\mathcal{N}=1$ supersymmetry. We will show that the moduli space for one membrane is naturally a quotient of Y by a dihedral group Θ_k, which is generated by the anti-involution and an abelian symmetry $\mathbb{Z}_{2k}$. This is a quite general construction of Chern-Simons theory with Spin(7) moduli space. The case of anti-involutions without fixed points, except the tip of the cone, is the most interesting one; in this case the near horizon background $AdS_4 \times H/\Theta_k$ is perfectly smooth. We will provide many examples of $\mathcal{N}=1$ Chern-Simons corresponding to anti-involutions without fixed points on the base.

The orbifolding and orientifolding procedure are standard for theories obtained from D branes in type II string theories but are less understood for M2 branes in M theory. From the string point of view, we take the attitude of considering our membrane theories as IR limit of D2 brane configurations in type IIA in the presence of fluxes. This point of view has been emphasized in [22, 23]. We motivate our construction by observing that all the involutions considered in this paper descend to an orientifold projection in type IIA. From the field theory point of view, we obtain the $\mathcal{N}=1$ Chern-Simon theory by projecting the parent Lagrangian. As for ordinary orbifolds and orientifolds of four-dimensional gauge theories, we expect that planar equivalence holds [24–26] and that the final theory will be conformal when the parent theory was.

Before finishing this introduction, let us explain in more details the geometric quotient which is at the base of all constructions in this paper. Recall that a Calabi-Yau four-fold is a four dimensional complex manifold with a real (1,1) form J and a complex (4,0) form ω satisfying

$$dJ = 0\,, \qquad\qquad d\omega = 0 \tag{1.1}$$

J is the Kahler form and ω the holomorphic four form. The manifold has holonomy SU(4), which is the group of transformations of the tangent bundle that leave J and ω invariant. On the other hand, Spin(7) manifolds are eight dimensional real manifolds with a globally defined self dual closed four form Ω_4,

$$\Omega_4 = *\Omega_4\,, \qquad\qquad d\Omega_4 = 0\,. \tag{1.2}$$

The manifold has holonomy Spin(7), which is the group of transformations of the tangent bundle that leave Ω_4 invariant.

Every Calabi Yau four-fold Y is also a Spin(7) manifold. Indeed, using the Kahler form J and the holomorphic volume form ω, we can define

$$\Omega_4 = \frac{1}{2} J \wedge J + \mathrm{Re}(\omega) \tag{1.3}$$

which is closed and self-dual. Of course the resulting Spin(7) manifold is not generic and actually it has holonomy SU(4). To obtain a pure Spin(7) manifold we can consider an antiholomorphic involution Θ on Y. It acts on ω and J as: $\omega \to \bar{\omega}$, $J \to -J$ and therefore it does not commute with the complex structure. The four form Ω_4 is invariant under Θ and the holonomy group is broken from SU(4) to Spin(7). The quotient $X = Y/\Theta$ is a Spin(7) manifold.

In the rest of this paper we construct $\mathcal{N} = 1$ Chern-Simons theories that naturally realize this construction on their moduli space. We start in section 2 by recalling the general structure of $\mathcal{N} = 2$ Chern-Simons theories dual to Calabi-Yau four-folds. In section 3 we discuss the orientifolding construction and, in section 4, we provide many examples of $\mathcal{N} = 1$ Chern-Simons theories with Spin(7) moduli spaces Y/Θ, with quotient groups Θ with and without fixed points (except the tip of the cone). We privilege $\mathcal{N} = 2$ Chern-Simons theories where the moduli space is a complete intersection, or a simple set of algebraic equations in some ambient space, but our construction is quite general. In section 5 we discuss generalizations of the orientifold procedure to the case where gauge groups are identified; a large number of other examples can be provided in this way. We end with conclusions and comments.

2 The $\mathcal{N} = 2$ theories

There are by now several examples of world-volume theories for M2 branes probing four-dimensional Calabi-Yau singularities. The recent attitude is to consider $\mathcal{N} = 2$ Chern-Simons gauge theories [5–7]. We consider quiver theories with U(N) gauge groups and adjoint and bifundamental chiral matter superfields X_{ab} interacting through a superpotential $W(X_{ab})$. There is no Yang-Mills kinetic term for the gauge groups but a Chern-Simons interaction with integer coefficients k_a, satisfying $\sum k_a = 0$. The Lagrangian in $\mathcal{N} = 2$ notations is reported in appendix A.

In a standard $\mathcal{N} = 2$ quiver with Yang-Mills interactions, the moduli space is obtained by solving the F and D term constraints[1]

$$\begin{aligned} &\partial_{X_{ab}} W = 0 \\ &\mathcal{D}_a(X) \equiv \sum_b X_{ab} X_{ab}^\dagger - \sum_c X_{ca}^\dagger X_{ca} + [X_{aa}, X_{aa}^\dagger] = 0 \end{aligned} \tag{2.1}$$

and dividing by the gauge group. These kind of quivers are utilized in four dimensions to describe the superconformal world-volume theory of D3 branes probing Calabi-Yau three-fold conical singularities in type IIB. In these cases the abelian moduli space, Z, is a Calabi-Yau three-fold.

When the same quiver is used as a three dimensional theory for membranes, the moduli space is bigger [5–7]. In $\mathcal{N} = 2$ supersymmetry in three dimensions the gauge vector has a scalar partner σ, which, in a Chern-Simons theory, has no kinetic term and is an auxiliary

[1] We use the same symbol to denote the scalar $\mathcal{N} = 2$ superfields and their lowest components.

field. The bosonic potential is

$$\sum_{X_{ab}} \mathrm{Tr}\left((\sigma_a X_{ab} - X_{ab}\sigma_b)(\sigma_a X_{ab} - X_{ab}\sigma_b)^\dagger + |\partial_{X_{ab}} W|^2 \right) . \tag{2.2}$$

where the auxiliary fields are determined by the constraints

$$\mathcal{D}_a(X) = \frac{k_a}{2\pi}\sigma_a. \tag{2.3}$$

The fields σ_a can be eliminated using these equations but sometimes we will find convenient to keep them in the Lagrangian. The potential is minimized by

$$\begin{aligned} \partial_{X_{ab}} W &= 0 \\ \sigma_a X_{ab} - X_{ab}\sigma_b &= 0 \end{aligned} \tag{2.4}$$

In the abelian case all $\sigma_a \equiv \sigma$ are equal and the equations $\mathcal{D}_a(X) = \frac{k_a}{2\pi}\sigma$ reduce to the standard D terms of an $\mathcal{N} = 2$ theory with a FI term depending on the Chern-Simons couplings. Since $\sum_a k_a = 0$ and $\sum_a \mathcal{D}_a(X) = 0$ by construction, one of these equations is redundant. Moreover, any linear combination of gauge groups with coefficient m_a orthogonal to the CS parameters $\sum_a k_a m_a = 0$ has a vanishing moment map. We are thus imposing $g-2$ D-term constraints, where g is the number of gauge groups. We can impose simultaneously the D-term constraints and the corresponding U(1) gauge transformations by modding out by the complexified gauge group. We do not need to impose the last D term condition since it determines the value of the auxiliary field σ. Moreover, the corresponding U(1) group, through its CS coupling with the overall gauge field, it is broken to $\mathbb{Z}_k$, where $k = \gcd(\{k_a\})$ [1, 27, 28]. As a result the abelian moduli space has a dimension of one unit bigger than Z and it has the general form $Y/\mathbb{Z}_k$ [5–7].

In general, Y is a $\mathbb{C}^*$ fibration over Z. Z is uniquely specified by the gauge group and matter field of the quiver, while Y is specified also by the choice of Chern-Simons couplings k_a: different values of the Chern-Simons levels correspond to different geometries Y. In the interesting case where Z was a three-fold, we obtain a four-dimensional manifold. For toric quiver based on Tilings, and some other generalization, one can explicitly show that the moduli space is still Calabi-Yau [7].

3 The orientifold construction and the $\mathcal{N} = 1$ theories

Our plan is now to find examples of theories probing Spin(7) cones $X = Y/\Theta$ by performing a quotient on the $\mathcal{N} = 2$ theory corresponding to the Calabi-Yau four-fold Y. The anti-holomorphic involution Θ is a real orbifold of Y and we are lead to consider both orbifold and orientifold projections. In the case of M2 brane theories the orbifold projection is complicated by the fact that there is no open string description of M theory branes. In particular, even in the case of $\mathcal{N} = 2$ holomorphic abelian orbifolds, the standard rules for performing a quotient on the Lagrangian give quivers whose moduli space is slightly more complicated than the geometric abelian quotient [2, 29]. We now show that the field-theoretic orientifold procedure for reducing the Lagrangian nicely reproduces Spin(7) cones of the required form. We first consider the cases where the number of gauge factors of the orientifolded theory is the same as in the parent theory. The case where there is a further identification of gauge groups and matter fields will be discussed in section 5.

We act on the Chern-Simons Lagrangian with an antiinvolution that conjugates the fields, thus breaking the holomorphic structure and $\mathcal{N} = 2$ supersymmetry. Here we summarize the transformations and the results referring to the explicit examples in the next section for details. The projection will be

$$\begin{aligned} A_\mu^a &\rightarrow -\Omega_a (A_\mu^a)^T \Omega_a^{-1} \\ X_{ab}^i &\rightarrow \Omega_a (X_{ab}^i)^* \Omega_b^{-1} \\ \sigma_a &\rightarrow \Omega_a (\sigma_a)^T \Omega_a^{-1} \end{aligned} \tag{3.1}$$

or, when there is more than one bi-fundamental field X_{ab}^i connecting the gauge groups a and b,

$$X_{ab}^i \rightarrow \eta_{ij} \Omega_a (X_{ab}^j)^* \Omega_b^{-1} \tag{3.2}$$

where η is a matrix satisfying $\eta^T \eta = \mathrm{Id}$. The projection commutes only with the gauge transformations that satisfy $\Omega g^* = g\Omega$ and thus breaks the $\mathrm{U}(2N)$ groups to $O(2N)$ for $\Omega = I$ and $USp(2N)$ for $\Omega = J$, where J is the $2N \times 2N$ symplectic matrix. The Chern-Simons coupling is $2k$ for $O(2N)$ and k for $USp(2N)$ [4, 28]. We obtain consistency conditions for the projection of matter fields by taking the square of the transformation. In particular, when we have two different X_{ab}^i connecting the group a and b, the choice $\eta_{ij} = \delta_{ij}$ (or a symmetric matrix of square one) requires $\Omega_a = \Omega_b$ and therefore same type of group, O or USp, on the two sides, while $\eta_{ij} = \epsilon_{ij}$ requires $\Omega_a \neq \Omega_b$ and two different groups in a and b. The projection is required to leave the bosonic potential and the full Lagrangian invariant. Note in particular that the D terms transform as $\mathcal{D}_a \rightarrow \Omega_a \mathcal{D}_a^T \Omega_a^{-1}$ consistently with the action on auxiliary fields and the constraint (2.3).

Since σ_a are $\mathcal{N} = 2$ superpartners of the gauge vector, it follows from the different transformations of A_μ^a and σ_a that the $\mathcal{N} = 2$ supersymmetry is broken. This is also obvious from the fact that the projection does not preserve the holomorphic structure of the Lagrangian. As explicitly shown in the appendix A, the projection preserves $\mathcal{N} = 1$ supersymmetry. This follows from the decomposition of $\mathcal{N} = 2$ into $\mathcal{N} = 1$ supermultiplets. The vector multiplet decomposes in an $\mathcal{N} = 1$ vector multiplet Γ_a^α and a real scalar superfield R_a whose lowest component is σ_a. The chiral multiplets become $\mathcal{N} = 1$ multiplets Y_{ab} each containing two real scalar superfields. The transformations (3.1) can be straightforwardly extended to an action on $\mathcal{N} = 1$ superfields

$$\begin{aligned} \Gamma_a^\alpha &\rightarrow -\Omega_a (\Gamma_a^\alpha)^T \Omega_a^{-1} \\ R_a &\rightarrow \Omega_a (R_a)^T \Omega_a^{-1} \\ Y_{ab}^i &\rightarrow \eta_{ij} \Omega_a (Y_{ab}^j)^* \Omega_b^{-1} \end{aligned} \tag{3.3}$$

that leaves the action invariant.

We are now ready to study the moduli space of the $\mathcal{N} = 1$ theory in the case of one membrane, $N = 1$. The theory is a real quiver with $O(2)$ and $USp(2) \equiv \mathrm{SU}(2)$

gauge groups. The projection identifies some components of the original fields that must now satisfy[2]

$$X^i_{ab} = \eta_{ij}\Omega_a(X^j_{ab})^*\Omega_b^{-1} \tag{3.4}$$

The vacuum conditions are obtained by projecting equations (2.4). We now make an ansatz for the vacuum configuration. The fields satisfying $X_{ab} = \Omega_a X^*_{ab}\Omega_b^{-1}$, which connect the same type of gauge group, can be chosen in the form

$$X_{ab} = \mathrm{Re}(x_{ab})I + \mathrm{Im}(x_{ab})J \tag{3.5}$$

where x_{ab} is a complex number.[3] Here and in the following I and $J = i\sigma_2$ denote the two by two identity and symplectic matrix, respectively. The fields satisfying $X^i_{ab} = \epsilon_{ij}\Omega_a(X^j_{ab})^*\Omega_b^{-1}$, which connect an $O(2)$ to an $USp(2)$ gauge group, can be chosen in the form

$$\begin{aligned} X^1_{ab} &= \frac{1}{\sqrt{2}}\big((\mathrm{Re}(x^1_{ab})I + \mathrm{Im}(x^1_{ab})J) + i(\mathrm{Re}(x^2_{ab})I + \mathrm{Im}(x^2_{ab})J)\big) \\ X^2_{ab} &= -\Omega_a(X^1_{ab})^*\Omega_b^{-1} \end{aligned} \tag{3.6}$$

with similar expressions for fields connecting $USp(2)$ to $O(2)$ gauge groups.

In many cases one can show that these configurations exhaust the vacuum space. Note that, with the above ansatz, we have the same number of complex fields and the same residual gauge symmetry as the original $\mathcal{N} = 2$ abelian quiver. Each gauge group is indeed broken to U(1); the SO(2) $\subset O(2)$ and the real section SO(2) $\subset$ SU(2) leave the ansatz invariant. In all our examples, D terms and F terms, restricted to the moduli space, reduce to

$$\mathcal{D}_a = \left(\sum_b |x_{ab}|^2 - \sum_c |x_{ca}|^2\right)\mathrm{Id}\,, \qquad\qquad \partial_{x_{ab}}W = 0\,. \tag{3.7}$$

The auxiliary fields are then determined by $\sigma_a = 2\pi\mathcal{D}_a/k_a$; the σ_a are diagonal and the remaining equations $\sigma_a X_{ab} = X_{ab}\sigma_b$ are trivially satisfied. Moreover the residual U(1) gauge symmetry acts on the fields x_{ab} exactly as in the parent $\mathcal{N} = 2$ theory. In particular, $\sum_a k_a \mathrm{U}(1)_a$ is broken to $\mathbb{Z}_{2k}$. At this point the moduli space would be $Y/\mathbb{Z}_{2k}$, similarly as the $\mathcal{N} = 2$ case. However, we now have an extra discrete transformation acting on the moduli space, obtained using the parity inversion $\sigma_3 \in O(2)$ and the element $i\sigma_3 \in$ SU(2). The transformation acts as

$$x_{ab} \to x^*_{ab} \tag{3.8}$$

on the fields (3.5), and

$$\begin{aligned} x^1_{ab} &\to \ \ (x^2_{ab})^* \\ x^2_{ab} &\to -(x^1_{ab})^* \end{aligned} \tag{3.9}$$

[2]We will use the symbol X to denote the $\mathcal{N} = 2$ scalar superfields and their lowest scalar components. We will use the symbol Y to denote the $\mathcal{N} = 1$ scalar superfields and we will use it when we want to emphasize the $\mathcal{N} = 1$ supersymmetry. The lowest component of the superfields X and Y is the same and it will be denoted X.

[3]The case with symmetric η_{ij} can be treated similarly

on the fields (3.6). This transformation preserves the vacuum conditions and can be lifted to an anti-involution Θ on Y. Combining with the $\mathbb{Z}_{2k}$ transformation we obtain a bigger group that we call Θ_k. The moduli space is Y/Θ_k and it is a Spin(7) cone.

In the following we will construct many explicit examples of Θ_k groups acting without fixed points, except the origin of the cone.

3.1 Descending to type IIA

We expect large quantum corrections to an $\mathcal{N}=1$ theory and its moduli space. We would like to have an explicit membrane realization of the previous construction, so that, by taking a near horizon geometry, we could argue for the superconformal invariance of the Chern-Simons theory and the form of the corresponding moduli space.

In order to obtain an open string description where we can explicitly compute the effect of an orbifold or orientifold projection we can use a duality with a type II string.

A general construction[4] involves shrinking the M theory circle and descending to type IIA. Various membrane theories have been explained in this way [22] and, although the construction does not explain all the proposed M2 theories and there are still few points to clarify, this construction is a good laboratory where to test our theories.

We already observed that that we can write Y as a $\mathbb{C}^*$ fibration. More precisely, we can see Y as a double fibration of the three dimensional CY Z over a real line $\mathbb{R}$, with coordinate σ, and a circle S^1, with coordinate ψ. The circle is the one acted upon by the discrete symmetry $\mathbb{Z}_{2k}$ and the one used for descending from eleven to ten dimensions. In type IIA the $\mathcal{N}=2$ Chern-Simons theory is realized on D2 branes probing a seven dimensional transverse space, which is Z fibered over a line. The Chern-Simons couplings on the D2 world-volume theory are induced by the RR fluxes generated by the dimensional reduction from M theory. The Chern-Simons couplings are assumed to dominate the IR physics and drive the D2 theory to an IR fixed point. In this picture, ψ parametrizes the M theory circle and σ is the value (FI) of the D term which is not imposed in the Chern-Simons theory.

In the type IIA description we are working with open strings and we can try to implement the projection to $\mathcal{N}=1$ on D2 branes. For this we need to understand how the action of Θ descends to type IIA. Locally the Calabi-Yau holomorphic four form ω will be given by

$$\omega \sim f(z_i)\; dz_1 \wedge dz_2 \wedge dz_3 \wedge (d\sigma + i d\psi)$$

where z_i are local holomorphic coordinates for the three-fold Z.

As we will discuss, all the anti-holomorphic involutions Θ considered in this paper, which conjugate ω and change sign to J, invert the sign of the M theory circle coordinate

$$\psi \to -\psi \tag{3.10}$$

and therefore can be interpreted as orientifolds in type IIA [30–33].

[4]The original ABJM model and few of its generalizations can be described by a web of D3 and NS branes in type IIB. However, for a general chiral quiver there is no such description in type IIB.

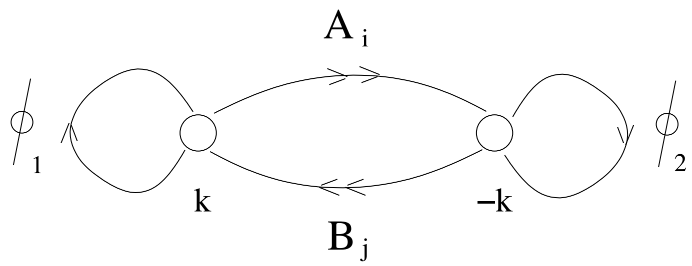

Figure 1. The quiver for $V_{5,2}$, $\mathbb{C}^4$ and $C(T^{1,1}) \times \mathbb{C}$.

The orientifold projection on D2 branes can be done with standard methods. Following this chain of dualities, we see that the $\mathcal{N} = 1$ orientifold theory flows in the IR to the world-volume theory of membranes probing Y/Θ.

4 Examples

In this section we provide many examples of orientifolded quivers with $\mathcal{N} = 1$ supersymmetry and Spin(7) moduli space, obtained as a quotient of a Calabi-Yau by a discrete group. By abuse of language, we will refer to actions that fix only the tip of the cone as actions without fixed points. We will provide examples both with and without fixed points. As discussed in the previous section, the construction is quite general and can be applied to a large class of quivers. Here we privilege, for simplicity, the cases where the Calabi-Yau can be written as a simple set of algebraic equations in some ambient space. Other cases can be handled with the well developed machinery of Tilings [34, 35] and the master space [36–38]. We also privilege the case where the quotient acts without fixed points.

4.1 Quivers with two groups

We consider the simple $\mathcal{N} = 2$ quiver theory studied in [16] and presented in figure 1. It has two gauge groups U(N), bi-fundamental fields A_i and B_j in the $(N, \bar{N})$ and $(\bar{N}, N)$, and two adjoints ϕ_1, ϕ_2 interacting with the superpotential

$$W = \frac{\phi_1^n}{n} + (-1)^{n+1}\frac{\phi_2^n}{n} + \phi_1(A_1B_1 + A_2B_2) + \phi_2(B_1A_1 + B_2A_2) \tag{4.1}$$

and Chern-Simons couplings $(k, -k)$. This example is particularly simple because we can consider the fundamental fields in the Lagrangian as embedding coordinates of Y, which is described as a complete intersection in $\mathbb{C}^5$.

4.1.1 The cone over $V_{5,2}$

Consider the cubic case, $n = 3$, studied in detail in [16]. To understand what is the corresponding Calabi-Yau four-fold we can study the abelian case. The moduli space of the theory is given by the solution of the F terms

$$\phi_1 = -\phi_2 = \phi, \qquad \phi^2 + A_1B_1 + A_2B_2 = 0 \tag{4.2}$$

With a change of variable the second equation can be rewritten as

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2 = 0 \tag{4.3}$$

which is the simplest example of Calabi Yau four-fold singularity, the quadric in $\mathbb{C}^5$. It is the real cone over the $V_{5,2}$ Sasaki-Einstein 7 manifold. It is the obvious generalization of the conifold singularity [39].

We still have to impose the D term constraints and mod by the gauge groups. One of the U(1)s acts trivially on matter fields and the corresponding D term vanishes. The auxiliary fields $\sigma_1 = \sigma_2 = \sigma$ are equal and we remain with the D term equation and gauge transformation

$$\sum_i |A_i|^2 - |B_i|^2 = \sigma\,, \qquad (A_i, B_i, \phi) \to (e^{i\psi}A_i, e^{-i\psi}B_i, \phi) \tag{4.4}$$

However, as discussed in the previous section, these conditions do not change the dimension of the moduli space. In fact the first equation fixes the value of the auxiliary field σ and the U(1) action is broken to $\mathbb{Z}_k$ by the Chern-Simons interaction. The moduli space is $C(V_{5,2})/\mathbb{Z}_k$. Note that the equations (4.4) allow to think the four-fold CY as a double fibration of the conifold over S^1 and $\mathbb{R}$ parametrized by ψ and σ.

We now construct orientifold models starting from the $\mathcal{N} = 2$ theory. According to the choice of projection we can have $O(2N)\times USp(2N)$ gauge groups (model I), $O(2N)\times O(2N)$ gauge groups (model II) and $USp(2N)\times USp(2N)$ gauge groups (model III). We will explain in details the construction for model I.

Orientifold model I. A natural antiholomorphic involution on the $\mathbb{C}^5$ complex coordinates that fixes only the origin of $Y = C(V_{5,2})$ is:

$$\phi \to \phi^*\,, \qquad A_i \to \epsilon_{ij} A_i^*\,, \qquad B_i \to \epsilon_{ij} B_i^* \tag{4.5}$$

It maps equations (4.2) into their complex conjugate and therefore preserves Y. The action squares to minus one and it is really a $\mathbb{Z}_4$ action. It is a $\mathbb{Z}_2$ action on $C(V_{5,2})/\mathbb{Z}_{2k}$. If we think of Y as a fibration over a CY_3 as in equation (4.4), we see that the antiholomorphic action inverts the M theory angle ψ and it corresponds to an orientifold action in the IIA stringy realization of the M2 brane theory. Since the action has no fixed points except the origin, we can see it as a generalization of an O2 plane.

We implement now the anti-involution (4.5) as an orientifold action on the $\mathcal{N} = 2$ $\mathrm{U}(2N) \times \mathrm{U}(2N)$ quiver with Chern-Simons couplings $(2k, -2k)$. We act as in (3.1) with $\Omega_1 = I$ and $\Omega_2 = J$ and $\eta_{ij} = \epsilon_{ij}$. Explicitly, the orientifold action on the quiver theory matter fields is

$$A_i \to -\epsilon_{ij} A_j^* J\,, \qquad B_i \to \epsilon_{ij} J B_j^*\,, \qquad \phi_1 \to \phi_1^*\,, \qquad \phi_2 \to -J\phi_2^* J \tag{4.6}$$

and on the auxiliary fields is

$$\sigma_1 \to (\sigma_1)^T\,, \qquad \sigma_2 \to -J(\sigma_2)^T J\,. \tag{4.7}$$

The action is a symmetry of the Lagrangian. The projection send W into its complex conjugate W^* and the bosonic potential is invariant. Moreover the D terms transforms as

$$D_1 \to (D_1)^T , \qquad\qquad D_2 \to -J(D_2)^T J \tag{4.8}$$

consistently with the auxiliary field constraints $D_a = \frac{k_a}{2\pi}\sigma_a$. It is then easy to see that the full Lagrangian is invariant. Note that the above action square to one on the matter fields. Any other choice of $\Omega_{1,2}$ when $\eta_{ij} = \epsilon_{ij}$ would lead to an action that square to minus one and would project out the A and B fields.

After orientifold projection, we obtain an $\mathcal{N} = 1$ theory with gauge group $O(2N) \times USp(2N)$. The Lagrangian is obtained by restricting to the fields configurations invariant under (4.6). The Chern-Simons coefficients become $2k$ for $O(2N)$ and k for $USp(2N)$. A reason for starting with coefficient $2k$ in the parent $\mathcal{N} = 2$ theory is that the final coefficient for $USp(2N)$, which results to be k, must be an integer [4, 28].

As a consistency check we compute the moduli space. Let us consider the case of just one M2 brane. The gauge group reduces to $O(2) \times \mathrm{SU}(2)$. The fields are two by two matrices. Let us consider the ansatz:

$$\begin{aligned} A_1 &= \frac{1}{\sqrt{2}}\Big(\big(\mathrm{Re}(a_1) + \mathrm{Im}(a_1)J\big) + i\big(\mathrm{Re}(a_2) + \mathrm{Im}(a_2)J\big)\Big) \qquad A_2 = A_1^* J \\ B_1 &= \frac{1}{\sqrt{2}}\Big(\big(\mathrm{Re}(b_1) + \mathrm{Im}(b_1)J\big) - i\big(\mathrm{Re}(b_2) + \mathrm{Im}(b_2)J\big)\Big) \qquad B_2 = -JB_1^* \\ -\phi_1 &= \phi_2 = \mathrm{Re}(\phi) + \mathrm{Im}(\phi)J \end{aligned} \tag{4.9}$$

with a_1, a_2, b_1, b_2, ϕ complex numbers. To find the vacua we can consider that the bosonic potential is the restriction of the $\mathcal{N} = 2$ one. In particular it is still a sum of squares minimized by

$$\begin{aligned} \partial_{X_{ab}} W &= 0 \\ \sigma_a X_{ab} - X_{ab}\sigma_b &= 0 \end{aligned} \tag{4.10}$$

where $\mathcal{D}_a(X) = \frac{k_a}{2\pi}\sigma_a$. On the ansatz the D terms are

$$\mathcal{D}_1 = -\mathcal{D}_2 = \left(\sum_{i=1}^{2} |a_i|^2 - |b_i|^2\right) \mathrm{Id} \tag{4.11}$$

It follows from $\mathcal{D}_a(X) = \frac{k_a}{2\pi}\sigma_a$ that $\sigma_1 = \sigma_2$ are diagonal; the second equation in (4.10) is then automatically satisfied. The F terms impose the complex constraint

$$a_1 b_1 + a_2 b_2 + \phi^2 = 0 \tag{4.12}$$

These is exactly the equation for the real cone over $V_{5,2}$.

We would have obtained the same result by studying the Lagrangian in an $\mathcal{N} = 1$ formalism which makes manifest the residual supersymmetry. As discussed in appendix A, the matter fields are combinations of real scalar multiplets interacting through an $\mathcal{N} = 1$

real superpotential. After elimination of the auxiliary multiplets R_a, the superpotential expressed in terms of A_1, B_1 and ϕ_i, reads

$$\begin{aligned} W_{\mathcal{N}=1} = & \frac{2}{3}\phi_1^3 + \frac{2}{3}\phi_2^3 + 2\phi_1(A_1B_1 + A_1^*B_1^*) + 2\phi_2(B_1A_1 - JB_1^*A_1^*J) \\ & + \frac{\pi}{k}\Big(\Big(A_1A_1^\dagger + A_1^*A_1^T - B_1^\dagger B_1 - B_1^TB_1^* + \phi_1\phi_1^T - \phi_1^T\phi_1\Big)^2 \\ & - \Big(B_1B_1^\dagger - JB_1^*B_1^TJ - A_1^\dagger A_1 + JA_1^TA_1^*J - \phi_2J\phi_2^TJ + J\phi_2^TJ\phi_2\Big)^2\Big) \end{aligned} \tag{4.13}$$

The vacua are obtained by minimizing the real superpotential and it is easy to see that we re-obtain the above result.

There is a residual gauge symmetry on the moduli space. The SO(2) groups contained in the $O(2)$ and the real subgroup SO(2) $\subset$ SU(2) factors preserve the ansatz. They act on the moduli space coordinates as

$$\begin{aligned} \mathrm{SO}(2) \subset O(2): \qquad & a_i \to e^{i\psi_1}a_i\,, \quad b_i \to e^{-i\psi_1}b_i \\ \mathrm{SO}(2) \subset \mathrm{SU}(2): \qquad & a_i \to e^{-i\psi_2}a_i\,, \quad b_i \to e^{i\psi_2}b_i \end{aligned} \tag{4.14}$$

as in the parent $\mathcal{N}=2$ theory. One of the two groups acts trivially on fields, while the other is broken to $\mathbb{Z}_{2k}$ by the Chern-Simons interaction and acts as $(a_i, b_i^*) \to e^{i\pi/k}(a_i, b_i^*)$ while leaving ϕ invariant. There is also a residual discrete symmetry given by the $\mathbb{Z}_2$ gauge symmetry acting as the inversion σ_3 inside the $O(2)$ and as $i\sigma_3$ inside SU(2). Its action on (4.9) reproduces exactly the antiholomorphic involution (4.5),

$$\phi \to \phi^*, \qquad a_i \to \epsilon_{ij}a_j^*, \qquad b_i \to \epsilon_{ij}b_j^* \tag{4.15}$$

The final result for the moduli space is $C(V_{5,2})/\Theta_k$, where Θ_k is the discrete group generated by $\mathbb{Z}_{2k}$ and the anti-involution.

Orientifold model II and III. we could have chosen anti-involutions with fixed points. The simplest one would send

$$a_i \to a_i^*\,, \qquad b_i \to b_i^*\,, \qquad \phi_i \to \phi_i^* \tag{4.16}$$

It leaves the equation for $C(V_{5,2})$ invariant but has a fixed point locus of real dimension four. The transformation can be implemented on the $\mathcal{N}=2$ theory as

$$A_i \to \Omega A_i^*\Omega^{-1}\,, \qquad B_i \to \Omega B_i^*\Omega^{-1}\,, \qquad \phi_i \to \Omega\phi_i^*\Omega^{-1} \tag{4.17}$$

with $\Omega = I$ or $\Omega = J$. The resulting $\mathcal{N}=1$ theories have $O(2N)\times O(2N)$ or $USp(2N)\times USp(2N)$ gauge groups. It is easy to see that the moduli space in the case of one membrane, $N=1$, correspond to the quotient of $C(V_{5,2})$ by a discrete group obtained by combining the usual abelian projection with the real involution (4.16). Note that the two types of theories with orthogonal or symplectic groups have the same moduli space. A similar phenomenon happens for D3 branes in type IIB [40]. The action descends to type IIA as an orientifold projection that fixes a six-dimensional plane and generalize an O6 plane. The two types of theories presumably correspond to two types of O6 action with the same geometrical lift to M theory.

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4.1.2 The case of $\mathbb{C}^4$

In the case $n=2$ we can integrate out the adjoint fields and obtain a quiver with fields A_i and B_j interacting with the $\mathcal{N}=2$ superpotential

$$W = \epsilon_{ij}\epsilon_{pq}A_iB_pA_jB_q \tag{4.18}$$

This is the original ABJM model [1] and describe membranes on $\mathbb{C}^4/\mathbb{Z}_k$. The complex coordinates A_i, B_j parametrize $\mathbb{C}^4$ and the $\mathbb{Z}_k$ action is $(A_i, B_j) \to (e^{2\pi i/k}A_i, e^{-2\pi i/k}B_j)$.

We can consider a simple anti-involution of $\mathbb{C}^4$ without fixed points

$$A_1 \to A_2^* \qquad A_2 \to -A_1^* \qquad B_1 \to B_2^* \qquad B_2 \to -B_1^* \tag{4.19}$$

and try to implement it as an orientifold projection on the $\mathcal{N}=2$ theory as we did for $V_{5,2}$

$$A_i \to -\epsilon_{ij}A_j^*J\,, \qquad\qquad B_i \to \epsilon_{ij}JB_j^* \tag{4.20}$$

The resulting theory has $O(2N)\times USp(2N)$ gauge groups.

Repeating the same analysis as done above for $V_{5,2}$ we will see that the one membrane moduli space is $\mathbb{C}^4/\mathcal{D}_k$ where $\mathcal{D}_k$ is a dihedral group obtained by combining the $\mathbb{Z}_{2k}$ action with the antiinvolution [41, 42].

$\mathcal{D}_k$ actually preserves a bigger $\mathcal{N}=5$ supersymmetry. The final Lagrangian has a hidden $\mathcal{N}=5$ supersymmetry and an $USp(4)$ global symmetry. The orientifold theory was indeed already constructed and discussed in [3, 4]. The ABJM model has a hidden $\mathcal{N}=6$ supersymmetry and SU(4) global symmetry rotating $(A_1, A_2, B_1^\dagger, B_2^\dagger)$. With an SU(4) transformation we can map the projection (4.20) into

$$A_i \to -\epsilon_{ij}B_j^TJ\,, \qquad\qquad B_i \to \epsilon_{ij}JA_j^T \tag{4.21}$$

where is manifest that at least an $\mathcal{N}=2$ supersymmetry is preserved.

We could obtain other real quotient of $\mathbb{C}^4$ with fixed points. For example

$$A_i \to A_i^* \qquad B_i \to B_i^* \tag{4.22}$$

is obtained by considering $O(2N)\times O(2N)$ or $USp(2N)\times USp(2N)$ orientifolds.

4.1.3 The case of the conifold times $\mathbb{C}$

The simplest model has $n=0$ [7]. In the $\mathcal{N}=2$ theory a single membrane probes a moduli space given by

$$A_1B_1 + A_2B_2 = 0 \tag{4.23}$$

with the ϕ unrestricted. The theory has various branches. As usual in the study of quivers, we restrict to the case $\phi_1=\phi_2$. We thus obtain a Calabi-Yau four-fold which is a conifold times $\mathbb{C}$ [7]. This cone is less interesting since it has a line of singularities. However we can still perform on the complex coordinates the anti-involution

$$A_1 \to A_2^* \qquad A_2 \to -A_1^* \qquad B_1 \to B_2^* \qquad B_2 \to -B_1^* \qquad \phi_i \to \phi_i^* \tag{4.24}$$

which acts with fixed points.

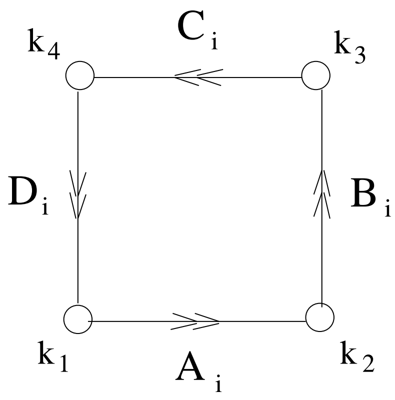

Figure 2. This is the quiver for $Q^{1,1,1}$ when the Chern-Simons couplings are $(0,1,0,-1)$ and for $Q^{2,2,2}$ when $(1,1,-1,-1)$.

We implement on the quiver fields the orientifold projection as above

$$A_i \to -\epsilon_{ij} A_j^* J\,, \qquad B_i \to \epsilon_{ij} J B_j^*\,, \qquad \phi_1 \to \phi_1^*\,, \qquad \phi_2 \to -J\phi_2^* J \tag{4.25}$$

obtaining an $O(2N) \times USp(2N)$ theory.

Repeating the same argument as above, we see that the single membrane moduli space is $C(T^{1,1}) \times \mathbb{C}$ divided by the discrete group generated by the anti-involution and the $\mathbb{Z}_{2k}$ action.

4.2 Anti-involution on $Q^{1,1,1}$ and its orbifolds

The cone over the manifold $Q^{1,1,1}$ and its $\mathbb{Z}_2$ orbifold $Q^{2,2,2}$ can be obtained by considering the chiral quiver in figure 2 with superpotential $W = \epsilon_{ij}\epsilon_{pq} A_i B_p C_j D_q$. The associated three-fold Z is the complex cone over F_0 and it is a $\mathbb{Z}_2$ quotient of the conifold. Z is relevant in the application of the quiver to D3 branes theories in type IIB. Here we consider the three-dimensional Chern-Simons theory with couplings (k_1, k_2, k_3, k_4) with $k_1 + k_2 + k_3 + k_4 = 0$. The moduli space is a Calabi-Yau four-fold (a different one for each choice of k_i) which is a fibration over Z. In the special case $(0,1,0,-1)$ we obtain $Q^{1,1,1}$ [22] and in the case $(1,1,-1,-1)$ we obtain $Q^{2,2,2}$ [13–15]. This model can be analyzed by descending in type IIA. We would obtain D2 branes transverse to a seven-dimensional manifold which is a fibration of Z over a real line.

4.2.1 The cone over $Q^{1,1,1}$

This case corresponds to the Chern-Simons couplings $(0,k,0,-k)$. The abelian F terms are

$$A_1 C_2 = A_2 C_1\,, \qquad B_1 D_2 = B_2 D_1 \tag{4.26}$$

The space of solution of the abelian F terms is called the master space [36–38] and it is useful to understand the M2 branes moduli space [7]. Here it has complex dimension six and it is the product of two conifolds.

To obtain the CY_4 from the six dimensional master space we need to impose the D term constraints and divide by the gauge groups. The relations $D_a = \frac{k_a}{2\pi}\sigma_a$ teach us that we need to impose the D term for the two gauge groups with zero Chern-Simons couplings and we need to mod by the corresponding U(1). As usual, we can dispose of both operations by dividing by the complexified gauge group. We thus quotient by the two complexified gauge groups with zero Chern-Simons. One is acting on C with charge $+1$ and on B with charge -1 and the other is acting on A with $+1$ and on D with -1. It is easy to get an algebraic description: the gauge invariants are

$$w_{ij} = B_i C_j \,, \qquad\qquad W_{ij} = D_i A_j \tag{4.27}$$

and satisfy nine quadratic constraints

$$\begin{aligned} w_{11}w_{22} &= w_{12}w_{21} \,, \qquad W_{11}W_{22} = W_{12}W_{21} \\ W_{i1}w_{j2} &= W_{i2}w_{j1} \,, \qquad W_{1i}w_{2j} = W_{2i}w_{1j} \qquad i,j = 1,2 \end{aligned} \tag{4.28}$$

where the first line comes from the definition of w and W and the second line from the F term equations. One of the eight equations in the second line is linearly dependent. Thus we obtain the description of Y as a set of nine quadrics in $\mathbb{C}^8$. These equations define the cone over $Q^{1,1,1}$ [43].

The remaining non-trivial gauge group acts as

$$\mathrm{U}(1)_2 - \mathrm{U}(1)_4 : \qquad \left(\frac{A \;\; B \;\; C \;\; D}{-1 \;\; 1 \;\; 1 \;\; -1} \right) \tag{4.29}$$

and it is broken to $\mathbb{Z}_k$ by the Chern-Simons interaction. The moduli space is then $C(Q^{1,1,1})/\mathbb{Z}_k$. Note that the $\mathbb{Z}_k$ action breaks the $\mathrm{SU}(2)^3$ isometry of $Q^{1,1,1}$ to $\mathrm{SU}(2)^2 \times \mathrm{U}(1)$ which is precisely the symmetry of the quiver.

By modding by $\mathrm{U}(1)_2 - \mathrm{U}(1)_4$ we would obtain the complex cone over F_0. We can thus see $Q^{1,1,1}$ as a $\mathbb{C}^*$ fibration over F_0. The leaves are copies of (resolutions of) F_0

$$\left\{ \sum_{i=1}^{2} |B_i|^2 + |C_i|^2 - |A_i|^2 - |D_i|^2 = \sigma \right\} / \mathrm{U}(1)_2 - \mathrm{U}(1)_4 \tag{4.30}$$

and we can choose local coordinates giving a point in F_0 and the value of σ and of the angle ψ parametrizing the U(1) orbit.

The orientifold projection. A possible anti-holomorphic action without fixed point on $Q^{1,1,1}/\mathbb{Z}_k$ is:

$$\begin{array}{llll} W_{11} \to \;\; W_{21}^* & W_{12} \to \;\; W_{22}^* & w_{12} \to \;\; w_{22}^* & w_{11} \to \;\; w_{21}^* \\ W_{21} \to -W_{11}^* & W_{22} \to -W_{12}^* & w_{22} \to -w_{12}^* & w_{21} \to -w_{11}^* \end{array} \tag{4.31}$$

It is actually a $\mathbb{Z}_4$ action, however it is a $\mathbb{Z}_2$ action on $Q^{1,1,1}/\mathbb{Z}_{2k}$. The action on the elementary coordinates is,

$$D_i \to \epsilon_{ij} D_j^* \qquad B_i \to \epsilon_{ij} B_j^* \qquad A_i \to A_i^* \qquad C_i \to C_i^* \tag{4.32}$$

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This anti-holomorphic projection breaks the $\mathrm{SU}(2)^2$ symmetry of the quiver theory to $\mathrm{SU}(2)\times\mathrm{U}(1)$. If we think of the four dimensional CY $C(Q^{1,1,1})/\mathbb{Z}_k$ as a double fibration of the complex cone over F_0 over $\mathbb{R}$ and S^1 as in (4.30), the projection (4.31) acts as an antiholomorphic involution on F_0, it leaves σ invariant and it changes the sign of ψ: $\psi\to-\psi$.

We can understand better this projection by descending in type IIA. In these coordinates, the type IIA limit corresponds to shrink the circle associated to ψ [22]. Following [30] the flip in the M theory circle will translate to an orientifold projection in type IIA.

We now realize the anti-holomorphic action as an orientifold projection on the Lagrangian. The final gauge groups are of type $O(2N)$ or $USp(2N)$. Consistency of the projection (4.32) requires that gauge groups 1 and 2 and gauge groups 3 and 4 are of same type, but the groups 1,2 are different from the groups 3,4. One possible choice is: O, O, USp, USp. The orientifold action is then,

$$D_i\to\epsilon_{ij}JD_j^* \qquad B_i\to-\epsilon_{ij}B_j^*J \qquad A_i\to A_i^* \qquad C_i\to-JC_i^*J \tag{4.33}$$

It is easy to check that this orientifold action is a symmetry of the original $\mathcal{N}=2$ theory.

The resulting field theory has O(2N)×O(2N)×USp(2N)×USp(2N) gauge group and $\mathcal{N}=1$ supersymmetry.

To test the proposal, we compute the moduli space for one membrane. The gauge group is $O(2)\times O(2)\times\mathrm{SU}(2)\times\mathrm{SU}(2)$. We parametrize elementary fields as,

$$\begin{aligned}
A_i &= Re(a_i)+JIm(a_i)\,, \qquad C_i = Re(c_i)+JIm(c_i)\\
B_1 &= \frac{1}{\sqrt{2}}\Big(\big(\mathrm{Re}(b_1)+\mathrm{Im}(b_1)J\big)+i\big(\mathrm{Re}(b_2)+\mathrm{Im}(b_2)J\big)\Big) \qquad B_2=B_1^*J\\
D_2 &= \frac{1}{\sqrt{2}}\Big(\big(\mathrm{Re}(d_2)+\mathrm{Im}(d_2)J\big)+i\big(\mathrm{Re}(d_1)+\mathrm{Im}(d_1)J\big)\Big) \qquad D_1=JD_2^*
\end{aligned}\tag{4.34}$$

where a_i, b_p, c_j, d_q are generic complex numbers. The four SO(2) gauge subgroups of the gauge groups act as the $\mathrm{U}(1)^4$ gauge group of the unorientifolded theory. The D term of the four gauge groups are proportional to the identity

$$\begin{aligned}
\mathcal{D}_1 &= \sum_i\left(|a_i|^2-|d_i|^2\right)\mathrm{Id}\,, \qquad \mathcal{D}_2=\sum_i\left(|b_i|^2-|a_i|^2\right)\mathrm{Id}\\
\mathcal{D}_3 &= \sum_i\left(|c_i|^2-|b_i|^2\right)\mathrm{Id}\,, \qquad \mathcal{D}_4=\sum_i\left(|d_i|^2-|a_i|^2\right)\mathrm{Id}
\end{aligned}\tag{4.35}$$

The vacuum conditions require $\sigma_a X_{ab}=X_{ab}\sigma_b$ with $D_a=\frac{k_a}{2\pi}\sigma_a$. The first condition is trivially satisfied while the second one requires

$$\sum_i|a_i|^2=\sum_i|d_i|^2\,, \qquad \sum_i|c_i|^2=\sum_i|b_i|^2 \tag{4.36}$$

which are the D term of the $\mathcal{N}=2$ theory. We should also mod by the SO(2) subgroups of the first and third gauge groups which has zero Chern-Simons coupling, exactly as in the parent theory. The F term equations give

$$a_1c_2=c_1a_2\,, \qquad b_1d_2=b_2d_1 \tag{4.37}$$

On top of these equations the CS interaction breaks the third SO(2) gauge group to a $\mathbb{Z}_{2k}$ action: $(a_i^*, b_p, c_j, d_p^*) \to e^{\pi i/k}(a_i^*, b_p, c_j, d_p^*)$. Taking the quotient with respect to this discrete action we obtain exactly $C(Q^{1,1,1})/\mathbb{Z}_{2k}$. There is an extra discrete $\mathbb{Z}_2$ symmetry on the moduli space acting as σ_3 in the $O(2)$ gauge groups and as $i\sigma_3$ in the SU(2) gauge groups. Its action on the coordinates of the moduli space is

$$(a_i, c_i) \to (a_i^*, c_i^*)\,, \qquad\qquad (b_i, d_i) \to \epsilon_{ij}(b_j^*, d_j^*) \tag{4.38}$$

which it is exactly the orientifold action we considered in the previous section. As a result, the moduli space is $C(Q^{1,1,1})$ divided by the discrete group generated by the anti-involution and the $\mathbb{Z}_{2k}$ action.

The $\mathcal{N} = 1$ description. We could have alternatively studied the moduli space using the $\mathcal{N} = 1$ superfield formalism. The original $\mathcal{N} = 2$ theory, written in $\mathcal{N} = 1$ formalism, contains four extra adjoint real superfields R^a. The original fields A_i, B_i, C_i, D_i are now considered as complexified $\mathcal{N} = 1$ superfields. As discussed in the appendix, the Chern-Simons contribution is (see (A.5))

$$\frac{k}{4\pi}\int d^2\theta_1 (R_4^2 - R_2^2) \tag{4.39}$$

There is no quadratic term for R_1 and R_3 since the corresponding Chern-Simons couplings are zero. Considering the linear interactions (A.6) and using the equation of motion for R_1, R_2, R_3, R_4, we obtain the constraints

$$A_i A_i^\dagger = D_i^\dagger D_i \qquad C_i C_i^\dagger = B_i^\dagger B_i \tag{4.40}$$

and the following contribution to the $\mathcal{N} = 1$ superpotential coming from the D terms,

$$\frac{\pi}{k}\int d^2\theta_1 (B_i B_i^\dagger - A_i^\dagger A_i)^2 - (D_i D_i^\dagger - C_i^\dagger C_i)^2 \tag{4.41}$$

which should be added to the contribution coming from the $\mathcal{N} = 2$ superpotential

$$\int d^2\theta_1 \epsilon_{ij}\epsilon_{pq}\ \mathrm{Tr}\ \left(A_i B_p C_j D_q + A_i^* B_p^* C_j^* D_q^*\right). \tag{4.42}$$

The $\mathcal{N} = 1$ theory can be obtained by restricting the $\mathcal{N} = 1$ superpotential to configurations invariant under (4.33). We can express the fields in terms of the independent ones $A_i, C_i, B_1, B_1^*, D_1, D_1^*$, where B_1 and D_1 are arbitrary complex matrices while A_i and C_i satisfy the reality conditions $A_i = A_i^*$ and $C_i = -JC_i^*J$. The $\mathcal{N} = 1$ superpotential comes from (4.40), (4.41), (4.42) and reads

$$\begin{aligned} W_{\mathcal{N}=1} = \frac{\pi}{k}&\left(\left(B_1B_1^\dagger + B_1^*B_1^T - A_i^TA_i\right)^2 - \left(D_1D_1^\dagger - JD_1^*D_1^TJ + JC_i^TJC_i\right)^2\right)\\ &-2\epsilon_{ij}\ \mathrm{Tr}\ \left(A_iB_1C_jJD_1^* + A_iB_1^*JC_jD_1\right)\\ &\alpha(A_iA_i^T - D_1^\dagger D_1 - D_1^TD_1^*) + \beta(-C_iJC_i^TJ - B_1^\dagger B_1 + JB_1^TB_1^*J) \end{aligned} \tag{4.43}$$

with α, β two Lagrange multipliers. The vacua can be obtained by minimizing the $\mathcal{N} = 1$ superpotential. The result agrees with that described above.

4.2.2 The cone over $Q^{2,2,2}$

$Q^{1,1,1}$ admits a supersymmetric orbifold $Q^{2,2,2}$ that reduces by half the length of the U(1) fiber. The field theory living on N M2 branes at the tip of the cone over $Q^{2,2,2}$ is described by the same quiver and superpotential of the $Q^{1,1,1}$ theory, but it has different values for the CS levels $(1,1,-1,-1)$ [14, 15].

Consider as usual the abelian case. One U(1) gauge group acts trivially on matter fields and another one is broken by the Chern-Simons interactions. The surviving U(1) actions

$$\left(\begin{array}{cccc} A & B & C & D \\ \hline 1 & 0 & -1 & 0 \\ 2 & -1 & 0 & -1 \end{array}\right) \tag{4.44}$$

can be paired with

$$\mathcal{D}_1 + \mathcal{D}_4 = \sum_{i=1}^{2} |A_i|^2 - |C_i|^2 = 0 \qquad \mathcal{D}_1 - \mathcal{D}_2 = \sum_{i=1}^{2} 2|A_i|^2 - |B_i|^2 - |D_i|^2 = 0 \tag{4.45}$$

(following from $\mathcal{D}_a = \frac{k_a}{2\pi}\sigma_a$) to give two complexified U(1). There are 27 invariants

$$\alpha_{ij\,pq} = A_i B_p C_j D_q\,, \qquad \beta_{ij\,pq} = A_i B_p C_j B_q\,, \qquad \gamma_{ij\,pq} = A_i D_p C_j D_q \tag{4.46}$$

totally symmetrized in the indices i,j and p,q by the F term equations (4.26). They satisfy 253 quadratic equations due to the F terms and give an algebraic description of $C(Q^{2,2,2})$ as a set of quadrics in $\mathbb{C}^{27}$. For generic k the moduli space is $C(Q^{2,2,2})/\mathbb{Z}_k$ with $\mathbb{Z}_k$ acting as

$$B_i \to e^{\frac{i\pi}{k}} B_i\,, \qquad D_i \to e^{-\frac{i\pi}{k}} D_i\,. \tag{4.47}$$

A natural antiholomorphic involution on $C(Q^{2,2,2})$ elementary coordinate fields is

$$A_i \to i\epsilon_{ij} A_j^* \qquad B_i \to B_i^* \qquad C_i \to -i\epsilon_{ij} C_j^* \qquad D_i \to D_i^* \tag{4.48}$$

It is an easy exercise to check that this action, which fixes a subspace of $\mathbb{C}^{27}$, has however no fixed points on the four-fold.

The $\mathcal{N}=1$ Chern-Simons theory $C(Q^{2,2,2})/\Theta_k$ where Θ_k is the semi-direct product of the anti-involution with $\mathbb{Z}_{2k}$ is obtained by projecting the quiver matter fields

$$A_i \to -i\epsilon_{ij} A_j^* J \qquad B_i \to -J B_i^* J \qquad C_i \to -i\epsilon_{ij} J C_j^* \qquad D_i \to D_i^*\,. \tag{4.49}$$

and it is a theory with gauge groups $O(2N) \times USp(2N) \times USp(2N) \times O(2N)$.

4.3 More general examples

The construction can be applied to the general $\mathcal{N}=2$ quiver. In particular, we can orientifold any quiver by the obvious anti-involution that conjugates all fields. The orientifold action $X_{ab} \to \Omega_a X_{ab}^* \Omega_b$ is consistent when all Ω_a are equal and we obtain theories with all orthogonal or all symplectic gauge groups. The one membrane moduli space will be in both cases a Spin(7) manifold of the form Y/Θ_k. This construction can be applied to both chiral and vectorial quivers and typically produces anti-involutions with a locus of fixed points. Actions which fix only the origin are more difficult to find and require particular choices. We will show in the next section how to find other examples by extending the orientifold construction.

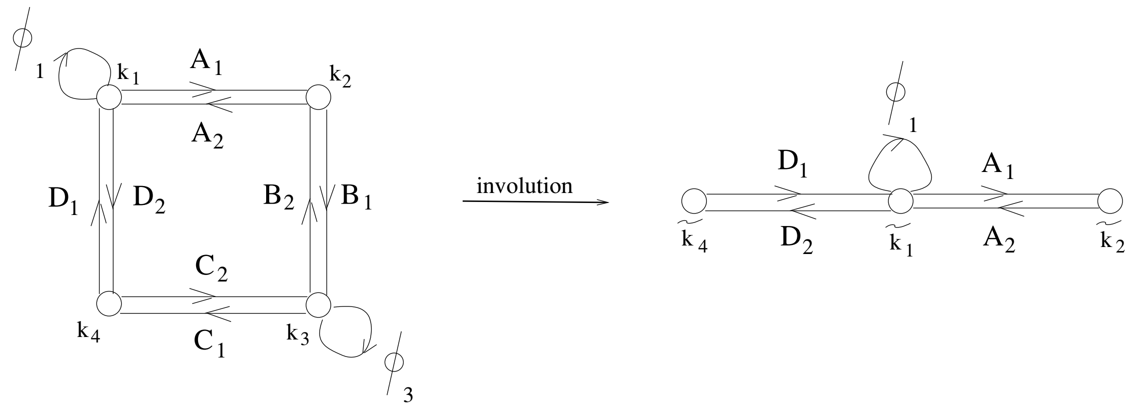

Figure 3. Quiver for $\widetilde{L^{2,2,2}}_{(1,-1,1,-1)}$ and its anti-involution.

5 Orientifolds with identification of gauge groups

We can perform more general orientifold projections using $\mathbb{Z}_2$ symmetries of the quiver. The orientifolded gauge theory is obtained from the parent theory by a certain $\mathbb{Z}_2$ identification of gauge groups and matter fields. This procedure has been utilized systematically in [44] to construct and classify $\mathcal{N}=1$ four-dimensional orientifolds theories based on Tilings. Here we combine the identification of gauge groups and matter fields with an anti-involution in order to obtain an $\mathcal{N}=1$ theory from a parent $\mathcal{N}=2$ Chern-Simons theory. We will not try to be exhaustive and classify all possible models but we restrict to a particular example which exemplifies the general construction.

5.1 A Chern-Simons quiver based on $L^{2,2,2}$

A particularly simple CY_4 is the intersection of two conifold singularities: $xy = wz = tv$. One candidate for the $\mathcal{N}=2$ theory living on N M2 branes at the tip of this cone is described by the first quiver on the left in figure 3 with superpotential

$$W = A_2A_1B_1B_2 - C_2C_1D_1D_2 + \phi_3(C_1C_2 - B_2B_1) + \phi_1(D_2D_1 - A_1A_2) \tag{5.1}$$

and Chern-Simons couplings $(1,-1,1,-1)$. The three-fold associated with this quiver and superpotential is part of the $C(L^{a,b,c})$ family [45–47]. We refer to the 3d conformal field theory with this specific choice of Chern-Simons as the $\widetilde{L^{2,2,2}}_{(1,-1,1,-1)}$ quiver theory. To check that the moduli space of the $\widetilde{L^{2,2,2}}_{(1,-1,1,-1)}$ theory is the intersection of two conifolds let us compute its abelian moduli space. The abelian F terms imply

$$\phi_1 = B_1B_2 = C_1C_2\,, \qquad \phi_3 = D_1D_2 = A_1A_2 \tag{5.2}$$

The $\mathcal{N}=2$ abelian moduli space is obtained by modding by the linear combinations of the (complexified) groups $\mathrm{U}(1)_1 + \mathrm{U}(1)_2$ and $\mathrm{U}(1)_1 - \mathrm{U}(1)_3$. The gauge invariants are

$$x_1 = A_1C_1\,,\quad x_2 = A_1A_2\,,\quad x_3 = B_1D_1\,,\quad x_4 = B_1B_2\,,\quad x_5 = A_2C_2\,,\quad x_6 = B_2D_2 \tag{5.3}$$

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and satisfy the two equations

$$x_2x_4 = x_1x_5 = x_3x_6 \tag{5.4}$$

which is exactly the Calabi-Yau fourfold Y we were looking for: a complete intersection of two quadrics in $\mathbb{C}^6$. For CS couplings $(k, -k, k, -k)$ the moduli space is obtained by modding Y by the remaining discrete group $\mathbb{Z}_k \in \mathrm{U}(1)_1 - \mathrm{U}(1)_2 + \mathrm{U}(1)_3 - \mathrm{U}(1)_4$.

The quiver has a $\mathbb{Z}_2$ symmetry across the diagonal connecting the groups 2 and 4. The symmetry preserves the choice of Chern-Simons couplings. We can therefore perform an orientifold projection where group 1 is identified with group 3, the fields B are identified with A and D with C. The resulting orientifolded quiver is shown in figure 3. The gauge groups 2 and 4 are projected with

$$A^a_\mu = -\Omega_a(A^a_\mu)^T\Omega_a^{-1} \qquad a = 2, 4$$

and become $O(2N)$ or $USp(2N)$ according to whether $\Omega = I$ or $\Omega = J$, respectively. The gauge groups 1 and 3 are identified by the orientifold projection

$$A^1_\mu = -\Omega(A^3_\mu)^T\Omega^{-1}$$

and remain unitary groups $\mathrm{U}(2N)$. The action on fields is

$$\begin{aligned} &A_1 \to \quad \Omega B_2^*\Omega_2^{-1}\,, \;\; A_2 \to \quad \Omega_2 B_1^*\Omega^{-1}\,, \;\;\; D_1 \to \quad \Omega_4 C_2^*\Omega^{-1}\,, \;\; D_2 \to \quad \Omega C_1^*\Omega_4^{-1} \\ &B_2 \to \pm\Omega A_1^*\Omega_2^{-1}\,, \;\; B_1 \to \pm\Omega_2 A_2^*\Omega^{-1}\,, \;\;\; C_2 \to \pm\Omega_4 D_1^*\Omega^{-1}\,, \;\; C_1 \to \pm\Omega D_2^*\Omega_4^{-1} \end{aligned} \tag{5.5}$$

where the choice of sign in the second line can be made independently on the pairs (A, B) and the pairs (C, D). Each choice implies some constraints on the relative choice of $\Omega_{2,4}$ and Ω.

We have three independent models: model I with gauge group $O(2N)_{-2k} \times \mathrm{U}(2N)_{2k} \times USp(2N)_{-k}$, model II with gauge group $O(2N)_{-2k} \times \mathrm{U}(2N)_{2k} \times O(2N)_{-2k}$, and model III with gauge group $USp(2N)_{-k} \times \mathrm{U}(2N)_{2k} \times USp(2N)_{-k}$.

Model I. it corresponds to an antiinvolution without fixed points. As usual we analize the case of one membrane, $N = 1$. For $N = 1$ we have $O(2) \times \mathrm{U}(2) \times \mathrm{SU}(2)$. The one membrane moduli space can be parameterized by the two-by-two matrices

$$A_{1,2} = \frac{a_{1,2} + b^*_{2,1}}{2} I + \frac{a_{1,2} - b^*_{2,1}}{2i} J, D_1 = \frac{d_1 + ic^*_2}{2} I + \frac{d_1 - ic^*_2}{2i} J, D_2 = \frac{d_2 - ic^*_1}{2} I + \frac{d_2 + ic^*_1}{2i} J \tag{5.6}$$

As usual, on the moduli space, the D term and F term constraints reduce to those of the parent $\mathcal{N} = 2$ theory. We also have four residual abelian groups $\mathrm{SO}(2) \subset O(2), \mathrm{SO}(2) \subset \mathrm{SU}(2), \mathrm{U}(1)\times\mathrm{U}(1) \subset \mathrm{U}(2)$ acting on the moduli space, and we can arrange for a basis where they act as in the $\mathcal{N} = 2$ theory. One abelian factor acts trivially, two factors are used to mod the moduli space, and the last abelian factor is broken to $\mathbb{Z}_{2k}$ by the Chern-Simons interactions. We have an extra discrete gauge symmetry $i\sigma_3 \in \mathrm{SU}(2), \mathrm{U}(2)$ and $\sigma_3 \in O(2)$ acting on the fields as

$$\begin{aligned} a_{1,2} &\to b^*_{2,1}\,, & d_{1,2} &\to \;\; c^*_{2,1} \\ b_{2,1} &\to a^*_{1,2}\,, & c_{2,1} &\to -d^*_{1,2} \end{aligned} \tag{5.7}$$

This transformation is lifted to an anti-involution on Y without fixed points

$$\begin{aligned} x_1 &\to -x_6^*\,, & x_5 &\to -x_3^*\,, & x_2 &\to x_4^* \\ x_6 &\to x_1^*\,, & x_3 &\to x_5^*\,, & x_4 &\to x_2^* \end{aligned} \tag{5.8}$$

The moduli space is the quotient of Y/Θ_k by the semi-direct product of the anti-involution with $\mathbb{Z}_{2k}$.

Model II and III. It is easy to see that they both correspond to the anti-involution

$$\begin{aligned} x_1 &\to x_6^*\,, & x_5 &\to x_3^*\,, & x_2 &\to x_4^* \\ x_6 &\to x_1^*\,, & x_3 &\to x_5^*\,, & x_4 &\to x_2^* \end{aligned} \tag{5.9}$$

which acts on Y with a fixed locus.

6 Conclusions and outlooks

In this paper we presented a large class of $\mathcal{N}=1$ (2+1) dimensional field theories that are supposed to live on the world volume of N M2 branes at the tip of Spin(7) cones. They are supersymmetric non-holomorphic field theories with gravity duals. Our procedure is based on an antiholomorphic involution on the geometrical side and on orientifold projection on the field theory side. It would be interesting to systematically study the possible $\mathcal{N}=1$ orientifolds of M2 brane theories. In particular it would be interesting to perform a general analysis of orientifold M2 theories based on Tilings as it was done in [44] for D3 branes. Another nice application would be to extend the ideas of toric/Seiberg duality studied in the context of $\mathcal{N}=2$ (2+1) dimensional field theories to the $\mathcal{N}=1$ cases [13–15, 48, 49]. In this paper, we studied real quotients of CY_4 and it would be interesting to see if there is a quiver realization of more general Spin(7) manifolds.

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A $\mathcal{N}=1$ CS theories

In this paper we studied an $\mathcal{N}=1$ orientifold projection of the $\mathcal{N}=2$ theory.

In this appendix we introduce the $\mathcal{N}=1$ (2+1) dimensional formalism and we explain how to pass from the $\mathcal{N}=2$ to the $\mathcal{N}=1$ notations.

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The low energy dynamics of M2 branes at Calabi Yau four-fold toric conical singularities is well described by (2+1) dimensional $\mathcal{N}=2$ Chern-Simons theories with gauge group $\prod_{i=1}^{G} U_i(N)$ and bifundamental or adjoint matter fields [6, 7]. The typical Lagrangian in $\mathcal{N}=2$ superspace notation is

$$\sum_a \frac{k_a}{8\pi} S^{\mathcal{N}=2}_{CS_a}(V_a) - \mathrm{Tr}\left(\int d^4\theta \sum_{X_{ab}} X^{\dagger}_{ab} e^{-V_a} X_{ab} e^{V_b}\right) + \int d^2\theta W(X_{ab}) + \int d^2\bar{\theta}\bar{W}(X^*_{ab}) \tag{A.1}$$

where V_a are the vector superfields, X_{ab} are bifundamental chiral superfields and $S^{\mathcal{N}=2}_{CS_a}$ is the $\mathcal{N}=2$ Chern-Simons action for the a-th vector superfield. The Chern-Simons couplings satisfy $\sum_a k_a = 0$. In the toric case, the superpotential $W(X_{ab})$ satisfies the following condition: every chiral superfield appears exactly twice, one time with plus sign and the other time with minus sign. Every V_a contains a gauge field, a two-component Dirac spinor and two real scalar fields, σ_a and D_a. Every X_{ab} contains two complex scalars and a two-dimensional Dirac spinor.

The three dimensional $\mathcal{N}=2$ field content is the dimensional reduction of the four dimensional $\mathcal{N}=1$ superfields. The Lagrangian (A.1) can be intuitively understood as the reduction from four to three dimensions of the field theory living on D3 branes probing a three-fold toric Calabi Yau cone. In the reduction we must substitute the kinetic term for the vector superfields with a Chern-Simons interaction.

In $\mathcal{N}=1$ notations, we need two kinds of superfields: the spinor superfield Γ_α and the scalar superfield Φ. The first contains a gauge vector field and a Majorana two-component spinor, while the second contains a Majorana two component spinor and two real scalars. The generic $\mathcal{N}=1$ quiver Chern-Simons theory Lagrangian is

$$\sum_a \frac{k_a}{8\pi} S^{\mathcal{N}=1}_{CS_a}(\Gamma^{\alpha}_{a}) - \int d^2\theta_1 \sum_{Y_{ab}} \mathrm{Tr}\Big((D^\alpha + i\Gamma^\alpha_b) Y^\dagger_{ab} (D_\alpha - i\Gamma^a_\alpha) Y_{ab}\Big) + W^{\mathcal{N}=1}(Y_{ab}, Y^*_{ab}) \tag{A.2}$$

where the first term is the $\mathcal{N}=1$ Chern-Simons action, the second is the kinetic term for the matter fields and the third is the $\mathcal{N}=1$ superpotential. It is important to note that the $\mathcal{N}=1$ superpotential is a real function of the scalar fields. The Ys are complex scalar $\mathcal{N}=1$ superfields: $Y=\Phi_1 + i\Phi_2$.

To pass from $\mathcal{N}=2$ to $\mathcal{N}=1$ formalism we need to decompose the $\mathcal{N}=2$ superspace in two copies of an $\mathcal{N}=1$ superspace [50–52],

$$\theta_\alpha = \theta_{1\alpha} + i\theta_{2\alpha}, \qquad D_\alpha = \frac{1}{2}(D_{1\alpha} + iD_{2\alpha}), \qquad \bar{D}_\alpha = \frac{1}{2}(D_{1\alpha} - iD_{2\alpha}) \tag{A.3}$$

and keep only the θ_1 component. The $\mathcal{N}=2$ superfields decompose as

$$V^a\big|_{\theta_2=0} = 0, \qquad D_{2\alpha}V^a\big|_{\theta_2=0} = \Gamma^a_\alpha, \qquad D_2^2 V^a\big|_{\theta_2=0} = R^a, \qquad X_{ab}\big|_{\theta_2=0} = Y_{ab}\,. \tag{A.4}$$

The $\mathcal{N}=2$ vector superfield V^a decomposes into an $\mathcal{N}=1$ spinor superfield Γ^a_α and an $\mathcal{N}=1$ real scalar superfield R^a, while the $\mathcal{N}=2$ chiral superfield X_{ab} decomposes into two real scalar $\mathcal{N}=1$ superfields $\mathrm{Re}(X_{ab})\big|_{\theta_2=0}$ and $\mathrm{Im}(X_{ab})\big|_{\theta_2=0}$ which combine into a complex scalar superfield Y_{ab}. The R^a are $\mathcal{N}=1$ auxiliary real scalar superfields that can be eliminated via their equations of motion.

When we write the original $\mathcal{N}=2$ Lagrangian (A.1) in $\mathcal{N}=1$ formalism the $\mathcal{N}=2$ Chern-Simons and the kinetic terms for scalars become the $\mathcal{N}=1$ Chern-Simons and kinetic terms. The $\mathcal{N}=1$ superpotential gets contributions from the $\mathcal{N}=2$ Chern-Simons action, the kinetic terms for chiral fields and the superpotential

$$\frac{k_a}{8\pi} S^{\mathcal{N}=2}_{CS_a} \;\rightarrow\; -\frac{k_a}{4\pi}\int d^2\theta_1 R_a^2 \tag{A.5}$$

$$-\int d^4\theta X^\dagger_{ab} e^{-V_a} X_{ab} e^{V_b} \;\rightarrow\; \int d^2\theta_1 \big(Y_{ab}Y^\dagger_{ab}R_a - Y^\dagger_{ab}Y_{ab}R_b\big) \tag{A.6}$$

$$\int d^2\theta W(X_{ab}) + \text{c.c} \;\rightarrow\; \int d^2\theta_1 W(Y_{ab}) + W(Y^*_{ab}) \tag{A.7}$$

Integrating out the $\mathcal{N}=1$ superfield R_a we get new interactions among the $\mathcal{N}=1$ scalar superfields Y_{ab} depending on the quiver and on the specific values of the CS levels,

$$\begin{aligned} W^{\mathcal{N}=1}(Y_{ab}, Y^*_{ab}) &= W(Y_{ab}) + W(Y^*_{ab}) + \sum_{a,k_a\neq 0} \frac{k_a}{4\pi} R_a^2 \\ \frac{k_a}{2\pi} R_a &= \sum_b Y_{ab}Y^\dagger_{ab} - \sum_c Y^\dagger_{ca}Y_{ca} \end{aligned} \tag{A.8}$$

supplemented with the constraints

$$\sum_b Y_{ab}Y^\dagger_{ab} - \sum_c Y^\dagger_{ca}Y_{ca} = 0 \tag{A.9}$$

from the gauge groups with zero Chern-Simons coupling.

In $\mathcal{N}=1$ notations the vacuum conditions are obtained by minimizing the superpotential with respect to all real superfields. These conditions contain both the F term and the D term constraints of the $\mathcal{N}=2$ notations. That the two ways of finding a vacuum coincide when the theory has $\mathcal{N}=2$ supersymmetry follows from the identity

$$\begin{aligned} \sum_{Y_{ab}} \mathrm{Tr}\Big|\partial_{Y_{ab}} W^{\mathcal{N}=1}\Big|^2 &= \sum_{Y_{ab}} \mathrm{Tr}\Big|\partial_{Y_{ab}} W^\dagger + \sum_b (\sigma_a Y_{ab} - Y_{ab}\sigma_b)\Big|^2 \\ &= \sum_{Y_{ab}} \mathrm{Tr}|\partial_{Y_{ab}} W|^2 + \sum_{Y_{ab}} \mathrm{Tr}|\sigma_a Y_{ab} - Y_{ab}\sigma_b|^2 \end{aligned} \tag{A.10}$$

valid for all $\mathcal{N}=2$ quivers. Indeed the first term on the left is the bosonic scalar potential obtained from the $\mathcal{N}=1$ formalism and the last term on the right is the scalar potential coming from the $\mathcal{N}=2$ formalism (2.2).

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