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Robust stability and H_∞-control of uncertain systems with impulsive perturbations

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Abstract

In this paper, the problems of robust stability, stabilization, and H_∞-control for uncertain systems with impulsive perturbations are investigated. The parametric uncertainties are assumed to be time-varying and norm-bounded. The sufficient conditions for the above problems are developed in terms of linear matrix inequalities. Numerical examples are given which illustrate the applicability of the theoretical results.

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Keywords: uncertain impulsive system; robust stability; robust stabilization; H_∞-control; linear matrix inequality (LMI)

1 Introduction

Many evolutionary processes are subject to short temporary perturbations that are negligible compared to the process duration. Thus the perturbations act instantaneously in the form of impulses. For example, biological phenomena involving thresholds, bursting rhythm models in pathology, optimal control of economic systems, frequency-modulated signal processing systems do exhibit impulse effects. Impulsive differential systems provide a natural description of observed evolutionary processes with impulse effects.

Problems with qualitative analysis of impulsive systems has been extensively studied in the literature, we refer to [1–7] and the references therein. Also, the control of impulsive or nonlinear systems received more recently researchers' special attention due to their applications; see, for example [8–11]. In [12], Guan *et al.* studied the H_∞ control problem for impulsive systems. In terms of the solutions to an algebraic Riccati equation, they obtained sufficient conditions for the existence of state feedback controllers guaranteeing asymptotic stability and prescribed H_∞ performance of the closed-loop system. But the result in [12] is based on the assumption that the state jumping at the impulsive time instant has a special form. This assumption is not satisfied for most impulsive systems. Therefore, the results in [12] are less applicable. Furthermore, the parameter uncertainties of impulsive systems were not considered in [12].

The goal of this paper is to study the robust stability, stabilization, and H_∞-control of uncertain impulsive systems under more general assumption on state jumping. Sufficient conditions for the existence of the solutions to the above problems are derived. Moreover, these sufficient conditions are all in linear matrix inequality (LMI) formalism, which makes their resolution easy.

The rest of this paper is organized as follows: Section 2 describes the system model; Section 3 addresses the robust stability and stabilization problems; Section 4 studies the robust H_∞ problem; Section 5 provides two examples to demonstrate the applicability of the proposed approach.

2 Problem statement

In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq)\, 0$ is used to denote a symmetric positive-definite (positive-semidefinite, negative, negative-semidefinite) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum and maximum eigenvalues of the corresponding matrix, respectively. $\|\cdot\|$ denotes the Euclidean norm for vectors or the spectral norm of matrices.

Consider uncertain linear impulsive systems described by the following state equation:

$$\begin{aligned} &\dot{x}(t) = A(t)x(t) + B_1(t)u(t) + H_1 w(t), \quad t \neq t_k, \\ &\Delta x(t) = C_k x(t_k), \quad t = t_k, \\ &z(t) = E(t)x(t) + B_2(t)u(t) + H_2 w(t), \\ &x(t_0) = x_0, \quad t_0 = 0, \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in \mathbb{R}^q$ is the controlled output. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ describes the state jumping at impulsive time instant $t = t_k$, $x(t_k^-) = x(t_k) = \lim_{h\to 0+} x(t_k - h)$, $x(t_k^+) = \lim_{h\to 0+} x(t_k + h)$, $k = 1, 2, \ldots$, and $0 < t_1 < t_2 < \cdots < t_k < \cdots$ ($t_k \to \infty$ as $t \to \infty$). $H_1 \in \mathbb{R}^{n\times p}$, $H_2 \in \mathbb{R}^{q\times p}$, $C_k \in \mathbb{R}^{n\times n}$, $k = 1, 2, \ldots$, are known constant matrices, and $A(t) \in \mathbb{R}^{n\times n}$, $B_1(t) \in \mathbb{R}^{n\times m}$, $E(t) \in \mathbb{R}^{q\times n}$, $B_2(t) \in \mathbb{R}^{q\times m}$ are matrix functions with time-varying uncertainties, that is,

$$\begin{aligned} &A(t) = A + \Delta A(t), \qquad B_1(t) = B_1 + \Delta B_1(t), \\ &E(t) = E + \Delta E(t), \qquad B_2(t) = B_2 + \Delta B_2(t), \end{aligned}$$

where $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{q\times n}$, $B_2 \in \mathbb{R}^{q\times m}$ are known real constant matrices, $\Delta A(t) \in \mathbb{R}^{n\times n}$, $\Delta B_1(t) \in \mathbb{R}^{n\times m}$, $\Delta E(t) \in \mathbb{R}^{q\times n}$, and $\Delta B_2(t) \in \mathbb{R}^{q\times m}$ are unknown matrices representing time-varying parameter uncertainties. We assume that the uncertainties are norm-bounded and can be described as

$$\begin{bmatrix} \Delta A(t) & \Delta B_1(t) \\ \Delta E(t) & \Delta B_2(t) \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F(t) [N \quad N_b], \tag{2.2}$$

where $D_1 \in \mathbb{R}^{n\times n_f}$, $D_2 \in \mathbb{R}^{q\times n_f}$, $N \in \mathbb{R}^{n_f\times n}$, $N_b \in \mathbb{R}^{n_f\times m}$ are known real constant matrices and $F(\cdot) \in \mathbb{R}^{n_f\times n_f}$ is an unknown matrix functions satisfying $F^T(t)F(t) \leq I$. It is assumed that the elements of $F(t)$ are Lebesgue measurable.

Throughout this paper, we shall use the following concepts of robust stability and robust performance for system (2.1).

Definition 2.1 System (2.1) with $u(t) = 0$ and $w(t) = 0$ is said to be robustly stable if the trivial solution of (2.1) with $u(t) = 0$ and $w(t) = 0$ is asymptotically stable for all admissible uncertainties satisfying (2.2).

Definition 2.2 Given a scalar $\gamma > 0$, the uncertain impulsive system (2.1) with $u(t) = 0$ is said to have robust stabilization with disturbance attenuation γ if it is robustly stable in the sense of Definition 2.1 and under zero initial conditions,

$$\int_0^\infty z^T(t)z(t)\,dt < \gamma^2 \int_0^\infty w^T(t)w(t)\,dt.$$

The following lemma is essential for the developments in the next sections.

Lemma 2.1 (see [13]) *For any vectors $x, y \in \mathbb{R}^n$, matrices $A, P \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times n_f}$, $E, N \in \mathbb{R}^{n_f\times n}$, and $D \in \mathbb{R}^{n\times n_f}$, $E, N \in \mathbb{R}^{n_f\times n}$, with $P > 0$, $\|F\| \le 1$, and scalar $\varepsilon > 0$, the following inequalities hold*:

(i) $DFN + N^T F^T D^T \le \varepsilon^{-1} DD^T + \varepsilon N^T N$;

(ii) *if* $\varepsilon I - EPE^T > 0$,
$(A + DF(t)E)P(A + DF(t)E)^T \le APA^T + APE^T(\varepsilon I - EPE^T)^{-1} EPA^T + \varepsilon DD^T$;

(iii) $2x^T y \le x^T P^{-1} x + y^T Py$;

(iv) *if* $P - \varepsilon DD^T > 0$, $(A + DF(t)E)^T P^{-1}(A + DF(t)E) \le A^T(P - \varepsilon DD^T)^{-1} A + \varepsilon^{-1} E^T E$.

3 Robust stability and robust stabilization

In this section, we restrict our study to the case of $w(t) = 0$ in system (2.1), *i.e.*

$$\begin{aligned} &\dot{x}(t) = A(t)x(t) + B_1(t)u(t), \quad t \ne t_k, \\ &\Delta x(t) = C_k x(t_k), \quad t = t_k, \\ &x(t_0) = x_0, \quad t_0 = 0. \end{aligned} \tag{3.1}$$

First, we present some sufficient conditions for robust stability of system (3.1) with $u(t) = 0$.

Theorem 3.1 *Assume that there exist $\alpha > 0$ and $\beta > 0$ such that $\|C_k\| \le \alpha$, $k = 1, 2, \dots$, $\beta = \inf_i\{t_{i+1} - t_i\}$. If for the prescribed scalars $\mu_1 > 0$ and $\mu_2 > 1$ satisfying $\ln(\mu_2) - \beta\mu_1 < 0$, there exist matrix $P > 0$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following linear matrix inequalities hold*:

$$\begin{bmatrix} \mu_1 P + PA + AP^T + \varepsilon_1 N^T N & PD_1 \\ D_1^T P & -\varepsilon_1 I \end{bmatrix} < 0, \tag{3.2}$$

$$\begin{bmatrix} -(\mu_2 - 1)P + \varepsilon_2 I & \alpha P \\ \alpha P & -\varepsilon_2 I + \alpha^2 P \end{bmatrix} < 0, \tag{3.3}$$

then system (3.1) *with $u(t) = 0$ is robustly asymptotically stable.*

Proof Take the Lyapunov function for system (3.1),

$$V(t) = x^T(t)Px(t). \tag{3.4}$$

For $t \in (t_k, t_{k+1}]$, the time derivative of $V(t)$ is

$$\begin{aligned}\dot{V}(t) &= x^T(t)\big(PA(t) + A^T(t)P\big)x(t) \\ &= x^T(t)\big(PA + A^T P\big)x(t) + 2x^T(t)P\Delta A(t)x(t).\end{aligned} \tag{3.5}$$

By (i) of Lemma 2.1, for any $\varepsilon > 0$, we get

$$2x^T(t)P\Delta A(t)x(t) \le x^T\big(\varepsilon^{-1}PD_1D_1^T P + \varepsilon N^T N\big)x(t). \tag{3.6}$$

By Schur complement, condition (3.2) is equivalent to

$$PA + A^T P + \varepsilon N^T N + \varepsilon^{-1}PD_1D_1^T P < -\mu_1 P. \tag{3.7}$$

Combining (3.4)-(3.7) yields

$$\dot{V}(t) < -\mu_1 V(t), \quad t \in (t_k, t_{k+1}],$$

which implies that

$$V(t) < V\big(t_k^+\big)e^{-\mu_1(t-t_k)}, \quad t \in (t_k, t_{k+1}]. \tag{3.8}$$

On the other hand, since $\|C_k\| \le \alpha$, it follows that C_k can be written as $C_k = DF_kE$ with $D = \alpha I$, $E = I$ and $\|F_k\| \le 1$. Using (ii) of Lemma 2.1, for any ε_2 satisfying $\varepsilon_2 I - \alpha^2 P > 0$, we get

$$\begin{aligned}V\big(t_k^+\big) &= x^T(t_k)(I + C_k)^T P(I + C_k)x(t_k) \\ &= x^T(t_k)(I + DF_kE)^T P(I + DF_kE)x(t_k) \\ &\le x^T(t_k)\big(P + \alpha^2 P\big(\varepsilon_2 I - \alpha^2 P\big)^{-1}P + \varepsilon_2 I\big)x(t_k).\end{aligned} \tag{3.9}$$

By Schur complement, condition (3.3) is equivalent to

$$P + \alpha^2 P\big(\varepsilon_2 I - \alpha^2 P\big)^{-1}P + \varepsilon_2 I < \mu_2 P.$$

Substituting the above inequality into (3.9) gives

$$V\big(t_k^+\big) < \mu_2 x^T(t_k)Px(t_k) = \mu_2 V(t_k). \tag{3.10}$$

On the basis of (3.8) and (3.10), we obtain

$$V(t) < \mu_2^k e^{-\mu_1(t-t_0)}V(t_0), \quad t \in (t_k, t_{k+1}].$$

By the assumption of $\beta = \inf_i\{t_{i+1} - t_i\}$, we get $t - t_0 \ge k\beta$. Noticing that $\mu_2 > 1$ and $\ln(\mu_2) - \beta\mu_1 < 0$, we deduce that

$$V(t) < \exp\left(\left(\frac{1}{\beta}\ln\mu_2 - \mu_1\right)(t - t_0)\right)V(t_0), \quad t \ge t_0. \tag{3.11}$$

It follows that system (2.1) with $u(t) = 0$ is robustly asymptotically stable. The proof is completed. □

Remark 3.1 When $\alpha = 0$, that is, there is no impulse jumping in the states, let $\varepsilon_2 \to 0^+$ and $\mu_1 \to 0^+$, $\mu_2 \to 1^+$, then the LMI conditions in Theorem 3.1 reduces to a single LMI

$$\begin{bmatrix} PA + AP^T + \varepsilon_1 N^T N & PD_1 \\ D_1^T P & -\varepsilon_1 I \end{bmatrix} < 0 \tag{3.12}$$

for some matrix $P > 0$ and some scalar $\varepsilon_1 > 0$. Condition (3.12) is exactly the sufficient condition for robust stability of continuous-time linear systems with norm-bounded uncertainty, for example, see [14].

Remark 3.2 From (3.11), we can show that

$$\|x(t)\| \le \sqrt{\frac{\lambda_1}{\lambda_0}} \exp\left(-\frac{1}{2}\left(\mu_1 - \frac{1}{\beta}\ln(\mu_2)\right)(t - t_0)\right)\|x_0\|, \quad t \ge t_0,$$

where $\lambda_1 = \lambda_{\max}(P)$, $\lambda_0 = \lambda_{\min}(P)$. It follows that under the conditions of Theorem 3.1, system (3.1) with $u(t) = 0$ is robustly exponentially stable with decay rate $\delta = \frac{1}{2}(\mu_1 - \frac{1}{\beta}\ln(\mu_2))$. For prescribed decay rate δ, we can choose $\mu_1 = 2\delta + \frac{1}{\beta}\ln(\mu_2)$ to find the feasible solution to LMIs (3.2) and (3.3) by tuning parameter μ_2.

Let us now design a memoryless state feedback controller of the following form:

$$u(t) = Kx(t) \tag{3.13}$$

to stabilize system (3.1), where $K \in \mathbb{R}^{m\times n}$ is a constant gain to be designed.

Substituting (3.13) into (3.1) yields the dynamics of the closed-loop system as follows:

$$\begin{aligned} &\dot{x}(t) = \big(A(t) + B_1(t)K\big)x(t), \quad t \neq t_k, \\ &\Delta x(t) = C_k x(t_k), \quad t = t_k, \\ &x(t_0) = x_0, \quad t_0 = 0. \end{aligned} \tag{3.14}$$

Theorem 3.2 *Assume that there exist $\alpha > 0$ and $\beta > 0$ such that $\|C_k\| \le \alpha$, $k = 1, 2, \ldots$, $\beta = \inf_i\{t_{i+1} - t_i\}$. If for prescribed scalars $\mu_1 > 0$ and $\mu_2 > 1$ satisfying $\ln(\mu_2) - \beta\mu_1 < 0$, there exist matrix $X > 0$, $\bar{K}$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following linear matrix inequalities hold*:

$$\begin{bmatrix} \mu_1 X + AX + XA^T + B_1\bar{K} + \bar{K}^T B_1^T + \varepsilon_1 D_1 D_1^T & \bar{K}^T N_b^T + XN^T \\ N_b\bar{K} + NX & -\varepsilon_1 I \end{bmatrix} < 0, \tag{3.15}$$

$$\begin{bmatrix} -\mu_2 X & X & X \\ X & -X + \alpha^2\varepsilon_2 I & 0 \\ X & 0 & -\varepsilon_2 I \end{bmatrix} < 0, \tag{3.16}$$

then the controller (3.13) *with $K = \bar{K}X^{-1}$ robustly stabilizes system* (3.1).

Proof From the proof of the Theorem 3.1, the sufficient condition for asymptotic stability of closed-loop system (3.14) is that there exist positive scalars μ_1, μ_2 satisfying $\ln(\mu_2) - \beta\mu_1 < 0$ such that the following two inequalities hold:

$$\dot{V}(t) < -\mu_1 V(t), \quad t \in (t_k, t_{k+1}], \tag{3.17}$$

$$\dot{V}(t_k^+) < \mu_2 V(t_k), \quad k = 1, 2, \ldots, \tag{3.18}$$

where $V(x) = x^T P x$ with $P > 0$.

Using the technique as in the proof of Theorem 3.1, it can easily be shown that if there exists a positive scalar ε_1 such that the following inequality is satisfied:

$$\begin{aligned} &\mu_1 P + P(A + B_1 K) + (A^T + K^T B_1^T)P + \frac{1}{\varepsilon_1}(K^T N_b^T + N^T)(N + N_b K) \\ &\quad + \varepsilon_1 P D_1 D_1^T P < 0, \end{aligned} \tag{3.19}$$

then (3.17) holds.

Now we consider the sufficient condition for inequality (3.18). As in the proof of Theorem 3.1, we represent C_k in the form of $C_k = DF_kE$ with $D = \alpha I$, $E = I$ and $\|F_k\| \leq 1$. Then using (iv) of Lemma 2.1, for any positive scalar ε_2 satisfying $P^{-1} - \alpha^2\varepsilon_2 I > 0$, we have

$$\begin{aligned} V(t_k^+) &= x^T(t_k)(I + C_k)^T P(I + C_k)x(t_k) \\ &= x^T(t_k)(I + DF_kE)^T P(I + DF_kE)x(t_k) \\ &\leq x^T(t_k)\big((P^{-1} - \alpha^2\varepsilon_2 I)^{-1} + \varepsilon_2^{-1} I\big)x(t_k). \end{aligned}$$

It follows that if

$$(P^{-1} - \alpha^2\varepsilon_2 I)^{-1} + \varepsilon_2^{-1} I < \mu_2 P, \tag{3.20}$$

then (3.18) holds. Thus, if (3.19) and (3.20) hold, then closed-loop system (3.14) is asymptotically stable. Define $X = P^{-1}$, $\bar{K} = KX$. Pre- and post-multiplying (3.19) by X yields

$$\mu_1 X + AX + B_1\bar{K} + XA^T + \bar{K}^T B_1^T + \frac{1}{\varepsilon_1}(\bar{K}^T N_b^T + XN^T)(NX + N_b\bar{K}) + \varepsilon_1 D_1 D_1^T < 0,$$

which combined with Schur complement leads to (3.15).

Pre- and post-multiplying (3.20) by X yields

$$-\mu_2 X + X(X - \alpha^2\varepsilon_2 I)^{-1} X + \varepsilon_2^{-1} X^2 \leq 0,$$

which combined with Schur complement leads to (3.16). The proof is completed. □

4 Robust H_∞-control

This section is devoted to studying the robust H_∞-control problem for system (2.1).

Theorem 4.1 *Assume that there exist* $\alpha > 0$ *and* $\beta > 0$ *such that* $\|C_k\| \leq \alpha$, $k = 1, 2, \ldots$, $\beta = \inf_i\{t_{i+1} - t_i\}$. *If for the prescribed scalars* $\mu_1 > 0$ *and* $\mu_2 > 1$ *satisfying* $2\ln(\mu_2) - \beta\mu_1 \leq 0$, *there exist matrix $X > 0$, $Q > 0$, $\bar{K}$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ such that* (3.15), (3.16), *and the following linear matrix inequalities hold*:

$$\begin{bmatrix} \Xi & H_1 & L & XN^T + \bar{K}^T N_b^T & \mu_2 I \\ H_1^T & -\gamma^2 I + \mu_2 H_1^T Q H_1 & H_2^T & 0 & 0 \\ L^T & H_2 & -I + \varepsilon_3 D_2 D_2^T & 0 & 0 \\ NX + N_b \bar{K} & 0 & 0 & -\varepsilon_3 I & 0 \\ \mu_2 I & 0 & 0 & 0 & -\mu_2 Q \end{bmatrix} < 0, \tag{4.1}$$

where $\Xi = AX + XA^T + B_1\bar{K} + \bar{K}^T B_1^T + \varepsilon_3 D_1 D_1^T$, $L = XE^T + \bar{K}^T B_2^T + \varepsilon_3 D_1 D_2^T$, *then system* (2.1) *has robust stabilization with disturbance attenuation γ. Moreover, the controller* (3.13) *with $K = \bar{K}X^{-1}$ robustly stabilizes system* (2.1).

Proof With the memoryless state feedback control law (3.13), system (2.1) becomes

$$\begin{aligned} &\dot{x}(t) = \big(A(t) + B_1(t)K\big)x(t) + H_1 w(t), \quad t \neq t_k, \\ &\Delta x(t) = C_k x(t_k), \quad t = t_k, \\ &z(t) = \big(E(t) + B_2(t)K\big)x(t) + H_2 w(t), \\ &x(t_0) = x_0, \quad t_0 = 0. \end{aligned} \tag{4.2}$$

By $2\ln(\mu_2) - \beta\mu_1 \le 0$ and $\mu_2 > 1$, it is easy to see that $\ln(\mu_2) - \beta\mu_1 < 0$. So, if $w(t) = 0$, then by (3.15) and (3.16) and Theorem 3.2, we can conclude that system (4.2) has robust stabilization. Next, we proceed to prove that system (4.2) verifies noise attenuation γ. To this end, we assume the zero initial condition, that is, $x(t) = 0$, for $t = 0$.

Applying Lyapunov function (3.4) with $P = X^{-1}$ to (4.2), for $t \in (t_k, t_{k+1}]$, the time derivative of $V(t)$ is

$$\dot{V}(t) = x^T(t)\big(P\big(A(t) + B_1(t)K\big) + \big(A(t) + B_1(t)K\big)^T P\big)x(t) + 2x^T(t)PH_1 w(t). \tag{4.3}$$

Using Lemma 2.1 and condition (3.15), we obtain

$$\dot{V}(t) \le -\mu_1 V(t) + 2x^T(t)PH_1 w(t), \quad t \in (t_k, t_{k+1}].$$

It follows that

$$V(t) \le e^{-\mu_1(t-t_k)} V\big(t_k^+\big) + 2\int_{t_k}^{t} e^{-\mu_1(t-s)} x^T(s)PH_1 w(s)\,ds, \quad t \in (t_k, t_{k+1}],$$

which yields

$$\begin{aligned} V(t_{k+1}) &\le e^{-\mu_1(t_{k+1}-t_k)} V\big(t_k^+\big) + 2\int_{t_k}^{t_{k+1}} e^{-\mu_1(t_{k+1}-s)} x^T(s)PH_1 w(s)\,ds \\ &\le e^{-\mu_1\beta} V\big(t_k^+\big) + 2\int_{t_k}^{t_{k+1}} \big|x^T(s)PH_1 w(s)\big|\,ds. \end{aligned} \tag{4.4}$$

From the proof of Theorem 3.2, condition (3.16) implies $V(t_{k+1}^+) \leq \mu_2 V(t_{k+1})$, $k = 0, 1, 2, \ldots$. Substituting this inequality into (4.4) gives

$$\begin{aligned} V(t_{k+1}^+) &\leq \mu_2^2 e^{-\mu_1 \beta} V(t_k) + 2\mu_2 \int_{t_k}^{t_{k+1}} \left| x^T(s) P H_1 w(s) \right| ds \\ &\leq V(t_k) + 2\mu_2 \int_{t_k}^{t_{k+1}} \left| x^T(s) P H_1 w(s) \right| ds. \end{aligned} \tag{4.5}$$

It follows that for $T \in (t_k, t_{k+1}]$

$$\begin{aligned} \int_0^T \dot{V}(t)\,dt &= \int_{t_k}^T \dot{V}(t)\,dt + \int_{t_{k-1}}^{t_k} \dot{V}(t)\,dt + \cdots + \int_{t_0}^{t_1} \dot{V}(t)\,dt \\ &= V(T) - V(t_k^+) + V(t_k) - V(t_{k-1}^+) + \cdots + V(t_1) - V(t_0) \\ &= V(T) + \sum_{i=1}^k V(t_i) - \sum_{i=1}^k V(t_i^+) \\ &\geq V(T) + \sum_{i=1}^k V(t_i) - \sum_{i=1}^k \left[V(t_{i-1}) + 2\mu_2 \int_{t_{i-1}}^{t_i} \left| x^T(s) H_1 w(s) \right| ds \right] \\ &\geq -2\mu_2 \int_0^T \left| x^T(s) P H_1 w(s) \right| ds. \end{aligned}$$

By (iii) of Lemma 2.1, for any $Q > 0$, we have

$$2\left| x^T(t) P H_1 w(t) \right| \leq x^T(t) P Q^{-1} P x(t) + w^T(t) H_1^T Q H_1 w(t).$$

Thus, for any $T > 0$, we have

$$\int_0^T \dot{V}(t)\,dt \geq -\mu_2 \int_0^T \left(x^T(t) P Q^{-1} P x(t) + w^T(t) H_1^T Q H_1 w(t) \right) dt. \tag{4.6}$$

Set $\tilde{E}(t) = [E(t) + B_2(t)K\ H_2]$, $\eta^T(t) = [x^T(t)\ w^T(t)]$, $\eta_e(t) = \tilde{E}(t)\eta(t)$, $\xi^T(t) = [\eta^T(t)\ \eta_e^T(t)]$. Define

$$J_T = \int_0^T \left[z^T(t) z(t) - \gamma^2 w^T(t) w(t) \right] dt.$$

Since $V(0) = 0$ by the zero initial condition, it follows from (4.6) and (4.3) that

$$\begin{aligned} J_T &= \int_0^T \left[z^T(t) z(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t) \right] dt - \int_0^T \dot{V}(t)\,dt \\ &\leq \int_0^T \Big\{ \eta^T(t) \tilde{E}^T(t) \tilde{E}(t) \eta(t) - \gamma^2 w^T(t) w(t) + 2x^T(t) P \big(A(t) + B_1(t) K \big) x(t) \\ &\quad + 2x^T(t) P H_1 w(t) + \mu_2 \big(x^T(t) P Q^{-1} P x(t) + w^T(t) H_1^T Q H_1 w(t) \big) \Big\} dt \\ &= \int_0^T \xi^T(t) \big(\Theta + \Gamma_D F(t) \Gamma_N + \Gamma_N^T F^T(t) \Gamma_D^T \big) \xi(t)\,dt, \end{aligned} \tag{4.7}$$

where $\Gamma_D^T = [D_1^T P \; 0 \; D_2^T]$, $\Gamma_N = [N + N_b K \; 0 \; 0]$, $\tilde{E} = [E + B_2 K \; H_2]$, and

$$\Theta = \begin{bmatrix} \begin{bmatrix} P(A + B_1 K) + (A^T + K^T B_1^T)P + \mu_2 P Q^{-1} P & PH_1 \\ H_1^T P & -\gamma^2 I + \mu_2 H_1^T Q H_1 \end{bmatrix} & \tilde{E}^T \\ \tilde{E} & -I \end{bmatrix}.$$

By (i) of Lemma 2.1, for any scalar $\varepsilon_3 > 0$,

$$\Gamma_D F(t) \Gamma_N + \Gamma_N^T F^T(t) \Gamma_D^T \leq \varepsilon_3 \Gamma_D \Gamma_D^T + \varepsilon_3^{-1} \Gamma_N^T \Gamma_N.$$

Thus, if the following inequality holds:

$$\Theta + \varepsilon_3 \Gamma_D \Gamma_D^T + \varepsilon_3^{-1} \Gamma_N^T \Gamma_N < 0, \tag{4.8}$$

then by (4.7), we have

$$\int_0^\infty z^T(t) z(t)\, dt < \gamma^2 \int_0^\infty w^T(t) w(t)\, dt,$$

so the proof will be completed.

Pre- and post-multiplying (4.8) by $\mathrm{diag}\{X, I, I\}$ and using Schur complement, it is easy to prove that (4.1) is equivalent to (4.8). The proof is completed. □

Remark 4.1 In [12], under the assumption that $C_k = c_k I$ and $c_k \in (-2, 0)$, sufficient condition for the existence of H_∞ state feedback controller was derived in terms of the Riccati equation. As compared to [12], our results can be used for a wider class of impulsive system. Moreover, Theorem 4.1 cast the existence problem of H_∞ state feedback controller into the feasibility problem of the LMIs (3.15), (3.16), and (4.1), the latter can be efficiently solved by the developed interior-point algorithm [15].

5 Numerical example

In this section, we shall give two numerical examples to demonstrate the effectiveness of the proposed results.

Example 1 Consider the linear uncertain impulsive system (3.1) with $u(t) = 0$. Assume that the system data are given as

$$A = \begin{bmatrix} -0.5 & 1 \\ -1.25 & -3 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad N = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \quad c \geq 0, \alpha = 0.1, \beta = 0.2.$$

If we select decay rate $\delta = 0.1$, then by Theorem 3.1, choosing $\mu_2 = 1.3$ and $\mu_1 = 2\delta + \frac{1}{\beta}\ln(\mu_2)$, the obtained maximum value of c such that the above system is robustly exponentially stable is $c = 0.53$. If we select the decay rate $\delta = 0.2$, by choosing the same values of μ_1 and μ_2, the corresponding maximum value of c is $c = 0.453$.

Example 2 Consider the uncertain impulsive system (2.1) with parameters as follows:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad E = [1 \quad 1],$$

$$B_2 = 1, \qquad H_2 = 0.1, \qquad D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad D_2 = [0.1 \quad 0.1],$$

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad N_b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

First we assume that $\alpha = 0.2$, $\beta = 0.8$. By Theorem 4.1, choosing $\mu_2 = 3.1$ and $\mu_1 = \frac{2}{\beta}\ln(\mu_2)$, it has been found that the smallest value of γ for the above system to have robust stabilization with disturbance attenuation γ is $\gamma = 0.86$. The corresponding stabilizing control law is given by $u(t) = [-32.9135\ 15.0960]x(t)$.

Next we assume that $\alpha = 0.1$, $\beta = 0.8$. By Theorem 4.1, choosing $\mu_2 = 1.6$ and $\mu_1 = \frac{2}{\beta}\ln(\mu_2)$, it has been found that the smallest value of γ is $\gamma = 0.29$ and the corresponding stabilizing control law is given by $u(t) = [-32.2988\ 8.6274]x(t)$.

6 Conclusion

Three problems for uncertain impulsive systems have been studied, namely, robust stability, robust stabilization, and robust H_∞-control. In each case, the sufficient conditions in terms of linear matrix inequalities have been established. Moreover the method to design a state feedback H_∞ controller is provided. Our method is helpful to improve the existing technologies used in the analysis and control for uncertain impulsive systems. Numerical examples have been provided to demonstrate the effectiveness and applicability of the proposed approach.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

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