Published for SISSA by Springer
Received: January 15, 2016
Revised: May 25, 2016
Accepted: June 8, 2016
Published: June 10, 2016

Supersymmetric Rényi entropy and Weyl anomalies in six-dimensional (2,0) theories

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Abstract: We propose a closed formula of the universal part of supersymmetric Rényi entropy S_q for $(2,0)$ superconformal theories in six-dimensions. We show that S_q across a spherical entangling surface is a cubic polynomial of $\gamma := 1/q$, with all coefficients expressed in terms of the newly discovered Weyl anomalies a and c. This is equivalent to a similar statement of the supersymmetric free energy on conic (or squashed) six-sphere. We first obtain the closed formula by promoting the free tensor multiplet result and then provide an independent derivation by assuming that S_q can be written as a linear combination of 't Hooft anomaly coefficients. We discuss a possible lower bound $\frac{a}{c} \geq \frac{3}{7}$ implied by our result.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory, Holography and condensed matter physics (AdS/CMT)

ArXiv ePrint: 1512.03008

doi:10.1007/JHEP06(2016)064

Contents

1 Introduction

Exact results in interacting quantum field theories are rare. Even less is known about the six-dimensional $(2,0)$ theories, although they are the local conformal field theories (CFTs) with maximal supersymmetry in the maximum number of dimensions [1, 2], which actually play important roles in understanding lower dimensional supersymmetric physics [3–7].

The main obstacle is that the proper formulation of the interacting theories is still lacking, for instance in the path integral formalism.[1] This also makes it challenging to study the theories in curved spaces. In particular it is unclear how to perform the supersymmetric localization [18–20] directly.

Recently alternative approaches to $6d$ $(2,0)$ theories, such as effective actions on the moduli space and the superconformal bootstrap, are advocated in [21, 22] and in [23, 24], respectively. In particular, the Weyl anomaly coefficients $a_{\mathfrak{g}}$ and $c_{\mathfrak{g}}$ have been determined for the $(2,0)$ superconformal field theory (SCFT) characterized by a Lie algebra $\mathfrak{g}$,[2]

$$\bar{a}_{\mathfrak{g}} := \frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}} = \frac{16}{7} h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}} + r_{\mathfrak{g}} \,, \quad \bar{c}_{\mathfrak{g}} := \frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}} = 4 h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}} + r_{\mathfrak{g}} \,, \tag{1.1}$$

where $r_{\mathfrak{g}}$, $d_{\mathfrak{g}}$ and $h^{\vee}_{\mathfrak{g}}$ are the rank, dimension and dual Coxeter number of the compact simply-laced Lie algebra $\mathfrak{g}$, respectively. a and c appear generally as coefficients of the anomalous trace of the stress tensor in a six-dimensional curved background [25, 26],

$$\langle T^{\mu}_{\mu} \rangle \sim a\, E_6 + \sum_{i=1}^{3} c_i \, I_i \,, \tag{1.2}$$

where E_6 is the Euler density while I_i are Weyl invariants. In the presence of $(2,0)$ superconformal symmetry, $c_{i=1,2,3}$ are proportional to a single coefficient c. One interesting fact is that both $\bar{a}_{\mathfrak{g}}$ and $\bar{c}_{\mathfrak{g}}$ will be uniquely fixed once we assume that they are linear combinations of the 't Hooft anomaly coefficients, $h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}}$ and $r_{\mathfrak{g}}$. This can be done by fitting to the large N values (from holography [27–30]) and the free tensor multiplet values [31, 32].

As robust observables, the 't Hooft anomalies of the continuous global symmetries in $6d$ $(2,0)$ theories have been worked out [33–39]. They are organized in an 8-form anomaly polynomial,

$$\mathcal{I}_8 = h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}} \frac{p_2(R)}{24} + r_{\mathfrak{g}} \, \mathcal{I}_{\mathfrak{u}(1)} \,, \tag{1.3}$$

where $p_2(R)$ is the second Pontryagin class of the field strength of the SO(5) R-symmetry background and $\mathcal{I}_{\mathfrak{u}(1)}$ is the anomaly polynomial of a free Abelian tensor multiplet.

As in other even dimensions, it is known that $a_{\mathfrak{g}}$ determines both the universal part[3] of the sphere partition function and the universal entanglement entropy associated with a spherical entangling surface (in flat space) [40]. On the other hand, it was pointed out that $c_{\mathfrak{g}}$ determines both the 2-point and the 3-point functions of the stress tensor in the vacuum in flat space [23, 24]. Due to the intrinsic relations between the flat space stress tensor correlators and the nearly-round sphere partition function, it is therefore attempting to ask whether one can fully determine the partition function on a branched (q-deformed) sphere,[4] which is directly related to the supersymmetric Rényi entropy S_q.

[1] For the attempts to write down a Lagrangian, see for instance [8–12] and for other field theoretical attempts, see [13–17].

[2] $\mathfrak{g} = \mathfrak{u}(1)$ corresponds to a free Abelian tensor multiplet.

[3] By "universal" we mean scheme-independent.

[4] A branched sphere is a sphere with a conical singularity with the deformation parameter $q-1$.

Supersymmetric Rényi entropy was first introduced in three-dimensions [41–43], and later studied in four-dimensions [44, 45, 47], five-dimensions [48, 49] and for free tensor multiplets in six-dimensions [50].[5] By turning on certain R-symmetry background fields (chemical potentials), one can calculate the partition function Z_q on a q-branched sphere $\mathbb{S}^d_q$, and define the supersymmetric Rényi entropy as

$$S_q = \frac{1}{1-q}\left[\log Z_q(\mu(q)) - q\log Z_1(0)\right] , \tag{1.4}$$

which is a supersymmetric refinement of the ordinary Rényi entropy (which is non-supersymmetric because of the conical singularity).[6] The quantities defined in (1.4) are UV divergent in general but one can extract universal parts free of ambiguities. For instance, for $\mathcal{N}=4$ SYM in four-dimensions, the log coefficient of S_q as a function of q and three chemical potentials μ_1, μ_2, μ_3 (corresponding to three independent R-symmetry Cartans) has been shown to be protected from the interactions [44]. It also receives a precise check from the holographic computation on the 5d BPS STU topological black holes [44]. Furthermore, there are universal relations between the Weyl anomaly coefficients a, c and the supersymmetric Rényi entropy in 4d $\mathcal{N}=1,2$ SCFTs, which provides a new way to understand the Hofman-Maldacena bounds [45].[7] The above facts indicate that the supersymmetric Rényi entropy may be used as a new robust observable to understand SCFTs.

1.1 Summary of results

The main result in this paper is the exact supersymmetric Rényi entropy of 6d (2,0) SCFTs. We show that, for theories characterized by simply-laced Lie algebra $\mathfrak{g}$, it is given by a cubic polynomial of $\gamma := \frac{1}{q}$

$$S_\gamma[\mathfrak{g}] = \sum_{n=0}^{3} s_n(\gamma-1)^n , \tag{1.5}$$

with four coefficients

$$s_0 = \frac{7}{12}\,\bar{a}_{\mathfrak{g}} , \quad s_1 = \frac{1+2r_1r_2}{12}\,\bar{c}_{\mathfrak{g}} , \quad s_2 = \frac{r_1r_2}{12}\,\bar{c}_{\mathfrak{g}} , \quad s_3 = \frac{r_1^2r_2^2}{12}\,\frac{7\bar{a}_{\mathfrak{g}}-3\bar{c}_{\mathfrak{g}}}{4} , \tag{1.6}$$

where $\bar{a}_{\mathfrak{g}}, \bar{c}_{\mathfrak{g}}$ are given by (1.1) and $r_{1,2}$ are background parameters denoting the weights of the two U(1) chemical potentials associated to the two R-symmetry Cartans, satisfying the supersymmetry constraint $r_1 + r_2 = 1$.[8] The basic ingredients in our argument are the following:

(A) S_γ has been computed for the free tensor multiplet by the author with collaborators [50]. The result takes the form (2.15)

$$S_\gamma[\mathfrak{u}(1)] = \frac{r_1^2r_2^2}{12}(\gamma-1)^3 + \frac{r_1r_2}{12}(\gamma-1)^2 + \frac{1+2r_1r_2}{12}(\gamma-1) + \frac{7}{12} . \tag{1.7}$$

This will be reviewed in section 2.

[5] The supersymmetric Rényi entropy was recently studied in two-dimensional (2,2) SCFTs [51] in a slightly different way.

[6] For CFTs, the Rényi entropy (or supersymmetric one) associated with a spherical entangling surface in flat space can be mapped to that on a sphere. Throughout this work we take the "regularized cone" boundary conditions.

[7] Some of a/c bounds by Hofman and Maldacena [46] coincide with Rényi entropy inequalities.

[8] We only consider non-negative weights of the chemical potentials, $r_1 \geq 0$ and $r_2 \geq 0$.

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(B) Based on (A) and (E)(F) below, a reasonable assumption is that the general form of supersymmetric Rényi entropy for $(2,0)$ theories is a cubic polynomial in $\gamma-1$. This assumption will be used in section 3. So far we do not have a sharp argument for this assumption.

(C) The first and second derivatives of S_γ at $\gamma=1$ can be expressed in terms of integrated two- and three-point functions of operators in the stress tensor multiplet, and so are proportional to $c_{\mathfrak{g}}$. This will be demonstrated in appendix A. Moreover their dependence on $r_{1,2}$ is seen to be universal. Because of this, one has

$$\left.\frac{\partial_\gamma S_\gamma[\mathfrak{g}]}{\partial_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1}=\left.\frac{\partial_\gamma^2 S_\gamma[\mathfrak{g}]}{\partial_\gamma^2 S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1}=\frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}}\ . \tag{1.8}$$

This will be used in section 3.

(D) The value of S_γ at $\gamma=1$ is the spherical entanglement entropy [41], which is proportional to $a_{\mathfrak{g}}$ [40]. As such, one has

$$\frac{S_{\gamma=1}[\mathfrak{g}]}{S_{\gamma=1}[\mathfrak{u}(1)]}=\frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}}\ . \tag{1.9}$$

This will also be used in section 3.

(E) The large γ behavior of the supersymmetric Rényi entropy is controlled by the "supersymmetric Casimir energy", which has been computed in [82].[9] This gives (4.26)

$$\lim_{\gamma\to\infty}\frac{S_\gamma[\mathfrak{g}]}{\gamma^3}=\frac{r_1^2r_2^2}{12}(r_{\mathfrak{g}}+h_{\mathfrak{g}}^\vee d_{\mathfrak{g}})\ . \tag{1.10}$$

This is our main result in section 4.

(F) S_γ can be computed for the A_{N-1} type $(2,0)$ theories at large N using the AdS/CFT correspondence, with the result appearing in (5.19):

$$\frac{S_\gamma[A_{N\to\infty}]}{N^3}=\frac{r_1^2r_2^2}{12}(\gamma-1)^3+\frac{r_1r_2}{3}(\gamma-1)^2+\frac{1+2r_1r_2}{3}(\gamma-1)+\frac{4}{3}\ . \tag{1.11}$$

This is our main result in section 5.

From (A)(B)(C)(D)(E) listed above, one can uniquely determine the general expression of $S_\gamma[\mathfrak{g}]$ given in (1.5), (1.6), as we do in section 3. The precise agreement between (F) and the large N limit of (1.5) for A_{N-1} type theories can be considered as a nontrivial test of our result. Except for (A), (B)-(F) are new as far as we know.

One may notice that both our result (1.5) and a,c anomalies (1.1) are linear combinations of $h_{\mathfrak{g}}^\vee d_{\mathfrak{g}}$ and $r_{\mathfrak{g}}$, which determine the anomaly polynomial (1.3). In fact, once such a relationship between the supersymmetric Rényi entropy and 't Hooft anomalies is assumed, (1.5) and (1.6) can be obtained by fitting to the known result of a free tensor multiplet [50] and the holographic result presented in section 5.

[9] Similar relation was first advertised in [45] in four-dimensions.

This paper is organized as follows. We begin with the review of the supersymmetric Rényi entropy of free tensor multiplets in section 2. Then we promote it to a general expression which works for general $(2,0)$ theories in section 3. In section 4 we demonstrate a relation between the $q \to 0$ behavior of supersymmetric Rényi entropy and supersymmetric Casimir energy, which is used to determine the remaining unfixed coefficient in the general formula in the previous section. We give a precise test of our result by comparing with the holographic results in section 5 and conjecture a lower bound for $\bar{a}/\bar{c}$ in section 6.

2 Review of abelian tensor multiplet

The six-dimensional $(2,0)$ superconformal algebra is $\mathfrak{osp}(8^*|4)$. While it is easy to identify a free Abelian tensor multiplet that realizes the $(2,0)$ superconformal symmetry, the existence of interacting $(2,0)$ theories was only inferred from decoupling limits of string constructions [62–64]. See for instance [65] for a review of various aspects of $6d$ $(2,0)$ theories.

Now we review the supersymmetric Rényi entropy of free tensor multiplets [50]. For free fields, the Rényi entropy associated with a spherical entangling surface in flat space can be computed by working on a hyperbolic space $\mathbb{S}^1_\beta \times \mathbb{H}^5$ and using heat kernel method.[10] A six-dimensional $(2,0)$ tensor multiplet includes 5 real scalars, 2 Weyl fermions and a 2-form field with self-dual strength. The 2-form field with self-dual strength can be considered as a chiral 2-form field with half of the degrees of freedom.

2.1 Heat kernel

The partition function of free fields on $\mathbb{S}^1_{\beta=2\pi q} \times \mathbb{H}^5$ can be obtained by heat kernel method,[11]

$$\log Z(\beta) = \frac{1}{2}\int_0^\infty \frac{dt}{t} K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t)\,, \tag{2.1}$$

where $K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t)$ is the heat kernel of the associated conformal Laplacian. The kernel can be factorized when the spacetime is a direct product,

$$K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t) = K_{\mathbb{S}^1_\beta}(t)\, K_{\mathbb{H}^5}(t)\ . \tag{2.2}$$

The kernel on a circle $K_{\mathbb{S}^1_\beta}(t)$ is known to be[12]

$$K_{\mathbb{S}^1_\beta}(t) = \frac{\beta}{\sqrt{4\pi t}} \sum_{n\neq 0,\in\mathbb{Z}} e^{\frac{-\beta^2 n^2}{4t}}\ . \tag{2.3}$$

In the presence of a chemical potential μ, it is twisted to be [55]

$$\widetilde{K}_{\mathbb{S}^1_\beta}(t) = \frac{\beta}{\sqrt{4\pi t}} \sum_{n\neq 0,\in\mathbb{Z}} e^{\frac{-\beta^2 n^2}{4t}+i2\pi n\mu+i\pi n f}\,, \tag{2.4}$$

[10] Six-dimensional $(2,0)$ theories have been studied in $AdS_5 \times S^1$ recently in the viewpoint of rigid holography [66].

[11] For Rényi entropy of free fields in other higher dimensions, see for instance [67–70].

[12] For fermions, the boundary conditions are anti-periodic.

where $f = 0$ for scalars and $f = 1$ for fermions. Finally the kernels on the hyperbolic space $K_{\mathbb{H}^5}(t)$ can be written as follows because $\mathbb{H}^5$ is homogeneous,

$$K_{\mathbb{H}^5}(t) = \int d^5x\sqrt{g}\; K_{\mathbb{H}^5}(x,x,t) = V_5\, K_{\mathbb{H}^5}(0,t)\ . \tag{2.5}$$

The regularized volume $V_5 = \pi^2 \log(\ell/\epsilon)$. ϵ is the UV cutoff of the theory in the original space[13] and ℓ is the curvature radius of $\mathbb{H}^5$. Note that the kernels $K_{\mathbb{H}^5}(0,t)$ for free fields with different spins are known. See [50] and references there.

2.2 Rényi entropy

The total Rényi entropy of a tensor multiplet can be obtained by summing up the contributions of 5 real scalars, 2 Weyl fermions and a chiral 2-form,

$$S_q^{\text{free}} = 5 \times \frac{S_q^s}{2} + 2S_q^f + \frac{S_q^v}{2}\,, \tag{2.6}$$

where the Rényi entropy for fields with different spins can be computed by using the corresponding heat kernels. For the details of this computation we refer to [50]. We will instead list the results here. The Rényi entropy of a $6d$ real scalar is

$$S_q^s = \frac{(q+1)\left(3q^2+1\right)\left(3q^2+2\right)}{15120q^5}\frac{V_5}{\pi^2}\,, \tag{2.7}$$

and the Rényi entropy of a $6d$ Weyl fermion is

$$S_q^f = \frac{(q+1)\left(1221q^4+276q^2+31\right)}{120960q^5}\frac{V_5}{\pi^2}\,, \tag{2.8}$$

and that of a $6d$ 2-from field is

$$S_q^v = \frac{(q+1)\left(37q^2+2\right)+877q^4+4349q^5}{5040q^5}\frac{V_5}{\pi^2}\ . \tag{2.9}$$

It is worth to mention that, to get the correct Rényi entropy for the two form field, one has to take into account a q-independent constant shift due to the edge modes [50], like what should be done for the gauge field in $4d$ [71, 72]. Finally the Rényi entropy for a free $(2,0)$ tensor multiplet is

$$S_q^{\text{free}} = \frac{(q+1)(28q^2+3)+313q^4+1305q^5}{2880q^5}\frac{V_5}{\pi^2}\ . \tag{2.10}$$

It has been checked that $\partial^0_{q=1}$, $\partial^1_{q=1}$ and $\partial^2_{q=1}$ of S_q^{free} are consistent [50] with the previous results about the tensor multiplet [31, 32, 73].

[13] This is the q-fold space with a conical singularity, which is used to compute Rényi entropy by replica trick.

2.3 S_q and S_γ

Before moving on, let us represent S_q^{free} in terms of

$$S_\gamma := \frac{\pi^2}{V_5} S_q\,, \text{ with } \gamma := 1/q\,,$$

$$S_\gamma^{\text{free}} = \frac{1}{960}(\gamma-1)^5 + \frac{1}{160}(\gamma-1)^4 + \frac{7}{288}(\gamma-1)^3 + \frac{1}{18}(\gamma-1)^2 + \frac{\gamma-1}{6} + \frac{7}{12}\,. \tag{2.11}$$

The reason why S_γ is convenient is that, the series expansion near $\gamma = 1$ has finite terms while the expansion of S_q near $q = 1$ has infinite terms. We will use S_γ instead of S_q to express Rényi entropy and supersymmetric Rényi entropy from now on. It is worth to note the relations between the derivatives with respect to q and the derivatives with respect to γ at $q = 1/\gamma = 1$,

$$\partial_\gamma S_\gamma = -\partial_q S_q\Big|_{q=1/\gamma=1} \cdot \frac{\pi^2}{V_5}\,, \quad \partial_\gamma^2 S_\gamma = \left(2\partial_q S_q + \partial_q^2 S_q\right)\Big|_{q=1/\gamma=1} \cdot \frac{\pi^2}{V_5}\,. \tag{2.12}$$

2.4 Supersymmetric Rényi entropy

The supersymmetric Rényi entropy of a free tensor multiplet can be computed by the twisted kernel (2.4) on the supersymmetric background. The R-symmetry group of $6d$ $(2,0)$ theories is SO(5), which has two U(1) Cartans. Therefore one can turn on two independent R-symmetry background gauge fields (chemical potentials) to twist the boundary conditions for scalars and fermions along the replica circle $\mathbb{S}^1_\beta$. A general analysis of the Killing spinor equation on the conic space ($\mathbb{S}^6_q$ or $\mathbb{S}^1_{\beta=2\pi q}\times\mathbb{H}^5$) leads to the solution of the R-symmetry chemical potential [50][14]

$$\mu(q) := k_i A^i = \frac{q-1}{2}\,, \tag{2.13}$$

with k_1 and k_2 being the R-charges of the Killing spinor under the two U(1) Cartans, respectively. We choose $k_1 = k_2 = \frac{1}{2}$ and the two background fields can be expressed as

$$A^1 = (q-1)\,r_1\,, \quad A^2 = (q-1)\,r_2\,, \quad \text{with } \ r_1 + r_2 = 1\,. \tag{2.14}$$

This is the most general background satisfying (2.13). For each component field in the tensor multiplet, one has to first figure out the Cartan charges k_1 and k_2 and then compute the chemical potential by $k_1A^1 + k_2A^2$. Then one can compute the free energy on $\mathbb{S}^1_\beta\times\mathbb{H}^5$ using the twisted heat kernel and get the supersymmetric Rényi entropy. For details, see [50].

After summing up all the component fields, the final supersymmetric Rényi entropy in terms of γ can be expressed as,[15]

$$S_\gamma[\mathfrak{u}(1)] = \frac{1}{12}r_1^2r_2^2(\gamma-1)^3 + \frac{1}{12}r_1r_2(\gamma-1)^2 + \frac{1}{12}(1+2r_1r_2)(\gamma-1) + \frac{7}{12}\,. \tag{2.15}$$

[14]The Killing spinors on round sphere have been explored in [74].

[15]Although the form of this expression is a series expansion, the result itself is complete.

It is worth to note that, for a single U(1) background, $r_1 = 1, r_2 = 0$, the result becomes

$$S_\gamma = \frac{1}{12}(\gamma + 6)\,, \tag{2.16}$$

while for two U(1) backgrounds with equal values, $r_1 = r_2 = \frac{1}{2}$, we have

$$S_\gamma = \frac{1}{192}(\gamma-1)^3 + \frac{1}{48}(\gamma-1)^2 + \frac{1}{8}(\gamma-1) + \frac{7}{12}\,. \tag{2.17}$$

3 Interacting (2,0) theories

Given the supersymmetric Rényi entropy (2.15) for a free tensor multiplet, now we promote it to a general form which works for general $(2,0)$ SCFTs,

$$S_\gamma[\mathfrak{g}] = \frac{r_1^2 r_2^2}{12}\cdot A\,(\gamma-1)^3 + \frac{r_1 r_2}{12}\cdot B\,(\gamma-1)^2 + \frac{1+2r_1 r_2}{12}\cdot C\,(\gamma-1) + \frac{7}{12}D\,, \tag{3.1}$$

where the coefficients $A\,, B\,, C\,, D$ will depend on the specific theory. As stated in the introduction, the assumption that $S_\gamma[\mathfrak{g}]$ is a cubic polynomial of $\gamma - 1$ is based on both the free multiplet result and the holographic result (as we will see in section 5).[16] Their dependence on $r_{1,2}$ is universal because $r_{1,2}$ originally come from the α_i ($\alpha_1 = r_1, \alpha_2 = r_2$) in (A.23), (A.27), which are background parameters independent of the specific theory. Later we will see that precisely the same $r_{1,2}$ dependence appears in the holographic supersymmetric Rényi entropy (5.19), which confirms this fact.

3.1 $S_{\gamma=1}$ and $a_\mathfrak{g}$

We would like to first determine the coefficient D in (3.1). Recall that the entanglement entropy associated with a spherical entangling surface, $S_{\gamma=1}$, is proportional to the a-type Weyl anomaly. This is true for general CFTs in even dimensions as shown in [40]. Therefore

$$\frac{S_{\gamma=1}[\mathfrak{g}]}{S_{\gamma=1}[\mathfrak{u}(1)]} = \frac{a_\mathfrak{g}}{a_{\mathfrak{u}(1)}}\,. \tag{3.2}$$

This allows us to fix

$$D = \frac{a_\mathfrak{g}}{a_{\mathfrak{u}(1)}} = \frac{16}{7}h^\vee_\mathfrak{g} d_\mathfrak{g} + r_\mathfrak{g}\,, \tag{3.3}$$

where we have used the a-type Weyl anomaly result in $6d$ $(2,0)$ theories [22].

[16]Similar thing happens in $\mathcal{N} = 4$ SYM. Here we see an essential difference between the ordinary Rényi entropy and the supersymmetric one, because the type of q scaling in the ordinary Rényi entropy is not protected [53, 75].

3.2 $\partial_\gamma S_{\gamma=1}$, $\partial^2_\gamma S_{\gamma=1}$ and $c_{\mathfrak{g}}$

The coefficients C and B in (3.1) are determined by the first and the second γ-derivatives of S_γ at $\gamma = 1$, respectively. γ-derivatives can be translated into q-derivatives. Taking q-derivatives is equal to taking derivatives with respect to background fields, therefore $\partial_\gamma S_{\gamma=1}$ and $\partial^2_\gamma S_{\gamma=1}$ are intrinsically related to the corresponding correlators. This is illustrated in appendix A.

More explicitly, the first γ-derivative (which is minus the q-derivative at $q = 1/\gamma = 1$) is determined by a linear combination of the integrated stress tensor 2-point function and the integrated R-current 2-point function. The first q-derivative at $q = 1$ is given by the equation (A.23),

$$S'_{q=1} = -V_{d-1}\left(\frac{\pi^{\frac{d}{2}+1}\Gamma\left(\frac{d}{2}\right)(d-1)}{(d+1)!}C_T - \alpha^2\frac{\pi^{\frac{d+3}{2}}}{2^{d-3}(d-1)\Gamma\left(\frac{d-1}{2}\right)}C_v\right), \tag{3.4}$$

which works for general SCFTs with conserved R-symmetries in d-dimensions.

Similarly the second γ-derivative at $\gamma = 1$ is related to q-derivatives by (2.12). The second q-derivative at $q = 1$ is determined by a linear combination of the integrated stress tensor 3-point function, the integrated R-current 3-point function and some mixed 3-point functions. This is given explicitly by (A.27)

$$S''_{q=1} = \frac{1}{6}I'''_{q=1} = \frac{4\pi^3}{3}\left[\langle\hat{E}\hat{E}\hat{E}\rangle^c - \alpha^3\langle\hat{Q}\hat{Q}\hat{Q}\rangle^c - 3\alpha\langle\hat{E}\hat{E}\hat{Q}\rangle^c + 3\alpha^2\langle\hat{E}\hat{Q}\hat{Q}\rangle^c\right]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}}, \tag{3.5}$$

which also works for general SCFTs with conserved R-symmetries in d-dimensions.

In the particular case of $6d$ $(2,0)$ SCFTs, the operators in the above two- and three-point functions stay in the same multiplet, the stress tensor multiplet. Therefore both the first and second derivative of S_γ at $\gamma = 1$ are proportional to the central charge $c_{\mathfrak{g}}$ (1.1), as discussed in detail in appendix A.[17] Because of this, we have

$$\left.\frac{\partial_\gamma S_\gamma[\mathfrak{g}]}{\partial_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1} = \left.\frac{\partial^2_\gamma S_\gamma[\mathfrak{g}]}{\partial^2_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1} = \frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}}. \tag{3.6}$$

This actually means we can fix

$$B = C = 4h^\vee_{\mathfrak{g}}d_{\mathfrak{g}} + r_{\mathfrak{g}}. \tag{3.7}$$

The remaining coefficient A will be fixed by (4.26) in section 4

$$A = h^\vee_{\mathfrak{g}}d_{\mathfrak{g}} + r_{\mathfrak{g}} \tag{3.8}$$

by studying the asymptotic $q := 1/\gamma \to 0$ behavior of the supersymmetric Rényi entropy. Obviously, the leading contribution in the limit $\gamma \to \infty$ is controlled only by A.

[17] This actually explains the universal ratio $4N^3$ between the explicit results on $\langle TT\rangle$, $\langle TTT\rangle$, $\langle JJ\rangle$, $\langle JJJ\rangle$ in holography and those in free tensor multiplets [32, 76].

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3.3 A closed formula

As a summary, we can uniquely determine a closed formula of supersymmetric Rényi entropy for $(2,0)$ SCFTs characterized by simply-laced Lie algebra $\mathfrak{g}$

$$
\begin{aligned}
S_\gamma[\mathfrak{g}] &= \frac{r_1^2 r_2^2}{12}(h^\vee_\mathfrak{g} d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1)^3 + \frac{r_1 r_2}{12}(4h^\vee_\mathfrak{g} d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1)^2 \\
&\quad + \frac{1+2r_1 r_2}{12}(4h^\vee_\mathfrak{g} d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1) + \left(\frac{4h^\vee_\mathfrak{g} d_\mathfrak{g}}{3} + \frac{7r_\mathfrak{g}}{12}\right), \qquad (3.9)\\
&= \frac{r_1^2 r_2^2}{48}(7\bar{a}_\mathfrak{g} - 3\bar{c}_\mathfrak{g})(\gamma-1)^3 + \frac{r_1 r_2}{12}\,\bar{c}_\mathfrak{g}\,(\gamma-1)^2 + \frac{1+2r_1 r_2}{12}\,\bar{c}_\mathfrak{g}\,(\gamma-1) + \frac{7}{12}\,\bar{a}_\mathfrak{g}\,, \qquad (3.10)
\end{aligned}
$$

where in the last line we have used the normalized Weyl anomalies defined in (1.1).

For a single U(1) chemical potential,

$$
r_1 = 1\,, \quad r_2 = 0\,, \tag{3.11}
$$

the result is simplified to be

$$
\begin{aligned}
S_\gamma[\mathfrak{g}] &= \frac{1}{12}\,\bar{c}_\mathfrak{g}\,(\gamma-1) + \frac{7}{12}\,\bar{a}_\mathfrak{g}\,, \qquad (3.12)\\
&= h^\vee_\mathfrak{g} d_\mathfrak{g}\left(\frac{1}{3}\gamma + 1\right) + r_\mathfrak{g}\frac{(\gamma+6)}{12}\,. \qquad (3.13)
\end{aligned}
$$

As for two U(1) chemical potentials with equal values,

$$
r_1 = r_2 = \frac{1}{2}\,, \tag{3.14}
$$

the result is simplified to be

$$
\begin{aligned}
S_\gamma[\mathfrak{g}] &= \frac{1}{192\times 4}(7\bar{a}_\mathfrak{g} - 3\bar{c}_\mathfrak{g})(\gamma-1)^3 + \frac{1}{48}\,\bar{c}_\mathfrak{g}\,(\gamma-1)^2 + \frac{1}{8}\,\bar{c}_\mathfrak{g}\,(\gamma-1) + \frac{7}{12}\,\bar{a}_\mathfrak{g}\,, \qquad (3.15)\\
&= \frac{175+67\gamma+13\gamma^2+\gamma^3}{192}h^\vee_\mathfrak{g} d_\mathfrak{g} + \frac{91+19\gamma+\gamma^2+\gamma^3}{192}r_\mathfrak{g}\,. \qquad (3.16)
\end{aligned}
$$

4 $q \to 0$ asymptotics

In this section we discuss the relation between the $q \to 0$ limit ($\gamma \to \infty$) of supersymmetric Rényi entropy S_q and supersymmetric Casimir energy. Recall the definition of S_q

$$
S_q = \frac{qI_1 - I_q}{1-q}\,. \tag{4.1}
$$

Assuming that in the limit $q \to 0$ the free energy behaves

$$
I_q = I_{(0)} q^{-\alpha} + \cdots\,, \tag{4.2}
$$

where $\alpha \geq 0$, one can easily get

$$
S_{q\to 0} = -I_{q\to 0} \tag{4.3}
$$

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in the leading order. This relation does not depend on which geometric background we are working on.

The idea is that, $\mathbb{S}^d_q$ can be conformally mapped to $\mathbb{H}^1 \times \mathbb{S}^{d-1}_q$, therefore the Rényi entropy (or supersymmetric) is invariant [40]. In the case with supersymmetry, one has to make sure that in the limit $q \to 0$, the background field on $\mathbb{S}^d_q$ coincides with that on $\mathbb{H}^1 \times \mathbb{S}^{d-1}_q$. If that is the case, the asymptotic supersymmetric Rényi entropy $S_{q\to 0}$ on $\mathbb{S}^d_q$ will coincide with the minus free energy on $\mathbb{H}^1 \times \mathbb{S}^{d-1}_{q\to 0}$. The latter is determined by the supersymmetric Casimir energy [77]. We will illustrate the details in the following.

4.1 From $\mathbb{S}^d_q$ to $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$

We start with the conformal transformation from conic sphere $\mathbb{S}^d_q$ to hyperbolic space $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$. Of course $\mathbb{S}^d_q$ can be considered as the special case of $p = d$.

In the particular case $p = 1$, the transformation is nothing but the Weyl transformation discussed in [40], which offers a convenient way to compute Rényi entropy of CFTs. In this case, the branched d-sphere is described as[18]

$$\mathrm{d}s^2 = \sin^2\theta q^2 \mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}^2\Omega_{d-2}\,, \tag{4.4}$$

with domains of coordinates given by

$$\tau \in [0, 2\pi)\,, \quad \theta \in \left[0, \frac{\pi}{2}\right]\,, \tag{4.5}$$

and Ω_{d-2} is a standard d-2-dimensional round sphere. The metric (4.4) can be written as

$$\mathrm{d}s^2 = \sin^2\theta \left(q^2\mathrm{d}\tau^2 + \frac{1}{\sin^2\theta}\mathrm{d}\theta^2 + \cot^2\theta \mathrm{d}^2\Omega_{d-2}\right)\,, \tag{4.6}$$

which can be related to the following space by dropping an overall factor $\sin^2\theta$ and using a coordinate transformation $\cot\theta = \sinh\eta$

$$\mathrm{d}s^2 = q^2\mathrm{d}\tau^2 + \mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}^2\Omega_{d-2}\,, \tag{4.7}$$

where $\eta \in [0, +\infty)$. This is the space of $\mathbb{H}^{d-1} \times \mathbb{S}^1_q$, which indeed fits the case of $p = 1$.

Now we consider the general cases, $1 \le p < d$. The key observation is that, the branched sphere can be presented in different forms. For instance, we can represent $\mathbb{S}^d_q$ as

$$\mathrm{d}s^2 = \sin^2\theta(\mathrm{d}\chi^2 + \sin^2\chi q^2\mathrm{d}\tau^2) + \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}^2\Omega_{d-3}\,, \tag{4.8}$$

with domains

$$\chi \in [0, \pi]\,, \quad \tau \in [0, 2\pi)\,, \quad \theta \in \left[0, \frac{\pi}{2}\right]\,, \tag{4.9}$$

and Ω_{d-3} is a standard d-3-dimensional round sphere. Again by dropping an overall factor $\sin^2\theta$ and using a coordinate transformation $\cot\theta = \sinh\eta$ for the metric (4.8), one obtains

$$\mathrm{d}s^2 = \mathrm{d}\chi^2 + \sin^2\chi q^2\mathrm{d}\tau^2 + \mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}^2\Omega_{d-3}\,, \tag{4.10}$$

[18] We normalize the radius as unit.

which is the space $\mathbb{H}^{d-2} \times \mathbb{S}^2_q$ with $p = 2$. One can follow the same way to eventually figure out the Weyl transformations between $\mathbb{S}^d_q$ and $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$ for any integer $1 \leq p < d$.

Since the Rényi entropy on $\mathbb{S}^d_q$ can not depend on which particular circle we choose to create the conical singularity, one eventually arrives at the conclusion by employing the same argument in [40]:[19]

The universal part of CFT$_d$ Rényi entropy is invariant on $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$ for different integer p, where $1 \leq p \leq d$.

For later purpose, let us discuss the particular case $p = d-1$. In this case we describe the branched sphere $\mathbb{S}^d_q$ as

$$\mathrm{d}s^2 = \sin^2\theta(\mathrm{d}\chi^2 + \sin^2\chi q^2 \mathrm{d}\tau^2 + \cos^2\chi \mathrm{d}^2\Omega_{d-3}) + \mathrm{d}\theta^2\,, \tag{4.11}$$

with domains

$$\chi \in \left[0, \frac{\pi}{2}\right]\,, \quad \tau \in [0, 2\pi)\,, \quad \theta \in [0, \pi]\ . \tag{4.12}$$

Again by dropping an overall factor $\sin^2\theta$ for the metric (4.11), one obtains

$$\mathrm{d}s^2 = \mathrm{d}\chi^2 + \sin^2\chi q^2 \mathrm{d}\tau^2 + \cos^2\chi \mathrm{d}^2\Omega_{d-3} + \mathrm{d}\eta^2\,, \tag{4.13}$$

where $\cot\theta = \sinh\eta$ and $\eta \in (-\infty, +\infty)$. This is the space $\mathbb{S}^{d-1}_q \times \mathbb{H}^1$. Here we use $\mathbb{H}^1$ instead of $\mathbb{R}^1$ to emphasize that the volume of $\mathbb{H}^d$ may be regularized. For free fields, one can compute the CFT Rényi entropy on $\mathbb{S}^{d-1}_q \times \mathbb{H}^1$ and show explicitly that the result agrees with that computed from $\mathbb{S}^d_q$ or $\mathbb{S}^1_q \times \mathbb{H}^{d-1}$. In consideration of supersymmetry, one has to add a background field A_τ along the replica τ circle inside $\mathbb{S}^{d-1}_q$, in order to find the agreement.

4.2 Coincidence of backgrounds

Our main concern is physical quantities for CFTs. For this purpose we can work on $\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{H}^1_{1/\sqrt{q}}$ instead of $\mathbb{S}^{d-1}_q \times \mathbb{H}^1$ because they are related by a scale transformation

$$\frac{1}{\sqrt{q}}[\mathbb{S}^{d-1}_q \times \mathbb{H}^1] = [\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{H}^1_{1/\sqrt{q}}]\ . \tag{4.14}$$

Furthermore, we focus on the limit $q \to 0$. For this purpose, one can instead consider $\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{S}^1_{1/\sqrt{q}}$ because it is equivalent to $\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{H}^1_{1/\sqrt{q}}$ in the limit $q \to 0$

$$\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{H}^1_{1/\sqrt{q}}\Big|_{q\to 0} = \mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{S}^1_{1/\sqrt{q}}\Big|_{q\to 0}\ . \tag{4.15}$$

In consideration of supersymmetry, one can use the squashed sphere $\widetilde{\mathbb{S}}^{d-1}_{\sqrt{q}}$ to replace the conic sphere $\mathbb{S}^{d-1}_{\sqrt{q}}$ in the right hand side of (4.15), because supersymmetric partition functions do not depend on the resolving factor [42, 49, 78–81].[20] Eq. (4.15) is useful in the

[19] Again by the universal part of Rényi entropy we refer to the scheme independent part.

[20] For this reason, we will not distinguish d-1-dimensional squashed sphere and conic sphere in the following unless it is necessary.

sense that it offers a way to compute the asymptotic supersymmetric Rényi entropy for interacting SCFTs. To do this, one has to make sure that the background gauge field on $\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{S}^1_{1/\sqrt{q}}$ agrees with that on the original space $\mathbb{S}^d_q$. Fortunately we have more knowledge about supersymmetric partition functions on $\mathbb{S}^{d-1} \times \mathbb{S}^1$ or its generalized version $\mathbb{S}^{d-1}_b \times \mathbb{S}^1_\beta$, where b is the squashing parameter.

4.3 Squashed Casimir energy

Now we make a connection between the asymptotic Rényi entropy and Casimir energy. It is known that the partition function Z on $\mathbb{S}^{d-1}_b \times \mathbb{S}^1_\beta$ is determined by the Casimir energy on $\mathbb{S}^{d-1}_b$ in the limit $\beta \to \infty$

$$E_c := -\lim_{\beta \to \infty} \partial_\beta \log Z(\beta)\,, \tag{4.16}$$

which is equivalent to say

$$\lim_{\beta \to \infty} \log Z(\beta) = -\beta E_c\ . \tag{4.17}$$

In this work, we concern the case with supersymmetry. In the particular case of $6d$ $(2,0)$ theories, the supersymmetric Casimir energy has been studied in [82],[21] where the authors considered a general 5-sphere with squashing parameters $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$. The squashing parameters are defined as parameters appearing in the Killing vector

$$K = \omega_1 \frac{\partial}{\partial \phi_1} + \omega_2 \frac{\partial}{\partial \phi_2} + \omega_3 \frac{\partial}{\partial \phi_3}\,, \tag{4.18}$$

where ϕ_1, ϕ_2, ϕ_3 are three circles representing $\mathrm{U}(1)^3$ isometries of $\mathbb{S}^5$. The supersymmetric Casimir energy of an interacting $(2,0)$ theory is [82]

$$E_{\mathfrak{g}} = r_{\mathfrak{g}} E_{\mathfrak{u}(1)} - d_{\mathfrak{g}} h^\vee_{\mathfrak{g}} \frac{\sigma_1^2 \sigma_2^2}{24\, \omega_1 \omega_2 \omega_3}\,, \tag{4.19}$$

where σ_1 and σ_2 are chemical potentials for the two Cartans of the SO(5) R-symmetry and $E_{\mathfrak{u}(1)}$ is given by

$$E_{\mathfrak{u}(1)} = -\frac{1}{48 \omega_1 \omega_2 \omega_3} \left[\sigma_1^2 \sigma_2^2 - \sum_{i<j} \omega_i^2 \omega_j^2 + \frac{1}{4} \left(\sum_j \omega_j^2 - \sigma_1^2 - \sigma_2^2 \right)^2 \right]\ . \tag{4.20}$$

For the particular case of $\mathbb{S}^5_q \times \mathbb{S}^1$ (which is equivalent to $\mathbb{S}^5_{\sqrt{q}} \times \mathbb{S}^1_{\frac{1}{\sqrt{q}}}$ for CFTs), we should identify the shape parameters as

$$\omega_1 = \omega_2 = 1\,, \quad \omega_3 = \frac{1}{q}\ . \tag{4.21}$$

In the limit $q \to 0$, in order to match our chemical potentials (2.14), we set σ_1 and σ_2 as[22]

$$\sigma_1^2 (q \to 0) = \frac{r_1^2}{q^2}\,, \quad \sigma_2^2 (q \to 0) = \frac{r_2^2}{q^2}\,, \quad \text{with} \ \ r_1 + r_2 = 1\ . \tag{4.22}$$

[21] For the $6d$ $(2,0)$ superconformal index, see [83–85].

[22] The q scalings in chemical potentials appear following the convention in [82].

Evaluating (4.19) we get

$$E_{\mathfrak{g}}\Big|_{q\to 0} = -\frac{1}{24}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee}) \ . \tag{4.23}$$

Therefore the free energy[23]

$$f[\mathbb{S}^5_{q\to 0}\times\mathbb{S}^1] = \frac{1}{\pi^3}\beta E_{\mathfrak{g}}\Big|_{q\to 0} = -\frac{1}{12\pi^2}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee})\,, \tag{4.24}$$

where we have divided a q-independent volume factor Vol $[\mathbb{D}^4\times\mathbb{S}^1] = \pi^3$. Due to (4.15), we have

$$f[\mathbb{S}^5_{q\to 0}\times\mathbb{S}^1] = f[\mathbb{S}^1_{q\to 0}\times\mathbb{H}^5]\,, \tag{4.25}$$

from which we obtain the asymptotic supersymmetric Rényi entropy on $\mathbb{S}^1_q\times\mathbb{H}^5$

$$S_{q\to 0}[\mathfrak{g}] = -I_{q\to 0}[\mathfrak{g}] = \frac{1}{12}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee}) \ . \tag{4.26}$$

This fixes the undetermined coefficient A in (3.1) as

$$A = r_{\mathfrak{g}} + h_{\mathfrak{g}}^{\vee} d_{\mathfrak{g}} \ . \tag{4.27}$$

Notice that the fact that the free limit of (4.26) precisely agrees with the leading large γ term of (2.15) by itself is nontrivial, which confirms the validity of (4.25) in the free case.

5 Large N limit

In the large N limit of the $(2,0)$ theory with $\mathfrak{g} = A_{N-1}$, the supersymmetric Rényi entropy (3.9) becomes

$$\begin{aligned}\frac{S_\gamma[A_{N\to\infty}]}{N^3} &= \frac{1}{12}r_1^2 r_2^2(\gamma-1)^3 + \frac{4}{12}r_1 r_2(\gamma-1)^2 \\ &\quad + \frac{4}{12}(1+2r_1 r_2)(\gamma-1) + \frac{4}{3} \ .\end{aligned} \tag{5.1}$$

We will demonstrate in this section that the above large N result precisely agrees with the holographic result from the seven-dimensional BPS topological black hole in gauged supergravity.

5.1 Gauged supergravity

The seven-dimensional gauged SO(5) supergravity can be obtained by Kaluza-Klein reduction of eleven-dimensional supergravity on $\mathbb{S}^4$. For our purpose, we consider a truncation where only the metic, two gauge fields associated to two Cartans of SO(5) and two scalars are retained. The seven-dimensional Lagrangian is given by [86]

$$\frac{1}{\sqrt{g}}\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{4}{L^2}V - \frac{1}{4}\sum_{i=1}^{2}\frac{1}{X_i^2}\left(F^i_{(2)}\right)^2 \,, \tag{5.2}$$

[23] $f := \frac{I}{V}$.

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where $\vec{\phi} = (\phi_1, \phi_2)$ are two scalars and

$$X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}, i = 1, 2 . \quad \vec{a}_1 = \left(\sqrt{2}, \sqrt{\frac{2}{5}}\right) , \quad \vec{a}_2 = \left(-\sqrt{2}, \sqrt{\frac{2}{5}}\right) . \tag{5.3}$$

The potential is given by

$$V = -4X_1X_2 - 2X_0X_1 - 2X_0X_2 + \frac{1}{2}X_0^2 , \quad X_0 = \frac{1}{X_1X_2} . \tag{5.4}$$

Note that for two equal scalars and two equal gauge strengths, the Lagrangian (5.2) can be further truncated. Turn to the CFT side, 6d (2, 0) theories have global SO(5) R-symmetry, which corresponds to the SO(5) gauge group in the bulk supergravity. Also there could be two U(1) background fields used to compensate the singularity on $\mathbb{S}^6_q$, which correspond to A^1, A^2 in the gauged supergravity.

5.2 Topological black hole

The 2-charge 7d AdS black hole solution for (5.2) was found in [86]

$$\begin{aligned} ds_7^2 &= -\frac{1}{[h_1h_2]^{\frac{4}{5}}} f(r)dt^2 + [h_1h_2]^{\frac{1}{5}} \left(\frac{dr^2}{f(r)} + r^2 d\Omega^2_{5,k}\right) \\ f(r) &= k - \frac{m}{r^4} + \frac{r^2}{L^2}h_1h_2 , \qquad h_i = 1 + \frac{q_i}{r^4} , \end{aligned} \tag{5.5}$$

together with scalars and gauge fields

$$X_i = \frac{[h_1h_2]^{\frac{2}{5}}}{h_i} , \quad A^i = \left[\sqrt{k}\left(\frac{1}{h_i} - 1\right) + \mu_i\right] dt . \tag{5.6}$$

$d\Omega^2_{5,k}$ is the metric on a unit $\mathbb{S}^5$, $\mathbb{T}^5$ or $\mathbb{H}^5$ corresponding to $k = 1, 0, -1$, respectively. Since our concern is the 6d SCFT on $\mathbb{S}^1 \times \mathbb{H}^5$, we are particularly interested in the extremal solution with hyperbolic foliation, where $m = 0$ and $k = -1$. We will first proceed in Lorentz signature and assume a well-defined Wick rotation.

The solution (5.5) is a BPS topological black hole with two charges. For convenience, define a rescaled charge

$$\kappa_i = \frac{q_i}{r_H^4} , \tag{5.7}$$

where the horizon r_H is the largest root of the equation

$$f(r_H) = 0 . \tag{5.8}$$

Then the horizon can be expressed in terms of κ_i

$$r_H = \frac{L}{\sqrt{(1+\kappa_1)(1+\kappa_2)}} . \tag{5.9}$$

The Hawking temperature of this black hole is

$$\begin{aligned} T &= \left.\frac{f'(r)}{4\pi\sqrt{h_1h_2}}\right|_{r=r_H} \\ &= \frac{1 - \kappa_1 - \kappa_2 - 3\kappa_1\kappa_2}{2\pi L(1+\kappa_1)(1+\kappa_2)} . \end{aligned} \tag{5.10}$$

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When all charges vanish, we get to the temperature of the uncharged black hole

$$T_0 = \frac{1}{2\pi L} \ . \tag{5.11}$$

The Bekenstein-Hawking entropy is given by the outer horizon area

$$S = \frac{V_5 L^5}{4G_7} \frac{1}{(1+\kappa_1)^2(1+\kappa_2)^2} \,, \tag{5.12}$$

where G_7 is the seven dimensional Newton constant and V_5 is the regularized volume of $\mathbb{H}^5$. The total charge Q_i can be computed by Gauss law

$$\begin{aligned} Q_i &= \frac{1}{16\pi G_7} \int_{r\to\infty} -\sqrt{g} F^{rt} = \frac{V_5}{4\pi G_7} i q_i \\ &= \frac{V_5 L^4}{4\pi G_7} \frac{i\kappa_i}{(1+\kappa_1)^2(1+\kappa_2)^2} \ . \end{aligned} \tag{5.13}$$

The chemical potential is

$$\mu_i = \frac{i}{\kappa_i^{-1}+1} \ . \tag{5.14}$$

5.3 Precise check

To match the background gauge fields of the boundary CFT, we set

$$\mu_1 = i(1-\gamma)\frac{r_1}{2} \,, \quad \mu_2 = i(1-\gamma)\frac{r_2}{2} \,, \quad \text{with} \quad r_1 + r_2 = 1 \ . \tag{5.15}$$

By using these inputs, we can solve κ_1 and κ_2 by (5.14). Then all physical quantities T, S, Q_i can be worked out explicitly. One can eventually compute the holographic supersymmetric Rényi entropy using the formula derived in [42]

$$S_q = \frac{q}{1-q} \int_q^1 \left(\frac{S(n)}{n^2} - \frac{Q_i(n)\mu_i'(n)}{T_0} \right) \mathrm{d}n \ . \tag{5.16}$$

Written in terms of $\gamma := 1/q$, the result is given by

$$S_\gamma = \frac{L^5\pi^2}{4G_7} \left[\frac{r_1^2 r_2^2 (\gamma-1)^3}{16} + \frac{(1+2r_1r_2)(\gamma-1)}{4} + \frac{(\gamma-1)^2 r_1 r_2}{4} + 1 \right] \ . \tag{5.17}$$

By identifying the bulk and boundary parameters,

$$\frac{L^5\pi^2}{4G_7} = \frac{4}{3} N^3 \,, \tag{5.18}$$

one can write the holographic result as

$$S_\gamma = N^3 \left(\frac{r_1^2 r_2^2 (\gamma-1)^3}{12} + \frac{(1+2r_1r_2)(\gamma-1)}{3} + \frac{(\gamma-1)^2 r_1 r_2}{3} + \frac{4}{3} \right) \ . \tag{5.19}$$

This precisely agrees with the field theory result (5.1).

6 A possible a/c bound

As what has been observed in $4d$ SCFTs [45], the Rényi entropy inequalities indicate a/c bounds in field theories[24]

$$\partial_q H_q \leq 0\,, \tag{6.1}$$

$$\partial_q \left(\frac{q-1}{q} H_q\right) \geq 0\,, \tag{6.2}$$

$$\partial_q ((q-1)H_q) \geq 0\,, \tag{6.3}$$

$$\partial_q^2 ((q-1)H_q) \leq 0\,, \tag{6.4}$$

where $H_q := S_q/S_1$. Imposing these conditions to our results (3.10)(3.12)(3.15), one obtains

$$0 < \frac{\bar{c}}{\bar{a}} \leq \frac{7}{3}\,, \tag{6.5}$$

or equivalently

$$\frac{\bar{a}}{\bar{c}} \geq \frac{3}{7}\,. \tag{6.6}$$

This lower bound can be derived alternatively by requiring a non-negative specific heat. In the limit $q \to 0$, the energy of the system can be read from (3.10)

$$E_{q\to 0} := \frac{1}{2\pi}\partial_q F_{q\to 0} = -\frac{1}{2\pi}\partial_q S_{q\to 0} = V_5 \frac{r_1^2 r_2^2}{32\pi^2}\frac{(7\bar{a}-3\bar{c})}{q^4} = V_5 \pi \frac{r_1^2 r_2^2}{2}(7\bar{a}-3\bar{c})\,T^4\,, \tag{6.7}$$

where $T = 1/\beta = \frac{1}{2\pi q}$. It follows from the stability of the ensemble that the specific heat must be non-negative, $\frac{\partial E}{\partial T} \geq 0$, which gives (6.6).

Note that all the a, c data of the currently known $6d$ $(2,0)$ SCFTs, listed in table 1 in appendix B, satisfy the inequality (6.5)(6.6). The lowest $\bar{a}/\bar{c}$ value in the current data, $4/7$, supported by the large N limits, is greater than our bound $3/7$. Note that the expression of supersymmetric Rényi entropy in terms of a, c anomalies could work for theories beyond the ADE type. It would be interesting to understand whether our bound implies new $(2,0)$ SCFTs. It would also be interesting to understand similar bounds in SCFTs with less supersymmetry. We leave these questions for future work.

Acknowledgments

The author is grateful for helpful discussions with Ofer Aharony, Thomas Dumitrescu, Igor Klebanov, Zohar Komargodski, Hong Liu, Mark Mezei, Jun Nian, Eric Perlmutter, Soo Jong Rey, Amit Sever, Cobi Sonnenschein and Xi Yin. The author would like to thank Princeton University and Harvard University for hospitality. This work was supported by "The PBC program of the Israel council of higher education" and in part by the Israel Science Foundation (grant 1989/14), the US-Israel bi-national fund (BSF) grant 2012383 and the German Israel bi-national fund GIF grant number I-244-303.7-2013.

[24] The validity of these inequalities for supersymmetric Rényi entropy is expected although a proof is still in preparation.

A Near $q = 1$ expansion

We study the perturbative expansion of supersymmetric Rényi entropy (associated with spherical entangling surface) around $q = 1$. This can be considered as an extension of the previous study of the ordinary Rényi entropy near $q = 1$. Although our main concern will be $6d$ $(2,0)$ SCFTs, we keep the discussions in this section valid for any SCFT with conserved R-symmetries in d-dimensions.

Following the way in [52, 53],[25] we consider the supersymmetric partition function on $\mathbb{S}^1_q \times \mathbb{H}^{d-1}$ with background gauge fields (R-symmetry chemical potentials), which can be used to compute the supersymmetric Rényi entropy across a spherical entangling surface, see $\mathbb{S}^{d-2}$, in flat space. We work in grand canonical ensemble. The partition function on $\mathbb{S}^1_{\beta=2\pi q} \times \mathbb{H}^{d-1}$ can be written as

$$Z[\beta, \mu] = \mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right) . \tag{A.1}$$

The state variables can be computed as follows

$$E = \left(\frac{\partial I}{\partial \beta}\right)_\mu - \frac{\mu}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_\beta , \tag{A.2}$$

$$S = \beta\left(\frac{\partial I}{\partial \beta}\right)_\mu - I , \tag{A.3}$$

$$Q = -\frac{1}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_\beta , \tag{A.4}$$

where $I := -\log Z$. Therefore we get energy expectation value by (A.2)

$$E = \frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,\hat{E}\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} , \tag{A.5}$$

and the charge expectation value by (A.4)

$$Q = \frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,\hat{Q}\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} . \tag{A.6}$$

In the presence of supersymmetry, both inverse temperature β and chemical potential μ are functions of a single variable q therefore I is considered as

$$I_q := I[\beta(q), \mu(q)] . \tag{A.7}$$

The supersymmetric Rényi entropy is defined as

$$S_q = \frac{qI_1 - I_q}{1-q} . \tag{A.8}$$

Consider the Taylor expansion around $q = 1$, with $\delta q = q - 1$,

$$S_q = S_{\mathrm{EE}} + \sum_{n=2}^{\infty} \frac{1}{n!}\left(\frac{\partial^n I_q}{\partial q^n}\right)_{q=1} \delta q^{n-1} . \tag{A.9}$$

[25]See [54] from the viewpoint of twisted operator.

A.1 $\partial_q I_q$

We will first consider $\partial_q I_q$. The first derivative with respect to q can be written as

$$\frac{dI_q}{dq} = \left(\frac{\partial I}{\partial \beta}\right)_\mu \beta'(q) + \left(\frac{\partial I}{\partial \mu}\right)_\beta \mu'(q) \; . \tag{A.10}$$

Using (A.2) and (A.4), we can rewrite it as

$$\frac{dI_q}{dq} = (E - \mu Q)\, \beta'(q) - \beta Q\, \mu'(q) \; . \tag{A.11}$$

The q-dependence of the temperature and the chemical potential can be read off from the supersymmetric background (including metric and R-symmetry gauge field),

$$\beta(q) = 2\pi q \, , \quad \mu(q) = \alpha \frac{q-1}{q} \, , \tag{A.12}$$

where $\beta(q)$ is determined by the geometric fact and $\mu(q)$ is solved from the Killing spinor equation on the background. α is some number which may be different in various rigid supersymmetric backgrounds.[26] The first q-derivative of I_q is simplified by using (A.12)

$$I'_q = 2\pi(E - \alpha Q) \; . \tag{A.13}$$

Notice that in general both E and Q are functions of q. Also E and Q here are expectation values rather than operators.

A.2 $S'_{q=1}$ and $I''_{q=1}$

From (A.9) we see that

$$S'_{q=1} = \frac{1}{2} I''_{q=1} \; . \tag{A.14}$$

Let us take one more derivative above on the first derivative (A.13) and take use of (A.5) and (A.6)

$$I''_q = -4\pi^2 \left(\frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\, (\hat{E}-\alpha\hat{Q})^2\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} - \frac{\left[\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\, (\hat{E}-\alpha\hat{Q})\right)\right]^2}{\left[\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)\right]^2} \right) \, , \tag{A.15}$$

which can be simplified in the limit $q \to 1$ by using $\mu = 0$ at $q = 1$

$$S'_{q=1} = -2\pi^2 \left(\frac{\mathrm{Tr}\left(e^{-\beta\hat{E}}\, (\hat{E}-\alpha\hat{Q})^2\right)}{\mathrm{Tr}\left(e^{-\beta\hat{E}}\right)} - \frac{\left[\mathrm{Tr}\left(e^{-\beta\hat{E}}\, (\hat{E}-\alpha\hat{Q})\right)\right]^2}{\left[\mathrm{Tr}\left(e^{-\beta\hat{E}}\right)\right]^2} \right)_{q=1} \; . \tag{A.16}$$

This can be rewritten as connected correlators

$$S'_{q=1} = -2\pi^2 \left[\langle \hat{E}\hat{E} \rangle^c + \alpha^2 \langle \hat{Q}\hat{Q} \rangle^c - 2\alpha \langle \hat{E}\hat{Q} \rangle^c \right]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}} \, , \tag{A.17}$$

[26] α characterizes the weight of the chemical potential. In the case of multiple chemical potentials, one should use $\alpha_{i=1,2\ldots R}$, where R denotes the number of U(1) R-symmetry Cartans. i should be summed over for $\alpha_i Q^i$.

where we have used $[\hat{E}, \hat{Q}] = 0$ to flip the order of $\hat{E}$ and $\hat{Q}$. Given that $\langle \hat{E}\hat{Q}\rangle^c = 0$ and $\langle \hat{E}\hat{E}\rangle^c$ has been computed in [52], we get

$$S'_{q=1} = -V_{d-1}\frac{\pi^{d/2+1}\Gamma(d/2)(d-1)}{(d+1)!}C_T - 2\pi^2\alpha^2\int_{\mathbb{H}^{d-1}}\int_{\mathbb{H}^{d-1}}\langle J_\tau(x)J_\tau(y)\rangle^c_{q=1} \,. \tag{A.18}$$

C_T is defined in the flat space correlator

$$\langle T_{ab}(x)T_{cd}(0)\rangle = \frac{C_T}{x^{2d}}I_{ab,cd}(x)\,, \tag{A.19}$$

where

$$I_{ab,cd}(x) = \frac{1}{2}\left(I_{ac}(x)I_{bd}(x) + I_{ad}(x)I_{bc}(x)\right) - \frac{1}{d}\delta_{ab}\delta_{cd}\,, \quad I_{ab}(x) = \delta_{ab} - 2\frac{x_a x_b}{x^2}\,. \tag{A.20}$$

Now the task is to compute the second term in (A.18). Following the way of computing $\langle TT\rangle$ on the hyperbolic space $\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}$, one can take use of the flat space correlators in the CFT vacuum. The result is[27]

$$\langle \hat{Q}\hat{Q}\rangle^c = -\frac{\pi^{\frac{d-1}{2}}V_{d-1}}{2^{d-2}(d-1)\Gamma(\frac{d-1}{2})}C_v\,, \tag{A.21}$$

where C_v is defined in the current correlator in flat space

$$\langle J_a(x)J_b(0)\rangle = \frac{C_v}{x^{2(d-1)}}I_{ab}(x)\,. \tag{A.22}$$

Then our final result of $S'_{q=1}$ becomes

$$S'_{q=1} = -V_{d-1}\left(\frac{\pi^{\frac{d}{2}+1}\Gamma(\frac{d}{2})(d-1)}{(d+1)!}C_T - \alpha^2\frac{\pi^{\frac{d+3}{2}}}{2^{d-3}(d-1)\Gamma(\frac{d-1}{2})}C_v\right)\,, \tag{A.23}$$

which tells us that the first q-derivative of supersymmetric Rényi entropy at $q = 1$ is given by a linear combination of C_T and C_v.[28] This is intuitively expected because in the presence of supersymmetry, taking the derivative with respect to q is equivalent to taking the derivative with respect to $g_{\tau\tau}$ and A_τ in the same time.[29] q-deformation can be often equivalent to the squashing $b := \sqrt{q}$, therefore this formula also shows the relation between $\partial^2_{b=1}$ of the free energy on squashed sphere and flat space correlators. It is clear from the above derivation that this formula works both for free theories and interacting SCFTs in general d-dimensions. In the particular case of $6d$ $(2,0)$ SCFTs, the 2-point function of the stress tensor is determined by the central charge $c_\mathfrak{g}$ in (1.1) [23, 24]. Therefore the integrated 2-point function is proportional to $c_\mathfrak{g}$. Furthermore, $S'_{q=1}$ is also proportional to $c_\mathfrak{g}$, because the stress tensor and the R-current in the right hand side of (A.23) live in the same multiplet.[30] The same thing happens in $\mathcal{N} = 4$ SYM [44].

[27] $\langle J\hat{Q}\rangle$ was first computed in [55].
[28] In another word, a linear combination of the integrated stress tensor 2-point function and the integrated R-current 2-point function.
[29] This was first suggested in [44].
[30] For $(2,0)$ tensor multiplet, this supermultiplet was studied explicitly in [56].

A.3 $S''_{q=1}$ and $I'''_{q=1}$

From (A.9) we see that

$$S''_{q=1} = \frac{1}{6} I'''_{q=1} \,. \tag{A.24}$$

One may go straightforward to compute I'''_q by taking one more derivative above on (A.15)

$$\frac{I'''_q}{8\pi^3} = \frac{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})^3\Big)}{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)} - 3\frac{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}(\hat{E}-\alpha\hat{Q})^2\Big)\,\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})\Big)}{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)\Big]^2}$$
$$+2\frac{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})\Big)\Big]^3}{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)\Big]^3}\,, \tag{A.25}$$

which may be simplified at $q=1$ where $\mu=0$

$$\frac{I'''_{q=1}}{8\pi^3} = \left(\frac{\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})^3\Big)}{\mathrm{Tr}e^{-\beta\hat{E}}} - 3\frac{\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})^2\Big)\,\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})\Big)}{\Big[\mathrm{Tr}e^{-\beta\hat{E}}\Big]^2}\right.$$
$$\left. +2\frac{\Big[\mathrm{Tr}(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q}))\Big]^3}{\Big[\mathrm{Tr}e^{-\beta\hat{E}}\Big]^3}\right)_{q=1}\,. \tag{A.26}$$

This can be further written in terms of connected correlation functions,

$$S''_{q=1} = \frac{1}{6}I'''_{q=1} = \frac{4\pi^3}{3}\left[\langle\hat{E}\hat{E}\hat{E}\rangle^c - \alpha^3\langle\hat{Q}\hat{Q}\hat{Q}\rangle^c - 3\alpha\langle\hat{E}\hat{E}\hat{Q}\rangle^c + 3\alpha^2\langle\hat{E}\hat{Q}\hat{Q}\rangle^c\right]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}}\,, \tag{A.27}$$

where we have used $[\hat{E},\hat{Q}]=0$ because $\hat{Q}$ is conserved charge. The integrated correlators in (A.27) can be computed by transforming the corresponding flat space correlators, $\langle TTT\rangle, \langle JJJ\rangle, \langle TTJ\rangle, \langle TJJ\rangle$ in the CFT vacuum.[31] These correlators in flat space can be determined up to some coefficients for general CFTs in d-dimensions by conformal Wald identities [57, 58]. In the presence of $6d$ $(2,0)$ superconformal symmetry, both the 2- and 3-point functions of the stress tensor multiplet are uniquely determined in terms of a single parameter, the central charge $c_{\mathfrak{g}}$ [23, 24]. And the right hand side of (A.27) should be proportional to $c_{\mathfrak{g}}$, because the stress tensor and the R-current belong to the same multiplet.[32] The same thing can be seen in $\mathcal{N}=4$ SYM [44].

[31] We leave the explicit computations of these correlators elsewhere.

[32] By representation theory, the stress tensor belongs to a half BPS multiplet. In superspace, the 2-, 3- and 4-point functions of all half BPS multiplets are known to admit a unique structure [59–61].

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B Data of simply-laced Lie algebra $\mathfrak{g}$

$\mathfrak{g}$	$r_\mathfrak{g}$	$h^\vee_\mathfrak{g}$	$d_\mathfrak{g}$	$\bar{a}_\mathfrak{g}$	$\bar{c}_\mathfrak{g}$	$\bar{a}_\mathfrak{g}/\bar{c}_\mathfrak{g}$
A_{n-1}	$n-1$	n	n^2-1	$\frac{16}{7}n^3-\frac{9}{7}n-1$	$4n^3-3n-1$	$\frac{3}{7(2n+1)^2}+\frac{4}{7}$
D_n	n	$2n-2$	$n(2n-1)$	$\frac{64}{7}n^3-\frac{96}{7}n^2+\frac{39}{7}n$	$16n^3-24n^2+9n$	$\frac{3}{7(3-4n)^2}+\frac{4}{7}$
E_6	6	12	78	$\frac{15018}{7}$	3750	~ 0.572114
E_7	7	18	133	5479	9583	~ 0.571742
E_8	8	30	248	$\frac{119096}{7}$	29768	$\sim 0.571544 > \frac{4}{7}$

Table 1. The rank $r_\mathfrak{g}$, dual Coxeter number $h^\vee_\mathfrak{g}$, dimension $d_\mathfrak{g}$ of the simply-laced Lie algebras and the normalized a, c anomalies for the associated $6d$ $(2,0)$ SCFTs [22].

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