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DOA estimation for wideband signals based on weighted Squared TOPS

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Abstract

This paper introduces a new direction-of-arrival (DOA) estimation method for wideband signal sources. The new method estimates the DOA of wideband signal sources based on squared test of orthogonality of projected subspaces (Squared TOPS) which is an improved method of TOPS. TOPS and Squared TOPS use the signal and noise subspaces of multiple frequency components of wideband signal sources. Although coherent wideband method, such as coherent signal subspace method (CSSM), performs high DOA estimation accuracy, it requires the initial estimate of signal source directions. On the other hand, TOPS and Squared TOPS can provide good performance of DOA estimation without the initial value of signal sources; however, some false peaks appear in spatial spectrum based on these methods. The proposed method, called weighted Squared TOPS (WS-TOPS), uses the modified squared matrix and selective weighted averaging process to improve DOA estimation performance. The performance of WS-TOPS is compared with those of TOPS, Squared TOPS, incoherent MUSIC, and test of orthogonality of frequency spaces (TOFS) through computer simulations. The simulation results show that WS-TOPS can suppress all false peaks in spatial spectrum and improve DOA estimation accuracy and also keep the same resolution performance as Squared TOPS.

Keywords: Array signal processing, Direction-of-arrival estimation, Wideband signals

1 Introduction

Direction-of-arrival (DOA) estimation for wideband signals has been attracting much attention for decades because wideband signals are commonly used in real world for such as signal source localization in wireless communication and radar systems. To improve degree-of-freedom (DOF) and accuracy of DOA estimation, many researches on wideband DOA estimation have also been introduced over several decades [1]. DOA estimation methods for narrowband signals cannot be applied directly to wideband signals because the phase difference between array antennas depends on not only the DOA of the signals but also the temporal frequency. Thus, the common pre-processing for wideband signal estimator decomposes a wideband signal into some narrowband signals using filter banks or a discrete Fourier transform (DFT). Based on the method, many algorithms have been introduced and they are categorized into two groups: incoherent signal subspace method (ISSM) [2, 3] and coherent signal subspace method (CSSM) [4].

ISSM is one of the simplest wideband DOA estimation methods. ISSM uses several narrowband signals decomposed from wideband signal incoherently [3]. In particular, the method applies narrowband DOA estimation techniques, such as MUSIC [5], independently to the narrowband signals. Then, these results are averaged to estimate the DOA of incoming wideband signal sources. Although ISSM provides better estimation accuracy in high signal-to-noise ratio (SNR) regions, the performance deteriorates when the SNR of some frequency bands are low. In other words, the poor estimates from some frequency bands will degrade the final estimation accuracy.

To overcome these disadvantages and to improve DOA estimation performance, CSSM was proposed [4]. In CSSM processing, the correlation matrix of each frequency band is focused by transformation matrices and the focused matrices are averaged to generate a new correlation matrix. Then, CSSM estimates the DOA of incoming wideband signal sources by applying a DOA estimation method for narrowband signals. The key point

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of CSSM algorithm is how to focus correlation matrices. Many techniques have been proposed to obtain a proper focusing matrix [6, 7]. However, each focusing technique requires the initial values, which means the pre-estimated direction of incoming signal sources, and the performance of CSSM is sensitive to the initial values [8]. The weighted average of signal subspaces (WAVES) [9] is also a well-known DOA estimation method for wideband signal sources. However, WAVES also needs the initial DOA estimates and its performance greatly depends on the accuracy of the initial values.

A novel wideband DOA estimation method, which is named test of orthogonality of projected subspaces (TOPS), was proposed [10]. TOPS uses the signal and noise subspaces of several frequency bands and provides good DOA estimation performance without requiring the initial values. However, the method has a drawback that the spatial spectrum calculated by TOPS algorithm has some false peaks and they make it difficult to estimate the true DOA of signal sources.

Squared TOPS was proposed as an improvement method of TOPS [11]. Squared TOPS improves DOA estimation performance by using the squared matrix for orthogonality test instead of the matrix to be tested in the signal processing of TOPS. The method provides higher DOA estimation accuracy and better resolution performance than those of TOPS. However, the undesirable false peaks in spatial spectrum remain.

The test of orthogonality of frequency subspaces (TOFS) was proposed as a new wideband DOA estimation method [12]. TOFS uses the noise subspaces of multiple frequency bands with the steering vector and shows high estimation accuracy when SNR is high. However, TOFS cannot resolve closely spaced signal sources when SNR is low.

Recently, Khatri-Rao (KR) subspace approach was proposed as the method to expand the array structure and to increase DOF [13]. Applying KR subspace approach to CSSM algorithm, some DOA estimation methods of wideband signal sources were proposed [14]. The methods achieve higher DOA estimation accuracy and resolution performance than the conventional CSSM even if there are fewer sensors or antennas than the incoming signal sources. However, it also requires the initial DOA estimate of each signal source.

Furthermore, sparse signal representation algorithms have also been received much attention, which can provide new approaches for wideband DOA estimation [15–17]. These DOA estimation methods based on the sparse signal representation perform higher resolution than the conventional methods without requiring the number of sources. However, there are some difficulties in selecting properly parameters to calculate optimal solutions.

In this paper, we propose a new DOA estimation method for wideband signals called weighted Squared TOPS (WS-TOPS) based on Squared TOPS. WS-TOPS also uses signal subspace and noise subspace of each frequency like Squared TOPS and does not require any initial values. Using modified squared matrix and selective weighted averaging process, WS-TOPS can suppress all false peaks in spatial spectrum and improve DOA estimation accuracy of wideband signal sources and also keep the same resolution performance as Squared TOPS.

This paper is organized as follows. In Section 2, the signal model is described, and the conventional DOA estimation algorithms are explained in Section 3. In Section 4, WS-TOPS is proposed. In Section 5, simulation results are presented, and conclusions are provided in Section 6.

Notation: We denote vectors and matrices by boldface lowercase and uppercase letters, respectively. The superscripts T and H are transpose and complex conjugate transpose, respectively. $E[\cdot]$ denotes the expectation operator.

2 Signal model

We consider estimating the DOA of L wideband signal sources using a uniform linear array that consists of M antennas. Assume that the number of signal sources L ($\leq M$) is either known or can be estimated [18, 19]. It is also assumed that all signals are uncorrelated with each other and exist in the bandwidth between w_L and w_H. Then, the received signal at mth antenna can be expressed as

$$x_m(t) = \sum_{l=1}^{L} s_l(t - v_m \sin\theta_l) + n_m(t), \tag{1}$$

where $s_l(t)$ is the lth signal source, $n_m(t)$ is additive white Gaussian noise at the mth antenna, $v_m = (m-1)d/c$, where d is the distance between adjacent antennas, and c is the speed of light. θ_l is the DOA to be estimated. Then, the received wideband signals are decomposed into K narrowband signals. The DFT of the signal received at mth antenna is

$$x_m(\omega) = \sum_{l=1}^{L} s_l(\omega) \exp\left(-j\omega v_m \sin\theta_l\right) + n_m(\omega). \tag{2}$$

Then, the output signals of the DFT can be written in vector form as follows:

$$\boldsymbol{x}(\omega_i) = \boldsymbol{A}(\omega_i, \boldsymbol{\theta})\boldsymbol{s}(\omega_i) + \boldsymbol{n}(\omega_i), i = 1, 2, \cdots, K, \tag{3}$$

where $\omega_L < \omega_i < \omega_H$ for $i = 1, 2, \cdots, K$,

$$\boldsymbol{A}(\omega_i, \boldsymbol{\theta}) = \left[\, \boldsymbol{a}(\omega_i, \theta_1) \;\; \boldsymbol{a}(\omega_i, \theta_2) \;\; \cdots \;\; \boldsymbol{a}(\omega_i, \theta_L) \,\right], \tag{4}$$

$$\boldsymbol{a}(\omega_i, \theta_l) = \left[1, \mathrm{e}^{-j\omega_i v_1 \sin\theta_l}, \cdots, \mathrm{e}^{-j\omega_i v_{M-1} \sin\theta_l}\right]^T. \tag{5}$$

For simplicity, hereafter, $\boldsymbol{A}(\omega_i, \boldsymbol{\theta})$ and $\boldsymbol{a}(\omega_i, \theta_l)$ will be represented as $\boldsymbol{A}_i(\boldsymbol{\theta})$ and $\boldsymbol{a}_i(\theta_l)$, respectively. The correlation matrix is calculated as follows:

$$\boldsymbol{R}_{xx}(\omega_i) = E\left[\boldsymbol{x}(\omega_i)\boldsymbol{x}^H(\omega_i)\right], \quad (6)$$

$$= \boldsymbol{A}_i(\boldsymbol{\theta})\boldsymbol{R}_{ss}(\omega_i)\boldsymbol{A}_i^H(\boldsymbol{\theta}) + \sigma_n^2\boldsymbol{I}, \quad (7)$$

where $\boldsymbol{R}_{ss}(\omega_i) = E[\boldsymbol{s}(\omega_i)\boldsymbol{s}^H(\omega_i)]$, σ_n^2 is the noise power, and $\boldsymbol{I}$ is the $M \times M$ unit matrix. Assuming the L signal sources are uncorrelated, $\boldsymbol{R}_{ss}(\omega_i)$ has full rank, then the signal subspace matrix $\boldsymbol{F}_i$ and the noise subspace matrix $\boldsymbol{W}_i$ at frequency ω_i can be formed from the eigenvalue decomposition (EVD) of the correlation matrix as

$$\boldsymbol{F}_i = [\,\boldsymbol{e}_{i,1}, \boldsymbol{e}_{i,2}, \cdots, \boldsymbol{e}_{i,L}\,], \quad (8)$$

$$\boldsymbol{W}_i = [\,\boldsymbol{e}_{i,L+1}, \boldsymbol{e}_{i,L+2}, \cdots, \boldsymbol{e}_{i,M}\,], \quad (9)$$

where $\boldsymbol{e}_{i,1}, \cdots, \boldsymbol{e}_{i,M}$ are the orthogonal eigenvectors of $\boldsymbol{R}_{xx}(\omega_i)$ indexed in descending order with respect to their corresponding eigenvalues as follows:

$$\lambda_{i,1} \geq \lambda_{i,2} \geq \cdots \geq \lambda_{i,L} > \lambda_{i,L+1} = \cdots = \lambda_{i,M} = \sigma_n^2. \quad (10)$$

3 Conventional wideband DOA estimation methods

In this section, we explain some conventional DOA estimation methods, which can estimate the DOA of incoming wideband signal sources without any initial value. In these methods, each wideband signal is decomposed into K narrowband signals by DFT as mentioned in the previous section.

3.1 Incoherent MUSIC (IMUSIC)

IMUSIC, which is one of the simplest DOA estimation methods for wideband signals, applies narrowband signal subspace methods (e.g., MUSIC) to each frequency band independently [2, 3]. Then, IMUSIC estimates the DOA of wideband signal sources by using the following equation:

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{K} \boldsymbol{a}_i^H(\theta)\boldsymbol{W}_i\boldsymbol{W}_i^H\boldsymbol{a}_i(\theta) \quad (11)$$

Since the DOAs estimated by Eq. (11) are averages of the result of each frequency band, the poor estimates from a single frequency band even degrades the final estimation accuracy.

3.2 Test of orthogonality of frequency subspaces (TOFS)

TOFS uses the noise subspace obtained from EVD of the correlation matrix of each frequency [12]. The DOA of each incoming wideband signal source is estimated by testing the orthogonality between the steering vector and the noise subspaces. If θ is the one DOA of incoming wideband signals, θ satisfies the following equation:

$$\boldsymbol{a}_i^H(\theta)\boldsymbol{W}_i\boldsymbol{W}_i^H\boldsymbol{a}_i(\theta) = 0 \quad (12)$$

Here, we define the vector $\boldsymbol{d}(\theta)$ as follows:

$$\boldsymbol{d}(\theta) = [\,\boldsymbol{a}_1^H(\theta)\boldsymbol{W}_1\boldsymbol{W}_1^H\boldsymbol{a}_1(\theta) \;\; \boldsymbol{a}_2^H(\theta)\boldsymbol{W}_2\boldsymbol{W}_2^H\boldsymbol{a}_2(\theta) \cdots \boldsymbol{a}_K^H(\theta)\boldsymbol{W}_K\boldsymbol{W}_K^H\boldsymbol{a}_K(\theta)\,] \quad (13)$$

All elements of the vector $\boldsymbol{d}(\theta)$ will be zero when θ is the DOA of incoming wideband signal sources. Then, we can estimate the DOAs by using the following equation:

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{||\boldsymbol{d}(\theta)||} \quad (14)$$

TOFS shows good DOA estimation accuracy in high SNR region by using the noise subspaces obtained from the correlation matrix of received signals. However, TOFS cannot resolve closely spaced signal sources when SNR is low.

3.3 Test of orthogonality of projected subspaces (TOPS)

TOPS uses both of the signal and noise subspaces of each frequency band to estimate the DOA of incoming wideband signal sources [10].

First of all, we obtain the signal subspace $\boldsymbol{F}_i$ and the noise subspace $\boldsymbol{W}_i$ from EVD of the correlation matrix of each frequency band, as described in the previous section.

Then, one frequency band ω_i should be selected and the signal subspace $\boldsymbol{F}_i$ of the selected frequency band is transformed into other frequencies. One of the advantages of TOPS over CSSM is that TOPS does not require the initial DOA estimates for the frequency transform process as follows.

TOPS uses a diagonal unitary transformation matrix. The mth term on the diagonal of the frequency transform matrix $\boldsymbol{\Phi}(\omega_i, \theta)$ is

$$[\,\boldsymbol{\Phi}(\omega_i, \theta)]_{(m,m)} = \exp(-j\omega_i \frac{md}{c}\sin\theta). \quad (15)$$

Using $\boldsymbol{\Phi}(\omega_i, \theta)$, the signal subspace $\boldsymbol{F}_i$ of the frequency band ω_i is transformed into the other frequency band ω_j, where we define the transformed signal subspace $\boldsymbol{U}_{ij}(\theta)$, as follows:

$$\boldsymbol{U}_{ij}(\theta) = \boldsymbol{\Phi}(\Delta\omega, \theta)\boldsymbol{F}_i, \;\; i \neq j, \quad (16)$$

where $\Delta\omega = \omega_j - \omega_i$. Eq. (16) can be expressed as

$$\boldsymbol{U}_{ij}(\theta) = \boldsymbol{\Phi}(\Delta\omega, \theta)\boldsymbol{A}_i(\boldsymbol{\theta})\boldsymbol{G}_i, \quad (17)$$

$$= \boldsymbol{A}_j(\hat{\boldsymbol{\theta}})\boldsymbol{G}_i, \quad (18)$$

where $\hat{\boldsymbol{\theta}}$ is the transformed $\boldsymbol{\theta}$ by using the frequency transform matrix $\boldsymbol{\Phi}(\omega_i, \theta)$, $\boldsymbol{G}_i$ is a full-rank square matrix that satisfies $\boldsymbol{F}_i = \boldsymbol{A}_i(\boldsymbol{\theta})\boldsymbol{G}_i$. The transformation process just transforms an array manifold at any frequency and DOA into another array manifold corresponding to another frequency. Therefore, the transformed matrix is a full rank matrix and could be used for the following test of orthogonality between transformed matrix and noise subspaces as discussed in detail in [10].

Assuming that selected frequency band is ω_1, the matrix $\boldsymbol{D}'(\theta)$ is defined as

$$\boldsymbol{D}'(\theta) = \left[\boldsymbol{U}_{12}^H(\theta)\boldsymbol{W}_2 \ \ \boldsymbol{U}_{13}^H(\theta)\boldsymbol{W}_3 \ \ \cdots \ \ \boldsymbol{U}_{1K}^H(\theta)\boldsymbol{W}_K\right]. \tag{19}$$

We can estimate the DOA of the incoming wideband signal sources from spatial spectrum calculated by the following equation since the rank of the matrix $\boldsymbol{D}'(\theta)$ also decreases when θ is the one DOA of incoming wideband signal sources same as TOFS.

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\sigma'_{\min}(\theta)} \tag{20}$$

where $\sigma'_{\min}(\theta)$ is the minimum singular value of the matrix $\boldsymbol{D}'(\theta)$.

DOA estimation performance depends on the accuracy of the estimated correlation matrix, which is basically determined by the number of snapshots and the SNR of received signal. In signal processing of TOPS, the subspace projection technique is applied to reduce the signal subspace component leakage in the estimated noise subspace. The projection matrix $\boldsymbol{P}_i(\theta)$ is defined as

$$\boldsymbol{P}_i(\theta) = \boldsymbol{I} - (\boldsymbol{a}_i^H(\theta)\boldsymbol{a}_i(\theta))^{-1}\boldsymbol{a}_i(\theta)\boldsymbol{a}_i^H(\theta), \tag{21}$$

where $\boldsymbol{I}$ is an $M \times M$ unit matrix. Then, we obtain the noise robust matrix $\boldsymbol{D}''(\theta)$ replacing the term $\boldsymbol{U}_{ij}(\theta)$ of Eq. (19) by a new transformed signal subspace matrix $\boldsymbol{U}'_{ij}(\theta)$.

$$\boldsymbol{U}'_{ij}(\theta) = \boldsymbol{P}_j(\theta)\boldsymbol{U}_{ij}(\theta) \tag{22}$$

$$\boldsymbol{D}''(\theta) = \left[\ \boldsymbol{U}'^H_{12}(\theta)\boldsymbol{W}_2 \ \ \boldsymbol{U}'^H_{13}(\theta)\boldsymbol{W}_3 \ \ \cdots \ \ \boldsymbol{U}'^H_{1K}(\theta)\boldsymbol{W}_K\ \right] \tag{23}$$

By using the following equation with $\boldsymbol{D}''(\theta)$, TOPS provides better performance since the estimation errors of subspaces are removed by the projection matrix $\boldsymbol{P}_i(\theta)$.

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\sigma''_{\min}(\theta)}, \tag{24}$$

where $\sigma''_{\min}(\theta)$ is the minimum singular value of the matrix $\boldsymbol{D}''(\theta)$.

The output signal of DFT or bandpass filter is not always a perfect narrowband signal. The filtered signals could degrade the DOA estimation accuracy. TOPS can reduce those degradations by using the certain signal subspace obtained from the estimated correlation matrix instead of the steering vector of the frequency band. It indicates that the method to select the frequency band, of which the signal subspace will be transformed to other frequency bands by Eq. (16), also affects the resulting DOA estimation. In contrast, the residual error of the subspaces causes some undesirable rank reductions of the matrix $\boldsymbol{D}''(\theta)$. Therefore, TOPS has the serious disadvantage that several false peaks appear in the spatial spectrum obtained by Eq. (24).

3.4 Squared TOPS

Squared TOPS applies two techniques to TOPS to improve the performance of DOA estimation [11]. One is the technique to select the frequency band of which the signal subspace will be used. The other is the technique to improve the sensitivity of rank decrease of the matrix $\boldsymbol{D}''(\theta)$ when θ is the one DOA of incoming wideband signal sources.

The reference frequency, which is defined as the frequency band of which the signal subspace will be used, should be the frequency band with the highest SNR. Squared TOPS uses the frequency band where the difference between the smallest signal eigenvalue $\lambda_{i,L}$ and the largest noise eigenvalue $\lambda_{i,L+1}$ is maximum as the reference frequency [11].

Then, the signal subspace of the reference frequency band is transformed into the other frequency bands by Eq. (16). Let us assume that the frequency band ω_i is selected and the signal subspace $\boldsymbol{F}_i$ is transformed to the other frequency bands ω_j. Using the transformed signal subspace matrix $\boldsymbol{U}'_{ij}(\theta)$ and the noise subspace matrix $\boldsymbol{W}_j$, we construct the matrix $\boldsymbol{Z}_i(\theta)$ for the test of orthogonality of projected subspaces as follows:

$$\boldsymbol{Z}_i(\theta) = \left[\ \cdots \ \boldsymbol{U}'^H_{ij}(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{U}'_{ij}(\theta) \ \cdots\ \right], \ \ i \neq j \tag{25}$$

Squared TOPS estimates the DOA of incoming wideband signal sources using the inverse of the minimum singular value $\sigma_{zi_{\min}}(\theta)$ of the matrix $\boldsymbol{Z}_i(\theta)$ as follows:

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\sigma_{zi_{\min}}(\theta)} \tag{26}$$

Both of the row and the column elements of the matrix $\boldsymbol{Z}_i(\theta)$ obtained by the squared operation should be close to zero when θ is the DOA of incoming wideband signal sources. It means that the operation improves the sensitivity to detect the rank reduction of the orthogonality evaluation matrix. Eventually, it provides improvement of the estimation performance of Squared TOPS. However, the undesirable false peaks in spatial spectrum remain.

4 Proposed method

In this section, we explain our proposed method named weighted Squared TOPS (WS-TOPS). WS-TOPS applies the following two approaches to Squared TOPS to improve DOA estimation performance. One is the modified squared matrix method, which is the algorithm to suppress the false peaks in the spatial spectrum of Squared TOPS. The other is the selective weighted averaging method, which is the algorithm to improve the DOA estimation accuracy by using the signal subspaces of multiple frequency bands. The details of these algorithms are shown in the following subsections.

4.1 Modified squared matrix (algorithm 1)

Although Squared TOPS and TOPS use the projection matrix $\boldsymbol{P}_i(\theta)$ to reduce the signal subspace component leakage in the estimated noise subspace, some false peaks in the spatial spectrum remain. This is the serious disadvantage of TOPS and Squared TOPS. The transformed signal subspace matrix $\boldsymbol{U}'_{ij}(\theta)$ has residual error, and it causes the undesirable rank decrease of the matrix $\boldsymbol{Z}_i(\theta)$. Thus, we propose the algorithm to suppress these false peaks by modifying the component of the matrix $\boldsymbol{Z}_i(\theta)$.

The steering vector $\boldsymbol{a}_i(\theta)$ is orthogonal to the noise subspaces only when θ is the DOA of the incoming wideband signal sources. Here, we define $b_j(\theta) = \boldsymbol{a}_j^H(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{a}_j(\theta)$. Then, we can avoid the undesirable rank decrease of the matrix $\boldsymbol{Z}_i(\theta)$ by adding the square matrix of which the diagonal elements are $b_j(\theta)$ of the frequency band ω_j. We, however, need to consider how we add $b_j(\theta)$ to the components of the matrix $\boldsymbol{Z}_i(\theta)$ because $b_j(\theta)$ is similar to the components of TOFS, and it would cause the degradation of the resolution performance of closely spaced signal sources.

$b_j(\theta)$ is calculated by using a steering vector and noise subspaces. As we can see from Eq. (5), $\boldsymbol{a}_j^H(\theta)\boldsymbol{a}_j(\theta)$ is M that is the number of antennas. Therefore, $b_j(\theta)$ changes between 0 and M. If the steering vector is orthogonal to all of noise subspaces, $b_j(\theta)$ is 0. If the steering vector is not orthogonal to noise subspaces, $b_j(\theta)$ comes close to M. On the other hand, the elements of $\boldsymbol{U}_{ij}^{'H}(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{U}_{ij}^{'}(\theta)$ are calculated by using the transformed signal subspaces and noise subspaces. Here, we define the lth column of $\boldsymbol{U}_{ij}^{'}(\theta)$ as $\boldsymbol{u}_{ijl}^{'}(\theta)$, which is a transformed signal subspace. As we can also see from Eqs. (16) and (22), $\boldsymbol{u}_{ijl}^{'H}(\theta)\boldsymbol{u}_{ijl}^{'}(\theta)$ is 1, thus each element of $\boldsymbol{U}_{ij}^{'H}(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{U}_{ij}^{'}(\theta)$ changes between 0 and 1. Therefore, we divide $b_j(\theta)$ by M to deal with the elements of $\boldsymbol{U}_{ij}^{'H}(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{U}_{ij}^{'}(\theta)$ and $b_j(\theta)$ as the same range. Based on the discussion, we modify the component of $\boldsymbol{Z}_i(\theta)$ as follows.

First, we obtain the matrix $\boldsymbol{C}_{ij}(\theta)$.

$$\boldsymbol{C}_{ij}(\theta) = \boldsymbol{U}_{ij}^{'H}(\theta)\boldsymbol{W}_j\boldsymbol{W}_j^H\boldsymbol{U}'_{ij}(\theta) + \boldsymbol{B}_j(\theta), \tag{27}$$

where $\boldsymbol{B}_j(\theta)$ is an $L \times L$ diagonal matrix and it can be expressed as

$$\boldsymbol{B}_j(\theta) = \frac{b_j(\theta)}{M}\boldsymbol{I}, \tag{28}$$

where $\boldsymbol{I}$ is an $L \times L$ unit matrix. The matrix $\boldsymbol{C}_{ij}(\theta)$ keeps the full rank except when θ is the DOA of incoming wideband signal sources even if the rank of $\boldsymbol{Z}_i(\theta)$ decreases undesirably. Thus, the algorithm can suppress false peaks in spatial spectrum. Then, we construct a new matrix $\boldsymbol{Z}'_i(\theta)$ using the matrix $\boldsymbol{C}_{ij}(\theta)$ as follows:

$$\boldsymbol{Z}'_i(\theta) = [\ \cdots\ \boldsymbol{C}_{ij}(\theta)\ \cdots\ ],\ \ i \neq j \tag{29}$$

Finally, we can estimate the DOA of incoming wideband signal sources using Eq. (30).

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\sigma'_{zi_{\min}}(\theta)}, \tag{30}$$

where $\sigma'_{zi_{\min}}(\theta)$ is the minimum singular value of the matrix $\boldsymbol{Z}'_i(\theta)$.

4.2 Selective weighted averaging (algorithm 2)

Squared TOPS uses only the single signal subspace $\boldsymbol{F}_i$ of the reference frequency band ω_i, where the difference between the smallest signal eigenvalue $\lambda_{i,L}$ and the largest noise eigenvalue $\lambda_{i,L+1}$ is maximum. This approach is reasonable in terms of computational complexity. However, there are signal subspaces of different frequency bands which could be exploited for further improvement of DOA estimation accuracy. Therefore, we introduce an algorithm using the signal subspaces of multiple frequency bands. We define the weight α_i using the smallest signal eigenvalue $\lambda_{i,L}$ and the largest noise eigenvalue $\lambda_{i,L+1}$ of the frequency band ω_i as follows:

$$\alpha_i = \lambda_{i,L}/\lambda_{i,L+1} = \lambda_{i,L}/\sigma_n^2 \tag{31}$$

The weight α_i can indicate the reliability of the frequency band ω_i because of the same reason Squared TOPS selects reference frequency. Then, using the weight α_i, the spatial spectrum of all frequency bands are combined as follows:

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\frac{1}{K}\sum_{i=1}^{K}\alpha_i\sigma_{zi_{\min}}(\theta)} \tag{32}$$

The spatial spectrum obtained from Eq. (32) is averaged with the weights α_i. Therefore, the algorithm can improve the DOA estimation accuracy of signal sources in high SNR region. However, the algorithm causes deteriorations of sharpness of spectrum peaks when each frequency band shows different peaks to each other, e.g., in low SNR region. Eventually, the algorithm degrades the resolution performance of closely spaced signal sources in low SNR region.

To prevent the deterioration caused by the process, we propose a selective averaging approach. By using only the frequency bands ω_i with weight α_i larger than a certain threshold α_{th}, which are the reliable frequency bands, we can improve DOA estimation performance and also reduce computational cost.

Finally, we obtain the spatial spectrum based on the algorithm 2 as follows:

$$\hat{\theta} = \arg\max_{\theta} \frac{K'}{\sum_{i=1} \alpha_i \sigma_{zi_{min}}(\theta)}, \{i \mid \alpha_i > \alpha_{th}\}, \quad (33)$$

where K' is the number of frequency bands with weight α_i larger than the threshold α_{th}. If there is no frequency band with weight α_i larger than the threshold α_{th}, we use the signal subspace of the frequency band with the largest weight α_i. For example, there are signal sources with low power and every α_i is smaller than α_{th}.

If we use only single frequency band, the proposed method keeps the same performance as Squared TOPS. In other words, the algorithm can provide better performance than that of Squared TOPS even in the case of all α_i that are smaller than the threshold α_{th}. In what follows, we set $\alpha_{\text{th}} = 9$, which implies that the signal power of the frequency band is larger than $(3\sigma_n)^2$ based on Eq. (31). α_{th} in the algorithm 2 determines the frequency bands of which spatial spectra are averaged based on SNR; therefore, the performance of the algorithm 2 depends on SNR of each frequency band.

4.3 Weighted Squared TOPS (WS-TOPS)

The algorithm 1 is effective to suppress the undesirable false peaks in the spatial spectrum. The algorithm 2 improves the DOA estimation accuracy in high SNR region. Therefore, we can achieve the further improvement of DOA estimation performance using the two algorithms simultaneously as follows:

$$\hat{\theta} = \arg\max_{\theta} \frac{K'}{\sum_{i=1} \alpha_i \sigma'_{zi_{\text{min}}}(\theta)}, \{i \mid \alpha_i > \alpha_{\text{th}}\}, \quad (34)$$

4.4 Computational complexity

The number of computations for an $M \times M$ SVD is $O(M^3)$ [20], which is the dominant factor of the computational complexity for the TOPS-based method (TOPS, Squared TOPS, and WS-TOPS). For example, the signal processing for TOPS requires an SVD of an $L \times (K-1)(M-L)$ matrix $\boldsymbol{D}''(\theta)$, where $(K-1)(M-L) > L^2$ because $2L \leq M$ and $K \geq L+1$ [10]. The calculation of the evaluation matrix ($\boldsymbol{D}, \boldsymbol{Z}$, and $\boldsymbol{Z}'$) for each method should also be in consideration. Table 1 lists the dominant factors of the computational complexity of each method, and Fig. 1 shows examples of the computational cost vs. the system parameters (M, L, and K). The proposed method, WS-TOPS, needs to repeat SVD calculations for several frequency bands. Thus, as we can see in Fig. 1, it requires K' times signal processing cost than that of Squared TOPS. However, considering the DOA estimation performance described in the following section, the proposed method can provide enough improvement to be applied.

Table 1 Computational complexity

Algorithm	Complexity: Calculation for $\boldsymbol{D}, \boldsymbol{Z}$, and $\boldsymbol{Z}'$	Calculation for SVD
TOPS	$O(LM(M-L)(K-1))$	$+O(L^2(M-L)(K-1))$
Squared TOPS	$O(\{2LM(M-L)+L^2(M-L)\}(K-1))$	$+O(L^3(K-1))$
WS-TOPS	$O(\{2LM(M-L)+L^2(M-L)\}(K-1)K')$	$+O(L^3(K-1)K')$

5 Numerical results

This section shows numerical simulation results to demonstrate the performance of WS-TOPS with respect to those of the conventional methods which do not require the initial value of DOA: IMUSIC, TOFS, TOPS, and Squared TOPS.

5.1 Simulation parameters

The received signals are divided into Q blocks with the number of samples in one block being equal to the number of DFT points. In this paper, we set Q to 100 and DFT points to 256. We use the frequency bands which are equally spaced K frequency bands between ω_L and ω_H from DFT output. We define the DFT output signal of frequency band ω_i is $\boldsymbol{x}_q(\omega_i), \{i \in 1 \sim K\}$ for the qth block. Then, the estimated correlation matrix of the frequency band ω_i is

$$\hat{\boldsymbol{R}}(\omega_i) = \frac{1}{Q} \sum_{q=0}^{Q-1} \boldsymbol{x}_q(\omega_i) \boldsymbol{x}_q^H(\omega_i). \quad (35)$$

Then, we calculate the signal subspace matrix $\boldsymbol{F}_i$ and the noise subspace matrix $\boldsymbol{W}_i$ from EVD of the correlation matrix $\hat{\boldsymbol{R}}(\omega_i)$ and estimate the DOA of incoming wideband signal sources by using WS-TOPS and each conventional method described in Section 4.

The statistical performance was evaluated by performing 500 Monte Carlo runs for each algorithm. The fixed simulation parameters to be used in the simulations are shown in Table 2. The number of antennas (M), the number of signal sources (L), and the number of frequency bands (K) are shown in the caption of each figure. λ is the wavelength corresponding to the highest frequency component of the received wideband signals. Note that the signal power on each frequency between ω_L and ω_H changes randomly for each simulation. This means that the efficient frequency band

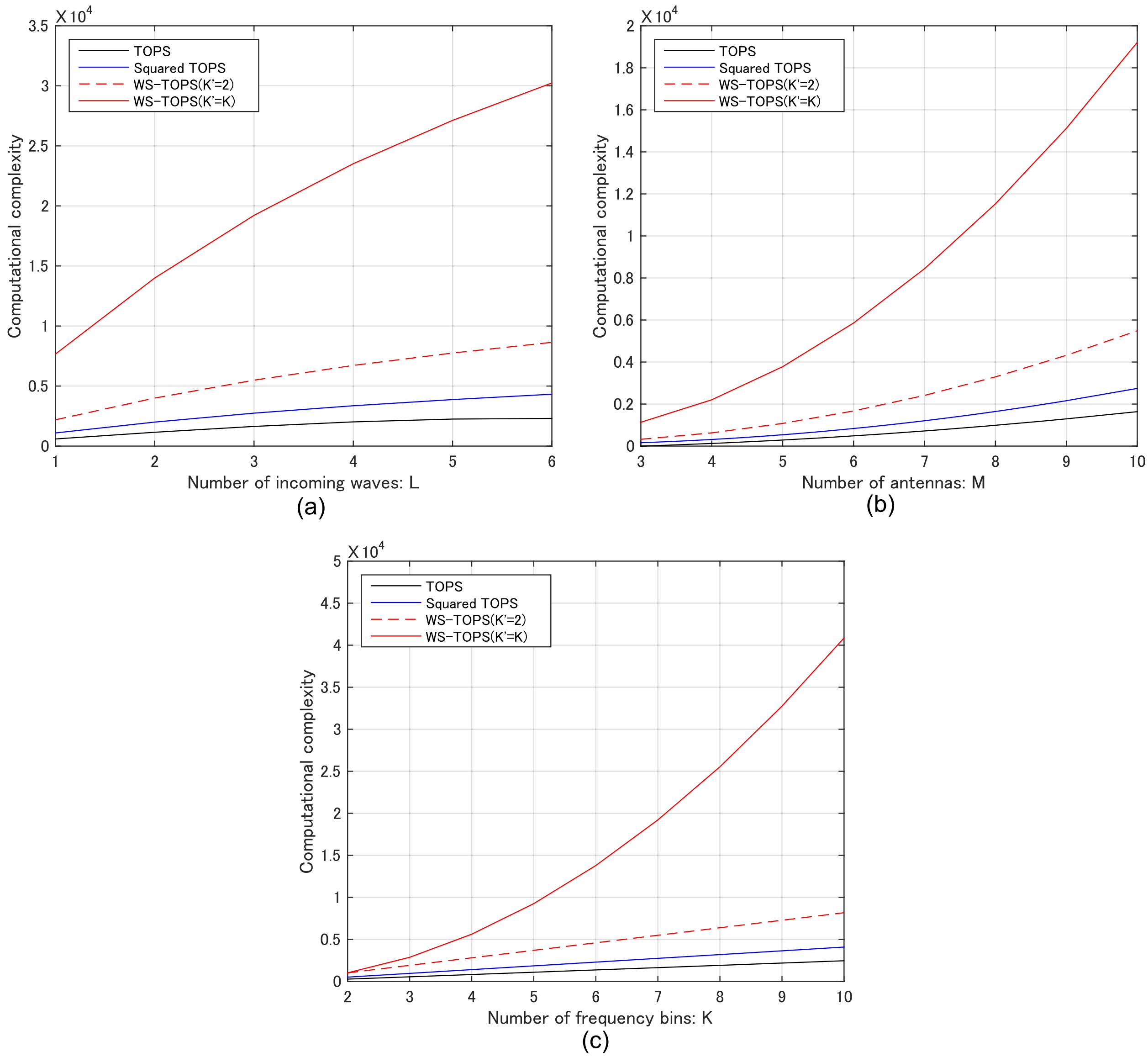

Fig. 1 Computational complexity. **a** Changing the number of waves L, where $M = 10$ and $K = 7$, **b** changing the number of antennas M, where $L = 3$ and $K = 7$, and **c** changing the number of frequency bins K, where $M = 10$ and $L = 3$

Table 2 Simulation parameters

Item	Symbol	Quantity	Remarks
Antenna spacing	d	$\lambda/2$	Uniform linear array
Frequency ω_L	ω_L	$\pi/3$	The lowest frequency of signal sources (ω domain)
Frequency ω_H	ω_H	$2\pi/3$	The highest frequency of signal sources (ω domain)
Parameter α_{th}	α_{th}	9	$3\sigma_n$

of each signal sources also changes randomly with each simulation.

5.2 Spatial spectrum

Figure 2 shows the spatial spectrum calculated by each method for four scenarios, where SNR of each incoming signal source is 5 dB. The details of the scenarios are described in the caption of the figure. The spatial spectrum of each method has some sharp peaks at the true directions, which are indicated as dotted lines in the figures. We can see that WS-TOPS can suppress all undesirable false peaks in the spatial

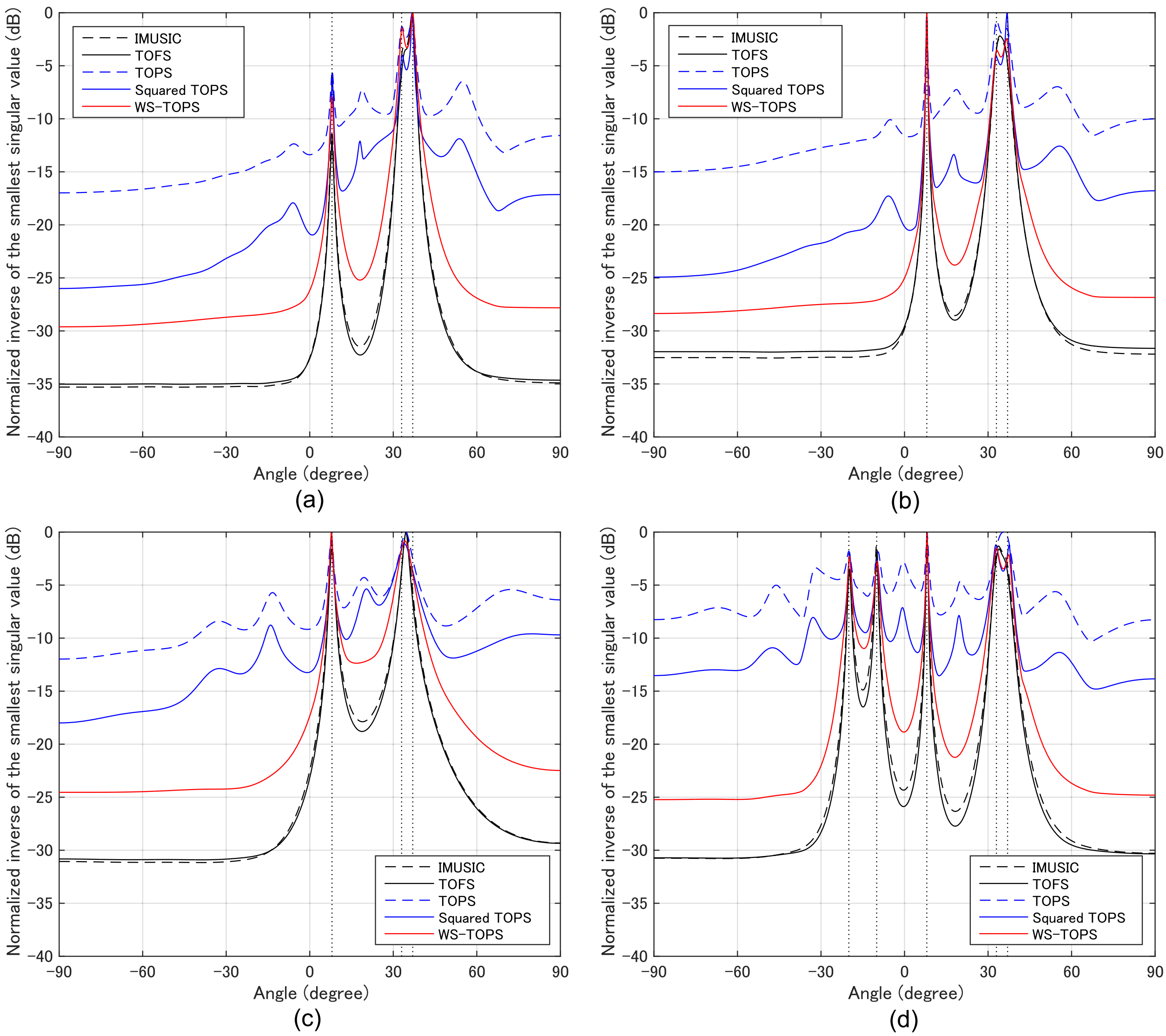

Fig. 2 Examples of spatial spectrum on **a** $M = 10, K = 7$, and $L = 3$ (8°, 33°, and 37°), **b** $M = 10, K = 15$, and $L = 3$ (8°, 33°, and 37°), **c** $M = 6, K = 7$, and $L = 3$ (8°, 33°, and 37°), and **d** $M = 10, K = 7$, and $L = 5$ (-20°, -10°, 8°, 33°, and 37°)

spectrum, while the spatial spectrum of TOPS and Squared TOPS have some false peaks. From Fig. 2, it is also found that WS-TOPS can detect closely spaced signal sources at 33° and 37°, while IMUSIC and TOFS cannot.

As shown in Fig. 2, WS-TOPS can hold the capability to suppress false peaks for all scenarios. The results prove that the WS-TOPS is robust to the system parameters, which are the number of antennas (M), the number of sources (L), and the number of frequency bins (K).

Regarding the computational complexity of WS-TOPS- and TOPS-based methods, we calculate the computational costs to obtain an inverse of the minimum singular value of each direction by using MATLAB. In the case of $M = 10, L = 3$, and $K = 7$, the averaged computation time of WS-TOPS ($K' = 7$) is 2.1 ms, that of WS-TOPS ($K' = 2$) is 0.59 ms, that of Squared TOPS is 0.21 ms, and that of TOPS is 0.17 ms. In the case of $M = 10, L = 3$, and $K = 15$, the averaged computation time of WS-TOPS ($K' = 7$) is 5.1 ms, that of WS-TOPS ($K' = 2$) is 1.47 ms, that of Squared TOPS is 0.44 ms, and that of TOPS is 0.42 ms. Although the actual computational times depend on the calculation system, the results show that the effective costs coincide with the computational complexity described in Fig. 1.

5.3 Probability of resolution

Figure 3 shows the probability of resolution of WS-TOPS and the conventional methods, where the simulation parameters are shown in the caption of the figure. The probability of resolution denotes the probability of successful detection of all signal sources. In other words, we consider a certain result as a successful detection only when all signal sources are detected. If the number of signal sources we detect is less than the actual number of incoming signal sources, we judge the result as a false one in terms of successful detection. As we can see from Fig. 3, the resolution performance of WS-TOPS is between those of Squared TOPS and TOPS. The results indicate that WS-TOPS can achieve better resolution than that of TOPS, TOFS, and IMUSIC, without dependence on the system parameters.

5.4 Root mean square error (RMSE) of estimated DOA

The RMSEs of the estimated DOA of the signal sources calculated by WS-TOPS and the conventional methods are shown in Figs. 4 and 5. For comparison purposes, the Cramér-Rao bound (CRB) [21] is also presented in each figure. Figure 4 shows DOA estimation accuracy of the signal source from 8°. where there is no closely spaced wideband signal sources. As we can see in Fig. 4, WS-TOPS can provide higher DOA estimation

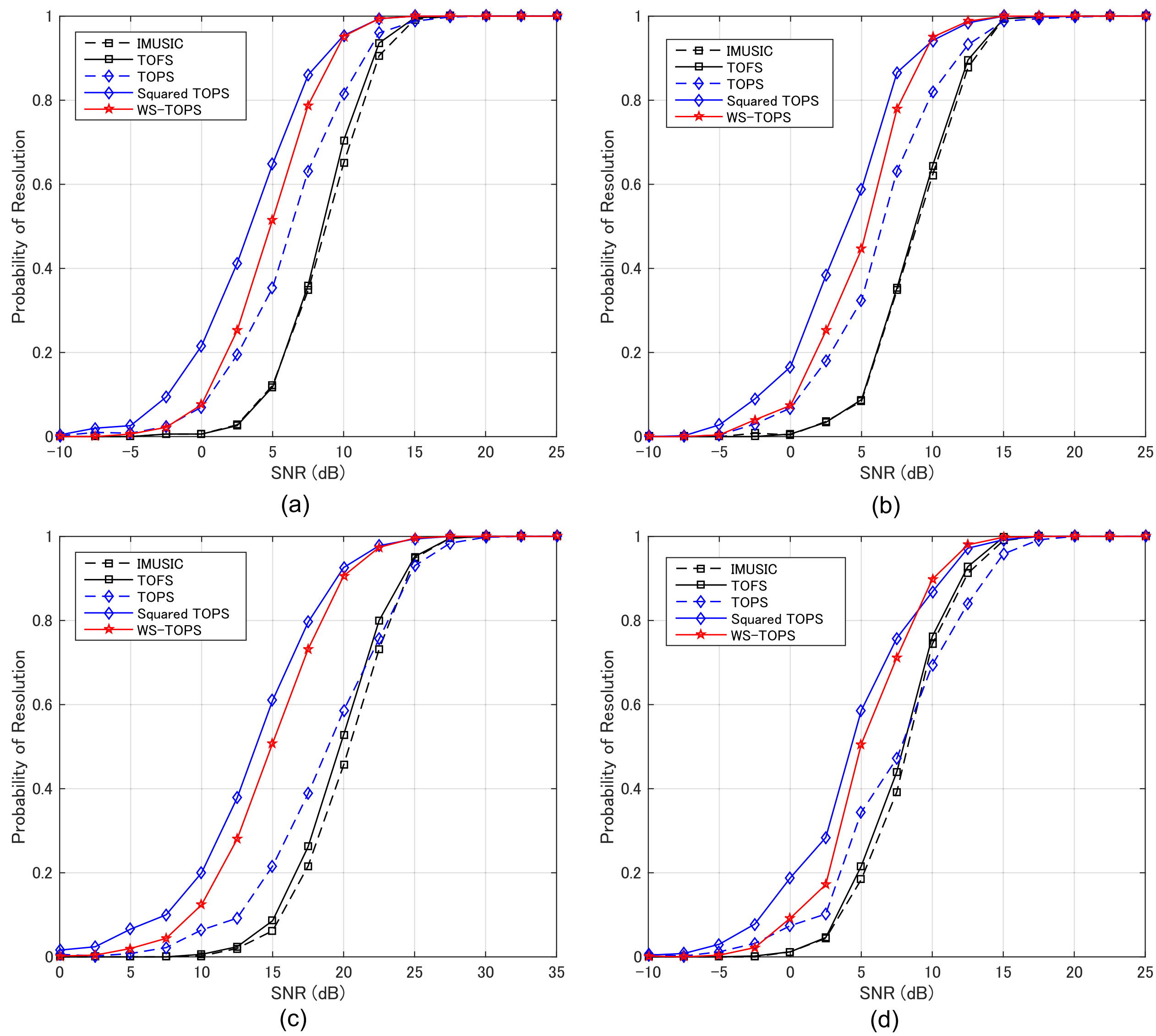

Fig. 3 Examples of resolution on **a** $M = 10, K = 7$, and $L = 3$ (8°, 33°, and 37°), **b** $M = 10, K = 15$,, and $L = 3$ (8°, 33°, and 37°), **c** $M = 6, K = 7$, and $L = 3$ (8°, 33°, and 37°), and **d** $M = 10, K = 7$, and $L = 5$ (-20°, -10°, 8°, 33°, and 37°)

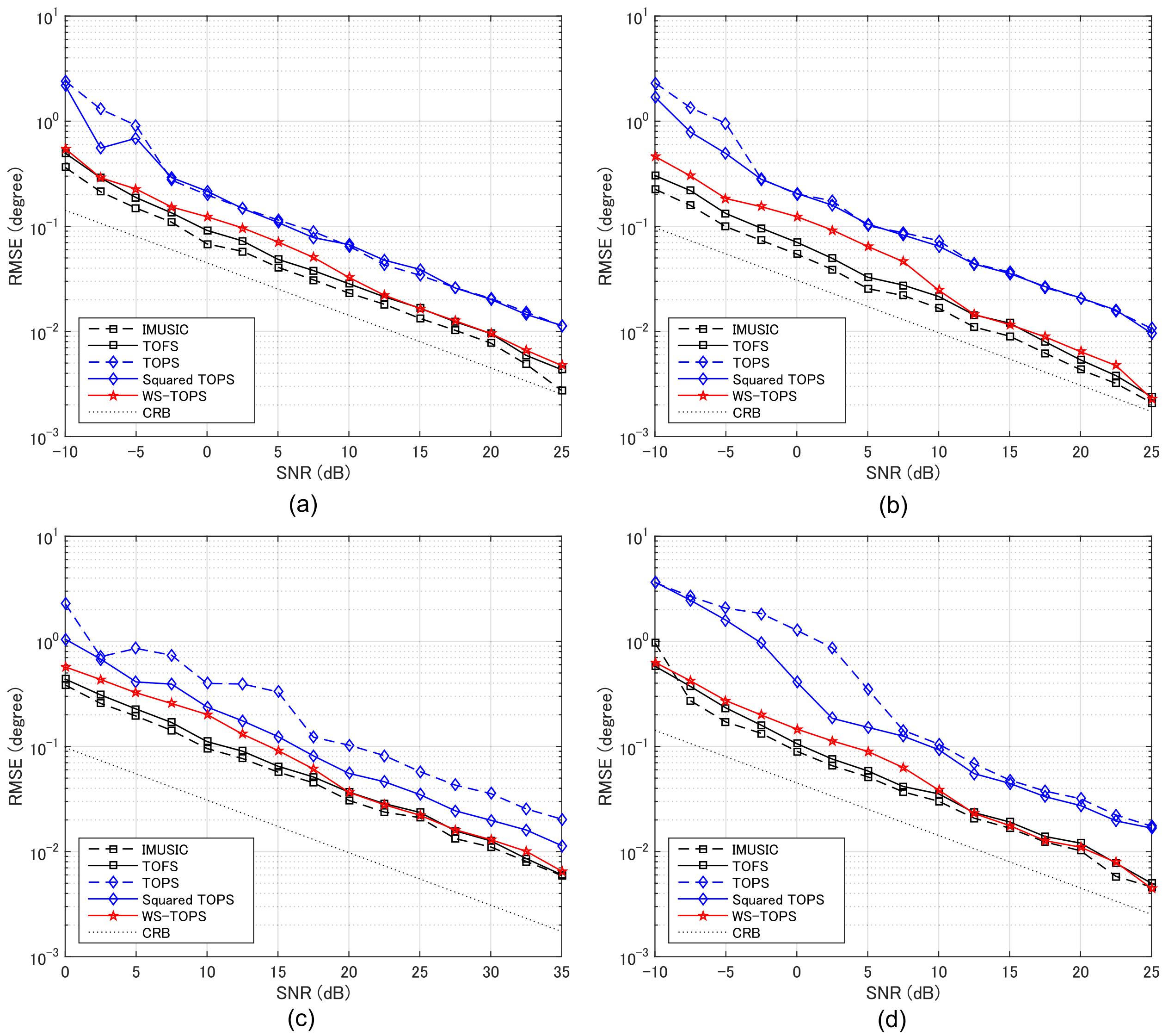

Fig. 4 Examples of RMSEs of estimated DOA of the signal source from 8° on **a** $M = 10, K = 7$, and $L = 3$ (8°, 33°, and 37°), **b** $M = 10, K = 15$, and $L = 3$ (8°, 33°, and 37°), **c** $M = 6, K = 7$, and $L = 3$ (8°, 33°, and 37°), and **d** $M = 10, K = 7$, and $L = 5$ (-20°, -10°, 8°, 33°, and 37°)

accuracy than that of TOPS and that of Squared TOPS in the full range of SNR. It is also found that WS-TOPS shows similar performance to TOFS and IMUSIC in high SNR region. The results that TOPS and Squared TOPS show lower accuracy of DOA estimation than that of IMUSIC coincide with the explanation in [10]. In contrast, the results show that WS-TOPS can improve DOA estimation accuracy and it comes close to that of IMUSIC and TOFS methods in high SNR region.

Figure 5 shows DOA estimation accuracy of the signal source from 33° where there is the closely spaced wideband signal source. From Fig. 5, it is found that WS-TOPS yields the best performance of DOA estimation accuracy for closely spaced wideband signal sources in full range of SNR. The results prove that the DOA estimation accuracy of WS-TOPS is better than the conventional methods and also show that the performance of WS-TOPS is robust to the system parameters.

6 Conclusions

In this paper, we propose a new DOA estimation method for wideband signals called WS-TOPS-based on Squared TOPS. WS-TOPS uses the selective weighted averaging method and the modified squared matrix method to

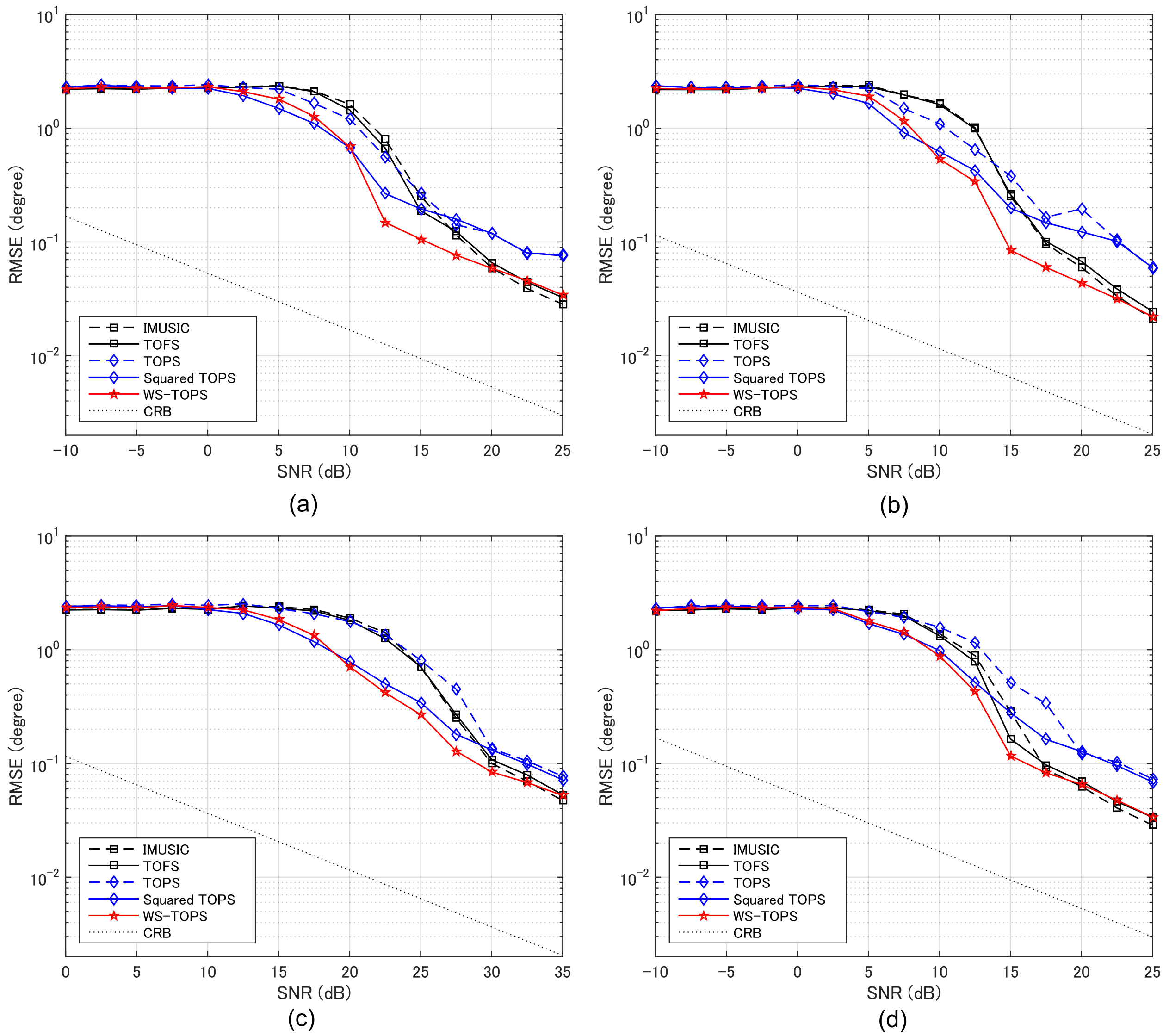

Fig. 5 Examples of RMSEs of estimated DOA of the signal source from 33° **a** $M = 10, K = 7$, and $L = 3$ (8°, 33°, and 37°), **b** $M = 10, K = 15$, and $L = 3$ (8°, 33°, and 37°), **c** $M = 6, K = 7$, and $L = 3$ (8°, 33°, and 37°), and **d** $M = 10, K = 7$, and $L = 5$ (-20°, -10°, 8°, 33°, and 37°)

improve DOA estimation performance. The simulation results show that WS-TOPS can suppress all false peaks in the spatial spectrum, while TOPS and Squared TOPS cannot. It is also shown that the DOA estimation accuracy and the resolution performance of WS-TOPS are better than those of the conventional methods. WS-TOPS can achieve the performance without requiring initial estimates. These results prove that WS-TOPS is effective in estimating the DOA of wideband signal sources.

Competing interests

The authors declare that they have no competing interests.

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