Published for SISSA by Springer
Received: *March 17, 2014*
Accepted: *July 9, 2014*
Published: *August 4, 2014*

KLWMIJ Reggeon field theory beyond the large N_c limit

Tolga Altinoluk,[a] **Néstor Armesto,**[a] **Alex Kovner,**[b] **Eugene Levin**[c,d] **and Michael Lublinsky**[e]

[a] *Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Galicia-Spain*
[b] *Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, CT 06269, U.S.A.*
[c] *Departemento de Física, Universidad Técnica Federico Santa María, and Centro Científico-Tecnológico de Valparaíso, Avda. Espana 1680, Casilla 110-V, Valparaíso, Chile*
[d] *Department of Particle Physics, Tel Aviv University, Tel Aviv 69978, Israel*
[e] *Physics Department, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel*

E-mail: tolga.altinoluk@usc.es, nestor.armesto@usc.es, kovner@phys.uconn.edu, leving@post.tau.ac.il, lublinm@bgu.ac.il

Abstract: We extend the analysis of KLWMIJ evolution in terms of QCD Reggeon fields beyond leading order in the $1/N_c$ expansion. We show that there is only one type of corrections to the leading order Hamiltonian discussed in [1]. These are terms linear in original Reggeons and quadratic in conjugate Reggeon operators. All of these have the interpretation as vertices of the "'merging"' type $2 \to 1$, where two Reggeons merge into one. Importantly, the triple Pomeron merging vertex does not emerge from the KLWMIJ Hamiltonian. We show that, although in the range of applicability of the KLWMIJ Hamiltonian these merging terms are subleading in N_c, in the dense-dense regime they all become of the same (leading) order in N_c. In this regime vertices involving higher Reggeons are enhanced by inverse powers of the coupling constant.

Keywords: 1/N Expansion, QCD

ArXiv ePrint: 1402.5936

doi:10.1007/JHEP08(2014)007

Contents

1 Introduction and conclusion

In recent papers [1, 2] we have studied in some detail the relation between the CGC [3–11] approach to the high energy evolution of hadronic observables in the guise of JIMWLK [3–11] and KLWMIJ [12] equations, and the Reggeon Field Theory (RFT) [13–19].

In [1] we have shown that the KLWMIJ Hamiltonian at leading order in the $1/N_c$ expansion can be written as a theory of interacting Reggeons: in addition to Pomerons, this theory contains other Reggeon states, at least some of which play an important role at high energy.

In [2] we have considered a more general Hamiltonian [20] that combines splitting (KLWMIJ) and merging (JIMWLK) vertices, in a way constructing Pomeron loops, and have shown that it defines a self dual [21] Reggeon Field Theory. It turns out, however, that the reliability of such construction is limited and, particularly, iterating these loops in the t-channel would take us beyond the applicability of the JIMWLK/KLWMIJ approximation.

The analysis of [1, 2] was performed in the large N_c limit, although as discussed in [2] the large N_c counting at high energy is peculiar, as the same Reggeon field is of different order in $1/N_c$ in different energy regimes.

In this paper we continue our analysis of the Reggeon Field Theory representation of KLWMIJ evolution, H^{RFT}_{KLWMIJ}, this time including all terms subleading in N_c. It is known since the early works on JIMWLK, that in the linearized approximation it reproduces the BKP formalism exactly at finite N_c. The same is true for the KLWMIJ evolution. When linearized, the Pomeron and quadrupole operators (see definitions below) can be identified with the 2-Reggeized gluon and 4-Reggeized gluon states respectively. Reference [1] completed the comparison by extending it to the question of $2 \to 4$ gluon vertex: it was demonstrated that the $1 \to 2$ Pomeron vertex which is read from the KLWMIJ Hamiltonian at leading N_c is exactly the triple Pomeron vertex of Bartels.[1]

Our aim in the present paper is to understand which type of Reggeon interactions are contained in the KLWMIJ Hamiltonian beyond leading order. This is important in order to understand whether the theory can be eventually cast into a form of reaction-diffusion process [23, 24]. Another reason for the study is to further clarify the evolution of higher order operators which are relevant for correlation-type observables, such as the ridge [25–27]. Importance of such corrections subeading in $1/N_c$ has been emphasized in [28].

Our results can be summarized in the following way. We show that no $1/N_c^2$ corrections arise to Reggeon propagator terms and to any of the $1 \to 2$ splitting vertices involving Reggeon operators. The nonvanishing corrections to the RFT Hamiltonian are all of order $1/N_c^2$, and all of them are of the type of $2 \to 1$ merging vertices. There is however no $2 \to 1$ triple Pomeron vertex which we normally associate with the JIMWLK Hamiltonian.

This large N_c counting pertains to the dilute-dense regime which is the appropriate regime for the application of H_{KLWMIJ}. It is nevertheless of some interest to rewrite the Hamiltonian in terms of the operators relevant to the dense-dense regime. This involves changing the basis from the Reggeon conjugates to dual Reggeon operators, as suggested in [2]. When this transformation is performed in H^{RFT}_{KLWMIJ} we find that all vertices (splitting and merging) become of the same, leading order at large N_c. We also find that the vertices involving higher Reggeons are enhanced by inverse powers of the 't Hooft coupling constant.

The plan of this paper is the following. In section 2 we recall the KLWMIJ Hamiltonian, and derive H^{RFT}_{KLWMIJ} including all orders in $1/N_c^2$, in terms of multipole operators and their conjugates. In section 3 we discuss the definition of n-point Reggeon operators in terms of n-point multipoles, generalizing the construction of [1]. In section 4 we discuss the N_c counting in the dense-dense limit. Appendices contain technical details of our derivations.

2 KLWMIJ Reggeon field theory beyond large N_c

The KLWMIJ evolution equation is a functional evolution equation for the weight functional $W[\delta/\delta\rho]$. This weight functional W represents the probability density to find a given configuration of charge density in the projectile. The expectation value of any observable

[1]A word about our nomenclature. When referring to $1/N_c^2$ counting, we mean the order in $1/N_c^2$ of the appropriate vertex in the RFT Hamiltonian written in terms of the Reggeon operators. This is not exactly the same as the $1/N_c^2$ counting for a vertex that appears in the evolution equation for the *average* value of the corresponding operator. For example, our definition of the triple Pomeron vertex is the coefficient V_T in the term $V_T P^\dagger PP$ in H^{RFT} (we omit the coordinate labels on the Pomeron operators for simplicity). On the other hand the standard definition of the Bartels triple Pomeron vertex is the coefficient in the evolution equation $\frac{\partial \langle P \rangle}{\partial Y} = \ldots + V_B \langle P \rangle \langle P \rangle$. The vertex V_T is of order one, and has no $1/N_c^2$ correction as we show below. The vertex V_B at leading order is exactly equal to V_T, but it also has a $1/N_c^2$ piece. This piece arises when one factorises the average of the square of the Pomeron operator $\langle PP \rangle = (1 + \frac{x}{N_c^2})\langle P \rangle \langle P \rangle$, when deriving the evolution of the average $\langle P \rangle$. As shown in [22], the standard Gaussian factorisation of the average yields precisely the correct $1/N_c^2$ term in V_B. In the present paper we will always mean order in $1/N_c^2$ of the coefficient in the Hamiltonian H^{RFT}.

$\mathcal{O}$ is given by

$$\langle \mathcal{O} \rangle = \langle \int d\rho \delta(\rho) W[\delta/\delta\rho] \mathcal{O}[\rho, \alpha] \rangle_\alpha \, . \tag{2.1}$$

Here, ρ is the color charge density of the projectile. We have allowed for the dependence of operator $\mathcal{O}$ on the color field of the target α, which also has to be averaged over. The averaging over the target field α is determined solely by the properties of the target, and is not directly relevant to our discussion, since we ascribe all evolution to the projectile wave function.

An example of an interesting observable is the S-matrix of a projectile dipole, given by

$$S = \left\langle \int d\rho \delta(\rho) \frac{1}{N_c} \operatorname{tr}[R_x R_y^\dagger] e^{i \int_z g^2 \rho(z) \alpha(z)} \right\rangle \, . \tag{2.2}$$

Here R_x is the unitary matrix in the fundamental representation, that represents the scattering amplitude of a quark at transverse point x,

$$R_x = e^{t^a \frac{\delta}{\delta \rho_x^a}} \, , \tag{2.3}$$

with t^a is the $\mathrm{SU(N_c)}$ generator in the fundamental representation. A general dilute projectile is not necessarily a single dipole but is rather composed of a small number of color neutral objects. The scattering matrix of such a projectile depends on dipole, quadrupole operators and so on. These are $SU_L(N_c) \times SU_R(N_c)$ invariant singlets. Other observables besides the scattering matrix, such as gluon production cross section, gluon correlations, $\ldots$, depend on the same color singlets.[2] The weight functional W is thus some functional of these color neutral objects:

$$W \to W[d_{12}, Q_{1234}, \ldots] \, , \tag{2.4}$$

where the dipoles and quadrupoles are defined as

$$\begin{aligned} d_{12} &= \frac{1}{N_c} \operatorname{tr}[R_1 R_2^\dagger] \, , \\ Q_{1234} &= \frac{1}{N_c} \operatorname{tr}[R_1 R_2^\dagger R_3 R_4^\dagger] \end{aligned} \tag{2.5}$$

and similarly for higher level operators. Here we introduced a shorthand notation for the transverse coordinates $x_1, x_2, \ldots, x_n \to 1, 2, \ldots, n$.

The evolution of any observable to higher energies in the KLWMIJ approximation is given by

$$\frac{d\langle \mathcal{O} \rangle}{dY} = \int d\rho \delta(\rho) W[\delta/\delta\rho] H_{KLWMIJ} \mathcal{O}[\rho] \, , \tag{2.6}$$

[2]This statement is not strictly precise. In general the gluon production cross section depends on objects which are symmetric only under the diagonal subgroup of the $SU_L(N_c) \times SU_R(N_c)$ symmetry of H_{KLWMIJ} and are not necessarily completely $SU_L(N_c) \times SU_R(N_c)$ symmetric, see for example [29]. Nevertheless, in many cases the evolution of $SU_L(N_c) \times SU_R(N_c)$ symmetric operators is sufficient to also describe the energy dependence of these more general observables [2].

JHEP08(2014)007

where the explicit form of the KLWMIJ Hamiltonian reads

$$H_{KLWMIJ} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} K_{xyz} \{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) R_A^{ab}(z) J_R^b(y) \} . \quad (2.7)$$

Here K_{xyz} is the Weizsäcker-Williams kernel

$$K_{xyz} = \frac{(x-z)_i}{(x-z)^2} \frac{(y-z)_i}{(y-z)^2} . \quad (2.8)$$

As long as we are interested in the action of the KLWMIJ Hamiltonian on gauge invariant quantities (invariant under the $SU_L(N_c) \times SU_R(N_c)$ transformation), the kernel K_{xyz} can be replaced by the dipole kernel

$$K_{xyz} \to -\frac{1}{2} M_{xyz} , \qquad M_{xyz} = \frac{(x-y)^2}{(x-z)^2 (y-z)^2} . \quad (2.9)$$

$J_L^a(x)$ and $J_R^a(x)$ are the left and right rotation generators

$$J_L^a(x) = \mathrm{tr} \left[\frac{\delta}{\delta R_x^T} t^a R_x \right] - \mathrm{tr} \left[\frac{\delta}{\delta R_x^*} R_x^\dagger t^a \right] , \quad (2.10)$$

$$J_R^a(x) = \mathrm{tr} \left[\frac{\delta}{\delta R_x^T} R_x t^a \right] - \mathrm{tr} \left[\frac{\delta}{\delta R_x^*} t^a R_x^\dagger \right] . \quad (2.11)$$

The action of the right and left rotation generators on the unitary matrix $R(x)$ are:

$$J_L^a(y) R_x = t^a R_x \delta(y-x) , \qquad J_L^a(y) R_x^\dagger = -R_x^\dagger t^a \delta(y-x) . \quad (2.12)$$

$$J_R^a(y) R_x = R_x t^a \delta(y-x) , \qquad J_R^a(y) R_x^\dagger = -t^a R_x^\dagger \delta(y-x) . \quad (2.13)$$

Our goal now is to represent the action of H_{KLWMIJ} on a functional which depends on general color singlet operators, similarly to what has been done in [30] for W which depends on dipoles only. We start off by assuming for simplicity, that W is a function of d and Q only.

The Hamiltonian is quadratic in the SU(N) rotation generators, and one should simply act with the two rotations sequentially. For simplicity we first consider only a dependence on d and Q. The action of the first rotation operator can be represented as

$$J_{L(R)}^a(y) W[d,Q] = \left[J_{L(R)}^a(y) d_{12} \right] \frac{\delta W}{\delta d_{12}} + \left[J_{L(R)}^a(y) Q_{1234} \right] \frac{\delta W}{\delta Q_{1234}} . \quad (2.14)$$

The integration over the coordinates $1,2$ and $1,2,3,4$ is implicitly assumed in eq. (2.14). Acting with the second generator we obtain, for example

$$\begin{aligned} J_R^a(x) J_L^a(y) W[d,Q] &= \left[J_R^a(x) J_L^a(y) d_{12} \right] \frac{\delta W}{\delta d_{12}} + \left[J_R^a(x) J_L^a(y) Q_{1234} \right] \frac{\delta W}{\delta Q_{1234}} \\ &+ \frac{1}{2} \Big(\left[J_R^a(x) d_{12} \right] \left[J_L^a(y) d_{34} \right] + \left[J_R^a(x) d_{34} \right] \left[J_L^a(y) d_{12} \right] \Big) \frac{\delta^2 W}{\delta d_{12} \delta d_{34}} \\ &+ \frac{1}{2} \Big(\left[J_L^a(y) d_{12} \right] \left[J_R^a(x) Q_{3456} \right] + \left[J_R^a(x) d_{12} \right] \left[J_L^a(y) Q_{3456} \right] \Big) \frac{\delta^2 W}{\delta d_{12} \delta Q_{3456}} \\ &+ \frac{1}{2} \Big(\left[J_R^a(x) Q_{1234} \right] \left[J_L^a(y) Q_{5678} \right] + \left[J_R^a(x) Q_{5678} \right] \left[J_L^a(y) Q_{1234} \right] \Big) \frac{\delta^2 W}{\delta Q_{1234} \delta Q_{5678}} , \quad (2.15) \end{aligned}$$

where again all coordinates appearing twice are integrated over.

The action of the left and right rotation operators on dipoles and quadrupoles is straightforwardly calculated with the help of the eqs. (2.12) and (2.13). To simplify the resulting expressions we use the completeness relation

$$t^a_{\alpha\beta}t^a_{\gamma\lambda} = \frac{1}{2}\left(\delta_{\alpha\lambda}\delta_{\beta\gamma} - \frac{1}{N_c}\delta_{\alpha\beta}\delta_{\gamma\lambda}\right) \tag{2.16}$$

and the identity

$$R^{ab}_z = 2\,\mathrm{tr}[t^a R_z t^b R^\dagger_z]\ . \tag{2.17}$$

After some algebra we find that the action of the KLWMIJ Hamiltonian on $W[d,Q]$ can be represented as the sum of the following terms:

$$H_{KLWMIJ} = H_d + H_Q + \frac{1}{N_c^2}\big(H_{dd} + H_{dQ} + H_{QQ}\big)\ . \tag{2.18}$$

The leading N_c terms reproduce and generalize the Mueller's dipole model [31] and have been derived in [1, 30]:

$$H_d = -\frac{\bar\alpha_s}{2\pi}\int_{z,12} M_{12z}\big(d_{1z}d_{z2} - d_{12}\big)\frac{\delta}{\delta d_{12}}\ , \tag{2.19}$$

$$\begin{aligned} H_Q = -\frac{\bar\alpha_s}{2\pi}\int_{z,1234}\Big\{ &-\big(M_{12z}+M_{34z}-L_{1342z}\big)Q_{1234} - L_{1432z}d_{12}d_{34} - L_{1234z}d_{14}d_{32} \\ &+L_{1214z}d_{1z}Q_{z234} + L_{1232z}d_{z2}Q_{1z34} + L_{3234z}d_{3z}Q_{12z4} + L_{1434z}d_{z4}Q_{123z}\Big\}\frac{\delta}{\delta Q_{1234}}\ , \end{aligned} \tag{2.20}$$

where $\bar\alpha_s \equiv \alpha_s N_c/\pi$ and

$$\begin{aligned} L_{xyuvz} &= \left[\frac{(x-z)_i}{(x-z)^2} - \frac{(y-z)_i}{(y-z)^2}\right]\left[\frac{(u-z)_i}{(u-z)^2} - \frac{(v-z)_i}{(v-z)^2}\right] \\ &= \frac{1}{2}\big[M_{yuz} + M_{xvz} - M_{yvz} - M_{xuz}\big]\ . \end{aligned} \tag{2.21}$$

The terms subleading in N_c account for color correlations and are given by

$$H_{dd} = -\frac{\bar\alpha_s}{2\pi}\int_{z1234} L_{1243z}\frac{1}{2}\Big\{\big[Q_{1234} - X_{12z43z}\big] + \big[Q_{1234} - X_{12z43z}\big]^T\Big\}\frac{\delta^2}{\delta d_{12}\delta d_{34}}, \tag{2.22}$$

$$\begin{aligned} H_{dQ} = -\frac{\bar\alpha_s}{2\pi}\int_{z123456} L_{1263z}\Big\{&\big[X_{123456} - \Sigma_{3456z21z}\big]\frac{\delta^2}{\delta d_{12}\delta Q_{3456}} \\ &+\big[X_{123456} - \Sigma_{3456z21z}\big]^T\frac{\delta^2}{\delta d_{12}\delta (Q_{3456})^T}\Big\}, \end{aligned} \tag{2.23}$$

$$\begin{aligned} H_{QQ} = -\frac{\bar\alpha_s}{2\pi}\int_{z12345678} 2L_{1485z}\Big\{&\big[\Sigma_{12345678} - Q_{1234}Q_{5678}\big]\frac{\delta^2}{\delta Q_{1234}\delta Q_{5678}} \\ &-\big[\Omega_{1234z8765z} - Q_{1234}(Q_{5678})^T\big]\frac{\delta^2}{\delta Q_{1234}\delta (Q_{5678})^T} \\ &+\big[\Sigma_{12345678} - Q_{1234}Q_{5678}\big]^T\frac{\delta^2}{\delta (Q_{1234})^T\delta (Q_{5678})^T} \\ &-\big[(\Omega_{1234z8765z})^T - (Q_{1234})^T Q_{5678}\big]\frac{\delta^2}{\delta (Q_{1234})^T\delta Q_{5678}}\Big\}, \end{aligned} \tag{2.24}$$

where we defined the higher multiplets as

$$X_{123456} = \frac{1}{N_c}\,\mathrm{tr}[R_1R_2^\dagger R_3R_4^\dagger R_5R_6^\dagger], \tag{2.25}$$

$$\Sigma_{12345678} = \frac{1}{N_c}\,\mathrm{tr}[R_1R_2^\dagger R_3R_4^\dagger R_5R_6^\dagger R_7R_8^\dagger], \tag{2.26}$$

$$\Omega_{12345\bar{1}\bar{2}\bar{3}\bar{4}\bar{5}} = \frac{1}{N_c}\,\mathrm{tr}[R_1R_2^\dagger R_3R_4^\dagger R_5R_{\bar{1}}^\dagger R_{\bar{2}}R_{\bar{3}}^\dagger R_{\bar{4}}R_{\bar{5}}^\dagger], \tag{2.27}$$

and have introduced the notation $Q^T_{1234} \equiv Q_{4321}$.

Two important properties stand out in the equations above. There are no $1/N_c^2$ corrections to the term linear in functional derivatives (conjugate momenta) in eq. (2.18). The only correction term is quadratic in functional derivatives, and it itself also has no further $1/N_c$ corrections. We now show that these properties remain true even if one includes higher Reggeons in the evolution.

The generalization to higher Reggeons is straightforward, albeit lengthy. We give details of the derivation in appendix 1, and here only present the final result. Consider a general probability density distribution,

$$W = W[\{O_{2n}\}], \qquad O_{2n} = \frac{1}{N_c}\,\mathrm{tr}[R_1R_2^\dagger \dots R_{2n-1}R_{2n}^\dagger]\,. \tag{2.28}$$

We show in appendix 1 that the action of H_{KLWMIJ} on such a functional can be written as

$$H_{KLWMIJ} = \sum_n H_{O_{2n}} + \frac{1}{N_c^2}\sum_{n,m} H_{O_{2n}O_{2m}} \tag{2.29}$$

where the two terms are linear and quadratic in functional derivatives respectively:

$$\begin{aligned}
H_{O_{2n}} = \frac{\bar{\alpha}_s}{2\pi}\int_{z,1,\dots,2n} \frac{\delta}{\delta O_{2n}(1,\dots,2n)}\Bigg\{ &\sum_{j=1}^{n}\frac{1}{2}\left[M_{2j-1,2j,z} + M_{2j-1,2j-2,z}\right] O_{2n}(1,\dots,2n) \\
+\sum_{j=2}^{n}\sum_{k=1}^{j-1} L_{2j-1,2j-2,2k-1,2k-2;z}\, & O_{2(k+n-j)}(1,\dots,2k-2,2j-1,\dots,2n) \\
&\times O_{2(j-k)}(2k-1,\dots,2j-2) \\
+\sum_{j=2}^{n}\sum_{k=1}^{j-1} L_{2j-1,2j,2k-1,2k;z}\, & O_{2(k+n-j)}(1,\dots,2k-1,2j,\dots,2n) \\
&\times O^T_{2(j-k)}(2k,\dots,2j-1) \\
-\sum_{j=1}^{n}\sum_{k=1}^{j} L_{2j-1,2j,2k-1,2k-2;z}\, & O_{2(k+n-j)}(1,\dots,2k-2,z,2j,\dots,2n) \\
&\times O_{2(j-k+1)}(2k-1,\dots,2j-1,z) \\
-\sum_{j=1}^{n}\sum_{k=j+1}^{n} L_{2j-1,2j,2k-1,2k-2;z}\, & O_{2(j+n-k+1)}(1,\dots,2j-1,z,2k-1,\dots,2n) \\
&\times O_{2(k-j)}(z,2j,\dots,2k-2)\Bigg\}\,,
\end{aligned} \tag{2.30}$$

$$H_{O_{2n}O_{2m}} = \frac{\bar{\alpha}_s}{4\pi}\int_{z,1\dots,2n,\bar{1},\dots,2\bar{m}} \frac{\delta^2}{\delta O_{2n}(1,\dots,2n)\delta O_{2\bar{m}}(\bar{1},\dots,2\bar{m})}\sum_{j=1}^{n}\sum_{k=\bar{1}}^{\bar{m}} \tag{2.31}$$
$$\times\Big\{L_{2j-1,2j-2,2k-\bar{1},2k-\bar{2};z}O_{2(n+\bar{m})}(1,\dots,2j-2,2k-\bar{1},\dots,2\bar{m},\bar{1},\dots,2k-\bar{2},2j-1,\dots,2n)$$
$$+L_{2j-1,2j,2k-\bar{1},2k-\bar{2};z}O_{2(n+\bar{m})}(1,\dots,2j-1,2k,\dots,2\bar{m},\bar{1},\dots,2k-\bar{1},2j,\dots,2n)$$
$$-L_{2j-1,2j-2,2k-\bar{1},2k;z}O_{2(n+\bar{m}+1)}(1,\dots,2j-2,z,2k,\dots,2\bar{m},\bar{1},\dots,2k-\bar{1},z,2j-1,\dots,2n)$$
$$-L_{2j-1,2j,2k-\bar{1},2k-\bar{2};z}O_{2(n+\bar{m}+1)}(1,l\dots,2j-1,z,2k-\bar{1},\dots,2\bar{m},\bar{1},\dots,2k-\bar{2},z,2j,\dots,2n)\Big\},$$

where $y_0 \equiv y_{2n}$. Eq. (2.30) generalizes the evolution of a single dipole (or single quadrupole) to arbitrary multipoles. In appendix 2, we present the explicit expressions for the evolution of 6-point function (X) and 8-point function (Σ).

3 Construction of Reggeons

In principle, eqs. (2.30) and (2.31) give the complete representation of H_{KLWMIJ} acting on $SU_L(N_c)\times SU_R(N_c)$ invariant operators. It is however useful to define Reggeon fields as irreducible representations of the discrete symmetries of the problem. This was done for the dipole and quadrupole in [1]. In this section we extend this discussion to include higher Reggeons.

As explained in detail in [1] and [32], in addition to the $SU_L(N_c)\times SU_R(N_c)$ continuous symmetry group, H_{KLWMIJ} also posesses the discrete signature Z_2 symmetry, $R\to R^\dagger$, and the discrete charge conjugation symmetry, $R\to R^*$. In [1] the Reggeon fields were constructed as singlets under $SU_L(N_c)\times SU_R(N_c)$ and either even or odd under signature and charge conjugation symmetries. In addition, in [1] we have constructed the Q^{++}-Reggeon[3] using a simple subtraction such that the evolution equation for the Q^{++}-Reggeon does not contain a single Pomeron source term. We generalize this subtraction procedure here.

The decomposition for the two point function is straightforward [1]. The Pomeron and the Odderon fields are defined as

$$P_{12} = \frac{1}{2}\big[2-d_{12}-d_{21}\big], \tag{3.1}$$
$$O_{12} = \frac{1}{2}\big[d_{12}-d_{21}\big]. \tag{3.2}$$

The Pomeron is even and the Odderon is odd under both signature and charge conjugation. The evolution equations for P and O are

$$\frac{d}{dY}P_{12} = \frac{\bar{\alpha}_s}{2\pi}\int_z M_{12z}\big[P_{1z}+P_{z2}-P_{12}-P_{1z}P_{z2}-O_{1z}O_{z2}\big], \tag{3.3}$$
$$\frac{d}{dY}O_{12} = \frac{\bar{\alpha}_s}{2\pi}\int_z M_{12z}\big[O_{1z}+O_{z2}-O_{12}-O_{1z}P_{z2}-P_{1z}O_{z2}\big]. \tag{3.4}$$

[3]The Q^{++}-Reggeon was called B-Reggeon in [1] and [2].

JHEP08(2014)007

Now let us consider the quadrupole. Decomposing it into eigenstates of discrete symmetries, we define Q^{++}, Q^{+-}, Q^{--} and Q^{-+} Reggeons:

$$Q^{++}_{1234} = \frac{1}{4}\big[4 - Q_{1234} - Q_{4123} - Q_{3214} - Q_{2143}\big] - \big[P_{12} - P_{13} + P_{14} + P_{23} - P_{24} + P_{34}\big], \quad (3.5)$$

$$Q^{+-}_{1234} = \frac{1}{4}\big[Q_{1234} + Q_{4123} - Q_{3214} - Q_{2143}\big], \quad (3.6)$$

$$Q^{--}_{1234} = \frac{1}{4}\big[Q_{1234} - Q_{4123} - Q_{3214} + Q_{2143}\big], \quad (3.7)$$

$$Q^{-+}_{1234} = \frac{1}{4}\big[Q_{1234} - Q_{4123} + Q_{3214} - Q_{2143}\big]. \quad (3.8)$$

Three independent Q^{++}-Reggeon and three independent Q^{+-}-Reggeon are

$$Q^{++}_{1234}, Q^{++}_{1243}, Q^{++}_{1342} \qquad \text{and} \qquad Q^{+-}_{1234}, Q^{+-}_{1243}, Q^{+-}_{1342}. \quad (3.9)$$

The $++$-Reggeon has quantum numbers of the Pomeron. It is signature and charge conjugation even. $+-$-Reggeon is signature even and charge conjugation odd, $-+$ signature odd, charge conjugation even and $--$ has the same quantum numbers as the Odderon - odd under both signature and charge conjugation. The subtraction of the Pomeron terms in Q^{++} achieves simultaneously two goals. First, when expanded in powers of $\delta/\delta\rho$, the field Q^{++} starts with the term $(\delta/\delta\rho)^4$ as opposed to Q whose expansion starts with $(\delta/\delta\rho)^2$. Second, the evolution equation for Q^{++} does not contain a term linear in P, which is in principle allowed due to the identical quantum numbers of P and Q^{++} [1]. This evolution equation for completeness is presented in appendix 2.

Clearly, the $+-$ and $-+$ Reggeons do not require any subtractions, since they do not mix with either the Pomeron or the Odderon. The $--$ Reggeon on the other hand does mix with the Odderon, and its evolution does contain a source due to the Odderon. However, it turns out to be impossible to find a local subtraction of the type of eq. (3.5) which eliminates this source term. In the following we will therefore only consider subtractions in the Pomeron channel.

Moving on to the six point function, we define the X^{++} Reggeon in the analogous way

$$\begin{aligned} X^{++}_{123456} = \frac{1}{4}\big[4 - X_{123456} - X_{612345} - X_{654321} - X_{543216}\big] \\ -\big[P_{12} - P_{13} + P_{14} - P_{15} + P_{16} + P_{23} - P_{24} + P_{25} - P_{26} + P_{34} - P_{35} + P_{36} + P_{45} - P_{46} + P_{56}\big] \\ -\big[Q^{++}_{1234} - Q^{++}_{1235} + Q^{++}_{1236} + Q^{++}_{1245} - Q^{++}_{1246} + Q^{++}_{1256} - Q^{++}_{1345} \\ + Q^{++}_{1346} - Q^{++}_{1356} + Q^{++}_{1456} + Q^{++}_{2345} - Q^{++}_{2346} + Q^{++}_{2356} - Q^{++}_{2456} + Q^{++}_{3456}\big]. \end{aligned} \quad (3.10)$$

As shown in appendix 2, this particular subtraction achieves the required effect, namely it eliminates source terms proportional to P and Q^{++} from the evolution equation for X^{++}. This equation can be found in appendix 2. The following ten X^{++} Reggeons associated with given six points, are independent:

$$X^{++}_{123456}, X^{++}_{123465}, X^{++}_{123546}, X^{++}_{124356}, X^{++}_{124365}, X^{++}_{125364}, X^{++}_{132456}, X^{++}_{132465}, X^{++}_{132546}, X^{++}_{142536}. \quad (3.11)$$

Analogously we define

$$
\begin{aligned}
X^{+-}_{123456} = & \frac{1}{4}\big[X_{123456} + X_{612345} - X_{654321} - X_{543216}\big] \\
& -\big[Q^{+-}_{1234} - Q^{+-}_{1235} + Q^{+-}_{1236} + Q^{+-}_{1245} - Q^{+-}_{1246} + Q^{+-}_{1256} - Q^{+-}_{1345} \\
& \quad + Q^{+-}_{1346} - Q^{+-}_{1356} + Q^{+-}_{1456} + Q^{+-}_{2345} - Q^{+-}_{2346} + Q^{+-}_{2356} - Q^{+-}_{2456} + Q^{+-}_{3456}\big],
\end{aligned} \tag{3.12}
$$

and ten independent X^{+-}-Reggeons are

$$
X^{+-}_{123456}, X^{+-}_{123465}, X^{+-}_{123546}, X^{+-}_{124356}, X^{+-}_{124365}, X^{+-}_{125364}, X^{+-}_{132456}, X^{+-}_{132465}, X^{+-}_{132546}, X^{+-}_{142536}. \tag{3.13}
$$

The signature odd 6-point Reggeons are given as

$$
\begin{aligned}
X^{-+}_{123456} = & \frac{1}{4}\big[X_{123456} - X_{612345} + X_{654321} - X_{543216}\big] \\
& -\big[Q^{-+}_{1234} - Q^{-+}_{1235} + Q^{-+}_{1236} + Q^{-+}_{1245} - Q^{-+}_{1246} + Q^{-+}_{1256} - Q^{-+}_{1345} \\
& \quad + Q^{-+}_{1346} - Q^{-+}_{1356} + Q^{-+}_{1456} + Q^{-+}_{2345} - Q^{-+}_{2346} + Q^{-+}_{2356} - Q^{-+}_{2456} + Q^{+-}_{3456}\big]
\end{aligned} \tag{3.14}
$$

and finally

$$
X^{--}_{123456} = \frac{1}{4}\big[X_{123456} - X_{612345} - X_{654321} + X_{543216}\big]. \tag{3.15}
$$

Note that we have included subtractions in the $+-$ and $-+$ channels. These eliminate sources proportional to Q^{+-} in the evolution of X^{+-} and similarly for X^{-+}. We do not bother with subtractions in the $--$ channel, since the Odderon source terms cannot be eliminated already in the evolution of Q^{--}.

The subtraction structure can be generalized to an arbitrary n-point Reggeon by inspection. Let us first define the unsubtracted signature and charge conjugation even multipole combinations:

$$
d^{SS}_{12} = \frac{1}{2}\Big[d_{12} + d_{21}\Big], \tag{3.16}
$$

$$
Q^{SS}_{1234} = \frac{1}{4}\Big[Q_{1234} + Q_{4123} + Q_{3214} + Q_{2143}\Big], \tag{3.17}
$$

$$
X^{SS}_{123456} = \frac{1}{4}\Big[X_{123456} + X_{612345} + X_{654321} + X_{543216}\Big] \tag{3.18}
$$

and, for an arbitrary $2n$ point function,

$$
\begin{aligned}
O^{SS}_{2n}(1,\ldots,2n) = & \frac{1}{4}\Big[O_{2n}(1,\ldots,2n) + O_{2n}(2n,1,\ldots,2n-1) \\
& + O^{T}_{2n}(1,\ldots,2n) + O^{T}_{2n}(2n,1,\ldots,2n-1)\Big].
\end{aligned} \tag{3.19}
$$

The generalisation of eqs. (3.5) and (3.10) for arbitrary n can then be written as

$$
O^{SS}_{2n}(1,\ldots,2n) = 1 - \left[\sum_{l=0}^{n-1} \sum_{\{i_1<i_2<\ldots<i_{2(n-l)}\}} (-1)^{n-l+\sum_{k=1}^{2(n-l)} i_k} O^{++}_{2(n-l)}(i_1, i_2, \ldots, i_{2(n-l)})\right]. \tag{3.20}
$$

Analogously, defining

$$
\begin{aligned}
O_{2n}^{SA}(1,\dots,2n) &= \frac{1}{4}\Big[O_{2n}(1,\dots,2n)+O_{2n}(2n,1,\dots,2n-1) \\
&\qquad -O_{2n}^{T}(1,\dots,2n)-O_{2n}^{T}(2n,1,\dots,2n-1)\Big], \\
O_{2n}^{AS}(1,\dots,2n) &= \frac{1}{4}\Big[O_{2n}(1,\dots,2n)-O_{2n}(2n,1,\dots,2n-1) \\
&\qquad +O_{2n}^{T}(1,\dots,2n)-O_{2n}^{T}(2n,1,\dots,2n-1)\Big], \\
O_{2}^{SA} &= O_{2}^{AS}\equiv 0,
\end{aligned}
\tag{3.21}
$$

we have

$$
\begin{aligned}
O_{2n}^{SA}(1,\dots,2n) &= 1-\left[\sum_{l=0}^{n-1}\sum_{\{i_1<i_2<\dots<i_{2(n-l)}\}}(-1)^{n-l+\sum_{k=1}^{2(n-l)}i_k}O_{2(n-l)}^{+-}(i_1,i_2,\dots,i_{2(n-l)})\right], \\
O_{2n}^{AS}(1,\dots,2n) &= 1-\left[\sum_{l=0}^{n-1}\sum_{\{i_1<i_2<\dots<i_{2(n-l)}\}}(-1)^{n-l+\sum_{k=1}^{2(n-l)}i_k}O_{2(n-l)}^{-+}(i_1,i_2,\dots,i_{2(n-l)})\right].
\end{aligned}
\tag{3.22}
$$

Expressing Reggeons in terms of multipoles requires some extra algebra. For $n=1,2,3$ this is straightforward, and we obtain

$$
P_{12} = 1-d_{12}^{SS}, \tag{3.23}
$$

$$
Q_{1234}^{++} = -1+\Big[d_{12}^{SS}-d_{13}^{SS}+d_{14}^{SS}+d_{23}^{SS}-d_{24}^{SS}+d_{34}^{SS}\Big]-Q_{1234}^{SS}, \tag{3.24}
$$

$$
\begin{aligned}
X_{123456}^{++} &= 1-\big[d_{12}^{SS}-d_{13}^{SS}+d_{14}^{SS}-d_{15}^{SS}+d_{16}^{SS}+d_{23}^{SS}-d_{24}^{SS}+d_{25}^{SS}-d_{26}^{SS}+d_{34}^{SS}-d_{35}^{SS} \\
&\qquad +d_{36}^{SS}+d_{45}^{SS}-d_{46}^{SS}+d_{56}^{SS}\big] \\
&\quad +\big[Q_{1234}^{SS}-Q_{1235}^{SS}+Q_{1236}^{SS}+Q_{1245}^{SS}-Q_{1246}^{SS}+Q_{1256}^{SS}-Q_{1345}^{SS}+Q_{1346}^{SS}-Q_{1356}^{SS}+Q_{1456}^{SS} \\
&\qquad +Q_{2345}^{SS}-Q_{2346}^{SS}+Q_{2356}^{SS}-Q_{2456}^{SS}+Q_{3456}^{SS}\big]-X_{123456}^{SS}.
\end{aligned}
\tag{3.25}
$$

It is obvious that these expressions lend themselves to a generalization for arbitrary n:

$$
O_{2n}^{++}(1,\dots,2n) = (-1)^{n+1}\left[\sum_{l=0}^{n-1}\sum_{\{i_1<i_2<\dots<i_{2(n-l)}\}}(-1)^{\sum_{k=1}^{2(n-l)}i_k}O_{2(n-l)}^{SS}(i_1,i_2,\dots,i_{2(n-l)})+1\right].
\tag{3.26}
$$

4 Merging vertices from the KLWMIJ Hamiltonian

Using eqs. (3.20) and (3.26) it is straigtforward to rewrite the Hamiltonian eqs. (2.30) and (2.31) in terms of the Reggeon fields. One expresses the multipole operators in terms of the Reggeons using eq. (3.20) and the functional derivatives via

$$
\begin{aligned}
\frac{\delta}{\delta O_{2n}(1,\dots,2n)} &= \sum_{l=0}^{\infty}\int_{2n+1,\dots,2(n+l)}\sum_{\{i_1,\dots,i_{2(n+l)}\}}\sum_{\alpha,\beta=\pm} \\
&\qquad \times\frac{\delta O_{2(n+l)}^{\alpha\beta}(i_1,\dots,i_{2(n+l)})}{\delta O_{2n}(1,\dots,2n)}\frac{\delta}{\delta O_{2(n+l)}^{\alpha\beta}(i_1,\dots,i_{2(n+l)})}.
\end{aligned}
\tag{4.1}
$$

We are not going to perform the substitution eq. (4.1) explicitly. We only note that it does not change the nature of the terms in the Hamiltonian. The one important result of this substitution is that it eliminates the terms of the form

$$O^{++}_{2(n-l)}\frac{\delta}{\delta O^{++}_{2n}},\quad O^{+-}_{2(n-l)}\frac{\delta}{\delta O^{+-}_{2n}},\quad O^{-+}_{2(n-l)}\frac{\delta}{\delta O^{-+}_{2n}} \tag{4.2}$$

which otherwise appear in eq. (2.30) when one of the multipole operators is taken to be unity. It thus leads to partial diagonalization of the equations, in the sense that the evolution of a level $2n$ Reggeon does not contain a term linear in level $2(n-k)$ Reggeons.

The structure of the Hamiltonian thus can be described as follows. The leading N_c terms are of two types. First, there are homogeneous terms of the type

$$O^{++}_{2n}\frac{\delta}{\delta O^{++}_{2n}},\qquad \text{etc.} \tag{4.3}$$

For $++$, $+-$ and $-+$ Reggeons these are diagonal, i.e. they mix only Reggeons on the same level n.

The second type of terms are $1\to 2$ splitting vertices, contained in eq. (2.30). These are akin to the $1\to 2$ Pomeron splitting vertices and generalize it to higher Reggeons. The most interesting such vertex corresponds to splitting of the Q^{++} Reggeon into two Pomerons, and contributes to the effective four Pomeron vertex as discussed in [2].

Finally the third type of terms is of order $1/N_c^2$, and is of a different nature. These are "merging" vertices of the $2\to 1$ type. Note that there is no vertex corresponding to transition from two Pomeron to one Pomeron. This is straightforward to see examining eqs. (2.22), (2.23), (2.24). In eq. (2.22) all the single Pomeron terms cancel in the factor that multiply the functional derivatives, while eqs. (2.23) and (2.24) do not contain a term with two functional derivatives with respect to the Pomeron. This is natural, since we expect to find this vertex in the JIMWLK Hamiltonian which describes the dense projectile regime, and not in the KLWMIJ Hamiltonian. On the other hand eqs. (2.22), (2.23), (2.24) do give rise, for example to a vertex of the type $PP\to Q^{++}$.

The order in $1/N_c$ assigned to the vertices is based on large N_c counting for the dilute projectile, dense target regime. In this regime all the Reggeons are close to saturation, and thus are of order one. The Reggeon conjugate operators $\delta/\delta O_{2n}$ are also assigned $O(1)$ at large N_c. However, as discussed in [2] the large N_c limit is tricky, and the order in N_c of a given operator depends on whether the target or the projectile is saturated, or both. For example, for a dense projectile a dipole scattering on it scatters with probability of order one. This means that the following dual Pomeron operator is of order one in the large N_c limit:

$$\bar{P}(x,y) = 1 - \frac{1}{2N_c}\mathrm{Tr}[S^{F\dagger}(x)S^F(y) + S^{F\dagger}(y)S^F(x)], \tag{4.4}$$

where

$$S^F(x) = e^{ig^2 t^a \alpha_P^a(x)} \tag{4.5}$$

and α_P is the color field created by the color charges of the projectile. The dual Pomeron operator is a nonlinear function of the projectile color sources. However, for weak fields it

JHEP08(2014)007

is related to the Pomeron conjugate operator by a simple linear relation

$$\frac{\delta}{\delta P(x,y)} = P^\dagger(x,y) = \frac{4}{g^4}\nabla_x^2\nabla_y^2\bar{P}(x,y) = \frac{N_c^2}{4\pi^4\bar{\alpha}_s^2}\nabla_x^2\nabla_y^2\bar{P}(x,y) \tag{4.6}$$

In the dense projectile limit $\bar{P}$ remains finite even at large N_c. Thus, if this relation can be taken as a guide of the correct N_c behaviour, in the limit of dense projectile, where $\bar{P} \sim O(1)$, one has $\frac{\delta}{\delta P(x,y)} \sim O(N_c^2)$. Similarly, we have for the 4 point Reggeon [2]

$$\begin{aligned} Q^{++\dagger}(x,y,u,v) &= \frac{N_c^2}{64\pi^8\bar{\alpha}_s^4}\nabla_x^2\nabla_y^2\nabla_u^2\nabla_v^2\bar{Q}^{++}(x,y,u,v), \\ Q^{+-\dagger}(x,y,u,v) &= \frac{N_c^2}{64\pi^8\bar{\alpha}_s^4}\nabla_x^2\nabla_y^2\nabla_u^2\nabla_v^2\bar{Q}^{+-}(x,y,u,v). \end{aligned} \tag{4.7}$$

The dual Reggeons with the nonvacuum quantum numbers are not important in the projectile saturation regime, since the limiting saturated value of Q^{+-} etc. is zero. However the $++$ conjugate Reggeon is again $O(N_c^2)$. The same is true for higher Reggeons. In general for a n-point $++$ Reggeon we have

$$O_{2n}^{++\dagger}(1,\dots,2n) = \frac{4N_c^2}{(4\pi^2)^{2n}\bar{\alpha}_s^{2n}}\nabla_1^2\dots\nabla_{2n}^2\bar{O}_{2n}^{++}(1,\dots 2n). \tag{4.8}$$

Substituting this into the expression for the Hamiltonian, we see that in the regime where both the projectile and the target are dense, all terms in the Hamiltonian are of the order N_c^2.

Another interesting feature of this regime is that the vertices involving higher Reggeons are in fact enhanced by inverse powers of the $\bar{\alpha}_s$, when 't Hooft coupling is small. The vertices scale as[4]

$$V^{n;m,k}_{\text{splitting}}\bar{O}_{2n}O_{2m}O_{2k} \propto \frac{N_c^2}{\bar{\alpha}_s^{2n-1}}, \qquad V^{n;m,k}_{\text{merging}}O_{2n}\bar{O}_{2m}\bar{O}_{2k} \propto \frac{N_c^2}{\bar{\alpha}_s^{2m+2k-1}}. \tag{4.9}$$

It is possible that eq. (4.8) is not correct parametrically in the dense limit, since strictly speaking it was deduced in the dilute regime. However if it holds, this suggests an interesting albeit complicated picture. At weak 't Hooft coupling, the higher Reggeons which are less important in the dilute-dilute and dense-dilute regime (since their value is small at small $\bar{\alpha}_s$) take over the quantum evolution in the dense-dense regime. To make sense of this regime one would then have to somehow resum the contributions of higher Reggeons. In other words one would have to deal with an essentially nonlinear regime of the Reggeon field theory, a level higher than nonlinear QCD which leads to the Reggeon field theory in the first place.

A The derivation of the KLWMIJ Reggeon theory

In this appendix, we derive the action of H_{KLWMIJ} on a generic functional of color singlet operators of the type

$$O_{2n} = \frac{1}{N_c}\,\mathrm{tr}[R_1R_2^\dagger\dots R_{2n-1}R_{2n}^\dagger]. \tag{A.1}$$

[4]Note that when the target and the projectile are saturated, $O_{2n} = O(1)$ for any n. This follows from their definition in terms of the appropriate multipoles, as all multipoles vanish at saturation.

The simplest such operator is the dipole. The dipole evolution operator H_d given in eq. (2.19), picks up a dipole from the weight functional W and evolves that dipole via the KLWMIJ Hamiltonian i.e. H_d can equivalently be written as

$$H_d = \int_{12} \left[H_{KLWMIJ} d_{12} \right] \frac{\delta}{\delta d_{12}}. \tag{A.2}$$

In the KLWMIJ evolution of a single dipole d_{12} the subleading piece in $1/N_c$ cancels between the real and virtual terms in the Hamiltonian and the exact KLWMIJ evolution of a dipole is all leading N_c. The same is true for the evolution of a quadrupole. The color correlators in eq. (2.18) that are subleading in N_c arise from the evolution of a product of two color neutral objects (two dipoles, one dipole and one quadruple and two quadrupoles). In this case, one of the two color rotation operators acts on one object, and the other acts on the second object generating the order $1/N_c^2$ term. The terms of order $1/N_c^3$, which a priori could be present, cancel in the final result between the real and virtual contributions of the KLWMIJ Hamiltonian. Hence, the expression given in eq. (2.18) is the result of exact color algebra. In order to generalise this expression one should show that a similar cancellation happens for the generic color neutral object O_{2n}.

A.1 KLWMIJ evolution of a generic single trace operator

First, we derive the evolution equation of a single trace operator with arbitrary number of entries and show that the large N_c approximation for the KLWMIJ evolution coincides with the exact evolution i.e. all the subleading N_c terms cancel in the action of H_{KLWMIJ} on a single trace operator. We write the generic single trace operator as

$$O_{2n} = \frac{1}{N_c} \operatorname{tr} \left(R_1 R_2^\dagger \ldots R_{2n-1} R_{2n}^\dagger \right) = \frac{1}{N_c} \operatorname{tr} \left[\prod_{i=1}^{n} R_{2i-1} R_{2i}^\dagger \right]. \tag{A.3}$$

In order to find the action of H_{KLWMIJ}, we need the calculate the action of right and left rotation generators on O_{2n}. For J_L we have

$$J_L^a(y) O_{2n} = \frac{1}{N_c} \sum_{j=1}^{n} \operatorname{tr} \left[\left(\prod_{i=1}^{j-1} R_{2i-1} R_{2i}^\dagger \right) t^a \left(\prod_{i=j}^{n} R_{2i-1} R_{2i}^\dagger \right) \right] \delta(y_{2j-1} - y_{2j-2}). \tag{A.4}$$

Here, we introduced a short hand notation for the δ-functions as

$$\delta(y_{2j-1} - y_{2j-2}) \equiv \delta[y - (2j-1)] - \delta[y - (2j-2)]. \tag{A.5}$$

Moreover, when writing eq. (A.4) we have adopted the following conventions:

$$y_0 \equiv y_{2n}, \tag{A.6}$$

$$\prod_{i=k}^{l} R_{2i-1} R_{2i}^\dagger \equiv 1 \qquad \text{for } l < k. \tag{A.7}$$

Similarly, one can write the action of J_R on O_{2n}:

$$J^a_R(y)O_{2n} = \frac{1}{N_c}\sum_{j=1}^{n} \mathrm{tr}\left[\left(\prod_{i=1}^{j-1} R_{2i-1}R^\dagger_{2i}\right) R_{2j-1}t^a R^\dagger_{2j}\left(\prod_{i=j+1}^{n} R_{2i-1}R^\dagger_{2i}\right)\right]\delta(y_{2j-1}-y_{2j}). \quad \text{(A.8)}$$

Now using eqs. (A.4) and (A.8) we obtain

$$\begin{aligned} J^a_L(x)J^a_L(y)O_{2n} = &\frac{1}{N_c}\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1} R_{2i-1}R^\dagger_{2i}\right) t^a t^a \left(\prod_{i=j}^{n} R_{2i-1}R^\dagger_{2i}\right)\right] \\ &\times\delta(y_{2j-1}-y_{2j-2})\delta(x_{2j-1}-x_{2j-2}) \\ +&\frac{1}{N_c}\sum_{j=2}^{n}\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R^\dagger_{2i}\right) t^a \left(\prod_{i=k}^{j-1} R_{2i-1}R^\dagger_{2i}\right) t^a \left(\prod_{i=j}^{n} R_{2i-1}R^\dagger_{2i}\right)\right] \\ &\times\left[\delta(x_{2j-1}-x_{2j-2})\delta(y_{2k-1}-y_{2k-2})+\delta(y_{2j-1}-y_{2j-2})\delta(x_{2k-1}-x_{2k-2})\right], \end{aligned} \quad \text{(A.9)}$$

$$\begin{aligned} J^a_R(x)J^a_R(y)O_{2n} = &\frac{1}{N_c}\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1} R_{2i-1}R^\dagger_{2i}\right) R_{2j-1}t^a t^a R^\dagger_{2j}\left(\prod_{i=j+1}^{n} R_{2i-1}R^\dagger_{2i}\right)\right] \\ &\times\delta(y_{2j-1}-y_{2j})\delta(x_{2j-1}-x_{2j}) \\ +&\frac{1}{N_c}\sum_{j=2}^{n}\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R^\dagger_{2i}\right) R_{2k-1}t^a R^\dagger_{2k}\left(\prod_{i=k+1}^{j-1} R_{2i-1}R^\dagger_{2i}\right) R_{2j-1}t^a R^\dagger_{2j}\left(\prod_{i=j+1}^{n} R_{2i-1}R^\dagger_{2i}\right)\right] \\ &\times\left[\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k})+\delta(y_{2k-1}-y_{2k})\delta(x_{2j-1}-x_{2j})\right], \end{aligned} \quad \text{(A.10)}$$

$$\begin{aligned} &J^a_L(x)J^b_R(y)O_{2n} = \\ &=\Bigg\{\frac{1}{N_c}\sum_{j=1}^{n}\sum_{k=1}^{j}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R^\dagger_{2i}\right) t^a \left(\prod_{i=k}^{j-1} R_{2i-1}R^\dagger_{2i}\right) R_{2j-1}t^b R^\dagger_{2j}\left(\prod_{i=j+1}^{n} R_{2i-1}R^\dagger_{2i}\right)\right] \\ &+\frac{1}{N_c}\sum_{j=1}^{n}\sum_{k=j+1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1} R_{2i-1}R^\dagger_{2i}\right) R_{2j-1}t^b R^\dagger_{2j}\left(\prod_{i=j+1}^{k-1} R_{2i-1}R^\dagger_{2i}\right) t^a \left(\prod_{i=k}^{n} R_{2i-1}R^\dagger_{2i}\right)\right]\Bigg\} \\ &\times\Big(\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k-2})\Big). \end{aligned} \quad \text{(A.11)}$$

These expressions can be simplified by using the color algebra relations

$$\mathrm{tr}[At^a t^a B] = \frac{N_c^2-1}{2N_c}\,\mathrm{tr}[AB], \quad \text{(A.12)}$$

$$\mathrm{tr}[At^a B t^a C] = \frac{1}{2}\left(\mathrm{tr}[AC]\,\mathrm{tr}[B] - \frac{1}{N_c}\,\mathrm{tr}[ABC]\right). \quad \text{(A.13)}$$

Then eqs. (A.12) and (A.13) can be rewritten as

$$
\begin{aligned}
J_L^a(x)J_L^a(y)O_{2n} &= \frac{1}{2}\,\mathrm{tr}\left[\prod_{i=1}^{n} R_{2i-1}R_{2i}^\dagger\right]\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j-2})\delta(x_{2j-1}-x_{2j-2}) \\
&\quad+\frac{1}{2N_c}\sum_{j=2}^{n}\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=j}^{n} R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\prod_{i=k}^{j-1} R_{2i-1}R_{2i}^\dagger\right] \\
&\qquad\times\Big(\delta(y_{2j-1}-y_{2j-2})\delta(x_{2k-1}-x_{2k-2})+\delta(y_{2k-1}-y_{2k-2})\delta(x_{2j-1}-x_{2j-2})\Big) \\
&\quad-\frac{1}{2N_c^2}\,\mathrm{tr}\left[\prod_{i=1}^{n} R_{2i-1}R_{2i}^\dagger\right]\left\{\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j-2})\delta(x_{2j-1}-x_{2j-2})\right. \\
&\left.+\sum_{j=2}^{n}\sum_{k=1}^{j-1}\Big(\delta(y_{2j-1}-y_{2j-2})\delta(x_{2k-1}-x_{2k-2})+\delta(y_{2k-1}-y_{2k-2})\delta(x_{2j-1}-x_{2j-2})\Big)\right\}. \qquad \text{(A.14)}
\end{aligned}
$$

Note that the N_c suppressed terms can be further simplified by using the identity

$$
\sum_{j=1}^{n} a_j b_j + \sum_{j=2}^{n}\sum_{k=1}^{j-1}(b_j a_k + b_k a_j) = \sum_{j=1}^{n} a_j \sum_{k=1}^{n} b_k. \qquad \text{(A.15)}
$$

The final expression for $J_L^a(x)J_L^a(y)O_{2n}$ is

$$
\begin{aligned}
J_L^a(x)J_L^a(y)O_{2n} &= \frac{1}{2}\,\mathrm{tr}\left[\prod_{i=1}^{n} R_{2i-1}R_{2i}^\dagger\right]\left\{\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j-2})\delta(x_{2j-1}-x_{2j-2})\right. \\
&\qquad\left.-\frac{1}{N_c^2}\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j-2})\sum_{k=1}^{n}\delta(x_{2k-1}-x_{2k-2})\right\} \\
&\quad+\frac{1}{2N_c}\sum_{j=2}^{n}\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=j}^{n} R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\prod_{i=k}^{j-1} R_{2i-1}R_{2i}^\dagger\right] \\
&\quad\times\Big(\delta(y_{2j-1}-y_{2j-2})\delta(x_{2k-1}-x_{2k-2})+\delta(y_{2k-1}-y_{2k-2})\delta(x_{2j-1}-x_{2j-2})\Big). \qquad \text{(A.16)}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
J_R^a(x)J_R^a(y)O_{2n} &= \frac{1}{2}\,\mathrm{tr}\left[\prod_{i=1}^{n} R_{2i-1}R_{2i}^\dagger\right]\left\{\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j})\delta(x_{2j-1}-x_{2j})\right. \\
&\qquad\left.-\frac{1}{N_c^2}\sum_{j=1}^{n}\delta(y_{2j-1}-y_{2j})\sum_{k=1}^{n}\delta(x_{2k-1}-x_{2k})\right\} \\
&\quad+\frac{1}{2N_c}\sum_{j=2}^{n}\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1} R_{2i-1}R_{2i}^\dagger\right)R_{2k-1}R_{2j}^\dagger\left(\prod_{i=j+1}^{n} R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[R_{2j-1}R_{2k}^\dagger\left(\prod_{i=k+1}^{j-1} R_{2i-1}R_{2i}^\dagger\right)\right] \\
&\quad\times\Big(\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k})+\delta(y_{2k-1}-y_{2k})\delta(x_{2j-1}-x_{2j})\Big). \qquad \text{(A.17)}
\end{aligned}
$$

The real term in the KLWMIJ Hamiltonian contains a factor of $(-2R_z^{ab})$ where the unitary matrix R_z^{ab} is in the adjoint representation. Thus, the color algebra is slightly different for this term. Again using the completeness relation and writing the adjoint matrix in terms of the product of two fundamental ones we get

$$-2R_z^{ab}\,\mathrm{tr}[At^aBt^bC] = -\left\{\mathrm{tr}[R_zCA]\,\mathrm{tr}[R_z^\dagger B] - \frac{1}{N_c}\,\mathrm{tr}[ABC]\right\}. \tag{A.18}$$

Using eq. (A.18),the real term can be written as

$$\begin{aligned}
-2J_L^a(x)R_z^{ab}J_R^b(y)O_{2n} &= \frac{1}{N_c^2}\,\mathrm{tr}\left[\prod_{i=1}^n R_{2i-1}R_{2i}^\dagger\right]\sum_{j=1}^n\delta(y_{2j-1}-y_{2j})\sum_{k=1}^n\delta(x_{2k-1}-x_{2k})\\
&-\frac{1}{N_c}\left\{\sum_{j=1}^n\sum_{k=1}^j\mathrm{tr}\left[\left(\prod_{i=1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\left(\prod_{i=k}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_z^\dagger\right]\right.\\
&\left.+\sum_{j=1}^n\sum_{k=j+1}^n\mathrm{tr}\left[R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_z^\dagger\left(\prod_{i=k}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\right\}\\
&\qquad\times\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k-2}).
\end{aligned} \tag{A.19}$$

Now, combining all the pieces eqs. (A.16), (A.17) and (A.19), one obtains

$$\begin{aligned}
H_{KLWMIJ}O_{2n} &= \frac{\bar{\alpha}_s}{2\pi}\int_{xyz}-\frac{1}{2}M_{xyz}\left\{\frac{1}{2N_c}\,\mathrm{tr}\left[\prod_{i=1}^n R_{2i-1}R_{2i}^\dagger\right]\right.\\
&\quad\times\sum_{j=1}^n\Big(\delta(y_{2j-1}-y_{2j})\delta(x_{2j-1}-x_{2j})+\delta(y_{2j-1}-y_{2j-2})\delta(x_{2j-1}-x_{2j-2})\Big)\\
&+\frac{1}{N_c^2}\sum_{j=2}^n\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\prod_{i=k}^{j-1}R_{2i-1}R_{2i}^\dagger\right]\\
&\qquad\times\delta(y_{2j-1}-y_{2j-2})\delta(x_{2k-1}-x_{2k-2})\\
&+\frac{1}{N_c^2}\sum_{j=2}^n\sum_{k=1}^{j-1}\mathrm{tr}\left[\left(\prod_{i=1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)R_{2k-1}R_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[R_{2j-1}R_{2k}^\dagger\left(\prod_{i=k+1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)\right]\\
&\qquad\times\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k})\\
&-\frac{1}{N_c^2}\sum_{j=1}^n\sum_{k=1}^j\mathrm{tr}\left[\left(\prod_{i=1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\left(\prod_{i=k}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_z^\dagger\right]\\
&\qquad\times\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k-2})\\
&-\frac{1}{N_c^2}\sum_{j=1}^n\sum_{k=j+1}^n\mathrm{tr}\left[R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{k-1}R_{2i-1}R_{2i}^\dagger\right)\right]\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_z^\dagger\left(\prod_{i=k}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\\
&\qquad\left.\times\delta(y_{2j-1}-y_{2j})\delta(x_{2k-1}-x_{2k-2})\right\}.
\end{aligned} \tag{A.20}$$

eq. (A.20) gives the complete KLWMIJ evolution of a generic single trace operator and it is equivalent to eq. (2.30). All the terms subleading in N_c cancel in this equation.

JHEP08(2014)007

A.2 KLWMIJ evolution of a generic double trace operator

We now derive the evolution equation of a double trace operator. Consider two generic operators O_{2n} and $O_{2\bar{m}}$:

$$O_{2n} = \frac{1}{N_c}\,\mathrm{tr}(R_1 R_2^\dagger \ldots R_{2n-1}R_{2n}^\dagger) = \frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=1}^{n} R_{2i-1}R_{2i}^\dagger\right], \tag{A.21}$$

$$O_{2\bar{m}} = \frac{1}{N_c}\,\mathrm{tr}(R_{\bar{1}} R_{\bar{2}}^\dagger \ldots R_{2\bar{m}-\bar{1}}R_{2\bar{m}}^\dagger) = \frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=\bar{1}}^{\bar{m}} R_{2i-\bar{1}}R_{2i}^\dagger\right]. \tag{A.22}$$

In the evolution of the product of these two objects, the two SU($\mathrm{N_c}$) rotation generators appearing in H_{KLWMIJ} can either act on the same object keeping the other one untouched, or one rotation generator can act on one object and the second one on the other object. Therefore, generically

$$H_{KLWMIJ}O_{2n}O_{2\bar{m}} = \left[H_{KLWMIJ}O_{2n}\right]O_{2\bar{m}} + O_{2n}\left[H_{KLWMIJ}O_{2\bar{m}}\right] + \chi_{\mathrm{mix}}, \tag{A.23}$$

with

$$\begin{aligned}\chi_{\mathrm{mix}} = \frac{\alpha_s}{2\pi^2}\int_{xyz} -\frac{1}{2}M_{xyz}\Big\{ &\left[J_L^a(x)O_{2n}\right]\left[J_L^a(y)O_{2\bar{m}}\right] + \left[J_R^a(x)O_{2n}\right]\left[J_R^a(y)O_{2\bar{m}}\right] \\ &- \left[J_L^a(x)O_{2n}\right]R_z^{ab}\left[J_R^b(y)O_{2\bar{m}}\right] - \left[J_L^a(x)O_{2\bar{m}}\right]R_z^{ab}\left[J_R^b(y)O_{2n}\right]\Big\}.\end{aligned} \tag{A.24}$$

By using eq. (2.12), the first term in eq. (A.24) can be written as

$$\begin{aligned}\left[J_L^a(x)O_{2n}\right]\left[J_L^a(y)O_{2\bar{m}}\right] = &\frac{1}{N_c^2}\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)t^a\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\delta(x_{2j-1}-x_{2j-2}) \\ &\times\sum_{k=\bar{1}}^{\bar{m}}\mathrm{tr}\left[\left(\prod_{i=\bar{1}}^{k-\bar{1}}R_{2i-\bar{1}}R_{2i}^\dagger\right)t^a\left(\prod_{i=k}^{\bar{m}}R_{2i-\bar{1}}R_{2i}^\dagger\right)\right]\delta(y_{2k-\bar{1}}-y_{2k-\bar{2}}),\end{aligned} \tag{A.25}$$

Further, using

$$\mathrm{tr}[At^aB]\,\mathrm{tr}[Ct^aD] = \frac{1}{2}\left\{\mathrm{tr}[BADC] - \frac{1}{N_c}\mathrm{tr}[AB]\,\mathrm{tr}[CD]\right\}, \tag{A.26}$$

we have

$$\begin{aligned}&\left[J_L^a(x)O_{2n}\right]\left[J_L^a(y)O_{2\bar{m}}\right] = \\ &= \frac{1}{2N_c^2}\sum_{j=1}^{n}\sum_{k=\bar{1}}^{\bar{m}}\Bigg\{\mathrm{tr}\left[\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=k}^{\bar{m}}R_{2i-\bar{1}}R_{2i}^\dagger\right)\left(\prod_{i=\bar{1}}^{k-\bar{1}}R_{2i-\bar{1}}R_{2i}^\dagger\right)\right] \\ &\quad -\frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=1}^{n}R_{2i-1}R_{2i}^\dagger\right]\mathrm{tr}\left[\prod_{i=\bar{1}}^{\bar{m}}R_{2i-\bar{1}}R_{2i}^\dagger\right]\Bigg\}\delta(y_{2k-\bar{1}}-y_{2k-\bar{2}})\delta(x_{2j-1}-x_{2j-2}).\end{aligned} \tag{A.27}$$

JHEP08(2014)007

By using eq. (2.13) the action of the virtual term can be written as

$$\begin{aligned}
&[J_R^a(x)O_{2n}]\,[J_R^a(y)O_{2\bar m}] \\
&= \frac{1}{N_c^2}\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}t^aR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\delta(x_{2j-1}-x_{2j}) \\
&\quad\times\sum_{k=\bar 1}^{\bar m}\mathrm{tr}\left[\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}t^aR_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\right]\delta(y_{2k-\bar 1}-y_{2k}). \qquad (A.28)
\end{aligned}$$

Performing the color algebra we get

$$\begin{aligned}
&[J_R^a(x)O_{2n}]\,[J_R^a(y)O_{2\bar m}] = \frac{1}{2N_c^2}\sum_{j=1}^{n}\sum_{k=\bar 1}^{\bar m}\Bigg\{ \\
&\times\mathrm{tr}\left[R_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}\right] \\
&-\frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=1}^{n}R_{2i-1}R_{2i}^\dagger\right]\mathrm{tr}\left[\prod_{i=\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right]\Bigg\}\delta(y_{2k-\bar 1}-y_{2k})\delta(x_{2j-1}-x_{2j}). \qquad (A.29)
\end{aligned}$$

The real term in H_{KLWMIJ} contributes to the "mixed" term:

$$\begin{aligned}
-R_z^{ab}\Big[J_L^a(x)O_{2n}\Big]\,\Big[J_R^b(y)O_{2\bar m}\Big] &= -\frac{R_z^{ab}}{N_c^2}\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)t^a\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\right] \qquad (A.30) \\
\times\delta(x_{2j-1}-x_{2j-2})\sum_{k=\bar 1}^{\bar m}&\mathrm{tr}\left[\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}t^bR_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{m}R_{2i-\bar 1}R_{2i}^\dagger\right)\right]\delta(y_{2k-\bar 1}-y_{2k}).
\end{aligned}$$

The relevant piece of color algebra here is

$$-R_z^{ab}\,\mathrm{tr}[At^aB]\,\mathrm{tr}[Ct^bD] = -\frac{1}{2}\left(\mathrm{tr}[R_z^\dagger BAR_zDC] - \frac{1}{N_c}\,\mathrm{tr}[AB]\,\mathrm{tr}[CD]\right). \qquad (A.31)$$

Using eq. (A.31), the real "mixed" term, eq. (A.30), can be written as

$$\begin{aligned}
-R_z^{ab}\Big[J_L^a(x)O_{2n}\Big]\,\Big[J_R^b(y)O_{2m}\Big] &= -\frac{1}{2N_c^2}\sum_{j=1}^{n}\sum_{k=\bar 1}^{\bar m}\Bigg\{\mathrm{tr}\Bigg[R_z^\dagger\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{n}R_{2i-1}R_{2i}^\dagger\right) \\
&\qquad\times R_zR_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}\Bigg] \\
-\frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=1}^{n}R_{2i-1}R_{2i}^\dagger\right]\mathrm{tr}&\left[\prod_{i=\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right]\Bigg\}\delta(x_{2j-1}-x_{2j-2})\delta(y_{2k-\bar 1}-y_{2k}). \qquad (A.32)
\end{aligned}$$

Similarly,

$$
\begin{aligned}
-R_z^{ab}\Big[J_L^a(x)O_{2\bar m}\Big]\Big[J_R^b(y)O_{2n}\Big] = &-\frac{R_z^{ab}}{N_c^2}\sum_{k=\bar 1}^{\bar m}\mathrm{tr}\left[\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)t^a\left(\prod_{i=k}^{\bar m}R_{2i-\bar 1}R^\dagger_{2i}\right)\right]\\
&\times\sum_{j=1}^{n}\mathrm{tr}\left[\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}t^bR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\right]\\
&\times\delta(x_{2k-\bar 1}-x_{2k-\bar 2})\delta(y_{2j-1}-y_{2j})
\end{aligned}
\tag{A.33}
$$

can be written as

$$
\begin{aligned}
-R_z^{ab}\Big[J_L^a(x)O_{2\bar m}\Big]\Big[J_R^b(y)O_{2n}\Big] = &-\frac{1}{2N_c^2}\sum_{j=1}^{n}\sum_{k=\bar 1}^{\bar m}\Bigg\{\mathrm{tr}\Bigg[R_z^\dagger\left(\prod_{i=k}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)\\
&\times R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}\Bigg]\\
&-\frac{1}{N_c}\,\mathrm{tr}\left[\prod_{i=1}^{n}R_{2i-1}R_{2i}^\dagger\right]\mathrm{tr}\left[\prod_{i=\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right]\Bigg\}\delta(x_{2k-\bar 1}-x_{2k-\bar 2})\delta(y_{2j-1}-y_{2j}).
\end{aligned}
\tag{A.34}
$$

Combining all the terms we see that all the order $1/N_c^3$ terms cancel. Hence,

$$
\begin{aligned}
\chi_{\rm mix} = &\frac{\alpha_s}{2\pi^2}\int_{xyz}-\frac{1}{2}M_{xyz}\frac{1}{2N_c^2}\sum_{j=1}^{n}\sum_{k=\bar 1}^{\bar m}\Bigg\{\\
&\times\mathrm{tr}\left[\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=k}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)\right]\\
&\times\delta(x_{2j-1}-x_{2j-2})\delta(y_{2k-\bar 1}-y_{2k-\bar 2})\\
&+\mathrm{tr}\left[R_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}R_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}\right]\\
&\times\delta(x_{2j-1}-x_{2j})\delta(y_{2k-\bar 1}-y_{2k})\\
&-\mathrm{tr}\left[R_z^\dagger\left(\prod_{i=j}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_zR_{2k}^\dagger\left(\prod_{i=k+\bar 1}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_{2k-\bar 1}\right]\\
&\times\delta(x_{2j-1}-x_{2j-2})\delta(y_{2k-\bar 1}-y_{2k})\\
&-\mathrm{tr}\left[R_z^\dagger\left(\prod_{i=k}^{\bar m}R_{2i-\bar 1}R_{2i}^\dagger\right)\left(\prod_{i=\bar 1}^{k-\bar 1}R_{2i-\bar 1}R_{2i}^\dagger\right)R_zR_{2j}^\dagger\left(\prod_{i=j+1}^{n}R_{2i-1}R_{2i}^\dagger\right)\left(\prod_{i=1}^{j-1}R_{2i-1}R_{2i}^\dagger\right)R_{2j-1}\right]\\
&\times\delta(x_{2k-\bar 1}-x_{2k-\bar 2})\delta(y_{2j-1}-y_{2j})\Bigg\}.
\end{aligned}
\tag{A.35}
$$

The delta functions can be now realized performing integrations over x and y. Combining eqs. (A.20) and (A.35), we arrive at eq. (2.29).

JHEP08(2014)007

B Evolution of multipoles

The evolution equation of a quadrupole is [33, 34]

$$\frac{d}{dY}Q_{1234} = \frac{\bar{\alpha_s}}{2\pi}\int_z -\frac{1}{2}\big(M_{12z}+M_{23z}+M_{34z}+M_{14z}\big)Q_{1234}-L_{1234z}d_{14}d_{32}-L_{1432z}d_{12}d_{34} \\ +L_{1214z}d_{1z}Q_{z234}+L_{1232z}d_{z2}Q_{1z34}+L_{3234z}d_{3z}Q_{12z4}+L_{1434z}d_{z4}Q_{123z}. \tag{B.1}$$

Using the definitions given in eqs. (3.5), (3.6), (3.7) and (3.8) one can calculate the evolution equation of 4-point Reggeons [1]

$$\begin{aligned}\frac{d}{dY}Q^{++}_{1234} = \frac{\bar{\alpha_s}}{2\pi}\int_z &-\frac{1}{2}\big(M_{12z}+M_{23z}+M_{34z}+M_{14z}\big)Q^{++}_{1234} \\ &+L_{1234z}\big[(P_{1z}-P_{4z})(P_{3z}-P_{2z})+P_{14}P_{32}+O_{14}O_{32}\big] \\ &+L_{1432z}\big[(P_{1z}-P_{2z})(P_{3z}-P_{z4})+P_{12}P_{34}+O_{12}O_{34}\big] \\ &+L_{1214z}\big[Q^{++}_{z234}(1-P_{1z})-Q^{--}_{z234}O_{1z}-P_{1z}(P_{23}-P_{24}+P_{34})\big] \\ &+L_{1232z}\big[Q^{++}_{1z34}(1-P_{z2})-Q^{--}_{1z34}O_{z2}-P_{z2}(P_{14}-P_{13}+P_{34})\big] \\ &+L_{3234z}\big[Q^{++}_{12z4}(1-P_{3z})-Q^{--}_{12z4}O_{3z}-P_{3z}(P_{12}-P_{24}+P_{14})\big] \\ &+L_{1434z}\big[Q^{++}_{123z}(1-P_{z4})-Q^{--}_{123z}O_{z4}-P_{z4}(P_{12}-P_{13}+P_{23})\big],\end{aligned} \tag{B.2}$$

$$\begin{aligned}\frac{d}{dY}Q^{+-}_{1234} = \frac{\bar{\alpha_s}}{2\pi}\int_z &-\frac{1}{2}\big(M_{12z}+M_{23z}+M_{34z}+M_{14z}\big)Q^{+-}_{1234} \\ &+L_{1214z}\big[Q^{+-}_{z234}(1-P_{1z})+Q^{-+}_{z234}O_{1z}\big]+L_{1232z}\big[Q^{+-}_{1z34}(1-P_{z2})+Q^{-+}_{1z34}O_{z2}\big] \\ &+L_{3234z}\big[Q^{+-}_{12z4}(1-P_{3z})+Q^{-+}_{12z4}O_{3z}\big]+L_{1434z}\big[Q^{+-}_{123z}(1-P_{z4})-Q^{-+}_{123z}O_{z4}\big].\end{aligned} \tag{B.3}$$

The evolution equation of a 6-point function can be calculated by taking the action of the KLWMIJ Hamiltonian. The resulting expression is

$$\begin{aligned}\frac{d}{dY}X_{123456} = \frac{\bar{\alpha_s}}{2\pi}\int_z &-\frac{1}{2}\big(M_{12z}+M_{34z}+M_{56z}+M_{16z}+M_{23z}+M_{45z}\big)X_{123456} \\ &+L_{1623z}d_{12}Q_{3456}+L_{2354z}d_{34}Q_{1256}+L_{1645z}d_{56}Q_{1234} \\ &+L_{1243z}d_{32}Q_{1456}+L_{1265z}d_{16}Q_{5234}+L_{3465z}d_{54}Q_{1236} \\ &-L_{1245z}Q_{1z56}Q_{z234}-L_{1643z}Q_{123z}Q_{z456}-L_{2356z}Q_{12z6}Q_{345z} \\ &+L_{1216z}d_{1z}X_{z23456}+L_{1232z}d_{z2}X_{1z3456}+L_{2343z}d_{3z}X_{12z456} \\ &+L_{3454z}d_{z4}X_{123z56}+L_{4565z}d_{5z}X_{1234z6}+L_{1656z}d_{z6}X_{12345z}.\end{aligned} \tag{B.4}$$

In the evolution of the 6-point function the terms with six points are either the hexapole or the breaking of the hexapole into two lower color singlet states which in this case it is a dipole and a quadrupole. The second structure which introduces two more points, breaks into two possible lower color singlet states; either a dipole and a hexapole or two quadrupoles. This is the same trend that was seen in the evolution of the 4-point function.

By using eq. (3.10), one can write the evolution equation of the 6-point ++-Reggeon as

$$\begin{aligned}
&\frac{d}{dY}X^{++}_{123456} = \frac{\bar{\alpha}_s}{2\pi}\int_z -\frac{1}{2}\big[M_{12z}+M_{23z}+M_{34z}+M_{45z}+M_{56z}+M_{61z}\big]X^{++}_{123456}\\
&+L_{1643z}\big[(P_{12}-P_{13}+P_{23})Q^{++}_{z456}+(P_{45}-P_{46}+P_{56})Q^{++}_{123z}+\bar{Q}_{123z}\bar{Q}_{z456}\big]\\
&+L_{3265z}\big[(P_{34}-P_{35}+P_{45})Q^{++}_{12z6}+(P_{12}-P_{26}+P_{16})Q^{++}_{z345}+\bar{Q}_{12z6}\bar{Q}_{z345}\big]\\
&+L_{1245z}\big[(P_{16}-P_{15}+P_{56})Q^{++}_{z234}+(P_{23}-P_{24}+P_{34})Q^{++}_{1z56}+\bar{Q}_{z234}\bar{Q}_{1z56}\big]\\
&+L_{1216z}\big\{(1-P_{1z})X^{++}_{z23456}-O_{1z}X^{--}_{z23456}-P_{1z}\big[Q^{++}_{2345}-Q^{++}_{2346}+Q^{++}_{2356}-Q^{++}_{2456}+Q^{++}_{3456}\big]\big\}\\
&+L_{2123z}\big\{(1-P_{z2})X^{++}_{1z3456}-O_{z2}X^{--}_{1z3456}-P_{z2}\big[Q^{++}_{1346}-Q^{++}_{1345}+Q^{++}_{1456}-Q^{++}_{1356}+Q^{++}_{3456}\big]\big\}\\
&+L_{2343z}\big\{(1-P_{3z})X^{++}_{12z456}-O_{3z}X^{--}_{12z456}-P_{3z}\big[Q^{++}_{1245}-Q^{++}_{1246}+Q^{++}_{1256}-Q^{++}_{2456}+Q^{++}_{1456}\big]\big\}\\
&+L_{3454z}\big\{(1-P_{z4})X^{++}_{123z56}-O_{z4}X^{--}_{123z56}-P_{z4}\big[Q^{++}_{1236}-Q^{++}_{1235}+Q^{++}_{1256}-Q^{++}_{1356}+Q^{++}_{2356}\big]\big\}\\
&+L_{4565z}\big\{(1-P_{5z})X^{++}_{1234z6}-O_{5z}X^{--}_{1234z6}-P_{5z}\big[Q^{++}_{1234}-Q^{++}_{1246}+Q^{++}_{1236}-Q^{++}_{2346}+Q^{++}_{1346}\big]\big\}\\
&+L_{1656z}\big\{(1-P_{z6})X^{++}_{12345z}-O_{z6}X^{--}_{12345z}-P_{z6}\big[Q^{++}_{1234}-Q^{++}_{1235}+Q^{++}_{1245}-Q^{++}_{1345}+Q^{++}_{2345}\big]\big\}\\
&+L_{1623z}\big\{(P_{2z}-P_{1z})\big[Q^{++}_{z345}-Q^{++}_{z346}-Q^{++}_{z456}+Q^{++}_{z356}\big]-P_{12}Q^{++}_{3456}-O_{12}Q^{--}_{3456}\big\}\\
&+L_{1243z}\big\{(P_{3z}-P_{2z})\big[Q^{++}_{z456}-Q^{++}_{1z45}-Q^{++}_{1z56}+Q^{++}_{1z46}\big]-P_{23}Q^{++}_{1456}-O_{32}Q^{--}_{1456}\big\}\\
&+L_{2354z}\big\{(P_{4z}-P_{3z})\big[Q^{++}_{12z5}-Q^{++}_{12z6}-Q^{++}_{z265}+Q^{++}_{1z56}\big]-P_{34}Q^{++}_{1256}-O_{34}Q^{--}_{1256}\big\}\\
&+L_{3465z}\big\{(P_{5z}-P_{4z})\big[Q^{++}_{12z6}-Q^{++}_{13z6}-Q^{++}_{123z}+Q^{++}_{z326}\big]-P_{45}Q^{++}_{1236}-O_{54}Q^{--}_{1236}\big\}\\
&+L_{1645z}\big\{(P_{6z}-P_{5z})\big[Q^{++}_{123z}-Q^{++}_{124z}-Q^{++}_{z234}+Q^{++}_{1z43}\big]-P_{56}Q^{++}_{1234}-O_{56}Q^{--}_{1234}\big\}\\
&+L_{1265z}\big\{(P_{1z}-P_{6z})\big[Q^{++}_{z234}-Q^{++}_{z235}-Q^{++}_{z345}+Q^{++}_{z245}\big]-P_{16}Q^{++}_{2345}-O_{16}Q^{--}_{2345}\big\},
\end{aligned}\tag{B.5}$$

where

$$\bar{Q}_{1234}\bar{Q}_{5678} = Q^{++}_{1234}Q^{++}_{5678}+Q^{+-}_{1234}Q^{+-}_{5678}+Q^{--}_{1234}Q^{--}_{5678}+Q^{-+}_{1234}Q^{-+}_{5678}\tag{B.6}$$

is introduced as a shorthand notation.

We have also calculated the evolution of the octupole

$$\begin{aligned}
&\frac{d}{dY}\Sigma_{12345678} = \frac{\bar{\alpha}_s}{2\pi}\int_z -\frac{1}{2}\big(M_{12z}+M_{34z}+M_{56z}+M_{78z}+M_{18z}+M_{23z}+M_{45z}+M_{67z}\big)\Sigma_{12345678}\\
&+L_{1823z}d_{12}X_{345678}+L_{3245z}d_{34}X_{125678}+L_{5467z}d_{56}X_{123478}+L_{1867z}d_{78}X_{123456}\\
&+L_{1287z}d_{18}X_{723456}+L_{1243z}d_{32}X_{145678}+L_{3465z}d_{54}X_{123678}+L_{5687z}d_{76}X_{123458}\\
&-L_{1854z}Q_{1234}Q_{5678}-L_{2367z}Q_{1278}Q_{3456}-L_{1256z}Q_{5234}Q_{1678}-L_{3478z}Q_{1238}Q_{5674}\\
&-L_{1245z}Q_{z234}X_{1z5678}-L_{1267z}Q_{1z78}X_{z23456}-L_{3481z}Q_{123z}X_{z45678}-L_{3467z}Q_{z456}X_{123z78}\\
&-L_{1865z}Q_{z678}X_{12345z}-L_{2356z}Q_{345z}X_{12z678}-L_{2378z}Q_{12z8}X_{34567z}-L_{4578z}Q_{567z}X_{1234z8}\\
&+L_{1218z}d_{1z}\Sigma_{z2345678}+L_{1232z}d_{z2}\Sigma_{1z345678}+L_{2343z}d_{3z}\Sigma_{12z45678}+L_{3454z}d_{z4}\Sigma_{123z5678}\\
&+L_{4565z}d_{5z}\Sigma_{1234z678}+L_{5676z}d_{z6}\Sigma_{12345z78}+L_{7678z}d_{7z}\Sigma_{123456z8}+L_{8781z}d_{z8}\Sigma_{1234567z}.
\end{aligned}\tag{B.7}$$

The evolution equation for 8-point ++-Reggeon is a long expression.

$$\frac{d}{dY}\Sigma^{++}_{12345678} = \frac{\bar{\alpha}_s}{2\pi}\int_z \Big[\chi^{(1)}+\chi^{(2)}+\chi^{(3)}+\chi^{(4)}+\chi^{(5)}\Big],\tag{B.8}$$

where $\chi^{(1)}$ is the term that arises from the direct Σ^{++} subtraction and comes with the same kernel as the first term in eq. (B.7). It is given as

$$\chi^{(1)} = -\frac{1}{2}\big[M_{12z}+M_{23z}+M_{34z}+M_{45z}+M_{56z}+M_{67z}+M_{78z}+M_{18z}\big]\Sigma^{++}_{12345678}. \tag{B.9}$$

$\chi^{(2)}$ is the term that arises from dipole times hexapole terms in the evolution for Σ whose explicit form reads

$$\begin{aligned}
\chi^{(2)} = {}& L_{1823z}\big[(P_{2z}-P_{1z})(X^{++}_{34567z}-X^{++}_{z34568}+X^{++}_{z34578}-X^{++}_{z34678}+X^{++}_{z35678}-X^{++}_{z45678}) \\
&\qquad -X^{++}_{345678}P_{12}-X^{--}_{345678}O_{12}\big] \\
&+L_{3245z}\big[(P_{4z}-P_{3z})(X^{++}_{12z567}-X^{++}_{12z568}+X^{++}_{12z578}-X^{++}_{12z678}+X^{++}_{1z5678}-X^{++}_{2z5678}) \\
&\qquad -X^{++}_{125678}P_{34}-X^{--}_{125678}O_{34}\big] \\
&+L_{5467z}\big[(P_{6z}-P_{5z})(X^{++}_{1234z7}-X^{++}_{1234z8}+X^{++}_{123z78}-X^{++}_{124z78}+X^{++}_{134z78}-X^{++}_{234z78}) \\
&\qquad -X^{++}_{123478}P_{56}-X^{--}_{123478}O_{56}\big] \\
&+L_{1867z}\big[(P_{8z}-P_{7z})(X^{++}_{12345z}-X^{++}_{12346z}+X^{++}_{12356z}-X^{++}_{12456z}+X^{++}_{13456z}-X^{++}_{z23456}) \\
&\qquad -X^{++}_{123456}P_{78}-X^{--}_{123456}O_{78}\big] \\
&+L_{1287z}\big[(P_{1z}-P_{8z})(X^{++}_{23456z}-X^{++}_{23457z}+X^{++}_{23467z}-X^{++}_{23567z}+X^{++}_{24567z}-X^{++}_{34567z}) \\
&\qquad -X^{++}_{234567}P_{18}-X^{--}_{234567}O_{18}\big] \\
&+L_{1243z}\big[(P_{2z}-P_{3z})(X^{++}_{1z4567}-X^{++}_{1z4568}+X^{++}_{1z4578}-X^{++}_{1z4678}+X^{++}_{1z5678}-X^{++}_{z45678}) \\
&\qquad -X^{++}_{145678}P_{32}-X^{--}_{145678}O_{32}\big] \\
&+L_{3465z}\big[(P_{4z}-P_{5z})(X^{++}_{123z67}-X^{++}_{123z68}+X^{++}_{123z78}-X^{++}_{12z678}+X^{++}_{13z678}-X^{++}_{23z678}) \\
&\qquad -X^{++}_{123678}P_{54}-X^{--}_{123678}O_{54}\big] \\
&+L_{5687z}\big[(P_{6z}-P_{7z})(X^{++}_{12345z}-X^{++}_{1234z8}+X^{++}_{1235z8}-X^{++}_{1245z8}+X^{++}_{1345z8}-X^{++}_{2345z8}) \\
&\qquad -X^{++}_{123458}P_{76}-X^{--}_{123458}O_{76}\big].
\end{aligned} \tag{B.10}$$

The $\chi^{(3)}$ part of the evolution of 8-point $++$-Reggeon originates from the quadrupole square terms in the evolution of Σ. Its explicit form is

$$\begin{aligned}
\chi^{(3)} = {}& L_{1854z}\big[\bar{Q}_{1234}\bar{Q}_{5678}+(Q^{++}_{123z}-Q^{++}_{124z}+Q^{++}_{1z43}-Q^{++}_{z234})(Q^{++}_{z567}-Q^{++}_{z568}+Q^{++}_{z578}-Q^{++}_{z678})\big] \\
&+L_{2367z}\big[\bar{Q}_{1278}\bar{Q}_{3456}+(Q^{++}_{z345}-Q^{++}_{z346}+Q^{++}_{z356}-Q^{++}_{z456})(Q^{++}_{1z78}-Q^{++}_{z287}+Q^{++}_{12z7}-Q^{++}_{12z8})\big] \\
&+L_{1256z}\big[\bar{Q}_{1678}\bar{Q}_{2345}+(Q^{++}_{z234}-Q^{++}_{z235}+Q^{++}_{z245}-Q^{++}_{z345})(Q^{++}_{z678}-Q^{++}_{1z67}+Q^{++}_{1z68}-Q^{++}_{1z78})\big] \\
&+L_{3478z}\big[\bar{Q}_{1238}\bar{Q}_{4567}+(Q^{++}_{z456}-Q^{++}_{z457}+Q^{++}_{z467}-Q^{++}_{z567})(Q^{++}_{z328}-Q^{++}_{13z8}+Q^{++}_{12z8}-Q^{++}_{123z})\big].
\end{aligned} \tag{B.11}$$

Here we use again the shorthand notation

$$\bar{Q}_{1234}\bar{Q}_{5678} = Q^{++}_{1234}Q^{++}_{5678}+Q^{+-}_{1234}Q^{+-}_{5678}+Q^{--}_{1234}Q^{--}_{5678}+Q^{-+}_{1234}Q^{-+}_{5678}. \tag{B.12}$$

$\chi^{(4)}$ originates from quadrupole times hexapole terms in the evolution of the 8-point ++- Reggeon.

$$\begin{aligned}\chi^{(4)} = \; & L_{1245z}\big[\bar{X}_{1z5678}\bar{Q}_{z234}+X^{++}_{1z5678}(P_{23}-P_{24}+P_{34}) \\ & \qquad +(Q^{++}_{1568}-Q^{++}_{1567}+Q^{++}_{1678}-Q^{++}_{1578}+Q^{++}_{5678})Q^{++}_{z234}\big] \\ & +L_{1267z}\big[\bar{X}_{23456z}\bar{Q}_{1z78}+X^{++}_{23456z}(P_{18}-P_{17}+P_{78}) \\ & \qquad +(Q^{++}_{2345}-Q^{++}_{2346}+Q^{++}_{2356}-Q^{++}_{2456}+Q^{++}_{3456})Q^{++}_{1z78}\big] \\ & +L_{3481z}\big[\bar{X}_{z45678}\bar{Q}_{123z}+X^{++}_{z45678}(P_{12}-P_{13}+P_{23}) \\ & \qquad +(Q^{++}_{4567}-Q^{++}_{4568}+Q^{++}_{4578}-Q^{++}_{4678}+Q^{++}_{5678})Q^{++}_{123z}\big] \\ & +L_{3467z}\big[\bar{X}_{123z78}\bar{Q}_{z456}+X^{++}_{123z78}(P_{45}-P_{46}+P_{56}) \\ & \qquad +(Q^{++}_{1238}-Q^{++}_{1237}+Q^{++}_{1278}-Q^{++}_{1378}+Q^{++}_{2378})Q^{++}_{z456}\big] \\ & +L_{1865z}\big[\bar{X}_{12345z}\bar{Q}_{z678}+X^{++}_{12345z}(P_{67}-P_{68}+P_{78}) \\ & \qquad +(Q^{++}_{1234}-Q^{++}_{1235}+Q^{++}_{1245}-Q^{++}_{1345}+Q^{++}_{2345})Q^{++}_{z678}\big] \\ & +L_{2356z}\big[\bar{X}_{12z678}\bar{Q}_{z345}+X^{++}_{12z678}(P_{34}-P_{35}+P_{45}) \\ & \qquad +(Q^{++}_{1267}-Q^{++}_{1268}+Q^{++}_{1278}-Q^{++}_{2678}+Q^{++}_{1678})Q^{++}_{z345}\big] \\ & +L_{2378z}\big[\bar{X}_{34567z}\bar{Q}_{12z8}+X^{++}_{34567z}(P_{12}-P_{28}+P_{18}) \\ & \qquad +(Q^{++}_{3456}-Q^{++}_{3457}+Q^{++}_{3467}-Q^{++}_{3567}+Q^{++}_{4567})Q^{++}_{12z8}\big] \\ & +L_{4578z}\big[\bar{X}_{1234z8}\bar{Q}_{z567}+X^{++}_{1234z8}(P_{56}-P_{57}+P_{67}) \\ & \qquad +(Q^{++}_{1234}+Q^{++}_{1238}-Q^{++}_{1248}+Q^{++}_{1348}-Q^{++}_{2348})Q^{++}_{z567}\big],\end{aligned} \tag{B.13}$$

where we introduced the following shorthand notation

$$\bar{X}_{12345z}\bar{Q}_{z678} = X^{++}_{12345z}Q^{++}_{z678} + X{+-}_{12345z}Q^{+-}_{z678} + X^{--}_{12345z}Q^{--}_{z678} + X^{-+}_{12345z}Q^{-+}_{z678}. \tag{B.14}$$

Finally, $\chi^{(5)}$ are the terms that come from the dipole times hexapole terms in the evolution of the 8-point function. Its explicit form reads

$$\begin{aligned}\chi^{(5)} = \; & L_{1218z}\big[P_{1z}(-X^{++}_{234567}+X^{++}_{234568}-X^{++}_{234578}+X^{++}_{234678}-X^{++}_{235678}+X^{++}_{245678}-X^{++}_{345678}) \\ & \qquad\qquad +(1-P_{1z})\Sigma^{++}_{z2345678}-O_{1z}\Sigma^{--}_{z2345678}\big] \\ & +L_{1232z}\big[P_{2z}(X^{++}_{134567}-X^{++}_{134568}+X^{++}_{134578}-X^{++}_{134678}+X^{++}_{135678}-X^{++}_{145678}-X^{++}_{345678}) \\ & \qquad\qquad +(1-P_{2z})\Sigma^{++}_{1z345678}-O_{z2}\Sigma^{--}_{1z345678}\big] \\ & +L_{2343z}\big[P_{3z}(-X^{++}_{124567}+X^{++}_{124568}-X^{++}_{124578}+X^{++}_{124678}-X^{++}_{125678}-X^{++}_{145678}+X^{++}_{245678}) \\ & \qquad\qquad +(1-P_{3z})\Sigma^{++}_{12z45678}-O_{3z}\Sigma^{--}_{12z45678}\big] \\ & +L_{3454z}\big[P_{4z}(X^{++}_{123567}-X^{++}_{123568}+X^{++}_{123578}-X^{++}_{123678}-X^{++}_{125678}+X^{++}_{135678}-X^{++}_{235678}) \\ & \qquad\qquad +(1-P_{4z})\Sigma^{++}_{123z5678}-O_{z4}\Sigma^{--}_{123z5678}\big] \\ & +L_{4565z}\big[P_{5z}(-X^{++}_{123467}+X^{++}_{123468}-X^{++}_{123478}-X^{++}_{123678}+X^{++}_{124678}-X^{++}_{134678}+X^{++}_{234678}) \\ & \qquad\qquad +\Sigma^{++}_{1234z678}(1-P_{5z})-O_{5z}\Sigma^{--}_{1234z678}\big] \\ & +L_{5676z}\big[P_{6z}(X^{++}_{123457}-X^{++}_{123458}-X^{++}_{123478}+X^{++}_{123578}-X^{++}_{124578}+X^{++}_{134578}-X^{++}_{234578})\end{aligned}$$

$$
\begin{aligned}
&+(1-P_{z6})\Sigma^{++}_{12345z78}-O_{z6}\Sigma^{--}_{12345z78}\big]\\
+L_{6787z}\big[&P_{7z}(-X^{++}_{123456}-X^{++}_{123458}+X^{++}_{123468}-X^{++}_{123568}+X^{++}_{124568}-X^{++}_{134568}+X^{++}_{234568})\\
&+(1-P_{7z})\Sigma^{++}_{123456z8}-O_{7z}\Sigma^{--}_{123456z8}\big]\\
+L_{1878z}\big[&P_{8z}(-X^{++}_{123456}+X^{++}_{123457}-X^{++}_{123467}+X^{++}_{123567}-X^{++}_{124567}+X^{++}_{134567}-X^{++}_{234567})\\
&+(1-P_{8z})\Sigma^{++}_{1234567z}-O_{z8}\Sigma^{--}_{1234567z}\big].
\end{aligned}
\tag{B.15}
$$

Acknowledgments

T.A. and M.L. thank the Physics Department of the University of Connecticut for hospitality during the visits while the work on this project was in progress. A.K. and T.A. thank the Physics Departments of the Ben-Gurion University of the Negev; A.K. also thanks Universidad Técnica Federico Santa María. The research was supported by the DOE grant DE-FG02-13ER41989; the BSF grant 2012124, Marie Curie Grant PIRG-GA-2009-256313; the ISRAELI SCIENCE FOUNDATION grant #87277111; the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programe FP7/2007-2013/ under REA grant agreement %318921; the Fondecyt (Chile) grants 1100648 and 1130549; European Research Council grant HotLHC ERC-2001- StG-279579; Ministerio de Ciencia e Innovacíon of Spain grants FPA2011-22776 and Consolider-Ingenio 2010 CPAN CSD2007-00042 and by FEDER.

References

[1] T. Altinoluk, C. Contreras, A. Kovner, E. Levin, M. Lublinsky et al., *QCD Reggeon Calculus From KLWMIJ/JIMWLK Evolution: Vertices, Reggeization and All*, *JHEP* **09** (2013) 115 [arXiv:1306.2794] [INSPIRE].

[2] T. Altinoluk, A. Kovner, E. Levin and M. Lublinsky, *Reggeon Field Theory for Large Pomeron Loops*, *JHEP* **04** (2014) 075 [arXiv:1401.7431] [INSPIRE].

[3] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The BFKL equation from the Wilson renormalization group*, *Nucl. Phys.* **B 504** (1997) 415 [hep-ph/9701284] [INSPIRE].

[4] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The Wilson renormalization group for low x physics: Towards the high density regime*, *Phys. Rev.* **D 59** (1998) 014014 [hep-ph/9706377] [INSPIRE].

[5] J. Jalilian-Marian, A. Kovner and H. Weigert, *The Wilson renormalization group for low x physics: Gluon evolution at finite parton density*, *Phys. Rev.* **D 59** (1998) 014015 [hep-ph/9709432] [INSPIRE].

[6] A. Kovner and J.G. Milhano, *Vector potential versus color charge density in low x evolution*, *Phys. Rev.* **D 61** (2000) 014012 [hep-ph/9904420] [INSPIRE].

[7] A. Kovner, J.G. Milhano and H. Weigert, *Relating different approaches to nonlinear QCD evolution at finite gluon density*, *Phys. Rev.* **D 62** (2000) 114005 [hep-ph/0004014] [INSPIRE].

[8] H. Weigert, *Unitarity at small Bjorken x*, *Nucl. Phys.* **A 703** (2002) 823 [hep-ph/0004044] [INSPIRE].

[9] E. Iancu, A. Leonidov and L.D. McLerran, *Nonlinear gluon evolution in the color glass condensate. 1.*, *Nucl. Phys.* **A 692** (2001) 583 [hep-ph/0011241] [INSPIRE].

[10] E. Iancu, A. Leonidov and L.D. McLerran, *The Renormalization group equation for the color glass condensate*, *Phys. Lett.* **B 510** (2001) 133 [hep-ph/0102009] [INSPIRE].

[11] E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, *Nonlinear gluon evolution in the color glass condensate. 2.*, *Nucl. Phys.* **A 703** (2002) 489 [hep-ph/0109115] [INSPIRE].

[12] A. Kovner and M. Lublinsky, *In pursuit of Pomeron loops: The JIMWLK equation and the Wess-Zumino term*, *Phys. Rev.* **D 71** (2005) 085004 [hep-ph/0501198] [INSPIRE].

[13] J. Bartels and M. Wuesthoff, *The Triple Regge limit of diffractive dissociation in deep inelastic scattering*, *Z. Physik* **C 66** (1995) 157.

[14] J. Bartels and C. Ewerz, *Unitarity corrections in high-energy QCD*, *JHEP* **09** (1999) 026 [hep-ph/9908454] [INSPIRE].

[15] J. Bartels, L.N. Lipatov and M. Wusthoff, *Conformal invariance of the transition vertex* $2 \to 4$ *gluons*, *Nucl. Phys.* **B 464** (1996) 298 [hep-ph/9509303] [INSPIRE].

[16] J. Bartels, L.N. Lipatov and G.P. Vacca, *Interactions of reggeized gluons in the Mobius representation*, *Nucl. Phys.* **B 706** (2005) 391 [hep-ph/0404110] [INSPIRE].

[17] M.A. Braun, *Nucleus-nucleus scattering in perturbative QCD with* $N_c \to \infty$, *Phys. Lett.* **B 483** (2000) 115 [hep-ph/0003004] [INSPIRE].

[18] M.A. Braun, *Nucleus nucleus interaction in the perturbative QCD*, *Eur. Phys. J.* **C 33** (2004) 113 [hep-ph/0309293] [INSPIRE].

[19] M.A. Braun, *Conformal invariant Pomeron interaction in the perurbative QCD with large-*N_c, *Phys. Lett.* **B 632** (2006) 297 [hep-ph/0512057] [INSPIRE].

[20] T. Altinoluk, A. Kovner, M. Lublinsky and J. Peressutti, *QCD Reggeon Field Theory for every day: Pomeron loops included*, *JHEP* **03** (2009) 109 [arXiv:0901.2559] [INSPIRE].

[21] A. Kovner and M. Lublinsky, *From target to projectile and back again: Selfduality of high energy evolution*, *Phys. Rev. Lett.* **94** (2005) 181603 [hep-ph/0502119] [INSPIRE].

[22] G.A. Chirilli, L. Szymanowski and S. Wallon, *Uncovering the triple Pomeron vertex from Wilson line formalism*, *Phys. Rev.* **D 83** (2011) 014020 [arXiv:1010.0285] [INSPIRE].

[23] S. Munier, *Quantum chromodynamics at high energy and statistical physics*, *Phys. Rept.* **473** (2009) 1 [arXiv:0901.2823] [INSPIRE].

[24] R.S. Kolevatov, K.G. Boreskov and L.V. Bravina, *All-Loop Calculation of the Reggeon Field Theory Amplitudes Via Stochastic Model*, *Eur. Phys. J.* **C 71** (2011) 1757 [arXiv:1105.3673] [INSPIRE].

[25] A. Kovner and M. Lublinsky, *Angular and long range rapidity correlations in particle*

production at high energy, Int. J. Mod. Phys. **E 22** (2013) 1330001 [arXiv:1211.1928] [INSPIRE].

[26] A. Dumitru, K. Dusling, F. Gelis, J. Jalilian-Marian, T. Lappi et al., *The Ridge in proton-proton collisions at the LHC*, Phys. Lett. **B 697** (2011) 21 [arXiv:1009.5295] [INSPIRE].

[27] E. Levin and A.H. Rezaeian, *The Ridge from the BFKL evolution and beyond*, Phys. Rev. **D 84** (2011) 034031 [arXiv:1105.3275] [INSPIRE].

[28] A. Dumitru and J. Jalilian-Marian, *Two-particle correlations in high energy collisions and the gluon four-point function*, Phys. Rev. **D 81** (2010) 094015 [arXiv:1001.4820] [INSPIRE].

[29] E. Levin and A. Prygarin, *Inclusive gluon production in the dipole approach: AGK cutting rules*, Phys. Rev. **C 78** (2008) 065202 [arXiv:0804.4747] [INSPIRE].

[30] A. Kovner and M. Lublinsky, *Remarks on high energy evolution*, JHEP **03** (2005) 001 [hep-ph/0502071] [INSPIRE].

[31] A.H. Mueller, *Small x Behavior and Parton Saturation: A QCD Model*, Nucl. Phys. **B 335** (1990) 115 [INSPIRE].

[32] A. Kovner and M. Lublinsky, *Odderon and seven Pomerons: QCD Reggeon field theory from JIMWLK evolution*, JHEP **02** (2007) 058 [hep-ph/0512316] [INSPIRE].

[33] J. Jalilian-Marian and Y.V. Kovchegov, *Inclusive two-gluon and valence quark-gluon production in DIS and pA*, Phys. Rev. **D 70** (2004) 114017 [*Erratum ibid.* **D 71** (2005) 079901] [hep-ph/0405266] [INSPIRE].

[34] A. Kovner, M. Lublinsky and H. Weigert, *Treading on the cut: Semi inclusive observables at high energy*, Phys. Rev. **D 74** (2006) 114023 [hep-ph/0608258] [INSPIRE].

JHEP08(2014)007