

Tri-bimaximal mixing from twisted Friedberg–Lee symmetry

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Abstract We investigate the Friedberg–Lee (FL) symmetry and its promotion to include the μ – τ symmetry, and call this the twisted FL symmetry. Based on the twisted FL symmetry, two possible schemes are presented toward the realistic neutrino mass spectrum and the tri-bimaximal mixing. In the first scheme, we suggest the semi-uniform translation of the FL symmetry. The second one is based on the S_3 permutation family symmetry. The breaking terms, which are twisted FL symmetric, are introduced. Some viable models in each scheme are also presented.

1 Introduction

The precision measurements of the neutrino oscillation have suggested that there are large mixings among three generations in the lepton sector unlike the quark sector. The current experimental data of mixing angles [1] is well approximated by the tri-bimaximal mixing (TBM) [2, 3], which is given by

$$V_{\text{TBM}} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (1)$$

The properties of this mixing matrix are that the second generation of the neutrino mass eigenstate is represented by trimaximal mixture of all flavor eigenstates, $\nu_2 = \sum_{\alpha} \nu_{\alpha}/\sqrt{3}$, and the third generation is bimaximal mixture of μ and τ neutrinos, $\nu_3 = (-\nu_{\mu} + \nu_{\tau})/\sqrt{2}$, in the diagonal basis of the charged leptons. Naively, it appears that (1) implies the following forms of the neutrino mass matrix in the flavor basis,

$$\mathcal{M}^{\nu} = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$+ \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad (2)$$

where m_i ($i = 1 \sim 3$) are the neutrino mass eigenvalues.

Such suggestive forms of the generation mixing and the neutrino mass matrices give us a strong motivation to look for a flavor structure of the lepton sector. In fact there exist μ – τ permutation symmetry in (2). The Majorana mass matrix having the μ – τ symmetry is written by [4–8]

$$\mathcal{M}^{\nu} = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}, \quad (3)$$

where we assume that A, B, C and D are real. This form of the neutrino mass matrix in the diagonal basis of the charged leptons can be diagonalized by the following mixing matrix,

$$V = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12}/\sqrt{2} & \cos \theta_{12}/\sqrt{2} & 1/\sqrt{2} \\ -\sin \theta_{12}/\sqrt{2} & \cos \theta_{12}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}. \quad (4)$$

This mixing matrix suggests that θ_{23} is maximal $\pi/4$, θ_{13} is vanishing and θ_{12} is undetermined. When there is a certain relation among the mass parameters in the matrix equation (3), which is $A + B = C + D$, the $\sin \theta_{12}$ is determined such as $\sin \theta_{12} = 1/\sqrt{3}$ and thus the mixing matrix becomes the TBM one. Indeed, each mass matrix in (2) possesses the relation $A + B = C + D$ as well as the μ – τ symmetry. What is the origin of such a special texture? A number of proposals based on a flavor symmetry to unravel it have been elaborated [9–27].

Recently, Friedberg and Lee have proposed a new type of family symmetry called Friedberg–Lee (FL) symmetry [28]. The FL symmetry is a translational hidden family symmetry and predicts one massless neutrino and the trimaximal mixture for the second generation neutrino. Several possible origins of the FL symmetry have been discussed in Ref. [29]. In

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addition to the symmetry, if we further impose the μ – τ symmetry, the model leads to the TBM [30–32] and can partly realize the structure of (2). That is, the FL symmetry can be the origin of the condition $A + B = C + D$. However, the model cannot reproduce the realistic neutrino mass spectrum as we shall explain later. Therefore, in order to construct a viable flavor model based on the FL symmetry, some breakings of the symmetry, additional symmetries and/or extensions are needed.

In this letter, we consider the μ – τ symmetric extension of the FL symmetry and refer to it as the twisted FL symmetry. Then we propose two possible schemes to obtain the experimentally favored neutrino mass spectrum and the TBM. The first scheme is based on the non-uniformity of the FL translation and its breaking. In the second scheme, we introduce the S_3 permutation symmetry as a family one. It is well known that the second term in (2), which is called a democratic form, can be realized by the S_3 symmetry [33–35]. The other terms of (2) can be generated by introducing the S_3 symmetry breaking terms which are governed by the twisted FL symmetry. In both schemes, the μ – τ symmetry remains intact and the FL symmetry (and S_3 symmetry) ensures the condition $A + B = C + D$ mentioned above.

The letter is organized as follows. In Sect. 2, we define the twisted FL symmetry and propose a realistic model for the Majorana neutrinos on the basis of the non-uniform translation. In Sect. 3, we focus on the S_3 permutation symmetry and propose models for both Dirac and Majorana neutrinos. Section 4 is devoted to a summary. Detailed derivations of some neutrino mass matrices based on the twisted FL symmetry are summarized in Appendix.

2 Twisted FL symmetry and uniformity breaking

We start with the ordinary (type-I) seesaw mechanism [36–38] with the three right-handed heavy Majorana neutrinos. After integrating out the right-handed Majorana neutrinos, the mass terms of the charged leptons and effective light Majorana neutrinos are written as

$$-\mathcal{L} = \bar{\ell}_{Li} M_{ij}^e \ell_{Rj} + \bar{\nu}_i^c \mathcal{M}_{ij}^{\nu} \nu_j + h.c., \quad (5)$$

where ℓ_{Li} and ℓ_{Ri} are the left- and right-handed charged leptons, and ν_i are the light Majorana neutrinos. The i and j stand for family indices. The charged lepton and effective Majorana neutrino mass matrices are denoted as M^e and \mathcal{M}^{ν} , respectively. In what follows, we take \mathcal{M}^{ν} as a real matrix since we do not discuss CP violation in this letter and the following discussions can be simply implied to a complex case. In the diagonal basis of the charged leptons, \mathcal{M}^{ν} is diagonalized by the Maki–Nakagawa–Sakata (MNS) matrix: $V_{\text{MNS}}^{\dagger} \mathcal{M}^{\nu} V_{\text{MNS}}^* = \text{diag}(m_1, m_2, m_3)$. In this basis,

we impose the following translational family symmetry as a hidden one only on the light Majorana neutrino mass term,

$$\nu_i \rightarrow \nu'_i = S_{ij} \nu_j + \eta_i \xi, \quad (6)$$

where ξ is a space-time independent Grassmann parameter, $\xi^2 = 0$, $\eta = (\eta_1, \eta_2, \eta_3)^T$ are c-numbers, and S is the permutation matrix between the second and third families:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (7)$$

This kind of family symmetry has been proposed by Friedberg and Lee in Ref. [28] for the first time, and is called Friedberg–Lee (FL) symmetry. In (6), we combine the FL symmetry with the μ – τ one and call it the twisted FL symmetry from now on.

In the following subsections, we propose two neutrino-flavor models on the basis of the twisted FL symmetry. The first model corresponds to a naive μ – τ symmetric extension of the original FL symmetry. We show that the model can partially realize the structure of (2) but should be improved to obtain the experimentally favored neutrino mass spectrum. Then we propose a realistic model.

2.1 Uniform translation

First, we consider the uniform translation, that is $\eta_1 = \eta_2 = \eta_3$ in (6). Such a translation leads to the following effective Majorana neutrino mass matrix,¹

$$\mathcal{M}^{\nu} = \frac{B}{2} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \left(A + \frac{B}{2} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \quad (8)$$

We find that the first and third terms in (2) are obtained. The mass matrix can be diagonalized by the TBM matrix equation (1) and leads to the neutrino masses

$$m_1 = 3B, \quad m_2 = 0, \quad m_3 = 2A + B. \quad (9)$$

Since $m_2 > m_1$ is suggested from the solar neutrino oscillation experiment, this model is inconsistent with the experimental data. In order to obtain the proper neutrino mass spectrum, we need to introduce the symmetry breaking terms in (8). However, it seems hard to derive $m_2 > m_1$ in this model because it may be natural to assume that the magnitude of the breaking terms should be smaller than that of the terms based on a symmetry. In that sense, this model cannot reproduce a realistic neutrino mass spectrum.

¹A detailed derivation is given in Appendix.

2.2 Semi-uniform translation

It has been shown that the twisted FL symmetry with the uniform translation cannot give the proper neutrino mass spectrum. Here, we consider a semi-uniform translation, $\eta_2 = \eta_3$ and $\sum \eta_i = 0$ ($i = 1 \sim 3$).² This translation leads to the effective Majorana mass matrix as³

$$\mathcal{M}^\nu = \frac{B}{2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \left(A + \frac{B}{2} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \quad (10)$$

We find that the second and third terms of (2) are obtained. This mass matrix can be diagonalized by the TBM matrix and leads to a massless neutrino, $m_1 = 0$. That is, we can easily realize a viable model in terms of the twisted FL symmetry with the semi-uniform translation.

The nonzero m_1 can be also obtained by introducing the symmetry breaking terms in (10). It is seen that if the breaking term has a particular structure such as the first term in (2),

$$c \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \quad (11)$$

the TBM and a massive m_1 are realized. Actually, such a form of a mass matrix can be obtained in the twisted FL scheme with non-uniform translation, $\eta_i \neq \eta_j$ ($i \neq j$) with $3\eta_1 = \sum \eta_i$.⁴ Consequently, the μ - τ symmetry and the condition $A + B = C + D$ mentioned in the introduction are remained. They are just the result of the twisted FL operation.

As a result, the whole mass matrix takes the same form as (2), and neutrino masses are written as

$$m_1 = 6c, \quad m_2 = \frac{3}{2}B, \quad m_3 = 2A + B. \quad (12)$$

The neutrino mass spectrum is the normal hierarchy. The smallness of m_1 is easily understood by the fact that the mass is induced from the symmetry breaking.

3 Democratic texture with twisted FL symmetry

In this section, we propose other realizations of the experimentally favored neutrino mass spectrum and the TBM in terms of the twisted FL symmetry. We take the S_3 symmetry as a fundamental flavor symmetry and introduce breaking terms which are twisted FL symmetric.

²This translation, equivalently $\eta_i \propto (2, -1, -1)$, corresponds to the case of $\kappa = -1/2$ in Ref. [30, 31].

³A detailed derivation is also given in Appendix.

⁴A detailed derivation is given in Appendix.

3.1 Dirac neutrinos

We focus on the Dirac neutrinos in this subsection. The mass terms of the charged leptons and Dirac neutrinos are written as

$$-\mathcal{L}_{\ell M} = \bar{\ell}_{Li} M_{ij}^e \ell_{Rj} + \bar{\nu}_{Li} M_{ij}^D \nu_{Rj} + h.c., \quad (13)$$

where ν_{Li} and ν_{Ri} are the left- and right-handed Dirac neutrinos, respectively. The Dirac mass matrix is denoted as M^D , and for simplicity we assume that M^D is real. Here we separately impose the S_3 permutation symmetry for the left- and right-handed neutrinos as a flavor symmetry. They are shown by S_{3L} and S_{3R} , respectively. It is well known that the minimal introduction of matter contents in the Higgs sector, which contains a single elementary scalar in the singlet representation of both S_{3L} and S_{3R} symmetries, leads to the following invariant Dirac mass matrix under $S_{3L} \times S_{3R}$ symmetry,⁵

$$M^D = C \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (14)$$

By comparing the mass matrix with (2), one can see that the resulting mass spectrum is $m_2 = 3C$ and $m_1 = m_3 = 0$ with the TBM. Thus the model is experimentally disfavored. The possible way to rescue the model is to obtain the non-vanishing m_1 or m_3 . We consider a case to have non-zero m_1 by introducing the symmetry breaking term based on the FL symmetry in what follows.⁶

Before introducing the breaking terms, we consider an extension of the twisted FL symmetry for the Dirac neutrinos. We extend the twisted FL symmetry for the Dirac neutrinos in the following way,

$$\nu_{Li} \rightarrow \nu'_{Li} = S_{ij}^L \nu_{Lj} + \xi_L, \quad (15)$$

$$\nu_{Ri} \rightarrow \nu'_{Ri} = S_{ij}^R \nu_{Rj} + \xi_R. \quad (16)$$

Under these transformations, the form of the Dirac mass matrix is constrained to the one given in (11) as discussed in Appendix. Note that the Dirac mass matrix is invariant under the independent transformations for the left- and

⁵By changing the basis of (14), the mass matrix has only the 3-3 element. Both the mass matrix and one in (14) have only one non-vanishing mass eigenvalue and the same mass spectrum. However, the flavor mixings are different. The mass matrix containing only the 3-3 element does not lead to any mixings clearly but the democratic form of matrix in (14) has large mixings. This has been pointed out in Ref. [39]. In this letter, we take the democratic basis in (14) for phenomenological interests.

⁶Some proposals of breaking of such a S_3 flavor symmetry, which leads to the democratic mass matrix, have been discussed in model including extended Higgs sector [33], $O(3)$ breaking mechanism [40], and considering perturbative effects [41].

right-handed neutrinos such as $(\nu_{Li}, \nu_{Ri}) \rightarrow (\nu'_{Li}, \nu_{Ri})$ or (ν_{Li}, ν'_{Ri}) .⁷ For the Dirac neutrinos, the twisted FL symmetry each for the left- and right-handed neutrinos can lead to the strange form of mass matrix given in (11) as preserving the uniformity. That is one of the interesting features of the twisted FL symmetry for the Dirac neutrinos.

We add the mass matrix equation (11) to (14) as the breaking terms for the $S_{3L} \times S_{3R}$ flavor symmetry. Consequently, the total Dirac neutrino mass matrix is written as

$$M^D = C \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \tag{17}$$

and the neutrino mass spectrum is given by

$$m_1 = 6c, \quad m_2 = 3C, \quad m_3 = 0, \tag{18}$$

which suggests the inverted neutrino mass hierarchy. The TBM can be also realized. This is a minimal scheme: the twisted FL symmetry for the Dirac neutrinos is introduced as the symmetry of the breaking term. We note that the whole mass matrix equation (17) violates the $S_{3L} \times S_{3R}$ symmetry but still preserves the μ - τ symmetry.

3.2 Majorana neutrinos

Here a similar model is presented for the Majorana neutrinos. The most general Majorana mass matrix based on the S_3 symmetry can be written as

$$\mathcal{M}^\nu = \begin{pmatrix} E & F & F \\ F & E & F \\ F & F & E \end{pmatrix}. \tag{19}$$

This matrix has two degenerate eigenvalues,

$$m_1 = E - F, \quad m_2 = E + 2F, \quad m_3 = E - F, \tag{20}$$

where we take $m_1 = m_3$ and assume that \mathcal{M}^ν is real. To obtain the mass difference between m_1 and m_3 , we introduce (8) as a symmetry breaking term. Then the total Majorana neutrino mass matrix is written as

$$\begin{aligned} \mathcal{M}^\nu = & \begin{pmatrix} E & F & F \\ F & E & F \\ F & F & E \end{pmatrix} + \frac{b}{2} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} \\ & + \left(a + \frac{b}{2}\right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \end{aligned} \tag{21}$$

⁷If we impose the same twisted FL symmetry on both left- and right-handed neutrinos, $\nu_{Li,Ri} \rightarrow \nu'_{Li,Ri} = S_{ij}\nu_{Lj,Rj} + \xi$, the form of the mass matrices given in (8) is obtained as the Dirac neutrino mass matrix.

This matrix can also be diagonalized by the TBM matrix, and mass eigenvalues are given by

$$\begin{aligned} m_1 &= E - F + 3b, \\ m_2 &= E + 2F, \\ m_3 &= m_1 + 2(a + b). \end{aligned} \tag{22}$$

It seems that a natural mass spectrum induced by this model is the quasi-degenerate one because the mass difference between m_1 and m_3 is determined by only the small symmetry breaking parameters a and b . For instance, in accordance with Ref. [41], if we require $F = -2E$ and $E > 0$, the breaking parameters must be $b < 0$ and $|a| \gg |b|$ to satisfy the recent neutrino oscillation update given in Ref. [1]

$$\Delta m_{21}^2 = (7.695 \pm 0.645) \times 10^{-5} \text{ eV}^2, \tag{23}$$

$$\Delta m_{31}^2 = (2.40_{-0.11}^{+0.12}) \times 10^{-3} \text{ eV}^2. \tag{24}$$

Then this model results in the degenerate mass spectrum.

4 Summary

We have considered the promotion of the FL symmetry to contain the μ - τ symmetry and call this the twisted FL symmetry. It has also been shown that the symmetry with the uniform translation for neutrinos can partially realize the desired forms of the neutrino mass matrix for the TBM but should be improved to obtain the experimentally favored neutrino mass spectrum. Then, we have presented two possible schemes which can achieve a realistic neutrino mass spectrum and the TBM.

The first scheme is based on the non-uniformity of the FL symmetry. We have discussed that for Majorana neutrinos and presented a viable model which predicts the normal mass hierarchy. In the second scheme, we have focused on the S_3 permutation family symmetry. The breaking terms as preserving the twisted FL symmetry have been introduced there. We have examined that for both Dirac and Majorana neutrinos. In the case of the Dirac neutrinos, the model predicts the massless third generation neutrino and the inverted mass hierarchy. The model for the Majorana neutrinos in this scheme results in the degenerate mass spectrum. All of the models lead to the TBM.

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Appendix: Twisted Friedberg–Lee symmetry

In this appendix, we give a detailed discussion how to obtain the neutrino mass matrices used in the main part from the twisted FL symmetry. Discussions are presented for both Majorana and Dirac neutrinos.

A.1 Majorana neutrinos

Let us consider the Majorana neutrino case and define the mass term as follows,

$$-\mathcal{L}_v = \bar{\nu}_i^c \mathcal{M}_{ij}^v \nu_j. \tag{A.1}$$

After operating the twisted FL transformation given in (6) on the neutrino fields, the mass term becomes

$$\bar{\nu}_i^c \mathcal{M}_{ij}^v \nu_j \rightarrow [\bar{\nu}_k^c S_{ki} + \bar{\eta}_i \xi] \mathcal{M}_{ij}^v [S_{jl} \nu_l + \eta_j \xi]. \tag{A.2}$$

Hence, the invariance of (6) requires the μ - τ symmetry, $S \mathcal{M}^v S = \mathcal{M}^v$, and the translational symmetry, $\bar{\eta} \mathcal{M}^v = \mathcal{M}^v \eta = 0$. Firstly, the μ - τ symmetry restricts the form of \mathcal{M}^v to that of (3). For convenience we adopt the following parameterization here:

$$\mathcal{M}^v = \begin{pmatrix} D & -2C & -2C \\ -2C & A+B & -A \\ -2C & -A & A+B \end{pmatrix}. \tag{A.3}$$

Secondly, from the translational symmetry, one finds the conditions

$$\mathcal{M}_{ij}^v \eta_j = \begin{pmatrix} D & -2C & -2C \\ -2C & A+B & -A \\ -2C & -A & A+B \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = 0. \tag{A.4}$$

The resulting form of the mass matrix depends on the correlation among η_i . The mass matrices used in the main part are corresponding to the following three classes of uniformity:

[Uniform: $\eta_1 = \eta_2 = \eta_3$]

$$\begin{aligned} \mathcal{M}^v &= \begin{pmatrix} 2B & -B & -B \\ -B & A+B & -A \\ -B & -A & A+B \end{pmatrix} \\ &= \frac{B}{2} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} \\ &\quad + \left(A + \frac{B}{2} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \end{aligned} \tag{A.5}$$

[Semi-uniform: $\eta_2 = \eta_3, \sum \eta_i = 0$]

$$\mathcal{M}^v = \begin{pmatrix} B/2 & B/2 & B/2 \\ B/2 & A+B & -A \\ B/2 & -A & A+B \end{pmatrix}$$

$$= \frac{B}{2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \left(A + \frac{B}{2} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{A.6}$$

[Shared-uniform: $3\eta_1 = \sum \eta_i, \eta_i \neq \eta_j (i \neq j)$]

$$\mathcal{M}^v = C \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}. \tag{A.7}$$

A.2 Dirac neutrinos

Next, let us consider the Dirac neutrino case

$$-\mathcal{L}_D = \bar{\nu}_{Li} M_{ij}^D \nu_{Rj} + h.c. \tag{A.8}$$

For Dirac neutrinos, in general, the twisted FL symmetry can be imposed on the left- and right-handed neutrinos separately as

$$\nu_{Li} \rightarrow \nu'_{Li} = S_{ij}^L \nu_{Lj} + \eta_{Lj} \xi, \tag{A.9}$$

$$\nu_{Ri} \rightarrow \nu'_{Ri} = S_{ij}^R \nu_{Rj} + \eta_{Rj} \xi. \tag{A.10}$$

Two independent μ - τ permutation symmetries make the Dirac mass matrix as

$$M^D = \begin{pmatrix} D & -2C & -2C \\ -2B & -A & -A \\ -2B & -A & -A \end{pmatrix}, \tag{A.11}$$

while the translational symmetries lead to the conditions

$$M_{ij}^D \eta_{Rj} = \begin{pmatrix} D & -2C & -2C \\ -2B & -A & -A \\ -2B & -A & -A \end{pmatrix} \begin{pmatrix} \eta_{R1} \\ \eta_{R2} \\ \eta_{R3} \end{pmatrix} = 0, \tag{A.12}$$

and

$$\begin{aligned} \eta_{Li} M_{ij}^D &= (\eta_{L1} \quad \eta_{L2} \quad \eta_{L3}) \begin{pmatrix} D & -2C & -2C \\ -2B & -A & -A \\ -2B & -A & -A \end{pmatrix} \\ &= 0. \end{aligned} \tag{A.13}$$

The resulting form of the mass matrix depends on the correlations among η_{Li} and η_{Ri} again. In this letter, we have assumed the uniform translation, that is $\eta_{Li} \propto (1, 1, 1)$ and $\eta_{Ri} \propto (1, 1, 1)$. Then, the mass matrix of the Dirac neutrino takes the form

$$M^D = C \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}. \tag{A.14}$$

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