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Research Article

Stability Criterion for Discrete-Time Systems

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This paper is concerned with the problem of delay-dependent stability analysis for discrete-time systems with interval-like time-varying delays. The problem is solved by applying a novel Lyapunov functional, and an improved delay-dependent stability criterion is obtained in terms of a linear matrix inequality.

1. Introduction

Recently, the problem of delay-dependent stability analysis for time-delay systems has received considerable attention, and lots of significant results have been reported; see, for example, Chen et al. [1], He et al. [2], Lin et al. [3], Park [4], and Xu and Lam [5], and the references therein. Among these references, we note that the delay-dependent stability problem for discrete-time systems with interval-like time-varying delays (i.e., the delay $d(k)$ satisfies $0 < d_m \leq d(k) \leq d_M$) has been studied by Fridman and Shaked [6], Gao and Chen [7], Gao et al. [8], and Jiang et al. [9], where some LMI-based stability criteria have been presented by constructing appropriate Lyapunov functionals and introducing free-weighting matrices. It should be pointed out that the Lyapunov functionals considered in these references are more restrictive due to the ignorance of the term $\sum_{j=-h_M}^{j=-h_m-1} \sum_{i=k+j}^{i=k-1} [x(i+1)-x(i)]^T R[x(i+1)-x(i)]$. Moreover, the term $\sum_{i=k-h_m}^{i=k-1} x(i)^T Q_2 x(i)$ is also ignored in Gao and Chen [7] and Gao et al. [8]. The ignorance of these terms may lead to considerable conservativeness.

On the other hand, in the study of stabilization for the discrete-time linear systems, traditional idea of the control schemes is to construct a control signal according to the current system state [10]. However, as pointed out by Xiong and Lam [11], in practice there is often a system that itself is not time-delayed but time-delayed may exist in a channel from system to controller. A typical example for the existence of such delays is the measurement and the network transmission of signals. In this case, a time-delayed controller is naturally taken into account. It is worth noting that the closed-loop system resulting from a delayed controller is actually a time-delay system. Therefore, stability results of time-delay systems could be applied to design time-delayed controller.

The present study, based on a new Lyapunov functional, an improved delay-dependent stability criterion for discrete-time systems with time-varying delays is presented in terms of LMIs. It is shown that the obtained result is less conservative than those by Fridman and Shaked [6], Gao and Chen [7], Gao et al. [8], Jiang et al. [9], and Zhang et al. [12].

2. Preliminaries

Fact 1. For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y. \tag{2.1}$$

Let us denote $V_\delta = \{x \in \mathbb{R}^n : \|x\| < \delta\}$.

Lemma 2.1 (see [13]). *The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x(k)) : \mathbb{R}^n \to \mathbb{R}^+$ such that*

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \le -\beta \|x(k)\|^2, \tag{2.2}$$

along the solution of the system. In the case the above condition holds for all $x(k) \in V_\delta$, say one that the zero solution is locally asymptotically stable.

Lemma 2.2 (see [13]). *For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M^T > 0$, scalar $s \in \mathbb{Z}^+/\{0\}$, vector function $W : [0, s] \to \mathbb{R}^n$, one has*

$$s \sum_{i=0}^{s-1} \left(w^T(i) M w(i) \right) \ge \left(\sum_{i=0}^{s-1} w(i) \right)^T M \left(\sum_{i=0}^{s-1} w(i) \right). \tag{2.3}$$

3. Improved Stability Criterion

In this section, we give a novel delay-dependent stability condition for discrete-time systems with interval-like time-varying delays. Now, consider the following system:

$$x(k+1) = Ax(k) + Bx(k - h(k)), \tag{3.1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, A and B are known constant matrices, and $h(k) > 0$ is a time-varying delay satisfying $0 < h_m \le h(k) \le h_M$, where h_m and h_M are positive integers representing the lower and upper bounds of the delay. For (3.1), we have the following result.

Theorem 3.1. *Give integers $h_m > 0$ and $h_M > 0$. Then, the discrete time-delay system (3.1) is asymptotically stable for any time delay $h(k)$ satisfying $h_m \le h(k) \le h_M$, if there exist symmetric positive definite matrices PGW satisfying the following matrix inequalities:*

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0, \tag{3.2}$$

where $(1,1) = A^TPA + \varepsilon A^TPPA + h(k)G + W - P$, *and* $(2,2) = B^TPB + \varepsilon^{-1}B^TB + \varepsilon_1^{-1}B^TB - W$, $(3,3) = -h(k)G$.

Proof. Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$\begin{aligned} V_1(y(k)) &= x^T(k)Px(k), \\ V_2(y(k)) &= \sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Gx(i), \\ V_3(y(k)) &= \sum_{i=k-h(k)}^{k-1} x^T(i)Wx(i), \end{aligned} \tag{3.3}$$

with PGW being symmetric positive definite solutions of (3.2) and $y(k) = [x(k), x(k-h)]$.

Then difference of $V(y(k))$ along trajectory of solution of (3.1) is given by

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)), \tag{3.4}$$

where

$$\begin{aligned} \Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [Ax(k) + Bx(k-h(k))]^T P[Ax(k) + Bx(k-h(k))] - x^T(k)Px(k) \\ &= x^T(k)\left[A^TPA - P\right]x(k) + x^T(k)A^TPBx(k-h(k)) + x^T(k-h(k))B^TPAx(k) \\ &\quad + x^T(k-h(k))B^TPBx(k-h(k)), \\ \Delta V_2(y(k)) &= \Delta\left(\sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Gx(i)\right) = h(k)x^T(k)Gx(k) - \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i), \end{aligned} \tag{3.5}$$

$$\Delta V_3(y(k)) = \Delta\left(\sum_{i=k-h(k)}^{k-1} x^T(i)Wx(i)\right) = x^T(k)Wx(k) - x^T(k-h(k))Wx(k-h(k)), \tag{3.6}$$

where Fact 1 is utilized in (3.6), respectively.

Note that

$$x^T(k)A^T PBx(k-h(k)) + x^T(k-h(k))B^T PAx(k) \\ \leq \varepsilon x^T(k)A^T PPAx(k) + \varepsilon^{-1}x^T(k-h(k))B^T Bx(k-h(k)), \tag{3.7}$$

and hence

$$\Delta V_1(y(k)) \leq x^T(k)\left[A^T PA + \varepsilon A^T PPA - P\right]x(k) \\ + x^T(k-h(k))\left[B^T PB + \varepsilon^{-1}B^T B\right]x(k-h(k)). \tag{3.8}$$

Then we have

$$\Delta V(y(k)) \leq x^T(k)\left[A^T PA + \varepsilon A^T PPA + h(k)G + W - P\right]x(k) \\ + x^T(k-h(k))\left[B^T PB + \varepsilon^{-1}B^T B - W\right]x(k-h(k)) - \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i). \tag{3.9}$$

Using Lemma 2.2, we obtain

$$\sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i) \geq \left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right)^T (h(k)G)\left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right). \tag{3.10}$$

From the above inequality it follows that

$$\begin{aligned}\Delta V(y(k)) &\leq x^T(k)\left[A^T PA + \varepsilon A^T PPA + h(k)G + W - P\right]x(k) \\ &\quad + x^T(k-h(k))\left[B^T PB + \varepsilon^{-1}B^T B - W\right]x(k-h(k)) \\ &\quad - \left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right)^T (h(k)G)\left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right) \\ &= \left(x^T(k), x^T(k-h(k)), \left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right)^T\right) \\ &\quad \times \begin{pmatrix}(1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3)\end{pmatrix}\begin{pmatrix} x(k) \\ x(k-h(k)) \\ \left(\frac{1}{h(k)}\sum_{i=k-h(k)}^{k-1} x(i)\right)\end{pmatrix} \\ &= y^T(k)\psi y(k),\end{aligned} \tag{3.11}$$

where $(1,1) = A^T PA + \varepsilon A^T PPA + h(k)G + W - P$, and $(2,2) = B^T PB + \varepsilon^{-1} B^T B - W$, and $(3,3) = -h(k)G$, and

$$y(k) = \begin{pmatrix} x(k) \\ x(k-h(k)) \\ \left(\dfrac{1}{h(k)} \displaystyle\sum_{i=k-h(k)}^{k-1} x(i) \right) \end{pmatrix}. \tag{3.12}$$

By condition (3.2), $\Delta V(y(k))$ is negative definite; namely, there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from Lemma 2.1. This completes the proof. □

Remark 3.2. Theorem 3.1 gives a sufficient condition for stability criterion for discrete-time systems (3.1). These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed in [14]. But Zhang et al. in [12] proved that these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed in [14].

4. Conclusions

In this paper, an improved delay-dependent stability condition for discrete-time linear systems with interval-like time-varying delays has been presented in terms of an LMI.

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