Dynamics and curl ratio of a curling stone

Norikazu Maeno

Abstract An attempt to develop a dynamic model for the motion of a curling stone is reported. Characteristic forms of the ice surface covered with many small pebbles and the narrow annular running band at the bottom of a stone lead to the increase in the force exerting on ice, resulting in the reduction of friction coefficient of ice and the production of ice fragments to increase the effective friction coefficient at the rear running band. In this paper, a numerical model is presented to compute the dynamic motion of a curling stone on the base of the evaporation–abrasion mechanism. The *friction magnification factor, A*, was introduced to evaluate the asymmetry that the friction coefficient of the rear running band is larger by a factor of A than that of the front. Numerical computations showed that curl distances and trajectories experienced in usual curling games were reproduced by magnitudes $5 \leq A \leq 20$. Another physical quantity, *curl ratio, C*, was introduced to describe quantitatively the amount of curl of a stone moving with definite translational and angular velocities. C specifies the direction to which a stone moves at any instant. It increases with decreasing translational velocity and increasing angular velocity. In other words, a stone curls more at smaller speeds and higher turns.

Keywords Curling · Curl ratio · Ice · Friction · Pebbles · Abrasion · Running band

N. Maeno (✉)
Professor Emeritus of Hokkaido University,
Hanakawa Minami 7-2-133, Ishikari,
Hokkaido 061-3207, Japan
e-mail: maenony@ybb.ne.jp

1 Introduction

Curling, which originated in Scotland in the sixteenth century, is now a popular sport in many northern countries and has been a regular sport in the Winter Olympic Games since 1998. It requires a combination of strategy and skill to control the motion of a curling stone on ice, and a number of sophisticated and varied techniques have been developed in the long history of curling. Unfortunately, however, most of them are neither based on quantitative measurements nor scientific evidence, but are empirical and qualitative. Understanding of the dynamics of a stone on ice is not complete, though it has been the subject of a number of studies [1–11].

In this paper, we first discuss the important relation between the friction property of ice and the characteristic forms of the curling stone bottom and ice surface, and derive a numerical model to calculate the dynamics of a curling stone. Then we show various results of the dynamics of a stone, which may be useful to playing and coaching strategies and ice making. It should be noted, however, that this is the first model to formulate the physical interaction of a curling stone and ice, and its validity should be checked and verified using various quantitative observations, which are extremely scarce as mentioned above.

2 Ice friction and curling

Ice friction is the most important property in curling since it determines the motion of a stone on ice. It is no exaggeration to say that the complex nature of ice friction has made the game of curling more varied, tactical and exciting. The complex nature arises because ice friction varies

sensitively with velocity, temperature, pressure, sliding material and so on.

A number of theoretical, experimental and practical studies of ice friction have been made and it has been recognized that two physical mechanisms work predominantly, namely, water lubrication and adhesive shear deformation [12–18]. Water lubrication due to frictional heating works at higher sliding velocities than roughly 10 mm s^{-1} at temperatures above about -10 °C where most curling games are played. Thin ice layer is melted by frictional heat and acts as lubricant, so that the friction coefficient is as small as 0.01. The water layer cannot be observed with naked eyes, since its thickness is of the order of 100 nm and it disappears immediately.

At lower velocities than roughly 10 mm s^{-1}, frictional heat is not enough to melt ice and the friction mechanism changes to the adhesion shear deformation of ice. The slide at these smaller velocities is undergone by the slow plastic deformation of adhered thin ice layer assisted by ice sintering [13, 18].

In the range of sliding velocities of usual curling games, <5 m s^{-1}, we can write the friction coefficient of ice (μ) as

$$\mu = \mu_{00} U^{-1/2} \tag{1}$$

where μ_{00} is a constant with dimension of (velocity)$^{1/2}$ and U is the sliding velocity relative to ice. When we discuss ice friction at different pressures, temperatures and velocities, we should use the following more general expression:

$$\mu = \mu_0 \left(\frac{P}{P_0}\right)^{-1/4} \left(\frac{T_m - T}{T_0}\right) \left(\frac{U}{U_0}\right)^{-1/2} \tag{2}$$

where μ_0, P_0, T_0 and U_0 are constants, T_m is the melting point of ice (273.16 K), and P and T are the pressure and temperature, respectively. This expression was derived by combining the results of theoretical and experimental researches [13–16, 18]. For convenience, we put $P_0 = 0.1$ MPa, $T_0 = 5$ K and $U_0 = 1$ m s^{-1}, then μ_0 is the friction coefficient of ice at $P = 0.1$ MPa, $T = 268$ K (-5 °C) and $U = 1$ m s^{-1}. It should be noted in Eq. (2) that ice friction coefficient decreases with increasing pressure, rising temperature and increasing velocity, and gives great effects on the dynamics of curling stones.

3 Pebbles and running band

The surface of an ice sheet used in curling games is covered with many tiny protrusions called ice pebbles, which are formed by sprinkling small water droplets on a smooth flat ice surface before start of games. Their average sizes are 1–2 mm in height and 3–10 mm in diameter, and their number density ranges from 2×10^4 to 5×10^4 m^{-2}. A typical curling stone is a granite rock, 20 kg in weight, and its bottom is concave at the center. It touches with ice at an annular band called running band, usually 130 mm in diameter and 3–8 mm in width.

These characteristic forms of both the ice surface and stone bottom give the two important physical effects. One is the reduction of friction coefficient due to the large force acting on ice pebbles. The nominal pressure a stone exerts on the ice surface is roughly 0.1–0.16 MPa, but the actual pressure on each pebble tip is much larger, amounting to 0.4–8.1 MPa. Accordingly, the magnitude of friction coefficient decreases according to Eq. (2) and it is the reason why curling stones slide so smoothly on the ice sheet.

Another effect is the mechanical deformation and fracture of ice pebbles, which occur because the pressure created by the stone becomes comparable or larger than the yield or fracture stress of ice, usually a few MPa depending on temperature and strain rate [17]. So the physical process of the stone slide is not only smooth friction, but also mechanical abrasion. Abrasion of ice pebbles takes place because of large pressures of several MPa, or several tens of kilograms of loads acting on square centimeter area. It is often noticed that pebbles are deformed and broken and fine ice fragments and debris are formed on the ice sheet. The important role of the ice fragments is discussed later.

4 Equations of motion of a curling stone

Regarding the motion of a curling stone sliding on an ice sheet as a two-dimensional motion of a rigid body and noting that it has three degrees of freedom, we can write the following three equations,

$$m \frac{dV_x}{dt} = \sum F_x \tag{3}$$

$$m \frac{dV_y}{dt} = \sum F_y \tag{4}$$

and

$$I \frac{d\omega}{dt} = \sum (r \times F) \tag{5}$$

where m and I are the mass and the moment of inertia of the stone, respectively, V (V_x, V_y) is the translational velocity, ω is the angular velocity (positive in counterclockwise direction), F (F_x, F_y) is the frictional force and r is the radius of the running band. The air drag is negligibly small compared with force due to ice friction in the
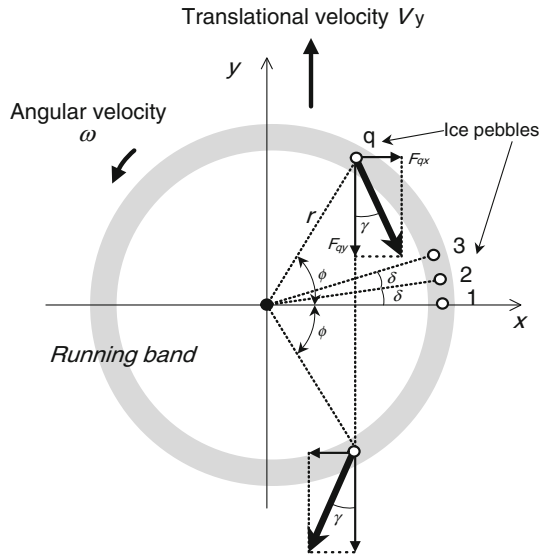
Fig. 1 Schematic diagram of a running band touching ice pebbles. Friction forces at the q-th pebble and those at the symmetric point are shown

velocity range of usual curling games. The total number of ice pebbles, J, the running band contacts at any instant is estimated as $J = 2\pi bnr = 40\text{--}100$ where b is the width of the running band and n the number density of ice pebbles. Taking $r = 130$ mm and $m = 20$ kg, the normal force exerted on each pebble is estimated as $f = mg/J = 2\text{--}5$ N where g is the acceleration of gravity.

We assume that J ice pebbles line up with angle space $\delta = 2\pi/J$ on the running band (Fig. 1). Then the total friction force working on the running band, F, is the sum of friction forces on each pebble, F_q:

$$F = \sum_{q=1}^{J} F_q = \sum_{q=1}^{J} \mu_q f \tag{6}$$

At the q-th pebble, the angle is $\varphi = (q-1)\,\delta$, the relative velocity between the running band and ice, U_q, is

$$U_q = \left[V_y^2 + 2r\omega V_y \cos\phi + (r\omega)^2 \right]^{1/2} \tag{7}$$

and the friction coefficient, μ_q, is given by Eqs. (1) or (2). Denoting the angle between U_q and y-axis as γ, the x- and y-components of the total friction force are, respectively,

$$F_x = \sum_{q=1}^{J} F_{qx} = \sum_{q=1}^{J} \mu_q f \sin\gamma \tag{8}$$

and

$$F_y = -\sum_{q=1}^{J} F_{qy} = -\sum_{q=1}^{J} \mu_q f \cos\gamma, \tag{9}$$

and the torque of rotation is

$$T = \sum (r \times F) = \sum_{q=1}^{J} \mu_q fr \cos(\gamma - \phi). \tag{10}$$

If we know the friction forces in Eqs. (6)–(10), we can compute the exact translational and angular velocities and positions at any instant using Eqs. (3)–(5).

Simple computation of Eq. (8) gives $F_x = 0$ because x-components of friction forces at the front running band ($q = 1,.., J/2$) and the rear band ($q = J/2 + 1,.., J$) are equal in magnitude and opposite in direction and cancel each other (Fig. 1). Actual arrangements of pebbles may not be completely symmetric, but possible displacement of pebbles gives <10 % change, so that there appear no effective lateral forces making a stone curl.

However, actually it is not the case, and the fact suggests that some mechanism works to produce net lateral forces to lead to curl. As a most simple case, we suppose that x-components of friction at the front running band and the rear band do not cancel each other, giving F_x a finite value. We assume for example that the friction coefficient of the rear running band (μ_q^R) is larger than that of the front (μ_q^F), and write as follows:

$$\mu_q^R = A\mu_q^F \tag{11}$$

where A is the *friction magnification factor*, $A \geq 1$. The assumption reasonably explains the actual behavior of a curling stone on ice, that is, a stone rotating clockwise curls toward the right-hand side and anticlockwise toward the left-hand side. However, other types of possibilities causing such asymmetric friction may not be excluded.

5 Evaporation–abrasion model and numerical computation

The asymmetric friction coefficient of ice at the front and rear running bands, $\mu_q^R \geq \mu_q^F$, has been noted by Johnston [2], Shegelski et al. [3, 5], Penner [6] and Denny [7], but none of them could give a reasonable physical mechanism for the occurrence of the asymmetry. Recently, Maeno [11] proposed the evaporation–abrasion model on the basis of the physical consideration of the interaction between the characteristic forms of ice pebbles and running band. The model includes two physical processes of ice: evaporation and abrasion.

When a stone slides on an ice sheet, each ice pebble in touch, rubbed and frictionally heated by the front running band will touch in the next moment the rear band. The evaporation during the time interval between the two contacts, roughly 10–100 ms, cools the tip of the pebble and increases the magnitude of friction coefficient according to Eq. (2). The decrease in temperature is small, but its effect on

the friction coefficient is appreciable. The magnitude of the ratio, $A = \mu_q^R/\mu_q^F$, at the symmetric points of angle φ and the translational velocity V (Fig. 1) are given as [11]

$$A = 1 + 0.264\sqrt{\frac{\sin\phi}{V}} \tag{12}$$

when the ice temperature is $-4\,°C$ and the difference in water vapor pressures of ice and environment is 173 Pa. Equation (12) does not include angular velocity, but the effect of rotation on the dynamics of a stone becomes significant through Eq. (8). Figure 2 gives a difference of rear and front lateral forces $(F_R–F_F)_x$ calculated by Eq. (8). It is clear that the difference, that is, the net lateral force increases as the angular velocity increases, and the total lateral force is larger at larger angular velocities.

Trajectories of a stone computed with A due to the evaporation effect only are shown in Fig. 3. In the calculation, ice friction coefficient was taken as $\mu_{00} = 0.008$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1) and the number density of ice pebbles $n = 5 \times 10^4$ m^{-2} so that the stone delivered with the initial translational velocity 2.0 m s^{-1} at the hog line stops around the tee line 28 m apart. Numerical computations through Eqs. (3)–(10) were repeated at intervals of 0.05 s or 0.1 s. Figure 3 shows that the curl distance increases with increasing angular velocities, but it is only 0.2 m at most because A does not exceed 1.26 at most as shown in Eq. (12).

The abrasion mechanism gives larger magnitudes of A and more realistic curl distances. As stated earlier, the actual pressure working on ice pebbles, on which a stone slides, is large and amounts to the yield or fracture stress of ice, so that abrasion or wear takes place. Ice pebbles are mechanically deformed and destroyed into ice fragments or debris. Ice fragments produced by the front running band will meet with the rear band and act as obstacles to the motion through complex processes such as slide over,
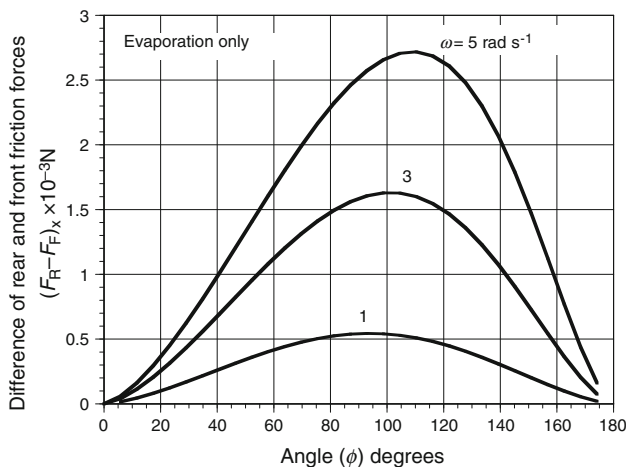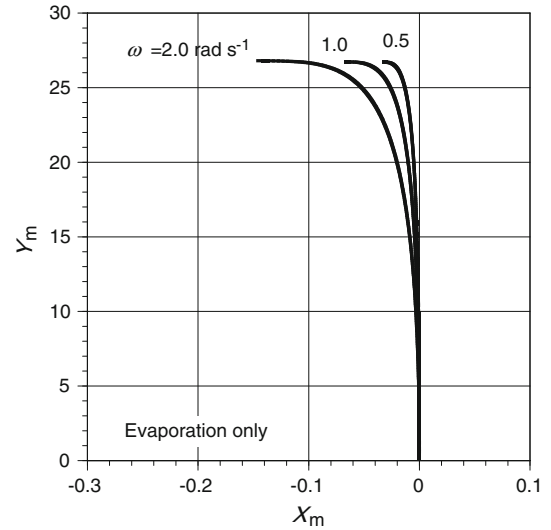
Fig. 3 Trajectories of a stone (evaporation mechanism). Initial translational velocity, 2.0 m s^{-1}, number density of ice pebbles, 5×10^4 m^{-2} and ice friction coefficient, Eq. (1) with $\mu_{00} = 0.008$ m$^{1/2}$ s$^{-1/2}$

mechanical deformation and fracture. The abrasion process is essentially an accidental phenomenon, but it is reasonable to consider that it takes place at every pebble touching with the running band and that the total effect on average is to produce the asymmetric friction forces of front and rear running bands. Greater asymmetry is expected at larger number density of ice pebbles and rougher running band, and also at higher temperatures since the fracture strength of ice is smaller [17]. The size and shape of running band, e.g., sharp edges, are also related with the asymmetry.

We assume that the asymmetry of friction coefficient of front and rear running bands is produced by the combination of evaporation and abrasion mechanisms and that the whole resultant effect is represented by the friction magnification factor, A. Though we have not found the functional relation of A to the sizes and mechanical properties of ice pebbles and running band, our next step for the present is to find suitable values of A which describe the realistic dynamics of a curling stone.

Trajectories of a curling stone calculated with various values of A are shown in Fig. 4. In the calculation, initial translational and angular velocities were taken as 2.0 m s^{-1} and 1.5 rad s^{-1}, respectively, and friction coefficients were chosen so that the stone stopped around 28 m; the adjustment was necessary because larger A makes the sliding distance shorter. It is noted in the figure that the range of magnitudes $5 \le A \le 20$ gives realistic curl distances found in usual games.

Figures 5 and 6 show details of the dynamics of a stone delivered with the initial translational and angular velocities of 2.0 m s^{-1} and 1.5 rad s^{-1}, respectively. A was taken as 10. It is clearly shown that the curl distance increases

Fig. 2 Net force or difference between the rear and front lateral friction forces, $(F_R–F_F)_x$, calculated using Eqs. (8), (11) and (12)
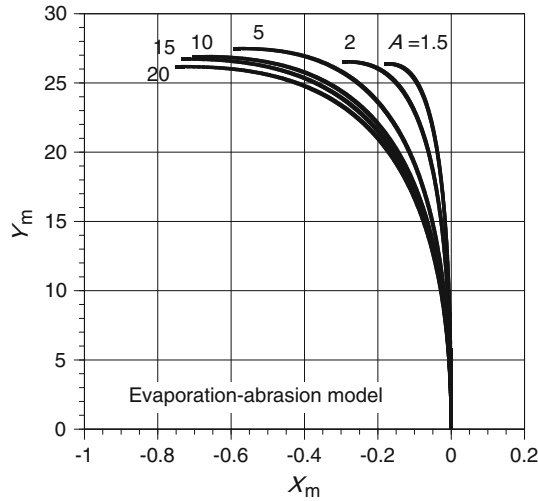
Fig. 4 Trajectories of a stone (evaporation–abrasion model). Initial translational velocity 2.0 m s^{-1}, angular velocity 1.5 rad s^{-1}, number density of ice pebbles 5×10^4 m^{-2} and ice friction coefficient $\mu_{00} = 0.007$, 0.0058, 0.0028, 0.00156, 0.00108 and 0.00084 m$^{1/2}$ s$^{-1/2}$ in Eq. (1) for $A = 1.5$, 2, 5, 10, 15 and 20, respectively

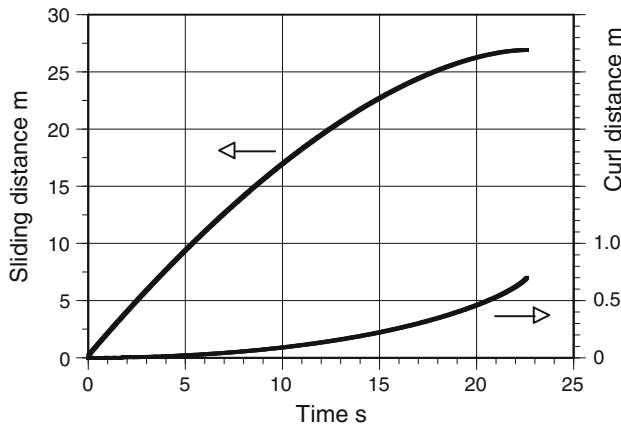**Fig. 5** Sliding and curl distances versus time. Initial translational velocity 2.0 m s^{-1}, angular velocity 1.5 rad s^{-1}, number density of ice pebbles 5×10^4 m^{-2}, ice friction coefficient $\mu_{00} = 0.00156$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1), and friction magnification factor $A = 10$

more rapidly near the end of the path. This corresponds to the rapid decrease of translational and angular velocities, as will be discussed in the next section.

6 Curl ratio

The amount of curl or lateral deflection is an extremely important element in the strategy of curling, but it has never been measured quantitatively or expressed suitably. Most expressions to describe the property are unclear and obscure; for example, the curl distance at stop is not satisfactory since it varies with different sliding distances, initial translational and angular velocities; moreover, it is a
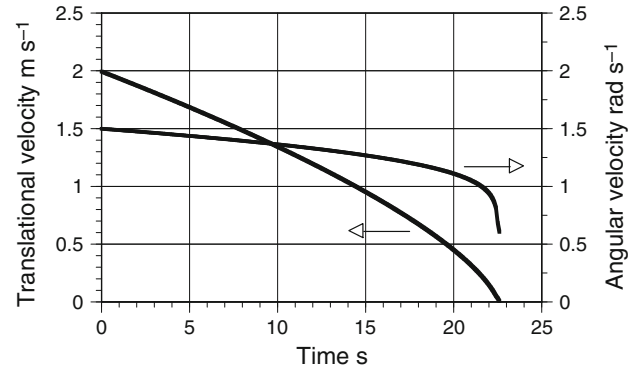

Fig. 6 Translational and angular velocities versus time. Initial translational velocity, angular velocity, number density of pebbles, ice friction coefficient and friction magnification factor are the same as in Fig. 5

total deflection of the whole path and does not give the amount of curl of a stone in motion. Relations between the curl distance and total turns were measured by Penner [6] and Jensen and Shegelski [8], but they did not give details of translational and angular velocities.

We propose a new physical quantity, *curl ratio*, to describe clearly and accurately the instantaneous amount of curl of a stone, which is defined as

$$C = \Delta X / \Delta Y \tag{13}$$

where ΔX and ΔY are the curl and sliding distances of the center of a curling stone, respectively, C is a quantity to be defined at any instant, and if we know friction forces working in a short time Δt, Eqs. (3) and (4) give the curl ratio:

$$C = \left(\frac{F_x \Delta t}{2m}\right) \Big/ \left(V + \frac{F_y \Delta t}{2m}\right) = \frac{mF_x}{2F_y^2}\left(-V \pm \sqrt{V^2 + \frac{2F_y L}{m}}\right)^2 \Big/ L \tag{14}$$

where L is the distance the stone slides in Δt, that is, $L = (V + F_y \Delta t/(2\,m))\Delta t$. We can calculate C at any translational and angular velocities by specifying appropriate values of L or Δt. L is more convenient to specify since C thus estimated can be compared to actual observations and experiences.

Figures 7 and 8 display the curl ratio calculated by Eqs. (8), (9) and (14). The parameters were as follows: number density of ice pebbles $n = 5 \times 10^4$ m^{-2}, ice friction coefficient $\mu_{00} = 0.007$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1), $A = 10$ and $L = 0.1$ m. The curl ratio decreases with the increase in translational velocity, and increases with the increase in angular velocity. In other words, a stone curls more at lower translational velocities and higher angular velocities.

Some trajectories of a stone delivered with a translational velocity 2.0 m s^{-1} are shown in Fig. 9. For example, the stone with the initial angular velocity -3.0 rad s^{-1} slides and stops at 28.0 m with a total curl distance of
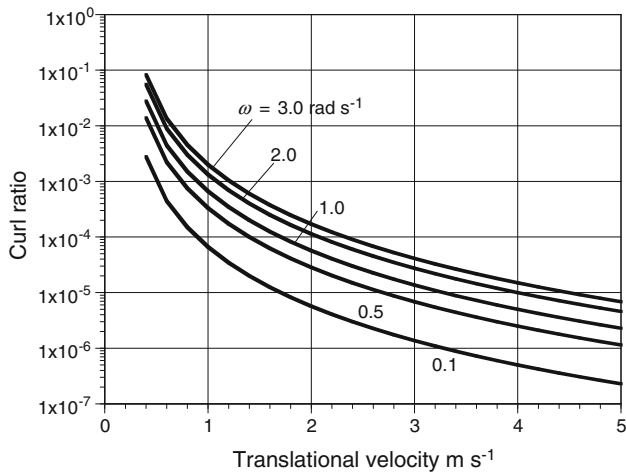
Fig. 7 Curl ratio versus velocity. Number density of ice pebbles 5×10^4 m^{-2}, ice friction coefficient $\mu_{00} = 0.007$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1), $A = 10$ and $L = 0.1$ m

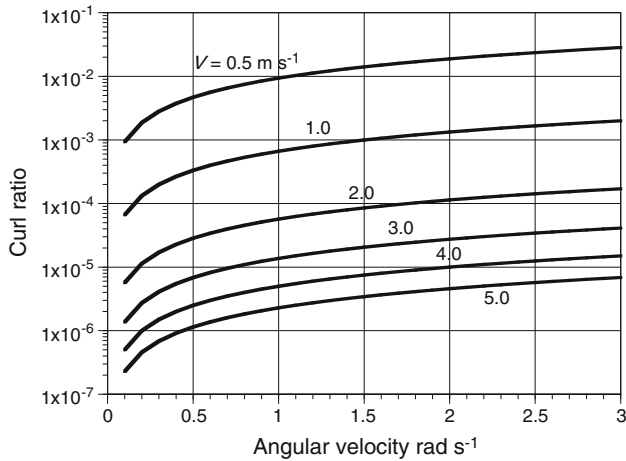**Fig. 8** Curl ratio versus angular velocity. Number density of ice pebbles 5×10^4 m^{-2}, ice friction coefficient $\mu_{00} = 0.007$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1), $A = 10$ and $L = 0.1$ m

Fig. 9 Trajectories at different angular velocities. Number density of ice pebbles 5×10^4 m^{-2}, ice friction coefficient $\mu_{00} = 0.00156$ m$^{1/2}$ s$^{-1/2}$ in Eq. (1) and $A = 10$. Initial translational velocity 2.0 m s^{-1}

Fig. 10 Measured curl ratio versus translational velocity, measured by Nittono et al. [19, 20]

1.6 m. The average curl ratio, that is the mean from 2.0 m s^{-1} to zero, is $C = 1.6/28.0 = 0.057$. At other angular velocities, -2.0, -1.0, -0.5 and -0.1 rad s^{-1}, $C = 0.039$ (=1.06/26.9), 0.019 (=0.52/26.8), 0.010 (=0.27/26.8) and 0.003 (=0.087/26.8), respectively.

Measurements of curl ratio were made recently by Nittono et al. [19, 20] by delivering a standard curling stone on an actual ice sheet maintained at -3.1 ± 0.3 °C. Images of the sliding stone were taken with two CCD cameras from above at an interval of 0.1 s and analyzed to give the translational position and angle of the center of mass of the stone with accuracies of ± 1.7 mm and $\pm 0.47°$ (0.0082 radian), respectively. Figure 10 gives some of the results obtained. Larger values of curl ratio are found at smaller translational velocities and larger angular velocities, which are all in harmony with the calculated results.
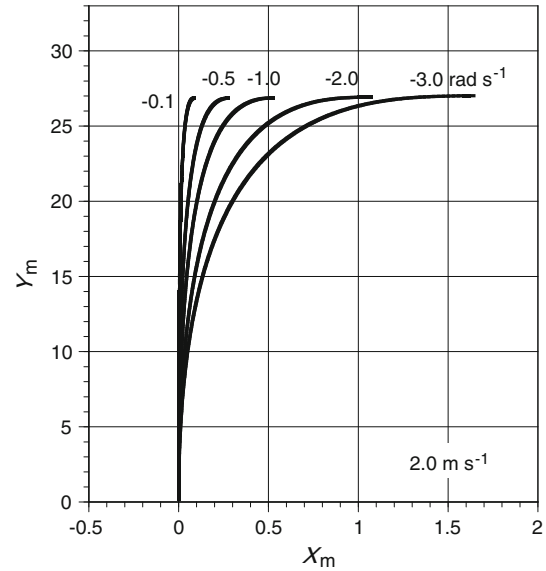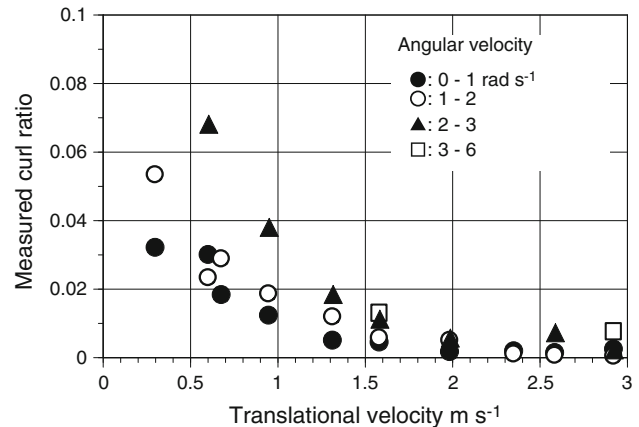
7 Effect of pebble density

The number of pebbles in contact with a running band is important because it determines the pressure acting on each pebble. The minimum number that can support a standard curling stone without plastic deformation or fracture of ice is estimated as roughly $J = 20$ pebbles, which corresponds to the number density of about $n = 1 \times 10^4$ m^{-2}. On the other hand, the maximum number depends on the sizes of pebbles and running band; if we assume that pebbles of 3 mm in diameter are closely arranged in a zigzag fashion on a standard running band, about $J = 280$ pebbles are estimated, which corresponds to $n = 1.4 \times 10^5$ m^{-2}. The estimate implies that the variation in the number density of
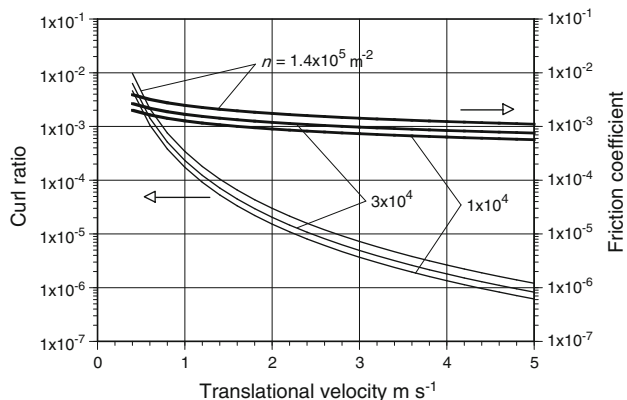
Fig. 11 Curl ratio and friction coefficient at different pebble densities. Angular velocity, 1.5 rad s^{-1}, $A = 10$ and $L = 0.1$ m. Ice friction coefficient $\mu_0 = 0.004$ and $(T_m–T)/T_0 = 1$ in Eq. (2)
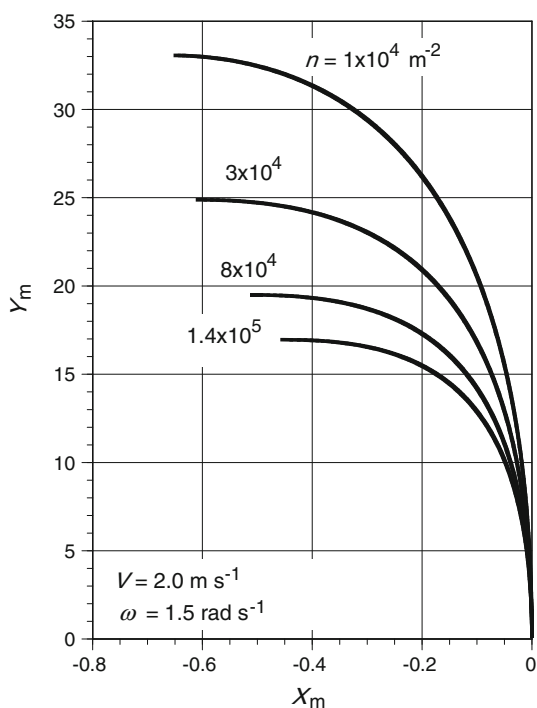

Fig. 12 Trajectories at different pebble densities. Parameters are the same as in Fig. 11. Initial translational velocity 2.0 m s^{-1}

pebbles has physical meaning only in the range from 1×10^4 to 1.4×10^5 m^{-2}.

Figure 11 gives the calculated friction coefficient and curl ratio at different number densities of pebbles. In the calculation, angular velocity was taken as 1.5 rad s^{-1}, $A = 10$, and ice friction coefficient $\mu_0 = 0.004$ and $(T_m–T)/T_0 = 1$ in Eq. (2). Increase of both friction coefficient and curl ratio with increasing number density of pebbles is clear.

Figure 12 gives travel paths of a stone delivered with the initial translational velocity 2.0 m s^{-1} and angular velocity 1.5 rad s^{-1}. At larger number densities, sliding distance is

shorter because of larger friction coefficients. However, the curl distance is longer; for example, at the sliding distance 15 m from the start it is 0.046, 0.074, 0.120 and 0.177 m for the number density 1×10^4, 3×10^4, 8×10^4 and 1.4×10^5 m^{-2}, respectively. Accordingly, the average curl ratio is 0.0030, 0.0049, 0.0080 and 0.0118, respectively.

The number of ice pebbles in contact depends on the size of a running band. It is smaller for thinner widths of a running band, and the effect is equivalent to that of smaller number density of ice pebbles as shown in Figs. 11 and 12.

8 Effect of sweeping

Sweeping ahead of a stone raises the surface temperature of ice momentarily and allows the stone to travel farther and curl less or go straighter. These well-known effects are reasonably explained by the reduction of friction coefficient and curl ratio caused by the temperature rise. Accurate measurements of the surface temperature of swept pebble tips are not possible, but we can apply varieties of techniques to measure the average temperature rise of 1–2 °C by vigorous sweeping on the actual ice surface. The laboratory-based rubbing experiment [9] showed the maximum temperature rise of about 0.8 °C at 2 mm below the ice surface.

Figure 13 displays variations of calculated friction coefficient and curl ratio by sweeping. The curves designated as sweeping-1, sweeping-2 and sweeping-3 correspond, respectively, to temperatures 0.5, 1.0 and 1.5 °C higher than no sweeping; the varieties of sweeping are results of different speed, frequency and pressure of the brush head movement curlers employed. We may consider the act of sweeping as switching μ and C curves to those at higher temperatures, such as from no sweeping to sweeping-1, sweeping-2 or sweeping-3.

Figure 14 shows more explicitly the effect of sweeping on trajectories of a stone delivered with the initial translational velocity 2.0 m s^{-1} and angular velocity 1.5 rad s^{-1}. The increase of travel distance by sweeping is clear, but the effect of sweeping on the curl distance needs some careful evaluation since the curl distance at the stop is longer for more vigorous sweeping.

Correct evaluation of sweeping can be made by comparison of the curl distance at the same travel distance. At 20 m in Fig. 14, for example, the curl distance without sweeping is 0.251 m, but it reduces to 0.182, 0.140 and 0.109 m for sweeping-1, sweeping-2 and sweeping-3, respectively. This explains clearly the usual experience that more vigorous sweeping makes a stone to curl less. By dividing with the sliding distance, 20 m, the effect can be more conveniently expressed by the average curl ratio: $C = 0.0125, 0.0091, 0.0070, 0.0054$ for no sweeping, sweeping-1, sweeping-2 and sweeping-3, respectively.
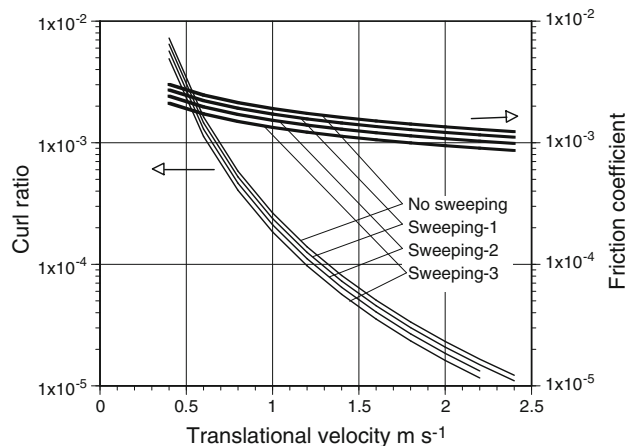
Fig. 13 Effect of sweeping on friction coefficient and curl ratio. Angular velocity, 1.5 rad s^{-1}, number density of ice pebbles, 5×10^4 m^{-2}, $A = 10$, $L = 0.1$ m and ice friction coefficient, $\mu_0 = 0.004$ and $P/P_0 = 1$ in Eq. (2). No sweeping, sweeping-1, sweeping-2 and sweeping-3 correspond to the calculations with $T_m - T = 5.0, 4.5, 4.0$ and 3.5 K, respectively, in Eq. (2)
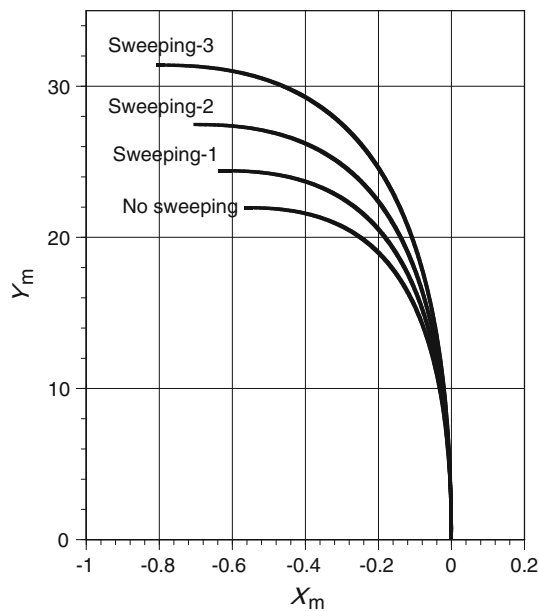

Fig. 14 Effect of sweeping on trajectory. Initial translational velocity 2.0 m s^{-1} and other parameters are the same as in Fig. 13

9 Conclusions

Characteristic forms of the ice surface (pebbles) and the bottom of a stone (running band) lead to the increase in the force exerted on ice, resulting in the reduction of friction coefficient of ice and production of ice fragments increasing the friction coefficient at the rear running band.

Based on the evaporation–abrasion model, the friction magnification factor, A, was introduced to evaluate the asymmetry that the friction coefficient of the rear running band is larger by a factor of A than that of the front. The magnitude in the range $5 \leq A \leq 20$ could reproduce curl distances and trajectories experienced in usual curling games.

Curl ratio, C, was introduced to describe quantitatively the amount of curl of a stone moving with definite translational and angular velocities. C specifies the direction to which a stone is to move at any instant.

Finally, the need for quantitative data about curling should be emphasized; especially, data of curl distance, curl ratio (C) and magnification factor (A) should be collected at different physical conditions such as various stones, ice sheets and temperatures. Such a set of data will be extremely valuable and useful for understanding the dynamics of a curling stone, playing and coaching strategies and ice making.

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