Competition in Markets with Intermediaries

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Chapter 1

Introduction

It is evident that wide areas of today's economy are dominated by a market structure that is characterized by intermediaries building a link between consumers and input suppliers. As exemplary illustrated figure 1.1, for their production process intermediaries procure input products from one or more suppliers, refine or combine them and offer the resulting final product or solution as the output to their customers. To get a broad understanding of the described intermediate goods market, *analyzing the market structure's impact on the participants' economic behavior and interactions is the main objective of this thesis.*

Within intermediate good markets, the existence of intermediaries is obligatory for two main reasons: a highly complex input market and the mutually non-awareness between customers and input suppliers. The input market's complexity results from suppliers usually providing an extensive amount of non-homogenous products which can be horizontally and/or vertically differentiated. Horizontal product differentiation refers to the fact that products can have different degrees of substitutability, complementarity or can be mutually independent. Vertical product differentiation considers different levels of product quality which can usually be increased by taking investments that foster a product's functionality or life period. Besides this the market may be dynamic, i.e., new input suppliers that offer distinguished products may enter, existing input suppliers may leave the market. As a result of the high and constantly changing variety of input goods, customers generally do not have the expertise and time to explore the market of suppliers directly. Hence, they have to approach an intermediary to satisfy their needs.

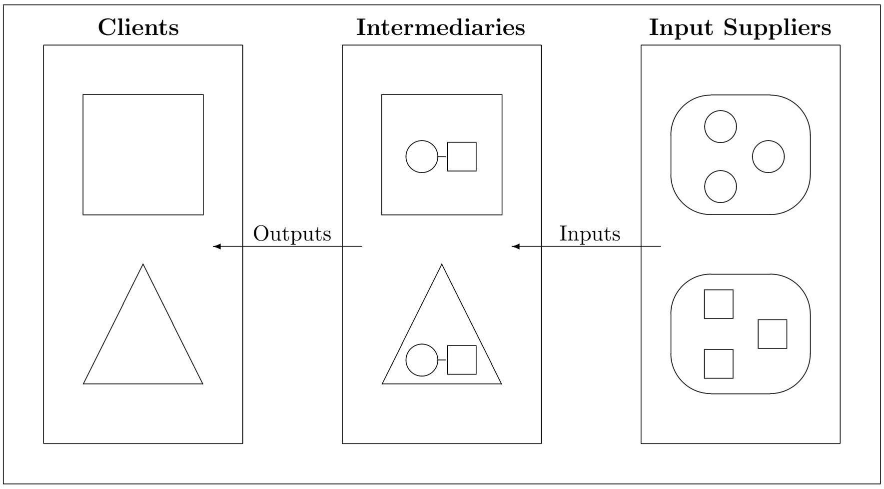


Figure 1.1: Intermediate goods market - exemplary

As a second reason, the presence of intermediaries is essential due to the fact that input suppliers are not necessarily aware of who the customers are and what they require in order to meet their expectations. Thus, a direct contact is unlikely and/or not promising, making suppliers dependent on a connecting instance which is the intermediary.

The challenges coming along with the described market structure are enormous. They arise due to its two-sided structure and the accompanied interdependencies, its highly dynamic nature, but also due to the presence of asymmetric information with respect to input goods' and final products' qualities.

When considering the market's two-sidedness, it can be observed that nearly any decision taken has an impact on all market participants and their behavior. For instance, when determining the sales price for the final product that is charged to the customers, the intermediary always needs to have the prices of input goods, his costs of procurement, in mind. Moreover, the customers' demand for final products on the sales side also has a direct impact on the intermediary's demand for input goods on the procurement side. Additional complexity arises due to the fact that intermediaries not just compete for the customers' demand on the sales side, but simultaneously face competition for input goods on the procurement side. Besides competition between intermediaries, competition may also take place between input suppliers influencing the interaction of all market participants.

In addition to horizontal differentiation input products can also be vertically differentiated, i.e., have different degrees of product quality. This does not only refer to the input goods but also to the final products. Therefore on the one hand the input supplier might be better informed about his product's quality and characteristics than the intermediary. On the other hand the intermediary might have more information about the quality and characteristics of the final good than his customer. In order to assure the functionality of the market, overcoming information asymmetries is a key issue.

Another challenge may arise due to the highly dynamic nature of the intermediate goods market. This dynamic refers to the input market as well as to the market where outputs are traded. For the input market with a permanently changing set of available inputs, the input selection process always needs to be reconsidered by intermediaries in a repeated game. A similar problem arises in a repeated trading scenario in which customers are exposed to a constantly changing market of intermediaries.

The main focus of this thesis is put on investigating the challenges which arise from the two-sided structure of the market as well as from the presence of asymmetric information. The described issues that are related to the dynamic nature of the market are not within scope of the current analysis. Thus, the intermediaries' input selection problem within a dynamic environment is left for further research. Moreover the competition between input suppliers is not covered in this thesis, a further issue that needs to be approached in future.

Within our analysis, methods of non-cooperative game theory are primarily applied, i.e., Nash equilibria as well as subgame perfect Nash equilibria are determined and analyzed and compared, amongst others.

The present work consists of six chapters. After having identified the research question of this thesis and discussed the key challenges of intermediate good markets in Chapter 1, the literature on oligopoly theory is discussed in Chapter 2. The subsequent Chapter 3 which is based on Brangewitz and Manegold (2016), Chapter 4 which is based on Manegold (2016) and Chapter 5 which is based on Brangewitz *et al.* (2014a) examine different variations of intermediate goods markets with different focuses. Finally an outlook is given in Chapter 6 summarizing the established results of this thesis as well identifying fields of further research. In the following, all chapters and their according focal points will be briefly introduced.

1.1 Literature on Oligopoly Theory

Chapter 2 discusses the literature on oligopoly theory which is of interest for our analysis in Chapter 3 and Chapter 4 and can be subdivided in two parts. The first part focuses on settings in which duopolists compete simultaneously for the demand of customers and set either prices or quantities. It will be shown that based on the fundamental contributions of Cournot (1838) and Bertrand (1883) a huge strand of literature developed over the years. For instance, in their seminal work, Singh and Vives (1984) establish a setting of a differentiated duopoly and compare the outcomes of price and quantity competition. Their approach is the basis of further extensions introducing and analyzing the choice of process or product innovation as for example considered by Motta (1993) or Symeonidis (2003). Discussing the impact of the presence of a strategic input supplier is another field that is for instance considered by Häckner (2003).

The second part of Chapter 2 refers to the modeling of Von Stackelberg (1934). It considers markets where duopolists do not compete simultaneously, but sequentially for the demand of customers by choosing production quantities for their product. The focus of a wide range of publications in this area is put on the analysis of duopolists' first- and second-mover advantages when competing. A first seminal contribution in this direction was done by Gal-Or (1985) and further extended. Those extensions go in the direction of differentiated duopolies, comparisons of simultaneous- and sequential-move games as well as the introduction of an input supplier amongst others. In contrast to Gal-Or (1985) in which a player's role, whether being the first- or second-mover was exogenously given, another strand of literature discusses a setting in which competitors may choose their according position. First steps in this area were taken by Hamilton and Slutsky (1990) and extended in directions of, e.g., asymmetric information between market participants.

Although literature covers a wide range of topics in terms of oligopoly theory, what is has not been conducted so far is the analysis of a two-sided horizontally and vertically differentiated market in which intermediaries simultaneously compete for the customer's demand and for input supplies. Especially the impact of a strategic input supplier which plays a key role in our setting has not been analyzed so far. This is true for both, intermediaries' simultaneous (Cournot and Bertrand competition) as well as sequential competition (Stackelberg competition). Hence, this contribution is a consequent continuation of the existing literature.

1.2 Simultaneous Competition of Intermediaries

Chapter 3 is based on Brangewitz and Manegold (2016) and considers a differentiated intermediate goods market. Emphasis is put on the impact of simultaneous competition for resources and customers on the market outcome. Moreover, the intermediaries' incentives to invest in product quality under Cournot and Bertrand competition is investigated.

The market's differentiation refers to horizontal and vertical product differentiation. Horizontal product differentiation describes the fact that products may have different degrees of substitutability or complementarity. Vertical product differentiation is considered by allowing for different product qualities of the intermediaries' products. Each of the intermediaries has the chance to foster his product's quality by investing in product innovation and thus potentially achieves a competitive advantage towards his competitor. Therefore, intermediaries must not necessarily be symmetric. Asymmetries may not just be present due to different product qualities, but also by reasons of different productivities of intermediaries. Productivities are exogenously given in our model and describe the amount of input goods necessary to produce a unit of output. Hence, it directly impacts the costs of procurement.

The basic setting discussed in Chapter 3 can be described as follows: two intermediaries procure inputs from a monopolistic input supplier, refine them within a production process and offer the final product to a representative customer. When competing for the customer's demand intermediaries both choose either sales prices or production quantities simultaneously. Moreover the profit maximizing input supplier is able to freely select his input price which is charged to both of the intermediaries. Depending on the price level chosen, the supplier has the chance to exclude one or both of the intermediaries from the market. Three different scenarios may arise: In a first scenario, given the chosen input price is sufficiently low both intermediaries are able to achieve non-negative profits and thus are willing to procure inputs resulting in a duopoly intermediate market. In a second case, given intermediaries are asymmetric with respect to their product quality and/or productivity and the input price has a certain intermediate level, the less competitive intermediary will be driven out of the market. In this scenario only one of the intermediaries procures inputs on the sales side and offers products on the procurement side, resulting in a monopoly. Finally, is the chosen input price sufficiently high, no intermediaries is willing to procure inputs and the customer will not be served.

Within our setting, decisions are taken within two main decision stages. Due to its nature as a long-term strategic decision, the intermediaries' choice of investing in product innovation is considered in stage 1, followed by the intermediaries' competing for the customer's demand in stage 2.

In stage 1 intermediaries take decisions about the level of investment which increases their product quality (product innovation). In general, we investigate the intermediaries' incentives to invest in product quality increasing innovation. In this context, we determine equilibrium conditions for product quality on the investment costs and compare them for Bertrand and Cournot competition. We find that when considering linear investment costs, in equilibrium product quality investments are always chosen at a minimum or maximum level of product quality. It turns out that there exist product qualities and degrees of horizontal product differentiation for complements such that asymmetric investment equilibria fail to exist. Moreover, there also exist product qualities and degrees of horizontal product differentiation for substitutes such that existence of asymmetric equilibria can be guaranteed if the investment costs are chosen accordingly.

In stage 2 intermediaries face simultaneous competition for resources and the demand of a representative customer. In this context we analyze the impact of intermediaries' asymmetries on the market outcome. Input suppliers choose their input price first, followed by either price or quantity competition of the intermediaries. It turns out that there exist product quality and productivity differences such that for quantity competition only one intermediary is willing to procure inputs from the input supplier, while for price competition both intermediaries are willing to purchase inputs. Besides this, we identify conditions in terms of horizontal and vertical product differentiation as well as productivities for which input prices within quantity competition are higher, lower or equal than input prices within price competition. Additionally, when comparing intermediaries' equilibrium choices of quantities, prices and the resulting profits we find that there exist asymmetries such that equilibrium prices and profits (for non-substitute products) under Cournot competition are lower and equilibrium quantities higher than under Bertrand competition. This result shows that the relations established in Singh and Vives (1984) must not necessarily be true in a model with endogenous input market and asymmetric intermediaries.

1.3 Sequential Competition of Intermediaries

Chapter 4 is based on Manegold (2016) and discusses a similar model as in Chapter 3 but changes the rules of interactions. Two intermediaries procure inputs from a supplier which are refined and finally offered to a representative customer. In contrast to Chapter 3, intermediaries compete for the customer's demand à la Stackelberg and choose production quantities sequentially. The type of competition in which sales prices are chosen is not considered. We again allow intermediaries to be asymmetric with respect to their productivity and product quality. The degree of productivity is exogenously given whereas product quality can be increased by taking an according investment. The input supplier strategically selects his input price and is able to either exclude one or both of the intermediaries from the market.

Within Chapter 4 we are interested in the impact of the two-sided market structure on the equilibrium outcome when intermediaries choose production quantities sequentially.

The contribution consists of two stages. In the first stage, we give a first outlook on the topic of innovation in which the incentives of intermediaries to invest in product quality is given. In this context we find that for linear costs of innovation intermediaries always choose a level of investment which results in either a minimum or maximum product quality. Besides this, we give a further outlook for product innovation which suggests a similar approach as discussed in Chapter 3.

In the second stage, we discuss the simultaneous competition of intermediaries for input supplies and the customer's demand. For a setting in which production quantities are selected sequentially, we are interested in the impact of simultaneous competition for resources and customers on the market outcome. Within the analysis of our model we determine subgame perfect Nash equilibria and compare the equilibrium choices of an intermediary when being the first-choosing Stackelberg leader with his equilibrium choices when being the second-choosing Stackelberg follower. Furthermore, we are interested in the input supplier's strategic behavior when selecting his input price and the resulting influence on the market outcome.

From previous contributions it is known that the introduction of a sequential game has an impact on the equilibrium production quantities which are offered to the representative customer. Furthermore, the successive approach influences the optimal price choice of the input supplier. Interestingly, we find that there exist asymmetries between intermediaries with respect to product quality and productivities such that given the less competitive intermediary is the Stackelberg leader, he will be driven out of the market whereas a less competitive Stackelberg follower is allowed to stay in the market, ceteris paribus.

Besides this, a comparison of intermediaries' equilibrium output quantities shows that in equilibrium a more competitive Stackelberg leader produces and offers a larger quantity than a more competitive Stackelberg follower. This relation is also true for substitute products when comparing the equilibrium production quantities of a less competitive leader and a less competitive follower. For complements, however, there may exist asymmetries for which the equilibrium production quantities of a less competitive follower are higher than the equilibrium production quantities of a less competitive leader. This outcome shows that the result of Von Stackelberg (1934) stating that the equilibrium production quantities of a Stackelberg leader are always at least as high as the equilibrium production quantities of a Stackelberg follower does not necessarily hold in a differentiated duopoly setting.

When analyzing the input supplier's equilibrium price choice with equal productivities, we obtain that if a more competitive intermediary is in the leading position the input price as well as the input supplier's profit is higher than if the intermediary was the more competitive follower.

1.4 Contract Design for Composed Services

The final Chapter 5 is based on Brangewitz *et al.* (2014a) and as well considers an intermediate goods market. In contrast to the previous chapters, a slightly different approach is used and focus is put. The market we have in mind is the market of IT services, characterized by a tremendous number of single services, in which clients usually search from a solution-oriented perspective.

As in previous chapters, the discussed model consists of three different types of market participants: clients, intermediaries and service providers. We focus on the contract design problem of one intermediary and two strategically interacting service providers. Essentially, the intermediary procures complementary services from both of the service providers, which can either be of high or of low quality. Producing high-quality goods requires higher costs of production than producing low-quality goods. The intermediary combines the procured services and offers the final product to his client. The quality of the service composition is directly impacted by its components and can also either be high or low. In our setting we assume clients to have a demand for both quality levels (usually at different prices).

By using a non-cooperative game-theoretic model, we analyze the incentives for high- and low-quality composed services to be an equilibrium outcome of the market. In this context we consider a one shot as well as a repeated game. Basically, the intermediary initially offers a contract to both of his service providers specifying the demanded product's quantity and quality. Furthermore transfer payments are defined which are depending on the quantity delivered and on the intermediary's quality report with respect to the composed service. In a further step, both providers take a decision about quantity and quality they would like to produce. The received inputs are combined by the intermediary and delivered to the clients as a final product. The intermediary strategically reports the final product's quality (not necessarily truthful) and pays the service providers accordingly.

It turns out that equilibria which result in low-quality products can be obtained in the short run and in the long run, whereas those with high quality can only be achieved in the long run. In our analysis we explicitly determine the intermediary's time preferences within an infinitely repeated game that are required to establish high quality in the market. Interestingly, it could be seen that even if selling high quality is profitable for the intermediary, the emergence of high-quality composed services on the market still crucially depends on the intermediary's discount factor. Additionally, we derive optimal contracts for implementing high- or low-quality composed services. Thus, we could finally show that in intermediate goods markets cooperation which leads to high quality can be established, although product qualities of input goods are not perfectly observable.

Chapter 2

Literature on Oligopoly Theory

The following section displays a literature overview that in the first part considers the classical approach of oligopoly theory in which actions are taken simultaneously while in the second part scenarios in which actions are taken sequentially are discussed. This literature overview primarily refers to the Chapters 3 and 4, in which Cournot and Bertrand competition as well as Stackelberg competition play a major role. We start with discussing the literature for oligopoly models in which players choose either quantities or prices simultaneously and introduce a sequential approach thereafter.

2.1 Oligopoly Theory - Simultaneous Competition

Two fundamental contributions in oligopoly theory were done by Cournot (1838) and Bertrand (1883), in which two firms produce a homogenous good and compete by selecting output quantities in the former and prices in the latter model. Their settings were continuously extended in different areas in the years after. Analyses of differentiated duopolies under Cournot and Bertrand competition date from the late 70's. Referring to Dixit (1979), in their seminal work Singh and Vives (1984) consider a linear customer demand structure with constant marginal costs allowing for complements and substitutes and compare the equilibrium outcomes of Bertrand and Cournot competition. It is shown that in equilibrium Bertrand competition is more efficient than Cournot competition, as it generates a lower price and a higher output level. According to Singh and Vives (1984) this result is independent of whether goods are substitutes or complements and regardless of demand structure's degree of symmetry. Thus, Bertrand competition implies a higher consumer and total welfare. Moreover they show that due to higher profits firms prefer to select quantities (prices), if goods are substitutes (complements). Cheng (1985) confirms the results of Singh and Vives (1984) within a more general environment. He uses a geometric analysis of duopoly in which no special form of demand structure needs to be assumed.

The classical approach of duopolies with firms producing homogeneous goods was later on extended by allowing for a number of n product varieties as for example discussed by Vives (1985), Okuguchi (1987), Häckner (2000) and Amir and Jin (2001). Within a differentiated oligopoly with substitute goods Vives (1985) shows that if demand is symmetric and Cournot as well as Bertrand equilibria are unique, Bertrand delivers smaller prices and profits, but larger quantities than Cournot. If the number of product varieties grows, Bertrand as well as Cournot equilibria converge to the efficient outcome in which prices equal marginal costs.

In contrast to Singh and Vives (1984), Okuguchi (1987) resigns the assumptions of products' substitutability and demand symmetry. For the more general setting within differentiated oligopolies they focus on the conditions that are necessary to make a clear-cut comparison between prices under Bertrand and Cournot competition. It is shown that if the set of products can be divided into two subsets such that products are substitutes within each subset and complements across subsets, the unique Bertrand price vector is lower than any Cournot price vector in equilibrium.

A slightly different model than discussed in the previously mentioned articles is analyzed by Häckner (2000), who not just considers horizontal (substitutes and complements) but also exogenously given vertical product differentiation (product quality). He points out that the results of Singh and Vives (1984) are sensitive towards the duopoly assumption. It is shown that in a setting with more than two firms prices may be lower under Cournot than under Bertrand competition, namely if goods are complements and quality differences are large. Moreover if goods are complements, Bertrand profits are higher than Cournot profits. If goods are substitutes and quality differences are large, high-quality firms might prefer to select prices rather than quantities.

The work of Amir and Jin (2001) goes in a similar direction and claims to provide further insight into the scope of the previous results in the literature. They support the view of Singh and Vives (1984) that the Bertrand equilibrium is more competitive than the Cournot equilibrium with lower prices and higher outputs, but also illustrate its limitations. By providing counter-examples they make clear that a consideration of strategic complementarity is necessarily required to make a clear-cut comparison of prices and quantities for both competition types. Two of their examples show that in an oligopoly with linear demand and complementarity of quantities (prices), one price (output) can be lower (higher) in Cournot equilibrium. In a third example in which the condition of strategic complementarity of either quantities or prices is removed, they show that one price is lower and one output is higher in Cournot equilibrium. Hence, is the key assumption of strategic complementarity is dropped, both the output and the price comparisons are ambiguous.

Tremblay and Tremblay (2011) state, that in the real world there exist markets in which some firms choose prices whereas other firms choose quantities. As claimed in Tremblay *et al.* (2013) such a behavior can be observed in a market for small cars where Saturn and Scion dealers set prices and Honda and Subaru dealers choose quantities. For this reason, they consider a setting of differentiated duopoly in which one firm competes in prices, the other firm in quantities. The analysis of Tremblay and Tremblay (2011) aims to investigate the impact of the degree of differentiation on the static Cournot-Bertrand equilibrium. They find that if on the one hand products are sufficiently differentiated, both firms survive and the equilibrium is stable. If on the other hand goods are homogenous, only the quantity-choosing firm survives. The according outcome in this scenario is perfectly competitive, i.e., prices equal marginal costs. Thus, the presence of a price-selecting firm ensures a competitive outcome in which only one firm is producing.

The work of Zanchettin (2006) discusses a differentiated duopoly with linear cost and demand functions. Similar to our approach within Chapter 3, he implements an exogenously given asymmetric productivity and product quality. He finds that when having strong asymmetries and/or products are weakly differentiated, the more productive firm's profit and industry profits are higher under Bertrand competition. This outcome is contradictory to the results of Singh and Vives (1984). Moreover with a declining degree of differentiation the productive firms' and industry's profits shrinks.

Within a similar model, Ledvina and Sircar (2011) have a focus on the number of active firms, i.e., firms producing a positive quantity in Bertrand Cournot equilibrium. It is shown that Cournot competition always yields a higher number of active firms compared to Bertrand competition. Moreover, a high degree of product differentiation results in more active firms compared to a setting with homogeneous goods. This is true for both types of competition.

The fundamental approach of differentiated oligopolies as displayed in the former paragraph was further extended by endogenous product as well as process innovation. A vertically differentiated duopoly in which endogenous product innovation leads to higher product quality is for example analyzed by Motta (1993) and Symeonidis (2003). Both contributions consider a two-stage game where a decision about product quality is carried out in stage one. In stage two, firms compete in either prices or quantities when selling their product. In contrast to Symeonidis (2003) the focus of Motta (1993) lies in the differentiation between fixed and variable innovation costs. He finds that for any of the competition types, firms always choose distinct qualities, where in general Bertrand competition induces more product differentiation than Cournot competition. In the case of variable costs innovation firms' profits are higher when competing in quantities. However, when assuming fixed costs firms' profits are higher under price competition, which is contradictory to the results of Singh and Vives (1984) and others. Finally, it is shown that total welfare is higher within price competition, supporting the conventional wisdom.

Differently, Symeonidis (2003) considers not just a vertical, but also a horizontal differentiation, i.e., goods that can either be substitutes to several degrees or mutually independent in his setting. He uses a quality augmented version of Singh and Vives (1984) customer utility function and allows for spillovers of product quality investments across firms. It is shown that under price competition, investments in product innovation, product prices and firms' profits are always higher, compared to quantity competition which is again contrary to the classical results. Additionally, if on the one hand spillovers are weak or products are sufficiently differentiated, the consumer's surplus as well as the total welfare is higher under Bertrand competition. If on the other hand spillovers are strong and products are not too much differentiated, Cournot competition delivers a higher outcome of the former mentioned.

In the previous section we have seen that firms may have the possibility to increase the customer demand by investing into their product's quality. Besides this, one can think about a further investment that fosters a firm's production technology and thus, its productivity. Such a productivity increasing process innovation has drawn attention to another branch of literature and is for example analyzed by Bester and Petrakis (1993) and Qiu (1997). Related to the approaches of Motta (1993) and Symeonidis (2003), they examine the outcome of Bertrand and Cournot competition in a two-stage game. In stage one, firms make a process innovation decision, in stage two, they compete on the market in either prices or quantities. In contrast to Qiu (1997), Bester and Petrakis (1993) only allow one firm to make an investment into innovation within the first stage. They show that on the one hand when goods are imperfect substitutes the investment of process innovation is inefficiently low in both types of competition. Furthermore Cournot competition yields a higher degree of innovation than Bertrand competition. If on the other hand goods are close substitutes, overinvestment may occur and Bertrand competition leads to higher innovation. Qiu (1997) introduces spillover effects of investments across firms. He finds, that in his model Cournot competition generates higher incentives for process innovation than Bertrand competition, confirming the results of Bester and Petrakis (1993) for imperfect substitutes. Additionally he shows that Bertrand yields a lower price, larger output and is more efficient.

In a setting with homogenous goods, Boone (2001) analyzes the relationship between the intensity of competition and the incentive to invest into process innovation for firms with different cost levels. One would expect that if competition increases, firms will improve their position by reducing costs through process innovation. In contrast to the conventional wisdom it turns out that the relation between competition intensity and the incentive to innovate must not necessarily be monotone. The result makes clear that the findings of Qiu (1997) may be overturned when considering asymmetric firms. Besides this Boone (2001) finds that given competition intensity is low, the low cost firm invests into innovations whereas if competition is high, the high cost firm is the innovator.

In a homogenous and symmetric oligopoly, Delbono and Denicolò (1990) compare equilibrium R&D investments under Bertrand and Cournot competition. They discuss a dynamic setting, in which for instance the realization of a cost-reducing technological improvement depends on the investment in R&D. It turns out that even if Bertrand competition yields higher R&D investments, social welfare net of R&D costs may be greater under Cournot competition. Delbono and Denicolò (1990) explain that in price competition too many resources may be invested in R&D, leading to inefficiencies in the dynamic setting.

There are several contributions in literature that consider both, product and process innovation and discuss their relation. First steps were done by Bonanno and Haworth (1998a) who analyze two major issues within a vertically differentiated duopoly, in which either a high- or a low-quality firm is able to innovate. Their first focus is on the relationship between competition type and profitability of process innovation, which was already covered by Bester and Petrakis (1993) in a horizontally differentiated duopoly. It is shown that independently of the degree of differentiation, process innovation is more profitable for firms when competing in quantities than when competing in prices. In contrast to Bester and Petrakis (1993), this result also holds for perfect substitutes. In a further step Bonanno and Haworth (1998a) not just allow for process, but also for product innovation. In this context they discuss the influence of competition type on the decision of whether investing in product or process innovation. It turns out that given process/product innovation is profitable and on the one hand the high-quality firm is the innovator three possible cases may arise. In a first scenario, in Bertrand as well as Cournot competition the same type of innovation is selected. In a second and third scenario, if the firm's choice is dependent on the competition type, Bertrand competition implies product innovation whereas Cournot competition leads to process innovation. Is on the other hand the low-quality firm the innovator the opposite is true.

A similar approach in which not just one, but both firms take decisions between process and product innovation simultaneously is considered by Filippini and Martini (2010), Lin and Saggi (2002) and Weiss (2003). Extending the results of Bonanno and Haworth (1998a), for a differentiated duopoly model with a low- and a high-quality firm in which decisions about process and product innovation are taken simultaneously, Filippini and Martini (2010) find that for both Bertrand and Cournot competition three equilibria may arise. Two symmetric equilibria in which both firms choose the same kind of innovation and one asymmetric equilibrium in which different innovations are selected can be obtained. For the asymmetric equilibrium, within Bertrand competition the high-quality firm selects product whereas the low-quality firm prefers process innovation. On the other hand Cournot competition yields the opposite effect, namely, if a firm produces high-quality products it chooses process, if it is a low-quality firms it prefers product innovation. Hence, the low-quality firm strives towards becoming the quality leader. A further result states that both firms rather tend to select product innovation under Cournot competition, showing that the result for a high-quality firm of Bonanno and Haworth (1998a) does not hold in a setting in which more than one firm is able to innovate.

The duopoly of Lin and Saggi (2002) do not pre-define a high- or low-quality firm. Within a three-stage game, in stage one and two the investment decision about product and process innovation takes place respectively. In the final stage firms compete in either prices or quantities, which is exogenously given. Their results show, that the incentives to invest in process innovation increases in the level of product differentiation and firms tend to invest more in product innovation, when also having the opportunity to invest in process innovation. In addition firms competing in quantities prefer process innovation, which supports the result of Qiu (1997).

Weiss (2003) investigates the relation between the degree of competition and the decision of a firm to invest into product and process innovation in a stage game. Under the assumptions that process and product innovations are strategic complements with diminishing returns of product innovations, she finds that in case of close substitute goods (fierce competition), firms prefer to invest in product innovation. If goods are highly differentiated (either vertically or horizontally) and thus competition is less serious, firms rather choose process innovation. In contrast to most of the works in this field of research, the type of competition is also a firm's choice variable.

Similar articles that are relevant in this area are for instance Rosenkranz (2003) and Battaggion and Tedeschi (2006).

One issue that has been neglected in the above mentioned contributions is the market power of an input supplier and is, with different focuses, addressed by Häckner (2003), López and Naylor (2004), Correa-López (2007), Pinopoulos (2011), Mukherjee *et al.* (2012) and Manasakis and Vlassis (2014), amongst others. Related to our modeling within Chapter 3 and Chapter 4, but with a different objective, Häckner (2003) considers a differentiated two-level market structure incorporating intermediaries and input suppliers and analyzes the impact of vertical mergers with an emphasis on social welfare. In a first step input suppliers compete in quantities and thus, by considering the inverse demand function, verify the according prices of their input goods. Given those input prices intermediary firms face Cournot competition thereafter. If a vertical merger takes place, the merging supplier exits the input market. It is found that the influence of vertical integration on welfare is depending on relative market shares. If the number of competitors within the input market is relatively higher than the number of competitors in the intermediary market and/or intermediaries' products are relatively close substitutes a vertical merger does not have a negative effect on social welfare.

In a downstream differentiated duopoly in which the input price is the result of a strategic bargaining process between downstream firm and upstream supplier López and Naylor (2004) compare Cournot and Bertrand equilibria. It is shown that when the differentiated duopoly game is played in imperfect substitutes, the standard results that under Bertrand competition profits are lower than under Cournot competition is reversible. Extending the approach of López and Naylor (2004), Correa-López (2007) focuses on a vertically differentiated duopoly, using a simplified version of the linear model by Dixit (1979) and Singh and Vives (1984). By introducing a more complex industrial structure, Correa-López (2007) assesses the robustness of the results of Singh and Vives (1984). The fundamental difference between both approaches is given by the incorporation of the market's supply side and the resulting endogenous costs of production. Besides this, in the contribution of Correa-López (2007) the type of competition is not exogenously given as in Singh and Vives (1984), but can be chosen by the duopolists. Thus, scenarios may arise in which one duopolist chooses prices whereas the other duopolist selects output quantities. The main question to be answered is, whether the introduction of an input market influences the results of Singh and Vives (1984) with respect to the duopolists' equilibrium profits in Bertrand and Cournot competition. Within a three-stage game, in stage one two intermediaries choose whether to compete in prices or quantities, followed by a stage in which input prices are bargained either centralized or decentralized with two input suppliers. In stage three the intermediary firms compete in the market according to the competition type selected in stage one. The analysis shows that if labor is the input factor and wages are the result of a decentralized bargaining process, choosing quantity competition is not necessarily a dominant strategy for firms, when producing substitutes. This indicates that the result of Singh and Vives (1984), claiming that Cournot competition always yields higher profits does not hold in a setting with input suppliers. If the wage bargaining process is however centrally organized, the result of Singh and Vives (1984) applies. Additionally for the more general case of profit maximizing input suppliers and exogenously given quality differences in the duopoly, the intermediaries may favor to compete both in quantities. Besides this, a scenario in which the high-quality firm chooses to compete in price and the low-quality firm selects output quantities, may also arise under certain conditions. Whether the first of second scenario arises is dependent on the product's degree of substitutability as well as the extent of vertical product differentiation and distribution of bargaining power when negotiating input prices. Welfare, nevertheless, is always higher when duopolists choose to compete in prices.

Differently, a focus on the input supplier's optimal price choice is put in the contribution of Pinopoulos (2011). Related to our modeling of Chapter 3 and Chapter 4, he considers a setting with an upstream market including a monopolistic supplier who provides an intermediate input good and an imperfectly competitive downstream market with retailers producing differentiated products. The article analyzes the supplier's optimal price choice depending on the downstream market structure, i.e., whether market entry is free or restricted. It is found that given market entry is free, the optimal input price is depending on the number of downstream firms. Furthermore, given the number of downstream firms is endogenously given (free entry) the supplier charges a lower input price compared to the case in which the number of downstream firms is determined exogenously (no-entry condition).

More recently Alipranti *et al.* (2014) analyze a vertically related market in which a monopolistic input supplier trades with two competing duopolists via two-part tariffs, i.e., a wholesale price and a fixed fee. Duopolists can either compete by setting prices or quantities which is exogenously given. When comparing the outcome of Bertrand and Cournot competition, it turns out that the standard results of Singh and Vives (1984) do not apply for the introduced model. It is shown that Cournot competition yields lower prices and higher output than Bertrand competition and therefore yields a more competitive market. They explain that the reversal arises from the fact that the monopolistic input supplier has stronger incentives to foster the intermediary firms' aggressiveness when they select quantities rather than prices. This leads to lower input prices for intermediaries and thus lower sales prices and higher outputs. Moreover Alipranti *et al.* (2014) found that Cournot competitions delivers higher intermediaries' profits as well as consumers' and total welfare.

Closely related to our modeling in Chapter 3 and Chapter 4, Mukherjee *et al.* (2012) analyze an intermediate goods market in which a profit-maximizing input supplier interacts with two unequally productive intermediaries who provide a homogeneous final good. The price-setting input supplier has the possibility of either discriminating intermediaries in prices or choosing a uniform price. In their contribution Mukherjee *et al.* (2012) emphasize the importance of the intermediaries' productivity differences as well as the input supplier's pricing strategy when comparing profits under Bertrand and Cournot competition. On the one hand, it is shown that under Bertrand competition the input supplier's profit, the aggregated profit of the input supplier and the intermediaries and the social welfare are always higher than under Cournot competition. It turns out that this outcome is regardless of whether the input supplier discriminates in prices or not. On the other hand Mukherjee *et al.* (2012) make clear that a comparison of the intermediary's profits across competition types can only be done conditional on intermediaries' productivity differences as well as on the input supplier's pricing strategy. Given that intermediaries are asymmetric in terms of productivity and the supplier charges uniform prices, their profits may be higher under price competition. Thus, the result of Singh and Vives (1984) in terms of profits does not necessarily hold in a vertical structured setting. Moreover it is verified that the results of López and Naylor (2004) and Correa-López (2007) cannot be confirmed if the input supplier charges a uniform price and intermediaries differ in productivity.

Differently, Manasakis and Vlassis (2014) discuss an oligopoly in which each of the two intermediaries is exclusively assigned to a supplier from whom he procures his inputs that are needed to produce a final product. The way intermediaries compete in the market (price or quantity competition) is endogenously determined in a first stage and fixed within a renegotiation-proof contract between intermediary and supplier. Moreover input prices are the result of a negotiation process. In the second stage two intermediaries compete in the market according to the type of competition that they committed to in stage one. In this context it is found that if goods are substitutes (complements) Cournot (Bertrand) competition is credibly sustained by the intermediary-supplier pairs. This outcome confirms the results of Singh and Vives (1984). As it holds independently of the extent of product differentiation and irrespectively of the distribution of bargaining power between the paired intermediary and supplier it however contradicts for instance López and Naylor (2004) and Correa-López (2007).

2.2 Oligopoly Theory - Sequential Competition

A new branch of literature originated from Von Stackelberg (1934) who considers a homogeneous product market in which identical duopolists select production quantities not simultaneously (as in Cournot (1838)), but one after the other. He finds that in equilibrium the first-choosing duopolist achieves higher profits than the firm taking decisions thereafter. The approach was later extended, e.g., towards differentiated product markets in which quantities and prices were chosen by duopolists. During the years two main strands of research (with some overlap) that refer to Stackelberg competition emerged.

The first strand of literature discusses the appearance of first- and second-mover advantages and has its foundations in contributions of Gal-Or (1985) and Dowrick

(1986). Within a sequential game in which two identical players with globally concave profit functions choose quantities or prices, the general results of Gal-Or (1985) show that if the player's best reply function is downwards (upwards) sloping the leader's profits are higher (lower) than the follower's profits.

Within a more general model Dowrick (1986) analyzes Nash (follower-follower) games, Stackelberg leadership and Stackelberg warfare with a focus on the comparison of Stackelberg leaders' and followers' profits. Similar to Gal-Or (1985), his analysis is based on the properties of reaction functions. The main result yields that if reaction functions are downward-sloping, duopolists will disagree over the choice of roles and both will prefer to be in the leading position. When having similar profit functions and the reaction functions are upward-sloping, duopolists will disagree about positions and prefer the other player to be the leader. Further contributions going in a similar direction were done by Schoonbeek (1990), Ono (1978) and Ono (1982).

As the impact of product differentiation plays a major role in their modeling, the work of Boyer and Moreaux (1987b) has a little different focus than Gal-Or (1985) and Dowrick (1986). When considering a differentiated duopoly in which competitors compete in quantities or prices, they address situations in which duopolists' decisions are taken simultaneously as well as sequentially and compare the according equilibrium outcomes. The results of Boyer and Moreaux (1987b) show, that for any role of a duopolist (Stackelberg leader, Stackelberg follower, Nash competitor), setting quantities (prices) always implies higher profits if goods are substitutes (complements). Furthermore they find that customer welfare is always maximal, if duopolists compete in prices. When considering total surpluses, the simultaneous Bertrand competition delivers highest outcomes in equilibrium, followed by price Stackelberg, mixed Nash, quantity Stackelberg and Cournot equilibria for complements and substitutes.

Asymmetric cross-price effects are introduced by Banerjee and Chatterjee (2014) who consider a horizontally differentiated duopoly in which quantities or prices are selected sequentially. They find that given goods are complements (substitutes) and the negative (positive) effect of the follower's price on the leader's quantity is larger (smaller) than the negative (positive) effect of the leader's price on the follower's quantity, the follower achieves higher profits than the leader. This is true as in the described cases, the follower is able to sell higher quantities for a higher price. As the negative (positive) effect of the follower's price as well as his profit vanishes, beyond a critical level the profit ordering reverses and the leader's profit exceeds the follower's profit.

In a slightly different direction with homogenous goods goes the work of Boyer and Moreaux (1987a) in which the impact of production costs on the preferred duopolists' role is discussed. In contrast to previous contributions, they extend the players' strategy space to price-quantity pairs and show that there will either be a conflict or an agreement over the role of leader and follower. If firms have identical or similar costs of production, both competitors would choose to be the Stackelberg follower, a result that is totally different to the results of a pure quantity selection. Sufficiently high cost differences between firms may result in two possible scenarios. Within a first scenario, the less efficient firm will take the position of the Stackelberg leader and sell a low quantity for a lower price whereas the more efficient firm serves the residual demand for a higher price afterwards. In a second possible case the more efficient firm will adopt the role of the Stackelberg leader and by selecting his prices accordingly, drives the less efficient firm out of the market. Boyer and Moreaux (1987a) state that as profits of both firms are higher in the first case, a coordination towards the role distribution in which the less efficient firm is the leader and the more efficient firm is the follower can be expected. This result differs from the results of Von Stackelberg (1934) in which the role of being the leader is always preferred.

Dastidar (2004) observed that a discussion of a homogenous product market with concave demand and strictly convex costs was neglected in previous literature. For such a setting, it is shown that when firms choose prices, there is a unique subgame-perfect Nash equilibrium in which the leader's price is lower than the follower's price whereas payoffs are equal. Hence, there is neither a first- nor a second-mover advantage, a result that contrasts former findings for price competition. He furthermore makes clear that games á la Cournot in which firms select output quantities are less competitive than games in which prices are selected, a result that may be reversed under certain conditions.

Similar to our approach, Lee *et al.* (2014) introduce a monopolistic input supplier trading with two intermediaries through two-part tariffs and address the issue of first- and second-mover advantages. It is shown that the standard results of Bertrand and Cournot competition must not necessarily hold when considering markets that are vertically related. This is due to the fact that a monopolistic input supplier is able to control first- and second-mover advantages by selecting his input price accordingly. By removing the first-mover (second-mover) advantage under Cournot (Bertrand) competition, he can optimize his profits. In contrast to Lee *et al.* (2014) within our approach the input supplier charges a unique input price and does not discriminate in prices.

The second literature strand addresses duopoly models in which the role of a duopolist, i.e., whether to move first or second, is not exogenously given. First steps in this field of research were done by Hamilton and Slutsky (1990) and Robson (1990a). Hamilton and Slutsky (1990) (see also Amir (1995)) make clear that the analysis of first- and second-mover advantages are of quite interest, but do not answer the question of endogenously determining who moves first. If both firms prefer to be in the leading or following position, neither can achieve this alone. Within their contribution Hamilton and Slutsky (1990) discuss an extended game in which the issue of moving simultaneously versus moving sequentially is analyzed. Their setting consists of two stages. In the first stage a duopolist has to commit to one of two time periods in which he takes action. In a second stage firms select their production quantity in the time period they committed to. They find that there exist only two pure strategy equilibria in undominated strategies, which are the Stackelberg outcomes of the underlying duopoly game. Hence, Hamilton and Slutsky (1990) conclude that a Stackelberg outcome will result in equilibrium.

A similar idea is discussed in Leininger (1993). He questions the modeling of Tullock's original problem which is for instance displayed in Tullock (2001) and supposes that rent seekers move simultaneously. The findings of Leininger (1993) show that when deciding whether to move simultaneously or sequentially, rent seekers agree to take actions in a specific sequential order.

By making use of the risk-dominance concept of Harsanyi *et al.* (1988), Van Damme and Hurkens (1999) go a step further than Hamilton and Slutsky (1990) and solve the equilibrium selection problem when firms compete in quantities. Their results show that the assignment of roles (Stackelberg leader or follower) may follow from risk considerations and clarify that a commitment is less risky for a low-cost firm which is therefore ending up in the preferred position of the Stackelberg leader. Addressing the same questions as in Van Damme and Hurkens (1999), price competition is considered in van Damme and Hurkens (2004). In the scenario of price selection, the role of the follower is the most preferred of both players. Surprisingly, as waiting is more risky for the low-cost firm, it will move first and the high-cost firm will end up in the preferred position as the follower.

A link between the literature that discusses first- and second-mover advantages and the literature analyzing endogenous timing is established by the article of Amir and Stepanova (2006). They consider first- and second-mover advantages, but allow for a general demand and consider asymmetric linear costs. In a differentiated-product Bertrand duopoly they target two objectives. At first they generalize the classic results of literature on first- and second-mover advantages with linear costs. This generalization is done by removing the standard assumptions of profits' concavity in own action as well as uniqueness of the equilibrium in Bertrand competition. The according analysis yields that given firms' costs are sufficiently asymmetric, the low-cost firm has a first-mover advantage. If firm's unit costs are similar, the low-cost firm has a second-mover advantage. Referring to the setting of Hamilton and Slutsky (1990), the second objective considers the issue of endogenous timing for the duopoly with asymmetric linear costs and linear prices. Close to the approach of Van Damme and Hurkens (1999) with similar results, it is shown that by using the equilibrium selection concept of Harsanyi *et al.* (1988) the unique equilibrium outcome yields a sequential play with the low-cost firm being the leader.

Further contributions in this field of research were for instance done by Amir and Grilo (1999), who give different sets of general conditions on the structures of demand and cost functions that yield all possible timing outcomes.

One of the first articles analyzing the impact of asymmetric information on first- and second-mover advantages with endogenous timing was done by Mailath (1993). He considers a duopoly in which one of the duopolists has superior information about demand. This duopolist is able to select his production quantities either as a first-mover or delay his choice such that decisions are taken simultaneously with the other duopolist. Mailath (1993) shows, that regardless of its private information, the more informed firm moves first in the unique stable outcome.

Referring to Mailath (1993), the work of Normann (2002) uses the setting of Hamilton and Slutsky (1990) and hence allows for the more informed firm to be in the position of the follower. Differently to Mailath (1993), it turns out that there is evidence for both endogenous Cournot and Stackelberg equilibria. A closely related modeling was for instance done by Normann (1997). In a slightly different direction goes Albaek (1990), who considers cost uncertainty of duopolists. In their model, firms are able to choose whether to be the leader or the follower if either quantities or prices are selected. When taking this decision, firms know the distribution, but not the realization of their own and their competitor's costs.

Albaek (1990) finds, that even if firms at some point of time know the realization of costs, they stick with their selection of role if competing in quantities. Hence, in this scenario one duopolist prefers the Nash equilibrium of a simultaneous game. This however is not true for Bertrand competition. Furthermore, their analysis yields that total expected welfare in a Stackelberg equilibrium is higher than in a Nash equilibrium.

A setting in which the move of the first decider reveals certain information is for instance analyzed by Mukhopadhyay *et al.* (2011). They have a focus on a duopoly market where two asymmetrically informed firms offer complementary products and compete within a sequential game. Firms can decide whether to share their private forecast information about the uncertain market demand. It is shown that if the follower firm shares information unconditionally, the leader firm profits from information sharing while the follower as well as the total system is worse off. Beyond this, they provide a "simple to implement" information sharing scheme from which both firms and the whole system is better off.

Another approach in which asymmetric information plays a key role is done by Rasmusen and Yoon (2012). They analyze the impact of asymmetric information about the profitability of new markets on the existence of a first- or second-mover advantage. They model a scenario with two firms deciding about which new market to enter and whether to take the decision in a first or second period. It is found that given a player has moderate superior information about the new markets' profitability, a first-mover advantage may arise if market foreclosing is possible. In this scenario the less-informed firm has no incentive to imitate the more-informed firm and both end up in different markets. If however information asymmetries are sufficiently high, a second-mover advantage may emerge. In this case the less-informed firm will imitate the more-informed firm and both will compete in the same market.

Other settings in which the first-mover's action reveals something about the state of the world is for example discussed by Bolton and Harris (1999), Hirokawa and Sasaki (2001) and Hoppe (2000).

Besides examining the impact of asymmetric information and uncertainty on the preferred roles of duopolists, the emergence of extensive literature with a focus on product innovation could be observed. Lambertini (1996) criticizes that previous contributions on duopoly theory do not take into account that firms are able to soften price competition by strategically choosing product characteristics. The proposed setting assumes firms to choose product qualities in a first stage and prices in a second stage. In both stages actions can be taken simultaneously or sequentially. It turns out that in case of full market coverage in which all customers are served, the unique subgame perfect equilibrium yields a simultaneous game in the quality stage and a sequential game in the price stage. Contrary to, e.g., Gal-Or (1985) both firms prefer to be the leader. A partial market however implies an optimal outcome where a simultaneous game in both stages takes place.

By referring to the modeling of Dutta *et al.* (1995), the work of Hoppe and Lehmann-Grube (2001) analyzes dynamic models with product innovation and focuses on situations in which second-mover advantages arise. They find that firms prefer to be a market's pioneer firm with payoff equalization in equilibrium, if costs of R&D are low. If however costs of R&D are high, being the second-mover delivers higher payoffs in equilibrium. Furthermore it is shown that the second-mover advantage monotonically increases in the costs of R&D. Note that in contrast to the former mentioned articles, the model of Hoppe and Lehmann-Grube (2001) restricts attention to the optimal timing of R&D. Similar discussions can be found in contributions of Aoki and Prusa (1997), Lehmann-Grube (1997), Reinganum (1985) and Wang (2003) amongst others.

Besides product innovation, another extension of Von Stackelberg (1934) considers settings in which more than two firms move sequentially as for instance discussed in Anderson and Engers (1992), Vives (1988) and Robson (1990b).

Another article that is quite interesting for our work was done by Noh and Moschini (2006). In a differentiated market, they analyze the potential entry of a new product. Their model supposes quality-dependent marginal costs of production where sequential quality choices by an incumbent and an entrant are considered. Decisions that are taken on entry-quality as well as the strategies for entry-deterrence are related to the fixed cost that is necessary to enter and to the degree of consumers' taste of quality. Noh and Moschini (2006) verify the conditions under which the incumbent firm deters entry by increasing its quality level. Due to quality-dependent marginal costs of production there as well exists the possibility of inferior-quality entry. It is finally made clear, that compared to deterrence, encouraging entry does not necessarily improve welfare.

Chapter 3

Simultaneous Competition of Intermediaries

3.1 Introduction

We consider a differentiated intermediate goods market in which two intermediaries compete for customers on the sales side as well as for inputs on the procurement side. Hereby, competition is on the sales side, either carried out by strategically choosing production quantities or by setting sales prices. In our context differentiation on the one hand refers to horizontal product differentiation, i.e., products offered at the market can either be substitutes or complements. On the other hand we allow for quality differences between the intermediaries' products and therefore products may also be vertically differentiated. In addition, asymmetries can also arise from efficiency differences of the intermediaries' production technologies. A simple example of such a differentiated intermediate goods market that we have in mind are companies who produce furniture. They need to procure identical resources such as wood in order to produce chairs or tables and, therefore, face competition on the input market. In addition, they sell their products on a competitive output market, on which they compete for customers. Hereby, the companies may be differentiated horizontally and either offer substitute products, i.e., both produce tables or both produce chairs, or complementary products, with one company offering chairs and the other one tables. Besides horizontal product differentiation, companies have the opportunity to invest in product quality. Therefore they foster vertical product differentiation which may give them a competitive advantage on the market for chairs or tables. A second example is the market for IT services. Consider two consulting companies that on the input market compete for graduate students in computer science, where input prices are represented by the entry-level salaries. Technological differences within the production process between the consulting companies may arise from different initial on-the-job trainings which support employees' working speed and, hence, the output quantity. Besides initial trainings, consulting companies may have the chance to increase their employees' working abilities by investing in further education, resulting in a higher output quality. The final IT services offered by the consulting companies may on the one hand be substitutable, if, for example, both firms offer accounting software services. On the other hand the companies' products are complementary, if, for instance, one firm offers accounting and the other one salary administration services.

In the literature, two fundamental contributions in oligopoly theory are made by *Cournot* and *Bertrand*, in which firms produce a homogenous good and select quantities in the former and prices in the latter model. In the years after, these settings were extended in various directions. Referring to Dixit (1979), in their seminal work Singh and Vives (1984) consider a differentiated duopoly allowing for complements and substitutes and compare the equilibrium outcomes of Bertrand and Cournot competition. Cheng (1985) generalizes their model such that no special form of demand structure needs to be assumed. The approach of duopolies was later on extended by allowing for n product varieties as for example by Vives (1985), Okuguchi (1987), Häckner (2000) and Amir and Jin (2001). Moreover, not just horizontal (substitutes vs. complements) but also vertical product differentiation (product quality) finds attention within the literature. A setting covering product differentiation in which product innovation leads to higher product quality is for example analyzed by Motta (1993) and Symeonidis (2003). Both present a vertically differentiated duopoly in which product quality can endogenously be chosen and compare equilibrium outcomes in quantity and price competition. By doing so, Motta (1993) differentiates between two versions of vertical product differentiation, one considering fixed and the other one variable costs of innovation. Extending the approach of Motta (1993), Symeonidis (2003) studies not just vertical, but also horizontal differentiation, i.e., goods that can either be substitutes or mutually independent in his setting. He uses a quality-augmented version of Singh and Vives' (1984) customer utility function and allows for spillovers of product quality investments across firms. Another branch of literature analyzes the effects of productivity-increasing process innovation as in Bester and Petrakis (1993), Qiu (1997) and Pauwels *et al.* (2014). Beyond this, settings in which both, process or product innovation, go along with each other are discussed by Bonanno and Haworth

(1998a), Rosenkranz (2003), Weiss (2003), Zanchettin (2006), Filippini and Martini (2010), Bacchiega *et al.* (2011), for instance. Closely related to our setting Zanchettin (2006) also has exogenously given asymmetric productivities and asymmetric product qualities. In a slightly other direction goes the analysis of Ledvina and Sircar (2011), who focus on the number of active firms, i.e., firms producing a positive quantity in Bertrand and Cournot equilibrium and consider homogeneous and differentiated products.

One issue that is neglected in the above-mentioned contributions but plays a major role in our article is the market power of an input supplier and is addressed by Häckner (2003), López and Naylor (2004), Correa-López (2007), Mukherjee *et al.* (2012) and Manasakis and Vlassis (2014), amongst others. Related to our model but with a different objective, Häckner (2003) considers a differentiated market with intermediaries and input suppliers and analyzes the impact of vertical mergers with an emphasis on social welfare. Correa-López (2007) focuses on a vertically differentiated duopoly, using a simplified version of the linear model by Dixit (1979) and Singh and Vives (1984). The fundamental difference to their approach is the incorporation of the market's supply side and the resulting endogenous costs of production. A vertical market structure that is similar to the one discussed in our work and incorporates the market's supply side is for example considered by Mukherjee *et al.* (2012). They analyze an intermediate goods market in which a profit-maximizing input supplier interacts with two unequally productive intermediaries who provide a homogeneous final good. The price-setting input supplier has the possibility of either discriminating intermediaries in prices or choosing a uniform price. In their contribution, Mukherjee *et al.* (2012) emphasize the importance of the intermediaries' technology differences as well as the input supplier's pricing strategy when comparing profits under Bertrand and Cournot competition. They show on the one hand that under Bertrand competition the input supplier's profit, the aggregated profit of the input supplier and the intermediaries and social welfare are always higher than under Cournot competition, regardless of the pricing strategy. On the other hand a comparison of the intermediary's profits across competition types can only be done conditional on technology differences between the intermediaries as well as the pricing strategy of the input supplier. Given that intermediaries are asymmetric in terms of productivity and the supplier charges uniform prices, their profit may be higher under price competition. Thus, the result of Singh and Vives (1984) in terms of profits does not necessarily hold in a vertically structured setting. Moreover it is shown that the results of López and Naylor (2004)

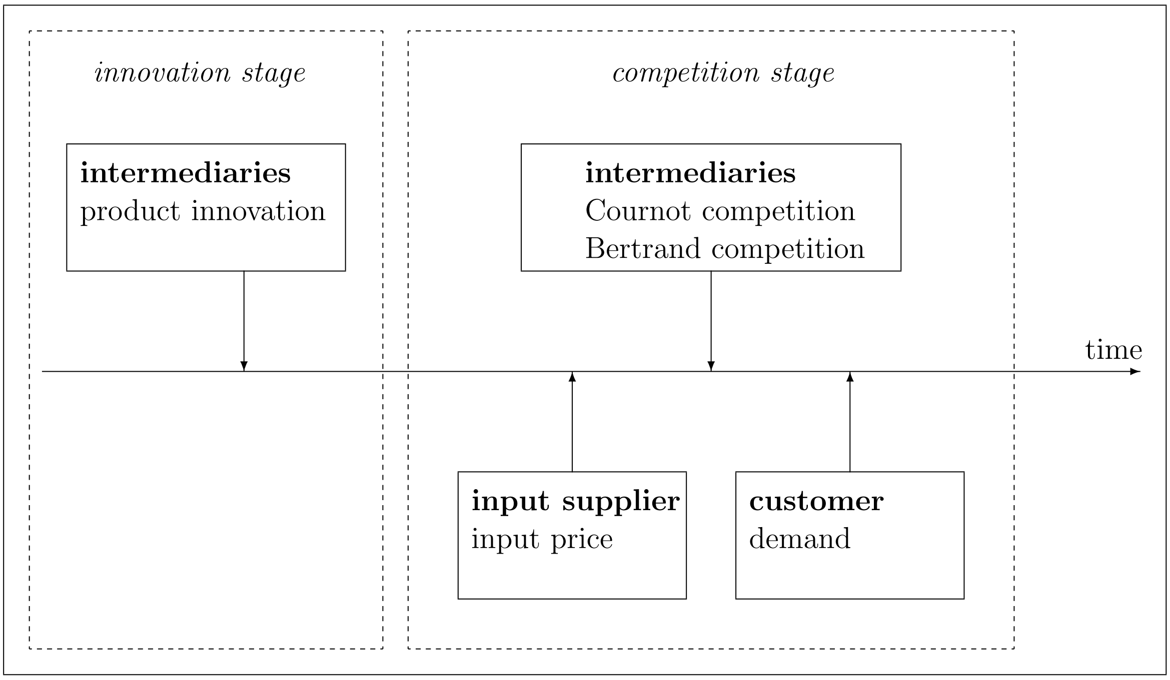


Figure 3.1: Timing of decisions

and Correa-López (2007) cannot be confirmed if the input supplier charges a uniform price and intermediaries differ in productivity.

While considering vertical and horizontal product differentiation, the focus of our analysis is put on the impact of simultaneous competition for resources and customers on the market outcome and the incentives intermediaries have to invest in product quality under Cournot and Bertrand competition. Our setting is based on two main decision stages: *innovation* and *competition*. The timing of decisions is illustrated in Fig. 3.1.

In stage 1, the two intermediaries take investment decisions to increase their product's quality (product innovation). For this innovation stage we concentrate on the analysis of investment equilibria with symmetric intermediaries with respect to the input productivity. We derive equilibrium conditions for product quality investments on the investment costs and compare them for price and quantity competition. It turns out that there exist product qualities such that for complementary products asymmetric investment equilibria do not exist, while existence can be guaranteed for close substitutes if the investment costs are chosen accordingly.

In stage 2, the intermediaries face competition for resources and customers. Within this stage the input prices are determined first and intermediaries compete by choosing quantities or prices afterwards. For the competition stage, we analyze the effect of the intermediaries' asymmetries on the market outcome. In this context we find that there exist quality and productivity differences such that for quantity competition only one intermediary is willing to procure inputs from the input supplier, while for price competition both intermediaries are willing to purchase inputs. The formal analysis in the forthcoming sections is backwards. We start with the competition stage for given product qualities and then investigate the innovation stage.

The analysis proceeds as follows: Section 2 analyzes the competition for resources and customers between the intermediaries for given differences in product qualities. We begin with the assumptions on the customer's demand, followed by the analysis of two different modes of competition between the intermediaries. As proposed in Häckner (2003) we explicitly consider an input market with a monopolistic input supplier. We start with quantity competition, investigate price competition and compare them afterwards. Section 3 considers the innovation stage with a focus on the intermediaries' decisions to invest in product quality for symmetric productivities. A final conclusion is made in section 4, whereas proofs can be found in the appendix.

3.2 Competition between Intermediaries

3.2.1 The customer

In order to allow for substitutable as well as for complementary products of the intermediaries, we assume that a representative customer has a utility function, which is of the following standard form as for instance in Singh and Vives (1984):

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + I. \tag{3.1}$$

Hereby, $q = (q_1, q_2)$ denotes the quantities the customer buys from each of the two intermediaries. The parameter $\alpha = (\alpha_1, \alpha_2)$ indicates the customer's valuation for the quality of the according product. For simplicity reasons we will denote α as product quality in the following. The variable $\gamma \in [-1, 1]$ represents the degree of horizontal product differentiation and defines whether products of the intermediaries are substitutable, complementary or independent. If $\gamma = -1$ the products are perfect complements, if $\gamma = 1$ they are perfect substitutes, while $\gamma = 0$ describes the case where products are independent and thus, both intermediaries are monopolists. We assume the customer to maximize his utility subject to the budget constraint $p_1q_1 + p_2q_2 + I \leq m$, where m is the customer's income and I his consumption of other goods. The customer's income is assumed to be sufficiently large such that utility maximization leads to an inner solution. Taking prices (p_1, p_2) as given, the customer optimally chooses to buy and consume those quantities (q_1, q_2) that maximize his utility function subject to his budget constraint, satisfying

$$\alpha_i - q_i - \gamma q_{3-i} - p_i = 0 \qquad \text{for } i = 1, 2. \tag{3.2}$$

As in Mukherjee *et al.* (2012), the two intermediaries may differ in their productivity to transform inputs to a final product, i.e., intermediary i needs $\lambda_i > 0$ input units to produce one unit of his final product ($i = 1, 2$). Therefore, asymmetries between intermediaries may arise from distinct product qualities and from differences in input productivities. To describe these asymmetries we refer to $\frac{\alpha_i}{\lambda_i}$ as the *relative quality* (with respect to the input productivity) for intermediary $i = 1, 2$.

3.2.2 Cournot competition

Within Cournot competition the two intermediaries compete by strategically choosing the quantities they produce. The price that intermediary $i \in \{1, 2\}$ charges his customer is not just depending on his own output quantity, but also on that of the other intermediary. From Eq. (3.2) we obtain the customer's demand function for intermediary i:

$$p_i(q_1, q_2) = \alpha_i - q_i - \gamma q_{3-i}, \tag{3.3}$$

where $p_i(q_1, q_2)$ is the market price which intermediary i charges in order to sell the quantities (q_1, q_2) that are produced. Intermediary i's profit function is given by the difference of the market price per unit on the sales side and the marginal input price on the procurement side, multiplied by the quantity he produces and sells:

$$\pi_i^C(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}) = (p_i(q_1, q_2) - \lambda_i c)\, q_i = (\alpha_i - q_i - \gamma q_{3-i} - \lambda_i c)\, q_i, \tag{3.4}$$

where λ_i is the input productivity and c the marginal input price charged by the input supplier. From Eq. (3.4) and the observation that negative profits can be avoided by not selling anything, we derive intermediary i's best reply function:

$$q_i(q_{3-i}) = \max\left\{\frac{\alpha_i - \gamma q_{3-i} - \lambda_i c}{2}, 0\right\}. \tag{3.5}$$

By using the best reply functions of both intermediaries we compute intermediary i's Nash equilibrium quantity q_i^C, which is given by

$$q_i^C = \begin{cases} \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i - \lambda_i c}{2} & \text{if } \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

The according equilibrium price p_i^C is

$$p_i^C = \begin{cases} q_i^C + \lambda_i c & \text{if } c < \min\left\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i + \lambda_i c}{2} & \text{if } \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.7}$$

In order to guarantee the quantity to be non-decreasing in "weighted" qualities given by $2\alpha_i - \gamma\alpha_{3-i}$ and to be non-increasing in the input price c we impose the following assumption on the parameter choices $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$:

Assumption 3.1. *We assume* $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2}$ *and* $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2}$.

We restrict our analysis to parameter choices $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ that satisfy Assumption 3.1. Note that

$$\min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}\right\} = \begin{cases} \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} & \text{if } \frac{\alpha_1}{\lambda_1} \leq \frac{\alpha_2}{\lambda_2}, \\ \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} & \text{if } \frac{\alpha_1}{\lambda_1} \geq \frac{\alpha_2}{\lambda_2}. \end{cases} \tag{3.8}$$

Taking the intermediaries' equilibrium quantities and prices as given, we next determine the optimal input price the monopolistic input supplier chooses. The total market demand on the input market is $q_I^C(c) = \lambda_1 q_1^C + \lambda_2 q_2^C$ and given by

$$q_I^C(c) = \begin{cases} \frac{2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)-2\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}\right\}, \\ \frac{\lambda_1(\alpha_1-\lambda_1 c)}{2} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \\ \frac{\lambda_2(\alpha_2-\lambda_2 c)}{2} & \text{if } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.9}$$

The monopolistic input supplier maximizes his profit $q_I^C(c)\,c$ where the supplier's production costs are normalized to zero. The next proposition states the impact of the input supplier's optimal decision on the market outcome for Cournot competition.

Proposition 3.1. *Suppose the intermediaries compete in choosing quantities. If the two intermediaries are sufficiently asymmetric, it is optimal for the input supplier to choose an input price such that he sells his inputs to just one intermediary and thus excludes the other intermediary from the input market.*

More precisely, if $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ *and*

$$\alpha_2 < \alpha_1\tau_1^C \left(\leq \alpha_1\frac{\lambda_2}{\lambda_1}\right) \tag{3.10}$$

with

$$\tau_1^C = \frac{\sqrt{(4-\gamma^2)\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)} - (2\lambda_1-\gamma\lambda_2)}{(2\lambda_2-\gamma\lambda_1)}, \tag{3.11}$$

then it is optimal for a profit-maximizing input supplier to only serve intermediary 1 and with analogous conditions to serve intermediary 2. For the remaining specifications of $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ *the input supplier optimally serves both intermediaries.*

Proposition 3.1 highlights the market power of the input supplier. If quality or productivity differences between the intermediaries are too large, which is $\alpha_2 < \alpha_1\tau_1^C$ (or $\alpha_1 < \alpha_2\tau_2^C$), then the input supplier has an incentive to charge a relatively high input price such that just one intermediary is willing to purchase inputs from the input market. As the input price is too high for the other intermediary to realize positive profits he prefers not to buy and thus, not to produce. This means even if in principal both intermediaries are willing to purchase positive quantities on the input market, one intermediary may be excluded from the market at the input supplier's profit-maximizing input price. In contrast, if the intermediaries are sufficiently symmetric, which is $\alpha_1 \tau_1^C \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$ (or $\alpha_2 \tau_2^C \leq \alpha_1 \leq \alpha_2 \frac{\lambda_1}{\lambda_2}$), then at the profit-maximizing input price both intermediaries purchase resources on the input market.

3.2.3 Bertrand competition

Within this section the two intermediaries compete by strategically choosing the prices of their products. The according quantities intermediaries are producing depend on the prices that both charge at the market. Given both intermediaries compete, by using Eq. (3.2) and assuming $\gamma \in (-1, 1)$ we obtain the customer's demand function

$$q_i(p_1, p_2) = \frac{\alpha_i - p_i + \gamma\,(p_{3-i} - \alpha_{3-i})}{(1-\gamma^2)} \tag{3.12}$$

for intermediary $i \in \{1, 2\}$. Thus, intermediary i's profit function is

$$\pi_i^B(p_i, p_{3-i}, c, \alpha_i, \alpha_{3-i}) = (p_i - \lambda_i c)\left(\frac{\alpha_i - p_i + \gamma\,(p_{3-i} - \alpha_{3-i})}{(1-\gamma^2)}\right). \tag{3.13}$$

By using Eq. (3.13) we compute intermediary i's best reply function. As above, negative profits can be avoided by not producing anything

$$p_i\,(p_{3-i}) = \max\left\{\frac{\alpha_i - \gamma\,(\alpha_{3-i} - p_{3-i}) + \lambda_i c}{2}, 0\right\}. \tag{3.14}$$

Hence, the Nash equilibrium price in Bertrand competition is given by

$$p_i^B = \begin{cases} \frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i}}{(2-\gamma^2)\lambda_i - \gamma\lambda_{3-i}}, \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i + \lambda_i c}{2} & \text{if } \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise,} \end{cases} \tag{3.15}$$

and the equilibrium quantity is given by

$$q_i^B = \begin{cases} \frac{p_i^B - \lambda_i c}{(1-\gamma^2)} & \text{if } c < \min\left\{\frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i}}{(2-\gamma^2)\lambda_i - \gamma\lambda_{3-i}}, \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i}\right\} \\ \frac{\alpha_i - \lambda_i c}{2} & \text{if } \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i} \\ 0 & \text{otherwise.} \end{cases} \tag{3.16}$$

For similar reasons as in Cournot competition we impose an analogue assumption on the parameter choices $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ for Bertrand competition:

Assumption 3.2. *We assume* $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2-\gamma^2}$ *and* $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2}$.

We restrict our analysis to parameter choices $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ that satisfy Assumption 3.2. Note that Assumption 3.2 is more restrictive on the parameter choices than Assumption 3.1. Similarly, as for Cournot competition, we have

$$\min\left\{\frac{(2-\gamma^2)\,\alpha_1 - \gamma\alpha_2}{(2-\gamma^2)\,\lambda_1 - \gamma\lambda_2}, \frac{(2-\gamma^2)\,\alpha_2 - \gamma\alpha_1}{(2-\gamma^2)\,\lambda_2 - \gamma\lambda_1}\right\} = \begin{cases} \frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2} & \text{if } \frac{\alpha_1}{\lambda_1} \geq \frac{\alpha_2}{\lambda_2} \\ \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} & \text{if } \frac{\alpha_1}{\lambda_1} \leq \frac{\alpha_2}{\lambda_2}. \end{cases} \tag{3.17}$$

The total market demand on the input market is $q_I^B(c) = \lambda_1 q_1^B + \lambda_2 q_2^B$ and given by

$$q_I^B(c) = \begin{cases} \frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)}{(4-\gamma^2)(1-\gamma^2)} & \\ \quad -\frac{\left[(2-\gamma^2)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)(1-\gamma^2)} & \text{if } c < \min\left\{\frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2}, \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}\right\}, \\ \frac{\lambda_1(\alpha_1-\lambda_1 c)}{2} & \text{if } \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \\ \frac{\lambda_2(\alpha_2-\lambda_2 c)}{2} & \text{if } \frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.18}$$

The monopolistic input supplier maximizes his profit $q_I^B(c)\,c$, where the supplier's production costs are normalized to zero. We obtain the analogue of Proposition 3.1 also for Bertrand competition.

Proposition 3.2. *Suppose the intermediaries compete in choosing prices. If the two intermediaries are sufficiently asymmetric, it is optimal for the input supplier to choose an input price such that he sells his inputs to just one intermediary and thus excludes the other intermediary from the input market.*

More precisely, if $\frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ *and*

$$\alpha_2 < \alpha_1\tau_1^B \left(\leq \alpha_1\frac{\lambda_2}{\lambda_1}\right) \tag{3.19}$$

with

$$\tau_1^B = \frac{\sqrt{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}-2\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right)}, \tag{3.20}$$

then it is optimal for a profit-maximizing input supplier to only serve intermediary 1 and with analogous conditions to serve intermediary 2. For the remaining specifications of parameters $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ *the input supplier optimally sells to both intermediaries.*

3.2.4 Comparison of Cournot and Bertrand competition

For the remaining part of this section we assume $\gamma \in (-1, 1)$ and suppose that there exists an inner solution for the monopolistic input price as assumed in Proposition 3.1 and 3.2. When at first considering the input market by comparing the results of Proposition 3.1 and 3.2 we observe the following:

Proposition 3.3 (Input Market)**.** *There exist parameters* $(\alpha_1, \lambda_1, \alpha_2, \lambda_2)$ *such that in Cournot competition one intermediary is excluded from the input market while, in Bertrand competition both intermediaries purchase inputs, i.e., we always have* $\tau_1^B \leq \tau_1^C$ *and* $\tau_2^B \leq \tau_2^C$.

Proposition 3.3 makes clear that there exist quality and productivity differences such that in Cournot competition only one intermediary is willing to procure inputs from the input supplier while in Bertrand competition still both intermediaries are willing to purchase inputs. This is illustrated in Fig. 3.2.
Hence, in Proposition 3.3 we establish that there is no situation in which Bertrand competition leads to an exclusion of one intermediary from the input market while Cournot competition does not. The reason is that the profits of the input supplier, given he sells to both intermediaries, are always higher when intermediaries compete in Bertrand compared to Cournot competition. Therefore, if the asymmetries between the two intermediaries increase, the input price of a profit-maximizing input supplier is raised earlier under Cournot competition than under Bertrand competition. This has the consequence that under Cournot competition the input price is earlier too high for the intermediary with quality and/or productivity disadvantages. Thus, he is no longer willing to purchase inputs. The next proposition explicitly compares the input prices for the three scenarios from Proposition 3.3 and Fig. 3.2.

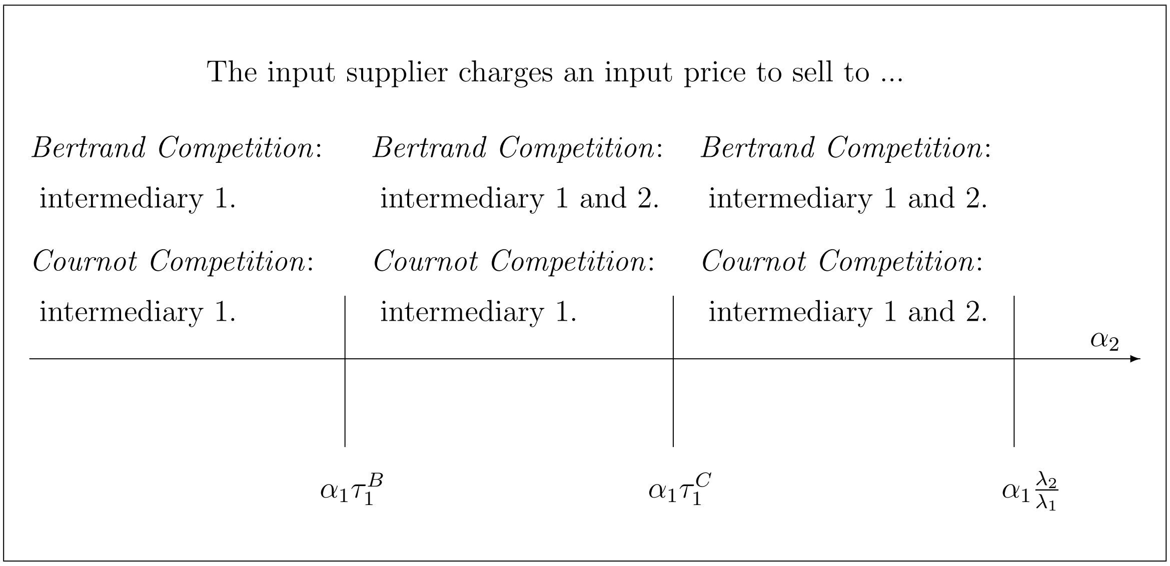


Figure 3.2: The input market for $\alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$

Proposition 3.4 (Input Prices)**.**

(i) Consider $\alpha_2 \leq \alpha_1\tau_1^B$ (or $\alpha_1 \leq \alpha_2\tau_2^B$). Input prices are equal under Cournot and Bertrand competition.

(ii) Consider $\alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1\tau_1^C$ (or $\alpha_2\tau_2^B \leq \alpha_1 \leq \alpha_2\tau_2^C$). Input prices are always higher under Cournot than under Bertrand competition.

(iii) Consider $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \leq \alpha_1 \leq \alpha_2\frac{\lambda_1}{\lambda_2}$).

If input productivities are equivalent ($\lambda_1 = \lambda_2$), both intermediaries act as monopolists ($\gamma = 0$) or both intermediaries produce identical relative qualities ($\frac{\alpha_1}{\lambda_1} = \frac{\alpha_2}{\lambda_2}$), then under Cournot and Bertrand competition the input prices are equal.

If the goods are substitutes ($\gamma > 0$) and the relative quality of the more productive intermediary is higher, then input prices are higher under Bertrand than under Cournot competition.

If the goods are complements ($\gamma < 0$) and the relative quality of the more productive intermediary is higher, then input prices are higher under Cournot than under Betrand competition.

Proposition 3.4 indeed confirms the intuition behind Proposition 3.3. The first statement for $\alpha_2 \leq \alpha_1\tau_1^B$ (or $\alpha_1 \leq \alpha_2\tau_2^B$) is obvious. As for Bertrand and Cournot competition the input supplier sets an input price that excludes one intermediary from the market and sells just to the other intermediary, there is no longer competition on the output market between the intermediaries. Thus, there is no difference in input prices. The second statement for $\alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1\tau_1^C$ (or $\alpha_2\tau_2^B \leq \alpha_1 \leq \alpha_2\tau_2^C$) confirms that the input price under Cournot competition is indeed too high for one intermediary to purchase positive quantities on the input market, while the input price is always lower under Bertrand competition. However, for the last constellation of parameters with $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \leq \alpha_1 \leq \alpha_2\frac{\lambda_1}{\lambda_2}$) the comparison of the input prices for Cournot and Bertrand competition depends on several parameters. In the proof of Proposition 3.4 we establish that for $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ the difference of the input prices $c^C - c^B \geq 0$ if and only if $\gamma^3(\lambda_1 - \lambda_2)(\lambda_1\alpha_2 - \lambda_2\alpha_1) \geq 0$. Thus, the products' substitutability as well as the differences in productivity and in relative qualities crucially impact the relation of the input prices for Cournot compared to Bertrand competition.

In order to analyze the intermediaries' incentives to invest in product quality, we now summarize their profits using Proposition 3.1 and 3.2. The profits for the scenarios identified in Proposition 3.3 and Fig. 3.2 can easily be derived accordingly. Consider $\alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$. The intermediaries profits are

$$\pi_1^C\left(q_1^C, q_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1\tau_1^C, \\ \frac{\left[2\alpha_1 - \gamma\alpha_2 - (2\lambda_1 - \gamma\lambda_2)c^C\right]^2}{(2-\gamma)^2(2+\gamma)^2} & \text{for } \alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}, \end{cases} \tag{3.21}$$

$$\pi_2^C\left(q_1^C, q_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1\tau_1^C, \\ \frac{\left[2\alpha_2 - \gamma\alpha_1 - (2\lambda_2 - \gamma\lambda_1)c^C\right]^2}{(2-\gamma)^2(2+\gamma)^2} & \text{for } \alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}, \end{cases} \tag{3.22}$$

with

$$c^C = \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right]}$$

and

$$\pi_1^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1\tau_1^B, \\ \frac{\left[(2-\gamma^2)\alpha_1 - \gamma\alpha_2 - \left((2-\gamma^2)\lambda_1 - \gamma\lambda_2\right)c^B\right]^2}{(2-\gamma)^2(2+\gamma)^2(1-\gamma^2)} & \text{for } \alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}, \end{cases} \tag{3.23}$$

$$\pi_2^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1 \tau_1^B, \\ \frac{\left[\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1 - \left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)c^B\right]^2}{(2-\gamma)^2(2+\gamma)^2(1-\gamma^2)} & \text{for } \alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}, \end{cases} \tag{3.24}$$

with

$$c^B = \frac{(2-\gamma^2)\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)}{2\left[(2-\gamma^2)\left(\lambda_1^2 + \lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]}.$$

The next two Propositions now compare the intermediaries' equilibrium prices, quantities and profits for Cournot and Bertrand competition.

Proposition 3.5. *Consider* $\lambda_1 = \lambda_2$ *and* $\alpha_1 = \alpha_2$. *Then,* $p_i^C \geq p_i^B$, $q_i^C \leq q_i^B$ *and, moreover,* $\pi_i^C \geq \pi_i^B$ *for* $\gamma \geq 0$ *and* $\pi_i^C \leq \pi_i^B$ *for* $\gamma \leq 0$ *for both intermediaries* $i = 1, 2$.

Proposition 3.6.

(i) There exist $(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma)$ *such that there is an intermediary* $i \in \{1, 2\}$ *with* $p_i^C < p_i^B$.

(ii) There exist $(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma)$ *such that there is an intermediary* $i \in \{1, 2\}$ *with* $q_i^C > q_i^B$.

(iii) There exist $(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma)$ *with* $\gamma \geq 0$ *such that* $\pi_i^C < \pi_i^B$ *for both intermediaries* $i = 1, 2$.

For our model with symmetric intermediaries, Proposition 3.5 confirms a prominent result from literature. Singh and Vives (1984) show for a linear demand structure that in equilibrium Bertrand competition is more efficient than Cournot competition, as it generates a lower price and a higher output level. They find that this result is independent of whether goods are substitutes or complements and regardless of the demand structure's degree of symmetry. Thus, Bertrand competition implies a higher consumer and total welfare. Furthermore, Singh and Vives (1984) show that firms prefer to select quantities (prices), if goods are substitutes (complements). However, Proposition 3.6 establishes that this must not always be true when the input market is explicitly modeled and asymmetries between the intermediaries are present. Zanchettin (2006), in a setting with quality and cost asymmetries, finds the same relations in terms of equilibrium prices and quantities as in Proposition 3.5. Nevertheless, in our model with an endogenous input market this relationship cannot always be guaranteed.

3.3 Product Innovation

3.3.1 Investments in product quality

We assume that the intermediaries can simultaneously choose to invest in product innovation, which increases their products' quality. The product quality is $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ with $0 < \underline{\alpha}_i < \overline{\alpha}_i$. Without investing intermediary i is assumed to produce the minimal product quality of $\underline{\alpha}_i$, whereas investing increases the product quality α_i from $\underline{\alpha}_i$ to maximally $\overline{\alpha}_i$ for $i = 1, 2$. The marginal costs of investing are assumed to be identical for both intermediaries and denoted by $k > 0$.

Proposition 3.7. *In Cournot as well as in Bertrand competition, the intermediaries' profits are strictly convex in own qualities and thus, Nash equilibrium strategies of the investment game with linear costs are in* $\{\underline{\alpha}_i, \overline{\alpha}_i\} \subset [\underline{\alpha}_i, \overline{\alpha}_i]$ *for* $i = 1, 2$.

Proposition 3.7 implies that if we are looking for Nash equilibrium investment levels for linear investment costs, we can restrict our attention to the minimal and maximal quality levels. Strict convexity of the intermediaries' profit functions in own qualities implies that there exists a quality level that minimizes the intermediaries' profits. Thus, in order to find the optimal level of quality within the interval $[\underline{\alpha}_i, \overline{\alpha}_i]$ it suffices to compare the profits at the according end points. This also holds if the minimum itself is not in the interval $[\underline{\alpha}_i, \overline{\alpha}_i]$. Therefore, if we assume that the minimal quality always needs to be provided, the actual decision may be simplified to either invest or not to invest.

The insights from the previous section, in particular from Proposition 3.3, imply that the mode of competition actually influences the incentives to invest in product quality. This means the profits of the intermediaries crucially depend on the degree of asymmetries and whether they compete in prices or quantities. Thus, besides the exogeneously given input productivities (λ_1, λ_2) the range of investment possibilities $(\underline{\alpha}_1, \overline{\alpha}_1, \underline{\alpha}_2, \overline{\alpha}_2)$ actually impacts the asymmetries between the intermediaries and with that the investment incentives. Suppose $\overline{\alpha}_2 \leq \underline{\alpha}_1 \frac{\lambda_2}{\lambda_1}$ and consider the example from Fig. 3.3. The productivities and therewith τ_1^B and τ_1^C as well as the range of investments are such that for the minimal quality levels intermediary 2 is excluded from the input market. In case of Bertrand competition, intermediary 2 is able to sufficiently lower the price on the input market by investing in product quality while this is not possible for Cournot competition. Whether an investment leads to positive profits for intermediary 2 crucially depends on the costs of investing in product quality. Moreover, for the equilibrium analysis we also have to take the investment incentives of intermediary 1 into account.

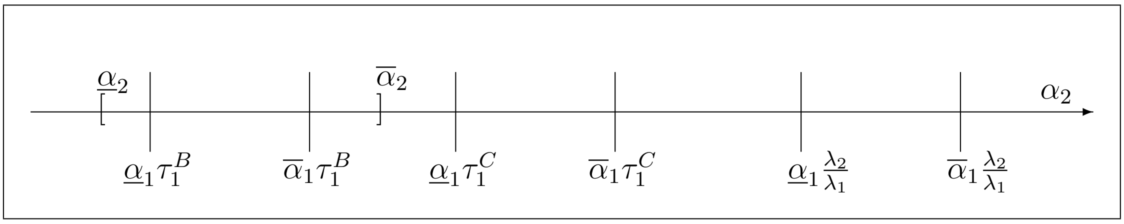


Figure 3.3: Example for investments into product quality

Fig. 3.3 indicates that there are various possibilities of how the productivity and quality parameters may relate to each other and influence the intermediaries' asymmetries. As we would like to put our focus on the investment incentives, we for the next subsection assume that the intermediaries have identical input productivities.

3.3.2 Nash equilibrium investments for symmetric productivities

We assume that the intermediaries can simultaneously choose to invest in quality-increasing product innovation. We analyze these investments for symmetric productivities, i.e., $\lambda_1 = \lambda_2$, which simplifies the expressions from the previous section to:

$$\tau_1^C = \sqrt{2+\gamma} - 1 \quad \text{and} \quad \tau_1^B = \sqrt{2+\gamma-\gamma^2} - 1.$$

Note that $\tau_1^B \leq \tau_1^C < 1$. The profits of the intermediaries for $\alpha_2 \leq \alpha_1$ are

$$\pi_1^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1\tau_1^C, \\ \frac{[\alpha_1(6+\gamma)-\alpha_2(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1, \end{cases}$$

$$\pi_2^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1\tau_1^C, \\ \frac{[\alpha_2(6+\gamma)-\alpha_1(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1, \end{cases}$$

and

$$\pi_1^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1\tau_1^B, \\ \frac{\left[\alpha_1(6+\gamma-3\gamma^2)-\alpha_2(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1, \end{cases}$$

$$\pi_2^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1 \tau_1^B, \\ \frac{\left[\alpha_2(6+\gamma-3\gamma^2)-\alpha_1(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \alpha_1 \tau_1^B \leq \alpha_2 \leq \alpha_1. \end{cases}$$

We use these profits to analyze the incentives to invest in product quality. Suppose $\underline{\alpha}_1 = \underline{\alpha}_2 = \underline{\alpha}$, $\overline{\alpha}_1 = \overline{\alpha}_2 = \overline{\alpha}$ and define $\kappa^{C/B} = k\left(\overline{\alpha} - \underline{\alpha}\right)$. Fig. 3.4 shows the investment game in normalform. Both intermediaries simultaneously decide whether to invest or not to invest in product quality.

	$\underline{\alpha}$	$\overline{\alpha}$
$\underline{\alpha}$	$\pi_1^{C/B}\left(\underline{\alpha}, \underline{\alpha}\right)$ $\pi_2^{C/B}\left(\underline{\alpha}, \underline{\alpha}\right)$	$\pi_1^{C/B}\left(\underline{\alpha}, \overline{\alpha}\right)$ $\pi_2^{C/B}\left(\underline{\alpha}, \overline{\alpha}\right) - \kappa^{C/B}$
$\overline{\alpha}$	$\pi_1^{C/B}\left(\overline{\alpha}, \underline{\alpha}\right) - \kappa^{C/B}$ $\pi_2^{C/B}\left(\overline{\alpha}, \underline{\alpha}\right)$	$\pi_1^{C/B}\left(\overline{\alpha}, \overline{\alpha}\right) - \kappa^{C/B}$ $\pi_2^{C/B}\left(\overline{\alpha}, \overline{\alpha}\right) - \kappa^{C/B}$

Figure 3.4: Normalform investment game

The payoffs shown in Fig. 3.4 are based on the investments chosen by the intermediaries. The effect an investment has on the profits and the market outcome depends on the quality difference between investing and not investing, which is $\overline{\alpha} - \underline{\alpha}$. If this difference is sufficiently large, an intermediary who does not invest while his competitor is investing may be excluded from the input market. Therefore, the decision to invest may actually change the situation on the input market. If an intermediary was excluded from the market while not investing, an investment may induce the input supplier to lower his input price and therefore to sell to both intermediaries.

We now continue with some general remarks on the Nash equilibrium conditions of the normalform investment game in Fig. 3.4. Due to symmetry reasons of the two intermediaries we may define an upper $\overline{\kappa}^{C/B}$ and a lower bound $\underline{\kappa}^{C/B}$ to describe the Nash equilibrium conditions on the investment costs $\kappa^{C/B}$. This is

$$\underline{\kappa}^{C/B} := \pi_1^{C/B}\left(\overline{\alpha}, \overline{\alpha}\right) - \pi_1^{C/B}\left(\underline{\alpha}, \overline{\alpha}\right) = \pi_2^{C/B}\left(\overline{\alpha}, \overline{\alpha}\right) - \pi_2^{C/B}\left(\overline{\alpha}, \underline{\alpha}\right), \tag{3.25}$$

$$\overline{\kappa}^{C/B} := \pi_1^{C/B}\left(\overline{\alpha}, \underline{\alpha}\right) - \pi_1^{C/B}\left(\underline{\alpha}, \underline{\alpha}\right) = \pi_2^{C/B}\left(\underline{\alpha}, \overline{\alpha}\right) - \pi_2^{C/B}\left(\underline{\alpha}, \underline{\alpha}\right). \tag{3.26}$$

The according equilibrium conditions are shown in Fig. 3.5. We explicitly determine the equilibrium profits and the conditions on the investment costs for Cournot competition in Appendix 3.5.8 and for Bertrand competition in Appendix 3.5.9.

strategy profile	*investment costs*
$(\underline{\alpha}, \underline{\alpha})$	$\kappa^{C/B} \in \left[\overline{\kappa}^{C/B}, \infty\right)$
$(\overline{\alpha}, \underline{\alpha})$, $(\underline{\alpha}, \overline{\alpha})$	$\kappa^{C/B} \in \left[\underline{\kappa}^{C/B}, \overline{\kappa}^{C/B}\right]$
$(\overline{\alpha}, \overline{\alpha})$	$\kappa^{C/B} \in \left(-\infty, \underline{\kappa}^{C/B}\right]$

Figure 3.5: Equilibrium conditions on the investment costs $\kappa^{C/B}$

The next proposition states that the existence of asymmetric Nash equilibria crucially depends on the substitutability and complementarity of the intermediaries' products. It turns out that asymmetric Nash equilibria may fail to exist.

Proposition 3.8 (Asymmetric investment equilibria, Cournot competition)**.**

(i) For $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$ *there are no asymmetric Nash equilibria if* $\gamma \in [-1, 0]$. *More precisely, there are no asymmetric Nash equilibria if* $\gamma(4+\gamma)\overline{\alpha}^2 - 4\underline{\alpha}^2 < 0$ *(which is bounded above by* $\gamma < 2\left(\sqrt{2}-1\right) \approx 0.83$*).*

(ii) For $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}$ *there are no asymmetric equilibria if* $\gamma \in \left[-1, -\frac{2}{3}\right] \approx [-1, -0.67]$.

Similarly, as in Cournot competition, asymmetric Nash equilibria may also fail to exist in case of Bertrand competition, as is indicated by the next proposition.

Proposition 3.9 (Asymmetric investment equilibria, Bertrand competition)**.**

(i) For $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right)$ *there are no asymmetric Nash equilibria if* $\gamma \in [-1, 0]$. *More precisely, there are no asymmetric Nash equilibria if* $\overline{\alpha}^2\gamma\left(4-3\gamma+\gamma^2\right) - 4\underline{\alpha}^2(1-\gamma) < 0$ *(which is bounded above for* γ *by* 0.61*).*

(ii) For $\overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha}$ *there are no asymmetric equilibria if* $\gamma \in [-1, -0.56]$.

Finally, we compare the upper and lower bounds for the investment costs for the two types of competition and the different scenarios that may arise depending on the effect of investments.

Proposition 3.10.

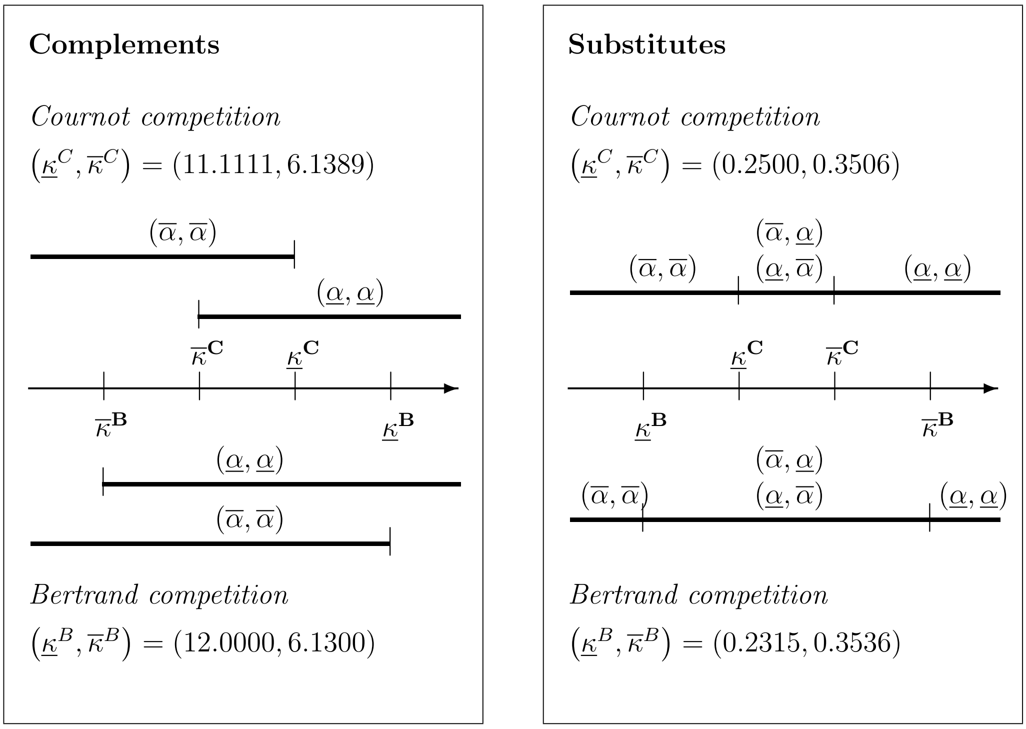
(i) *Consider* $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right)$. *Then, for complements with* $\gamma \in (-1,0]$ *we have* $\underline{\kappa}^C \leq \underline{\kappa}^B$ *and* $\overline{\kappa}^B \leq \overline{\kappa}^C$. *For substitutes with* $\gamma \in [0,1)$ *we have* $\underline{\kappa}^B \leq \underline{\kappa}^C$ *and* $\overline{\kappa}^C \leq \overline{\kappa}^B$.

(ii) *Consider* $\overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$. *Then, for substitutes with* $\gamma \in [0,1)$ *we have* $\underline{\kappa}^B \leq \underline{\kappa}^C$. *The relationship of* $\overline{\kappa}^B$ *and* $\overline{\kappa}^C$ *depends on the values of* $\underline{\alpha}$ *and* $\overline{\alpha}$. *The same is true for complements for the lower and upper bounds.*

(iii) *Consider* $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}$. *Then, for complements with* $\gamma \in (-1,0]$ *we always have* $\underline{\kappa}^C \leq \underline{\kappa}^B$ *and if and only if* $\overline{\alpha}\,(3+5\gamma) \geq \underline{\alpha}\,(7+\gamma)$, *then we also have* $\overline{\kappa}^B \leq \overline{\kappa}^C$. *For substitutes with* $\gamma \in [0,1)$ *we always have* $\underline{\kappa}^B \leq \underline{\kappa}^C$ *and if and only if* $\overline{\alpha}\,(3+5\gamma) \geq \underline{\alpha}\,(7+\gamma)$, *then we also have* $\overline{\kappa}^C \leq \overline{\kappa}^B$.

Proposition 3.10 compares the incentives to invest in product quality for Cournot and Bertrand competition. The lower and upper bounds for the investment costs are conditions on the Nash equilibria of the investment game. According to the first statement in Proposition 3.10(i), for complementary products the lower bound on the investment costs in Cournot competition is less or equal to the lower bound for Bertrand competition, $\underline{\kappa}^C \leq \underline{\kappa}^B$ for $\underline{\alpha} \leq \overline{\alpha}\tau_1^B$. This means that if the intermediaries compete by choosing quantities, the range of costs for which it is a Nash equilibrium for both intermediaries to invest, is contained in the according interval for price competition. Hence, there may exist investment costs for which in price competition both intermediaries invest in product quality, while these strategies are not a Nash equilibrium if the intermediaries compete in choosing quantities. The upper bound on the investment costs determine the scenarios in which not investing is a Nash equilibrium. When considering the upper bound we observe for complementary products that $\overline{\kappa}^B \leq \overline{\kappa}^C$ for $\underline{\alpha} \leq \overline{\alpha}\tau_1^B$. Thus, there are investment costs such that not investing is a Nash equilibrium for Bertrand competition, while for Cournot competition it is not. This is the case as the investment costs are still sufficiently low for at least one intermediary to deviate and invest. For substitutable products we have according to Proposition 3.10(i) the reverse relationship between Cournot and Bertrand competition.

Fig. 3.6 exemplarily summarizes the results of Propositions 3.8, 3.9 and 3.10 graphically for complements and substitutes if the quality difference between investing and not investing is sufficiently large. For complements we observe in Fig. 3.6(a) that in



(a) $\gamma = -\frac{1}{2}$ and $(\overline{\alpha}, \underline{\alpha}) = (10, 1)$

(b) $\gamma = \frac{1}{2}$ and $(\overline{\alpha}, \underline{\alpha}) = \left(\frac{5}{2}, 1\right)$

Figure 3.6: Examples for Nash equilibria of the investment game for $\lambda_1 = \lambda_2$

case of Bertrand competition we have a larger range of investment costs for which the two symmetric Nash equilibria of the investment game coexist. For substitutes in Fig. 3.6(b) the interval of investment costs for which asymmetric Nash equilibria exist is larger for price than for quantity competition. Note that the intervals in Fig. 3.6 are schematic and not drawn to scale. However, the numerical values for the upper and lower bounds on the investment costs of the examples in Fig. 3.6(a) and 3.6(b) are explicitly mentioned in the according figures. These examples illustrate that if investing in product quality has an sufficient impact on the asymmetries between the intermediaries, we observe a range of cost parameters in which both symmetric investment Nash equilibria coexist when products are complements. For substitutable products, however, asymmetric Nash equilibria also exist. This observation confirms the intuition that for complementary products the investment decisions of the intermediaries are coordinated, while for substitutable products there are also equilibria with asymmetric investment strategies. These effects are in a sense stronger for price competition compared to quantity competition.

We finish the analysis of the investment game with a last remark on the special case of independent products, where $\gamma = 0$ and $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2} - 1\right)$. Here, the situation is similar to the one described in Fig. 3.6(a), but with identical intervals in which the symmetric equilibria exist within Cournot and Bertrand competition. For $\gamma = 0$ and $\overline{\alpha}\left(\sqrt{2}-1\right) \leq \underline{\alpha}$ the intervals resemble the one in Fig. 3.6(b), also with identical intervals for Cournot and Bertrand competition. The according mathematics can be found after Proof of Proposition 3.10 in Appendix 3.5.10.

3.4 Conclusion

We analyzed the influence of asymmetries between intermediaries on an intermediate goods market with horizontal and vertical product differentiation. The interaction between the intermediaries was divided into two different decision stages: innovation and competition. The according analysis proceeded backwards.

For the *competition stage*, we established that introducing a strategically acting input supplier may lead to exclusion of one intermediary from the input market if the asymmetries are sufficiently large (Propositions 3.1 and 3.2). There exists a range of asymmetries where in Cournot competition the input prices are too high for one intermediary, while in Bertrand competition both intermediaries demand positive quantities on the input market (Propositions 3.3 and 3.4). Compared to Zanchettin (2006), we excluded by Assumption 3.1 and Assumption 3.2 the range of parameters for which asymmetries drive the inefficient firm immediately out of the market (Zanchettin, 2006, Eq. 8). However, due to the explicit inclusion of the input market, we make no initial assumption on the relation between qualities and input prices as done in Zanchettin (2006). Nevertheless, in our model with a strategically acting input supplier and thus, an endogenously determined input price, this exclusion from the input market even survives for a more restrictive range of asymmetries. Our bound on the asymmetries is established for substitutable as well as for complementary products with $\gamma \in (-1, 1)$. Related to our approach, the model in Mukherjee *et al.* (2012) is also focused on the presence of an input market. However, in order to point out the effect of efficiency differences and also analyzing the impact of uniform prices compared to price discrimination, the quality of the intermediaries' products is assumed to be identical. This distinguishes from our goal to examine the impact of a strategically acting input supplier on the intermediaries' decisions to purchase resources on the input market for their products and their incentives for product innovation. The analysis of price discrimination is indeed one direction to further extend our model. An additional objective for future research is the investigation of competition on the input market.

In Appendix 3.6 we already indicate the similarities and differences compared to the monopolistic input market. However, the comprehensive analysis of the relation between the number of suppliers and the optimal decisions on the input market is left for future research. Additionally we also compared the intermediaries' equilibrium prices, quantities and profits for Cournot and Bertrand competition (Propositions 3.5 and 3.6). We observed that the relations established in Singh and Vives (1984) cannot always be guaranteed in our model with an endogenous input market and asymmetric intermediaries.

For the *innovation stage*, we analyzed the intermediaries' incentives to improve the quality of their products. Using the intermediaries' equilibrium profits from the competition stage and taking the assumption of linear investment costs into account, we first established that the intermediaries' Nash equilibrium investments into product quality are always at the minimum or maximum product quality (Proposition 3.7). Afterwards, we investigated the Nash equilibrium investments for the case of symmetric productivities in more detail and compared them for Cournot and Bertrand competition (Propositions 3.8, 3.9 and 3.10). Taking the results from the competition stage into consideration, the focus of our analysis for the innovation stage was on the equilibrium conditions for the investments. We compared those conditions for the different scenarios derived for the input market as well as for the substitutability between the intermediaries' products. In contrast to the model in Symeonidis (2003), who modifies the approach of Motta (1993) by additionally allowing for technological spillovers, we investigated the direct impact of investments on product quality. In our model, the function that links quality to innovation expenditures is simply the identity whereas in Symeonidis (2003) this is assumed to be a fourth root function. This allows Symeonidis (2003) to analyze interior symmetric investment equilibria while our equilibrium investments are always at the minimum or maximum product quality, which is a consequence of the direct influence of investments on product quality. Besides the above-mentioned investigation of a price discriminating input supplier, the examination of more complex investment effects, in particular related to technological spillovers for quality investments, is also considered to be an objective for further research, which is beyond the scope of our current analysis.

Acknowledgments

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3.5 Appendix A: Proofs

3.5.1 Proof of Proposition 3.1

Proof of Proposition 3.1. The proof proceeds in two steps. In a first step we determine the prices the input supplier charges. In doing so we distinguish different cases depending on the number of intermediaries demanding positive quantities. Afterwards the according profits are compared in step two.

Step 1 (Input Prices and Profits)

Case 1: The input price is chosen such that both intermediaries produce strictly positive quantities. Therefore, the following must hold:

$$c < \min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}\right\}. \tag{3.27}$$

Maximizing the profit of the input supplier $q_I^C(c)\, c$ yields an input price

$$c^C = \frac{2\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)}{4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right]}. \tag{3.28}$$

Note that for this input price both intermediaries indeed purchase positive quantities. Suppose $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$, then $\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} > \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}$. Using $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2}$ and $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2}$ we have

$$0 < \left(4 - \gamma^2\right)\left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right) \Leftrightarrow \frac{2\alpha_2 - \gamma\alpha_1}{2\alpha_1 - \gamma\alpha_2} < \frac{2\lambda_2 - \gamma\lambda_1}{2\lambda_1 - \gamma\lambda_2}$$

and thus,

$$\begin{aligned} &2\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] \\ &= \lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(2\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(2\lambda_2 - \gamma\lambda_1\right)}{\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right)} \end{aligned}$$

$$> \lambda_1 (2\lambda_1 - \gamma\lambda_2) + \lambda_1 (2\lambda_1 - \gamma\lambda_2) \frac{\lambda_2 (2\alpha_2 - \gamma\alpha_1)}{\lambda_1 (2\alpha_1 - \gamma\alpha_2)}.$$

By using

$$4 \left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] > 2 \left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] \quad \text{for } \lambda_1, \lambda_2 > 0$$

this implies that

$$4 \left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] > \lambda_1 (2\lambda_1 - \gamma\lambda_2) + \lambda_1 (2\lambda_1 - \gamma\lambda_2) \frac{\lambda_2 (2\alpha_2 - \gamma\alpha_1)}{\lambda_1 (2\alpha_1 - \gamma\alpha_2)}.$$

Rearranging yields

$$\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} > \frac{2 (\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma (\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4 [\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]}$$

showing that the input price c^C is indeed sufficiently small to sell to both intermediaries. The profit of the input supplier is

$$q_I^C(c^C)\, c^C = \frac{[2 (\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma (\lambda_1\alpha_2 + \lambda_2\alpha_1)]^2}{8 (4 - \gamma^2) [\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]}. \tag{3.29}$$

Case 2: The input price is chosen such that intermediary 1 produces a strictly positive quantity while intermediary 2 produces zero. Thus, we require

$$\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \tag{3.30}$$

which implies $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$. Maximizing the profit of the input supplier $q_I^C(c)\, c$ and taking a sufficiently high input price into account yields an input price

$$c^C = \max \left\{ \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1} \right\}. \tag{3.31}$$

Note that by definition this input price is indeed too high for intermediary 2 to purchase positive quantities. The analogous argument holds if intermediary 2 produces a strictly positive quantity while intermediary 1 produces zero. The profit of the input supplier is

$$q_I^C\left(c^C\right)c^C = \begin{cases} \frac{\alpha_1^2}{8} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}, \\ \frac{\lambda_1(2\alpha_2-\gamma\alpha_1)(\alpha_1\lambda_2-\alpha_2\lambda_1)}{(2\lambda_2-\gamma\lambda_1)^2} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \geq \frac{\alpha_1}{2\lambda_1}. \end{cases} \tag{3.32}$$

Case 3: The input price is too high for at least one intermediary to purchase strictly positive quantities.

Step 2 (Comparing Profits)
An input price as suggested in Case 3 cannot be optimal. In this scenario the input price is too high for the intermediaries to demand goods and the input supplier's profit is zero. In contrast, Case 1 as well as Case 2 yield positive profits. Therefore, we analyze the input supplier's profits for Cases 1 and 2. We compare the two monopoly profits for selling to intermediary 1 with the duopoly profits for selling to both intermediaries. Thus, suppose $\alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$. The remaining part when only selling to intermediary 2 follows analogously.

Step 2.1: We compare the two monopoly profits for selling to intermediary 1 and obtain by observing

$$\left(\alpha_1\left(2\lambda_2+\gamma\lambda_1\right)-4\lambda_1\alpha_2\right)^2 \geq 0 \tag{3.33}$$

that

$$\frac{\alpha_1^2}{8} \geq \frac{\lambda_1\left(2\alpha_2-\gamma\alpha_1\right)\left(\alpha_1\lambda_2-\alpha_2\lambda_1\right)}{\left(2\lambda_2-\gamma\lambda_1\right)^2}. \tag{3.34}$$

Step 2.2: We compare the duopoly profit with the monopoly profit for selling to intermediary 1. If the duopoly profit is greater than or equal to both monopoly profits, then the input supplier decides to sell to both intermediaries. Therefore, we obtain that if

$$\alpha_2 \geq \alpha_1\tau_1^C \tag{3.35}$$

with

$$\tau_1^C = \left(\frac{\sqrt{\left(4-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)}-\left(2\lambda_1-\gamma\lambda_2\right)}{\left(2\lambda_2-\gamma\lambda_1\right)}\right), \tag{3.36}$$

then

$$\frac{\alpha_1^2}{8} \leq \frac{\left[2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{8\left(4-\gamma^2\right)\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \tag{3.37}$$

holds. Thus, whenever condition (3.35) holds, we know that a profit-maximizing input supplier prefers to sell in any case to both intermediaries. Thus, we know that for

$$\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$$

the input supplier prefers to set a price for which both intermediaries procure inputs. Note that the following is always true:

$$\tau_1^C \leq \frac{\lambda_2}{\lambda_1}. \tag{3.38}$$

Step 2.1 and Step 2.2 show that for $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ the the claim of Proposition 3.1 holds. Note that requiring $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ is equivalent to choosing $\alpha_2 \leq \alpha_1\left(\frac{2\lambda_2+\gamma\lambda_1}{4\lambda_1}\right)$. □

3.5.2 Proof of Proposition 3.2

Proof of Proposition 3.2. The proof proceeds in two steps. In a first step we determine the prices the input supplier charges. In doing so we distinguish different cases depending on the number of intermediaries demanding positive quantities. Afterwards the according profits are compared in step two.

Step 1 (Input Prices and Profits)

Case 1: The input price is chosen such that both intermediaries produce strictly positive quantities. Therefore the following must hold:

$$c < \min\left\{\frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2}, \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1}\right\}. \tag{3.39}$$

Maximizing the profit of the input supplier yields

$$c^B = \frac{\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}. \tag{3.40}$$

Note that for this input price both intermediaries indeed purchase positive quantities. Suppose $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$, then $\frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2} > \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}$. Using $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2}$ and $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2-\gamma^2}$ we have

$$0 < \left(2-\gamma^2\right)^2\left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right) \Leftrightarrow \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2} < \frac{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1}{\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2}$$

and thus,

$$\begin{aligned}
&\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2 \\
&= \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)} \\
&> \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2\right)}
\end{aligned}$$

which implies when using

$$2\left(\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right) > \left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2 \quad \text{for } \lambda_1, \lambda_2 > 0$$

that

$$\begin{aligned}
&2\left(\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right) \\
&> \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2\right)}.
\end{aligned}$$

Rearranging yields

$$\frac{(2-\gamma^2)\alpha_1 - \gamma\alpha_2}{(2-\gamma^2)\lambda_1 - \gamma\lambda_2} > \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2]}$$

showing that the input price c^B is indeed sufficiently small to sell to both intermediaries. The profit of the input supplier is

$$q_I^B(c^B)\,c^B = \frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]}. \tag{3.41}$$

Case 2: The input price is chosen such that intermediary 1 produces a strictly positive quantity while intermediary 2 produces zero. Thus, we require

$$\frac{(2-\gamma^2)\,\alpha_2 - \gamma\alpha_1}{(2-\gamma^2)\,\lambda_2 - \gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \tag{3.42}$$

which implies $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$. Given a sufficiently high input price, maximizing the profit of the input supplier yields

$$c^B = \max\left\{\frac{(2-\gamma^2)\,\alpha_2 - \gamma\alpha_1}{(2-\gamma^2)\,\lambda_2 - \gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\}. \tag{3.43}$$

Note that by definition this input price is indeed too high for intermediary 2 to purchase positive quantities. The analogous argument holds if intermediary 2 produces a strictly positive quantity while intermediary 1 produces zero. The profit of the input supplier is

$$q_I^B(c^B)\,c^B = \begin{cases} \frac{\alpha_1^2}{8} & \text{if } \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1} \\ \frac{\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right](\alpha_1\lambda_2-\alpha_2\lambda_1)}{2[(2-\gamma^2)\lambda_2-\gamma\lambda_1]^2} & \text{if } \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \geq \frac{\alpha_1}{2\lambda_1}. \end{cases} \tag{3.44}$$

Case 3: The input price is too high for at least one intermediary to purchase strictly positive quantities.

Step 2 (Comparing Profits)

As in Cournot competition, it suffices to compare the profits of the input supplier for Cases 1 and 2. We compare the two monopoly profits for selling to intermediary 1 with the duopoly profits for selling to both intermediaries. Thus, suppose $\alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$. The remaining part when selling to intermediary 2 follows analogously.

Step 2.1: We compare the two monopoly profits for selling to intermediary 1. We obtain by observing

$$\left[\alpha_1\left(\left(2-\gamma^2\right)\lambda_2+\gamma\lambda_1\right)-4\left(2-\gamma^2\right)\alpha_2\lambda_1\right]^2 \geq 0 \tag{3.45}$$

that

$$\frac{\alpha_1^2}{8} \geq \frac{\lambda_1\,(2-\gamma^2)\,[(2-\gamma^2)\,\alpha_2-\gamma\alpha_1]\,(\alpha_1\lambda_2-\alpha_2\lambda_1)}{2\,[(2-\gamma^2)\,\lambda_2-\gamma\lambda_1]^2}. \tag{3.46}$$

Step 2.2: We compare the duopoly profit with the monopoly profit for selling to intermediary 1. If the duopoly profit is greater than or equal to both monopoly profits, then the input supplier decides to sell to both intermediaries. Therefore, we obtain that if

$$\alpha_2 \geq \alpha_1 \tau_1^B \tag{3.47}$$

with

$$\tau_1^B = \left(\frac{\sqrt{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} - 2\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \right), \tag{3.48}$$

then

$$\frac{\alpha_1^2}{8} \leq \frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} \tag{3.49}$$

holds. Thus, whenever condition (3.47) is true, we know that a profit-maximizing input supplier prefers to sell only to intermediary 1. Thus, we know that for

$$\alpha_1 \tau_1^B \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$$

the input supplier prefers to set a price to sell to both intermediaries. Note that the following is always true:

$$\tau_1^B \leq \frac{\lambda_2}{\lambda_1}. \tag{3.50}$$

Step 2.1 and Step 2.2 show that for $\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ the the claim of Proposition 3.1 holds. Note that requiring $\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ is equivalent to choosing $\alpha_2 \leq \alpha_1 \left(\frac{\left(2-\gamma^2\right)\lambda_2+\gamma\lambda_1}{2\left(2-\gamma^2\right)\lambda_1} \right)$. □

3.5.3 Proof of Proposition 3.3

Proof of Proposition 3.3. The proof proceeds in two steps. First we compare the profits of the input supplier in the case of intermediaries competing in quantities with the setting in which intermediaries compete in prices.

Step 1 (Duopoly Profit of the Input Supplier)
Suppose the input supplier sells inputs to both intermediaries, which is

$$\max\left\{\alpha_1\tau_1^B, \alpha_1\tau_1^C\right\} \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}. \tag{3.51}$$

We show that

$$q_I^B(c^B)\,c^B \geq q_I^C(c^C)\,c^C. \tag{3.52}$$

The nominator of

$$\begin{aligned}
&q_I^B(c^B)\,c^B - q_I^C(c^C)\,c^C \\
&= \frac{\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{4(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]} \\
&\quad - \frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{8(4-\gamma^2)\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
&= \frac{2\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]}{8(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
&\quad - \frac{(1-\gamma^2)\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]}{8(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]}
\end{aligned}$$

can be rewritten as

$$\begin{aligned}
&2\left[\left(2-\gamma^2\right)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] \\
&\quad - \left(1-\gamma^2\right)\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right] \\
&= \gamma^2\lambda_1(2\lambda_1-\gamma\lambda_2)\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right]\left[[\lambda_1-\gamma\lambda_2](\alpha_1-\alpha_2)^2+2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\right] \\
&\quad + \gamma^2\lambda_2(2\lambda_2-\gamma\lambda_1)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]\left[[\lambda_2-\gamma\lambda_1](\alpha_1-\alpha_2)^2+2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\right].
\end{aligned}$$

Assumption 3.1 and 3.2, which is

$$\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2} \quad \text{and} \quad \min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2},$$

implies for $\lambda_1 \geq \lambda_2$ that

$$\begin{aligned}
0 &\leq 2\lambda_2-\gamma\lambda_1 \leq 2\lambda_1-\gamma\lambda_2, \\
0 &\leq \left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1 \leq \left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2.
\end{aligned}$$

Thus, we obtain

$$\gamma^2\lambda_1(2\lambda_1-\gamma\lambda_2)\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right]\left[\left[\lambda_1-\gamma\lambda_2\right](\alpha_1-\alpha_2)^2+2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\right]$$
$$+\gamma^2\lambda_2(2\lambda_2-\gamma\lambda_1)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]\left[\left[\lambda_2-\gamma\lambda_1\right](\alpha_1-\alpha_2)^2+2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\right]$$
$$\geq\gamma^2(2\lambda_2-\gamma\lambda_1)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]$$
$$\left[\left((\lambda_1-\lambda_2)^2+2(1-\gamma)\lambda_1\lambda_2\right)(\alpha_1-\alpha_2)^2+2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\right]$$
$$\geq 0.$$

The analogous argument holds for $\lambda_1\leq\lambda_2$. Note that we have for $q_I^C(c^C)c^C$ with Assumption 3.1

$$\frac{\partial\left[q_I^C(c^C)c^C\right]}{\partial\alpha_2}=2\frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]\left[2\lambda_2-\gamma\lambda_1\right]}{8\left(4-\gamma^2\right)\left[(\lambda_1^2+\lambda_2^2)-\gamma\lambda_1\lambda_2\right]}\geq 0, \tag{3.53}$$

and with Assumption 3.2 for $q_I^B(c^B)c^B$

$$\frac{\partial\left[q_I^B(c^B)c^B\right]}{\partial\alpha_2}=2\frac{\left[\left(2-\gamma^2\right)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]}\geq 0. \tag{3.54}$$

Step 2 (Comparing τ_1^C and τ_1^B)

Note that τ_1^B and τ_1^C was deduced within the proof of Proposition 3.1 and Proposition 3.2 by comparing the profit to sell to both intermediaries with the profit to sell only to intermediary 1. This is stated in Ineq. (3.37) and (3.46). When considering the difference between those profits and using *step 1*, we therefore have

$$\begin{aligned}&\frac{\alpha_1^2}{8}-\frac{\left[\left(2-\gamma^2\right)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]}\\&\quad\leq\frac{\alpha_1^2}{8}-\frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{8\left(4-\gamma^2\right)\left[(\lambda_1^2+\lambda_2^2)-\gamma\lambda_1\lambda_2\right]}.\end{aligned} \tag{3.55}$$

With the observation that the input supplier's profit when selling to both intermediaries is non-decreasing in α_2, Ineq. (3.53) and (3.54) and thus, the right- and left-hand side of Ineq. (3.55) are non-increasing in α_2. This implies that the zero of the left-hand side is less or equal than the zero of the right-hand side. Therefore,

$$\tau_1^B\leq\tau_1^C \tag{3.56}$$

and thus, $\max\left\{\alpha_1\tau_1^B, \alpha_1\tau_1^C\right\} = \alpha_1\tau_1^C$ and $\min\left\{\alpha_1\tau_1^B, \alpha_1\tau_1^C\right\} = \alpha_1\tau_1^B$.

Similarly, we have $\alpha_2\tau_2^B \le \alpha_2\tau_2^C$. □

3.5.4 Proof of Proposition 3.4

Proof of Proposition 3.4.

Case 1: $\alpha_2 \le \alpha_1\tau_1^B$ (or $\alpha_1 \le \alpha_2\tau_2^B$)

In Bertrand as well as in Cournot competition intermediary 2 is excluded from the input market. As the input demand of the intermediary is identical for both types of competition, the input price is identical also.

Case 2: $\alpha_1\tau_1^B \le \alpha_2 \le \alpha_1\tau_1^C$ (or $\alpha_2\tau_2^B \le \alpha_1 \le \alpha_2\tau_2^C$)

In Bertrand competition both intermediaries purchase inputs, while in Cournot competition only intermediary 1 procures on the input market. We have

$$\begin{aligned} c^C - c^B &= \frac{\alpha_1}{2\lambda_1} - \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\ &= \frac{(\lambda_2\alpha_1 - \lambda_1\alpha_2)\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]}. \end{aligned} \tag{3.57}$$

Therefore, $c^C - c^B \ge 0$ as $\alpha_2 \le \alpha_1\tau_1^C \le \alpha_1\frac{\lambda_2}{\lambda_1}$ and per assumption $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2}$.

Case 3: $\alpha_1\tau_1^C \le \alpha_2 \le \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \le \alpha_1 \le \alpha_2\frac{\lambda_1}{\lambda_2}$)

Both intermediaries purchase inputs on the input market in Bertrand as well as in Cournot competition. We have

$$\begin{aligned} & c^C - c^B \\ &= \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4\left[(\lambda_1^2+\lambda_2^2) - \gamma\lambda_1\lambda_2\right]} - \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\ &= \frac{\gamma^3(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2)(\lambda_1\alpha_2 - \lambda_2\alpha_1)}{4(\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2)\left((2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right)}. \end{aligned} \tag{3.58}$$

Therefore, $c^C - c^B \ge 0$ if and only if $\gamma^3(\lambda_1 - \lambda_2)(\lambda_1\alpha_2 - \lambda_2\alpha_1) \ge 0$.

□

3.5.5 Proof of Proposition 3.5

Proof of Proposition 3.5. For $\lambda_1 = \lambda_2$ and $\alpha_1 = \alpha_2$ we have

$$p_i^C - p_i^B = \frac{(3+\gamma)\,\alpha_i}{2\,(2+\gamma)} - \frac{(3-2\gamma)\,\alpha_i}{2\,(2-\gamma)} = \frac{\gamma^2\alpha_i}{2\,(4-\gamma^2)}, \tag{3.59}$$

$$q_i^C - q_i^B = \frac{\alpha_i}{2\,(2+\gamma)} - \frac{\alpha_i}{2\,(2-\gamma)\,(1+\gamma)} = -\frac{\gamma^2\alpha_i}{2\,(4-\gamma^2)\,(1+\gamma)}, \tag{3.60}$$

$$\pi_i^C - \pi_i^B = \frac{\alpha_i^2}{4\,(2+\gamma)^2} - \frac{(1-\gamma)\,\alpha_i^2}{4\,(2-\gamma)^2\,(1+\gamma)} = \frac{\gamma^3\alpha_i^2}{2\,(4-\gamma^2)^2\,(1+\gamma)}. \tag{3.61}$$

Thus, $p_i^C \geq p_i^B$ and $q_i^C \leq q_i^B$. Moreover, $\pi_i^C \geq \pi_i^B$ for $\gamma \geq 0$ and $\pi_i^C \leq \pi_i^B$ for $\gamma \leq 0$ for $i = 1, 2$. □

3.5.6 Proof of Proposition 3.6

Proof of Proposition 3.6. For this proof consider $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \leq \alpha_1 \leq \alpha_2\frac{\lambda_1}{\lambda_2}$) and $\lambda_1 = \lambda_2$.

(i) We have

$$p_i^C - p_i^B = \frac{\gamma^2\,(3\alpha_i - \alpha_{3-i})}{4\,(4-\gamma^2)}. \tag{3.62}$$

Thus, for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(4, 1, 1, 1, -\frac{1}{2}\right)$$

it can easily be verified that Assumptions 3.1 and 3.2 as well as $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ hold and we have

$$p_2^C = \frac{89}{60} < \frac{3}{2} = p_2^B.$$

(ii) We have

$$q_i^C - q_i^B = -\frac{\gamma^2\,[(3+\gamma)\,\alpha_i - (1+3\gamma)\,\alpha_{3-i}]}{4\,(4-\gamma^2)\,(1-\gamma^2)}. \tag{3.63}$$

Thus, it can easily be verified for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(\frac{3}{2}, 1, 1, 1, \frac{1}{2}\right)$$

that Assumptions 3.1 and 3.2 as well as $\alpha_1 \tau_1^C \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$ hold and we have

$$q_2^C = \frac{1}{12} > \frac{7}{90} = q_2^B.$$

(iii) We have

$$\begin{aligned} &\pi_i^C \left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) - \pi_i^B \left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) \\ &= \frac{\gamma^3 \left[4\alpha_i \alpha_{3-i} (1-\gamma) - (\alpha_i - \alpha_{3-i})^2 (3+5\gamma)\right]}{8 (4-\gamma^2)^2 (1-\gamma^2)}. \end{aligned} \tag{3.64}$$

Thus, for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(\frac{3}{2}, 1, 1, 1, \frac{3}{4}\right)$$

it can easily be verified that Assumptions 3.1 and 3.2 as well as $\alpha_1 \tau_1^C \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$ hold and we have

$$\begin{aligned} \pi_1^C \left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) &= \frac{2209}{12100} < \frac{625}{3388} = \pi_1^B \left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right), \\ \pi_2^C \left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) &= \frac{9}{12100} < \frac{9}{3388} = \pi_2^B \left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right). \end{aligned}$$

□

3.5.7 Proof of Proposition 3.7

Proof of Proposition 3.7. We consider the different cases from the previous propositions separately and show that the intermediaries' profit functions are convex in own qualities.

Case 1: $\alpha_2 \leq \alpha_1 \tau_1^B$ (or $\alpha_1 \leq \alpha_2 \tau_2^B$)

We obtain

$$\pi_1^C (\alpha_1, \alpha_2) = \pi_1^B (\alpha_1, \alpha_2) = \frac{\alpha_1^2}{16}.$$

Thus, for all $\alpha_1 \in [\underline{\alpha}_1, \overline{\alpha}_1]$ we have

$$\frac{\partial^2 \pi_1^C(\alpha_1, \alpha_2)}{\partial \alpha_1^2} = \frac{\partial^2 \pi_1^C(\alpha_1, \alpha_2^2)}{\partial \alpha_1^2} = \frac{2}{16} > 0.$$

Case 2: $\alpha_1 \tau_1^B \le \alpha_2 \le \alpha_1 \tau_1^C$ (or $\alpha_2 \tau_2^B \le \alpha_1 \le \alpha_2 \tau_2^C$)

We already computed the profit for Cournot competition in Case 1, the result for Bertrand competition is shown in Case 3.

Case 3: $\alpha_1 \tau_1^C \le \alpha_2 \le \alpha_1 \frac{\lambda_2}{\lambda_1}$ (or $\alpha_2 \tau_2^C \le \alpha_1 \le \alpha_2 \frac{\lambda_1}{\lambda_2}$)

First observe that for Cournot as well as for Bertrand competition the profit of intermediary i is quadratic in α_i for $i = 1, 2$. Consider intermediary i. In the case of Cournot competition we obtain the following second derivative of the profit function using Assumption 3.1

$$\begin{aligned}
&\frac{\partial^2 \pi_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2} \\
&= \frac{\partial^2 \left[\left(p_i^C(\alpha_i, \alpha_{3-i}) - \lambda_i c^C(\alpha_i, \alpha_{3-i})\right) q_i^C(\alpha_i, \alpha_{3-i})\right]}{\partial \alpha_i^2} \\
&= 2\left(\frac{\partial p_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} - \lambda_i \frac{\partial c^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right) \frac{\partial q_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \\
&= 2\left(\frac{\partial q_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right)^2 \\
&= 2\left(\frac{2}{(4-\gamma^2)} - \frac{(2\lambda_i - \gamma\lambda_{3-i})^2}{(4-\gamma^2)\, 4\left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i}\right)}\right)^2 \\
&= \frac{\left((4-\gamma^2)\lambda_{3-i}^2 + 4\left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i}\right)\right)^2}{8(4-\gamma^2)^2\left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i}\right)^2} > 0
\end{aligned}$$

for all $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ for $i = 1, 2$. Similarly, we obtain for Bertrand competition with $\gamma \in (-1, 1)$ using Assumption 3.2

$$\begin{aligned}
&\frac{\partial^2 \pi_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2} \\
&= \frac{\partial^2 \left[\left(p_i^B(\alpha_i, \alpha_{3-i}) - \lambda_i c^B(\alpha_i, \alpha_{3-i})\right) q_i^B(\alpha_i, \alpha_{3-i})\right]}{\partial \alpha_i^2} \\
&= 2\left(\frac{\partial p_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} - \lambda_i \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right) \frac{\partial q_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}
\end{aligned}$$

$$= \frac{\left((2-\gamma^2)\,2\left[(2-\gamma^2)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right]-\left((2-\gamma^2)\,\lambda_i-\gamma\lambda_{3-i}\right)^2\right)^2}{(4-\gamma^2)^2\,(1-\gamma^2)\,2\left[(2-\gamma^2)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right]^2}$$

$$= \frac{\left((4-\gamma^2)\,(1-\gamma^2)\,\lambda_{3-i}^2+(2-\gamma^2)\left((2-\gamma^2)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right)\right)^2}{2\,(4-\gamma^2)^2\,(1-\gamma^2)\left((2-\gamma^2)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right)^2} > 0$$

for all $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ for $i = 1, 2$. Hence, for Cournot as well as for Bertrand competition any zero of the first-order condition is a minimum which is independent of the other intermediary's investment choice. Thus, the candidates for optimal investment choices are at the corner points of the according interval.

□

3.5.8 Proof of Proposition 3.8

For Cournot competition, the payoffs of the non-cooperative game are

$$\pi_1^C\left(\underline{\alpha},\underline{\alpha}\right) = \pi_2^C\left(\underline{\alpha},\underline{\alpha}\right) = \frac{\underline{\alpha}^2}{4\,(2+\gamma)^2}, \tag{3.65}$$

$$\pi_1^C\left(\overline{\alpha},\underline{\alpha}\right) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.66}$$

$$\pi_2^C\left(\overline{\alpha},\underline{\alpha}\right) = \begin{cases} 0 & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.67}$$

$$\pi_1^C\left(\underline{\alpha},\overline{\alpha}\right) = \begin{cases} 0 & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.68}$$

$$\pi_2^C\left(\underline{\alpha},\overline{\alpha}\right) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)]^2}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.69}$$

$$\pi_1^C\left(\overline{\alpha},\overline{\alpha}\right) = \pi_2^C\left(\overline{\alpha},\overline{\alpha}\right) = \frac{\overline{\alpha}^2}{4\,(2+\gamma)^2}, \tag{3.70}$$

where the upper and lower bound for the investment costs are given by

$$\underline{\kappa}^C = \begin{cases} \frac{\overline{\alpha}^2}{4(2+\gamma)^2} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)]}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.71}$$

$$\overline{\kappa}^C = \begin{cases} \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)]}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}. \end{cases} \tag{3.72}$$

Proof of Proposition 3.8.

(i) Consider $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$. If $\overline{\alpha}^2\gamma(4+\gamma) - 4\underline{\alpha}^2 < 0$, then

$$4\overline{\alpha}^2 > [\overline{\alpha}(2+\gamma) - 2\underline{\alpha}][\overline{\alpha}(2+\gamma) + 2\underline{\alpha}],$$

and hence, $\underline{\kappa}^C > \overline{\kappa}^C$. The zeros of $\overline{\alpha}^2\gamma(4+\gamma) - 4\underline{\alpha}^2$ are

$$\begin{aligned} \gamma \in & \left\{ \frac{2\sqrt{\overline{\alpha}(\overline{\alpha}+\underline{\alpha})}-2\underline{\alpha}}{\overline{\alpha}}, -\frac{2\sqrt{\overline{\alpha}(\overline{\alpha}+\underline{\alpha})}+2\underline{\alpha}}{\overline{\alpha}} \right\} \\ & = \left\{ 2\left(\sqrt{1+\frac{\underline{\alpha}}{\overline{\alpha}}}-1\right), -2\left(\sqrt{1+\frac{\underline{\alpha}}{\overline{\alpha}}}+1\right) \right\}. \end{aligned}$$

(ii) Consider $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}$. If $\gamma < -\frac{2}{3}$, then

$$\overline{\alpha}(2-5\gamma) + \underline{\alpha}(6+\gamma) > \overline{\alpha}(6+\gamma) + \underline{\alpha}(2-5\gamma),$$

and hence, $\underline{\kappa}^C > \overline{\kappa}^C$.

□

3.5.9 Proof of Proposition 3.9

The payoffs of the non-cooperative game for Bertrand competition are

$$\pi_1^B(\underline{\alpha},\underline{\alpha}) = \pi_2^B(\underline{\alpha},\underline{\alpha}) = \frac{\underline{\alpha}^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)}, \tag{3.73}$$

$$\pi_1^B(\overline{\alpha},\underline{\alpha}) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha}, \end{cases} \tag{3.74}$$

$$\pi_2^B(\overline{\alpha},\underline{\alpha}) = \begin{cases} 0 & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left[\underline{\alpha}(6+\gamma-3\gamma^2)-\overline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha}, \end{cases} \tag{3.75}$$

$$\pi_1^B(\underline{\alpha},\overline{\alpha}) = \begin{cases} 0 & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left[\underline{\alpha}(6+\gamma-3\gamma^2)-\overline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha}, \end{cases} \tag{3.76}$$

$$\pi_2^B(\underline{\alpha},\overline{\alpha}) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.77}$$

$$\pi_1^B(\overline{\alpha},\overline{\alpha}) = \pi_2^C(\overline{\alpha},\overline{\alpha}) = \frac{\overline{\alpha}^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)}. \tag{3.78}$$

Moreover the upper and lower bounds for the investment costs are given by

$$\underline{\kappa}^B = \begin{cases} \frac{\overline{\alpha}^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left(6+\gamma-3\gamma^2\right)[\overline{\alpha}-\underline{\alpha}]\left[\overline{\alpha}\left(2-5\gamma-\gamma^2\right)+\underline{\alpha}\left(6+\gamma-3\gamma^2\right)\right]}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \le \underline{\alpha}, \end{cases} \tag{3.79}$$

$$\overline{\kappa}^B = \begin{cases} \frac{\overline{\alpha}^2(2-\gamma)^2(1+\gamma)-4\underline{\alpha}^2(1-\gamma)}{16(2-\gamma)^2(1+\gamma)} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\left(6+\gamma-3\gamma^2\right)[\overline{\alpha}-\underline{\alpha}]\left[\overline{\alpha}\left(6+\gamma-3\gamma^2\right)+\underline{\alpha}\left(2-5\gamma-\gamma^2\right)\right]}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \le \underline{\alpha}. \end{cases} \tag{3.80}$$

Proof of Proposition 3.9.

(i) Consider $\underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right)$. If $\overline{\alpha}^2\gamma(4-3\gamma+\gamma^2)-4\underline{\alpha}^2(1-\gamma)<0$, then

$$4\overline{\alpha}^2(1-\gamma) > \overline{\alpha}^2(2-\gamma)^2(1+\gamma) - 4\underline{\alpha}^2(1-\gamma),$$

and hence, $\underline{\kappa}^B > \overline{\kappa}^B$. Note that $\overline{\alpha}^2\gamma(4-3\gamma+\gamma^2)-4\underline{\alpha}^2(1-\gamma)$ is increasing in γ for $\gamma \in [0,1]$, which can be seen as follows

$$\begin{aligned} \frac{\partial\left(\overline{\alpha}^2\gamma(4-3\gamma+\gamma^2)-4\underline{\alpha}^2(1-\gamma)\right)}{\partial\gamma} &= \overline{\alpha}^2\left(4-6\gamma+3\gamma^2\right)+4\underline{\alpha}^2 \\ &= \overline{\alpha}^2\left(3(1-\gamma)^2+1\right)+4\underline{\alpha}^2 > 0, \end{aligned}$$

and decreasing in the multiplier ξ, if we write $\underline{\alpha}=\xi\overline{\alpha}$ with $\xi\in(0,1]$, which is

$$\frac{\partial\left(\overline{\alpha}^2\gamma(4-3\gamma+\gamma^2)-4\xi^2\overline{\alpha}^2(1-\gamma)\right)}{\partial\gamma} = -8\xi\overline{\alpha}^2(1-\gamma)<0 \quad \text{for } \gamma\in[0,1).$$

These two observations imply that the zero in γ of $\overline{\alpha}^2\gamma(4-3\gamma+\gamma^2)-4\underline{\alpha}^2(1-\gamma)$ is shifted in the direction of the origin if the difference between $\overline{\alpha}$ and $\underline{\alpha}$ is increased. Thus, we need to look at $\overline{\alpha}=\underline{\alpha}$ to find an upper bound γ for which asymmetric Nash equilibria do not exist. This means we need to find the zeros of

$$4-8\gamma+3\gamma^2-\gamma^3.$$

The only real zero is at

$$\gamma = \frac{\left(2\sqrt{2}\sqrt{19} - 3\sqrt{3}\right)^{\frac{2}{3}} + \sqrt{3}\left(2\sqrt{2}\sqrt{19} - 3\sqrt{3}\right)^{\frac{1}{3}} - 5}{\sqrt{3}\left(2\sqrt{2}\sqrt{19} - 3\sqrt{3}\right)^{\frac{1}{3}}} \approx 0.6117.$$

(ii) Consider $\overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2} - 1\right) \leq \underline{\alpha}$. If $\gamma < -\frac{\sqrt{17}-3}{2} \approx -0.56$, then

$$\overline{\alpha}\left(2 - 5\gamma - \gamma^2\right) + \underline{\alpha}\left(6 + \gamma - 3\gamma^2\right) > \overline{\alpha}\left(6 + \gamma - 3\gamma^2\right) + \underline{\alpha}\left(2 - 5\gamma - \gamma^2\right),$$

and hence, $\underline{\kappa}^B > \overline{\kappa}^B$.

□

3.5.10 Proof of Proposition 3.10

Proof of Proposition 3.10. We have

$$\underline{\kappa}^C - \underline{\kappa}^B$$
$$= \begin{cases} \frac{\gamma^3\overline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2} - 1\right), \\ \frac{8\gamma^3(1-\gamma)\overline{\alpha}^2 + \left[\underline{\alpha}(6+\gamma-3\gamma^2) - \overline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2} - 1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma} - 1\right), \\ \frac{\gamma^3[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(7+\gamma) - \underline{\alpha}(3+5\gamma)]}{8(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma} - 1\right) \leq \underline{\alpha}, \end{cases}$$

and

$$\overline{\kappa}^C - \overline{\kappa}^B$$
$$= \begin{cases} -\frac{\gamma^3\underline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2} - 1\right), \\ \frac{\overline{\alpha}^2}{16} - \frac{8\gamma^3(1-\gamma)\underline{\alpha}^2 + \left[\overline{\alpha}(6+\gamma-3\gamma^2) - \underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2} - 1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma} - 1\right), \\ -\frac{\gamma^3[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(3+5\gamma) - \underline{\alpha}(7+\gamma)]}{8(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma} - 1\right) \leq \underline{\alpha}. \end{cases}$$

From these expressions for $\underline{\kappa}^C - \underline{\kappa}^B$ and $\overline{\kappa}^C - \overline{\kappa}^B$ we immediately derive the statements of Proposition 3.10.

□

For the special case of $\gamma = 0$ the lower and upper bounds of investment are given by

$$\underline{\kappa}^{C/B} = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2}-1\right), \\ \frac{3[\overline{\alpha}-\underline{\alpha}][3\overline{\alpha}+\underline{\alpha}]}{64} & \text{for } \overline{\alpha}\left(\sqrt{2}-1\right) \leq \underline{\alpha}, \end{cases} \tag{3.81}$$

$$\overline{\kappa}^{C/B} = \begin{cases} \frac{[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}+\underline{\alpha}]}{16} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2}-1\right), \\ \frac{3[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}+3\underline{\alpha}]}{64} & \text{for } \overline{\alpha}\left(\sqrt{2}-1\right) \leq \underline{\alpha}. \end{cases} \tag{3.82}$$

Hence, the expressions from the proof of Proposition 3.10 simplify to

$$\overline{\kappa}^{C/B} - \underline{\kappa}^{C/B} = \begin{cases} -\frac{[\overline{\alpha}-\underline{\alpha}][5\overline{\alpha}-\underline{\alpha}]}{64} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2}-1\right), \\ \frac{[\overline{\alpha}-3\underline{\alpha}]^2}{64} & \text{for } \overline{\alpha}\left(\sqrt{2}-1\right) \leq \underline{\alpha}. \end{cases} \tag{3.83}$$

3.6 Appendix B: Competing Input Suppliers

So far the input market was assumed to be monopolistic. In this section we relax this assumption and analyze an input market with $m > 1$ competing input suppliers. The total input demands of the intermediaries were determined in Eq. (3.9) for Cournot competition and in Eq. (3.18) for Bertrand competition. Let Q denote the total input demand, then in both cases the total input demand is of the form

$$Q\left(c\right) = \psi - \vartheta c \quad \text{with } \psi, \vartheta > 0. \tag{3.84}$$

This implies that the inverse demand function for the input market is

$$c\left(Q\right) = \frac{\psi - Q}{\vartheta}. \tag{3.85}$$

We suppose that the input suppliers deliver a homogenous product and compete by choosing quantities.[1] The total quantity provided on the input market is the sum of individual quantities of all input suppliers. This is

$$Q = \sum_{r=1}^{m} Q_r. \tag{3.86}$$

[1]Note that price competition for homogeneous goods implies pricing at marginal costs and thus, with the assumption that marginal costs are zero, the input price is zero. We therefore analyze quantity competition between the intermediaries.

Assuming zero marginal costs, each input supplier ℓ chooses his production quantity Q_ℓ to maximize

$$\left(\frac{\psi - \sum_{r=1}^{m} Q_r}{\vartheta}\right) Q_\ell. \tag{3.87}$$

The first-order condition of input supplier ℓ is

$$\frac{\psi - \sum_{r\neq\ell}^{m} Q_r - 2Q_\ell}{\vartheta} = 0. \tag{3.88}$$

As the input suppliers are symmetric, we look for the symmetric Nash equilibrium with $Q_\ell = Q_r$ for $1 \leq r, \ell \leq m$. Thus, the first-order condition becomes

$$\frac{\psi - (m+1)\, Q_\ell}{\vartheta} = 0 \tag{3.89}$$

and therefore,

$$Q_\ell = \frac{\psi}{m+1} \quad \text{and} \quad c\,(Q) = \frac{\psi - mQ_\ell}{\vartheta} = \frac{\psi}{(m+1)\,\vartheta}$$

with a profit of

$$c\,(Q)\, Q_\ell = \frac{\psi^2}{(m+1)^2\,\vartheta}.$$

Hence, we just have to insert the according expressions for ψ and ϑ from Eq. (3.9) and (3.18) to obtain the input prices for an input market with m input suppliers. The input prices for selling to both intermediaries are

$$c^C = \frac{2\,(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\,(m+1)\,[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]}, \tag{3.90}$$

$$c^B = \frac{(2-\gamma^2)\,(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{(m+1)\,[(2-\gamma^2)\,(\lambda_1^2 + \lambda_2^2) - 2\gamma\lambda_1\lambda_2]}, \tag{3.91}$$

and if $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$, for selling to intermediary 1 the input prices are

$$c^C = \max\left\{\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}, \frac{\alpha_1}{(m+1)\,\lambda_1}\right\}, \tag{3.92}$$

$$c^B = \max\left\{\frac{(2-\gamma^2)\,\alpha_2 - \gamma\alpha_1}{(2-\gamma^2)\,\lambda_2 - \gamma\lambda_1}, \frac{\alpha_1}{(m+1)\,\lambda_1}\right\}. \tag{3.93}$$

These input prices indicate that introducing competition on the input market requires additional comparisons of the input suppliers' profits beyond those considered in the proof of Proposition 3.1 (step 2.1) and 3.2 (step 2.1). The reason is that the importance of the corner solution can no longer be denied when selling to just one intermediary, given the number of input suppliers grows.

Mathematically, if we assume that the profit of the corner solution is always less than or equal to the profit of the interior solution for selling to just one intermediary, then comparing these profits and rearranging, we have for Cournot competition the condition

$$\frac{m}{(m+1)^2} \geq \frac{2\lambda_1 (2\alpha_2 - \gamma\alpha_1)(\alpha_1\lambda_2 - \alpha_2\lambda_1)}{\alpha_1^2 (2\lambda_2 - \gamma\lambda_1)^2}, \tag{3.94}$$

and for Bertrand competition

$$\frac{m}{(m+1)^2} \geq \frac{\lambda_1 (2-\gamma^2)((2-\gamma^2)\alpha_2 - \gamma\alpha_1)(\alpha_1\lambda_2 - \alpha_2\lambda_1)}{\alpha_1^2 ((2-\gamma^2)\lambda_2 - \gamma\lambda_1)^2}. \tag{3.95}$$

However, as $\lim_{m\to\infty} \frac{m}{(m+1)^2} = 0$, this is not always possible. This implies that there exists an m such that in a symmetric situation the input suppliers have to charge the price for the corner solution in order to sell to just one intermediary. Thus, including competition on the input market may influence the decisions on the input market and hence, impact our results. Therefore, a comprehensive analysis of the relation between the number of suppliers and the optimal decisions on the input market is left for future research.

3.7 Appendix C: Computations

Computation of Equation 3.6

By using the best reply functions of both intermediaries we compute intermediary i's Nash equilibrium quantity q_i^C, which is given by

$$q_i^C = \begin{cases} \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i - \lambda_i c}{2} & \text{if } \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise.} \end{cases}$$

The equilibrium quantity q_i^C for a sufficiently low input price with $c < \min\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\}$ is determined as follows:

$$\begin{aligned}
& q_i^C = \frac{\alpha_i - \gamma\left(\frac{\alpha_{3-i} - \gamma q_i^C - \lambda_{3-i}c}{2}\right) - \lambda_i c}{2} \\
\Leftrightarrow \quad & q_i^C = \frac{2\alpha_i - \gamma\left(\alpha_{3-i} - \gamma q_i^C - \lambda_{3-i}c\right) - 2\lambda_i c}{4} \\
\Leftrightarrow \quad & q_i^C - \frac{\gamma^2 q_i^C}{4} = \frac{2\alpha_i - \gamma\alpha_{3-i} + \gamma\lambda_{3-i}c - 2\lambda_i c}{4} \\
\Leftrightarrow \quad & q_i^C \left(1 - \frac{\gamma^2}{4}\right) = \frac{2\alpha_i - \gamma\alpha_{3-i} + \gamma\lambda_{3-i}c - 2\lambda_i c}{4} \\
\Leftrightarrow \quad & q_i^C \left(\frac{4 - \gamma^2}{4}\right) = \frac{2\alpha_i - \gamma\alpha_{3-i} + \gamma\lambda_{3-i}c - 2\lambda_i c}{4} \\
\Leftrightarrow \quad & q_i^C = \frac{2\alpha_i - \gamma\alpha_{3-i} + \gamma\lambda_{3-i}c - 2\lambda_i c}{4 - \gamma^2} \\
\Leftrightarrow \quad & q_i^C = \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(2\lambda_i - \gamma\lambda_{3-i}\right)c}{(4 - \gamma^2)}.
\end{aligned}$$

Computations of Equation 3.7

The according equilibrium price p_i^C is

$$p_i^C = \begin{cases} q_i^C + \lambda_i c & \text{if } c < \min\left\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i + \lambda_i c}{2} & \text{if } \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise.} \end{cases}$$

The equilibrium price for a sufficiently low input price is determined as follows:

$$\begin{aligned}
p_i^C &= p_i(q_1^C, q_2^C) \\
&= \alpha_i - \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(2\lambda_i - \gamma\lambda_{3-i}\right)c}{(2-\gamma)(2+\gamma)} - \gamma\frac{2\alpha_{3-i} - \gamma\alpha_i - \left(2\lambda_{3-i} - \gamma\lambda_i\right)c}{(2-\gamma)(2+\gamma)} \\
&= \alpha_i - \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(2\lambda_i - \gamma\lambda_{3-i}\right)c + 2\gamma\alpha_{3-i} - \gamma^2\alpha_i - \left(2\gamma\lambda_{3-i} - \gamma^2\lambda_i\right)c}{(2-\gamma)(2+\gamma)} \\
&= \alpha_i - \frac{\left(2 - \gamma^2\right)\alpha_i + \gamma\alpha_{3-i} - \left(\left(2 - \gamma^2\right)\lambda_i + \gamma\lambda_{3-i}\right)c}{(2-\gamma)(2+\gamma)} \\
&= \frac{\left(4 - \gamma^2\right)\alpha_i - \left(2 - \gamma^2\right)\alpha_i - \gamma\alpha_{3-i} + \left(\left(2 - \gamma^2\right)\lambda_i + \gamma\lambda_{3-i}\right)c}{(2-\gamma)(2+\gamma)} \\
&= \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(\left(\gamma^2 - 2\right)\lambda_i - \gamma\lambda_{3-i}\right)c}{(2-\gamma)(2+\gamma)}
\end{aligned}$$

$$= \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c - (\gamma^2 - 4)\, \lambda_i c}{(2-\gamma)(2+\gamma)}$$
$$= \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c + (4 - \gamma^2)\, \lambda_i c}{(2-\gamma)(2+\gamma)}$$
$$= \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c}{(2-\gamma)(2+\gamma)} + \lambda_i c$$
$$= q_i^C + \lambda_i c.$$

Computations of Assumption 3.1

Consider the equilibrium quantity q_i^C with $c < \min\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}\}$. For the Cournot equilibrium quantity q_i^C to be non-decreasing in "weighted" qualities we demand

$$2\alpha_i - \gamma\alpha_{3-i} > 0$$
$$2\alpha_i > \gamma\alpha_{3-i}$$
$$\frac{\alpha_i}{\alpha_{3-i}} > \frac{\gamma}{2}.$$

For q_i^C to be non-increasing in the input price c we need

$$2\lambda_i - \gamma\lambda_{3-i} > 0$$
$$2\lambda_i > \gamma\lambda_{3-i}$$
$$\frac{\lambda_i}{\lambda_{3-i}} > \frac{\gamma}{2}.$$

Computations of Equation 3.9

Taking the intermediaries' equilibrium quantities as given, the total market demand on the input market is $q_I^C(c) = \lambda_1 q_1^C + \lambda_2 q_2^C$ and given by

$$q_I^C(c) = \begin{cases} \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1) - 2\left(\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right)c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}\right\}, \\ \frac{\lambda_1(\alpha_1 - \lambda_1 c)}{2} & \text{if } \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \\ \frac{\lambda_2(\alpha_2 - \lambda_2 c)}{2} & \text{if } \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}, \\ 0 & \text{otherwise.} \end{cases}$$

The total market demand for a sufficiently small input price $c < \min\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}\}$ is determined as follows:

$$\begin{aligned}
& \lambda_1\left(\frac{2\alpha_1-\gamma\alpha_2-(2\lambda_1-\gamma\lambda_2)\,c}{(4-\gamma^2)}\right)+\lambda_2\left(\frac{2\alpha_2-\gamma\alpha_1-(2\lambda_2-\gamma\lambda_1)\,c}{(4-\gamma^2)}\right)\\
\Leftrightarrow\quad & \frac{\lambda_1\left(2\alpha_1-\gamma\alpha_2-(2\lambda_1-\gamma\lambda_2)\,c\right)+\lambda_2\left(2\alpha_2-\gamma\alpha_1-(2\lambda_2-\gamma\lambda_1)\,c\right)}{(4-\gamma^2)}\\
\Leftrightarrow\quad & \frac{2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)-2\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c}{(4-\gamma^2)}.
\end{aligned}$$

Computations of Proof of Proposition 3.1

Proof of Proposition 3.1. The proof proceeds in two steps. In a first step we determine the prices the input supplier charges. In doing so we distinguish different cases depending on the number of intermediaries demanding positive quantities. Afterwards the according profits are compared in step two.

Step 1 (Input Prices and Profits)

Case 1: The input price is chosen such that both intermediaries produce strictly positive quantities. Therefore, the following must hold:

$$c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}\right\}.$$

Maximizing the profit of the input supplier $q_I^C(c)\,c$ yields an input price

$$c^C = \frac{2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]}.$$

This can be seen as follows:

$$\begin{aligned}
& \frac{\partial\left[\frac{2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)-2\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c}{(4-\gamma^2)}c\right]}{\partial c} \overset{!}{=} 0\\
\Leftrightarrow\quad & \frac{2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)-4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c^C}{(4-\gamma^2)} \overset{!}{=} 0\\
\Leftrightarrow\quad & \frac{2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{(4-\gamma^2)} = \frac{4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c^S}{(4-\gamma^2)}\\
\Leftrightarrow\quad & \frac{2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)} = c^C
\end{aligned}$$

and

$$\frac{\partial^2 \left[\frac{2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)-2\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)c}{(4-\gamma^2)}c\right]}{\partial c^2} = \frac{-4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)}{(4-\gamma^2)} \leq 0.$$

Note that for this input price both intermediaries indeed purchase positive quantities. Suppose $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$, then $\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} > \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}$. Using $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2}$ and $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2}$ we have

$$0 < \left(4-\gamma^2\right)\left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right) \Leftrightarrow \frac{2\alpha_2-\gamma\alpha_1}{2\alpha_1-\gamma\alpha_2} < \frac{2\lambda_2-\gamma\lambda_1}{2\lambda_1-\gamma\lambda_2},$$

which can be seen as follows:

$$\begin{aligned}
& \frac{2\alpha_2-\gamma\alpha_1}{2\alpha_1-\gamma\alpha_2} < \frac{2\lambda_2-\gamma\lambda_1}{2\lambda_1-\gamma\lambda_2} \\
\Leftrightarrow\quad & (2\alpha_2-\gamma\alpha_1)(2\lambda_1-\gamma\lambda_2) < (2\lambda_2-\gamma\lambda_1)(2\alpha_1-\gamma\alpha_2) \\
\Leftrightarrow\quad & 4\alpha_2\lambda_1 - 2\gamma\alpha_2\lambda_2 - 2\gamma\alpha_1\lambda_1 + \gamma^2\alpha_1\lambda_2 < 4\alpha_1\lambda_2 - 2\gamma\alpha_2\lambda_2 - 2\gamma\alpha_1\lambda_1 + \gamma^2\alpha_2\lambda_1 \\
\Leftrightarrow\quad & 4\alpha_2\lambda_1 + \gamma^2\alpha_1\lambda_2 < 4\alpha_1\lambda_2 + \gamma^2\alpha_2\lambda_1 \\
\Leftrightarrow\quad & 0 < \left(4-\gamma^2\right)(\alpha_1\lambda_2 - \alpha_2\lambda_1) \\
\Leftrightarrow\quad & 0 < \left(4-\gamma^2\right)\left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right),
\end{aligned}$$

and thus,

$$\begin{aligned}
& 2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] \\
& = \lambda_1(2\lambda_1-\gamma\lambda_2) + \lambda_1(2\lambda_1-\gamma\lambda_2)\frac{\lambda_2(2\lambda_2-\gamma\lambda_1)}{\lambda_1(2\lambda_1-\gamma\lambda_2)} \\
& > \lambda_1(2\lambda_1-\gamma\lambda_2) + \lambda_1(2\lambda_1-\gamma\lambda_2)\frac{\lambda_2(2\alpha_2-\gamma\alpha_1)}{\lambda_1(2\alpha_1-\gamma\alpha_2)}.
\end{aligned}$$

By using

$$4\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] > 2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] \quad \text{for } \lambda_1, \lambda_2 > 0$$

this implies that

$$4\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] > \lambda_1(2\lambda_1-\gamma\lambda_2) + \lambda_1(2\lambda_1-\gamma\lambda_2)\frac{\lambda_2(2\alpha_2-\gamma\alpha_1)}{\lambda_1(2\alpha_1-\gamma\alpha_2)}.$$

Rearranging yields

$$\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} > \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]}$$

showing that the input price c^C is indeed sufficiently small to sell to both intermediaries. Alternatively, this can be seen as follows:
We have

$$\begin{aligned}
& c^C < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \\
\Leftrightarrow \quad & \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]} < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \\
\Leftrightarrow \quad & \frac{\lambda_1(2\alpha_1 - \gamma\alpha_2) + \lambda_2(2\alpha_2 - \gamma\alpha_1)}{4[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]} < \frac{\lambda_1(2\alpha_1 - \gamma\alpha_2)}{\lambda_1(2\lambda_1 - \gamma\lambda_2)} \\
\Leftrightarrow \quad & \lambda_1(2\lambda_1 - \gamma\lambda_2) + \lambda_1(2\lambda_1 - \gamma\lambda_2)\frac{\lambda_2(2\alpha_2 - \gamma\alpha_1)}{\lambda_1(2\alpha_1 - \gamma\alpha_2)} < 4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right]
\end{aligned}$$

and, therefore, using $\frac{2\alpha_2 - \gamma\alpha_1}{2\alpha_1 - \gamma\alpha_2} < \frac{2\lambda_2 - \gamma\lambda_1}{2\lambda_1 - \gamma\lambda_2}$ it suffices to show

$$\begin{aligned}
& \lambda_1(2\lambda_1 - \gamma\lambda_2) + \lambda_1(2\lambda_1 - \gamma\lambda_2)\frac{\lambda_2(2\lambda_2 - \gamma\lambda_1)}{\lambda_1(2\lambda_1 - \gamma\lambda_2)} < 4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] \\
\Leftrightarrow \quad & \lambda_1(2\lambda_1 - \gamma\lambda_2) + \lambda_2(2\lambda_2 - \gamma\lambda_1) < 4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] \\
\Leftrightarrow \quad & 2\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right] < 4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right],
\end{aligned}$$

which is always true for $\lambda_1, \lambda_2 > 0$. The profit of the input supplier is

$$q_I^C(c^C)\,c^C = \frac{[2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)]^2}{8(4-\gamma^2)[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2]}.$$

Case 2: The input price is chosen such that intermediary 1 produces a strictly positive quantity while intermediary 2 produces zero. Thus, we require

$$\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1},$$

which implies $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$. Maximizing the profit of the input supplier $q_I^C(c)\,c$ and taking a sufficiently high input price into account yields an input price

$$c^C = \max\left\{\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\}.$$

This can be seen as follows:

$$\begin{aligned}
&\frac{\partial\left[\frac{\lambda_i(\alpha_i-\lambda_i c)}{2}c\right]}{\partial c} \stackrel{!}{=} 0\\
\Leftrightarrow\quad &\frac{\lambda_i\alpha_i - 2\lambda_i^2 c}{2} \stackrel{!}{=} 0\\
\Leftrightarrow\quad &\lambda_i\alpha_i = 2\lambda_i^2 c\\
\Leftrightarrow\quad &\frac{\alpha_i}{2\lambda_i} = c.
\end{aligned}$$

Note that by definition this input price is indeed too high for intermediary 2 to purchase positive quantities. The analogous argument holds if intermediary 2 produces a strictly positive quantity while intermediary 1 produces zero. The profit of the input supplier is

$$q_I^C(c^C)\,c^C = \begin{cases} \frac{\alpha_1^2}{8} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1},\\ \frac{\lambda_1(2\alpha_2-\gamma\alpha_1)(\alpha_1\lambda_2-\alpha_2\lambda_1)}{(2\lambda_2-\gamma\lambda_1)^2} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \geq \frac{\alpha_1}{2\lambda_1}. \end{cases}$$

Suppose $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \geq \frac{\alpha_1}{2\lambda_1}$, then $c^C = \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}$,

$$\begin{aligned}
q_I^C(c^C) &= \frac{\lambda_1(\alpha_1-\lambda_1 c^C)}{2}\\
&= \frac{\lambda_1(\alpha_1-\lambda_1\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1})}{2}\\
&= \frac{\lambda_1(\frac{\alpha_1(2\lambda_2-\gamma\lambda_1)-\lambda_1(2\alpha_2-\gamma\alpha_1)}{2\lambda_2-\gamma\lambda_1})}{2}\\
&= \frac{\lambda_1(\alpha_1(2\lambda_2-\gamma\lambda_1)-\lambda_1(2\alpha_2-\gamma\alpha_1))}{2(2\lambda_2-\gamma\lambda_1)},
\end{aligned}$$

and

$$\begin{aligned}
q_I^C(c^C)c^C =& \frac{\lambda_1(\alpha_1(2\lambda_2-\gamma\lambda_1)-\lambda_1(2\alpha_2-\gamma\alpha_1))(2\alpha_2-\gamma\alpha_1)}{2(2\lambda_2-\gamma\lambda_1)^2}\\
=& \frac{\lambda_1((2\alpha_1\alpha_2-\gamma\alpha_1^2)(2\lambda_2-\gamma\lambda_1)-(2\lambda_1\alpha_2-\gamma\lambda_1\alpha_1)(2\alpha_2-\gamma\alpha_1))}{2(2\lambda_2-\gamma\lambda_1)^2}\\
=& \frac{\lambda_1(4\lambda_2\alpha_1\alpha_2-2\gamma\lambda_1\alpha_1\alpha_2-2\gamma\lambda_2\alpha_1^2+\gamma^2\lambda_1\alpha_1^2-4\lambda_1\alpha_2^2}{2(2(2\lambda_2-\gamma\lambda_1))^2}\\
&+\frac{2\gamma\lambda_1\alpha_1\alpha_2+2\gamma\lambda_1\alpha_1\alpha_2-\gamma^2\lambda_1\alpha_1^2)}{2(2\lambda_2-\gamma\lambda_1)^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\lambda_1(2\lambda_2\alpha_1\alpha_2 - \gamma\lambda_2\alpha_1^2 - 2\lambda_1\alpha_2^2 + \gamma\lambda_1\alpha_1\alpha_2)}{(2\lambda_2 - \gamma\lambda_1)^2} \\
&= \frac{\lambda_1(\lambda_2\alpha_1(2\alpha_2 - \gamma\alpha_1) - \lambda_1\alpha_2(2\alpha_2 - \gamma\alpha_1))}{(2\lambda_2 - \gamma\lambda_1)^2} \\
&= \frac{\lambda_1((2\alpha_2 - \gamma\alpha_1)(\lambda_2\alpha_1 - \lambda_1\alpha_2))}{(2\lambda_2 - \gamma\lambda_1)^2}.
\end{aligned}
$$

Note that if $\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} = \frac{\alpha_1}{2\lambda_1}$, then

$$
\begin{aligned}
& \frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1} = \frac{\alpha_1}{2\lambda_1} \\
\Leftrightarrow \quad & 2\lambda_1\left(2\alpha_2 - \gamma\alpha_1\right) = \alpha_1\left(2\lambda_2 - \gamma\lambda_1\right) \\
\Leftrightarrow \quad & 4\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 = 2\alpha_1\lambda_2 - \gamma\alpha_1\lambda_1 \\
\Leftrightarrow \quad & 4\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 - 2\alpha_1\lambda_2 + \gamma\alpha_1\lambda_1 = 0 \\
\Leftrightarrow \quad & 2\left(\alpha_2\lambda_1 - \alpha_1\lambda_2\right) + \lambda_1\left(2\alpha_2 - \gamma\alpha_1\right) = 0 \\
\Leftrightarrow \quad & \lambda_1\left(2\alpha_2 - \gamma\alpha_1\right) = 2\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)
\end{aligned}
$$

and thus,

$$
\begin{aligned}
q_I^C(c^C)c^C &= \frac{\lambda_1((2\alpha_2 - \gamma\alpha_1)(\lambda_2\alpha_1 - \lambda_1\alpha_2))}{(2\lambda_2 - \gamma\lambda_1)^2} \\
&= \lambda_1 \frac{\alpha_1}{2\lambda_1} \frac{(\lambda_2\alpha_1 - \lambda_1\alpha_2)}{(2\lambda_2 - \gamma\lambda_1)} \\
&= \frac{\alpha_1}{2} \frac{2(\lambda_2\alpha_1 - \lambda_1\alpha_2)}{2(2\lambda_2 - \gamma\lambda_1)} \\
&= \frac{\alpha_1}{2} \frac{\lambda_1\left(2\alpha_2 - \gamma\alpha_1\right)}{2(2\lambda_2 - \gamma\lambda_1)} \\
&= \frac{\alpha_1}{2} \frac{\lambda_1}{2} \frac{\alpha_1}{2\lambda_1} \\
&= \frac{\alpha_1^2}{8}.
\end{aligned}
$$

Case 3: The input price is too high for at least one intermediary to purchase strictly positive quantities.

Summing up, given the input demand the monopolistic input supplier charges a price of

$$c^C = \begin{cases} \frac{2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)}{4\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} & \text{for selling to both intermediaries,} \\ \max\left\{\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\} & \text{for selling only to intermediary 1 and } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\ \max\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{\alpha_2}{2\lambda_2}\right\} & \text{for selling only to intermediary 2 and } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds,} \end{cases}$$

and obtains a profit of

$$q_I^C(c^C)\,c^C = \begin{cases} \frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{8(4-\gamma^2)\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} & \text{for selling to both intermediaries} \\ \frac{\alpha_1^2}{8} & \text{for selling only to intermediary 1 and } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\ \frac{\lambda_1(2\alpha_2-\gamma\alpha_1)(\alpha_1\lambda_2-\alpha_2\lambda_1)}{(2\lambda_2-\gamma\lambda_1)^2} & \text{for selling only to intermediary 1 and } \frac{\alpha_1}{2\lambda_1} \leq \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\ \frac{\alpha_2^2}{8} & \text{for selling only to intermediary 2 and } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq \frac{\alpha_2}{2\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds,} \\ \frac{\lambda_2(2\alpha_1-\gamma\alpha_2)(\alpha_2\lambda_1-\alpha_1\lambda_2)}{(2\lambda_1-\gamma\lambda_2)^2} & \text{for selling only to intermediary 2 and } \frac{\alpha_2}{2\lambda_2} \leq \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds.} \end{cases}$$

Step 2 (Comparing Profits)
An input price as suggested in Case 3 cannot be optimal. In this scenario the input price is too high for the intermediaries to demand goods and the input supplier's profit is zero. In contrast, Case 1 as well as Case 2 yield positive profits. Therefore, we analyze the input supplier's profits for Cases 1 and 2. We compare the two monopoly profits for selling to intermediary 1 with the duopoly profits for selling to both intermediaries. Thus, suppose $\alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$. The remaining part when only selling to intermediary 2 follows analogously.

Step 2.1: We compare the two monopoly profits for selling to intermediary 1 and obtain by observing

$$(\alpha_1(2\lambda_2 + \gamma\lambda_1) - 4\lambda_1\alpha_2)^2 \geq 0$$

that

$$\frac{\alpha_1^2}{8} \geq \frac{\lambda_1 \left(2\alpha_2 - \gamma\alpha_1\right)\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)}{\left(2\lambda_2 - \gamma\lambda_1\right)^2}.$$

This can be seen as follows:

$$\begin{aligned}
& \frac{\alpha_1^2}{8} \geq \frac{\lambda_1 \left(2\alpha_2 - \gamma\alpha_1\right)\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)}{\left(2\lambda_2 - \gamma\lambda_1\right)^2} \\
\Leftrightarrow\quad & \alpha_1^2 \left(2\lambda_2 - \gamma\lambda_1\right)^2 \geq 8\lambda_1 \left(2\alpha_2 - \gamma\alpha_1\right)\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right) \\
\Leftrightarrow\quad & \alpha_1^2 \left(4\lambda_2^2 - 4\gamma\lambda_1\lambda_2 + \gamma^2\lambda_1^2\right) \geq 8\lambda_1 \left(2\alpha_2\alpha_1\lambda_2 - 2\alpha_2^2\lambda_1 - \gamma\alpha_1^2\lambda_2 + \gamma\alpha_1\alpha_2\lambda_1\right) \\
\Leftrightarrow\quad & \alpha_1^2 \left(4\lambda_2^2 - 4\gamma\lambda_1\lambda_2 + \gamma^2\lambda_1^2\right) \geq 16\alpha_2\alpha_1\lambda_2\lambda_1 - 16\alpha_2^2\lambda_1^2 - 8\gamma\alpha_1^2\lambda_1\lambda_2 + 8\gamma\alpha_1\alpha_2\lambda_1^2 \\
\Leftrightarrow\quad & \alpha_1^2(4\lambda_2^2 - 4\gamma\lambda_1\lambda_2 + 8\gamma\lambda_1\lambda_2 + \gamma^2\lambda_1^2) \geq 16\lambda_1\lambda_2\alpha_1\alpha_2 - 16\lambda_1^2\alpha_2^2 + 8\gamma\lambda_1^2\alpha_1\alpha_2 \\
\Leftrightarrow\quad & \alpha_1^2(4\lambda_2^2 + 4\gamma\lambda_1\lambda_2 + \gamma^2\lambda_1^2) - 16\lambda_1\lambda_2\alpha_1\alpha_2 + 16\lambda_1^2\alpha_2^2 - 8\gamma\lambda_1^2\alpha_1\alpha_2 \geq 0 \\
\Leftrightarrow\quad & \alpha_1^2 \left(2\lambda_2 + \gamma\lambda_1\right)^2 - 16\alpha_2\alpha_1\lambda_2\lambda_1 + 16\alpha_2^2\lambda_1^2 - 8\gamma\alpha_1\alpha_2\lambda_1^2 \geq 0 \\
\Leftrightarrow\quad & \alpha_1^2 \left(2\lambda_2 + \gamma\lambda_1\right)^2 - 16\lambda_1\alpha_2\alpha_1\lambda_2 - 8\gamma\lambda_1^2\alpha_1\alpha_2 + 16\lambda_1^2\alpha_2^2 \geq 0 \\
\Leftrightarrow\quad & \alpha_1^2 \left(2\lambda_2 + \gamma\lambda_1\right)^2 - 8\lambda_1\alpha_2\alpha_1 \left(2\lambda_2 + \gamma\lambda_1\right) + 16\lambda_1^2\alpha_2^2 \geq 0 \\
\Leftrightarrow\quad & \left(\alpha_1 \left(2\lambda_2 + \gamma\lambda_1\right) - 4\lambda_1\alpha_2\right)^2 \geq 0.
\end{aligned}$$

Step 2.2: We compare the duopoly profit with the monopoly profit for selling to intermediary 1. If the duopoly profit is greater than or equal to both monopoly profits, then the input supplier decides to sell to both intermediaries. Therefore, we obtain that if

$$\alpha_2 \geq \alpha_1 \tau_1^C$$

with

$$\tau_1^C = \left(\frac{\sqrt{\left(4-\gamma^2\right)\left(\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right)} - \left(2\lambda_1 - \gamma\lambda_2\right)}{\left(2\lambda_2 - \gamma\lambda_1\right)} \right),$$

then

$$\frac{\alpha_1^2}{8} \leq \frac{\left[2\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)\right]^2}{8\left(4-\gamma^2\right)\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right]}$$

holds. This can be seen as follows:

$$
\begin{aligned}
& \frac{(2(\alpha_1\lambda_1+\alpha_2\lambda_2)-\gamma(\alpha_1\lambda_2+\alpha_2\lambda_1))^2}{8\,(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)} \geq \frac{\alpha_1^2}{8} \\
\Leftrightarrow\quad & \frac{(2(\alpha_1\lambda_1+\alpha_2\lambda_2)-\gamma(\alpha_1\lambda_2+\alpha_2\lambda_1))^2}{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)} \geq \alpha_1^2 \\
\Leftrightarrow\quad & (2(\alpha_1\lambda_1+\alpha_2\lambda_2)-\gamma(\alpha_1\lambda_2+\alpha_2\lambda_1))^2 \geq \alpha_1^2(\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)) \\
\Leftrightarrow\quad & \left(\alpha_1(2\lambda_1-\gamma\lambda_2)+\alpha_2(2\lambda_2-\gamma\lambda_1)\right)^2 \geq \alpha_1^2\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2) \\
\Leftrightarrow\quad & \alpha_1(2\lambda_1-\gamma\lambda_2)+\alpha_2(2\lambda_2-\gamma\lambda_1) \geq \alpha_1\sqrt{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)} \\
\Leftrightarrow\quad & \alpha_2 \geq \alpha_1\left(\frac{\sqrt{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)}-(2\lambda_1-\gamma\lambda_2)}{(2\lambda_2-\gamma\lambda_1)}\right).
\end{aligned}
$$

Thus, whenever condition (3.35) holds, we know that a profit-maximizing input supplier prefers to sell in any case to both intermediaries. Thus, we know that for

$$\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$$

the input supplier prefers to set a price for which both intermediaries procure inputs. Note that the following is always true:

$$\tau_1^C \leq \frac{\lambda_2}{\lambda_1}.$$

This can be seen as follows:

$$
\begin{aligned}
& \frac{\sqrt{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)}-(2\lambda_1-\gamma\lambda_2)}{(2\lambda_2-\gamma\lambda_1)} \leq \frac{\lambda_2}{\lambda_1} \\
\Leftrightarrow\quad & \lambda_1\sqrt{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)}-\lambda_1\,(2\lambda_1-\gamma\lambda_2) \leq \lambda_2\,(2\lambda_2-\gamma\lambda_1) \\
\Leftrightarrow\quad & \lambda_1\sqrt{(4-\gamma^2)\,(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2)} \leq \lambda_2\,(2\lambda_2-\gamma\lambda_1)+\lambda_1\,(2\lambda_1-\gamma\lambda_2) \\
\Leftrightarrow\quad & \lambda_1^2\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2) \leq (\lambda_2\,(2\lambda_2-\gamma\lambda_1)+\lambda_1\,(2\lambda_1-\gamma\lambda_2))^2 \\
\Leftrightarrow\quad & \lambda_1^2\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2) \leq \left(2\lambda_2^2-\gamma\lambda_1\lambda_2+2\lambda_1^2-\gamma\lambda_1\lambda_2\right)^2 \\
\Leftrightarrow\quad & \lambda_1^2\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2) \leq 4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)^2 \\
\Leftrightarrow\quad & 0 \leq 4\left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)^2-\lambda_1^2\left(4-\gamma^2\right)(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2) \\
\Leftrightarrow\quad & 0 \leq \left(\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right)\left(4\lambda_1^2+4\lambda_2^2-4\gamma\lambda_1\lambda_2-\lambda_1^2\left(4-\gamma^2\right)\right)
\end{aligned}
$$

$$\Leftrightarrow \quad 0 \leq \left(\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right)\left(4\lambda_2^2 - 4\gamma\lambda_1\lambda_2 + \lambda_1^2\gamma^2\right)$$
$$\Leftrightarrow \quad 0 \leq \left(\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right)\left(2\lambda_2 - \gamma\lambda_1\right)^2.$$

Step 2.1 and Step 2.2 show that for $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ the the claim of Proposition 3.1 holds. Note that requiring $\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq \frac{\alpha_1}{2\lambda_1}$ is equivalent to choosing $\alpha_2 \leq \alpha_1\left(\frac{2\lambda_2+\gamma\lambda_1}{4\lambda_1}\right)$. □

Computations of Equation 3.12

Given both intermediaries compete, by using Eq. (3.2) and assuming $\gamma \in (-1, 1)$ we obtain the customer's demand function

$$q_i(p_1, p_2) = \frac{\alpha_i - p_i + \gamma\left(p_{3-i} - \alpha_{3-i}\right)}{(1-\gamma^2)}$$

for intermediary $i \in \{1, 2\}$. This can be seen as follows: we have $q_i = \alpha_i - \gamma q_{3-i} - p_i$ and $q_{3-i} = \alpha_{3-i} - \gamma q_i - p_{3-i}$. Therefore,

$$q_i = \alpha_i - \gamma\left(\alpha_{3-i} - \gamma q_i - p_{3-i}\right) - p_i$$
$$\Leftrightarrow \quad q_i = \alpha_i - \gamma\alpha_{3-i} + \gamma^2 q_i + \gamma p_{3-i} - p_i$$
$$\Leftrightarrow \quad q_i - \gamma^2 q_i = \alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} - p_i$$
$$\Leftrightarrow \quad q_i\left(1-\gamma^2\right) = \alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} - p_i$$
$$\Leftrightarrow \quad q_i = \frac{\alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} - p_i}{(1-\gamma^2)}.$$

Computations of Equation 3.14

By using Eq. (3.13) we compute intermediary i's best reply function. As above, negative profits can be avoided by not producing anything

$$p_i\left(p_{3-i}\right) = \max\left\{\frac{\alpha_i - \gamma\left(\alpha_{3-i} - p_{3-i}\right) + \lambda_i c}{2}, 0\right\}.$$

This can be seen as follows:

$$\frac{\partial\pi_i^B(p_i, p_{3-i}, c, \alpha_i, \alpha_{3-i})}{\partial p_i} \stackrel{!}{=} 0$$
$$\Leftrightarrow \quad \frac{1}{1-\gamma^2}\left(\alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} - 2p_i + \lambda_i c\right) \stackrel{!}{=} 0$$
$$\Leftrightarrow \quad \alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} - 2p_i + \lambda_i c = 0$$

$$\begin{aligned}
&\Leftrightarrow \quad 2p_i = \alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} + \lambda_i c \\
&\Leftrightarrow \quad p_i = \frac{\alpha_i - \gamma\alpha_{3-i} + \gamma p_{3-i} + \lambda_i c}{2} \\
&\Leftrightarrow \quad p_i = \frac{\alpha_i - \gamma\left(\alpha_{3-i} - p_{3-i}\right) + \lambda_i c}{2}.
\end{aligned}$$

Computations of Equation 3.15 and Equation 3.16

The Nash equilibrium price in Bertrand competition is given by

$$p_i^B = \begin{cases} \frac{\left(2-\gamma^2\right)\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})c}{(4-\gamma^2)} & \text{if } c < \min\left\{\frac{\left(2-\gamma^2\right)\alpha_i - \gamma\alpha_{3-i}}{(2-\gamma^2)\lambda_i - \gamma\lambda_{3-i}}, \frac{\left(2-\gamma^2\right)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i}\right\}, \\ \frac{\alpha_i + \lambda_i c}{2} & \text{if } \frac{\left(2-\gamma^2\right)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i}, \\ 0 & \text{otherwise,} \end{cases}$$

which can be seen as follows:

$$\begin{aligned}
& p_i^B = \frac{\alpha_i - \gamma\alpha_{3-i} + \gamma\left(\frac{\alpha_{3-i} - \gamma\alpha_i + \gamma p_i^B + \lambda_{3-i}c}{2}\right) + \lambda_i c}{2} \\
\Leftrightarrow \quad & p_i^B = \frac{1}{2}\left(\alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\left(\alpha_{3-i} - \gamma\alpha_i + \gamma p_i^B + \lambda_{3-i}c\right) + \lambda_i c\right) \\
\Leftrightarrow \quad & 2p_i^B = \alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\left(\alpha_{3-i} - \gamma\alpha_i + \gamma p_i^B + \lambda_{3-i}c\right) + \lambda_i c \\
\Leftrightarrow \quad & 2p_i^B - \frac{1}{2}\gamma^2 p_i^B = \alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\left(\alpha_{3-i} - \gamma\alpha_i + \lambda_{3-i}c\right) + \lambda_i c \\
\Leftrightarrow \quad & p_i^B\left(2 - \frac{1}{2}\gamma^2\right) = \alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\left(\alpha_{3-i} - \gamma\alpha_i + \lambda_{3-i}c\right) + \lambda_i c \\
\Leftrightarrow \quad & p_i^B = \frac{\alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\left(\alpha_{3-i} - \gamma\alpha_i + \lambda_{3-i}c\right) + \lambda_i c}{2 - \frac{1}{2}\gamma^2} \\
\Leftrightarrow \quad & p_i^B = \frac{\alpha_i - \gamma\alpha_{3-i} + \frac{1}{2}\gamma\alpha_{3-i} - \frac{1}{2}\gamma^2\alpha_i + \frac{1}{2}\gamma\lambda_{3-i}c + \lambda_i c}{2 - \frac{1}{2}\gamma^2} \\
\Leftrightarrow \quad & p_i^B = \frac{\alpha_i\left(1 - \frac{1}{2}\gamma^2\right) - \frac{1}{2}\gamma\alpha_{3-i} + \frac{1}{2}\gamma\lambda_{3-i}c + \lambda_i c}{2 - \frac{1}{2}\gamma^2} \\
\Leftrightarrow \quad & p_i^B = \frac{\alpha_i\left(2 - \gamma^2\right) - \gamma\alpha_{3-i} + \gamma\lambda_{3-i}c + 2\lambda_i c}{4 - \gamma^2} \\
\Leftrightarrow \quad & p_i^B = \frac{(2 - \gamma^2)\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})c}{4 - \gamma^2} \\
\Leftrightarrow \quad & p_i^B = \frac{\left(2 - \gamma^2\right)\alpha_i - \gamma\left(\alpha_{3-i} - \lambda_{3-i}c\right) + 2\lambda_i c}{4 - \gamma^2}.
\end{aligned}$$

The equilibrium quantity is denoted by

$$q_i^B = \begin{cases} \frac{p_i^B - \lambda_i c}{(1-\gamma^2)} & \text{if } c < \min\left\{\frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i}}{(2-\gamma^2)\lambda_i - \gamma\lambda_{3-i}}, \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i}\right\} \\ \frac{\alpha_i - \lambda_i c}{2} & \text{if } \frac{(2-\gamma^2)\alpha_{3-i} - \gamma\alpha_i}{(2-\gamma^2)\lambda_{3-i} - \gamma\lambda_i} \leq c < \frac{\alpha_i}{\lambda_i} \\ 0 & \text{otherwise.} \end{cases}$$

This can be seen as follows: using

$$\begin{aligned} p_i^B &= \frac{\alpha_i (2-\gamma^2) - \gamma(\alpha_{3-i} - \lambda_{3-i} c) + 2\lambda_i c}{4-\gamma^2} \\ p_{3-i}^B &= \frac{\alpha_{3-i} (2-\gamma^2) - \gamma(\alpha_i - \lambda_i c) + 2\lambda_{3-i} c}{4-\gamma^2} \end{aligned}$$

we have

$$\begin{aligned} q_i^B =& \frac{\alpha_i - \left(\frac{\alpha_i(2-\gamma^2) - \gamma(\alpha_{3-i} - \lambda_{3-i}c) + 2\lambda_i c}{4-\gamma^2}\right) - \gamma\left(\alpha_{3-i} - \left(\frac{\alpha_{3-i}(2-\gamma^2) - \gamma(\alpha_i - \lambda_i c) + 2\lambda_{3-i}c}{4-\gamma^2}\right)\right)}{1-\gamma^2} \\ =& \frac{\alpha_i - \left(\frac{\alpha_i(2-\gamma^2) - \gamma(\alpha_{3-i} - \lambda_{3-i}c) + 2\lambda_i c - \gamma\alpha_{3-i}(2-\gamma^2) + \gamma^2(\alpha_i - \lambda_i c) - 2\gamma\lambda_{3-i}c}{4-\gamma^2} + \gamma\alpha_{3-i}\right)}{1-\gamma^2} \\ =& \frac{\alpha_i + \frac{-\alpha_i(2-\gamma^2) + \gamma(\alpha_{3-i} - \lambda_{3-i}c) - 2\lambda_i c + \gamma\alpha_{3-i}(2-\gamma^2) - \gamma^2(\alpha_i - \lambda_i c) + 2\gamma\lambda_{3-i}c}{4-\gamma^2} - \gamma\alpha_{3-i}}{1-\gamma^2} \\ =& \frac{(4-\gamma^2)\alpha_i - \alpha_i(2-\gamma^2) + \gamma(\alpha_{3-i} - \lambda_{3-i}c) - 2\lambda_i c + \gamma\alpha_{3-i}(2-\gamma^2)}{(1-\gamma^2)(4-\gamma^2)} \\ &+ \frac{-\gamma^2(\alpha_i - \lambda_i c) + 2\gamma\lambda_{3-i}c - \gamma(4-\gamma^2)\alpha_{3-i}}{(1-\gamma^2)(4-\gamma^2)} \\ =& \frac{(2-\gamma^2)(\alpha_i - \lambda_i c) + \gamma(\alpha_{3-i} - \lambda_{3-i}c) + \gamma\alpha_{3-i}(2-\gamma^2)}{(1-\gamma^2)(4-\gamma^2)} \\ &+ \frac{2\gamma\lambda_{3-i}c - \gamma(4-\gamma^2)\alpha_{3-i}}{(1-\gamma^2)(4-\gamma^2)} \\ =& \frac{(2-\gamma^2)(\alpha_i - \lambda_i c) + \gamma(\alpha_{3-i} - \lambda_{3-i}c) - 2\gamma\alpha_{3-i} + 2\gamma\lambda_{3-i}c}{(1-\gamma^2)(4-\gamma^2)} \\ =& \frac{(2-\gamma^2)(\alpha_i - \lambda_i c) - \gamma(\alpha_{3-i} - \lambda_{3-i}c)}{(4-\gamma^2)(1-\gamma^2)} \\ =& \frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i} - ((2-\gamma^2)\lambda_i - \gamma\lambda_{3-i})c}{(4-\gamma^2)(1-\gamma^2)} \\ =& \frac{(2-\gamma^2)\alpha_i - \gamma\alpha_{3-i}}{(4-\gamma^2)(1-\gamma^2)} - \frac{((2-\gamma^2)\lambda_i - \gamma\lambda_{3-i})c}{(4-\gamma^2)(1-\gamma^2)} \end{aligned}$$

$$
\begin{aligned}
&+\frac{(2\lambda_i+\gamma\lambda_{3-i})\,c}{(4-\gamma^2)(1-\gamma^2)}-\frac{(2\lambda_i+\gamma\lambda_{3-i})\,c}{(4-\gamma^2)(1-\gamma^2)}\\
=&\frac{(2-\gamma^2)\,\alpha_i-\gamma\alpha_{3-i}+(2\lambda_i+\gamma\lambda_{3-i})\,c}{(4-\gamma^2)(1-\gamma^2)}\\
&-\frac{((2-\gamma^2)\,\lambda_i-\gamma\lambda_{3-i})\,c+(2\lambda_i+\gamma\lambda_{3-i})\,c}{(4-\gamma^2)(1-\gamma^2)}\\
=&\frac{(2-\gamma^2)\,\alpha_i-\gamma\alpha_{3-i}+(2\lambda_i+\gamma\lambda_{3-i})\,c}{(4-\gamma^2)(1-\gamma^2)}-\frac{(4-\gamma^2)\,\lambda_i c}{(4-\gamma^2)(1-\gamma^2)}\\
=&\frac{p_i^B}{(1-\gamma^2)}-\frac{\lambda_i c}{(1-\gamma^2)}\\
=&\frac{p_i^B-\lambda_i c}{(1-\gamma^2)}.
\end{aligned}
$$

Computations of Equation 3.18

The total market demand on the input market is $q_I^B(c)=\lambda_1 q_1^B+\lambda_2 q_2^B$ and given by

$$
q_I^B(c)=\begin{cases}
\frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)}{(4-\gamma^2)(1-\gamma^2)} & \\
\quad-\frac{\left[(2-\gamma^2)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)(1-\gamma^2)} & \text{if } c<\min\left\{\frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2},\frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}\right\},\\
\frac{\lambda_1(\alpha_1-\lambda_1 c)}{2} & \text{if } \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}\le c<\frac{\alpha_1}{\lambda_1},\\
\frac{\lambda_2(\alpha_2-\lambda_2 c)}{2} & \text{if } \frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2}\le c<\frac{\alpha_2}{\lambda_2},\\
0 & \text{otherwise.}
\end{cases}
$$

The total market demand for a sufficiently small input price $c<\min\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2},\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}\}$ is determined as follows:

$$
\begin{aligned}
q_I^B(c)=&\lambda_1\left(\frac{(2-\gamma^2)\,\alpha_1-\gamma\alpha_2-((2-\gamma^2)\,\lambda_1-\gamma\lambda_2)\,c}{(4-\gamma^2)(1-\gamma^2)}\right)\\
&+\lambda_2\left(\frac{(2-\gamma^2)\,\alpha_2-\gamma\alpha_1-((2-\gamma^2)\,\lambda_2-\gamma\lambda_1)\,c}{(4-\gamma^2)(1-\gamma^2)}\right)\\
\Leftrightarrow\quad&\frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)}{(4-\gamma^2)(1-\gamma^2)}\\
&-\frac{\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)(1-\gamma^2)}.
\end{aligned}
$$

Computations of Proof of Proposition 3.2

Proof of Proposition 3.2. The proof proceeds in two steps. In a first step we determine the prices the input supplier charges. In doing so we distinguish different cases depending on the number of intermediaries demanding positive quantities. Afterwards the according profits are compared in step two.

Step 1 (Input Prices and Profits)

Case 1: The input price is chosen such that both intermediaries produce strictly positive quantities. Therefore the following must hold:

$$c < \min\left\{\frac{(2-\gamma^2)\,\alpha_1 - \gamma\alpha_2}{(2-\gamma^2)\,\lambda_1 - \gamma\lambda_2}, \frac{(2-\gamma^2)\,\alpha_2 - \gamma\alpha_1}{(2-\gamma^2)\,\lambda_2 - \gamma\lambda_1}\right\}.$$

Maximizing the profit of the input supplier yields

$$c^B = \frac{(2-\gamma^2)\,(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)\,(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]}.$$

This can be seen as follows:

$$\begin{aligned}
&\frac{\partial\left[\frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)-\left[(2-\gamma^2)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)(1-\gamma^2)}c\right]}{\partial c} \overset{!}{=} 0\\
\Leftrightarrow\quad &\frac{(2-\gamma^2)\,(\lambda_1\alpha_1+\lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2+\lambda_2\alpha_1)}{(4-\gamma^2)\,(1-\gamma^2)}\\
&-\frac{2\left[(2-\gamma^2)\,(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)\,(1-\gamma^2)} \overset{!}{=} 0\\
\Leftrightarrow\quad &\frac{(2-\gamma^2)\,(\lambda_1\alpha_1+\lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2+\lambda_2\alpha_1)}{(4-\gamma^2)\,(1-\gamma^2)}\\
=&\frac{2\left[(2-\gamma^2)\,(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)\,(1-\gamma^2)}\\
\Leftrightarrow\quad &\frac{(2-\gamma^2)\,(\lambda_1\alpha_1+\lambda_2\alpha_2) - \gamma\,(\lambda_1\alpha_2+\lambda_2\alpha_1)}{2\left[(2-\gamma^2)\,(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} = c
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\partial^2\left[\frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)-\left[(2-\gamma^2)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]c}{(4-\gamma^2)(1-\gamma^2)}c\right]}{\partial c^2}\\
=&\frac{-2\left[(2-\gamma^2)\,(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]}{(4-\gamma^2)\,(1-\gamma^2)} \leq 0.
\end{aligned}$$

Note that for this input price both intermediaries indeed purchase positive quantities. Suppose $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$, then $\frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2} > \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}$. Using $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2}$ and $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} > \frac{\gamma}{2-\gamma^2}$ we have

$$0 < \left(2-\gamma^2\right)^2 \left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right) \Leftrightarrow \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2} < \frac{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1}{\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2}$$

which can be seen as follows:

$$\begin{aligned}
&\frac{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1}{\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2} > \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2} \\
\Leftrightarrow\quad & \left((2-\gamma^2)\alpha_1 - \gamma\alpha_2\right)\left((2-\gamma^2)\lambda_2 - \gamma\lambda_1\right) \\
> & \left((2-\gamma^2)\alpha_2 - \gamma\alpha_1\right)\left((2-\gamma^2)\lambda_1 - \gamma\lambda_2\right) \\
\Leftrightarrow\quad & \left(2\alpha_1 - \gamma^2\alpha_1 - \gamma\alpha_2\right)\left(2\lambda_2 - \gamma^2\lambda_2 - \gamma\lambda_1\right) \\
> & \left(2\alpha_2 - \gamma^2\alpha_2 - \gamma\alpha_1\right)\left(2\lambda_1 - \gamma^2\lambda_1 - \gamma\lambda_2\right) \\
\Leftrightarrow\quad & 4\lambda_2\alpha_1 - 2\gamma^2\lambda_2\alpha_1 - 2\gamma\lambda_1\alpha_1 - 2\gamma^2\lambda_2\alpha_1 + \gamma^4\lambda_2\alpha_1 \\
& + \gamma^3\lambda_1\alpha_1 - 2\gamma\lambda_2\alpha_2 + \gamma^3\lambda_2\alpha_2 + \gamma^2\lambda_1\alpha_2 \\
> & 4\lambda_1\alpha_2 - 2\gamma^2\lambda_1\alpha_2 - 2\gamma\lambda_2\alpha_2 - 2\gamma^2\lambda_1\alpha_2 + \gamma^4\lambda_1\alpha_2 \\
& + \gamma^3\lambda_2\alpha_2 - 2\gamma\lambda_1\alpha_1 + \gamma^3\lambda_1\alpha_1 + \gamma^2\lambda_1\alpha_2 \\
\Leftrightarrow\quad & 4\lambda_2\alpha_1 - 4\gamma^2\lambda_2\alpha_1 + \gamma^4\lambda_2\alpha_1 + 4\gamma^2\lambda_1\alpha_2 - 4\lambda_1\alpha_2 - \gamma^4\lambda_1\alpha_2 > 0 \\
\Leftrightarrow\quad & \lambda_2\alpha_1\left(4 - 4\gamma^2 + \gamma^4\right) - \lambda_1\alpha_2\left(4 - 4\gamma^2 - \gamma^4\right) > 0 \\
\Leftrightarrow\quad & \left(4 - 4\gamma^2 + \gamma^4\right)\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right) > 0 \\
\Leftrightarrow\quad & \left(2-\gamma^2\right)^2\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right) > 0 \\
\Leftrightarrow\quad & \left(2-\gamma^2\right)^2\left(\frac{\alpha_1}{\alpha_2} - \frac{\lambda_1}{\lambda_2}\right) > 0,
\end{aligned}$$

and thus,

$$\begin{aligned}
&\left(2-\gamma^2\right)\left(\lambda_1^2 + \lambda_2^2\right) - 2\gamma\lambda_1\lambda_2 \\
&= \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)} \\
&> \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1 - \gamma\alpha_2\right)},
\end{aligned}$$

which implies when using

$$2\left(\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right) > \left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2 \quad \text{for } \lambda_1,\lambda_2>0$$

that

$$\begin{aligned}&2\left(\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right)\\&>\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)+\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2\right)}.\end{aligned}$$

Rearranging yields

$$\frac{(2-\gamma^2)\alpha_1-\gamma\alpha_2}{(2-\gamma^2)\lambda_1-\gamma\lambda_2} > \frac{(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)}{2[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2]}$$

showing that the input price c^B is indeed sufficiently small to sell to both intermediaries. Alternatively, this can be seen as follows: we have

$$\begin{aligned}&c^B<\frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2}\\
\Leftrightarrow\quad&\frac{\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}<\frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2}\\
\Leftrightarrow\quad&\frac{2\lambda_1\alpha_1+2\lambda_2\alpha_2-\gamma^2\lambda_1\alpha_1-\gamma^2\lambda_2\alpha_2-\gamma\lambda_1\alpha_2-\gamma\lambda_2\alpha_1}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}<\frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2}\\
\Leftrightarrow\quad&\frac{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right)}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}<\frac{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2\right)}{\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)}\\
\Leftrightarrow\quad&\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)+\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2\right)}\\
&\quad<2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\end{aligned}$$

and therefore, using $\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\alpha_1-\gamma\alpha_2}<\frac{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1}{(2-\gamma^2)\lambda_1-\gamma\lambda_2}$ it suffices to show

$$\begin{aligned}&\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\lambda_2\gamma\right)+\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)\frac{\lambda_2\left(\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right)}{\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\lambda_2\gamma\right)}\\
&\quad<2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\\
\Leftrightarrow\quad&\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\lambda_2\gamma\right)+\lambda_2\left(\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right)\\
&\quad<2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\\
\Leftrightarrow\quad&\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2<2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\end{aligned}$$

which is always true for $\lambda_1, \lambda_2 > 0$. The profit of the input supplier is

$$q_I^B(c^B)\,c^B = \frac{\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{4(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]}.$$

Case 2: The input price is chosen such that intermediary 1 produces a strictly positive quantity while intermediary 2 produces zero. Thus, we require

$$\frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \le c < \frac{\alpha_1}{\lambda_1},$$

which implies $\frac{\alpha_1}{\alpha_2} > \frac{\lambda_1}{\lambda_2}$. Given a sufficiently high input price, maximizing the profit of the input supplier yields

$$c^B = \max\left\{\frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\}.$$

This can be seen as follows:

$$\begin{aligned} & \frac{\partial\left[\frac{\lambda_i(\alpha_i-\lambda_i c)}{2}c\right]}{\partial c} \overset{!}{=} 0 \\ \Leftrightarrow\; & \frac{\lambda_i\alpha_i - 2\lambda_i^2 c}{2} \overset{!}{=} 0 \\ \Leftrightarrow\; & \lambda_i\alpha_i = 2\lambda_i^2 c \\ \Leftrightarrow\; & \frac{\alpha_i}{2\lambda_i} = c. \end{aligned}$$

Note that by definition this input price is indeed too high for intermediary 2 to purchase positive quantities. The analogous argument holds if intermediary 2 produces a strictly positive quantity while intermediary 1 produces zero. The profit of the input supplier is

$$q_I^B(c^B)\,c^B = \begin{cases} \frac{\alpha_1^2}{8} & \text{if } \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \le \frac{\alpha_1}{2\lambda_1} \\ \frac{\lambda_1(2-\gamma^2)\left[(2-\gamma^2)\alpha_2-\gamma\alpha_1\right](\alpha_1\lambda_2-\alpha_2\lambda_1)}{2\left[(2-\gamma^2)\lambda_2-\gamma\lambda_1\right]^2} & \text{if } \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \ge \frac{\alpha_1}{2\lambda_1}. \end{cases}$$

Suppose $\frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1} \ge \frac{\alpha_1}{2\lambda_1}$, then $c^B = \frac{(2-\gamma^2)\alpha_2-\gamma\alpha_1}{(2-\gamma^2)\lambda_2-\gamma\lambda_1}$,

$$q_I^B\left(c^B\right) = \lambda_1\frac{\alpha_1-\lambda_1 c^B}{2}$$

$$
\begin{aligned}
&= \lambda_1 \frac{\alpha_1 - \lambda_1 \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1}}{2} \\
&= \lambda_1 \frac{\alpha_1 \left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) - \left(2-\gamma^2\right)\lambda_1\alpha_2 + \gamma\lambda_1\alpha_1}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \\
&= \lambda_1 \frac{\lambda_2\alpha_1 \left(2-\gamma^2\right) - \gamma\lambda_1\alpha_1 - \lambda_1\alpha_2\left(2-\gamma^2\right) + \gamma\lambda_1\alpha_1}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}
\end{aligned}
$$

and

$$
\begin{aligned}
q_I^B\left(c^B\right)c^B &= \lambda_1 \frac{\left(2-\gamma^2\right)\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)\left(2-\gamma^2\right)\alpha_2 - \left(2-\gamma^2\right)\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)\gamma\alpha_1}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)^2} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left[\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)\left(2-\gamma^2\right)\alpha_2 - \left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)\gamma\alpha_1\right]}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)^2} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left[2\lambda_2\alpha_1\alpha_2 - 2\lambda_1\alpha_2^2 - \gamma^2\lambda_2\alpha_1\alpha_2 + \gamma^2\lambda_1\alpha_2^2 - \gamma\lambda_2\alpha_1^2 + \gamma\lambda_1\alpha_1\alpha_2\right]}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)^2} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left[\alpha_1\lambda_2\left(2\alpha_2 - \gamma^2\alpha_2 - \gamma\alpha_1\right) - \alpha_2\lambda_1\left(2\alpha_2 - \gamma^2\alpha_2 - \gamma\alpha_1\right)\right]}{\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)^2} \\
&= \lambda_1 \frac{\left(2-\gamma^2\right)\left[\left(\lambda_2\alpha_1 - \lambda_1\alpha_2\right)\left(2\alpha_2 - \gamma^2\alpha_2 - \gamma\alpha_1^2\right)\right]}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)^2}.
\end{aligned}
$$

Note that if $\frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1} = \frac{\alpha_1}{2\lambda_1}$, then

$$
\begin{aligned}
& \frac{\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1} = \frac{\alpha_1}{2\lambda_1} \\
\Leftrightarrow \quad & 2\lambda_1\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right) = \alpha_1\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) \\
\Leftrightarrow \quad & 2\left(2-\gamma^2\right)\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 = \left(2-\gamma^2\right)\alpha_1\lambda_2 - \gamma\alpha_1\lambda_1 \\
\Leftrightarrow \quad & 2\left(2-\gamma^2\right)\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 - \left(2-\gamma^2\right)\alpha_1\lambda_2 + \gamma\alpha_1\lambda_1 = 0 \\
\Leftrightarrow \quad & \left(2-\gamma^2\right)\left(\alpha_2\lambda_1 - \alpha_1\lambda_2\right) + \lambda_1\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right) = 0 \\
\Leftrightarrow \quad & \lambda_1\left(\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right) = \left(2-\gamma^2\right)\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)
\end{aligned}
$$

and thus,

$$
q_I^B(c^B)c^B = \frac{\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right]\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2}
$$

$$
\begin{aligned}
&= \lambda_1 \frac{\alpha_1}{2\lambda_1} \frac{\left(2-\gamma^2\right)\left(\alpha_1\lambda_2-\alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]} \\
&= \frac{\alpha_1}{2} \frac{\left(2-\gamma^2\right)\left(\alpha_1\lambda_2-\alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]} \\
&= \frac{\alpha_1}{2} \frac{\lambda_1\left[\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right]}{2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]} \\
&= \frac{\alpha_1}{2}\frac{\lambda_1}{2}\frac{\alpha_1}{2\lambda_1} \\
&= \frac{\alpha_1^2}{8}.
\end{aligned}
$$

Case 3: The input price is too high for at least one intermediary to purchase strictly positive quantities.

Summing up, given the input demand the monopolistic input supplier charges a price of

$$
c^B = \begin{cases}
\frac{\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} & \text{for selling to both intermediaries,} \\
\max\left\{\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\} & \text{for selling only to intermediary 1 and } \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\
\max\left\{\frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2}, \frac{\alpha_2}{2\lambda_2}\right\} & \text{for selling only to intermediary 2 and } \frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds,}
\end{cases}
$$

and obtains a profit of

$$
q_I^B\left(c^B\right)c^B = \begin{cases}
\frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} & \text{for selling to both intermediaries,} \\
\frac{\alpha_1^2}{8} & \text{for selling only to intermediary 1 and } \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\
\frac{\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1\right]\left(\alpha_1\lambda_2-\alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]^2} & \text{for selling only to intermediary 1 and } \frac{\alpha_1}{2\lambda_1} \le \frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1} < \frac{\alpha_1}{\lambda_1} \text{ holds,} \\
\frac{\alpha_2^2}{8} & \text{for selling only to intermediary 2 and } \frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2} \le \frac{\alpha_2}{2\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds,} \\
\frac{\lambda_2\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2\right]\left(\alpha_2\lambda_1-\alpha_1\lambda_2\right)}{2\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right]^2} & \text{for selling only to intermediary 2 and } \frac{\alpha_2}{2\lambda_2} \le \frac{\left(2-\gamma^2\right)\alpha_1-\gamma\alpha_2}{\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2} < \frac{\alpha_2}{\lambda_2} \text{ holds.}
\end{cases}
$$

Step 2 (Comparing Profits)
As in Cournot competition, it suffices to compare the profits of the input supplier for Cases 1 and 2. We compare the two monopoly profits for selling to intermediary 1 with the duopoly profits for selling to both intermediaries. Thus, suppose $\alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$. The remaining part when selling to intermediary 2 follows analogously.

Step 2.1: We compare the two monopoly profits for selling to intermediary 1. We obtain by observing

$$\left[\alpha_1 \left(\left(2-\gamma^2\right) \lambda_2 + \gamma\lambda_1\right) - 4\left(2-\gamma^2\right) \alpha_2\lambda_1\right]^2 \geq 0$$

that

$$\frac{\alpha_1^2}{8} \geq \frac{\lambda_1 \left(2-\gamma^2\right) \left[\left(2-\gamma^2\right) \alpha_2 - \gamma\alpha_1\right] \left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2}.$$

This can be seen as follows:

$$\begin{aligned}
& \frac{\alpha_1^2}{8} \geq \frac{\lambda_1 \left(2-\gamma^2\right) \left[\left(2-\gamma^2\right) \alpha_2 - \gamma\alpha_1\right] \left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right)}{2\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2} \\
\Leftrightarrow \quad & 2\alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2 \geq 8\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right]\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right) \\
\Leftrightarrow \quad & 2\alpha_1^2 \left[\left(2-\gamma^2\right)^2 \lambda_2^2 - 2\left(2-\gamma^2\right)\lambda_2\gamma\lambda_1 + \gamma^2\lambda_1^2\right] \\
& \quad \geq 8\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2 - \gamma\alpha_1\right]\left(\alpha_1\lambda_2 - \alpha_2\lambda_1\right) \\
\Leftrightarrow \quad & 2\alpha_1^2 \left[\left(2-\gamma^2\right)^2 \lambda_2^2 - 2\left(2-\gamma^2\right)\lambda_2\gamma\lambda_1 + \gamma^2\lambda_1^2\right] \\
& \quad \geq 8\lambda_1\left(2-\gamma^2\right)\left[\left(2-\gamma^2\right)\alpha_2\alpha_1\lambda_2 - \left(2-\gamma^2\right)\alpha_2^2\lambda_1 - \gamma\alpha_1^2\lambda_2 + \gamma\alpha_1\alpha_2\lambda_1\right] \\
\Leftrightarrow \quad & 2\alpha_1^2 \left[\left(2-\gamma^2\right)^2 \lambda_2^2 - 2\left(2-\gamma^2\right)\lambda_2\gamma\lambda_1 + \gamma^2\lambda_1^2\right] \\
& \quad \geq 8\left(2-\gamma^2\right)^2 \alpha_2\alpha_1\lambda_1\lambda_2 - 8\left(2-\gamma^2\right)^2\alpha_2^2\lambda_1^2 - 8\gamma\left(2-\gamma^2\right)\alpha_1^2\lambda_1\lambda_2 \\
& \qquad + 8\gamma\left(2-\gamma^2\right)\alpha_1\alpha_2\lambda_1^2 \\
\Leftrightarrow \quad & 2\alpha_1^2 \left[\left(2-\gamma^2\right)^2 \lambda_2^2 + 4\gamma\left(2-\gamma^2\right)\lambda_1\lambda_2 + \gamma^2\lambda_1^2\right] \\
& \quad \geq 8\left(2-\gamma^2\right)^2 \alpha_2\alpha_1\lambda_1\lambda_2 - 8\left(2-\gamma^2\right)^2\alpha_2^2\lambda_1^2 + 8\gamma\left(2-\gamma^2\right)\alpha_1\alpha_2\lambda_1^2 \\
\Leftrightarrow \quad & \alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_2 + \gamma\lambda_1\right]^2 \\
& \quad \geq 4\left(2-\gamma^2\right)^2 \alpha_2\alpha_1\lambda_1\lambda_2 - 4\left(2-\gamma^2\right)^2\alpha_2^2\lambda_1^2 + 4\gamma\left(2-\gamma^2\right)\alpha_1\alpha_2\lambda_1^2 \\
\Leftrightarrow \quad & \alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_2 + \gamma\lambda_1\right]^2 \\
& \quad - 4\left(2-\gamma^2\right)^2 \alpha_2\alpha_1\lambda_1\lambda_2 + 4\left(2-\gamma^2\right)^2\alpha_2^2\lambda_1^2 - 4\gamma\left(2-\gamma^2\right)\alpha_1\alpha_2\lambda_1^2 \geq 0
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad & \alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_2 + \gamma\lambda_1\right]^2 \\
& \quad + 4\left(2-\gamma^2\right)^2 \alpha_2^2\lambda_1^2 - 4\left(2-\gamma^2\right)\alpha_1\alpha_2\lambda_1\left(\left(2-\gamma^2\right)\lambda_2 + \gamma\lambda_1\right) \geq 0 \\
\Leftrightarrow \quad & \left[\alpha_1\left(\left(2-\gamma^2\right)\lambda_2 + \gamma\lambda_1\right) - 4\left(2-\gamma^2\right)\alpha_2\lambda_1\right]^2 \geq 0.
\end{aligned}$$

Step 2.2: We compare the duopoly profit with the monopoly profit for selling to intermediary 1. If the duopoly profit is greater than or equal to both monopoly profits, then the input supplier decides to sell to both intermediaries. Therefore, we obtain that if

$$\alpha_2 \geq \alpha_1 \tau_1^B$$

with

$$\tau_1^B = \left(\frac{\sqrt{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]} - 2\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}\right),$$

then

$$\frac{\alpha_1^2}{8} \leq \frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]}$$

holds. This can be seen as follows:

$$\begin{aligned}
& \frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]} \geq \frac{\alpha_1^2}{8} \\
\Leftrightarrow \quad & \frac{2\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]} \geq \alpha_1^2 \\
\Leftrightarrow \quad & 2\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2 \\
& \quad \geq \alpha_1^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right] \\
\Leftrightarrow \quad & 2\left[\alpha_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \alpha_2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)\right]^2 \\
& \quad \geq \alpha_1^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right] \\
\Leftrightarrow \quad & \sqrt{2}\left[\alpha_1\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) + \alpha_2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)\right] \\
& \quad \geq \alpha_1\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]} \\
\Leftrightarrow \quad & \alpha_2\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) \\
& \quad \geq \alpha_1\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]}
\end{aligned}$$

$$
\begin{aligned}
&\quad - \alpha_1\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) \\
\Leftrightarrow \quad & \alpha_2\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) \\
&\geq \alpha_1\Big(\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} \\
&\quad - \sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)\Big) \\
\Leftrightarrow \quad & \alpha_2 \geq \alpha_1\Big(\frac{\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}}{\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \\
&\quad + \frac{-\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}\Big) \\
\Leftrightarrow \quad & \alpha_2 \geq \alpha_1\Big(\frac{\sqrt{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \\
&\quad + \frac{-2\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)}\Big).
\end{aligned}
$$

Thus, whenever condition (3.47) is true, we know that a profit-maximizing input supplier prefers to sell only to intermediary 1. Thus, we know that for

$$
\alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}
$$

the input supplier prefers to set a price to sell to both intermediaries. Note that the following is always true:

$$
\tau_1^B \leq \frac{\lambda_2}{\lambda_1}.
$$

This can be seen as follows:

$$
\begin{aligned}
& \frac{\sqrt{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} - 2\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{2\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \leq \frac{\lambda_2}{\lambda_1} \\
\Leftrightarrow \quad & \frac{\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} - \sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right)}{\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right)} \leq \frac{\lambda_2}{\lambda_1} \\
\Leftrightarrow \quad & \lambda_1\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} - \lambda_1\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) \\
&\quad \leq \lambda_2\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) \\
\Leftrightarrow \quad & \lambda_1\sqrt{\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} \\
&\quad \leq \lambda_2\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right) + \lambda_1\sqrt{2}\left(\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right) \\
\Leftrightarrow \quad & \lambda_1^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]
\end{aligned}
$$

$$
\begin{aligned}
&\leq 2\left(\lambda_2\left(\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right)+\lambda_1\left(\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right)\right)^2\\
\Leftrightarrow\quad &\lambda_1^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\\
&\leq 2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]^2\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\right.\\
&\left.-\lambda_1^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right]\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left(2-\gamma^2\right)\lambda_2^2-4\gamma\lambda_1\lambda_2\right.\\
&\left.+\left[2\left(2-\gamma^2\right)-\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right]\lambda_1^2\right]\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left(2-\gamma^2\right)\lambda_2^2-4\gamma\lambda_1\lambda_2+\gamma^2\left(3-\gamma^2\right)\lambda_1^2\right]\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left(2-\gamma^2\right)^2\lambda_2^2-4\left(2-\gamma^2\right)\gamma\lambda_1\lambda_2\right.\\
&\left.+\gamma^2\left(2-\gamma^2\right)\left(3-\gamma^2\right)\lambda_1^2\right]\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left(2-\gamma^2\right)^2\lambda_2^2-4\left(2-\gamma^2\right)\gamma\lambda_1\lambda_2\right.\\
&\left.+2\gamma^2\lambda_1^2+\left(\left(2-\gamma^2\right)\left(3-\gamma^2\right)-2\right)\gamma^2\lambda_1^2\right]\\
\Leftrightarrow\quad &0\leq\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\left[2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]^2\right.\\
&\left.+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\gamma^2\lambda_1^2\right].
\end{aligned}
$$

Step 2.1 and Step 2.2 show that for $\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1}\leq\frac{\alpha_1}{2\lambda_1}$ the the claim of Proposition 3.1 holds. Note that requiring $\frac{\left(2-\gamma^2\right)\alpha_2-\gamma\alpha_1}{\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1}\leq\frac{\alpha_1}{2\lambda_1}$ is equivalent to choosing $\alpha_2\leq\alpha_1\left(\frac{\left(2-\gamma^2\right)\lambda_2+\gamma\lambda_1}{2\left(2-\gamma^2\right)\lambda_1}\right)$. □

Computations of Proof of Proposition 3.3

Proof of Proposition 3.3. The proof proceeds in two steps. First we compare the profits of the input supplier in the case of intermediaries competing in quantities with the setting in which intermediaries compete in prices.

Step 1 (Duopoly Profit of the Input Supplier)
Suppose the input supplier sells inputs to both intermediaries, which is

$$\max\left\{\alpha_1\tau_1^B,\alpha_1\tau_1^C\right\}\leq\alpha_2\leq\alpha_1\frac{\lambda_2}{\lambda_1}.$$

We show that

$$q_I^B\left(c^B\right)c^B\geq q_I^C\left(c^C\right)c^C.$$

The nominator of

$$
\begin{aligned}
& q_I^B(c^B)\,c^B - q_I^C(c^C)\,c^C \\
={}& \frac{\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{4(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]} \\
& - \frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{8(4-\gamma^2)\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
={}& \frac{2\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]}{8(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
& - \frac{(1-\gamma^2)\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]}{8(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]}
\end{aligned}
$$

can be rewritten as

$$
\begin{aligned}
& 2\left[\left(2-\gamma^2\right)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] \\
& \quad - \left(1-\gamma^2\right)\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right] \\
& = \gamma^2\lambda_1(2\lambda_1-\gamma\lambda_2)\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right]\Big[\left[\lambda_1-\gamma\lambda_2\right](\alpha_1-\alpha_2)^2 \\
& \quad + 2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\Big] \\
& \quad + \gamma^2\lambda_2(2\lambda_2-\gamma\lambda_1)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]\Big[\left[\lambda_2-\gamma\lambda_1\right](\alpha_1-\alpha_2)^2 \\
& \quad + 2(1-\gamma)\alpha_1\alpha_2(\lambda_1+\lambda_2)\Big].
\end{aligned}
$$

This can be seen as follows: comparing Cournot and Bertrand profits yields

$$
\begin{aligned}
& \frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{8(2-\gamma)(2+\gamma)\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
& \leq \frac{\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{4(1-\gamma^2)(4-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]} \\
\Leftrightarrow\quad & \frac{\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right]} \\
& \leq \frac{\left[(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2}{(1-\gamma^2)\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2)-2\gamma\lambda_1\lambda_2\right]} \\
\Leftrightarrow\quad & \left(1-\gamma^2\right)\left[2(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right] \\
& \quad \leq 2\left[\left(2-\gamma^2\right)(\lambda_1\alpha_1+\lambda_2\alpha_2)-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\right]^2\left[\lambda_1^2+\lambda_2^2-\gamma\lambda_1\lambda_2\right] \\
\Leftrightarrow\quad & \left(1-\gamma^2\right)\left[\alpha_1(2\lambda_1-\gamma\lambda_2)+\alpha_2(2\lambda_2-\gamma\lambda_1)\right]^2\Big[\lambda_1\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right] \\
& + \lambda_2\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]\Big]
\end{aligned}
$$

$$
\begin{aligned}
&\leq \left[\alpha_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] + \alpha_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right]^2 \\
&\quad \left[\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right)\right] \\
\Leftrightarrow \quad & \left(1-\gamma^2\right) \left[\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] + \lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right] \\
&\quad \left[\alpha_1^2 \left(2\lambda_1 - \gamma\lambda_2\right)^2 + 2\alpha_1\alpha_2 \left(2\lambda_1 - \gamma\lambda_2\right)\left(2\lambda_2 - \gamma\lambda_1\right) + \alpha_2^2 \left(2\lambda_2 - \gamma\lambda_1\right)^2\right] \\
&\leq \left[\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right)\right] \\
&\quad \left[\alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]^2 + 2\alpha_1\alpha_2 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]\right. \\
&\quad \left.\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] + \alpha_2^2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2\right] \\
\Leftrightarrow \quad & 0 \leq \left[\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right)\right] \\
&\quad \left[\alpha_1^2 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]^2 + 2\alpha_1\alpha_2 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right. \\
&\quad \left. + \alpha_2^2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]^2\right] \\
&\quad - \left(1-\gamma^2\right) \left[\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] + \lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right] \\
&\quad \left[\alpha_1^2 \left(2\lambda_1 - \gamma\lambda_2\right)^2 + 2\alpha_1\alpha_2 \left(2\lambda_1 - \gamma\lambda_2\right)\left(2\lambda_2 - \gamma\lambda_1\right) + \alpha_2^2 \left(2\lambda_2 - \gamma\lambda_1\right)^2\right].
\end{aligned}
$$

We look first at the coefficient of α_1^2 and obtain

$$
\begin{aligned}
&\left[\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right)\right] \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]^2 \\
&\quad - \left(1-\gamma^2\right) \left[\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] + \lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right] \left(2\lambda_1 - \gamma\lambda_2\right)^2 \\
&= \lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]^2 - \left(1-\gamma^2\right)\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left(2\lambda_1 - \gamma\lambda_2\right)^2 \\
&\quad + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right]^2 - \left(1-\gamma^2\right)\lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \left(2\lambda_1 - \gamma\lambda_2\right)^2 \\
&= \lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] - \left(1-\gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)\right] \\
&\quad + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] - \left(1-\gamma^2\right)\left(2\lambda_2 - \gamma\lambda_1\right)\right] \\
&= \gamma^2\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\lambda_1 - \gamma\lambda_2\right] + \gamma^2\lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right) \\
&\quad \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\lambda_2 - \gamma\lambda_1\right].
\end{aligned}
$$

For the coefficient of α_2^2 we have analogously

$$
\begin{aligned}
&\gamma^2\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\lambda_1 - \gamma\lambda_2\right] \\
&\quad + \gamma^2\lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\lambda_2 - \gamma\lambda_1\right],
\end{aligned}
$$

and for the coefficient of $2\alpha_1\alpha_2$ this is

$$
\begin{aligned}
&\left[\lambda_1 \left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2 \left(2\lambda_2 - \gamma\lambda_1\right)\right] \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\quad - \left(1-\gamma^2\right) \left[\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] + \lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right]\right] \left(2\lambda_1 - \gamma\lambda_2\right)\left(2\lambda_2 - \gamma\lambda_1\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\quad - \left(1-\gamma^2\right)\lambda_1 \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] (2\lambda_1 - \gamma\lambda_2)(2\lambda_2 - \gamma\lambda_1) \\
&\quad + \lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\quad - \left(1-\gamma^2\right)\lambda_2 \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] (2\lambda_1 - \gamma\lambda_2)(2\lambda_2 - \gamma\lambda_1) \\
&= \lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \left[\left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] - \left(1-\gamma^2\right)(2\lambda_2 - \gamma\lambda_1)\right] \\
&\quad + \lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \left[\left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] - \left(1-\gamma^2\right)(2\lambda_1 - \gamma\lambda_2)\right] \\
&= \gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_2 - \gamma\lambda_1] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] [\lambda_1 - \gamma\lambda_2].
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\alpha_1^2 \big[\gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_1 - \gamma\lambda_2] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_2 - \gamma\lambda_1]\big] \\
&\quad + 2\alpha_1\alpha_2 \big[\gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_2 - \gamma\lambda_1] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] [\lambda_1 - \gamma\lambda_2]\big] \\
&\quad + \alpha_2^2 \big[\gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_1 - \gamma\lambda_2] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] [\lambda_2 - \gamma\lambda_1]\big] \\
&= \gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \big[\alpha_1^2 [\lambda_1 - \gamma\lambda_2] \\
&\quad + 2\alpha_1\alpha_2 [\lambda_2 - \gamma\lambda_1] + \alpha_2^2 [\lambda_1 - \gamma\lambda_2]\big] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\qquad \big[\alpha_1^2 [\lambda_2 - \gamma\lambda_1] + 2\alpha_1\alpha_2 [\lambda_1 - \gamma\lambda_2] + \alpha_2^2 [\lambda_2 - \gamma\lambda_1]\big] \\
&= \gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \big[[\lambda_1 - \gamma\lambda_2] (\alpha_1 - \alpha_2)^2 + 2\alpha_1\alpha_2 [\lambda_2 - \gamma\lambda_1] \\
&\quad + 2\alpha_1\alpha_2 [\lambda_1 - \gamma\lambda_2]\big] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\qquad \big[[\lambda_2 - \gamma\lambda_1] (\alpha_1 - \alpha_2)^2 + 2\alpha_1\alpha_2 [\lambda_1 - \gamma\lambda_2] + 2\alpha_1\alpha_2 [\lambda_2 - \gamma\lambda_1]\big] \\
&= \gamma^2\lambda_1 (2\lambda_1 - \gamma\lambda_2) \left[\left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2\right] \\
&\qquad \big[[\lambda_1 - \gamma\lambda_2] (\alpha_1 - \alpha_2)^2 + 2(1-\gamma)\alpha_1\alpha_2 (\lambda_1 + \lambda_2)\big] \\
&\quad + \gamma^2\lambda_2 (2\lambda_2 - \gamma\lambda_1) \left[\left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1\right] \\
&\qquad \big[[\lambda_2 - \gamma\lambda_1] (\alpha_1 - \alpha_2)^2 + 2(1-\gamma)\alpha_1\alpha_2 (\lambda_1 + \lambda_2)\big].
\end{aligned}
$$

Assumption 3.1 and 3.2, which is

$$\min\left\{\frac{\lambda_1}{\lambda_2},\frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2} \quad \text{and} \quad \min\left\{\frac{\lambda_1}{\lambda_2},\frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2},$$

implies for $\lambda_1 \geq \lambda_2$ that

$$\begin{aligned} 0 \leq 2\lambda_2 - \gamma\lambda_1 &\leq 2\lambda_1 - \gamma\lambda_2, \\ 0 \leq \left(2-\gamma^2\right)\lambda_2 - \gamma\lambda_1 &\leq \left(2-\gamma^2\right)\lambda_1 - \gamma\lambda_2. \end{aligned}$$

Thus, we obtain

$$\begin{aligned} &\gamma^2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)\left[\left(2-\gamma^2\right)\lambda_1-\gamma\lambda_2\right]\left[\left[\lambda_1-\gamma\lambda_2\right]\left(\alpha_1-\alpha_2\right)^2+2\left(1-\gamma\right)\alpha_1\alpha_2\left(\lambda_1+\lambda_2\right)\right] \\ &\quad+\gamma^2\lambda_2\left(2\lambda_2-\gamma\lambda_1\right)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]\Big[\left[\lambda_2-\gamma\lambda_1\right]\left(\alpha_1-\alpha_2\right)^2 \\ &\quad+2\left(1-\gamma\right)\alpha_1\alpha_2\left(\lambda_1+\lambda_2\right)\Big] \\ &\geq \gamma^2\left(2\lambda_2-\gamma\lambda_1\right)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right] \\ &\qquad\Big[\lambda_1\left[\lambda_1-\gamma\lambda_2\right]\left(\alpha_1-\alpha_2\right)^2+2\left(1-\gamma\right)\lambda_1\alpha_1\alpha_2\left(\lambda_1+\lambda_2\right) \\ &\quad+\lambda_2\left[\lambda_2-\gamma\lambda_1\right]\left(\alpha_1-\alpha_2\right)^2+2\left(1-\gamma\right)\lambda_2\alpha_1\alpha_2\left(\lambda_1+\lambda_2\right)\Big] \\ &= \gamma^2\left(2\lambda_2-\gamma\lambda_1\right)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right] \\ &\quad\left[\left(\left(\lambda_1-\lambda_2\right)^2+2\left(1-\gamma\right)\lambda_1\lambda_2\right)\left(\alpha_1-\alpha_2\right)^2+2\left(1-\gamma\right)\alpha_1\alpha_2\left(\lambda_1+\lambda_2\right)\right] \\ &\geq 0. \end{aligned}$$

The analogous argument holds for $\lambda_1 \leq \lambda_2$. Note that we have for $q_I^C(c^C)c^C$ with Assumption 3.1

$$\frac{\partial\left[q_I^C(c^C)c^C\right]}{\partial\alpha_2} = 2\frac{\left[2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]\left[2\lambda_2-\gamma\lambda_1\right]}{8\left(4-\gamma^2\right)\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]} \geq 0,$$

and with Assumption 3.2 for $q_I^B(c^B)c^B$

$$\frac{\partial\left[q_I^B(c^B)c^B\right]}{\partial\alpha_2} = 2\frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]} \geq 0.$$

Step 2 (Comparing τ_1^C and τ_1^B)
Note that τ_1^B and τ_1^C was deduced within the proof of Proposition 3.1 and Proposition 3.2 by comparing the profit to sell to both intermediaries with the profit to sell only to intermediary 1. This is stated in Ineq. (3.37) and (3.46). When considering the difference between those profits and using *step 1*, we therefore have

$$\frac{\alpha_1^2}{8} - \frac{\left[\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{4\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$\leq \frac{\alpha_1^2}{8} - \frac{\left[2\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right]^2}{8\left(4-\gamma^2\right)\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]}.$$

With the observation that the input supplier's profit when selling to both intermediaries is non-decreasing in α_2, Ineq. (3.53) and (3.54) and thus, the right- and left-hand side of Ineq. (3.55) are non-increasing in α_2. This implies that the zero of the left-hand side is less or equal than the zero of the right-hand side. Therefore,

$$\tau_1^B \leq \tau_1^C$$

and thus, $\max\left\{\alpha_1\tau_1^B, \alpha_1\tau_1^C\right\} = \alpha_1\tau_1^C$ and $\min\left\{\alpha_1\tau_1^B, \alpha_1\tau_1^C\right\} = \alpha_1\tau_1^B$.

Similarly, we have $\alpha_2\tau_2^B \leq \alpha_2\tau_2^C$. □

Computations of Proof of Proposition 3.4

Proof of Proposition 3.4.

Case 1: $\alpha_2 \leq \alpha_1\tau_1^B$ (or $\alpha_1 \leq \alpha_2\tau_2^B$)
In Bertrand as well as in Cournot competition intermediary 2 is excluded from the input market. As the input demand of the intermediary is identical for both types of competition, the input price is identical also.

Case 2: $\alpha_1\tau_1^B \leq \alpha_2 \leq \alpha_1\tau_1^C$ (or $\alpha_2\tau_2^B \leq \alpha_1 \leq \alpha_2\tau_2^C$)
In Bertrand competition both intermediaries purchase inputs, while in Cournot competition only intermediary 1 procures on the input market. We have

$$c^C - c^B = \frac{\alpha_1}{2\lambda_1} - \frac{\left(2-\gamma^2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)-\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\left(\lambda_2\alpha_1-\lambda_1\alpha_2\right)\left[\left(2-\gamma^2\right)\lambda_2-\gamma\lambda_1\right]}{2\lambda_1\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}.$$

This can be seen as follows:

$$
\begin{aligned}
c^C - c^B &= \frac{\alpha_1}{2\lambda_1} - \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&= \frac{\alpha_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right] - \lambda_1\big[\,(2-\gamma^2)(\lambda_1\alpha_1+\lambda_2\alpha_2)}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&\quad + \frac{-\gamma(\lambda_1\alpha_2+\lambda_2\alpha_1)\,\big]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&= \frac{\alpha_1\left[\lambda_1\left[(2-\gamma^2)\lambda_1 - \gamma\lambda_2\right] + \lambda_2\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right]\right]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&\quad - \frac{\lambda_1\left[\alpha_1\left[(2-\gamma^2)\lambda_1 - \gamma\lambda_2\right] + \alpha_2\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right]\right]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&= \frac{\lambda_2\alpha_1\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right] - \lambda_1\alpha_2\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&= \frac{(\lambda_2\alpha_1 - \lambda_1\alpha_2)\left[(2-\gamma^2)\lambda_2 - \gamma\lambda_1\right]}{2\lambda_1\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]}.
\end{aligned}
$$

Therefore, $c^C - c^B \geq 0$ as $\alpha_2 \leq \alpha_1\tau_1^C \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ and per assumption $\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} > \frac{\gamma}{2-\gamma^2}$.

Case 3: $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \leq \alpha_1 \leq \alpha_2\frac{\lambda_1}{\lambda_2}$)

Both intermediaries purchase inputs on the input market in Bertrand as well as in Cournot competition. We have

$$
\begin{aligned}
&c^C - c^B \\
&= \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4\left[(\lambda_1^2+\lambda_2^2) - \gamma\lambda_1\lambda_2\right]} \\
&\quad - \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]} \\
&= \frac{\gamma^3(\lambda_1 - \lambda_2)(\lambda_1+\lambda_2)(\lambda_1\alpha_2 - \lambda_2\alpha_1)}{4(\lambda_1^2+\lambda_2^2 - \gamma\lambda_1\lambda_2)((2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2)}.
\end{aligned}
$$

This can be seen as follows:

$$
\begin{aligned}
&c^C - c^B \\
&= \frac{2(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{4\left[(\lambda_1^2+\lambda_2^2) - \gamma\lambda_1\lambda_2\right]} \\
&\quad - \frac{(2-\gamma^2)(\lambda_1\alpha_1 + \lambda_2\alpha_2) - \gamma(\lambda_1\alpha_2 + \lambda_2\alpha_1)}{2\left[(2-\gamma^2)(\lambda_1^2+\lambda_2^2) - 2\gamma\lambda_1\lambda_2\right]}
\end{aligned}
$$

$$= \frac{\left(4\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]-\left(2-\gamma^2\right)4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$+ \frac{\left(4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]-2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]\right)\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\left(-8\gamma\lambda_1\lambda_2+4\left(2-\gamma^2\right)\gamma\lambda_1\lambda_2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$+ \frac{\left(4\lambda_1^2+4\lambda_2^2-2\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)\right)\gamma\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\left(-4\gamma^3\lambda_1\lambda_2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$+ \frac{2\gamma^3\left(\lambda_1^2+\lambda_2^2\right)\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\left(-4\gamma^3\lambda_1\lambda_2\right)\left(\lambda_1\alpha_1+\lambda_2\alpha_2\right)+2\gamma^3\left(\lambda_1^2+\lambda_2^2\right)\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\gamma^3\left(-4\lambda_1^2\lambda_2\alpha_1-4\lambda_1\lambda_2^2\alpha_2+2\lambda_1^3\alpha_2+2\lambda_1^2\lambda_2\alpha_1+2\lambda_1\lambda_2^2\alpha_2+2\lambda_2^3\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\gamma^3\left(-2\lambda_1^2\lambda_2\alpha_1-2\lambda_1\lambda_2^2\alpha_2+2\lambda_1^3\alpha_2+2\lambda_2^3\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{2\gamma^3\left(-\lambda_1^2\lambda_2\alpha_1-\lambda_1\lambda_2^2\alpha_2+\lambda_1^3\alpha_2+\lambda_2^3\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{2\gamma^3\left(\lambda_1^2\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)-\lambda_2^2\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{2\gamma^3\left(\lambda_1-\lambda_2\right)\left(\lambda_1+\lambda_2\right)\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{2\gamma^3\left(\lambda_1^2-\lambda_2^2\right)\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]2\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}$$
$$= \frac{\gamma^3\left(\lambda_1^2-\lambda_2^2\right)\left(\lambda_1\alpha_2+\lambda_2\alpha_1\right)}{4\left[\left(\lambda_1^2+\lambda_2^2\right)-\gamma\lambda_1\lambda_2\right]\left[\left(2-\gamma^2\right)\left(\lambda_1^2+\lambda_2^2\right)-2\gamma\lambda_1\lambda_2\right]}.$$

Therefore, $c^C - c^B \geq 0$ if and only if $\gamma^3\left(\lambda_1-\lambda_2\right)\left(\lambda_1\alpha_2-\lambda_2\alpha_1\right) \geq 0$.

□

Computations of Equation 3.21 and of Equation 3.23

The intermediaries profits are

$$\pi_1^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1\tau_1^C, \\ \frac{\left[2\alpha_1-\gamma\alpha_2-\left(2\lambda_1-\gamma\lambda_2\right)c^C\right]^2}{(2-\gamma)^2(2+\gamma)^2} & \text{for } \alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}, \end{cases}$$

$$\pi_2^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1 \tau_1^C, \\ \frac{\left[2\alpha_2 - \gamma\alpha_1 - (2\lambda_2 - \gamma\lambda_1)c^C\right]^2}{(2-\gamma)^2(2+\gamma)^2} & \text{for } \alpha_1 \tau_1^C \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}, \end{cases}$$

with

$$c^C = \frac{2\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)}{4\left[\lambda_1^2 + \lambda_2^2 - \gamma\lambda_1\lambda_2\right]}$$

and

$$\pi_1^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\alpha_1^2}{16} & \text{for } \alpha_2 \leq \alpha_1 \tau_1^B, \\ \frac{\left[(2-\gamma^2)\alpha_1 - \gamma\alpha_2 - \left((2-\gamma^2)\lambda_1 - \gamma\lambda_2\right)c^B\right]^2}{(2-\gamma)^2(2+\gamma)^2(1-\gamma^2)} & \text{for } \alpha_1 \tau_1^B \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}, \end{cases}$$

$$\pi_2^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \begin{cases} 0 & \text{for } \alpha_2 \leq \alpha_1 \tau_1^B, \\ \frac{\left[(2-\gamma^2)\alpha_2 - \gamma\alpha_1 - \left((2-\gamma^2)\lambda_2 - \gamma\lambda_1\right)c^B\right]^2}{(2-\gamma)^2(2+\gamma)^2(1-\gamma^2)} & \text{for } \alpha_1 \tau_1^B \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}, \end{cases}$$

with

$$c^B = \frac{(2-\gamma^2)\left(\lambda_1\alpha_1 + \lambda_2\alpha_2\right) - \gamma\left(\lambda_1\alpha_2 + \lambda_2\alpha_1\right)}{2\left[(2-\gamma^2)\left(\lambda_1^2 + \lambda_2^2\right) - 2\gamma\lambda_1\lambda_2\right]}.$$

This can be seen as follows:

Cournot Competition: $\alpha_2 \leq \alpha_1 \tau_1^C$

$$\begin{aligned} \pi_1^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) &= \left(\frac{\alpha_1 + \lambda_1 c^C}{2} - \lambda_1 c^C\right)\frac{\alpha_1 - \lambda_1 c^C}{2} \\ &= \left(\frac{\alpha_1 - \lambda_1 c^C}{2}\right)^2 \\ &= \left(\frac{\alpha_1 - \lambda_1 \frac{\alpha_1}{2\lambda_1}}{2}\right)^2 \\ &= \left(\frac{\frac{\alpha_1}{2}}{2}\right)^2 \\ &= \left(\frac{\alpha_1}{4}\right)^2 \\ &= \frac{\alpha_1^2}{16} \quad \text{and} \quad \pi_2^C\left(\alpha_1, \alpha_2\right) = 0. \end{aligned}$$

Cournot Competition: $\alpha_1 \tau_1^C \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$

$$\begin{aligned}
&\pi_i^C \left(p_i^C, p_{3-i}^C, c^C, \alpha_i, \alpha_{3-i}\right) \\
&= \left(p_i^C - \lambda_i c^C\right) q_i^C \\
&= \left(q_i^C + \lambda_i c^C - \lambda_i c^C\right) q_i^C \\
&= \left(q_i^C\right)^2 \\
&= \left(\frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c^C}{(2-\gamma)(2+\gamma)}\right)^2 \\
&= \frac{\left[2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c^C\right]^2}{(2-\gamma)^2 (2+\gamma)^2} \quad \text{for } i = 1, 2.
\end{aligned}$$

Bertrand Competition: $\alpha_2 \leq \alpha_1 \tau_1^B$

$$\begin{aligned}
\pi_1^B \left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) &= \left(\frac{\alpha_1 + \lambda_1 c^B}{2} - \lambda_1 c^C\right) \frac{\alpha_1 - \lambda_1 c^B}{2} \\
&= \left(\frac{\alpha_1 - \lambda_1 c^B}{2}\right)^2 \\
&= \left(\frac{\alpha_1 - \lambda_1 \frac{\alpha_1}{2\lambda_1}}{2}\right)^2 \\
&= \left(\frac{\frac{\alpha_1}{2}}{2}\right)^2 \\
&= \left(\frac{\alpha_1}{4}\right)^2 \\
&= \frac{\alpha_1^2}{16} \quad \text{and} \quad \pi_2^B (\alpha_1, \alpha_2) = 0.
\end{aligned}$$

Bertrand Competition: $\alpha_1 \tau_1^B \leq \alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$

$$\begin{aligned}
&\pi_i^B \left(p_i^B, p_{3-i}^B, c^B, \alpha_i, \alpha_{3-i}\right) \\
&= \left(p_i^B - \lambda_i c^B\right) q_i^B \\
&= \left(\frac{(2-\gamma^2)\,\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)} - \lambda_i c^B\right) \\
&\quad \left(\frac{(2-\gamma^2)\,\alpha_i - \gamma\alpha_{3-i} - ((2-\gamma^2)\,\lambda_i - \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)\,(1-\gamma^2)}\right) \\
&= \left(\frac{(2-\gamma^2)\,\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})\, c^B - (4-\gamma^2)\,\lambda_i c^B}{(4-\gamma^2)}\right)
\end{aligned}$$

$$
\begin{aligned}
&\left(\frac{\left(2-\gamma^2\right)\alpha_i-\gamma\alpha_{3-i}-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)c^B}{\left(4-\gamma^2\right)\left(1-\gamma^2\right)}\right)\\
=&\left(\frac{\left(2-\gamma^2\right)\alpha_i-\gamma\alpha_{3-i}-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)c^B}{\left(4-\gamma^2\right)}\right)\\
&\left(\frac{\left(2-\gamma^2\right)\alpha_i-\gamma\alpha_{3-i}-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)c^B}{\left(4-\gamma^2\right)\left(1-\gamma^2\right)}\right)\\
=&\frac{\left[\left(2-\gamma^2\right)\alpha_i-\gamma\alpha_{3-i}-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)c^B\right]^2}{\left(2-\gamma\right)^2\left(2+\gamma\right)^2\left(1-\gamma^2\right)} \quad \text{for } i=1,2.
\end{aligned}
$$

Computations of Proof of Proposition 3.5

Proof of Proposition 3.5. For $\lambda_1 = \lambda_2$ and $\alpha_1 = \alpha_2$ we have

$$
\begin{aligned}
p_i^C - p_i^B &= \frac{\left(3+\gamma\right)\alpha_i}{2\left(2+\gamma\right)} - \frac{\left(3-2\gamma\right)\alpha_i}{2\left(2-\gamma\right)} = \frac{\gamma^2\alpha_i}{2\left(4-\gamma^2\right)},\\
q_i^C - q_i^B &= \frac{\alpha_i}{2\left(2+\gamma\right)} - \frac{\alpha_i}{2\left(2-\gamma\right)\left(1+\gamma\right)} = -\frac{\gamma^2\alpha_i}{2\left(4-\gamma^2\right)\left(1+\gamma\right)},\\
\pi_i^C - \pi_i^B &= \frac{\alpha_i^2}{4\left(2+\gamma\right)^2} - \frac{\left(1-\gamma\right)\alpha_i^2}{4\left(2-\gamma\right)^2\left(1+\gamma\right)} = \frac{\gamma^3\alpha_i^2}{2\left(4-\gamma^2\right)^2\left(1+\gamma\right)}.
\end{aligned}
$$

Thus, $p_i^C \geq p_i^B$ and $q_i^C \leq q_i^B$. Moreover, $\pi_i^C \geq \pi_i^B$ for $\gamma \geq 0$ and $\pi_i^C \leq \pi_i^B$ for $\gamma \leq 0$ for $i = 1, 2$. □

$$
\begin{aligned}
c^C &= \frac{2\left(\lambda_i\alpha_i+\lambda_i\alpha_i\right)-\gamma\left(\lambda_i\alpha_i+\lambda_i\alpha_i\right)}{4\left(\lambda_i^2+\lambda_i^2-\gamma\lambda_i^2\right)}\\
&= \frac{2\lambda_i\alpha_i\left(2-\gamma\right)}{4\lambda_i^2\left(2-\gamma\right)}\\
&= \frac{\alpha_i}{2\lambda_i}
\end{aligned}
$$

$$
\begin{aligned}
q_i^C &= \frac{2\alpha_i-\gamma\alpha_i-\left(2\lambda_i-\gamma\lambda_i\right)c^C}{\left(4-\gamma^2\right)}\\
&= \frac{\alpha_i\left(2-\gamma\right)-\left(2-\gamma\right)\lambda_i c^C}{\left(4-\gamma^2\right)}\\
&= \frac{\left(2-\gamma\right)\left(\alpha_i-\lambda_i c^C\right)}{\left(4-\gamma^2\right)}\\
&= \frac{\left(\alpha_i-\lambda_i c^C\right)}{\left(2+\gamma\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\alpha_i - \frac{\alpha_i}{2\lambda_i}\lambda_i}{(2+\gamma)} \\
&= \frac{2\alpha_i - \alpha_i}{2\,(2+\gamma)} \\
&= \frac{\alpha_i}{2\,(2+\gamma)}
\end{aligned}
$$

$$
\begin{aligned}
p_i^C &= q_i^C + \lambda_i c^C \\
&= \frac{\alpha_i}{2\,(2+\gamma)} + \lambda_i \frac{\alpha_i}{2\lambda_i} \\
&= \frac{\alpha_i + \alpha_i\,(2+\gamma)}{2\,(2+\gamma)} \\
&= \frac{\alpha_i\,(3+\gamma)}{2\,(2+\gamma)}
\end{aligned}
$$

$$
\begin{aligned}
\pi_i^C &= \left(\frac{\alpha_i\,(3+\gamma)}{2\,(2+\gamma)} - \lambda_i \frac{\alpha_i}{2\lambda_i}\right)\frac{\alpha_i}{2\,(2+\gamma)} \\
&= \left(\frac{\alpha_i\,(3+\gamma) - \alpha_i\,(2+\gamma)}{2\,(2+\gamma)}\right)\frac{\alpha_i}{2\,(2+\gamma)} \\
&= \frac{\alpha_i^2}{4\,(2+\gamma)^2}
\end{aligned}
$$

$$
\begin{aligned}
c^B &= \frac{(2-\gamma^2)\,(\lambda_i\alpha_i + \lambda_i\alpha_i) - \gamma\,(\lambda_i\alpha_i + \lambda_i\alpha_i)}{2[(2-\gamma^2)\,(\lambda_i^2 + \lambda_i^2) - 2\gamma\lambda_i^2]} \\
&= \frac{2\,(2-\gamma^2)\,\lambda_i\alpha_i - 2\gamma\lambda_i\alpha_i}{2[2\lambda_i^2\,(2-\gamma^2) - 2\gamma\lambda_i^2]} \\
&= \frac{2\lambda_i\alpha_i\,((2-\gamma^2)-\gamma)}{4\lambda_i^2[(2-\gamma^2)-\gamma]} \\
&= \frac{\alpha_i}{2\lambda_i}
\end{aligned}
$$

$$
\begin{aligned}
p_i^B &= \frac{\alpha_i\,(2-\gamma-\gamma^2) + (2+\gamma)\,\lambda_i c}{(4-\gamma^2)} \\
&= \frac{\alpha_i\,(2-\gamma-\gamma^2) + (2+\gamma)\,\frac{\alpha_i}{2}}{(4-\gamma^2)} \\
&= \frac{\alpha_i\,(4-2\gamma-2\gamma^2+2+\gamma)}{2\,(4-\gamma^2)} \\
&= \frac{\alpha_i\,(6-\gamma-2\gamma^2)}{2\,(4-\gamma^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\alpha_i (3 - 2\gamma)(2 + \gamma)}{2(2 - \gamma)(2 + \gamma)} \\
&= \frac{\alpha_i (3 - 2\gamma)}{2(2 - \gamma)}
\end{aligned}
$$

$$
\begin{aligned}
q_i^B &= \frac{p_i^B - \lambda_i c}{(1 - \gamma^2)} \\
&= \frac{\frac{\alpha_i(3-2\gamma)}{2(2-\gamma)} - \lambda_i \frac{\alpha_i}{2\lambda_i}}{(1 - \gamma^2)} \\
&= \frac{\alpha_i (3 - 2\gamma) - \alpha_i (2 - \gamma)}{2(2 - \gamma)(1 - \gamma)(1 + \gamma)} \\
&= \frac{\alpha_i (3 - 2\gamma - 2 + \gamma)}{2(2 - \gamma)(1 - \gamma)(1 + \gamma)} \\
&= \frac{\alpha_i (1 - \gamma)}{2(2 - \gamma)(1 - \gamma)(1 + \gamma)} \\
&= \frac{\alpha_i}{2(2 - \gamma)(1 + \gamma)}
\end{aligned}
$$

$$
\begin{aligned}
\pi_i^B &= \left(p_i^B - \lambda_i c^B\right) q_i^B \\
&= \left(\frac{\alpha_i (3 - 2\gamma)}{2(2 - \gamma)} - \lambda_i \frac{\alpha_i}{2\lambda_i}\right) \frac{\alpha_i}{2(2 - \gamma)(1 + \gamma)} \\
&= \left(\frac{\alpha_i (3 - 2\gamma)}{2(2 - \gamma)} - \frac{\alpha_i}{2}\right) \frac{\alpha_i}{2(2 - \gamma)(1 + \gamma)} \\
&= \left(\frac{\alpha_i (3 - 2\gamma) - \alpha_i (2 - \gamma)}{2(2 - \gamma)}\right) \frac{\alpha_i}{2(2 - \gamma)(1 + \gamma)} \\
&= \frac{\alpha_i (1 - \gamma)}{2(2 - \gamma)} \cdot \frac{\alpha_i}{2(2 - \gamma)(1 + \gamma)} \\
&= \frac{\alpha_i^2 (1 - \gamma)}{4(2 - \gamma)^2 (1 + \gamma)}
\end{aligned}
$$

$$
\begin{aligned}
p_i^C - p_i^B &= \frac{(3 + \gamma)\alpha_i}{2(2 + \gamma)} - \frac{(3 - 2\gamma)\alpha_i}{2(2 - \gamma)} \\
&= \frac{(3 + \gamma)(2 - \gamma)\alpha_i - (3 - 2\gamma)(2 + \gamma)\alpha_i}{} \\
&= \frac{(6 - 3\gamma + 2\gamma - \gamma^2 - 6 - 3\gamma + 4\gamma + 2\gamma^2)\alpha_i}{2(4 - \gamma^2)} \\
&= \frac{\gamma^2 \alpha_i}{2(4 - \gamma^2)}
\end{aligned}
$$

$$\begin{aligned}
q_i^C - q_i^B &= \frac{\alpha_i}{2\left(2+\gamma\right)} - \frac{\alpha_i}{2\left(2-\gamma\right)\left(1+\gamma\right)} \\
&= \frac{\alpha_i\left(2-\gamma\right)\left(1+\gamma\right) - \alpha_i\left(2+\gamma\right)}{2\left(4-\gamma^2\right)\left(1+\gamma\right)} \\
&= \frac{\alpha_i\left(2+2\gamma-\gamma-\gamma^2-2-\gamma\right)}{2\left(4-\gamma^2\right)\left(1+\gamma\right)} \\
&= -\frac{\alpha_i\gamma^2}{2\left(4-\gamma^2\right)\left(1+\gamma\right)}
\end{aligned}$$

$$\begin{aligned}
\pi_i^C - \pi_i^B &= \frac{\alpha_i^2}{4\left(2+\gamma\right)^2} - \frac{\left(1-\gamma\right)\alpha_i^2}{4\left(2-\gamma\right)^2\left(1+\gamma\right)} \\
&= \frac{\alpha_i^2\left(2-\gamma\right)^2\left(1+\gamma\right) - \alpha_i^2\left(1-\gamma\right)\left(2+\gamma\right)^2}{4\left(2+\gamma\right)^2\left(2-\gamma\right)^2\left(1+\gamma\right)} \\
&= \frac{\alpha_i\left(\left(4-4\gamma+\gamma^2\right)\left(1+\gamma\right) - \left(1-\gamma\right)\left(4+4\gamma+\gamma^2\right)\right)}{4\left(2+\gamma\right)^2\left(2-\gamma\right)^2\left(1+\gamma\right)} \\
&= \frac{\alpha_i^2\left(4-4\gamma+\gamma^2+4\gamma-4\gamma^2+\gamma^3-4-4\gamma-\gamma^2+4\gamma+4\gamma^2+\gamma^3\right)}{4\left(2+\gamma\right)^2\left(2-\gamma\right)^2\left(1+\gamma\right)} \\
&= \frac{2\alpha_i^2\gamma^3}{4\left(2+\gamma\right)^2\left(2-\gamma\right)^2\left(1+\gamma\right)} \\
&= \frac{\alpha_i^2\gamma^3}{2\left(2+\gamma\right)^2\left(2-\gamma\right)^2\left(1+\gamma\right)}
\end{aligned}$$

Computations of Proof of Proposition 3.6

Proof of Proposition 3.6. For this proof consider $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ (or $\alpha_2\tau_2^C \leq \alpha_1 \leq \alpha_2\frac{\lambda_1}{\lambda_2}$) and $\lambda_1 = \lambda_2$.

(i) We have

$$p_i^C - p_i^B = \frac{\gamma^2\left(3\alpha_i - \alpha_{3-i}\right)}{4\left(4-\gamma^2\right)}.$$

This can be seen as follows: We have

$$\begin{aligned}
&p_i^C - p_i^B \\
&= \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(2\lambda_i - \gamma\lambda_{3-i}\right)c^C + \left(4-\gamma^2\right)\lambda_i c^C}{\left(4-\gamma^2\right)} \\
&\quad - \frac{\left(2-\gamma^2\right)\alpha_i - \gamma\alpha_{3-i} + \left(2\lambda_i + \gamma\lambda_{3-i}\right)c^B}{\left(4-\gamma^2\right)} \\
&= \frac{\gamma^2\left(\alpha_i - \lambda_i c^C\right) + \left(c^C - c^B\right)\left(2\lambda_i + \gamma\lambda_{3-i}\right)}{\left(4-\gamma^2\right)}.
\end{aligned}$$

and

$$\begin{aligned}
&p_i^C - p_i^B \\
&= \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c^C + (4-\gamma^2)\,\lambda_i c^C}{(4-\gamma^2)} \\
&\quad - \frac{(2-\gamma^2)\,\alpha_i - \gamma\alpha_{3-i} + (2\lambda_i + \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)} \\
&= \frac{\gamma^2\alpha_i - \gamma^2\lambda_i c^C - (2\lambda_i - \gamma\lambda_{3-i})\, c^C + 4\lambda_i c^C - \gamma\lambda_{3-i}c^B - 2\lambda_i c^B}{(4-\gamma^2)} \\
&= \frac{\gamma^2\left(\alpha_i - \lambda_i c^C\right) + 2\lambda_i c^C + \gamma\lambda_{3-i}c^C - \gamma\lambda_{3-i}c^B - 2\lambda_i c^B}{(4-\gamma^2)} \\
&= \frac{\gamma^2\left(\alpha_i - \lambda_i c^C\right) + 2\lambda_i\left(c^C - c^B\right) + \gamma\lambda_{3-i}\left(c^C - c^B\right)}{(4-\gamma^2)} \\
&= \frac{\gamma^2\left(\alpha_i - \lambda_i c^C\right) + \left(c^C - c^B\right)(2\lambda_i + \gamma\lambda_{3-i})}{(4-\gamma^2)}.
\end{aligned}$$

For $\lambda_1 = \lambda_2$ we have $c^C = c^B$ and $c^C = \frac{\alpha_1+\alpha_2}{4\lambda_1}$ and, thus,

$$\begin{aligned}
&p_i^C - p_i^B \\
&= \frac{\gamma^2\left(\alpha_i - \lambda_i \frac{\alpha_i+\alpha_{3-i}}{4\lambda_i}\right)}{(4-\gamma^2)} \\
&= \frac{\gamma^2\,(4\alpha_i - \alpha_i - \alpha_{3-i})}{4\,(4-\gamma^2)} \\
&= \frac{\gamma^2\,(3\alpha_i - \alpha_{3-i})}{4\,(4-\gamma^2)}.
\end{aligned}$$

Thus, for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(4, 1, 1, 1, -\frac{1}{2}\right)$$

it can easily be verified that Assumptions 3.1 and 3.2 as well as $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ hold and we have

$$p_2^C = \frac{89}{60} < \frac{3}{2} = p_2^B.$$

This can be seen as follows: Intermediary 2's equilibrium prices are

$$p_2^C = \frac{89}{60} \approx 1.4833$$
$$p_2^B = \frac{3}{2} = 1.5$$

and the according conditions are

$$\tau_1^C = \sqrt{2 - \frac{1}{2}} - 1 = \sqrt{\frac{3}{2}} - 1 \approx 0.2247,$$

$$\text{Condition } \alpha_2 \leq \alpha_1 \tfrac{\lambda_2}{\lambda_1}\text{:} \quad \alpha_1 \frac{\lambda_2}{\lambda_1} - \alpha_2 = 4 - 1 = 3 > 0,$$

$$\text{Condition } \alpha_1 \tau_1^C \leq \alpha_2\text{:} \quad \alpha_2 - \tau_1^C \alpha_1 = 1 - \left(4\sqrt{\frac{3}{2}} - 4\right) = 5 - 4\sqrt{\frac{3}{2}} \approx 0.1010 > 0,$$

$$\text{Assumption 3.1:} \quad \min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} - \frac{\gamma}{2} = \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{2} = 0.5 > 0,$$

$$\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} - \frac{\gamma}{2} = 1 - \left(-\frac{1}{4}\right) = \frac{5}{4} = 1.25 > 0,$$

$$\text{Assumption 3.2:} \quad \min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} - \frac{\gamma}{2 - \gamma^2} = \frac{1}{4} - \left(\frac{-\frac{1}{2}}{\frac{7}{4}}\right) = \frac{1}{4} - \left(-\frac{2}{7}\right) = \frac{15}{28} \approx 0.5357 > 0,$$

$$\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} - \frac{\gamma}{2 - \gamma^2} = 1 - \left(-\frac{2}{7}\right) = \frac{9}{7} \approx 1.2857 > 0.$$

(ii) We have

$$q_i^C - q_i^B = -\frac{\gamma^2 \left[(3+\gamma)\alpha_i - (1+3\gamma)\alpha_{3-i}\right]}{4\left(4-\gamma^2\right)\left(1-\gamma^2\right)}.$$

This can be seen as follows: We have

$$\begin{aligned} &q_i^C - q_i^B \\ &= \frac{2\alpha_i - \gamma\alpha_{3-i} - \left(2\lambda_i - \gamma\lambda_{3-i}\right)c^C}{\left(4-\gamma^2\right)} - \frac{\left(2-\gamma^2\right)\left(\alpha_i - \lambda_i c^B\right) - \gamma\left(\alpha_{3-i} - \lambda_{3-i}c^B\right)}{\left(4-\gamma^2\right)\left(1-\gamma^2\right)} \\ &= -\frac{\gamma^2\left[\alpha_i - \lambda_i c^C - \gamma\left(\alpha_{3-i} - \lambda_{3-i}c^C\right)\right] + \left(c^C - c^B\right)\left(2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i}\right)}{\left(4-\gamma^2\right)\left(1-\gamma^2\right)}. \end{aligned}$$

$$
\begin{aligned}
&q_i^C - q_i^B \\
&= \frac{2\alpha_i - \gamma\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c^C}{(4-\gamma^2)} - \frac{(2-\gamma^2)\left(\alpha_i - \lambda_i c^B\right) - \gamma\left(\alpha_{3-i} - \lambda_{3-i} c^B\right)}{(4-\gamma^2)(1-\gamma^2)} \\
&= \frac{(1-\gamma^2)(2\alpha_i - \gamma\alpha_{3-i}) - (1-\gamma^2)(2\lambda_i - \gamma\lambda_{3-i})\, c^C}{(4-\gamma^2)(1-\gamma^2)} \\
&\quad - \frac{(2-\gamma^2)\left(\alpha_i - \lambda_i c^B\right) - \gamma\left(\alpha_{3-i} - \lambda_{3-i} c^B\right)}{(4-\gamma^2)(1-\gamma^2)} \\
&= \frac{-\gamma^2\alpha_i + \gamma^3\alpha_{3-i} - (1-\gamma^2)(2\lambda_i - \gamma\lambda_{3-i})\, c^C + (2-\gamma^2)\lambda_i c^B - \gamma\lambda_{3-i} c^B}{(4-\gamma^2)(1-\gamma^2)} \\
&= \frac{-\gamma^2\alpha_i + \gamma^3\alpha_{3-i} - (1-\gamma^2)(2\lambda_i - \gamma\lambda_{3-i})\, c^C + (2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)(1-\gamma^2)} \\
&= \frac{-\gamma^2\alpha_i + \gamma^3\alpha_{3-i} - (2\lambda_i - \gamma\lambda_{3-i})\, c^C + \gamma^2(2\lambda_i - \gamma\lambda_{3-i})\, c^C}{(4-\gamma^2)(1-\gamma^2)} \\
&\quad + \frac{(2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)(1-\gamma^2)} \\
&= \frac{-\gamma^2\alpha_i + \gamma^3\alpha_{3-i} - (2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i})\, c^C + \gamma^2(\lambda_i - \gamma\lambda_{3-i})\, c^C}{(4-\gamma^2)(1-\gamma^2)} \\
&\quad + \frac{(2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i})\, c^B}{(4-\gamma^2)(1-\gamma^2)} \\
&= -\frac{\gamma^2\left[\alpha_i - \lambda_i c^C - \gamma\left(\alpha_{3-i} - \lambda_{3-i} c^C\right)\right] + \left(c^C - c^B\right)(2\lambda_i - \gamma^2\lambda_i - \gamma\lambda_{3-i})}{(4-\gamma^2)(1-\gamma^2)}.
\end{aligned}
$$

For $\lambda_1 = \lambda_2$ we have $c^C = c^B$ and $c^C = \frac{\alpha_1+\alpha_2}{4\lambda_1}$ and, thus,

$$
\begin{aligned}
q_i^C - q_i^B &= -\frac{\gamma^2\left[\alpha_i - \lambda_i c^C - \gamma\left(\alpha_{3-i} - \lambda_{3-i} c^C\right)\right]}{(4-\gamma^2)(1-\gamma^2)} \\
&= -\frac{\gamma^2\left[3\alpha_i - \alpha_{3-i} - \gamma(3\alpha_{3-i} - \alpha_i)\right]}{4(4-\gamma^2)(1-\gamma^2)} \\
&= -\frac{\gamma^2\left[(3+\gamma)\alpha_i - (1+3\gamma)\alpha_{3-i}\right]}{4(4-\gamma^2)(1-\gamma^2)}.
\end{aligned}
$$

Thus, it can easily be verified for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(\frac{3}{2}, 1, 1, 1, \frac{1}{2}\right)$$

that Assumptions 3.1 and 3.2 as well as $\alpha_1\tau_1^C \leq \alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$ hold and we have

$$q_2^C = \frac{1}{12} > \frac{7}{90} = q_2^B.$$

This can be seen as follows: Intermediary 2's equilibrium quantities are

$$q_2^C = \frac{1}{12} \approx 0.0833$$
$$q_2^B = \frac{7}{90} \approx 0.0778$$

and the according conditions are

$$\tau_1^C = \sqrt{2+\frac{1}{2}} - 1 = \sqrt{\frac{5}{2}} - 1 \approx 0.5811,$$

Condition $\alpha_2 \leq \alpha_1 \frac{\lambda_2}{\lambda_1}$: $\alpha_1 \frac{\lambda_2}{\lambda_1} - \alpha_2 = \frac{3}{2} - 1 = \frac{1}{2} = 0.5 > 0,$

Condition $\alpha_1 \tau_1^C \leq \alpha_2$: $\alpha_2 - \tau_1^C \alpha_1 = 1 - \left(\frac{3}{2}\sqrt{\frac{5}{2}} - \frac{3}{2}\right) = \frac{5}{2} - \frac{3}{2}\sqrt{\frac{5}{2}} \approx 0.1283 > 0,$

Assumption 3.1: $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} - \frac{\gamma}{2} = \frac{2}{3} - \frac{1}{4} = \frac{5}{12} \approx 0.4167 > 0,$

$$\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} - \frac{\gamma}{2} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75 > 0,$$

Assumption 3.2: $\min\left\{\frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1}\right\} - \frac{\gamma}{2-\gamma^2} = \frac{2}{3} - \frac{\frac{1}{2}}{\frac{7}{4}} = \frac{2}{3} - \frac{2}{7} = \frac{8}{21} \approx 0.3810 > 0,$

$$\min\left\{\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}\right\} - \frac{\gamma}{2-\gamma^2} = 1 - \frac{2}{7} = \frac{5}{7} \approx 0.7142 > 0.$$

(iii) We have

$$\pi_i^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) - \pi_i^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \frac{\gamma^3\left[4\alpha_i\alpha_{3-i}(1-\gamma) - (\alpha_i - \alpha_{3-i})^2(3+5\gamma)\right]}{8(4-\gamma^2)^2(1-\gamma^2)}.$$

This can be seen as follows: We show

$$\pi_2^C\left(p_1^C, p_2^C, c^C, \alpha_1, \alpha_2\right) - \pi_2^B\left(p_1^B, p_2^B, c^B, \alpha_1, \alpha_2\right) = \frac{\gamma^3\left[4\alpha_1\alpha_2(1-\gamma) - (\alpha_1 - \alpha_2)^2(3+5\gamma)\right]}{8(4-\gamma^2)^2(1-\gamma^2)}.$$

Consider

$$\frac{\left[\alpha_2(6+\gamma)-\alpha_1(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2}-\frac{\left[\alpha_2(6+\gamma-3\gamma^2)-\alpha_1(2+3\gamma-\gamma^2)\right]^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=\frac{\left[\alpha_2^2\left(6+\gamma\right)^2-2\alpha_1\alpha_2\left(6+\gamma\right)\left(2+3\gamma\right)+\alpha_1^2\left(2+3\gamma\right)^2\right]}{16\left(4-\gamma^2\right)^2}$$
$$-\frac{\left[\alpha_2^2\left(6+\gamma-3\gamma^2\right)^2-2\alpha_1\alpha_2\left(6+\gamma-3\gamma^2\right)\left(2+3\gamma-\gamma^2\right)+\alpha_1^2\left(2+3\gamma-\gamma^2\right)^2\right]^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=\frac{\alpha_2^2\left[-2\gamma^3\left(3+5\gamma\right)\right]-2\alpha_1\alpha_2\left[-2\gamma^3\left(5+3\gamma\right)\right]+\alpha_1^2\left[-2\gamma^3\left(3+5\gamma\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=-\gamma^3\frac{\alpha_2^2\left(3+5\gamma\right)-2\alpha_1\alpha_2\left(5+3\gamma\right)+\alpha_1^2\left(3+5\gamma\right)}{8\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=-\gamma^3\frac{\left[\left(\alpha_1-\alpha_2\right)^2\left(3+5\gamma\right)-2\alpha_1\alpha_2\left(5+3\gamma\right)+2\alpha_1\alpha_2\left(3+5\gamma\right)\right]}{8\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=-\gamma^3\frac{\left[\left(\alpha_1-\alpha_2\right)^2\left(3+5\gamma\right)+2\alpha_1\alpha_2\left(-2+2\gamma\right)\right]}{8\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=-\gamma^3\frac{\left[\left(\alpha_1-\alpha_2\right)^2\left(3+5\gamma\right)-4\alpha_1\alpha_2\left(1-\gamma\right)\right]}{8\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$
$$=\gamma^3\frac{\left[4\alpha_1\alpha_2\left(1-\gamma\right)-\left(\alpha_1-\alpha_2\right)^2\left(3+5\gamma\right)\right]}{8\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$

The second equality follows from

$$\left(1-\gamma^2\right)\left(6+\gamma\right)^2-\left(6+\gamma-3\gamma^2\right)^2$$
$$=\left(1-\gamma^2\right)\left(6+\gamma\right)^2-\left[\left(6+\gamma\right)^2-6\gamma^2\left(6+\gamma\right)+9\gamma^4\right]$$
$$=-\gamma^2\left(6+\gamma\right)^2+6\gamma^2\left(6+\gamma\right)-9\gamma^4$$
$$=-\gamma^2\left[\left(6+\gamma\right)^2-6\left(6+\gamma\right)+9\gamma^2\right]$$
$$=-\gamma^2\left[36+12\gamma+\gamma^2-36-6\gamma+9\gamma^2\right]$$
$$=-2\gamma^3\left(3+5\gamma\right),$$

$$\left(1-\gamma^2\right)\left(2+3\gamma\right)^2-\left(2+3\gamma-\gamma^2\right)^2$$
$$=\left(1-\gamma^2\right)\left(2+3\gamma\right)^2-\left[\left(2+3\gamma\right)^2-2\left(2+3\gamma\right)\gamma^2+\gamma^4\right]$$
$$=-\gamma^2\left(2+3\gamma\right)^2+2\left(2+3\gamma\right)\gamma^2-\gamma^4$$
$$=-\gamma^2\left[\left(2+3\gamma\right)^2-2\left(2+3\gamma\right)+\gamma^2\right]$$

$$= -\gamma^2 \left[4 + 12\gamma + 9\gamma^2 - 4 - 6\gamma + \gamma^2\right]$$
$$= -2\gamma^3 (3 + 5\gamma),$$

$$\left(1-\gamma^2\right)(6+\gamma)(2+3\gamma) - \left(6+\gamma-3\gamma^2\right)\left(2+3\gamma-\gamma^2\right)$$
$$= \left(1-\gamma^2\right)(6+\gamma)(2+3\gamma) - \left[(6+\gamma)(2+3\gamma) - \gamma^2(6+\gamma) - 3\gamma^2(2+3\gamma) + 3\gamma^4\right]$$
$$= -\gamma^2 (6+\gamma)(2+3\gamma) + \gamma^2(6+\gamma) + 3\gamma^2(2+3\gamma) - 3\gamma^4$$
$$= -\gamma^2 \left[(6+\gamma)(2+3\gamma) - (6+\gamma) - 3(2+3\gamma) + 3\gamma^2\right]$$
$$= -\gamma^2 \left[12 + 18\gamma + 2\gamma + 3\gamma^2 - 6 - \gamma - 6 - 9\gamma + 3\gamma^2\right]$$
$$= -\gamma^3 (5 + 3\gamma).$$

Thus, for

$$(\alpha_1, \alpha_2, \lambda_1, \lambda_2, \gamma) = \left(\frac{3}{2}, 1, 1, 1, \frac{3}{4}\right)$$

it can easily be verified that Assumptions 3.1 and 3.2 as well as $\alpha_1 \tau_1^C \le \alpha_2 \le \alpha_1 \frac{\lambda_2}{\lambda_1}$ hold and we have

$$\pi_1^C\left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) = \frac{2209}{12100} < \frac{625}{3388} = \pi_1^B\left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right),$$
$$\pi_2^C\left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) = \frac{9}{12100} < \frac{9}{3388} = \pi_2^B\left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right).$$

This can be seen as follows: The intermediaries' equilibrium profits are

$$\pi_1^C\left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) = \frac{47^2}{2^2 \cdot 5^2 \cdot 11^2} = \frac{2209}{12100} \approx 0.1826,$$
$$\pi_1^B\left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right) = \frac{5^4}{2^2 \cdot 7 \cdot 11^2} = \frac{625}{3388} \approx 0.1845,$$
$$\pi_2^C\left(p_1^C, p_2^C, c^C, \frac{3}{2}, 1\right) = \frac{3^2}{2^2 \cdot 5^2 \cdot 11^2} = \frac{9}{12100} \approx 0.0007,$$
$$\pi_2^B\left(p_1^B, p_2^B, c^B, \frac{3}{2}, 1\right) = \frac{3^2}{2^2 \cdot 7 \cdot 11^2} = \frac{9}{3388} \approx 0.0027.$$

and the according conditions are

$$\tau_1^C = \sqrt{2 + \frac{3}{4}} - 1 = \frac{\sqrt{11}}{2} - 1 \approx 0.6583,$$

$$\text{Condition } \alpha_2 \le \alpha_1 \tfrac{\lambda_2}{\lambda_1}\text{:} \quad \alpha_1 \frac{\lambda_2}{\lambda_1} - \alpha_2 = \frac{3}{2} - 1 = \frac{1}{2} = 0.5 > 0,$$

$$\text{Condition } \alpha_1 \tau_1^C \le \alpha_2\text{:} \quad \alpha_2 - \tau_1^C \alpha_1 = 1 - \left(\frac{3}{2} \frac{\sqrt{11}}{2} - \frac{3}{2} \right) = \frac{5}{2} - \frac{3}{4}\sqrt{11}$$

$$\approx 0.0125 > 0,$$

$$\text{Assumption 3.1:} \quad \min\left\{ \frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1} \right\} - \frac{\gamma}{2} = \frac{2}{3} - \frac{3}{8} = \frac{7}{24} \approx 0.2917 > 0,$$

$$\min\left\{ \frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1} \right\} - \frac{\gamma}{2} = 1 - \frac{3}{8} = \frac{5}{8} = 0.625 > 0,$$

$$\text{Assumption 3.2:} \quad \min\left\{ \frac{\alpha_1}{\alpha_2}, \frac{\alpha_2}{\alpha_1} \right\} - \frac{\gamma}{2 - \gamma^2} = \frac{2}{3} - \frac{\frac{3}{4}}{\frac{23}{16}} = \frac{2}{3} - \frac{12}{23} = \frac{10}{69}$$

$$\approx 0.1450 > 0,$$

$$\min\left\{ \frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1} \right\} - \frac{\gamma}{2 - \gamma^2} = 1 - \frac{12}{23} = \frac{11}{23} \approx 0.4783 > 0.$$

□

Computations of Proof of Proposition 3.7

Proof of Proposition 3.7. We consider the different cases from the previous propositions separately and show that the intermediaries' profit functions are convex in own qualities.

Case 1: $\alpha_2 \le \alpha_1 \tau_1^B$ (or $\alpha_1 \le \alpha_2 \tau_2^B$)

We obtain

$$\pi_1^C(\alpha_1, \alpha_2) = \pi_1^B(\alpha_1, \alpha_2) = \frac{\alpha_1^2}{16}.$$

Thus, for all $\alpha_1 \in [\underline{\alpha}_1, \overline{\alpha}_1]$ we have

$$\frac{\partial^2 \pi_1^C(\alpha_1, \alpha_2)}{\partial \alpha_1^2} = \frac{\partial^2 \pi_1^C(\alpha_1, \alpha_2^2)}{\partial \alpha_1^2} = \frac{2}{16} > 0.$$

Case 2: $\alpha_1 \tau_1^B \le \alpha_2 \le \alpha_1 \tau_1^C$ (or $\alpha_2 \tau_2^B \le \alpha_1 \le \alpha_2 \tau_2^C$)

We already computed the profit for Cournot competition in Case 1, the result for Bertrand competition is shown in Case 3.

Case 3: $\alpha_1 \tau_1^C \le \alpha_2 \le \alpha_1 \frac{\lambda_2}{\lambda_1}$ (or $\alpha_2 \tau_2^C \le \alpha_1 \le \alpha_2 \frac{\lambda_1}{\lambda_2}$)

First observe that for Cournot as well as for Bertrand competition the profit of intermediary i is quadratic in α_i for $i = 1, 2$. Consider intermediary i. In the case of Cournot competition we obtain the following second derivative of the profit function using Assumption 3.1

$$\begin{aligned}
&\frac{\partial^2 \pi_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2}\\
&= \frac{\partial^2 \left[\left(p_i^C(\alpha_i, \alpha_{3-i}) - \lambda_i c^C(\alpha_i, \alpha_{3-i}) \right) q_i^C(\alpha_i, \alpha_{3-i}) \right]}{\partial \alpha_i^2}\\
&= 2 \left(\frac{\partial p_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} - \lambda_i \frac{\partial c^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \right) \frac{\partial q_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\\
&= 2 \left(\frac{\partial q_i^C(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \right)^2\\
&= 2 \left(\frac{2}{(4-\gamma^2)} - \frac{(2\lambda_i - \gamma\lambda_{3-i})^2}{(4-\gamma^2)\, 4 \left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i} \right)} \right)^2\\
&= \frac{\left((4-\gamma^2)\, \lambda_{3-i}^2 + 4 \left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i} \right) \right)^2}{8 (4-\gamma^2)^2 \left(\lambda_i^2 + \lambda_{3-i}^2 - \gamma\lambda_i\lambda_{3-i} \right)^2} > 0
\end{aligned}$$

for all $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ for $i = 1, 2$. Similarly, we obtain for Bertrand competition with $\gamma \in (-1, 1)$ using Assumption 3.2

$$\begin{aligned}
&\frac{\partial^2 \pi_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2}\\
&= \frac{\partial^2 \left[\left(p_i^B(\alpha_i, \alpha_{3-i}) - \lambda_i c^B(\alpha_i, \alpha_{3-i}) \right) q_i^B(\alpha_i, \alpha_{3-i}) \right]}{\partial \alpha_i^2}\\
&= 2 \left(\frac{\partial p_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} - \lambda_i \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \right) \frac{\partial q_i^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\\
&= 2 \left(\frac{(2-\gamma^2) + (2\lambda_i + \gamma\lambda_{3-i}) \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}}{(4-\gamma^2)} - \lambda_i \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \right)\\
&\quad \left(\frac{(2-\gamma^2) - (2\lambda_i - \gamma\lambda_{3-i}) \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}}{(4-\gamma^2)(1-\gamma^2)} \right)\\
&= 2 \left(\frac{(2-\gamma^2) - ((2-\gamma^2)\, \lambda_i - \gamma\lambda_{3-i}) \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}}{(4-\gamma^2)} \right)\\
&\quad \left(\frac{(2-\gamma^2) - ((2-\gamma^2)\, \lambda_i - \gamma\lambda_{3-i}) \frac{\partial c^B(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}}{(4-\gamma^2)(1-\gamma^2)} \right)
\end{aligned}$$

$$= 2\frac{\left(\left(2-\gamma^2\right)-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)\frac{\partial c^B(\alpha_i,\alpha_{3-i})}{\partial\alpha_i}\right)^2}{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$

$$= 2\frac{\left(\left(2-\gamma^2\right)-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)\frac{\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}}{2\left[\left(2-\gamma^2\right)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right]}\right)^2}{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}$$

$$= \frac{\left(\left(2-\gamma^2\right)2\left[\left(2-\gamma^2\right)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right]-\left(\left(2-\gamma^2\right)\lambda_i-\gamma\lambda_{3-i}\right)^2\right)^2}{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)2\left[\left(2-\gamma^2\right)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right]^2}$$

$$= \frac{\left(\left(4-\gamma^2\right)\left(1-\gamma^2\right)\lambda_{3-i}^2+\left(2-\gamma^2\right)\left(\left(2-\gamma^2\right)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right)\right)^2}{2\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\left(\left(2-\gamma^2\right)\left(\lambda_i^2+\lambda_{3-i}^2\right)-2\gamma\lambda_i\lambda_{3-i}\right)^2} > 0$$

for all $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ for $i = 1, 2$. Hence, for Cournot as well as for Bertrand competition any zero of the first-order condition is a minimum which is independent of the other intermediary's investment choice. Thus, the candidates for optimal investment choices are at the corner points of the according interval.

□

Computations of Proof of Proposition 3.8

For Cournot competition, the payoffs of the non-cooperative game are

$$\pi_1^C\left(\underline{\alpha},\underline{\alpha}\right) = \pi_2^C\left(\underline{\alpha},\underline{\alpha}\right) = \frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2},$$

$$\pi_1^C\left(\overline{\alpha},\underline{\alpha}\right) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases}$$

$$\pi_2^C\left(\overline{\alpha},\underline{\alpha}\right) = \begin{cases} 0 & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases}$$

$$\pi_1^C\left(\underline{\alpha},\overline{\alpha}\right) = \begin{cases} 0 & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases}$$

$$\pi_2^C\left(\underline{\alpha},\overline{\alpha}\right) = \begin{cases} \frac{\overline{\alpha}^2}{16} & \text{for } \underline{\alpha} \le \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \le \underline{\alpha}, \end{cases}$$

$$\pi_1^C\left(\overline{\alpha},\overline{\alpha}\right) = \pi_2^C\left(\overline{\alpha},\overline{\alpha}\right) = \frac{\overline{\alpha}^2}{4\left(2+\gamma\right)^2},$$

The profits are derived as follows:

$$\begin{aligned}\frac{\left[\underline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} &= \frac{\left[\underline{\alpha}(4-2\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \\ &= \frac{4\underline{\alpha}^2(2-\gamma)^2}{16\left(4-\gamma^2\right)^2} \\ &= \frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2}\end{aligned}$$

The equilibrium conditions on the investment costs κ^C are shown in Table 3.1. These

strategy profile	$\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$
$(\underline{\alpha}, \underline{\alpha})$	$\left[\frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2}, \infty\right)$
$(\overline{\alpha}, \underline{\alpha})$, $(\underline{\alpha}, \overline{\alpha})$	$\left[\frac{\overline{\alpha}^2}{4(2+\gamma)^2}, \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2}\right]$
$(\overline{\alpha}, \overline{\alpha})$	$\left[0, \frac{\overline{\alpha}^2}{4(2+\gamma)^2}\right]$
strategy profile	$\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}$
$(\underline{\alpha}, \underline{\alpha})$	$\left[\frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)]}{16(4-\gamma^2)^2}, \infty\right)$
$(\overline{\alpha}, \underline{\alpha})$, $(\underline{\alpha}, \overline{\alpha})$	$\left[\frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)]}{16(4-\gamma^2)^2}, \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)]}{16(4-\gamma^2)^2}\right]$
$(\overline{\alpha}, \overline{\alpha})$	$\left[0, \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)]}{16(4-\gamma^2)^2}\right]$

Table 3.1: Equilibrium conditions on the investment costs κ^C

conditions are derived as follows: Consider the strategy profile $(\underline{\alpha}, \underline{\alpha})$. For this to be a Nash equilibrium we require for $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$

$$\begin{aligned} & \frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2} \geq \frac{\overline{\alpha}^2}{16} - \kappa^C \\ \Leftrightarrow \quad & \kappa^C \geq \frac{\overline{\alpha}^2}{16} - \frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2} \\ \Leftrightarrow \quad & \kappa^C \geq \frac{\overline{\alpha}^2\left(2+\gamma\right)^2 - 4\underline{\alpha}^2}{16\left(2+\gamma\right)^2} \\ \Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}\left(2+\gamma\right)-2\underline{\alpha}\right]\left[\overline{\alpha}\left(2+\gamma\right)+2\underline{\alpha}\right]}{16\left(2+\gamma\right)^2}\end{aligned}$$

and for $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}$

$$\begin{aligned}
& \frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2} \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2}-\kappa^C \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2}-\frac{\underline{\alpha}^2}{4\left(2+\gamma\right)^2} \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2}-\frac{4\underline{\alpha}^2\left(2-\gamma\right)^2}{16\left(4-\gamma^2\right)^2} \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)\right]^2-4\underline{\alpha}^2\left(2-\gamma\right)^2}{16\left(4-\gamma^2\right)^2} \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)-2\underline{\alpha}\left(2-\gamma\right)\right]\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}(2+3\gamma)+2\underline{\alpha}\left(2-\gamma\right)\right]}{16\left(4-\gamma^2\right)^2} \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left[\overline{\alpha}(6+\gamma)-\underline{\alpha}\left(6+\gamma\right)\right]\left[\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)\right]}{16\left(4-\gamma^2\right)^2} \\
\Leftrightarrow \quad & \kappa^C \geq \frac{\left(6+\gamma\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)\right]}{16\left(4-\gamma^2\right)^2}.
\end{aligned}$$

Consider the strategy profile $(\overline{\alpha},\overline{\alpha})$. For this to be a Nash equilibrium we require for $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$

$$\begin{aligned}
& \frac{\overline{\alpha}^2}{4\left(2+\gamma\right)^2}-\kappa^C \geq 0 \\
\Leftrightarrow \quad & \frac{\overline{\alpha}^2}{4\left(2+\gamma\right)^2} \geq \kappa^C
\end{aligned}$$

and for $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}$

$$\begin{aligned}
& \frac{\overline{\alpha}^2}{4\left(2+\gamma\right)^2}-\kappa^C \geq \frac{\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \\
\Leftrightarrow \quad & \frac{\overline{\alpha}^2}{4\left(2+\gamma\right)^2}-\frac{\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \geq \kappa^C \\
\Leftrightarrow \quad & \frac{4\overline{\alpha}^2\left(2-\gamma\right)^2}{16\left(4-\gamma^2\right)^2}-\frac{\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \geq \kappa^C \\
\Leftrightarrow \quad & \frac{4\overline{\alpha}^2\left(2-\gamma\right)^2-\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \geq \kappa^C \\
\Leftrightarrow \quad & \frac{4\overline{\alpha}^2\left(2-\gamma\right)^2-\left[\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]^2}{16\left(4-\gamma^2\right)^2} \geq \kappa^C
\end{aligned}$$

$$\Leftrightarrow \quad \frac{\left[2\overline{\alpha}\,(2-\gamma)-\underline{\alpha}(6+\gamma)+\overline{\alpha}(2+3\gamma)\right]\left[2\overline{\alpha}\,(2-\gamma)+\underline{\alpha}(6+\gamma)-\overline{\alpha}(2+3\gamma)\right]}{16\,(4-\gamma^2)^2} \geq \kappa^C$$

$$\Leftrightarrow \quad \frac{\left[\overline{\alpha}\,(6+\gamma)-\underline{\alpha}(6+\gamma)\right]\left[\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)\right]}{16\,(4-\gamma^2)^2} \geq \kappa^C$$

$$\Leftrightarrow \quad \frac{(6+\gamma)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)\right]}{16\,(4-\gamma^2)^2} \geq \kappa^C.$$

For symmetry reasons these are also the bounds for asymmetric Nash equilibria. where the upper and lower bound for the investment costs are given by

$$\underline{\kappa}^C = \begin{cases} \frac{\overline{\alpha}^2}{4(2+\gamma)^2} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma)+\underline{\alpha}(6+\gamma)]}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}, \end{cases}$$

$$\overline{\kappa}^C = \begin{cases} \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{(6+\gamma)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)]}{16(4-\gamma^2)^2} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}. \end{cases}$$

Computations of Proof of Proposition 3.10

Proof of Proposition 3.10. We have

$$\underline{\kappa}^C - \underline{\kappa}^B$$

$$= \begin{cases} \frac{\gamma^3\overline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{8\gamma^3(1-\gamma)\overline{\alpha}^2+\left[\underline{\alpha}(6+\gamma-3\gamma^2)-\overline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ \frac{\gamma^3[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(7+\gamma)-\underline{\alpha}(3+5\gamma)]}{8(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}, \end{cases}$$

and

$$\overline{\kappa}^C - \overline{\kappa}^B$$

$$= \begin{cases} -\frac{\gamma^3\underline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)} & \text{for } \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right), \\ \frac{\overline{\alpha}^2}{16} - \frac{8\gamma^3(1-\gamma)\underline{\alpha}^2+\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right), \\ -\frac{\gamma^3[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(3+5\gamma)-\underline{\alpha}(7+\gamma)]}{8(4-\gamma^2)^2(1-\gamma^2)} & \text{for } \overline{\alpha}\left(\sqrt{2+\gamma}-1\right) \leq \underline{\alpha}. \end{cases}$$

From these expressions for $\underline{\kappa}^C - \underline{\kappa}^B$ and $\overline{\kappa}^C - \overline{\kappa}^B$ we immediately derive the statements of Proposition 3.10.

(i) Consider $\underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right)$. We have for

$$\begin{aligned}
&\underline{\kappa}^C - \underline{\kappa}^B \\
&= \frac{\overline{\alpha}^2}{4(2+\gamma)^2} - \frac{\overline{\alpha}^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)} \\
&= \frac{\overline{\alpha}^2(2-\gamma)^2(1+\gamma) - \overline{\alpha}^2(1-\gamma)(2+\gamma)^2}{4(4-\gamma^2)^2(1+\gamma)} \\
&= \frac{2\overline{\alpha}\gamma^3}{4(4-\gamma^2)^2(1+\gamma)} \\
&= \frac{\gamma^3\overline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)}
\end{aligned}$$

and for

$$\begin{aligned}
&\overline{\kappa}^C - \overline{\kappa}^B \\
&= \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2} - \frac{\overline{\alpha}^2(2-\gamma)^2(1+\gamma)-4\underline{\alpha}^2(1-\gamma)}{16(2-\gamma)^2(1+\gamma)} \\
&= \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}](2-\gamma)^2(1+\gamma)}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&\quad - \frac{\left[\overline{\alpha}^2(2-\gamma)^2(1+\gamma)-4\underline{\alpha}^2(1-\gamma)\right](2+\gamma)^2}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= \frac{\left[\overline{\alpha}^2(2+\gamma)^2-4\underline{\alpha}^2\right](2-\gamma)^2(1+\gamma)}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&\quad - \frac{\left[\overline{\alpha}^2(2-\gamma)^2(1+\gamma)-4\underline{\alpha}^2(1-\gamma)\right](2+\gamma)^2}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= \frac{\left[-4\underline{\alpha}^2\left((2-\gamma)^2(1+\gamma)-(2+\gamma)^2(1-\gamma)\right)\right]}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= \frac{[-4\underline{\alpha}^2((4-4\gamma+\gamma^2)(1+\gamma)-(4+4\gamma+\gamma^2)(1-\gamma))]}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= \frac{-8\underline{\alpha}^2\gamma^3}{16(2+\gamma)^2(2-\gamma)^2(1+\gamma)} \\
&= -\frac{\gamma^3\underline{\alpha}^2}{2(4-\gamma^2)^2(1+\gamma)}
\end{aligned}$$

(ii) Consider $\overline{\alpha}\left(\sqrt{2+\gamma-\gamma^2}-1\right) \leq \underline{\alpha} \leq \overline{\alpha}\left(\sqrt{2+\gamma}-1\right)$. Note that

$$\underline{\kappa}^B = \frac{4\overline{\alpha}^2(2+\gamma)^2(1-\gamma)^2 - [\underline{\alpha}(6+\gamma-3\gamma^2) - \overline{\alpha}(2+3\gamma-\gamma^2)]^2}{16(4-\gamma^2)^2(1-\gamma^2)},$$
$$\overline{\kappa}^B = \frac{[\overline{\alpha}(6+\gamma-3\gamma^2) - \underline{\alpha}(2+3\gamma-\gamma^2)]^2 - 4\underline{\alpha}^2(2+\gamma)^2(1-\gamma)^2}{16(4-\gamma^2)^2(1-\gamma^2)}.$$

We have for

$$\begin{aligned}
&\underline{\kappa}^C - \underline{\kappa}^B \\
&= \frac{\overline{\alpha}^2}{4(2+\gamma)^2} - \frac{(6+\gamma-3\gamma^2)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma-\gamma^2)+\underline{\alpha}(6+\gamma-3\gamma^2)]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{4\overline{\alpha}^2(1-\gamma^2)(2-\gamma)^2 - (6+\gamma-3\gamma^2)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(2-5\gamma-\gamma^2)+\underline{\alpha}(6+\gamma-3\gamma^2)]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{4\overline{\alpha}^2(1-\gamma^2)(2-\gamma)^2 - 4\overline{\alpha}^2(2+\gamma)^2(1-\gamma)^2 + [\underline{\alpha}(6+\gamma-3\gamma^2)-\overline{\alpha}(2+3\gamma-\gamma^2)]^2}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{8\gamma^3(1-\gamma)\overline{\alpha}^2 + [\underline{\alpha}(6+\gamma-3\gamma^2)-\overline{\alpha}(2+3\gamma-\gamma^2)]^2}{16(4-\gamma^2)^2(1-\gamma^2)},
\end{aligned}$$

and for

$$\begin{aligned}
&\overline{\kappa}^C - \overline{\kappa}^B \\
&= \frac{[\overline{\alpha}(2+\gamma)-2\underline{\alpha}][\overline{\alpha}(2+\gamma)+2\underline{\alpha}]}{16(2+\gamma)^2} \\
&\quad - \frac{(6+\gamma-3\gamma^2)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma-3\gamma^2)+\underline{\alpha}(2-5\gamma-\gamma^2)]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{(2-\gamma)^2(1-\gamma^2)\left[\overline{\alpha}^2(2+\gamma)^2-4\underline{\alpha}^2\right]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&\quad + \frac{-(6+\gamma-3\gamma^2)[\overline{\alpha}-\underline{\alpha}][\overline{\alpha}(6+\gamma-3\gamma^2)+\underline{\alpha}(2-5\gamma-\gamma^2)]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{(2-\gamma)^2(1-\gamma^2)\left[\overline{\alpha}^2(2+\gamma)^2-4\underline{\alpha}^2\right]}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&\quad + \frac{-[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)]^2 + 4\underline{\alpha}^2(2+\gamma)^2(1-\gamma)^2}{16(4-\gamma^2)^2(1-\gamma^2)} \\
&= \frac{(4-\gamma^2)^2(1-\gamma^2)\overline{\alpha}^2 - [\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)]^2}{16(4-\gamma^2)^2(1-\gamma^2)}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{4\underline{\alpha}^2\left(2+\gamma\right)^2\left(1-\gamma\right)^2-4\left(2-\gamma\right)^2\left(1-\gamma^2\right)\underline{\alpha}^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\overline{\alpha}^2-\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2-8\gamma^3\left(1-\gamma\right)\underline{\alpha}^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\overline{\alpha}^2-\left[8\gamma^3\left(1-\gamma\right)\underline{\alpha}^2+\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\overline{\alpha}^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}-\frac{8\gamma^3\left(1-\gamma\right)\underline{\alpha}^2+\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\overline{\alpha}^2}{16}-\frac{8\gamma^3\left(1-\gamma\right)\underline{\alpha}^2+\left[\overline{\alpha}(6+\gamma-3\gamma^2)-\underline{\alpha}(2+3\gamma-\gamma^2)\right]^2}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}.
\end{aligned}
$$

(iii) Consider $\overline{\alpha}\left(\sqrt{2+\gamma}-1\right)\leq\underline{\alpha}$. We have for

$$
\begin{aligned}
&\underline{\kappa}^C-\underline{\kappa}^B\\
=&\frac{\left(6+\gamma\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(2-5\gamma\right)+\underline{\alpha}\left(6+\gamma\right)\right]}{16\left(4-\gamma^2\right)^2}\\
&-\frac{\left(6+\gamma-3\gamma^2\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(2-5\gamma-\gamma^2\right)+\underline{\alpha}\left(6+\gamma-3\gamma^2\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\left(6+\gamma\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(2-5\gamma\right)+\underline{\alpha}\left(6+\gamma\right)\right]\left(1-\gamma^2\right)}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
&-\frac{\left(6+\gamma-3\gamma^2\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(2-5\gamma-\gamma^2\right)+\underline{\alpha}\left(6+\gamma-3\gamma^2\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(14\gamma^3+2\gamma^4\right)-\underline{\alpha}\left(6\gamma^3-10\gamma^4\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{2\gamma^3\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(7+\gamma\right)-\underline{\alpha}\left(3-5\gamma\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\\
=&\frac{\gamma^3\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(7+\gamma\right)-\underline{\alpha}\left(3+5\gamma\right)\right]}{8\left(4-\gamma^2\right)^2\left(1+\gamma\right)\left(1-\gamma\right)}
\end{aligned}
$$

and for

$$
\begin{aligned}
&\overline{\kappa}^C-\overline{\kappa}^B\\
=&\frac{\left(6+\gamma\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)\right]}{16\left(4-\gamma^2\right)^2}\\
&-\frac{\left(6+\gamma-3\gamma^2\right)\left[\overline{\alpha}-\underline{\alpha}\right]\left[\overline{\alpha}\left(6+\gamma-3\gamma^2\right)+\underline{\alpha}\left(2-5\gamma-\gamma^2\right)\right]}{16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(6+\gamma)\,[\overline{\alpha}-\underline{\alpha}]\,[\overline{\alpha}(6+\gamma)+\underline{\alpha}(2-5\gamma)]\,(1-\gamma^2)}{16\,(4-\gamma^2)^2\,(1-\gamma^2)} \\
&\quad - \frac{(6+\gamma-3\gamma^2)\,[\overline{\alpha}-\underline{\alpha}]\,[\overline{\alpha}\,(6+\gamma-3\gamma^2)+\underline{\alpha}\,(2-5\gamma-\gamma^2)]}{16\,(4-\gamma^2)^2\,(1-\gamma^2)} \\
&= \frac{[\overline{\alpha}-\underline{\alpha}]\,[\overline{\alpha}\,(-6\gamma^3-10\gamma^4)-\underline{\alpha}\,(-14\gamma^3-2\gamma^4)]}{16\,(4-\gamma^2)^2\,(1-\gamma^2)} \\
&= -\frac{2\gamma^3\,[\overline{\alpha}-\underline{\alpha}]\,[\overline{\alpha}\,(3+5\gamma)-\underline{\alpha}\,(7+\gamma)]}{16\,(4-\gamma^2)^2\,(1-\gamma^2)} \\
&= -\frac{\gamma^3\,[\overline{\alpha}-\underline{\alpha}]\,[\overline{\alpha}\,(3+5\gamma)-\underline{\alpha}\,(7+\gamma)]}{8\,(4-\gamma^2)^2\,(1+\gamma)\,(1-\gamma)}.
\end{aligned}
$$

□

Chapter 4

Sequential Competition of Intermediaries

4.1 Introduction

Referring to the model used within Chapter 3, we consider a differentiated two-sided market in which two intermediaries face competition for the demand of a representative customer on the sales side and for the supply of input goods on the procurement side. We analyze the impact of the market structure on the equilibrium outcome when intermediaries choose production quantities sequentially. In this context we are particularly interested in the equilibrium decisions taken by intermediaries when choosing first and when choosing second.

4.1.1 Basic setup

Basically, intermediaries procure homogenous products from a monopolistic input supplier, refine them and offer the resulting final product to a representative customer. On the sales side, by choosing production quantities intermediaries compete for the customer's demand. Hereby decisions are taken sequentially. Consequently there is an intermediary, the *Stackelberg leader*, who takes a strategic decisions first, followed by an intermediary, *Stackelberg follower*, choosing product quantities afterwards. In our context an intermediaries' role, whether being in the leading or following position will be exogenously given.

Intermediaries must not necessarily be symmetric. Asymmetries between intermediaries may be present as a result of product differentiation as well as due to distinct production technologies that may be used.

When considering differentiation we distinguish between horizontal and vertical product differentiation. Horizontal product differentiation describes the fact that final products can either be substitutes, complements or mutually independent. For instance, within the white good industry two manufacturers may procure a homogenous input, e.g. steal, from a monopolistic supplier. This input can be used to either produce washing machines or laundry dryers. In the case that both manufacturers decide to produce washing machines, provided goods are substitutes. Thus, competition between manufacturers is rather fierce. Given one firm produces washing machines whereas the other firm produces laundry dryers, products are (imperfect) complements and competition is less intense. In contrast to the given example, horizontal product differentiation will exogenously given in our setting.

Besides horizontal differentiation, products can be vertically differentiated, i.e., may have different levels of product quality. Within our model intermediaries are able to take an investment decision and thus foster their product's quality. This quality enhancement increases market demand and may lead to asymmetries, given one intermediary invests while the other does not.

As mentioned, asymmetries may not just arise due to product differentiation, but also by reasons of distinct productivities. An intermediary's productivity describes the number of input goods needed in order to produce a unit of output. Hence, the productivity directly determines the intermediaries' costs of procurement. Intermediaries' productivities are exogenously given in our setting.

The timing of decisions is displayed in Fig. 4.1. Our model distinguished between two stages, the *innovation stage* and the *competition stage*. In the innovation stage intermediaries decide about their investment in product innovation, which increases their product quality and has a positive effect on the customer's demand for their product. The choice of product innovation can be seen as a long-term decision and is therefore taken first. In the competition stage the monopolistic input supplier strategically decides about the price he is demanding for his input goods. Given the input price, intermediaries compete by choosing their production quantities sequentially. Intermediary i as the Stackelberg leader chooses his output quantity first, whereas intermediary $3-i$ as the Stackelberg follower chooses his quantity thereafter, for $i = 1, 2$. According to

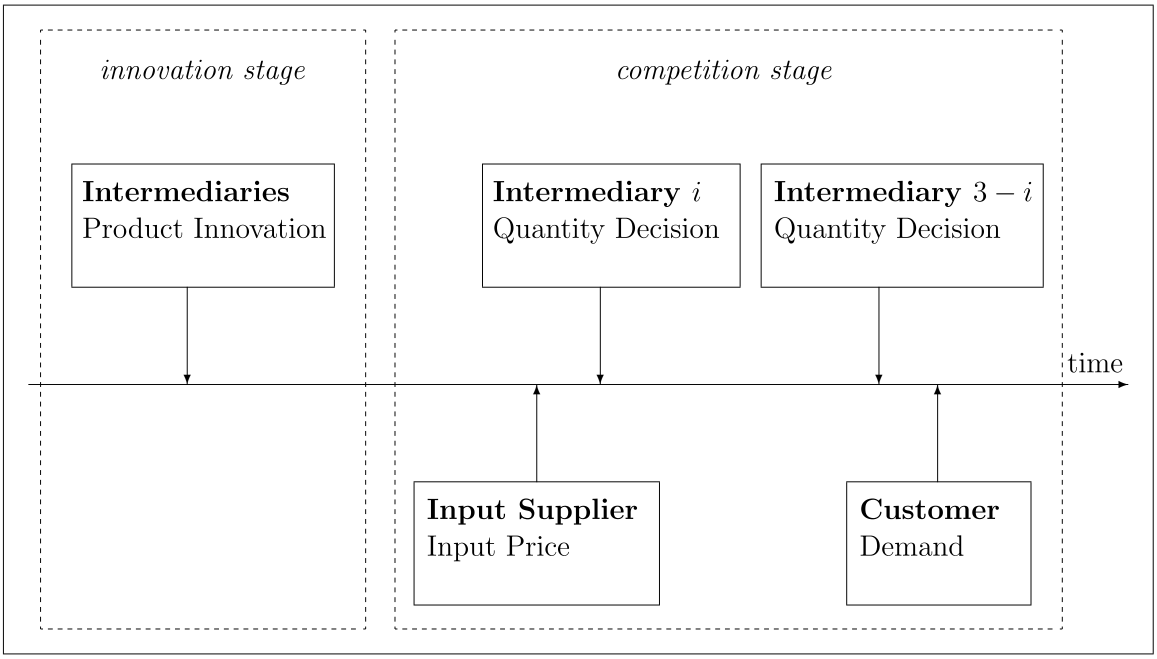

Figure 4.1: Timing of Decisions when intermediaries move sequentially

the equilibrium quantity choices, input units are procured from the supplier. Finally, intermediaries refine the inputs and offer their products to a representative customer on the sales side. In order to determine subgame perfect Nash equilibria our analysis proceeds via backwards induction.

Close to the approach of Chapter 3, in a first step we analyze the impact of a two-sided market structure on the equilibrium outcome when intermediaries choose production quantities sequentially. We find that if asymmetries between intermediaries are sufficiently high, the input supplier has an incentive to exclude the less competitive intermediary from the market. This result is independent of an intermediary's role, i.e., whether he is in the leading or following position.

For equal productivities, our main result refers to the input supplier's incentives to exclude an intermediary from the market. It can be shown that there exist conditions such that an intermediary being the Stackelberg leader will be excluded, but will be accepted in the market when taking the role of the Stackelberg follower, ceteris paribus.

When comparing the equilibrium outcomes of the Stackelberg leader and the Stackelberg follower it turns out that given an intermediary is more competitive, his equilibrium output quantity is higher when being the leader than when being the follower.

This outcome is independent of the degree of horizontal product differentiation and still holds for a less competitive intermediary, given goods are substitutes. If the intermediary is less competitive and goods are complements, there may exist asymmetries such that an intermediary being in the position of the Stackelberg follower offers higher output quantities in equilibrium than when being in the position of the Stackelberg leader.

Considering the input supplier's equilibrium prices and profits within a duopoly market, we observe that for a more competitive intermediary being the Stackelberg leader input prices are higher compared to the situation in which he is the Stackelberg follower. In addition, the supplier's duopoly profit is higher if the more competitive intermediary is choosing quantities first than if he is choosing second.

Besides analyzing the stage of competition, in a second step, we will give an outlook in which intermediaries' incentives to invest in product quality is discussed. A first result shows that given costs of investment are linear, intermediaries always either choose a maximum or minimum level of investment.

4.1.2 Literature

The foundations of today's classical oligopoly theory were established within two seminal works, the contribution of Cournot (1838) discussing a model in which duopolists compete by setting quantities and of Bertrand (1883) considering competition in which prices are selected.

Over the years, the standard simultaneous-move models were further extended in different directions. First steps in the area of differentiated duopolies were taken by the fundamental work of Singh and Vives (1984), who focus on simultaneous-move games and discuss the impact of horizontal product differentiation on the outcome of Bertrand and Cournot competition. By allowing for more general demand functions, Cheng (1985) generalizes the approach of Singh and Vives (1984), whereas Vives (1985), Okuguchi (1987), Häckner (2000) and Amir and Jin (2001) allow for n product varieties with different focuses of analysis.

Another branch of literature considers vertical product differentiation including works of Motta (1993) and Symeonidis (2003). Productivity-increasing process innovation finds attention in the articles of for example Bester and Petrakis (1993), Qiu (1997) and Pauwels *et al.* (2014). Quite similar to our modeling, Zanchettin (2006) discusses a model of horizontal and vertical product differentiation with exogenously given asymmetric product qualities and asymmetric duopolists' productivities. In contrast to our work, he restricts attention to substitute goods with exogenous input prices. Further contributions with both types of differentiation, going in distinctive directions are of Bonanno and Haworth (1998b), Weiss (2003), Filippini and Martini (2010) and Bacchiega *et al.* (2011) amongst others.

The aforementioned articles did indeed consider differentiated duopolies, but neglect the market power of an input supplier which is considered in Häckner (2003), Correa-López (2007), Mukherjee *et al.* (2012) and Manasakis and Vlassis (2014), for instance. Closest to our modeling is the work of Mukherjee *et al.* (2012) in which two unequally productive intermediaries procure their inputs from a monopolistic input supplier and offer a homogenous final good at the market. Differently to us, the input supplier is able to discriminate in prices, having the main objective to compare the market participants' profits for Bertrand and Cournot competition.

The works described above share the feature of duopolists selecting either prices or quantities simultaneously. Criticizing the simultaneous-move approach of Cournot (1838), the work of Von Stackelberg (1934) introduces a homogenous goods market in which firms take decisions about production quantities sequentially. In his model focus is put on the market's outcome in a subgame perfect Nash equilibrium. It turns out that first-moving firm must make at least as much profits as in a simultaneous move Cournot equilibrium. For identical firms, concave demand and convex costs, it could be shown that there exists a first-mover advantage, i.e., the leader's profit is higher than the follower's profit in equilibrium.

The crucial work of Von Stackelberg (1934) was afterwards extended by models allowing firms to set prices instead of quantities. In her seminal contribution, Gal-Or (1985) found that the classical result of Von Stackelberg (1934) showing the existence of a first-mover advantage does not hold if prices and not quantities are chosen by duopolists.

Beyond this, further extensions were conducted by Boyer and Moreaux (1987b) or Banerjee and Chatterjee (2014), who analyze the impact of horizontal product differentiation on the equilibrium outcome when decisions on prices or quantities are taken sequentially. Lee *et al.* (2014) introduce an input supplier and investigate for first- and second-mover advantages. They show that the standard results established by for Bertrand and Cournot competition must not necessarily hold when considering vertically related markets. This outcome can be explained by the monopolistic input supplier controlling first- and second-mover advantages by selecting input prices accordingly. He optimizes his profit by removing the first-mover (second-mover) advantage under Cournot (Bertrand) competition. Differently than Lee *et al.* (2014) within our approach the input supplier charges a unique input price and is thus not able to discriminate in prices.

Another stream of literature goes in a slightly different direction, namely, settings in which the role of a firm, whether to be in the leading or following position is endogenous. First steps in this area were taken by Hamilton and Slutsky (1990) and continuously extended over the years of for instance Van Damme and Hurkens (1999) and van Damme and Hurkens (2004). Further contributions consider asymmetries and uncertainty with Motta (1993) and Albaek (1990) as well as product innovation which is treated by for instance Lambertini (1996) and Hoppe and Lehmann-Grube (2001).

Although differently than the aforementioned works, but similarly to our ideas, Noh and Moschini (2006) discuss a differentiated market for which the potential entry of a new product is analyzed. They consider a game including an incumbent and a potential entry firm and investigate the players' strategic investment choice in product quality. This kind of modeling goes in the direction of the discussion we have in the last section of this paper.

For a more detailed literature overview see Chapter 2. To the best of our knowledge, settings of intermediate goods markets in which choices of intermediaries are taken sequentially were not considered in literature yet.

The remainder of this chapter is organized as follows: In section 4.2 we discuss the competition stage of our model. In this context we introduce the customer's utility function and determine his according demand for products of both intermediaries. Furthermore, the intermediaries' equilibrium production quantities as well as the input supplier's optimal choice of input price are determined and discussed. The section closes with a comparison of the equilibria evolving for the Stackelberg leader and follower. Section 4.3 considers the innovation stage and gives an outlook for investigating the intermediaries' optimal investment decisions in product innovation when productivities are symmetric. We finally conclude in section 4.4 and identify directions for future research.

4.2 Competition between Intermediaries

In the following section we introduce the players of our model, i.e., the customer, the intermediaries and the input supplier, and analyze the decisions they take. As we proceed backwards, we start with the customer and the analysis of his utility function, followed by an investigation of the competition between intermediaries as well as the input supplier's optimal price decision.

4.2.1 The customer

The representative customer is able to purchase quantities of two different products $q = (q_1, q_2)$ which are supplied by intermediary 1 and intermediary 2 respectively. The product of intermediary i has a quality of $\alpha_i > 0$ for $i = 1, 2$. Products are assumed to be horizontally differentiated, i.e., different degrees of substitutability, complementarity or mutually independence are possible. The degree of horizontal product differentiation is formalized by the parameter $\gamma \in [-1, 1]$, where for substitutes $\gamma \in (0, 1]$, for complements $\gamma \in [-1, 0)$ and for independent products $\gamma = 0$ holds. If $\gamma = 1$, goods are perfect substitutes and competition is rather intense, whereas if $\gamma = -1$ goods are perfect complements and competition is less fierce. The customer's utility function is similar to the one used by Singh and Vives (1984) and given by

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + I. \tag{4.1}$$

The customer is able to obtain utility from consuming a quantity q_1 offered by intermediary 1 and a quantity q_2 offered by intermediary 2. Utility increases in the according product qualities $\alpha = (\alpha_1, \alpha_2)$. Hence, the customer prefers to consume high-quality products over consuming products with low quality. Within Eq. (4.1) it can easily be seen that if on the one hand goods are complements and therefore $\gamma < 0$, the customer profits from consuming quantities of both products, q_1 and q_2, resulting in an additional utility. If on the other hand goods are substitutes and thus $\gamma > 0$, consuming quantities of both goods decreases utility. The factor I represents the quantity of an outside good which the customer is able to consume besides the quantities of q_1 and q_2. The utility function U is linear and separable in the consumption of good I and prevents income effects on the duopolistic sector.

In the following, we will focus on the competition between intermediaries within the intermediate goods market and determine their according subgame perfect Nash equilibrium strategies. To do so, the customer's demand for quantities $q = (q_1, q_2)$ need to be determined. Taken the prices $p = (p_1, p_2)$ of intermediary 1's and intermediary 2's product as given, the customer chooses to purchase the quantity q_i that maximize his utility function, respecting his budget constraint of $m \geq p_i q_i + p_{3-i} q_{3-i} + I$, with m being the overall budget. In order to ensure an inner solution we assume m to be sufficiently large. The utility maximizing quantity decision for product i with $i = 1, 2$, subject to the budget constraint satisfies

$$\alpha_i - q_i - p_i - \gamma q_{3-i} = 0. \tag{4.2}$$

From Eq. (4.2) we obtain the inverse demand function for the product of intermediary i, which is

$$p_i \left(q_i, q_{3-i}\right) = \alpha_i - q_i - \gamma q_{3-i} \quad \text{for } i = 1, 2. \tag{4.3}$$

Given intermediary i chooses to produce quantity q_i and intermediary $3 - i$ chooses to produce quantity q_{3-i}, the resulting market price for product i is given by p_i as displayed in Eq. (4.3). It can be seen that intermediary i is able to demand a higher price p_i for a product of high quality than for product of low quality. In addition, if goods are complements and thus $\gamma < 0$, p_i is increasing in q_{3-i}. The opposite is true, if goods are substitutes ($\gamma > 0$). Finally, if goods are independent ($\gamma = 0$), there is no relation between p_i and q_{3-i}.

Intermediary i's profit function π_i is given by

$$\pi_i \left(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}\right) = \left(p_i \left(q_i, q_{3-i}\right) - \lambda_i c\right) q_i = \left(\alpha_i - q_i - \gamma q_{3-i} - \lambda_i c\right) q_i. \tag{4.4}$$

On the sales side, intermediary i achieves returns of $p_i q_i$ whereas on the procurement side, he suffers total input costs of $\lambda_i q_i c$. As in Mukherjee *et al.* (2012) intermediaries can have different degrees of productivity described by the factor $\lambda_i > 0$. λ_i represents the number of inputs that are needed to produce one unit of output. If λ_i increases, intermediary i's productivity decreases as more input units are needed to produce the same quantity of outputs. Hence, the costs of procurement increase in λ_i. Within our model we assume λ_i to be exogenously given for $i = 1, 2$. Besides this, c represents the input price which is charged by the input supplier.

4.2.2 The intermediaries

In contrast to Chapter 3 we do not consider a game in which decisions are taken simultaneously. In our approach intermediaries choose their production quantities one after the other. The *Stackelberg leader* is the first player to choose quantities whereas the *Stackelberg follower* decides thereafter. For the upcoming section, without loss of generality we assume intermediary 1 to be the Stackelberg leader and intermediary 2 to be the Stackelberg follower.

In the following we will first specify the equilibrium quantities produced by the Stackelberg leader and the Stackelberg follower. Afterwards we will derive the stated results for the different market scenarios of duopoly and monopoly market. The equilibrium quantities of the Stackelberg leader are given by

$$q_1^{SL} = \begin{cases} \frac{(2\alpha_1-\gamma\alpha_2)-(2\lambda_1-\gamma\lambda_2)c}{2(2-\gamma^2)} & \text{if } c < \min\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\} \\ \frac{\alpha_1-\lambda_1 c}{2} & \text{if } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1} \\ 0 & \text{otherwise,} \end{cases} \tag{4.5}$$

whereas equilibrium quantities of the Stackelberg follower are given by

$$q_2^{SF} = \begin{cases} \frac{\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)-\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)c}{4(2-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\right\} \\ \frac{\alpha_2-\lambda_2 c}{2} & \text{if } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2} \\ 0 & \text{otherwise.} \end{cases} \tag{4.6}$$

Eq. (4.5) and (4.6) show, that equilibrium quantities of the Stackelberg leader and the Stackelberg follower depend on the supplier's input price selection c. Within the first case, for a sufficiently low input price $c < \min\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\}$, we obtain a duopoly market in which both intermediaries procure inputs and offer products on the market. For the second case, c is too high for one of the intermediaries, but still low enough for the other intermediary to achieve non-negative profits, a monopoly situation occurs in equilibrium.

We now derive the above stated equilibrium outcomes and start our analysis by considering the case of a *duopoly* market in which intermediary 1 (Stackelberg leader) as well as intermediary 2 (Stackelberg follower) procure input goods from the input supplier and sell products to the customer ($q_1 > 0$ and $q_2 > 0$). For such a market situation, the input price c must be sufficiently low for both intermediaries to achieve non-negative profits. Given the requirements of a duopoly market are satisfied, we derive the above displayed equilibrium quantities q_1^{SL} and q_2^{SF} via backwards induction. Thus, we start with the second-moving intermediary 2.

From intermediary 2's profit function π_2 as well as the fact that negative profits can be avoided by not producing, we obtain the following best reply function

$$q_2(q_1) = \max\left\{\frac{\alpha_2 - \gamma q_1 - \lambda_2 c}{2}, 0\right\}. \tag{4.7}$$

The best reply function of intermediary 2 specifies the quantity choice that maximizes his profit for a given quantity of intermediary 1. Anticipating $q_2(q_1)$, intermediary 1 chooses the profit maximizing equilibrium quantity. For the first case of $q_2(q_1) > 0$ we obtain

$$q_1^{SL} = \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\, c}{2(2-\gamma^2)}. \tag{4.8}$$

With q_1^{SL} and intermediary 2's best reply function $q_2(q_1)$ we are now able to determine intermediary 2's equilibrium quantity, which is given by

$$q_2^{SF} = \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\, c}{4(2-\gamma^2)}. \tag{4.9}$$

When considering the second case of Eq. (4.7) with $q_2(q_1) = 0$, intermediary 2 chooses not to produce a positive quantity and we obtain a monopoly scenario with intermediary 1 as the monopolist.

In a next step we derive conditions on the input price c such that both intermediaries have an incentive to demand inputs on the procurement side and offer positive quantities on the sales side. For the Stackelberg leader, in order to assure $q_1^{SL} > 0$ we need $c < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$, for intermediary 2 to assure $q_2^{SF} > 0$ the input price must satisfy $c < \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}$. Thus, if the input price satisfies

$$c < \min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}\right\}, \tag{4.10}$$

both intermediaries procure inputs and offer positive quantities on the market. If c exceeds the minimum of the above values, there will either be a monopoly market or no production. It can easily be seen, that the "critical" input price of intermediary 1 increases in its own product quality α_1 and decreases in the productivity λ_1. Hence, for a given input price c an intermediary producing a low-quality product and/or being unproductive may leave the market whereas an intermediary producing a high-quality product and/or being productive stays. The influence of intermediary 2's quality α_2 and λ_2 on the critical input price of intermediary 1 is dependening on whether products are complements or substitutes. If goods are complements, the critical input price of intermediary 1 is increasing in α_2 and decreasing in λ_2. In this case intermediary 1 profits from a high product quality or productivity of intermediary 2. If goods are substitutes, the critical input price of intermediary 1 is decreasing in α_2 and increasing in λ_2. In this scenario, a high product quality as well as a high productivity of intermediary 2 has a negative effect on intermediary 1's critical input price. Similar effects as above can be observed when considering intermediary 2's critical input price.

When comparing critical input prices of the Stackelberg leader and the Stackelberg follower, we get Eq. (4.11). Intermediary i is the potential monopolist if and only if he is more competitive than intermediary $3-i$ and vice versa. More precisely,

$$\min\left\{\frac{2\alpha_i-\gamma\alpha_{3-i}}{2\lambda_i-\gamma\lambda_{3-i}},\frac{4\alpha_{3-i}-\gamma^2\alpha_{3-i}-2\gamma\alpha_i}{4\lambda_{3-i}-\gamma^2\lambda_{3-i}-2\gamma\lambda_i}\right\}=\begin{cases}\frac{4\alpha_{3-i}-\gamma^2\alpha_{3-i}-2\gamma\alpha_i}{4\lambda_{3-i}-\gamma^2\lambda_{3-i}-2\gamma\lambda_i} & \text{iff } \frac{\alpha_i}{\lambda_i}>\frac{\alpha_{3-i}}{\lambda_{3-i}},\\ \frac{2\alpha_i-\gamma\alpha_{3-i}}{2\lambda_i-\gamma\lambda_{3-i}} & \text{iff } \frac{\alpha_i}{\lambda_i}<\frac{\alpha_{3-i}}{\lambda_{3-i}}\end{cases} \tag{4.11}$$

holds. For the case that $\frac{\alpha_i}{\lambda_i}>\frac{\alpha_{3-i}}{\lambda_{3-i}}$ is satisfied, intermediary i has a competitive advantage towards intermediary $3-i$. This is due to the fact that on the one hand for a high product quality α_i intermediary i faces a higher costumer demand for his product. On the other hand, when being rather productive with low λ_i his costs of procurement low. Eq. (4.11) shows that given intermediary 1 is more competitive than intermediary 2, with a rising input price c intermediary 2 is the first one to drop out of the market. Therefore, if c is sufficiently high, intermediary 2 will leave the market and intermediary 1 will be a monopolist. The same holds vice versa.

After having considered the duopoly market we now analyze the case of a *monopoly* market, in which either intermediary 1 or intermediary 2 can be the monopolist. In order to determine intermediary i's equilibrium quantity within a monopoly, suppose $\frac{\alpha_i}{\lambda_i}>\frac{\alpha_{3-i}}{\lambda_{3-i}}$ for $i=1,2$. Intermediary i is more competitive than intermediary $3-i$ and thus the potential monopolist. With $q_{3-i}=0$ and maximizing intermediary i's profit function with respect to q_i yields the monopolist's equilibrium quantity of

$$q_i^{SL/SF} = \frac{\alpha_i - \lambda_i c}{2}. \tag{4.12}$$

As stated above, a monopoly market only arises if the input price c is small enough for intermediary i to achieve non-negative profits, but high enough to exclude intermediary $3-i$ from the market.

Suppose $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$ and the input price satisfying $c \geq \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}$. In this setting intermediary 2 is excluded from the market. By using Eq. (4.12), we obtain that given $q_2^{SF} = 0$, if and only if $c < \frac{\alpha_1}{\lambda_1}$, intermediary 1 has an incentive to produce a positive amount with $q_1^{SL} > 0$. Hence, an input price satisfying

$$\frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}, \tag{4.13}$$

results in a monopoly situation with intermediary 1 as the monopolist.

For the opposite case with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$ the Stackelberg follower is more competitive and thus the potential monopolist. An input price such that

$$\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2} \tag{4.14}$$

guarantees a monopoly with intermediary 2 as the monopolist. As $c \geq \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$ holds, the input price is too high for intermediary 1 to have a positive demand for inputs. Given $q_1^{SL} = 0$ and using Eq. (4.12) we obtain that for $c < \frac{\alpha_2}{\lambda_2}$ intermediary 2 always has an incentive to offer products on the market resulting in a monopoly.

In the final case, *no production* occurs if the input price c is too high for intermediaries to achieve non-negative profits. In this scenario neither intermediary 1 nor intermediary 2 produces and therefore $q_1^{SL} = q_2^{SF} = 0$ holds.

In the following we impose some technical assumptions on the product qualities α_i and productivities λ_i for $i = 1, 2$. We demand the equilibrium quantities q_1^{SL} and q_2^{SF} to increase in "weighted" qualities and decrease in the input price c. Note that the assumptions on q_1^{SL} and q_2^{SF} to increase in "weighted" qualities excludes a range of quality-asymmetries that immediately forces one of the intermediary out of the market.

Assumption 4.1. *Given intermediary* 1 *is the Stackelberg leader and intermediary* 2 *the Stackelberg follower, we assume* $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} \in \left[\frac{\gamma}{2}, \frac{4-\gamma^2}{2\gamma}\right]$ *for* $\gamma > 0$.

Given equilibrium quantities q_1^{SL} and q_2^{SF} as well as the customer's inverse demand function are as displayed in Eq. (4.3), we are now able to determine the equilibrium prices of intermediary 1, given by

$$p_1^{SL} = p_1^{SL}(q_1^{SL}, q_2^{SF}) = \begin{cases} \left(\frac{2-\gamma^2}{2}\right) q_1^{SL} + \lambda_1 c & \text{if } c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\right\} \\ \frac{\alpha_1+\lambda_1 c}{2} & \text{if } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1} \\ 0 & \text{otherwise,} \end{cases} \tag{4.15}$$

and of intermediary 2, given by

$$p_2^{SF}(q_1^{SL}, q_2^{SF}) = \begin{cases} q_2^{SF} + \lambda_2 c & \text{if } c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\right\} \\ \frac{\alpha_2+\lambda_2 c}{2} & \text{if } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2} \\ 0 & \text{otherwise.} \end{cases} \tag{4.16}$$

The total equilibrium demand for input products which will be needed within the next section is depending on the intermediaries' equilibrium production quantities q_1^{SL} and q_2^{SF} as well as their according input productivities λ_i for $i = 1, 2$. For a duopoly and a monopoly with either intermediary 1 or intermediary 2 producing the total demand is given by

$$\begin{aligned} q_I^S(c) &= \lambda_1 q_1^{SL} + \lambda_2 q_2^{SF} \\ &= \begin{cases} \frac{\left(2\lambda_1(2\alpha_1-\gamma\alpha_2)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)}{4(2-\gamma^2)} \\ \quad -\frac{\left(2\lambda_1(2\lambda_1-\gamma\lambda_2)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)c}{4(2-\gamma^2)} & \text{if } c < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\right\} \\ \frac{\lambda_1(\alpha_1-\lambda_1 c)}{2} & \text{if } \frac{4\alpha_2-\alpha_2\gamma^2-2\alpha_1\gamma}{4\lambda_2-\lambda_2\gamma^2-2\lambda_1\gamma} \leq c < \frac{\alpha_1}{\lambda_1} \\ \frac{\lambda_2(\alpha_2-\lambda_2 c)}{2} & \text{if } \frac{2\alpha_1-\alpha_2\gamma}{2\lambda_1-\lambda_2\gamma} \leq c < \frac{\alpha_2}{\lambda_2} \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{4.17}$$

4.2.3 The input supplier

In the following we determine the supplier's equilibrium input price decision and derive his according profits. Within our analysis, we distinguish between the cases in which the supplier serves a duopoly, a monopoly or no production occurs. In general, the monopolistic input supplier selects the input price that maximizes his profit $\pi_I(c) = q_I^S(c)c$, where his production costs are normalized to zero.

First, consider a *duopoly* market. When using the input supplier's profit function $\pi_I(c)$ as well as the total equilibrium input demand of Eq. (4.17) for a duopoly, we get the following profit maximizing input price:

$$c^* = \frac{2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_2 (4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1)}{2(2\lambda_1 (2\lambda_1 - \gamma\lambda_2) + \lambda_2 (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1))}. \tag{4.18}$$

The input price c^* is a unique maximizer as $\pi_I(c)$ is quadratic in c (see Eq. (4.17)) and Assumption 4.1 ensures concavity. Note that c^* is the optimal input price for a duopoly market if and only if $c^* < \min\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\}$ is satisfied.

In the following we derive conditions such that c^* suffices the above inequality. The analysis is done for the scenarios in which intermediary 1 and in which intermediary 2 is more competitive.

First suppose intermediary 1 to be more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$. According to Eq. (4.11), in such a setting we obtain a duopoly market if $c^* < \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$ is satisfied. The input price c^* is indeed the maximizer of a duopoly market if and only if intermediary 2's product quality α_2 is sufficiently high compared to the weighted product quality α_1 of intermediary 1. More precisely, we need

$$\alpha_2 > \alpha_1 \frac{2((4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)(2\lambda_1 + \gamma\lambda_2) + 4\gamma\lambda_1(2\lambda_1 - \gamma\lambda_2))}{(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1) + 4\lambda_1(4-\gamma^2)(2\lambda_1 - \gamma\lambda_2)}. \tag{4.19}$$

Thus, is Ineq. (4.19) satisfied we get an *interior solution* with the optimal input price $c^S = c^*$. Given α_2 is too small and Ineq. (4.19) is not satisfied we face a *corner solution* with the optimal input price of $c^S = \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$. This input price guarantees that both intermediaries produce in equilibrium. It is indeed the maximizer, as the supplier's profit function $\pi_I(c)$ is concave in c and monotonically increasing in the interval $[0, c^*]$ with $\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \in [0, c^*]$. Hence, the input supplier's optimal input price is given by

$$c^S = \min\left\{c^*, \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}\right\}. \tag{4.20}$$

Now, suppose the opposite case in which intermediary 2 is more competitive than intermediary 1 with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$. According to Eq. (4.11), for c^* being the maximizing input price within a duopoly $c^* < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$ must be satisfied. This is true if and only if the product quality α_1 of intermediary 1 is sufficiently large compared to the weighted product quality of intermediary 2. More precisely, we need

$$\alpha_1 > \alpha_2 \frac{(2\lambda_1 - \gamma\lambda_2)(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1) + 2\gamma\lambda_2(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)}{2\left((2\lambda_1 - \gamma\lambda_2)(2\lambda_1 + \gamma\lambda_2) + 2\lambda_2(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\right)}. \tag{4.21}$$

Analogous to the case above, is Ineq. (4.21) satisfied we obtain an interior solution with $c^S = c^*$. Is Ineq. (4.21) not satisfied we get a corner solution with $c^S = \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$ guaranteeing that both intermediaries produce in equilibrium. The input price of $c^S = \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$ is indeed the maximizer, as $\pi_I(c)$ is monotonically increasing in the interval $[0, c^*]$ with $\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \in [0, c^*]$. Thus, in a duopoly intermediate goods market the input supplier optimally chooses an input price of

$$c^S = \min\left\{c^*, \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}\right\}. \tag{4.22}$$

Summarized, within a *duopoly* market the input supplier's optimal input prices are

$$c^S = \begin{cases} \frac{2\lambda_1(2\alpha_1 - \gamma\alpha_2) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)}{2(2\lambda_1(2\lambda_1 - \gamma\lambda_2) + \lambda_2(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1))} & \text{if } c^* < \min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}\right\} \\ \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} & \text{if } c^* > \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} \\ & \text{and } \frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2} \\ \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} & \text{if } c^* > \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \\ & \text{and } \frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}. \end{cases}$$

Given the above optimal input prices, the supplier's profits when serving a *duopoly* are given by

$$\pi_I\left(c^S\right) = \begin{cases} \frac{\lambda_1(2\alpha_1-\gamma\alpha_2)c^S}{4(2-\gamma^2)} & \text{if } c^* < \min\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\right\} \\ & \text{with } c^S = c^* \\ \frac{2\lambda_1(\alpha_1\lambda_2-\alpha_2\lambda_1)c^S}{(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)} & \text{if } c^* > \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \\ & \text{and } \frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2} \text{ with } c^S = \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}, \\ \frac{\lambda_2(\alpha_2\lambda_1-\alpha_1\lambda_2)c^S}{(2\lambda_1-\gamma\lambda_2)} & \text{if } c^* > \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \\ & \text{and } \frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2} \text{ with } c^S = \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}. \end{cases} \tag{4.23}$$

After having analyzed the supplier's optimal input price choices for a duopoly market, we now consider a *monopolistic* intermediate goods market. In a first case we assume the more competitive intermediary 1 to be the monopolist, in a second case we consider intermediary 2 to be the more competitive monopolist.

Assume intermediary 1 to be more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$. The input supplier's profit maximizing input price with intermediary 1 as the monopolist is given by

$$c^* = \frac{\alpha_1}{2\lambda_1}. \tag{4.24}$$

For c^* to create a monopoly market it must on the one hand be sufficiently low for intermediary 1 to achieve non-negative profits. This is true for $c^* < \frac{\alpha_1}{\lambda_1}$. On the other hand, c^* must be high enough to exclude intermediary 2 from the market. This is satisfied if $c^* > \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$ is satisfied. The input price c^* of Eq. (4.24) satisfies the above inequalities if and only if the product quality α_2 is sufficiently small. More precisely

$$\alpha_2 < \alpha_1 \frac{4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1}{2\lambda_1\left(4-\gamma^2\right)} \tag{4.25}$$

must hold. Hence, is Ineq. (4.25) satisfied we obtain an *interior solution* with $c^S = c^*$. Is α_2 too large, we get a *corner solution* with the input price of $c^S = \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$. The corner solution indeed maximizes the input supplier's profit, as $\pi_I(c)$ is concave in c. Hence, the optimal input price for a monopoly with intermediary 1 being the monopolist is given by

$$c^S = \max\left\{\frac{\alpha_1}{2\lambda_1}, \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}\right\}.$$

The resulting input supplier's equilibrium profits for the cases interior and corner solutions are given by

$$\pi_I\left(c^S\right) = \begin{cases} \frac{\alpha_1^2}{8} & \text{if } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1} \\ & \text{and } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} < \frac{\alpha_1}{2\lambda_1} \\ \frac{\lambda_1(\alpha_1\lambda_2-\alpha_2\lambda_1)\left(4-\gamma^2\right)}{2(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)} \cdot c^S & \text{if } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1} \\ & \text{and } \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} > \frac{\alpha_1}{2\lambda_1} \\ & \text{with } c^S = \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}. \end{cases} \tag{4.26}$$

Now suppose intermediary 2 to be more competitive than intermediary 1 with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$. Intermediary 2 will be the potential monopolist in this scenario. We have shown above, that a monopoly market arises, if the input supplier selects an input price such that $\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}$. The input price that maximizes the supplier's profit function for intermediary 2 as the monopolist is given by

$$c^* = \frac{\alpha_2}{2\lambda_2}. \tag{4.27}$$

The input price c^* of Eq. (4.27) satisfies $\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}$ if and only if the product quality α_1 is sufficiently small. More precisely we need

$$\alpha_1 < \alpha_2 \frac{2\lambda_1 + \gamma\lambda_2}{4\lambda_2}. \tag{4.28}$$

Thus, is Ineq. (4.28) satisfied we obtain an *interior solution* with the input price of $c^S = c^*$. Is α_1 too large, we get a *corner solution* with the input price of $\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}$. Note that c^S indeed maximizes the input supplier's profit, as the input supplier's monopoly profit function is concave in c. The input supplier's optimal input price for a monopoly in which intermediary 2 produces is therefore given by

$$c^S = \left\{\frac{\alpha_2}{2\lambda_2}, \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}\right\}. \tag{4.29}$$

The resulting input supplier's equilibrium profits for the cases of interior and corner solutions are given by

$$\pi_I\left(c^S\right) = \begin{cases} \frac{\alpha_2^2}{8} & \text{if } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2} \\ & \text{and } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} < \frac{\alpha_2}{2\lambda_2} \\ \frac{\lambda_2(\alpha_2\lambda_1-\alpha_1\lambda_2)}{(2\lambda_1-\gamma\lambda_2)} \cdot c^S & \text{if } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2} \\ & \text{and } \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} > \frac{\alpha_2}{2\lambda_2} \\ & \text{with } c^S = \frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}. \end{cases} \tag{4.30}$$

Besides duopoly and monopoly, the final case of *no production* occurs, if the input price is too high for both of the intermediaries to produce. In this case $q_1^{SL} = q_2^{SF} = 0$ holds and therefore the total input demand is $q_I^S = 0$ with the input supplier's profit $\pi_I\left(c\right) = 0$.

In the above section we have seen that by choosing the input price accordingly, the supplier is able to influence the type of market, i.e., duopoly, monopoly or no production. While focusing on interior solutions within monopoly and duopoly, the next Proposition discusses the supplier's incentives to exclude one of the intermediaries from the market.

Proposition 4.1. *Suppose an interior solution within monopoly and duopoly market. If intermediaries are sufficiently asymmetric in favor of the more competitive Stackelberg leader, it is optimal for the input supplier to choose an input price c such that he sells his inputs only to the Stackelberg leader excluding the Stackelberg follower from the market.*

More precisely, if

$$\alpha_2 < \alpha_1 \tau_1^{SL} \tag{4.31}$$

with

$$\tau_1^{SL} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)} - 2\left(2\lambda_1-\gamma\lambda_2\right)}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1},$$

then it is optimal for the input supplier to only serve intermediary 1*. The same holds if intermediary* 1 *is the Stackelberg follower and intermediary* 2 *is the Stackelberg leader.*

Note that τ_1^{SL} can be rewritten as

$$\tau_1^{SL} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\frac{\lambda_1}{\lambda_2}\left(2\frac{\lambda_1}{\lambda_2}-\gamma\right)+\left(4-\gamma^2-2\gamma\frac{\lambda_1}{\lambda_2}\right)\right)}-2\left(2\frac{\lambda_1}{\lambda_2}-\gamma\right)}{4-\gamma^2-2\gamma\frac{\lambda_1}{\lambda_2}}$$

and is therefore homogeneous of degree zero in λ_1 and λ_2.

Within Proposition 4.1 we analyze the scenarios for which the input supplier's profit when serving a monopoly is higher than when serving a duopoly, given intermediary 1 is more competitive than intermediary 2. It could be seen that if there are high asymmetries of product qualities with $\alpha_2 < \alpha_1 \tau_1^{SL}$ the input supplier selects the input price $c^S = \frac{\alpha_1}{2\lambda_1}$, excluding intermediary 2 from the market and hence resulting in a monopoly with only intermediary 1 producing. For the opposite case with $\alpha_2 \geq \alpha_1 \tau_1^{SL}$, the input supplier favors to serve a duopoly with the input price of $c^S = \frac{2\lambda_1(2\alpha_1-\gamma\alpha_2)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{2(2\lambda_1(2\lambda_1-\gamma\lambda_2)+\lambda_2(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1))}$.

The next Proposition considers the case in which the Stackelberg follower is more competitive than the Stackelberg leader with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$. It states the impact of the input supplier's optimal input price decision on the type of market for interior monopoly and duopoly solutions. Differently to Proposition 4.1, Proposition 4.2 analyzes the input supplier's incentive to exclude the less competitive Stackelberg leader from the market.

Proposition 4.2. *Suppose an interior solution within monopoly and duopoly market where intermediary* 2 *is the more competitive Stackelberg follower. If intermediaries are sufficiently asymmetric in favor of the more competitive Stackelberg follower, it is optimal for the input supplier to choose an input price c such that he sells his inputs only to intermediary* 2 *excluding intermediary* 1 *from the market.*

More precisely, if

$$\alpha_1 < \alpha_2 \tau_2^{SF} \tag{4.32}$$

with

$$\tau_2^{SF} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}-\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_2\right)},$$

then it is optimal for the input supplier to only serve intermediary 2*. The same holds if intermediary 1 is the more competitive Stackelberg follower and intermediary* 2 *is the Stackelberg leader.*

The multiplier τ_2^{SF} is homogeneous of degree zero in λ_1 and λ_2 and can be rewritten as

$$\tau_2^{SF} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\frac{\lambda_2}{\lambda_1}\right)+\frac{\lambda_2}{\lambda_1}\left(4\frac{\lambda_2}{\lambda_1}-\gamma^2\frac{\lambda_2}{\lambda_1}-2\gamma\right)\right)}-\left(4\frac{\lambda_2}{\lambda_1}-\gamma^2\frac{\lambda_2}{\lambda_1}-2\gamma\right)}{2\left(2-\gamma\frac{\lambda_2}{\lambda_1}\right)}.$$

Within Proposition 4.2 we analyze the scenarios for which the input supplier's profit when serving a monopoly is higher than when serving a duopoly, given intermediary 2 is more competitive than intermediary 1. If product qualities are sufficiently asymmetric with $\alpha_1 < \alpha_2\tau_2^{SF}$ the input supplier selects the input price $c^S = \frac{\alpha_2}{2\lambda_2}$, which excludes intermediary 1 from the market and therefore results in a monopoly with only intermediary 2 producing. Supposing the opposite case with $\alpha_1 \geq \alpha_2\tau_2^{SF}$, the input supplier prefers to serve a duopoly and selects the duopoly input price of $c^S = \frac{2\lambda_1(2\alpha_1-\gamma\alpha_2)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{2(2\lambda_1(2\lambda_1-\gamma\lambda_2)+\lambda_2(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1))}$.

Notice that by choosing his input price c, the input supplier is not able to impact whether intermediary 1 or intermediary 2 is producing in a monopoly. This is solely determined by the intermediaries' competitiveness, which can be increased by investing in product innovation.

4.2.4 Comparison between Stackelberg leader and Stackelberg follower

In this section we compare the equilibrium quantities of a Stackelberg leader and a Stackelberg follower. Moreover, we analyze the input supplier's incentives for driving a Stackelberg follower and a Stackelberg leader out of the market.

In the previous sections we considered intermediary 1 to be the Stackelberg leader and intermediary 2 to be the Stackelberg follower. For the upcoming analysis the equilibrium outcome for the reversed role distribution, i.e., intermediary 1 as the Stackelberg follower and intermediary 2 as the Stackelberg leader, is required. We have a focus on intermediary 1 and compare his equilibrium quantity q_1^{SL} when being in the leading and q_1^{SF} when being in the following position. A similar comparison will be done between τ_1^{SL} and τ_1^{SF}.

For the above analysis, supposing intermediary 1 to be the follower, we derive his equilibrium quantities q_1^{SF} for a duopoly and monopoly market which are analogous to the quantities q_2^{SF} displayed in Eq. (4.5). Furthermore, we determine the multiplier

τ_1^{SF} which is closely related to τ_2^{SF} of Proposition 4.2. Note that a similar analysis can be carried out when considering intermediary 2.

As the customer's utility function is symmetric in the product quantities of intermediary 1 and intermediary 2, intermediary 1's equilibrium quantity when being the Stackelberg follower is similar to the quantity in which intermediary 2 is the follower, as displayed in Eq. (4.6). Thus, it is given by

$$q_1^{SF} = \begin{cases} \frac{\left(4\alpha_1-\gamma^2\alpha_1-2\gamma\alpha_2\right)-\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_2\right)c}{2(4-2\gamma^2)} & \text{if } c < \min\{\frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1}, \frac{4\alpha_1-\gamma^2\alpha_1-2\gamma\alpha_2}{4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_2}\} \\ \frac{\alpha_1-\lambda_1 c}{2} & \text{if } \frac{2\alpha_2-\gamma\alpha_1}{2\lambda_2-\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1} \\ 0 & \text{otherwise.} \end{cases} \tag{4.33}$$

Analogous to the previous section, we need to impose technical assumptions for intermediary 1's equilibrium quantities q_1^{SF} and for intermediary 2's equilibrium quantities q_2^{SL} to increase in "weighted" qualities and decrease in the input price c. Therefore we obtain Assumption 4.2.

Assumption 4.2. *Given intermediary* 1 *is the Stackelberg follower and intermediary* 2 *the Stackelberg leader, we assume* $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} \in \left[\frac{2\gamma}{4-\gamma^2}, \frac{2}{\gamma}\right]$ *for* $\gamma > 0$.

When comparing intermediary 1's equilibrium quantities as a Stackelberg leader with his equilibrium quantities as a Stackelberg follower, we get Proposition 4.3.

Proposition 4.3.

(i) Suppose intermediary i to be more competitive than intermediary $3-i$ for $i = \{1,2\}$. Within a duopoly, intermediary i's equilibrium quantities are always higher as a Stackelberg leader than as a Stackelberg follower: $q_i^{SL} > q_i^{SF}$.

(ii) Suppose intermediary $3-i$ to be more competitive than intermediary i for $i = \{1,2\}$. Within a duopoly, if goods are substitutes, intermediary i's equilibrium quantities are always higher as a Stackelberg leader than as a Stackelberg follower: $q_i^{SL} > q_i^{SF}$.

(iii) Suppose intermediary $3-i$ to be more competitive than intermediary i for $i = \{1,2\}$. Within a duopoly, if goods are complements, there may exist asymmetries such that intermediary i's equilibrium quantities are higher as a Stackelberg follower than as a Stackelberg leader with $q_i^{SL} < q_i^{SF}$.

Part (*i*) of Proposition 4.3 shows that in a duopoly intermediary i's equilibrium quantities when being the more competitive Stackelberg leader are higher than his equilibrium quantities when being the more competitive Stackelberg follower. The outcome holds for substitutes, complements as well as for mutually independent products and confirms the classic results for homogenous products of Von Stackelberg (1934). Part (*ii*) and (*iii*) of Proposition 4.3 consider the opposite case in which for both positions intermediary i is less competitive. Thus, intermediary $3-i$ will be the potential monopolist. For (*ii*), given goods are substitutes with $\gamma > 0$ the same result as in (*i*) holds. Thus, within a duopoly the equilibrium quantities of intermediary i as the less competitive Stackelberg leader are higher than his equilibrium quantities as the less competitive Stackelberg follower. According to (*iii*), if goods are complements, this must not necessarily be true. For a duopoly, if asymmetries are chosen accordingly, the equilibrium quantity of the Stackelberg follower is higher than the equilibrium quantity of the Stackelberg leader. An explanation for this outcome might be that when having complements, competition between intermediaries is rather low. Hence, the first-mover advantage of the Stackelberg leader which is indeed present for substitutes vanishes.

Within the next section we compare the input supplier's incentives to exclude a less competitive Stackelberg follower with his incentives to exclude a less competitive Stackelberg leader from the market. For the upcoming analysis, we assume intermediary 1 to be more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$. Therefore, if the input supplier has an incentive to exclude an intermediary, it will always be intermediary 2 leaving the market.

Analogous to Ineq. (4.28), the condition ensuring an interior monopoly solution is given by

$$\alpha_2 < \alpha_1 \frac{2\lambda_2 + \gamma\lambda_1}{4\lambda_1}, \tag{4.34}$$

whereas related to Ineq. (4.21) the condition yielding an interior duopoly solution is denoted by

$$\alpha_2 > \alpha_1 \frac{(2\lambda_2 - \gamma\lambda_1)(4\lambda_1 - \gamma^2\lambda_1 + 2\gamma\lambda_2) + 2\gamma\lambda_1(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2)}{2((2\lambda_2 - \gamma\lambda_1)(2\lambda_2 + \gamma\lambda_1) + 2\lambda_1(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2))}. \tag{4.35}$$

Related to Proposition 4.2, supposing an interior solution within monopoly and duopoly market, the input supplier prefers to only serve intermediary 1, the Stackelberg follower, if his product quality α_1 is sufficiently high compared to intermediary 2's product quality α_2. More precisely, if

$$\alpha_2 < \alpha_1 \tau_1^{SF} \tag{4.36}$$

with

$$\tau_1^{SF} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_2\left(2\lambda_2-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_2\right)\right)}-\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_2\right)}{2\left(2\lambda_2-\gamma\lambda_1\right)} \tag{4.37}$$

holds, the input supplier has an incentive to exclude intermediary 2, the Stackelberg leader, from the market.

In the following analysis we compare τ_1^{SL} and τ_1^{SF} for a given range of asymmetries and degree of horizontal product differentiation. We get the following Proposition 4.4.

Proposition 4.4.

(i) Given intermediaries have the same productivity with $\lambda_1 = \lambda_2$ and products are mutually independent with $\gamma = 0$, the input supplier's incentive to exclude an intermediary from the market is the same for the Stackelberg leader and for the Stackelberg follower. More precisely:

$$\tau_1^{SL} = \tau_1^{SF}$$

holds.

(ii) Given intermediaries have the same productivity with $\lambda_1 = \lambda_2$ the input supplier's incentive to exclude the less competitive Stackelberg leader from the market is higher than his incentive to exclude the less competitive Stackelberg follower. More precisely:

$$\tau_1^{SL} \leq \tau_1^{SF}$$

holds.

Proposition 4.4 makes clear, that for given characteristics of $(\gamma, \alpha_1, \alpha_2, \lambda_1, \lambda_2)$ the input supplier rather prefers to exclude a less competitive Stackelberg leader over

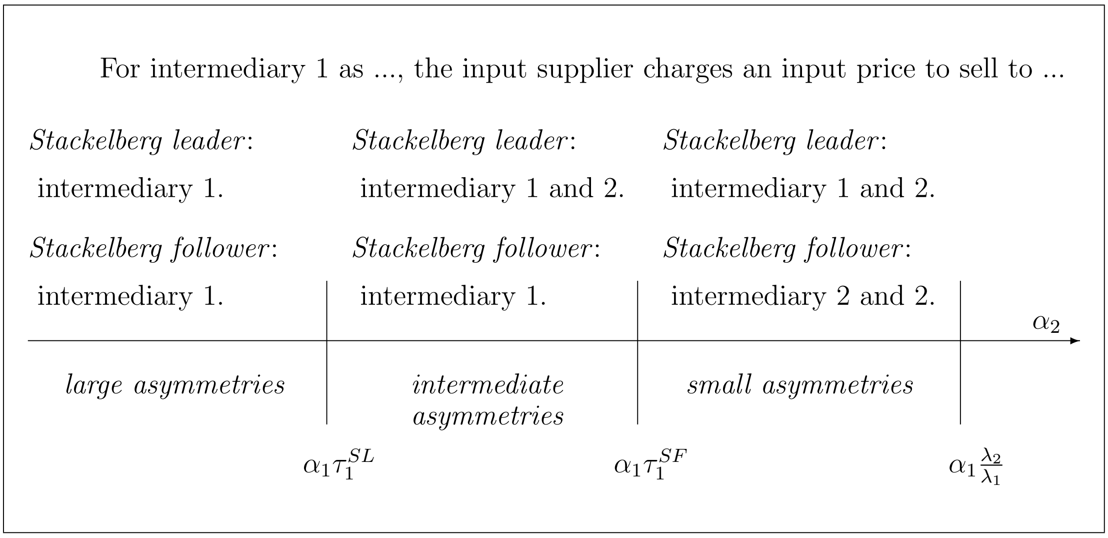

Figure 4.2: The input market for $\alpha_2 \leq \alpha_1\frac{\lambda_2}{\lambda_1}$

excluding a less competitive Stackelberg follower from the market. The described scenario is illustrated in Fig. 4.2. When considering the area of *large asymmetries* in which intermediary 2's product quality α_2 is smaller than $\alpha_1\tau_1^{SL}$ and smaller than $\alpha_1\tau_1^{SF}$, he will be excluded from the market, no matter if he is the Stackelberg leader or follower. For the most interesting case of *intermediate asymmetries* with $\alpha_1\tau_1^{SL} < \alpha_2 < \alpha_1\tau_1^{SF}$, intermediary 2 will produce in equilibrium as a Stackelberg follower, but will be driven out of the market as a Stackelberg leader. For the case of *small asymmetries*, the input supplier selects his input price c sufficiently small such that both intermediaries produce in equilibrium, independently of whether intermediary 2 is the Stackelberg leader or follower.

The input supplier's behavior when facing intermediate asymmetries may be explained by his equilibrium profits that he is able to achieve with intermediary 1 in the leading and in the following position. A comparison of input prices and resulting profits for intermediary 1 as a Stackelberg leader and as a Stackelberg follower results in Proposition 4.5.

Proposition 4.5. *Suppose intermediary* 1 *is more competitive than intermediary* 2*, with equal productivities* $\lambda_1 = \lambda_2$ *as well as the conditions for interior solutions being satisfied.*

(i) The input supplier's equilibrium duopoly input price $c^S = c^*$ *is higher when intermediary* 1 *is the Stackelberg leader than the duopoly equilibrium input price when intermediary* 1 *is the Stackelberg follower.*

(ii) The input supplier's equilibrium duopoly profit is higher when intermediary 1 *is the Stackelberg leader than his duopoly profit when intermediary* 1 *is the Stackelberg follower.*

As the input supplier's equilibrium duopoly profit is higher if intermediary 1 is in the leading and intermediary 2 in the following position he is willing to accept higher asymmetries between product qualities and select an input price c such that intermediary 2 may stay in the market.

Something similar as described for the supplier's profits can be observed for the interior solution equilibrium input price. The input price c^* is higher, if the more competitive intermediary 1 is the Stackelberg leader compared to the case in which he is the more competitive Stackelberg follower.

Within the upcoming section, we will give a first analysis of the intermediaries' incentives to invest in product quality. To do so, we need to verify the intermediaries' profits.

Suppose intermediary 1 to be the more competitive Stackelberg leader. With the equilibrium quantities q_1^{SL} and q_2^{SF}, the input supplier's optimal interior solution-input price c^S as well as the results of Proposition 4.1 and Proposition 4.2, intermediaries' equilibrium profits are given by

$$\pi_1^{SL}\left(q_1, q_2, c^S, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\left((2\alpha_1-\gamma\alpha_2)-(2\lambda_1-\gamma\lambda_2)c^S\right)^2}{8(2-\gamma^2)} & \text{for } \alpha_1\tau_1^{SL} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1}, \\ \frac{\alpha_1^2}{16} & \text{for } \alpha_2 < \alpha_1\tau_1^{SL}, \end{cases} \tag{4.38}$$

$$\pi_2^{SF}\left(q_1, q_2, c^S, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\left(\left(4\alpha_2-2\gamma\alpha_1-\gamma^2\alpha_1\right)-\left(4\lambda_2-2\gamma\lambda_1-\gamma^2\lambda_2\right)c^S\right)^2}{16(2-\gamma^2)^2} & \text{for } \alpha_1\tau_1^{SL} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1}, \\ 0 & \text{for } \alpha_2 < \alpha_1\tau_1^{SL}, \end{cases} \tag{4.39}$$

with

$$c^S = \frac{2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}.$$

Note, that the condition $\alpha_1\tau_1^{SL} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1}$ implies a duopoly market whereas $\alpha_2 < \alpha_1\tau_1^{SL}$ results in a monopoly market in which intermediary 1 is the monopolist.

For the opposite case, suppose intermediary 1 to be the more competitive Stackelberg follower. With q_1^{SF} and q_2^{SL}, the input supplier's optimal interior solution-input price c^S as well as the results of Propositions 4.1 and Proposition 4.2, intermediaries' equilibrium profits are denoted by

$$\pi_1^{SF}\left(q_1, q_2, c^S, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\left(\left(4\alpha_1 - 2\gamma\alpha_2 - \gamma^2\alpha_2\right) - \left(4\lambda_1 - 2\gamma\lambda_2 - \gamma^2\lambda_1\right)c^S\right)^2}{16(2-\gamma^2)^2} & \text{for } \alpha_1\tau_1^{SF} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1}, \\ \frac{\alpha_1^2}{16} & \text{for } \alpha_2 < \alpha_1\tau_1^{SF}, \end{cases} \tag{4.40}$$

$$\pi_2^{SL}\left(q_1, q_2, c^S, \alpha_1, \alpha_2\right) = \begin{cases} \frac{\left((2\alpha_2 - \gamma\alpha_1) - (2\lambda_2 - \gamma\lambda_1)c^S\right)^2}{8(2-\gamma^2)} & \text{for } \alpha_1\tau_1^{SF} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1} \\ 0 & \text{for } \alpha_2 < \alpha_1\tau_1^{SF}, \end{cases} \tag{4.41}$$

with

$$c^S = \frac{2\lambda_2\left(2\alpha_2 - \gamma\alpha_1\right) + \lambda_1\left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right)}{2\left(2\lambda_2\left(2\lambda_2 - \gamma\lambda_1\right) + \lambda_1\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2\right)\right)}.$$

Similar to the previous case, $\alpha_1\tau_1^{SF} < \alpha_2 < \alpha_1\frac{\lambda_2}{\lambda_1}$ results in a duopoly market and $\alpha_2 < \alpha_1\tau_1^{SF}$ implies a monopoly market in which intermediary 1 is the monopolist.

4.3 Product Innovation

4.3.1 Basic setup

Within this section we assume intermediaries being able to simultaneously choose a level of investment to foster product quality. The product quality is given by $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$, with $0 < \underline{\alpha}_i < \overline{\alpha}_i$ for $i = 1, 2$. If intermediary i decides not to invest, his product quality is assumed to be $\underline{\alpha}_i$, whereas a full investment results in $\overline{\alpha}_i$. We assume the costs of investment $k > 0$ to be identical for both intermediaries.

An investment in product quality may have two positive effects for the intermediary investing. First, it increases the own product's demand on the sales side and depending on the costs of investment may result in higher profits. Second, there is also a strategic component when deciding about investing in product innovation. Suppose one intermediary is excluded from the market. An investment may reduce asymmetries between the own and the other intermediary's product quality. If the resulting asymmetries are sufficiently small, the input supplier has an incentive to reduce the input price and allow for a duopoly in the market. In another scenario in which a duopoly is present, an intermediary may strategically increase his product quality such that asymmetries arise. If those asymmetries are sufficiently large, the supplier increases his input price and thus excludes the less competitive intermediary from the market. Although suffering a higher input price, the fact of being the monopolist may result in higher profits for the more competitive intermediary.

When analyzing the intermediaries' profit functions $\pi_i^{SL}\left(q_i, q_{3-i}, c^S, \alpha_i, \alpha_{3-i}\right)$ and $\pi_{3-i}^{SF}\left(q_i, q_{3-i}, c^S, \alpha_i, \alpha_{3-i}\right)$ with respect to product quality α_i we obtain the following Lemma 4.1 as well as Proposition 4.6.

Lemma 4.1. *The Stackelberg leader's profit $\pi_i^{SL}\left(q_i, q_{3-i}, c^S, \alpha_i, \alpha_{3-i}\right)$ and the Stackelberg follower's profit $\pi_{3-i}^{SF}\left(q_i, q_{3-i}, c^S, \alpha_i, \alpha_{3-i}\right)$ (for $i = 1, 2$) are piecewise strictly convex in its own qualities.*

Due to the result of Lemma 4.1 we obtain the Proposition 4.6 which is similar to the result established in Chapter 3.

Proposition 4.6. *Nash equilibrium strategies of the investment game with linear costs are in $\{\underline{\alpha}_i, \overline{\alpha}_i\} \subset [\underline{\alpha}_i, \overline{\alpha}_i]$ for $i = 1, 2$.*

Proposition 4.6 shows that when determining Nash equilibrium investment levels with linear costs of investment, attention can be restricted to the minimum and maximum quality levels $\underline{\alpha}_i$ and $\overline{\alpha}_i$. Due to piecewise strict convexity of the intermediaries' profit function in own qualities, there exists a profit-minimizing product quality. Therefore the product quality level that delivers an intermediary's maximal profit will be obtained at one of the end points of the interval $[\underline{\alpha}_i, \overline{\alpha}_i]$. This is true for the case in which the profit minimizing product quality is within as well as outside the interval $[\underline{\alpha}_i, \overline{\alpha}_i]$. Thus, given a certain minimum quality level of $\underline{\alpha}_i$ must be provided for instance due to legal requirements we are able to simplify the intermediaries' decision of either investing or not investing.

4.3.2 Outlook

In the previous sections we have seen that given intermediary 1 is more competitive than intermediary 2, it is indeed crucial whether intermediary 1 is the Stackelberg leader or Stackelberg follower. Proposition 4.4 yielded that there exist product qualities $\alpha = (\alpha_1, \alpha_2)$ for which the input supplier drives a Stackelberg leader out of the market, but prefers to accept a Stackelberg follower, ceteris paribus. Moreover, Proposition 4.5 has established that for equal productivities the input supplier's duopoly profit is higher if the more competitive intermediary is in the leading position.

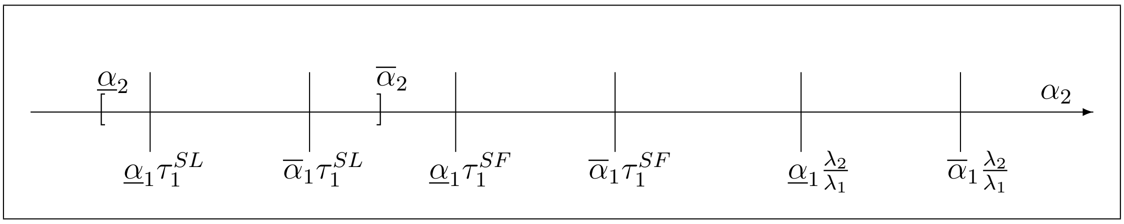

Figure 4.3: Example for investments in product quality

Consider the example displayed in Figure 4.3 in which intermediary 1 is always more competitive than intermediary 2. This is true even if intermediary 2 has invested with $\overline{\alpha}_2$ and intermediary 1 has not invested with $\underline{\alpha}_1$. Therefore, $\overline{\alpha}_2 < \underline{\alpha}_1\frac{\lambda_2}{\lambda_1}$ or equivalently $\frac{\underline{\alpha}_1}{\lambda_1} > \frac{\overline{\alpha}_2}{\lambda_2}$ is satisfied and intermediary 2 never has the chance to be more competitive than intermediary 1. This setting implies that when having a monopoly, intermediary 1 will be the monopolist.

Suppose intermediary 2 does not invest and therefore offers a product quality of $\underline{\alpha}_2$. As displayed in Figure 4.3 intermediary 2's product quality with $\underline{\alpha}_2 < \underline{\alpha}_1\tau_1^{SL} < \underline{\alpha}_1\tau_1^{SF}$ is too small such that in equilibrium the input supplier selects an input price which drives him out of the market. Note that intermediary 2 will be excluded in both cases, i.e., when being the leader and when being the follower. Now assume intermediary 2 decides to invest in product quality which results in $\overline{\alpha}_2$. Figure 4.3 shows that given intermediary 1 is the Stackelberg leader, the input supplier has no incentive to drive intermediary 2 out of the market which results in a duopoly. This is true as $\overline{\alpha}_2 > \overline{\alpha}_1\tau_1^{SL}$ holds. Even if intermediary 1 also invests in product quality with $\overline{\alpha}_1$, the asymmetries between $\overline{\alpha}_1$ and $\overline{\alpha}_2$ are sufficiently low such that the input supplier prefers to serve a duopoly. However, is intermediary 1 in the position of the follower and intermediary 2 in the position of the leader, we observe $\overline{\alpha}_2 < \underline{\alpha}_1\tau_1^{SF}$. Hence, the input supplier selects an input price c such that intermediary 2 has no incentive to produce in equilibrium and we have a monopoly market. This result is independent on whether intermediary 1 decides to invest or not to invest.

For the given example, when determining the Nash equilibrium investment decision, the cost of investment k also needs to be considered. We have seen that when being the Stackelberg follower, the investment of intermediary 2 and the according reduction of asymmetries in product quality led to an input price reduction of the supplier. However, whether an investment has a positive or negative effect on intermediary 2's profit is always dependent on the costs of investment. In case that the costs of investment are too high such that an investment results in a loss, investing will never be a Nash equilibrium strategy.

A simplified example which describes the strategic component of investing in product quality can be given as follows: Within a monopoly market there is only one highly competitive firm procuring his inputs from a monopolistic input supplier and offering his products to his customers. Besides this, there exists a firm, let's say an innovator thinking about entering the market. Without an investment in product quality the firm is not able to overcome market entry barriers. If and only if the potential entrant firm is able to decrease asymmetries between product qualities of the incumbent firm and itself, it will be able to enter the market and, under certain circumstances, make positive profits. By investing itself, the incumbent firm tries to prevent the other firm to enter the market defending its positions as a monopolist. Are the costs of investment higher for the incumbent than for the entrant firm, it can be expected that in long-term, a duopoly or even a monopoly with the entering innovator will arise.

4.4 Conclusion

We studied a differentiated duopoly in which two intermediaries compete for the customer's demand on the sales side and for input goods on the procurement side. In a first stage of competition we determined the equilibrium outcome of intermediaries when choosing production quantities sequentially. We were interested in decisions taken by the first moving Stackelberg leader and by the second moving Stackelberg follower, whereas positions of intermediaries were exogenously given. Besides this we focused on optimal input prices selected by a strategic input supplier. In a second stage that had a focus on innovation we gave a first investigation on the intermediaries' incentives to invest in product quality. Hereby, our analysis was primarily discussed by means of a simplifying example.

Similar to Chapter 3, we have seen that the presence of a strategic input supplier may lead to an exclusion of one or both intermediaries resulting in a monopoly intermediate goods market or no production. It turned out that given intermediaries are sufficiently asymmetric, the input supplier has an incentive to exclude the less competitive player from the market. Furthermore, we derived that there exists asymmetries between intermediaries such that the input supplier is willing to exclude a less competitive Stackelberg leader whereas within the same environment a less competitive Stackelberg follower is accepted in the market (Proposition 4.4).

When comparing the equilibrium output quantities of intermediaries, we established that given the Stackelberg leader is more competitive than the Stackelberg follower, he always delivers higher production quantities. However, if the Stackelberg follower is more competitive than his opponent, the above is only true for substitutes. If goods are complements, there may arise scenarios in which the more competitive Stackelberg follower delivers higher output quantities (Proposition 4.3) than his competitor.

When considering the input supplier's equilibrium price choices, it turned out that if the Stackelberg leader is the more competitive intermediary, the input supplier selects higher equilibrium prices and achieves higher profits than in the opposite case (Proposition 4.5).

When analyzing intermediaries' incentives to invest in product innovation, we found that given costs of investment are linear, attention can be restricted to the investments resulting in a minimum and maximum level of product quality (Proposition 4.6). By giving an example we have seen that an intermediary's investment in innovation does not just foster the customer's demand, but also has a strategic component as asymmetries may be established or increased. Furthermore, we verified the impact of the role of an intermediary on the incentives to invest in product quality.

Beyond the above analysis, future research should be focusing on the investigation of intermediaries' incentives to invest in product innovation. For such an analysis, we have a simultaneous two-player non-cooperative investment game in mind as it was already discussed in Chapter 3. Moreover, within our current work we made a rather restrictive assumption allowing the input supplier only to charge a uniform price for both intermediaries. In order to further corroborate the input supplier's market power and strategic behavior when selecting prices, a further approach needs to allow for price discrimination in which the robustness of recent results is checked.

Besides this, literature has shown that beyond Von Stackelberg (1934) who considers a sequential game in which quantities are selected by duopolists, it can also be thought of a setting in which players set prices as in Gal-Or (1985), for instance. Going in this direction and determining equilibrium outcomes for price competition, a comparison between the according results can be undertaken.

In our current work, positions of players were exogenously given. A consequent next step goes in the direction of an intermediate goods market in which the role of a player, i.e., whether to move first or second, is endogenously chosen, as for instance in Hamilton and Slutsky (1990). Due to the fact that the approach of Chapter 3 is quite similar to our setting, it is obvious that a further consideration will compare the outcomes of the game in which decisions are take simultaneously with the results when decisions are taken sequentially.

Acknowledgments

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4.5 Appendix A: Proofs

4.5.1 Proof of Proposition 4.1

Proof of Proposition 4.1. Suppose intermediary 1 is more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$. When determining τ_1^{SL}, we are aiming for conditions on asymmetries between α_1 and α_2 such that the input supplier's profit when serving intermediary 1 (monopoly) is at least as high as his profit when serving both intermediaries (duopoly). Therefore we obtain

$$\frac{\left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)^2}{16\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)} \leq \frac{\alpha_1^2}{8}$$

$$\Leftrightarrow \quad \alpha_2 \leq \alpha_1 \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_2\right)}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$$

$$= \quad \alpha_2 \leq \alpha_1\tau_1.$$

□

4.5.2 Proof of Proposition 4.2

Proof of Proposition 4.2. Suppose intermediary 2 is more competitive than intermediary 1 with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$. When determining τ_2^{SF} we are aiming for conditions on asymmetries between α_1 and α_2 such that the input supplier's monopoly profit when serving intermediary 2 (monopoly) is at least as high as his profit when serving both intermediaries (duopoly). Therefore we obtain

$$\begin{aligned} & \frac{\left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)^2}{16\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)} \leq \frac{\alpha_2^2}{8} \\ \Leftrightarrow \quad & \alpha_1 \leq \alpha_2 \Bigg(\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}}{2\left(2\lambda_1-\gamma\lambda_2\right)} \\ & \qquad - \frac{\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_2\right)} \Bigg) \\ = \quad & \alpha_1 \leq \alpha_2\tau_2. \end{aligned}$$

□

4.5.3 Proof of Proposition 4.3

Proof of Proposition 4.3. From Eq. (4.5) we know that intermediary i's equilibrium quantity as a Stackelberg leader within a duopoly is given by

$$q_i^{SL} = \frac{\left(2\alpha_i-\gamma_{3-i}\right)-\left(2\lambda_i-\gamma\lambda_{3-i}\right)c}{2\left(2-\gamma^2\right)}$$

if

$$c < \min\left\{\frac{2\alpha_i-\gamma\alpha_{3-i}}{2\lambda_i-\gamma\lambda_{3-i}}, \frac{4\alpha_{3-i}-\gamma^2\alpha_{3-i}-2\gamma\alpha_i}{4\lambda_{3-i}-\gamma^2\lambda_{3-i}-2\gamma\lambda_i}\right\}. \tag{4.42}$$

From Eq. (4.33) we know that intermediary i's equilibrium quantity as a Stackelberg follower is given by

$$q_i^{SF} = \frac{\left(4\alpha_i-\gamma^2\alpha_i-2\gamma\alpha_{3-i}\right)-\left(4\lambda_i-\gamma^2\lambda_i-2\gamma\lambda_{3-i}\right)c}{4\left(2-\gamma^2\right)}$$

if

$$c < \min \left\{ \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}, \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} \right\}. \tag{4.43}$$

The equilibrium quantity of intermediary i as a Stackelberg leader q_i^{SL} is larger than his equilibrium quantity as a Stackelberg follower q_i^{SF} if and only if $\frac{\alpha_i}{\lambda_i} > c$. This can be seen as follows:

$$\begin{aligned} & \frac{(2\alpha_i - \gamma\alpha_{3-i}) - (2\lambda_i - \gamma\lambda_{3-i})\, c}{4 - 2\gamma^2} \\ & > \frac{(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}) - (4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i})\, c}{2\,(4 - 2\gamma^2)} \\ \Leftrightarrow \quad & \frac{\alpha_i}{\lambda_i} > c. \end{aligned}$$

First, suppose that intermediary i is more competitive than intermediary $3 - i$ and therefore $\frac{\alpha_i}{\lambda_i} > \frac{\alpha_{3-i}}{\lambda_{3-i}}$. Thus, independently of whether he is the Stackelberg leader or the Stackelberg follower, intermediary i's critical input price is larger than the critical input price of intermediary $3 - i$ and he will always be the potential monopolist. Therefore, if intermediary i is in the leading position, we have

$$\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} > \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i},$$

whereas if intermediary i takes the role of the follower we get

$$\frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}.$$

The conditions on the input price c that ensure a duopoly market are therefore given by

$$c < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i}$$

for the first and

$$c < \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}$$

for the second case. It can easily be seen that the following always holds:

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i} < \frac{\alpha_i}{\lambda_i} \end{aligned}$$

as well as

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}. \end{aligned}$$

Therefore the conditions which ensure a duopoly market also imply the condition for $q_i^{SL} > q_i^{SF}$. The input price c satisfies

$$c < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}$$

for the case in which intermediary i is in the leading and

$$c < \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}$$

for the case intermediary i is in the following position. Hence, if the conditions for a duopoly market with a sufficiently small c are satisfied, $q_i^{SL} > q_i^{SF}$ holds.

Now suppose intermediary $3-i$ to be more competitive than intermediary i and therefore $\frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i}$. Independently of whether he is the leader or the follower, intermediary i's critical input price is smaller than the critical input price of intermediary $3-i$. Thus intermediary $3-i$ is the potential monopolist in both cases. Supposing intermediary i is the Stackelberg leader, we have

$$\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i},$$

given he is the Stackelberg follower we get

$$\frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} > \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}.$$

The conditions that ensure a sufficiently low input price c for a duopoly therefore demand

$$c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}$$

for the first and

$$c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}$$

for the second case. It can be seen that for the given scenario, if goods are substitutes with $\gamma > 0$ the following inequalities are true:

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i} \end{aligned}$$

and

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}. \end{aligned}$$

Therefore, given the conditions that guarantee a duopoly market are satisfied, $q_i^{SL} > q_i^{SF}$ also holds. This is true as

$$c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}$$

and

$$c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}$$

are satisfied. However, when considering complements with $\gamma < 0$, the duopoly condition is not necessarily sufficient for $c < \frac{\alpha_i}{\lambda_i}$. It is obvious that

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \end{aligned}$$

and

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i}. \end{aligned}$$

holds. Given intermediary i is the Stackelberg leader with an input price satisfying

$$\frac{\alpha_i}{\lambda_i} < c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}$$

as well as intermediary i as the Stackelberg follower with an input price satisfying

$$\frac{\alpha_i}{\lambda_i} < c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}},$$

the equilibrium quantity of the follower will be higher than the equilibrium quantity of the leader with $q_i^{SF} > q_i^{SL}$. Thus, there exist input prices for which the equilibrium quantities of a more competitive Stackelberg leader must not necessarily be higher than the equilibrium quantities of a more competitive Stackelberg follower. □

4.5.4 Proof of Proposition 4.4 (i)

Proof of Proposition 4.4. Suppose intermediaries' productivities are identical with $\lambda_1 = \lambda_2$. When comparing τ_1^{SL} and τ_1^{SF} we obtain

$$\begin{aligned} & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_1\right)}{4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1} \\ & -\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}-\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_1\right)} \\ & =0 \\ \Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)}-2\left(2-\gamma\right)}{\left(4-\gamma^2-2\gamma\right)} \\ & -\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)}-\left(4-\gamma^2-2\gamma\right)}{2\left(2-\gamma\right)} \\ & =0, \end{aligned}$$

which is true for mutually independent goods ($\gamma = 0$). Thus, $\tau_1^{SL} = \tau_1^{SF}$ holds. □

4.5.5 Proof of Proposition 4.4 (ii)

Proof of Proposition 4.4. Suppose intermediary 1 is more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$, where both intermediaries have the same productivity with $\lambda_1 = \lambda_2$. When comparing τ_1^{SL} with intermediary 1 as the leader and τ_1^{SF} with intermediary 1 as the follower, we get

$$
\begin{aligned}
& \tau_1^{SL} - \tau_1^{SF} \leq 0 \\
\Rightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_1\right)}{4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1} \\
& - \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}-\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_1\right)} \\
& \leq 0 \\
\Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)}-2\left(2-\gamma\right)}{4-\gamma^2-2\gamma} \\
& - \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)}-\left(4-\gamma^2-2\gamma\right)}{2\left(2-\gamma\right)} \leq 0 \\
\Leftrightarrow \quad & \gamma^2\sqrt{2\left(2-\gamma^2\right)\left(8-4\gamma-\gamma^2\right)} \leq \gamma^2\left(8-\gamma^2-4\gamma\right) \\
\Leftrightarrow \quad & \sqrt{2\left(2-\gamma^2\right)\left(8-4\gamma-\gamma^2\right)} \leq \left(8-\gamma^2-4\gamma\right) \\
\Leftrightarrow \quad & 0 \leq 4\left(1-\gamma\right)+\gamma^2,
\end{aligned}
$$

which is always true. Hence, given intermediary 1 is more competitive than intermediary 2 and productivities are equal

$$\tau_1^{SL} \leq \tau_1^{SF}$$

always holds. □

4.5.6 Proof of Proposition 4.5

Proof of Proposition 4.5. Within Proposition 4.5 we suppose that intermediaries have identical productivities with $\lambda_1 = \lambda_2$ and intermediary 1 being more competitive than intermediary 2. Additionally, we assume the conditions for interior solution input prices to be satisfied.

Given intermediary 1 is the Stackelberg leader and intermediary 2 the Stackelberg follower, we demand condition (4.19)

$$\alpha_2 > \alpha_1 \frac{2\left(\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(2\lambda_1 + \gamma\lambda_2\right) + 4\gamma\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right)\right)}{\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1\right) + 4\lambda_1\left(4 - \gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)}$$

which for a duopoly yields the interior solution equilibrium input price

$$c^S = c^* = \frac{2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)}$$

and the input supplier's equilibrium profit of

$$\begin{aligned}\pi_I\left(c^S\right) =& \frac{\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right)c^S}{4\left(2 - \gamma^2\right)} \\ =& \frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)^2}{16\left(2 - \gamma^2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)}.\end{aligned}$$

If intermediary 1 is the Stackelberg follower and intermediary 2 the Stackelberg leader an interior solution will, analogous to condition (4.21), be obtained if

$$\alpha_1 > \alpha_2 \frac{2\left(\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2\right)\left(2\lambda_2 + \gamma\lambda_1\right) + 4\gamma\lambda_2\left(2\lambda_2 - \gamma\lambda_1\right)\right)}{\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2\right)\left(4\lambda_1 - \gamma^2\lambda_1 + 2\gamma\lambda_2\right) + 4\lambda_2\left(4 - \gamma^2\right)\left(2\lambda_2 - \gamma\lambda_1\right)}$$

holds. This results in the duopoly equilibrium input price of

$$c^S = c^* = \frac{2\lambda_2\left(2\alpha_2 - \gamma\alpha_1\right) + \lambda_1\left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right)}{2\left(2\lambda_2\left(2\lambda_2 - \gamma\lambda_1\right) + \lambda_1\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2\right)\right)}$$

and the input supplier's equilibrium profit within a duopoly of

$$\begin{aligned}\pi_I\left(c^S\right) =& \frac{\lambda_2\left(2\alpha_2 - \gamma\alpha_1\right)c^S}{2\left(4 - 2\gamma^2\right)} \\ =& \frac{\left(2\lambda_2\left(2\alpha_2 - \gamma\alpha_1\right) + \lambda_1\left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right)\right)^2}{16\left(2 - \gamma^2\right)\left(2\lambda_2\left(2\lambda_2 - \gamma\lambda_1\right) + \lambda_1\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2\right)\right)}.\end{aligned}$$

Part (i) of Proposition 4.5 yields that for a duopoly the input supplier's equilibrium input price is higher if the more competitive intermediary 1 is the Stackelberg leader. When comparing the above displayed input prices with identical productivities for both intermediaries with $\lambda_1 = \lambda_2$, this can be seen by

$$\frac{2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_1\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_1\right) + \lambda_2\left(4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_1\right)\right)}$$

$$> \frac{2\lambda_1 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 (4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2)}{2 (2\lambda_1 (2\lambda_1 - \gamma\lambda_1) + \lambda_1 (4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_1))}$$
$$\Leftrightarrow \quad \alpha_1 > \alpha_2,$$

which is true for $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$.

Part (ii) of Proposition 4.5 claims that the input supplier's duopoly equilibrium profits are higher when the more competitive intermediary 1 is the Stackelberg leader compared to the scenario in which he is the Stackelberg follower. This can be seen in the following. With $\lambda_1 = \lambda_2$ we get

$$\frac{(2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_1 (4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1))^2}{16 (2-\gamma^2) (2\lambda_1 (2\lambda_1 - \gamma\lambda_1) + \lambda_1 (4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_1))}$$
$$> \frac{(2\lambda_2 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 (4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2))^2}{16 (2-\gamma^2) (2\lambda_2 (2\lambda_2 - \gamma\lambda_1) + \lambda_1 (4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2))}$$
$$\Leftrightarrow \quad \alpha_1 > \alpha_2,$$

which is true for $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$.

□

4.5.7 Proof of Lemma 4.1

Proof of Lemma 4.1. When analyzing the characteristics of the intermediaries' profit functions, we will discuss the areas of large, intermediate and small asymmetries as displayed in figure 4.2.

Case 1 (large asymmetries): $\alpha_{3-i} < \alpha_i \tau_i^{SL}$ (or $\alpha_i < \alpha_{3-i}\tau_{3-i}^{SL}$).
We obtain

$$\pi_i^{SL}(\alpha_i, \alpha_{3-i}) = \pi_i^{SF}(\alpha_i, \alpha_{3-i}) = \frac{\alpha_i^2}{16}.$$

Hence, for all $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$ we get

$$\frac{\partial^2 \pi_i^{SL}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2} = \frac{\partial^2 \pi_i^{SF}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2} = \frac{1}{8} > 0.$$

Case 2 (intermediate asymmetries): $\alpha_i \tau_i^{SL} < \alpha_{3-i} < \alpha_i \tau_i^{SF}$ (or $\alpha_{3-i}\tau_{3-i}^{SL} < \alpha_i < \alpha_{3-i}\tau_{3-i}^{SF}$).

The profit of the more competitive monopolistic Stackelberg follower was already considered in the previous case, the profit of the Stackelberg leader as a duopolist will be determined in Case 3.

Case 3 (small asymmetries): $\alpha_i \tau_i^{SF} < \alpha_{3-i} < \alpha_i \frac{\lambda_{3-i}}{\lambda_i}$ (or $\alpha_{3-i}\tau_{3-i}^{SF} < \alpha_i < \alpha_{3-i}\frac{\lambda_i}{\lambda_{3-i}}$).
For the Stackelberg leader we obtain

$$\frac{\partial^2 \pi_i^{SL}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2}$$
$$= \frac{\partial^2 \left[\left(p_i^{SL}(\alpha_i, \alpha_{3-i}) - \lambda_i c^S(\alpha_i, \alpha_{3-i})\right) q_i^{SL}(\alpha_i, \alpha_{3-i})\right]}{\partial \alpha_i^2}$$
$$= \left(2-\gamma^2\right)\left(\frac{\partial q_i(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right)^2$$
$$= \frac{1}{4\left(2-\gamma^2\right)}\left(2 - \frac{\left(2\lambda_i - \gamma\lambda_{3-i}\right)^2}{\left(2\lambda_i\left(2\lambda_i - \gamma\lambda_{3-i}\right) + \lambda_{3-i}\left(4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i\right)\right)}\right)^2 > 0,$$

for all $\alpha_i \in \left[\underline{\alpha_i}, \overline{\alpha_i}\right]$ for $i = 1, 2$.

For the Stackelberg follower within a duopoly we have

$$\frac{\partial^2 \pi_i^{SF}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i^2}$$
$$= \frac{\partial^2 \left[\left(p_i^{SF}(\alpha_i, \alpha_{3-i}) - \lambda_i c^S(\alpha_i, \alpha_{3-i})\right) q_i^{SF}(\alpha_i, \alpha_{3-i})\right]}{\partial \alpha_i^2}$$
$$2\left(\frac{\partial q_i^{SF}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right)^2$$
$$= \frac{1}{8\left(2-\gamma^2\right)^2}\left(\left(4-\gamma^2\right) - \frac{\left(4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}\right)^2}{2\left(2\lambda_{3-i}\left(2\lambda_{3-i} - \gamma\lambda_i\right) + \lambda_i\left(4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}\right)\right)}\right)^2$$
$$> 0,$$

for all $\alpha_i \in \left[\underline{\alpha_i}, \overline{\alpha_i}\right]$ for $i = 1, 2$.

□

4.5.8 Proof of Proposition 4.6

Proof of Proposition 4.6. It follows directly from Lemma 4.1. □

4.6 Appendix B: Computations

Computations of Equation 4.2 and of Equation 4.3

The demand function of the customer displayed in Eq. (4.2) can be determined by taking the first derivative of the customer's utility function U with respect to q_i for $i = 1, 2$. We obtain

$$\max_{q_i} U(q_i, q_{3-i}) \quad \text{s.t. } m \geq p_i q_i + p_{3-i} q_{3-i} + I \quad \text{for} \quad i = 1, 2,$$

which yields

$$\begin{aligned} & \frac{\partial U}{\partial q_i} = \alpha_i - q_i - \gamma q_{3-i} - p_i \stackrel{!}{=} 0 \\ \Leftrightarrow \quad & p_i = \alpha_i - q_i - \gamma q_{3-i}. \end{aligned}$$

Computations of Equations 4.5 and Equation 4.6

In order to determine the equilibrium quantities q_1^{SL} and q_2^{SF} for a duopoly with a sufficiently small input price with $c < \min\{\frac{2\alpha_2 - \gamma\alpha_1}{2\lambda_2 - \gamma\lambda_1}, \frac{4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2}{4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2}\}$, we need to maximize the intermediaries' profit functions and establish the best reply function of intermediary 2. A profit maximization of intermediary i results in

$$\begin{aligned} & \frac{\partial \pi_i \left(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}\right)}{\partial q_i} = \alpha_i - 2q_i - \gamma q_{3-i} - \lambda_i c \stackrel{!}{=} 0 \\ \Leftrightarrow \quad & q_i = \frac{\alpha_i - \gamma q_{3-i} - \lambda_i c}{2}. \end{aligned}$$

As negative profits can be avoided by not producing, intermediary 2's best reply function is given by

$$q_2\left(q_1\right) = \max\left\{\frac{\alpha_2 - \gamma q_1 - \lambda_2 c}{2}, 0\right\}.$$

When considering the case in which intermediary 2 produces a positive quantity and therefore $\frac{\alpha_2 - \gamma q_1 - \lambda_2 c}{2} > 0$, intermediary 1's profit function when considering intermediary 2's best reply is given as:

$$\begin{aligned} \pi_1\left(q_1, q_2, c, \alpha_1, \alpha_2\right) &= \left(p_1 - \lambda_1 c\right) q_1 \\ &= \left(\alpha_1 - q_1 - \gamma q_2 - \lambda_1 c\right) q_1 \end{aligned}$$

$$= \left(\alpha_1 - q_1 - \gamma\left(\frac{\alpha_2 - \gamma q_1 - \lambda_2 c}{2}\right) - \lambda_1 c\right) q_1$$
$$= \alpha_1 q_1 - q_1^2 - \frac{1}{2}\left(\gamma\alpha_2 q_1 - \gamma^2 q_1^2 - \gamma\lambda_2 q_1 c\right) - \lambda_1 q_1 c.$$

A maximization of π_1 with respect to q_1 yields intermediary 1's equilibrium quantity choice

$$\begin{aligned}
&\frac{\partial \pi_1 (q_1, q_2, c, \alpha_1, \alpha_2)}{\partial q_1} = \\
&\alpha_1 - 2q_1^{SL} - \frac{1}{2}\gamma\alpha_2 + \gamma^2 q_1^{SL} + \frac{1}{2}\gamma\lambda_2 c - \lambda_1 c \stackrel{!}{=} 0 \\
\Leftrightarrow \quad & 2q_1^{S} - \gamma^2 q_1^{SL} = \alpha_1 - \frac{1}{2}\gamma\alpha_2 + \frac{1}{2}\gamma\lambda_2 c - \lambda_1 c \\
\Leftrightarrow \quad & q_1^{SL}\left(2 - \gamma^2\right) = \alpha_1 - \frac{1}{2}\gamma\alpha_2 + \frac{1}{2}\gamma\lambda_2 c - \lambda_1 c \\
\Leftrightarrow \quad & 2q_1^{SL} = \frac{2\alpha_1 - \gamma\alpha_2 + \gamma\lambda_2 c - 2\lambda_1 c}{2 - \gamma^2} \\
\Leftrightarrow \quad & 2q_1^{SL} = \frac{2(\alpha_1 - \lambda_1 c) - \gamma(\alpha_2 - \lambda_2 c)}{2 - \gamma^2} \\
\Leftrightarrow \quad & q_1^{SL} = \frac{2(\alpha_1 - \lambda_1 c) - \gamma(\alpha_2 - \lambda_2 c)}{4 - 2\gamma^2} \\
\Leftrightarrow \quad & q_1^{SL} = \frac{2\alpha_1 - \gamma\alpha_2 - 2\lambda_1 c + \gamma\lambda_2 c}{4 - 2\gamma^2} \\
\Leftrightarrow \quad & q_1^{SL} = \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2) c}{2(2 - \gamma^2)}.
\end{aligned}$$

With q_1^{SL} and intermediary 2's best reply function $R^2(q_1) = q_2(q_1)$ we are now able to determine intermediary 2's equilibrium quantity, which is denoted by

$$\begin{aligned}
& q_2^{SF} = \frac{\alpha_2 - \gamma\left(\frac{2(\alpha_1 - \lambda_1 c) - \gamma(\alpha_2 - \lambda_2 c)}{4 - 2\gamma^2}\right) - \lambda_2 c}{2} = R^2\left(q_1^{SL}\right) \\
\Leftrightarrow \quad & q_2^{SF} = \frac{\alpha_2(4 - 2\gamma^2) - 2\gamma(\alpha_1 - \lambda_1 c) + \gamma^2(\alpha_2 - \lambda_2 c) - \lambda_2 c(4 - 2\gamma^2)}{8 - 4\gamma^2} \\
\Leftrightarrow \quad & q_2^{SF} = \frac{4\alpha_2 - 2\gamma^2\alpha_2 - 2\gamma(\alpha_1 - \lambda_1 c) + \gamma^2(\alpha_2 - \lambda_2 c) - 4\lambda_2 c + 2\gamma^2\lambda_2 c}{8 - 4\gamma^2} \\
\Leftrightarrow \quad & q_2^{SF} = \frac{4(\alpha_2 - \lambda_2 c) - 2\gamma^2(\alpha_2 - \lambda_2 c) - 2\gamma(\alpha_1 - \lambda_1 c) + \gamma^2(\alpha_2 - \lambda_2 c)}{8 - 4\gamma^2} \\
\Leftrightarrow \quad & q_2^{SF} = \frac{4(\alpha_2 - \lambda_2 c) - \gamma^2(\alpha_2 - \lambda_2 c) - 2\gamma(\alpha_1 - \lambda_1 c)}{8 - 4\gamma^2}
\end{aligned}$$

$$\Leftrightarrow \quad q_2^{SF} = \frac{4(\alpha_2 - \lambda_2 c) - \gamma(\gamma(\alpha_2 - \lambda_2 c) + 2(\alpha_1 - \lambda_1 c))}{8 - 4\gamma^2}$$
$$\Leftrightarrow \quad q_2^{SF} = \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\, c}{4(2 - \gamma^2)}.$$

When considering the case of the best reply function with $\frac{\alpha_2 - \gamma q_1 - \lambda_2 c}{2} > 0$, intermediary 2 chooses not to produce a positive quantity and we obtain a monopoly scenario with intermediary 1 as the monopolist.

Suppose the case of monopoly in which in the input price satisfies $\frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}$ for a more competitive Stackelberg leader with $(\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2})$. Fur such an input price c, intermediary 2 as the Stackelberg follower will not produce in equilibrium and therefore $q_2^{SF} = 0$. By using the best reply function of intermediary 1

$$q_1 = \frac{\alpha_1 - \gamma q_2 - \lambda_1 c}{2}$$

we obtain with $q_2^{SF} = 0$ intermediary 1's equilibrium quantity within a monopoly

$$q_1^{SL} = \frac{\alpha_1 - \lambda_1 c}{2}.$$

Assuming intermediary 2 to be more competitive than intermediary 1 and the input price satisfies $\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}$, by using the best reply function of intermediary 2 with $q_1^{SL} = 0$ we get

$$q_2^{SL} = \frac{\alpha_2 - \lambda_2 c}{2}.$$

Computations of Equation 4.11

Suppose intermediary 1 is more competitive than intermediary 2 and therefore $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$. It can easily be seen that the critical input price of the less competitive intermediary 2 is smaller than the critical input price of the more competitive intermediary 2. Remember that the critical input price of intermediary 1 as the Stackelberg leader is given by $\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$, the critical input price of intermediary 2 as the Stackelberg follower is denoted by $\frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}$.

We get

$$
\begin{aligned}
& \frac{\alpha_2}{\lambda_2} < \frac{\alpha_1}{\lambda_1} \\
\Leftrightarrow \quad & \alpha_2\lambda_1 < \alpha_1\lambda_2 \\
\Leftrightarrow \quad & \alpha_2\lambda_1 - \alpha_1\lambda_2 < 0 \\
\Leftrightarrow \quad & (\alpha_2\lambda_1 - \alpha_1\lambda_2)\left(4\left(2-\gamma^2\right)\right) < 0 \\
\Leftrightarrow \quad & 4\alpha_2\lambda_1\left(2-\gamma^2\right) - 4\alpha_1\lambda_2\left(2-\gamma^2\right) < 0 \\
\Leftrightarrow \quad & 8\alpha_2\lambda_1 - 8\alpha_1\lambda_2 + 4\gamma^2\alpha_1\lambda_2 - 4\gamma^2\alpha_2\lambda_1 < 0 \\
\Leftrightarrow \quad & 8\alpha_2\lambda_1 - 4\gamma\alpha_2\lambda_2 - 2\gamma^2\alpha_2\lambda_1 + \gamma^3\alpha_2\lambda_2 - 4\gamma\alpha_1\lambda_1 + 2\gamma^2\alpha_1\lambda_2 \\
& < 8\alpha_1\lambda_2 - 4\gamma\alpha_2\lambda_2 - 2\gamma^2\alpha_1\lambda_2 + \gamma^3\alpha_2\lambda_2 - 4\gamma\alpha_1\lambda_1 + 2\gamma^2\alpha_2\lambda_1 \\
\Leftrightarrow \quad & \left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)(2\lambda_1 - \gamma\lambda_2) < \left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)(2\alpha_1 - \gamma\alpha_2) \\
\Leftrightarrow \quad & \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}.
\end{aligned}
$$

Computations of Equation 4.10

We now derive conditions on the input price c such that both intermediaries have incentives to demand inputs on the procurement side and offer positive quantities on the sales side. For the Stackelberg leader, with $q_1^{SL} > 0$ and rearranging towards c we get

$$
\begin{aligned}
& q_1^{SL} = \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c}{4 - 2\gamma^2} > 0 \\
\Leftrightarrow \quad & (2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c > 0 \\
\Leftrightarrow \quad & -(2\lambda_1 - \gamma\lambda_2)\,c > -(2\alpha_1 - \gamma\alpha_2) \\
\Leftrightarrow \quad & c < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}.
\end{aligned}
$$

and for intermediary 2 with $q_2^{SF} > 0$ we obtain

$$
\begin{aligned}
& q_2^{SF} = \frac{(4\alpha_2 - 2\gamma\alpha_1 - \gamma^2\alpha_2) - (4\lambda_2 - 2\gamma\lambda_1 - \gamma^2\lambda_2)\,c}{8 - 4\gamma^2} > 0 \\
\Leftrightarrow \quad & \left(4\alpha_2 - 2\gamma\alpha_1 - \gamma^2\alpha_2\right) - \left(4\lambda_2 - 2\gamma\lambda_1 - \gamma^2\lambda_2\right)c > 0 \\
\Leftrightarrow \quad & -\left(4\lambda_2 - 2\gamma\lambda_1 - \gamma^2\lambda_2\right)c > -4\alpha_2 - 2\gamma\alpha_1 - \gamma^2\alpha_2 \\
\Leftrightarrow \quad & c < \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}.
\end{aligned}
$$

Thus, if the input price satisfies

$$c < \min\left\{\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}, \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}\right\},$$

both intermediaries procure inputs and offer positive quantities at the market.

Computations of Assumption 4.1

Assumption (4.1) demands that given intermediary 1 is the Stackelberg leader and intermediary 2 the Stackelberg follower, we assume $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} > \frac{\gamma}{2}$ for all γ, $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} > \frac{4-\gamma^2}{2\gamma}$ for $\gamma < 0$ and $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} < \frac{4-\gamma^2}{2\gamma}$ for $\gamma \geq 0$.

Assumption (4.1) is derived in the following. First, suppose $\gamma < 0$. For q_1^S to increase in "weighted" qualities we need

$$\begin{aligned} & 2\alpha_1 - \gamma\alpha_2 > 0 \\ \Leftrightarrow \quad & 2\alpha_1 > \gamma\alpha_2 \\ \Leftrightarrow \quad & \frac{\alpha_1}{\alpha_2} > \frac{\gamma}{2}, \end{aligned}$$

for q_2^S to increase in "weighted" qualities we demand

$$\begin{aligned} & 4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1 > 0 \\ \Leftrightarrow \quad & -2\gamma\alpha_1 > -\alpha_2\left(4 - \gamma^2\right) \\ \Leftrightarrow \quad & \frac{\alpha_1}{\alpha_2} > \frac{4 - \gamma^2}{2\gamma}. \end{aligned}$$

Furthermore, in order to assure that q_1^S is decreasing in c we have

$$\begin{aligned} & 2\lambda_1 - \gamma\lambda_2 > 0 \\ \Leftrightarrow \quad & 2\lambda_1 > \gamma\lambda_2 \\ \Leftrightarrow \quad & \frac{\lambda_1}{\lambda_2} > \frac{\gamma}{2}, \end{aligned}$$

for q_2^S decreasing in c we need

$$4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1 > 0$$

$$\begin{aligned}
\Leftrightarrow \quad & \lambda_2\left(4-\gamma^2\right)-2\gamma\lambda_1>0 \\
\Leftrightarrow \quad & -2\gamma\lambda_1>-\lambda_2\left(4-\gamma^2\right) \\
\Leftrightarrow \quad & \frac{\lambda_1}{\lambda_2}>\frac{4-\gamma^2}{2\gamma}.
\end{aligned}$$

Note that the above inequalities are always satisfied. For $\gamma < 0$ we know that $\frac{\gamma}{2} < 0$ and $\frac{4-\gamma^2}{2\gamma} < 0$. As $\alpha_1, \alpha_2, \lambda_1, \lambda_2 > 0$ holds, $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} > 0$ is always true.

Now suppose $\gamma \geq 0$. The assumptions for q_1^S to increase in "weighted" qualities and decrease in c are given as above with

$$\frac{\alpha_1}{\alpha_2} > \frac{\gamma}{2}$$

and

$$\frac{\lambda_1}{\lambda_2} > \frac{\gamma}{2}.$$

In order to assure that q_2^S increases in "weighted" qualities we demand

$$\begin{aligned}
& 4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1>0 \\
\Leftrightarrow \quad & -2\gamma\alpha_1>-\alpha_2\left(4-\gamma^2\right) \\
\Leftrightarrow \quad & 2\gamma\alpha_1<\alpha_2\left(4-\gamma^2\right) \\
\Leftrightarrow \quad & \frac{\alpha_1}{\alpha_2}<\frac{4-\gamma^2}{2\gamma},
\end{aligned}$$

and to decrease in c we impose

$$\begin{aligned}
& 4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1>0 \\
\Leftrightarrow \quad & -2\gamma\lambda_1>-\lambda_2\left(4-\gamma^2\right) \\
\Leftrightarrow \quad & 2\gamma\lambda_1<\lambda_2\left(4-\gamma^2\right) \\
\Leftrightarrow \quad & \frac{\lambda_1}{\lambda_2}<\frac{4-\gamma^2}{2\gamma}.
\end{aligned}$$

Computations of Equation 4.15 and Equation 4.16

The intermediaries' equilibrium prices within a duopoly are determined by using intermediary 1's and intermediary 2's inverse demand function as well as the equilibrium quantities q_1^{SL} and q_2^{SF}. Intermediary 1's equilibrium price is denoted by

$$
\begin{aligned}
p_1^{SL} = {} & \alpha_1 - q_1^{SL} - \gamma q_2^{SF} \\
= & \alpha_1 - \left(\frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\, c}{4 - 2\gamma^2} \right) \\
& - \gamma \left(\frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\, c}{2\,(4 - 2\gamma^2)} \right) \\
= & \frac{\alpha_1\,(8 - 4\gamma^2) - 2\,(2\alpha_1 - \gamma\alpha_2) + 2\,(2\lambda_1 - \gamma\lambda_2)\, c}{2\,(4 - 2\gamma^2)} \\
& + \frac{-\gamma\,(4\alpha_2 - 2\gamma\alpha_1 - \gamma^2\alpha_2) + \gamma\,(4\lambda_2 - 2\gamma\lambda_1 - \gamma^2\lambda_2)\, c}{2\,(4 - 2\gamma^2)} \\
= & \frac{8\alpha_1 - 4\gamma^2\alpha_1 - 4\alpha_1 + 2\gamma\alpha_2 + 4\lambda_1 c - 2\gamma\lambda_2 c - 4\gamma\alpha_2}{2\,(4 - 2\gamma^2)} \\
& + \frac{2\gamma^2\alpha_1 + \gamma^3\alpha_2 + 4\gamma\lambda_2 c - 2\gamma^2\lambda_1 c - \gamma^3\lambda_2 c}{2\,(4 - 2\gamma^2)} \\
= & \frac{4\alpha_1 - 2\gamma^2\alpha_1 - 2\gamma\alpha_2 + 4\lambda_1 c + 2\gamma\lambda_2 c + \gamma^3\alpha_2 - 2\gamma^2\lambda_1 c - \gamma^3\lambda_2 c}{2\,(4 - 2\gamma^2)} \\
= & \frac{4\,(\alpha_1 + \lambda_1 c) - 2\gamma^2\,(\alpha_1 + \lambda_1 c) - 2\gamma\,(\alpha_2 - \lambda_2 c) + \gamma^3\,(\alpha_2 - \lambda_2 c)}{2\,(4 - 2\gamma^2)} \\
= & \frac{2\,(2 - \gamma^2)\,(\alpha_1 + \lambda_1 c) - \gamma\,(2 - \gamma^2)\,(\alpha_2 - \lambda_2 c)}{4\,(2 - \gamma^2)} \\
= & \frac{2\,(\alpha_1 + \lambda_1 c) - \gamma\,(\alpha_2 - \lambda_2 c)}{4} \\
= & \frac{(2\alpha_1 - \gamma\alpha_2) + (2\lambda_1 + \gamma\lambda_2)\, c}{4}.
\end{aligned}
$$

It can easily be seen that

$$
\begin{aligned}
p_1^{SL} &= \left(\frac{2 - \gamma^2}{2} \right) q_1^{SL} + \lambda_1 c \\
&= \frac{2 - \gamma^2}{2} \left(\frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\, c}{2\,(2 - \gamma^2)} \right) + \lambda_1 c \\
&= \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\, c}{4} + \lambda_1 c \\
&= \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\, c}{4} + \frac{4\lambda_1 c}{4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c + 4\lambda_1 c}{4} \\
&= \frac{(2\alpha_1 - \gamma\alpha_2) + (2\lambda_1 + \gamma\lambda_2)\,c}{4}
\end{aligned}
$$

holds. Intermediary 2's equilibrium price is given by:

$$
\begin{aligned}
p_2^{SF} =& \alpha_2 - q_2^{SF} - \gamma q_1^{SL} \\
=& \alpha_2 - \left(\frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)}\right) \\
&- \gamma\left(\frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c}{2\,(2-\gamma^2)}\right) \\
=& \alpha_2 - \left(\frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)}\right) \\
&- 2\gamma\left(\frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c}{2\,(4-2\gamma^2)}\right) \\
=& \frac{2\alpha_2\,(4-2\gamma^2) - (4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) + (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)} \\
&+ \frac{-2\gamma\,(2\alpha_1 - \gamma\alpha_2) + 2\gamma\,(2\lambda_1 - \gamma\lambda_2)\,c}{2\,(4-2\gamma^2)} \\
=& \frac{8\alpha_2 - 4\gamma^2\alpha_2 - 4\alpha_2 + \gamma^2\alpha_2 + 2\gamma\alpha_1 + 4\lambda_2 c - \gamma^2\lambda_2 c - 2\gamma\lambda_1 c - 4\gamma\alpha_1}{4\,(2-\gamma^2)} \\
&+ \frac{2\gamma^2\alpha_2 + 4\gamma\lambda_1 c - 2\gamma^2\lambda_2 c}{4\,(2-\gamma^2)} \\
=& \frac{4\alpha_2 - \gamma^2\alpha_2 + 4\lambda_2 c - 3\gamma^2\lambda_2 c - 2\gamma\alpha_1 + 2\gamma\lambda_1 c}{4\,(2-\gamma^2)} \\
=& \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) + (4\lambda_2 - 3\gamma^2\lambda_2 + 2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)} \\
=& \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) + ((4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1) + 2\gamma\,(2\lambda_1 - \gamma\lambda_2))\,c}{4\,(2-\gamma^2)}.
\end{aligned}
$$

It can be seen that

$$
\begin{aligned}
p_2^{SF} &= q_2^{SF} + \lambda_2 c \\
&= \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)} + \lambda_2 c \\
&= \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1)\,c + 4\lambda_2\,(2-\gamma^2)\,c}{4\,(2-\gamma^2)} \\
&= \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) - (4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1 - 8\lambda_2 + 4\gamma^2\lambda_2)\,c}{4\,(2-\gamma^2)}
\end{aligned}
$$

$$= \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) + (-4\lambda_2 + \gamma^2\lambda_2 + 2\gamma\lambda_1 + 8\lambda_2 - 4\gamma^2\lambda_2)\,c}{4\,(2-\gamma^2)}$$
$$= \frac{(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1) + ((4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1) + 2\gamma\,(2\lambda_1 - \gamma\lambda_2))\,c}{4\,(2-\gamma^2)}$$

is satisfied.

The equilibrium price of intermediary 1 within a monopoly with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$ and $\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \leq c < \frac{\alpha_1}{\lambda_1}$ is determined by using the customer's inverse demand function for intermediary 1's product as well as the according monopolist equilibrium production quantity q_1^{SL}. Therefore we get

$$\begin{aligned} p_1^{SL} &= \alpha_1 - \left(\frac{\alpha_1 - \lambda_1 c}{2}\right) \\ &= \frac{2\alpha_1 - \alpha_1 + \lambda_1 c}{2} \\ &= \frac{\alpha_1 + \lambda_1 c}{2}, \end{aligned}$$

and for intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$ and $\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2} \leq c < \frac{\alpha_2}{\lambda_2}$ and using the according monopolist equilibrium quantity q_2^{SF}:

$$\begin{aligned} p_2^{SF} &= \alpha_2 - \left(\frac{\alpha_2 - \lambda_2 c}{2}\right) \\ &= \frac{2\alpha_2 - \alpha_2 + \lambda_2 c}{2} \\ &= \frac{\alpha_2 + \lambda_2 c}{2}. \end{aligned}$$

Computations of Equation 4.17

The total equilibrium demand for input products is dependent on the intermediaries' equilibrium production quantities q_1^{SL} and q_2^{SF} as displayed in Eq. (4.5) and Eq. (4.6) as well as their according productivities λ_i for $i = 1, 2$. The total quantity demanded at the input market in the duopoly scenario with a sufficiently low input price c is given as

$$\begin{aligned} q_I^S =& \lambda_1 q_1^{SL} + \lambda_2 q_2^{SF} \\ q_I^S =& \lambda_1 \left(\frac{(2\alpha_1 - \gamma\alpha_2) - (2\lambda_1 - \gamma\lambda_2)\,c}{(4 - 2\gamma^2)}\right) \end{aligned}$$

$$
\begin{aligned}
&+\lambda_2\left(\frac{(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)-(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)\,c}{2\,(4-2\gamma^2)}\right)\\
=&\frac{2\lambda_1\,(2\alpha_1-\gamma\alpha_2)-2\lambda_1\,(2\lambda_1-\gamma\lambda_2)\,c+\lambda_2\,(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)}{4\,(2-\gamma^2)}\\
&+\frac{-\lambda_2\,(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)\,c}{4\,(2-\gamma^2)}\\
=&\frac{(2\lambda_1\,(2\alpha_1-\gamma\alpha_2)+\lambda_2\,(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1))-\big(2\lambda_1\,(2\lambda_1-\gamma\lambda_2)}{4\,(2-\gamma^2)}\\
&\frac{+\lambda_2\,(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)\,\big)c}{4\,(2-\gamma^2)}.
\end{aligned}
$$

The input demand within a monopoly in which only intermediary 1 or intermediary 2 is producing, is straightforward.

Computations of Equation 4.18

When using the input supplier's profit function $\pi_I(c)$ as well as the total equilibrium input demand of Eq. (4.17) for a duopoly, we get the following profit maximizing input price:

$$
\begin{aligned}
&\frac{\partial\pi_I(c)}{\partial c}=\\
&\frac{\lambda_2\,((\gamma^2\lambda_2+2\gamma\lambda_1-4\lambda_2)\,c+4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)}{4\gamma^2-8}\overset{!}{=}0
\end{aligned}
$$

$$
c^*=\frac{2\lambda_1\,(2\alpha_1-\gamma\alpha_2)+\lambda_2\,(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)}{2\,(2\lambda_1\,(2\lambda_1-\gamma\lambda_2)+\lambda_2\,(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1))}.
$$

which can be seen as follows:

$$
\begin{aligned}
\pi_I(c)=&\frac{(2\lambda_1\,(2\alpha_1-\gamma\alpha_2)+\lambda_2\,(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1))\,c}{4\,(2-\gamma^2)}\\
&-\frac{(2\lambda_1\,(2\lambda_1-\gamma\lambda_2)+\lambda_2\,(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1))\,c^2}{4\,(2-\gamma^2)}\\
&\frac{\partial\pi_I(q_1,q_2,c,\alpha_1,\alpha_2)}{\partial c}\overset{!}{=}0\\
\Rightarrow\quad&\frac{(2\lambda_1\,(2\alpha_1-\gamma\alpha_2)+\lambda_2\,(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1))}{4\,(2-\gamma^2)}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)c^*}{4\left(2-\gamma^2\right)} \overset{!}{=} 0 \\
\Leftrightarrow \quad & \frac{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)c^*}{2\left(4-2\gamma^2\right)} \\
&=\frac{\left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)}{2\left(4-2\gamma^2\right)} \\
\Leftrightarrow \quad & 2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)c^* \\
&=2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right) \\
\Leftrightarrow \quad & c^*=\frac{2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}.
\end{aligned}
$$

The input price c^* is a unique maximizer as, due to Assumption 4.1, $\pi_I(c)$ is concave in c with

$$
\frac{\partial^2\left[q_I^S(c)c\right]}{\partial c^2}=-\frac{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}{2\left(4-2\gamma^2\right)} \leq 0.
$$

Computations of Equation 4.19

First, suppose intermediary 1 to be more competitive with $\frac{\alpha_1}{\lambda_1}>\frac{\alpha_2}{\lambda_2}$ where

$$
\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}>\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}
$$

holds. When charging an input price of c^* a duopoly arises if $c^*<\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$ is satisfied. Therefore, we need

$$
\begin{aligned}
& \frac{2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}<\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \\
\Leftrightarrow \quad & \frac{2\alpha_1\left(2\lambda_1-\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}{2\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}<\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1} \\
\Leftrightarrow \quad & \left(2\alpha_1\left(2\lambda_1-\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right) \\
&<\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\left(4\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+2\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right) \\
\Leftrightarrow \quad & 2\alpha_1\left(2\lambda_1-\gamma\lambda_2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)^2 \\
&<\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\left(4\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+2\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right) \\
\Leftrightarrow \quad & 2\alpha_1\Big(\left(2\lambda_1-\gamma\lambda_2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right) \\
&\quad+\gamma\left(4\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+2\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)\Big) \\
&<\alpha_2\Big(\left(4-\gamma^2\right)\left(4\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+2\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&- \left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)^2 \Big) \\
\Leftrightarrow \quad & \alpha_2 > \alpha_1 \frac{2\left(\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(2\lambda_1 + \gamma\lambda_2\right) + 4\gamma\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right)\right)}{\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1\right) + 4\lambda_1\left(4 - \gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)}.
\end{aligned}
$$

Computation of Equation 4.21

Suppose intermediary 2 is more competitive with $\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$ where

$$
\frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} < \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}
$$

holds. For c^* being the maximizing input price within a duopoly $c^* < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$ must be satisfied. Hence, we need

$$
\begin{aligned}
& \frac{2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)}{2\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)} < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \\
\Leftrightarrow \quad & \frac{2\alpha_1\left(2\lambda_1 - \gamma\lambda_2\right) + \alpha_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)}{4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} < \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2} \\
\Leftrightarrow \quad & \left(2\alpha_1\left(2\lambda_1 - \gamma\lambda_2\right) + \alpha_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)\left(2\lambda_1 - \gamma\lambda_2\right) \\
& < \left(4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)\left(2\alpha_1 - \gamma\alpha_2\right) \\
\Leftrightarrow \quad & 2\alpha_1\left(\left(2\lambda_1 - \gamma\lambda_2\right)^2 - 4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) - 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right) \\
& < -\alpha_2\Big(\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(2\lambda_1 - \gamma\lambda_2\right) \\
& + \gamma\left(4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)\Big) \\
\Leftrightarrow \quad & 2\alpha_1\left(4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right) - \left(2\lambda_1 - \gamma\lambda_2\right)^2\right) > \\
& \alpha_2\Big(\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\left(2\lambda_1 - \gamma\lambda_2\right) \\
& + \gamma\left(4\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)\Big) \\
\Leftrightarrow \quad & \alpha_1 > \alpha_2 \frac{\left(2\lambda_1 - \gamma\lambda_2\right)\left(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1\right) + 2\gamma\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)}{2\left(\left(2\lambda_1 - \gamma\lambda_2\right)\left(2\lambda_1 + \gamma\lambda_2\right) + 2\lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)}.
\end{aligned}
$$

Computations of Equation 4.23

Given the duopoly interior solution with $c^S = c^*$ the input supplier's profit is denoted by:

$$
\begin{aligned}
& \pi_I\left(c^S\right) = q_I^S\left(c^S\right)c^S \\
= & \frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)}{4\left(2 - \gamma^2\right)}
\end{aligned}
$$

$$+\frac{-\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)c^S}{4\left(2-\gamma^2\right)}\cdot c^S$$
$$=\frac{2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{4\left(2-\gamma^2\right)}$$
$$+\frac{-\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)-\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{4\left(2-\gamma^2\right)}\cdot c^S$$
$$=\frac{\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)c^S}{2\left(4-2\gamma^2\right)}.$$

For the corner solution in which intermediary 1 is the potential monopolist with

$$\frac{\alpha_1}{\lambda_1}>\frac{\alpha_2}{\lambda_2}$$

and

$$c^*>\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$$

the input supplier's profit is given by:

$$\pi_I\left(c^S\right)=q_I^S\left(c^S\right)c^S$$
$$=\frac{\alpha_1\left(4\lambda_1-2\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}{4\left(2-\gamma^2\right)}$$
$$+\frac{-\left(4\lambda_1^2+4\lambda_2^2-4\gamma\lambda_1\lambda_2-\gamma^2\lambda_2^2\right)c^S}{4\left(2-\gamma^2\right)}\cdot c^S$$
$$=\frac{\alpha_1\left(4\lambda_1-2\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}{4\left(2-\gamma^2\right)}$$
$$+\frac{-\left(4\lambda_1^2+4\lambda_2^2-4\gamma\lambda_1\lambda_2-\gamma^2\lambda_2^2\right)\cdot\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}}{4\left(2-\gamma^2\right)}\cdot c^S$$
$$=\frac{\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\left(\alpha_1\left(4\lambda_1-2\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}{4\left(2-\gamma^2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}$$
$$+\frac{-\left(4\lambda_1^2+4\lambda_2^2-4\gamma\lambda_1\lambda_2-\gamma^2\lambda_2^2\right)\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)}{4\left(2-\gamma^2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}\cdot c^S$$
$$=\frac{16\alpha_1\lambda_1\lambda_2+8\gamma^2\alpha_2\lambda_1^2-16\alpha_2\lambda_1^2-8\gamma^2\alpha_1\lambda_1\lambda_2}{\left(8-4\gamma^2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}\cdot c^S$$
$$=\frac{2\alpha_1\lambda_1\lambda_2\left(8-4\gamma^2\right)-2\alpha_2\lambda_1^2\left(8-4\gamma^2\right)}{\left(8-4\gamma^2\right)\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}\cdot c^S$$
$$=\frac{2\lambda_1\left(\alpha_1\lambda_2-\alpha_2\lambda_1\right)}{\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)}\cdot c^S.$$

For the corner solution in which intermediary 2 is the potential monopolist with

$$\frac{\alpha_1}{\lambda_1} < \frac{\alpha_2}{\lambda_2}$$

and

$$c^* > \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$$

the input supplier's profit is given by:

$$\begin{aligned}
&\pi_I\left(c^S\right) = q_I^S\left(c^S\right)c^S \\
=&\frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)}{4\left(2-\gamma^2\right)} \\
&+ \frac{-\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)c^S}{4\left(2-\gamma^2\right)} \cdot c^S \\
=&\frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)}{4\left(2-\gamma^2\right)} \\
&+ \frac{-\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}}{4\left(2-\gamma^2\right)} \cdot c^S \\
=&\frac{\left(2\lambda_1 - \gamma\lambda_2\right)\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)}{2\left(4-2\gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
&- \frac{\left(2\alpha_1 - \gamma\alpha_2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)}{2\left(4-2\gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
=&\frac{8\alpha_2\lambda_1\lambda_2 - 4\gamma^2\alpha_2\lambda_1\lambda_2 - 8\alpha_1\lambda_2^2 + 4\gamma^2\alpha_1\lambda_2^2}{2\left(4-2\gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
=&\frac{2\alpha_2\lambda_1\lambda_2\left(4-2\gamma^2\right) - 2\alpha_1\lambda_2^2\left(4-2\gamma^2\right)}{2\left(4-2\gamma^2\right)\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
=&\frac{\alpha_2\lambda_1\lambda_2 - \alpha_1\lambda_2^2}{\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
=&\frac{\lambda_2\left(\alpha_2\lambda_1 - \alpha_1\lambda_2\right)}{\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S.
\end{aligned}$$

Computations of Equation 4.24

For a monopoly in which the more competitive intermediary 1 produces, the input supplier's profit maximizing input price is given by

$$\frac{\partial\left[\frac{\lambda_1(\alpha_1-\lambda_1)}{2}c\right]}{\partial c} \stackrel{!}{=} 0$$

$$\begin{aligned}
&\Rightarrow \quad \frac{\alpha_1\lambda_1 - 2\lambda_1^2 c^*}{2} \stackrel{!}{=} 0 \\
&\Leftrightarrow \quad \alpha_1\lambda_1 = 2\lambda_1^2 c^* \\
&\Leftrightarrow \quad c^* = \frac{\alpha_1}{2\lambda_1}.
\end{aligned}$$

Computations of Equation 4.25

For c^* to create a monopoly market in which intermediary 1 is more competitive, we need $c^* = \frac{\alpha_1}{2\lambda_1} > \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}$. Thus,

$$\begin{aligned}
&\frac{\alpha_1}{2\lambda_1} > \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1} \\
\Leftrightarrow\quad & \alpha_1\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right) > 2\lambda_1\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right) \\
\Leftrightarrow\quad & \alpha_1\lambda_2\left(4-\gamma^2\right) - 2\gamma\alpha_1\lambda_1 + 4\gamma\alpha_1\lambda_1 > 2\alpha_2\lambda_1\left(4-\gamma^2\right) \\
\Leftrightarrow\quad & \alpha_1\lambda_2\left(4-\gamma^2\right) + 2\gamma\alpha_1\lambda_1 > 2\alpha_2\lambda_1\left(4-\gamma^2\right) \\
\Leftrightarrow\quad & \alpha_1\left(4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1\right) < \alpha_2\left(2\lambda_1\left(4-\gamma^2\right)\right) \\
\Leftrightarrow\quad & \alpha_1 > \alpha_2 \frac{2\lambda_1\left(4-\gamma^2\right)}{\lambda_2\left(4-\gamma^2\right) + 2\gamma\lambda_1} \\
\Leftrightarrow\quad & \alpha_2 < \alpha_1 \frac{4\lambda_2 - \gamma^2\lambda_2 + 2\gamma\lambda_1}{2\lambda_1\left(4-\gamma^2\right)}
\end{aligned}$$

must hold.

Computation of Equation 4.26

Suppose a monopoly with intermediary 1 producing and the optimal input price of $c^S = \max\left\{\frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}, \frac{\alpha_1}{2\lambda_1}\right\}$. The input supplier's equilibrium profits with the input price of

$$c^S = \frac{\alpha_1}{2\lambda_1}$$

is denoted by:

$$\begin{aligned}
&\pi_I\left(q_1, q_2, c, \alpha_1, \alpha_2\right) = q_I^S(c^S)c^S \\
&= \frac{\alpha_1\lambda_1 - \lambda_1^2 c^S}{2} c^S \\
&= \frac{\alpha_1\lambda_1 - \lambda_1^2 \cdot \frac{\alpha_1}{2\lambda_1}}{2} \cdot \frac{\alpha_1}{2\lambda_1}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\alpha_1\lambda_1^2 - \alpha_1\lambda_1^2}{4\lambda_1} \cdot \frac{\alpha_1}{2\lambda_1} \\
&= \frac{\alpha_1^2\lambda_1^2}{8\lambda_1^2} \\
&= \frac{\alpha_1^2}{8}.
\end{aligned}
$$

The input supplier's equilibrium profit for the corner monopoly solution with intermediary 1 as the monopolist and the input price

$$
c^S = \frac{4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1}{4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1}
$$

is denoted by:

$$
\begin{aligned}
&\pi_I\left(c^S\right) = q_I^S(c^S)c^S \\
&= \frac{\alpha_1\lambda_1 - \lambda_1^2 \cdot c^S}{2} \cdot c^S \\
&= \frac{\alpha_1\lambda_1 - \lambda_1^2 \cdot \frac{4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}}{2} \cdot c^S \\
&= \frac{\alpha_1\lambda_1\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right) - \lambda_1^2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)}{2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} \cdot c^S \\
&= \frac{4\alpha_1\lambda_1\lambda_2 - \gamma^2\alpha_1\lambda_1\lambda_2 - 2\gamma\alpha_1\lambda_1^2 - 4\alpha_2\lambda_1^2 + \gamma^2\alpha_2\lambda_1^2 + 2\gamma\alpha_1\lambda_1^2}{2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} \cdot c^S \\
&= \frac{4\alpha_1\lambda_1\lambda_2 - \gamma^2\alpha_1\lambda_1\lambda_2 - 4\alpha_2\lambda_1^2 + \gamma^2\alpha_2\lambda_1^2}{2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} \cdot c^S \\
&= \frac{\alpha_1\lambda_1\lambda_2\left(4 - \gamma^2\right) - \alpha_2\lambda_1^2\left(4 - \gamma^2\right)}{2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} \cdot c^S \\
&= \frac{\lambda_1\left(\alpha_1\lambda_2 - \alpha_2\lambda_1^2\right)\left(4 - \gamma^2\right)}{2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)} \cdot c^S.
\end{aligned}
$$

Computation of Equation 4.27

The input price that maximizes the supplier's profit function, given intermediary 2 is the monopolist, is given by

$$\begin{aligned}
&\frac{\partial\left[\frac{\lambda_2(\alpha_2-\lambda_2)}{2}c\right]}{\partial c} \overset{!}{=} 0\\
\Rightarrow\quad &\frac{\lambda_2\alpha_2 - 2\lambda_2^2 c^*}{2} = 0\\
\Leftrightarrow\quad &\lambda_2\alpha_2 = 2\lambda_2^2 c^*\\
\Leftrightarrow\quad &c^* = \frac{\alpha_2}{2\lambda_2}.
\end{aligned}$$

Computation of Equation 4.28

For c^* to be the optimal input price for a monopoly in which intermediary 2 produces, we need

$$\begin{aligned}
&\frac{\alpha_2}{2\lambda_2} > \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}\\
\Leftrightarrow\quad &\alpha_2\left(2\lambda_1 - \gamma\lambda_2\right) > 2\lambda_2\left(2\alpha_1 - \gamma\alpha_2\right)\\
\Leftrightarrow\quad &2\alpha_2\lambda_1 - \gamma\alpha_2\lambda_2 > 4\alpha_1\lambda_2 - 2\gamma\alpha_2\lambda_2\\
\Leftrightarrow\quad &2\alpha_2\lambda_1 + \gamma\alpha_2\lambda_2 > 4\alpha_1\lambda_2\\
\Leftrightarrow\quad &\alpha_1 < \alpha_2\frac{2\lambda_1 + \gamma\lambda_2}{4\lambda_2}.
\end{aligned}$$

Computations of Equation 4.26

Suppose a monopoly with intermediary 2 producing and the optimal input price of $c^S = \max\left\{\frac{2\alpha_1-\gamma\alpha_2}{2\lambda_1-\gamma\lambda_2}, \frac{\alpha_2}{2\lambda_2}\right\}$. The input supplier's equilibrium profit for the interior monopoly solution with intermediary 2 as the monopolist and the input price

$$c^S = \frac{\alpha_2}{2\lambda_2}$$

is denoted by:

$$\begin{aligned}
\pi_I\left(c^S\right) &= q_I^S(c^S)c^S\\
&= \frac{\lambda_2\left(\alpha_2 - \lambda_2 c^S\right)}{2}\cdot c^S
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\lambda_2 \left(\alpha_2 - \lambda_2 \cdot \frac{\alpha_2}{2\lambda_2}\right)}{2} \cdot \frac{\alpha_2}{2\lambda_2} \\
&= \frac{\alpha_2\lambda_2 - \frac{\alpha_2\lambda_2}{2}}{2} \cdot \frac{\alpha_2}{2\lambda_2} \\
&= \frac{\alpha_2\lambda_2}{4} \cdot \frac{\alpha_2}{2\lambda_2} \\
&= \frac{\alpha_2^2}{8}.
\end{aligned}
$$

The input supplier's equilibrium profit for the corner monopoly solution with intermediary 2 as the monopolist and the input price

$$c^S = \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}$$

is given by:

$$
\begin{aligned}
\pi_I\left(c^S\right) &= q_I^S(c^S)c^S \\
&= \frac{\lambda_2\left(\alpha_2 - \lambda_2 c^S\right)}{2} \cdot c^S \\
&= \frac{\lambda_2\left(\alpha_2 - \lambda_2 \cdot \frac{2\alpha_1 - \gamma\alpha_2}{2\lambda_1 - \gamma\lambda_2}\right)}{2} \cdot c^S \\
&= \frac{\lambda_2\left(2\alpha_2\lambda_1 - \gamma\alpha_2\lambda_2 - 2\alpha_1\lambda_2 + \gamma\alpha_2\lambda_2\right)}{2\left(2\lambda_1 - \gamma\lambda_2\right)} \cdot c^S \\
&= \frac{\lambda_2\left(\alpha_2\lambda_1 - \alpha_1\lambda_2\right)}{2\lambda_1 - \gamma\lambda_2} c^S.
\end{aligned}
$$

Computations of Proof of Proposition 4.1

Proof of Proposition 4.1. When determining the multiplier τ_1^{SL} for $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$ we investigate for conditions on α_2 for which the input supplier's monopoly profit is at least as high as his duopoly profit. Therefore we get

$$
\begin{aligned}
& \frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)^2}{16\left(2 - \gamma^2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)} \leq \frac{\alpha_1^2}{8} \\
\Leftrightarrow \quad & \frac{\left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)^2}{2\left(2 - \gamma^2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)} \leq \alpha_1^2 \\
\Leftrightarrow \quad & \left(2\lambda_1\left(2\alpha_1 - \gamma\alpha_2\right) + \lambda_2\left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)^2 \\
& \leq 2\alpha_1^2\left(2 - \gamma^2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)
\end{aligned}
$$

$$\begin{aligned}
\Leftrightarrow\quad & 2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\\
& \leq \alpha_1\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\\
\Leftrightarrow\quad & 4\alpha_1\lambda_1-2\gamma\alpha_2\lambda_1+4\alpha_2\lambda_2-\gamma^2\alpha_2\lambda_2-2\gamma\alpha_1\lambda_2\\
& \leq \alpha_1\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\\
\Leftrightarrow\quad & 2\alpha_1\left(2\lambda_1-\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\\
& \leq \alpha_1\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\\
\Leftrightarrow\quad & \alpha_2\leq\alpha_1\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_2\right)}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}\\
=\quad & \alpha_2\leq\alpha_1\tau_1
\end{aligned}$$

with

$$\tau_1^{SL}=\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_2\right)}{4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1}.$$

□

Computations of Proof of Proposition 4.2

Proof of Proposition 4.2. When determining the multiplier τ_2^{SF} for $\frac{\alpha_1}{\lambda_1}<\frac{\alpha_2}{\lambda_2}$ we investigate for conditions for which the input supplier's monopoly profit is at least as high as his duopoly profit. Therefore we get

$$\begin{aligned}
& \frac{\left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)^2}{16\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\leq\frac{\alpha_2^2}{8}\\
\Leftrightarrow\quad & \frac{\left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)^2}{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\leq\alpha_2^2\\
\Leftrightarrow\quad & \left(2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\right)^2\\
& \leq 2\alpha_2^2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)\\
\Leftrightarrow\quad & 2\lambda_1\left(2\alpha_1-\gamma\alpha_2\right)+\lambda_2\left(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1\right)\\
& \leq \alpha_2\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\\
\Leftrightarrow\quad & 2\alpha_1\left(2\lambda_1-\gamma\lambda_2\right)+\alpha_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\\
& \leq \alpha_2\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}\\
\Leftrightarrow\quad & \alpha_1\leq\alpha_2\Bigg(\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_2\right)+\lambda_2\left(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1\right)\right)}}{2\left(2\lambda_1-\gamma\lambda_2\right)}
\end{aligned}$$

$$- \frac{\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)}{2\left(2\lambda_1 - \gamma\lambda_2\right)}\Bigg)$$

$$= \quad \alpha_1 \leq \alpha_2\tau_2$$

with

$$\tau_2^{SF} = \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1 - \gamma\lambda_2\right) + \lambda_2\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)\right)}}{2\left(2\lambda_1 - \gamma\lambda_2\right)} - \frac{\left(4\lambda_2 - \gamma^2\lambda_2 - 2\gamma\lambda_1\right)}{2\left(2\lambda_1 - \gamma\lambda_2\right)}.$$

□

Computations of Assumption 4.2

Given intermediary 1 is the Stackelberg follower and intermediary 2 the Stackelberg leader, we assume $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} > \frac{2\gamma}{4-\gamma^2}$ for all γ, $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} < \frac{2}{\gamma}$ for $\gamma \geq 0$ and $\frac{\alpha_1}{\alpha_2}, \frac{\lambda_1}{\lambda_2} > \frac{2}{\gamma}$ for $\gamma < 0$.

For q_1^{SF} to increase in "weighted" qualities we get:

$$\begin{aligned} 4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2 &> 0 \\ \alpha_1\left(4-\gamma^2\right) &> 2\gamma\alpha_2 \\ \frac{\alpha_1}{\alpha_2} &> \frac{2\gamma}{4-\gamma^2} \end{aligned}$$

and for q_1^{SF} to decrease in the input price c we need:

$$\begin{aligned} 4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_2 &> 0 \\ \lambda_1\left(4-\gamma^2\right) &> 2\gamma\lambda_2 \\ \frac{\lambda_1}{\lambda_2} &> \frac{2\gamma}{4-\gamma^2}. \end{aligned}$$

For intermediary 2's equilibrium quantity q_2^{SL} to increase in "weighted" qualities we need

$$\begin{aligned} 2\alpha_2 - \gamma\alpha_1 &> 0 \\ 2\alpha_2 &> \gamma\alpha_1 \\ \frac{\alpha_1}{\alpha_2} &< \frac{2}{\gamma}, \end{aligned}$$

if $\gamma \geq 0$ and

$$\begin{aligned} 2\alpha_2 - \gamma\alpha_1 &> 0 \\ 2\alpha_2 &> \gamma\alpha_1 \\ \frac{\alpha_1}{\alpha_2} &> \frac{2}{\gamma}, \end{aligned}$$

if $\gamma < 0$. Furthermore, for q_2^{SL} to decrease in the input price c we demand

$$\begin{aligned} 2\lambda_2 - \gamma\lambda_1 &> 0 \\ 2\lambda_2 &> \gamma\lambda_1 \\ \frac{\lambda_1}{\lambda_2} &< \frac{2}{\gamma} \end{aligned}$$

for $\gamma \geq 0$ and

$$\begin{aligned} 2\lambda_2 - \gamma\lambda_1 &> 0 \\ 2\lambda_2 &> \gamma\lambda_1 \\ \frac{\lambda_1}{\lambda_2} &> \frac{2}{\gamma} \end{aligned}$$

for $\gamma < 0$.

Computations of Proof of Proposition 4.3

Proof of Proposition 4.3. From Eq. (4.5) we know that intermediary i's equilibrium quantity as a Stackelberg leader within a duopoly is given by

$$q_i^{SL} = \frac{(2\alpha_i - \gamma_{3-i}) - (2\lambda_i - \gamma\lambda_{3-i})\, c}{4 - 2\gamma^2}$$

if

$$c < \min\left\{\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}, \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i}\right\}$$

and from equation (4.33) we know that intermediary i's equilibrium quantity as a Stackelberg follower is given by

$$q_i^{SF} = \frac{(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}) - (4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i})\,c}{2\,(4-2\gamma^2)}$$

if

$$c < \min\left\{\frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}, \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}\right\}.$$

The equilibrium quantity of intermediary i as a leader q_i^{SL} is larger than his equilibrium quantity as a follower q_i^{SF} if and only if $\frac{\alpha_i}{\lambda_i} > c$, which can be seen as follows:

$$\begin{aligned}
& \frac{(2\alpha_i - \gamma\alpha_{3-i}) - (2\lambda_i - \gamma\lambda_{3-i})\,c}{4-2\gamma^2} \\
& > \frac{(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}) - (4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i})\,c}{2\,(4-2\gamma^2)} \\
\Leftrightarrow \quad & \frac{2\,(2\alpha_i - \gamma\alpha_{3-i}) - 2\,(2\lambda_i - \gamma\lambda_{3-i})\,c}{2\,(4-2\gamma^2)} \\
& > \frac{(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}) - (4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i})\,c}{2\,(4-2\gamma^2)} \\
\Leftrightarrow \quad & 2\,(2\alpha_i - \gamma\alpha_{3-i}) - 2\,(2\lambda_i - \gamma\lambda_{3-i})\,c \\
& > \left(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}\right) - \left(4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}\right)c \\
\Leftrightarrow \quad & 2\,(2\alpha_i - \gamma\alpha_{3-i}) - \left(4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}\right) \\
& - 2\,(2\lambda_i - \gamma\lambda_{3-i})\,c + \left(4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}\right)c > 0 \\
\Leftrightarrow \quad & \gamma^2\alpha_i - \gamma^2\lambda_i c > 0 \\
\Leftrightarrow \quad & \gamma^2\alpha_i > \gamma^2\lambda_i c \\
\Leftrightarrow \quad & \alpha_i > \lambda_i c \\
\Leftrightarrow \quad & \frac{\alpha_i}{\lambda_i} > c.
\end{aligned}$$

First, suppose that intermediary i is more competitive than intermediary $3-i$ and therefore $\frac{\alpha_i}{\lambda_i} > \frac{\alpha_{3-i}}{\lambda_{3-i}}$. Thus, independently of whether he is the Stackelberg leader or the Stackelberg follower, intermediary i's critical input price is larger than the critical input price of intermediary $3-i$. If intermediary i is in the leading position, we have

$$\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} > \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i},$$

if he takes the role of the follower we get

$$\frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}.$$

Thus, intermediary i is the potential monopolist when being the Stackelberg leader and follower. The conditions that ensure a sufficiently low input price c for a duopoly therefore demand

$$c < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i}$$

and

$$c < \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i}.$$

It can easily be seen that the following always holds:

$$\begin{aligned}
& \frac{\alpha_{3-i}}{\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i < \alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i \left(4 - \gamma^2\right) < \alpha_i\lambda_{3-i} \left(4 - \gamma^2\right) \\
\Leftrightarrow \quad & 4\alpha_{3-i}\lambda_i - \gamma^2\alpha_{3-i}\lambda_i - 2\gamma\alpha_i\lambda_i < 4\alpha_i\lambda_{3-i} - \gamma^2\alpha_i\lambda_{3-i} - 2\gamma\alpha_i\lambda_i \\
\Leftrightarrow \quad & \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}
\end{aligned}$$

as well as

$$\begin{aligned}
& \frac{\alpha_{3-i}}{\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i < \alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & 2\alpha_{3-i}\lambda_i - \gamma\alpha_i\lambda_i < 2\alpha_i\lambda_{3-i} - \gamma\alpha_i\lambda_i \\
\Leftrightarrow \quad & \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}
\end{aligned}$$

Therefore the conditions which ensure a duopoly market also imply the condition for $q_i^{SL} > q_i^{SF}$:

$$c < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}$$

and

$$c < \frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} < \frac{\alpha_i}{\lambda_i}.$$

Now suppose intermediary $3-i$ to be more competitive than intermediary i and therefore $\frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i}$. Independently of whether he is the Stackelberg leader or the Stackelberg follower, intermediary i's critical input price is smaller than the critical input price of intermediary $3-i$. If intermediary i is in the leading position, we have

$$\frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{4\alpha_{3-i} - \gamma^2\alpha_{3-i} - 2\gamma\alpha_i}{4\lambda_{3-i} - \gamma^2\lambda_{3-i} - 2\gamma\lambda_i},$$

if intermediary i takes the role of the follower we get

$$\frac{2\alpha_{3-i} - \gamma\alpha_i}{2\lambda_{3-i} - \gamma\lambda_i} > \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}.$$

Therefore, in both scenarios intermediary $3-i$ is the potential monopolist. The conditions that ensure a sufficiently low input price c for a duopoly therefore demand

$$c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}$$

for the first case and

$$c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}$$

for the second case. It can be seen that for the given scenario with $\gamma > 0$ the following inequalities are true:

$$\begin{aligned} & \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\ \Leftrightarrow \quad & \alpha_{3-i}\lambda_i > \alpha_i\lambda_{3-i} \\ \Leftrightarrow \quad & \gamma\alpha_{3-i}\lambda_i > \gamma\alpha_i\lambda_{3-i} \end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad & -\gamma\alpha_{3-i}\lambda_i < -\gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & 2\alpha_i\lambda_i - \gamma\alpha_{3-i}\lambda_i < 2\alpha_i\lambda_i - \gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i > \alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & -2\gamma\alpha_{3-i}\lambda_i < -2\gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & 4\alpha_i\lambda_i - \gamma^2\alpha_i\lambda_i - 2\gamma\alpha_{3-i}\lambda_i < 4\alpha_i\lambda_i - \gamma^2\alpha_i\lambda_i - 2\gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}.
\end{aligned}$$

Therefore, given the conditions that guarantee a duopoly market are satisfied, $q_i^{SL} > q_i^{SF}$ also holds:

$$c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}$$

and

$$c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} < \frac{\alpha_i}{\lambda_i}.$$

With $\gamma < 0$, however, the duopoly condition is not necessarily sufficiently for $c < \frac{\alpha_i}{\lambda_i}$. It is obvious that

$$\begin{aligned}
& \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i > \alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \gamma\alpha_{3-i}\lambda_i < \gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i}
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\alpha_{3-i}}{\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i} \\
\Leftrightarrow \quad & \alpha_{3-i}\lambda_i > \alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \gamma\alpha_{3-i}\lambda_i < \gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & -2\gamma\alpha_{3-i}\lambda_i > -2\gamma\alpha_i\lambda_{3-i} \\
\Leftrightarrow \quad & \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}} > \frac{\alpha_i}{\lambda_i}.
\end{aligned}$$

Given intermediary i is the Stackelberg leader with an input price satisfying

$$\frac{\alpha_i}{\lambda_i} < c < \frac{2\alpha_i - \gamma\alpha_{3-i}}{2\lambda_i - \gamma\lambda_{3-i}}$$

as well as intermediary i as the Stackelberg follower with an input price satisfying

$$\frac{\alpha_i}{\lambda_i} < c < \frac{4\alpha_i - \gamma^2\alpha_i - 2\gamma\alpha_{3-i}}{4\lambda_i - \gamma^2\lambda_i - 2\gamma\lambda_{3-i}}$$

the equilibrium quantity of the Stackelberg leader will be higher than the equilibrium of the Stackelberg follower. □

Computations of Proof of Proposition 4.4

Part (i) of Proposition 4.4 shows that when assuming $\lambda_1 = \lambda_2$, we obtain

$$\begin{aligned}
& \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}-2\left(2\lambda_1-\gamma\lambda_1\right)}{4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1} \\
& -\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}}{2\left(2\lambda_1-\gamma\lambda_1\right)} \\
& -\frac{\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_1\right)} = 0 \\
\Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1^2\left(2-\gamma\right)+\lambda_1^2\left(4-\gamma^2-2\gamma\right)\right)}-2\lambda_1\left(2-\gamma\right)}{\lambda_1\left(4-\gamma^2-2\gamma\right)} \\
& -\frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1^2\left(2-\gamma\right)+\lambda_1^2\left(4-\gamma^2-2\gamma\right)\right)}-\lambda_1\left(4-\gamma^2-2\gamma\right)}{2\lambda_1\left(2-\gamma\right)} = 0 \\
\Leftrightarrow \quad & \frac{\lambda_1\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)}-2\lambda_1\left(2-\gamma\right)}{\lambda_1\left(4-\gamma^2-2\gamma\right)}
\end{aligned}$$

$$
\begin{aligned}
&- \frac{\lambda_1 \sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - \lambda_1\left(4-\gamma^2-2\gamma\right)}{2\lambda_1\left(2-\gamma\right)} = 0 \\
\Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - 2\left(2-\gamma\right)}{\left(4-\gamma^2-2\gamma\right)} \\
&- \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - \left(4-\gamma^2-2\gamma\right)}{2\left(2-\gamma\right)} = 0.
\end{aligned}
$$

With products being mutually independent and therefore $\gamma = 0$ we get

$$
\frac{\sqrt{20}-4}{4} - \frac{\sqrt{20}-4}{4} = 0,
$$

which is true.

Part (ii) of Proposition 4.4 compares the input supplier's incentives to exclude either a Stackelberg leader or a Stackelberg follower. Suppose intermediary 1 is more competitive than intermediary 2 with $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$, where both intermediaries have the same productivity with $\lambda_1 = \lambda_2$. When comparing τ_1^{SL} with intermediary 1 as the Stackelberg leader and τ_1^{SF} with intermediary 1 as the Stackelberg follower we obtain

$$
\begin{aligned}
& \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)} - 2\left(2\lambda_1-\gamma\lambda_1\right)}{4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1} \\
&- \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1\left(2\lambda_1-\gamma\lambda_1\right)+\lambda_1\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)\right)}}{2\left(2\lambda_1-\gamma\lambda_1\right)} \\
&+ \frac{-\left(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1\right)}{2\left(2\lambda_1-\gamma\lambda_1\right)} < 0 \\
\Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1^2\left(2-\gamma\right)+\lambda_1^2\left(4-\gamma^2-2\gamma\right)\right)} - 2\lambda_1\left(2-\gamma\right)}{\lambda_1\left(4-\gamma^2-2\gamma\right)} \\
&- \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\lambda_1^2\left(2-\gamma\right)+\lambda_1^2\left(4-\gamma^2-2\gamma\right)\right)} - \lambda_1\left(4-\gamma^2-2\gamma\right)}{2\lambda_1\left(2-\gamma\right)} < 0 \\
\Leftrightarrow \quad & \frac{\lambda_1\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - 2\lambda_1\left(2-\gamma\right)}{\lambda_1\left(4-\gamma^2-2\gamma\right)} \\
&- \frac{\lambda_1\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - \lambda_1\left(4-\gamma^2-2\gamma\right)}{2\lambda_1\left(2-\gamma\right)} < 0 \\
\Leftrightarrow \quad & \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - 2\left(2-\gamma\right)}{\left(4-\gamma^2-2\gamma\right)} \\
&- \frac{\sqrt{2\left(2-\gamma^2\right)\left(2\left(2-\gamma\right)+\left(4-\gamma^2-2\gamma\right)\right)} - \left(4-\gamma^2-2\gamma\right)}{2\left(2-\gamma\right)} < 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \frac{2(2-\gamma)\sqrt{2(2-\gamma^2)(2(2-\gamma)+(4-\gamma^2-2\gamma))}-4(2-\gamma)^2}{2(4-\gamma^2-2\gamma)(2-\gamma)} \\
& -\frac{(4-\gamma^2-2\gamma)\sqrt{2(2-\gamma^2)(2(2-\gamma)+(4-\gamma^2-2\gamma))}}{2(4-\gamma^2-2\gamma)(2-\gamma)} \\
& -\frac{(4-\gamma^2-2\gamma)^2}{2(4-\gamma^2-2\gamma)(2-\gamma)}<0 \\
\Leftrightarrow \quad & 2(2-\gamma)\sqrt{2(2-\gamma^2)(2(2-\gamma)+(4-\gamma^2-2\gamma))}-4(2-\gamma)^2 \\
& -\left(4-\gamma^2-2\gamma\right)\sqrt{2(2-\gamma^2)(2(2-\gamma)+(4-\gamma^2-2\gamma))} \\
& +\left(4-\gamma^2-2\gamma\right)^2<0 \\
\Leftrightarrow \quad & \sqrt{2(2-\gamma^2)(2(2-\gamma)+(4-\gamma^2-2\gamma))}\left(2(2-\gamma)-\left(4-\gamma^2-2\gamma\right)\right) \\
& -4(2-\gamma)^2+\left(4-\gamma^2-2\gamma\right)^2<0 \\
\Leftrightarrow \quad & \gamma^2\sqrt{2(2-\gamma^2)(8-4\gamma-\gamma^2)}-4(2-\gamma)^2+\left(4-\gamma^2-2\gamma\right)^2<0 \\
\Leftrightarrow \quad & \gamma^2\sqrt{2(2-\gamma^2)(8-4\gamma-\gamma^2)}-\gamma^2\left(8-\gamma^2-4\gamma\right)<0 \\
\Leftrightarrow \quad & \gamma^2\sqrt{2(2-\gamma^2)(8-4\gamma-\gamma^2)}<\gamma^2\left(8-\gamma^2-4\gamma\right) \\
\Leftrightarrow \quad & \sqrt{2(2-\gamma^2)(8-4\gamma-\gamma^2)}<\left(8-\gamma^2-4\gamma\right) \\
\Leftrightarrow \quad & 2\left(2-\gamma^2\right)\left(8-4\gamma-\gamma^2\right)<\left(8-\gamma^2-4\gamma\right)^2 \\
\Leftrightarrow \quad & 2\left(2-\gamma^2\right)<8-\gamma^2-4\gamma \\
\Leftrightarrow \quad & 4-2\gamma^2<8-\gamma^2-4\gamma \\
\Leftrightarrow \quad & 0<4+\gamma^2-4\gamma \\
\Leftrightarrow \quad & 0<4(1-\gamma)+\gamma^2,
\end{aligned}
$$

which is always satisfied.

Computations of Proof of Proposition 4.5

Part (i) of Proposition 4.5 claims that the input supplier's equilibrium duopoly input price is higher if intermediary 1 is the Stackelberg leader compared to the scenario in which he takes the role of the follower. With identical productivities for both intermediaries with $\lambda_1 = \lambda_2$, we can be seen by

$$
\begin{aligned}
& \frac{2\lambda_1(2\alpha_1-\gamma\alpha_2)+\lambda_1(4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)}{2(2\lambda_1(2\lambda_1-\gamma\lambda_1)+\lambda_2(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1))} \\
> & \frac{2\lambda_1(2\alpha_2-\gamma\alpha_1)+\lambda_1(4\alpha_1-\gamma^2\alpha_1-2\gamma\alpha_2)}{2(2\lambda_1(2\lambda_1-\gamma\lambda_1)+\lambda_1(4\lambda_1-\gamma^2\lambda_1-2\gamma\lambda_1))}
\end{aligned}
$$

$$\begin{aligned}
\Leftrightarrow\quad & 2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_1 \left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right) \\
& > 2\lambda_1 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 \left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right) \\
\Leftrightarrow\quad & 2 (2\alpha_1 - \gamma\alpha_2) + \left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right) \\
& > 2 (2\alpha_2 - \gamma\alpha_1) + \left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right) \\
\Leftrightarrow\quad & 4\alpha_1 - 2\gamma\alpha_2 - 4\alpha_1 + \gamma^2\alpha_1 + 2\gamma\alpha_2 \\
& 4\alpha_2 - 2\gamma\alpha_1 - 4\alpha_2 + \gamma^2\alpha_2 + 2\gamma\alpha_1 \\
\Leftrightarrow\quad & \gamma^2\alpha_1 > \gamma^2\alpha_2 \\
\Leftrightarrow\quad & \alpha_1 > \alpha_2,
\end{aligned}$$

which is true for $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$.

Part (ii) of Proposition 4.5 yields that for a duopoly the input supplier's equilibrium duopoly input price is higher if intermediary 1 is the Stackelberg leader. When comparing the input prices with identical productivities for both intermediaries ($\lambda_1 = \lambda_2$), this can be seen by

$$\begin{aligned}
& \frac{\left(2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_1 (4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1)\right)^2}{16 (2 - \gamma^2) (2\lambda_1 (2\lambda_1 - \gamma\lambda_1) + \lambda_1 (4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_1))} \\
> & \frac{\left(2\lambda_2 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 (4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2)\right)^2}{16 (2 - \gamma^2) (2\lambda_1 (2\lambda_1 - \gamma\lambda_1) + \lambda_1 (4\lambda_1 - \gamma^2\lambda_1 - 2\gamma\lambda_1))} \\
\Leftrightarrow\quad & \left(2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_1 \left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right)\right)^2 \\
> & \left(2\lambda_1 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 \left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right)\right)^2 \\
\Leftrightarrow\quad & 2\lambda_1 (2\alpha_1 - \gamma\alpha_2) + \lambda_1 \left(4\alpha_2 - \gamma^2\alpha_2 - 2\gamma\alpha_1\right) \\
> & 2\lambda_1 (2\alpha_2 - \gamma\alpha_1) + \lambda_1 \left(4\alpha_1 - \gamma^2\alpha_1 - 2\gamma\alpha_2\right) \\
\Leftrightarrow\quad & 4\alpha_1\lambda_1 - 2\gamma\alpha_2\lambda_1 + 4\alpha_2\lambda_1 - \gamma^2\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 \\
> & 4\alpha_2\lambda_1 - 2\gamma\alpha_1\lambda_1 + 4\alpha_1\lambda_1 - \gamma^2\alpha_1\lambda_1 - 2\gamma\alpha_2\lambda_1 \\
\Leftrightarrow\quad & -\gamma^2\alpha_2\lambda_1 > -\gamma^2\alpha_1\lambda_1 \\
\Leftrightarrow\quad & \alpha_1 > \alpha_2,
\end{aligned}$$

which is true for $\frac{\alpha_1}{\lambda_1} > \frac{\alpha_2}{\lambda_2}$.

Computations of Equation 4.38

$$\begin{aligned}
\pi_1^{SL}(q_1,q_2,c,\alpha_1,\alpha_2) &= \left(p_1^{SL}-\lambda_1 c^S\right) q_1^{SL} \\
&= \left(\left(\frac{2-\gamma^2}{2}\right) q_1^{SL} + \lambda_1 c^S - \lambda_1 c^S\right) q_1^{SL} \\
&= \left(\frac{2-\gamma^2}{2}\right)\left(q_1^{SL}\right)^2 \\
\Rightarrow \quad \pi_1^{SL}(q_1,q_2,c,\alpha_1,\alpha_2) &= \left(\frac{2-\gamma^2}{2}\right)\left(\frac{(2\alpha_1-\gamma\alpha_2)-(2\lambda_1-\gamma\lambda_2)c}{2(2-\gamma^2)}\right)^2 \\
&= \frac{\left((2\alpha_1-\gamma\alpha_2)-(2\lambda_1-\gamma\lambda_2)c\right)^2}{8(2-\gamma^2)}
\end{aligned}$$

Computations of Equation 4.39

$$\begin{aligned}
\pi_2^{SF}(q_1,q_2,c,\alpha_1,\alpha_2) &= \left(p_2^{SF}-\lambda_2 c^S\right) q_2^{SF} \\
&= \left(q_2^{SF}+\lambda_2 c-\lambda_2 c\right) q_2^{SF} \\
&= \left(q_2^{SF}\right)^2 \\
\Rightarrow \quad \pi_2^{SF}(q_1,q_2,c,\alpha_1,\alpha_2) &= \frac{\left((4\alpha_2-\gamma^2\alpha_2-2\gamma\alpha_1)-(4\lambda_2-\gamma^2\lambda_2-2\gamma\lambda_1)c\right)^2}{16(2-\gamma^2)^2}
\end{aligned}$$

Computations of Proof of Lemma 4.1

Proof of Lemma 4.1.

Case 3 (small asymmetries): $\alpha_i\tau_i^{SF} < \alpha_{3-i} < \alpha_i\frac{\lambda_{3-i}}{\lambda_i}$ (or $\alpha_{3-i}\tau_{3-i}^{SF} < \alpha_i < \alpha_{3-i}\frac{\lambda_i}{\lambda_{3-i}}$).

For the Stackelberg leader we obtain

$$\begin{aligned}
&\frac{\partial^2\pi_i^{SL}(q_i,q_{3-i},c,\alpha_i,\alpha_{3-i})}{\partial\alpha_i^2} \\
=&\frac{\partial^2\left[\left(p_i^{SL}(\alpha_i,\alpha_{3-i})-\lambda_i c^S(\alpha_i,\alpha_{3-i})\right) q_i^{SL}(\alpha_i,\alpha_{3-i})\right]}{\partial\alpha_i^2} \\
=&2\left(\frac{\partial p_i^{SL}(\alpha_i,\alpha_{3-i})}{\partial\alpha_i}-\lambda_i\frac{\partial c^S(\alpha_i,\alpha_{3-i})}{\partial\alpha_i}\right)\frac{\partial q_i^{SL}(\alpha_i,\alpha_{3-i})}{\partial\alpha_i}.
\end{aligned}$$

As we know that

$$\frac{\partial p_i^{SL}(\alpha_i,\alpha_{3-i})}{\partial\alpha_i} = \left(\frac{2-\gamma^2}{2}\right)\frac{\partial q_i(\alpha_i,\alpha_{3-i})}{\partial\alpha_i} + \lambda_i\frac{\partial c^S(\alpha_i,\alpha_{3-i})}{\partial\alpha_i}$$

we get

$$\begin{aligned}
&\frac{\partial^2 \pi_i^{SL}\left(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i^2} \\
=&2\Big(\left(\frac{2-\gamma^2}{2}\right)\frac{\partial q_i\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} + \lambda_i \frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} \\
&- \lambda_i \frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i}\Big) \cdot \frac{\partial q_i^{SL}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} \\
=&\left(2-\gamma^2\right)\left(\frac{\partial q_i\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i}\right)^2 \\
=&\left(2-\gamma^2\right)\left(\frac{1}{2\left(2-\gamma^2\right)}\left(2-\left(2\lambda_i-\gamma\lambda_{3-i}\right)\frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\alpha_i}\right)\right)^2 \\
=&\frac{1}{4\left(2-\gamma^2\right)}\left(2-\left(2\lambda_i-\gamma\lambda_{3-i}\right)\frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\alpha_i}\right)^2 \\
=&\frac{1}{4\left(2-\gamma^2\right)}\left(2-\frac{\left(2\lambda_i-\gamma\lambda_{3-i}\right)^2}{\left(2\lambda_i\left(2\lambda_i-\gamma\lambda_{3-i}\right)+\lambda_{3-i}\left(4\lambda_{3-i}-\gamma^2\lambda_{3-i}-2\gamma\lambda_i\right)\right)}\right)^2 > 0
\end{aligned}$$

for all $\alpha_i \in \left[\underline{\alpha_i}, \overline{\alpha_i}\right]$ for $i = 1, 2$.

For the Stackelberg follower within a duopoly we have

$$\begin{aligned}
&\frac{\partial^2 \pi_i^{SF}\left(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i^2} \\
=&\frac{\partial^2\left[\left(p_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)-\lambda_i c^S\left(\alpha_i, \alpha_{3-i}\right)\right) q_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)\right]}{\partial \alpha_i^2} \\
=&2\left(\frac{\partial p_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} - \lambda_i \frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i}\right)\frac{\partial q_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i}.
\end{aligned}$$

As we know that

$$\frac{\partial p_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} = \frac{\partial q_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} + \lambda_i \frac{\partial c^S}{\partial \alpha_i}$$

we obtain

$$\begin{aligned}
&\frac{\partial^2 \pi_i^{SF}\left(q_i, q_{3-i}, c, \alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i^2} \\
=&\left(\frac{\partial q_i^{SF}\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} + \lambda_i \frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i} - \lambda_i \frac{\partial c^S\left(\alpha_i, \alpha_{3-i}\right)}{\partial \alpha_i}\right)
\end{aligned}$$

$$
\begin{aligned}
&\cdot \frac{\partial q_i^{SF}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i} \\
=&2\left(\frac{\partial q_i^{SF}(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right)^2 \\
=&\frac{1}{8\left(2-\gamma^2\right)^2}\left(\left(4-\gamma^2\right)-\left(4\lambda_i-\gamma^2\lambda_i-2\gamma\lambda_{3-i}\right)\frac{\partial c^S(\alpha_i, \alpha_{3-i})}{\partial \alpha_i}\right)^2 \\
=&\frac{1}{8\left(2-\gamma^2\right)^2}\left(\left(4-\gamma^2\right)-\frac{\left(4\lambda_i-\gamma^2\lambda_i-2\gamma\lambda_{3-i}\right)^2}{2\left(2\lambda_{3-i}\left(2\lambda_{3-i}-\gamma\lambda_i\right)+\lambda_i\left(4\lambda_i-\gamma^2\lambda_i-2\gamma\lambda_{3-i}\right)\right)}\right)^2 \\
&>0
\end{aligned}
$$

for all $\alpha_i \in \left[\underline{\alpha_i}, \overline{\alpha_i}\right]$ for $i = 1, 2$.

□

Chapter 5

Contract Design for Composed Services

5.1 Introduction

On the worldwide market for IT services with a huge number of single services clients often search from a solution-oriented point of view. Moreover, the evolution of the "cloud" as an infrastructure with flexible and on demand access to applications has the effect that "[...] the computing world is rapidly transforming towards developing software for millions to consume as a service, rather than to run on their individual computers." (Buyya *et al.*, 2009, p. 599). Our article analyzes the interaction between providers of such single services and an intermediary. The intermediary combines these services and sells them as a composed service on the market. We contribute to a broader research agenda called "On-The-Fly (OTF) Computing" which investigates the economic and technical challenges for dynamic automated service composition in a cloud computing environment. The main goal of OTF computing is to configure and provide individual IT services in a flexible way to overcome inefficiencies related to traditional software solutions. On such a market an intermediary is indispensable, since clients usually do not have the necessary knowledge and expertise to find the desired services themselves and, moreover, to combine them flexibly. The intermediary's task is to create a link between clients and service providers producing single services. He proposes composed services as solutions that combine different single services to a new product. The vision of OTF computing is briefly presented in Happe *et al.* (2013). As Sundareswaran *et al.* (2012) points out "Aggregating service providers is very challenging in the Cloud due to complex relationship among Cloud service providers that are built via subcontracting."

More specifically, in this article we focus on the contract design problem of one intermediary and two service providers who strategically interact on the market. We use a non-cooperative game-theoretic approach for our analysis. The service providers deliver complementary services which they can choose to produce in either high or low quality. For services in a cloud computing environment this quality decision may be an important factor. It crucially influences the performance of the service composition and results in additional costs or effort for the service providers. Examples are the bandwidth provided, resources made available for the execution of a service as well as the priority given to the inquiry of the intermediary. Besides choosing the quality, the service providers may deliver their single services in various quantities, such as number of licenses to use the provided service, number of instances, utilization time available for execution, or the amount of storage space. The intermediary receives the single services and combines them to a composed service. The quality of the composed service depends on the inputs' qualities and can also be either high or low. The intermediary strategically reports the quality of the composed service to his input suppliers and pays them accordingly. We assume that he is only able to observe the quality of the composed service, but is not aware of the two single services' qualities. A typical feature of a composed service is that its quality properties are dependent on those of the single services and their interaction when the composed service is executed. Quality properties of the single services can only be tested to a limited extent prior to composition. For example, in a cloud computing environment it may technically be difficult or too costly to individually test the quality of a software service or of a hardware service that is used for execution. On the demand side we suppose that clients have a demand for both composed services of high and low quality (usually at different prices).

Besides considering a one-shot game we analyze a repeated interaction between the intermediary and his suppliers. Here, the intermediary initially offers contracts to the sellers in which he specifies the payment he is willing to make, depending on the reported quality of the composed service. While in the short run producing low-quality composed services is always a Nash equilibrium (Proposition 5.1) and producing high-quality composed services is never a Nash equilibrium (Proposition 5.2), the long run situation is more promising. We identify conditions on the time preferences (discount factors) under which high quality is a subgame-perfect equilibrium on the market in the long run. The main finding is that even if selling high quality is profitable for the intermediary, the emergence of high-quality composed services on the market still crucially depends on the intermediary's discount factor (Proposition 5.3). We identify optimal payments of the intermediary to implement high- or low-quality composed services (Propositions 5.4 and 5.5).

5.2 Related Literature

Our model is related to the literature on principal-agent relationships as well as on reputation and repeated interaction. We use this section to describe similarities and differences in comparison to our approach.

The price model in the procurement situation that we study is related to the modeling of contracts in principal-agent relationships. For example, if the agent's contribution to the firm's value is not directly observable, Baker *et al.* (1994) addresses how to design compensation contracts consisting of a base salary and a bonus payment in a repeated setting. A similar procurement problem with respect to price and quality considerations is also discussed by Asker and Cantillon (2010). They derive the optimal procurement mechanism and compare it with alternatives such as scoring auctions and (sequential) bargaining. The procurement situation in Laussel and Long (2011) is related to the one we assume here, but the focus of the analysis is different. Having several intermediate-input suppliers they study the incentives of vertical integration in a dynamic model. In comparison to the approaches just mentioned, even if the relationship between the intermediary and its service providers resembles a principal-agent model, we additionally incorporate the influence of the clients' demand on the intermediary's profit function. Therefore, the profit is influenced simultaneously by two aspects: on the one hand the effort or quality choice of the service provider and on the other hand the resulting client's demand in dependence on the quality of the composed service. Moreover, by considering composed services we are interested in the right coordination of the service providers' quality choices.

In a setting with repeated interaction, experiences that were made with a certain product in the past have an influence on its future evaluations as well as future sales opportunities. Dellarocas (2005) refers to this as "reputation". Similarly, already Shapiro (1983) argues:

> "When product attributes are difficult to observe prior to purchase, consumers may plausibly use the quality of products produced by the firm in the past as an indicator of present or future quality. In such cases a firm's decision to produce high quality items is a dynamic one: the benefits of

> doing so accrue in the future via the effect of building up a reputation. In this sense, reputation formation is a type of signaling activity: the quality of items produced in previous periods serves as a signal of the quality of those produced during the current period."

We follow this definition of reputation for our analysis. Similar to our approach, Dana and Fong (2011) analyzes the influence of the market structure on the incentives to produce high quality in a repeated game setting. While they focus on the comparison of different market structures, our analysis highlights the interaction and contract design of an intermediary and its suppliers. In addition, we relax the assumption that customers have a unit-demand. Tang and Cheng (2005) consider a market in which an intermediary offers a composed service. In their setting, a user has to decide to buy two complementary web services either from a web service intermediary or directly from its providers. Optimal location and price choices of the web service intermediary are analyzed in a spatial model, more precisely in a linear and circular city model. Contrary to their approach we focus on the optimal contract design between the intermediary and its service providers.

Reputation systems for online markets are explicitly addressed by Dellarocas (2005). He studies the mechanism design problem of a reputation system where buyers and sellers directly interact with each other. A monopolistic seller exerts an effort that influences the quality of the product. After having experienced the quality of the good the buyer reports his satisfaction of the product. These reports are summarized in the reputation profile of the seller. A repeated game setting is used to analyze efficiency and seller payoffs by varying the number of observations in the reputation profile and by extending his model to multi-values including incomplete reports and multiple competing sellers. Our analysis, however, does not include a reputation system. We assume that the seller is always able to sell his products at the market by adapting prices to demand. From an architectural perspective, including reputation information in the OTF computing process has been discussed in Brangewitz *et al.* (2014b). Intermediaries selling composed services are also analyzed in the literature on Cloud Services Brokerage as for example in Sundareswaran *et al.* (2012), Giovanoli *et al.* (2014), Jula *et al.* (2014). We complement previous works by considering contract design issues from an economic perspective.

The analysis proceeds as follows: First, we present the model. Then we analyze the different decisions considering the trade of services and the contract design. Finally, we conclude and comment on further extensions.

5.3 The Model

In our setting we consider three types of market participants: clients, intermediaries, and service providers. Our focus is on modeling the strategic interaction of one intermediary and two service providers. The intermediary chooses a long run contract that he offers to the service providers. Then the service providers strategically choose the quality and quantity of the service they would like to provide. The intermediary afterwards reports (not necessarily truthful) the quality of the composed service he produced from the service providers' inputs. The clients are assumed to be non-strategic.

5.3.1 Long run contracts

Before services are actually traded the intermediary makes a decision on the quality he is willing to deliver to the market and the long run contracts he offers to the service providers. He privately observes his clients' demands for the composed service which is demanded in low (L) and high quality (H). Furthermore, he is aware of the service providers' cost functions for producing (single) services in high or low quality. Therefore, he is able to determine his expected profit possibilities for the "two" markets. He compares them and afterwards chooses the quality he prefers to deliver to his clients.

Based on this strategic quality choice, the intermediary designs contracts specifying the way the service providers are paid. These contracts are of the following form: for delivering a service a service provider receives a base payment independently of the quality of his service. This payment is equivalent to the payment for a low-quality service. Furthermore, the intermediary *promises* him an additional payment, if the observed quality of the composed service, which is dependent on the quality of both inputs, is high. For the interaction with his client the intermediary takes the client's demand functions as given and determines the sales prices as well as the sales quantities that maximize his profits. Then, using the contract design and the optimal sales quantities, he explicitly determines the transfer payments and offers them in the form of contracts to the service providers. More precisely, the transfer payments towards a service provider depend on the observable quantity of the single service he delivered and the quality of the composed service reported by the intermediary.

5.3.2 Market interaction

In the next step, the services are traded on the market. We start to describe the *procurement side*. For simplicity, we suppose that the intermediary needs one unit of each service to produce one unit of composed service. In every period, each service provider decides on which of the demanded quantities, the optimal sales quantity for either high or low quality, he delivers and if the service is of quality L or H. Service provider i's cost function is denoted by $C^i_{\theta^i} : \mathbb{R}_+ \to \mathbb{R}_+$ for quality $\theta^i \in \{H, L\}$ and $i \in \{1, 2\}$. We assume convex cost functions and higher cost to deliver high quality services, $C^i_H(\cdot) > C^i_L(\cdot)$. Convexity means that marginal costs are non-decreasing, i.e., providing an additional instance (or unit) is at least as expensive as the previous one. The quality of the composed service depends on the quality choices of the service providers and is defined as follows[1]

$$\theta^0 : \{L, H\}^2 \to \{L, H\} \quad \text{with } \theta^0\left(\theta^1, \theta^2\right) = \begin{cases} H & \text{if } (\theta^1, \theta^2) = (H, H) \\ L & \text{otherwise.} \end{cases} \tag{5.1}$$

The intermediary is able to directly observe the delivered quantities. The suppliers' quality choices are not directly observable and can only be uniquely determined in certain cases by considering the quality of the composed service. The intermediary decides whether to report this quality truthfully or not. The reported quality is defined by $\hat{\theta}^0 : \{L, H\} \to \{L, H\}$ such that $\theta^0 \mapsto \hat{\theta}^0(\theta^0)$. According to the report he rewards the service providers with a transfer payment of $T^i_H, T^i_L : \mathbb{R}_+ \to \mathbb{R}_+$ for $i \in \{1, 2\}$ specified in the long run contract.[2] We assume convex transfer payments that are differentiable at 0 and suppose $T^i_H(\cdot) \geq T^i_L(\cdot) \geq C^i_L(\cdot)$. This reflects the idea, that in any case the intermediary pays $T^i_L(\cdot)$ and if he reports high quality the service providers receive $T^i_H(\cdot) - T^i_L(\cdot)$ additionally as a bonus. The actions of the service providers, i.e., choosing the quantity and quality of the delivered services, as well as of the intermediary whether to report truthfully or not are assumed to be chosen simultaneously.

On the *demand side* the intermediary is facing a client who has a certain demand for both composed services of quality H and of quality L. The client's demand functions (for low and high quality) assign to any positive price P the demanded quantities

[1] The superscript 0 indicates that the quality of the composed service is observed by the intermediary, denoted in the following as player 0.

[2] Note that the subscript relates to the reported quality $\hat{\theta}^0\left(\theta^0\right) \in \{H, L\}$.

and are given by $D_{\theta^0} : \mathbb{R}_+ \to \mathbb{R}_+$ with $P \mapsto D_{\theta^0}(P)$ for quality $\theta^0 \in \{L, H\}$ of the composed service. Thus, when charging a price of P for the composed service, the intermediary faces a demand of his client of $D_{\theta^0}(P)$ units of the composed service with quality θ^0. For our subsequent analysis we make some technical assumptions related to the client's demand functions. Suppose $D_{\theta^0}(P)$ is twice continuously differentiable, non-increasing, strictly decreasing whenever $D_{\theta^0}(P) > 0$ and satisfies $D''_{\theta^0}(P)D_{\theta^0}(P) - 2\left(D'_{\theta^0}(P)\right)^2 < 0$. The inverse demand function assigning a price to a given demand is denoted by $P_{\theta^0}(D)$.[3] Suppose for a given price P we always have $D_H(P) > D_L(P)$. Moreover, demand and transfer payments are assumed to satisfy $P_{\theta^0}(0) > T'_{\theta^0}(0)$.

Our focus for the succeeding analysis is on the information asymmetries between the intermediary and the service providers. We assume complete information on the demand side, the intermediary delivers the demanded quality of the final product truthfully to his clients.

The remainder of this section is used to explain the assumptions we made by means of an example illustrated in Fig. 5.1. Suppose a client has an own program routine and wants to analyze a huge data set by running simulations. As the computations are complex and resource-intense, the client is not able to execute his program within his own infrastructure. Therefore, he decides to make use of cloud services. He approaches an OTF intermediary who is offering a composed service that is able to process the client's demand. The OTF intermediary flexibly combines a service for storage with a service for computing that he purchases from other service providers. For a given price per instance, the quantity demanded by the client is the number of instances he is willing to book for the execution of his simulations. Clearly, a higher price triggers a lower demanded quantity. Quality properties of the single services are the access time for the storage and the time needed by the computing service. The quality of the composed service is the overall time per instance to run the client's simulations. If the composed service is of high quality, e.g. the total process time is short, the client has a high willingness to pay. Accordingly, in case of a low-quality composed service he accepts a longer processing time while having a lower willingness to pay. A comprehensive analytical example for specific demand and cost functions to illustrate our results of the next sections can be found in Appendix 5.7.

[3] The third assumption on the demand function is related to its curvature and can be equivalently expressed in terms of elasticities as $D''_{\theta^0}(P)P/D'_{\theta^0}(P) > 2D'_{\theta^0}(P)P/D_{\theta^0}(P)$. This assumption ensures that $DP_{\theta^0}(D)$ is strictly concave and is derived from $DP''_{\theta^0}(D) + 2P'_{\theta^0}(D) < 0$, using the derivative of the inverse demand function. The three assumptions on the demand function are also made in Dana (2001), for example.

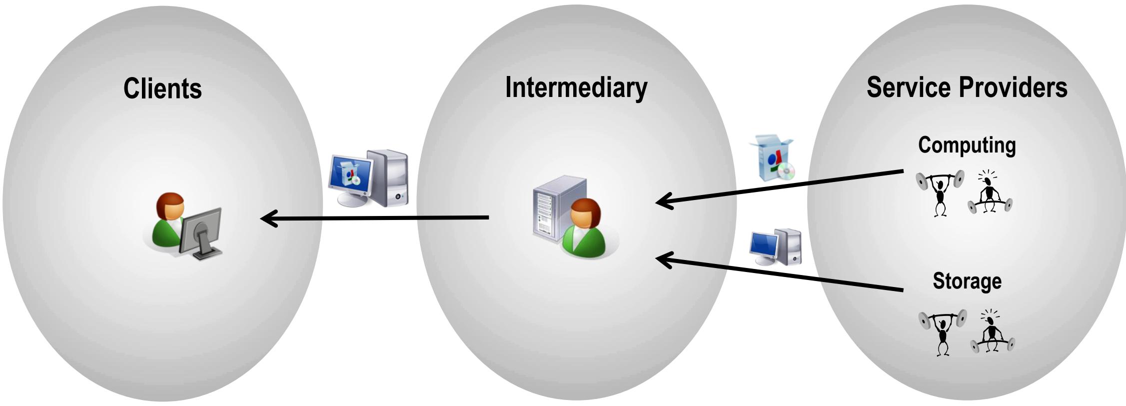


Figure 5.1: Composed services consisting of hardware and software services

5.4 Trading Services

5.4.1 Optimal sales prices and quantities

Given the contract $T = (T^i_L(\cdot), T^i_H(\cdot))_{i=1,2}$ the intermediary determines the optimal quantity $D^{T*}_{\theta^0}$ he is willing to sell to his client for a fixed quality θ^0 of the composed service. This is the client's demanded quantity that maximizes the intermediary's profit. With this optimal demand the optimal sales price denoted by $P^{T*}_{\theta^0} := P_{\theta^0}(D^{T*}_{\theta^0})$ can be computed. Hereby, the intermediary assumes that both service providers deliver him services of the same quality $\theta^1 = \theta^2$ and therefore the resulting quality of the composed service is $\theta^0 = \theta^1 = \theta^2$. Formally, the intermediary $D^{T*}_{\theta^0} = \text{argmax}_{D \in \mathbb{R}_+} DP_{\theta^0}(D) - \sum_{i=1}^{2} T^i_{\theta^0}(D)$. Note that due to our assumptions a maximizer $D^{T*}_{\theta^0}$ exists and is unique. The function $DP_{\theta^0}(D)$ is strictly concave and the transfer payments $T^i_\theta(\cdot)$ are convex. Therefore, the objective function is strictly concave. Moreover, we have $P_{\theta^0}(0) > T'_{\theta^0}(0)$ ensuring positive profits.Using the contract design and the optimal sales quantities the intermediary offers the following contracts to the service providers $\left(T^i_L\left(D^{T*}_L\right), T^i_L\left(D^{T*}_H\right), T^i_H\left(D^{T*}_L\right), T^i_H\left(D^{T*}_H\right)\right)_{i=1,2}$. The service providers decide whether to choose a quantity D^{T*}_L or D^{T*}_H as well as the quality, either L or H, they deliver to the intermediary.

5.4.2 Trading once

We use a non-cooperative game in strategic form with three players, which we first consider as a one-shot game and in the next subsection as a repeated game. We denote the intermediary as player 0 and the two service providers as players 1 and 2. The strategy sets are $\{(L, D_L^{T*}), (L, D_H^{T*}) (H, D_L^{T*}), (H, D_H^{T*})\}$ for the service providers and $\{\text{truthful}, \text{not truthful}\}$ for the intermediary. The intermediary's strategy "truthful" indicates that he reports the composed service's quality honestly and "not truthful" means that he always pretends to have seen low quality. He chooses the strategy to be *truthful* if $\hat{\theta}^0(L) = L$ and $\hat{\theta}^0(H) = H$. If he reports $\hat{\theta}^0(L) = L$ and $\hat{\theta}^0(H) = L$, we say that he is *not truthful*. From now on we refer to the composed service's quality θ^0 and reported quality $\hat{\theta}^0$ for short. For a delivered quantity $D^i \in \{D_L^{T*}, D_H^{T*}\}$ of service provider $i \in \{1, 2\}$ we define $D_{\min} := \min_{i \in \{1,2\}} D^i$. Given a strategy profile the payoff of service provider $i \in \{1, 2\}$ is his profit $\Pi^{T,i}\left(\hat{\theta}^0, (\theta^1, D^1), (\theta^2, D^2)\right) = T_{\hat{\theta}^0}^i(D^i) - C_{\theta^i}^i(D^i)$. With the quantities the intermediary receives from the service providers he is able to produce maximally $D_{\min}$ units of the composed service. Therefore, the intermediary needs to reconsider his revenue maximization problem. He chooses the sales price of the composed service such that his profits are maximized given the capacity constraint $D_{\min}$, that is $P_{\theta^0}^{T**} = P_{\theta^0}\left(D_{\theta^0}^{T**}\right)$ with $D_{\theta^0}^{T**} = \text{argmax}_{D_{\theta^0}^T \leq D_{\min}} P_{\theta^0}\left(D_{\theta^0}^T\right) D_{\theta^0}^T - \sum_{i=1}^2 T_{\hat{\theta}^0}^i(D^i)$ yielding a profit of $\Pi^{T,0}\left(\hat{\theta}^0, (\theta^1, D^1), (\theta^2, D^2)\right) = P_{\theta^0}^{T**} D_{\theta^0}^{T**} - \sum_{i=1}^2 T_{\hat{\theta}^0}^i(D^i)$ with $P_{\theta^0}^{T**} := P_{\theta^0}\left(D_{\theta^0}^{T**}\right)$.[4]

The most well-known solution concept for non-cooperative games is the Nash equilibrium Nash (1951). A Nash equilibrium is a collection of strategy choices of the OTF provider and the service providers such that no one has an incentive to unilaterally change his strategy given the others' strategies. For the OTF market this means that in an equilibrium the OTF provider cannot increase his profit from changing his report for given quality choices of the service providers and each service provider cannot increase his profit from unilaterally changing the produced quality or quantity. We observe the following equilibrium result, when services are traded just once.

Proposition 5.1. *Given the other service provider delivers low-quality services, then producing the correct quantity of* ***low quality*** *services is a Nash equilibrium strategy if and only if* $T_L^i\left(D_L^{T*}\right) - C_L^i\left(D_L^{T*}\right) \geq T_L^i\left(D_H^{T*}\right) - C_L^i\left(D_H^{T*}\right)$ *for* $i = 1, 2$.

Proof of Proposition 5.1. First, note that the intermediary is indifferent in being truthful or not. The condition in Proposition 5.1 is immediately derived from comparing the profits of the service providers in case of a deviation from $\left(L, D_L^{T*}\right)$. □

Proposition 5.1 demonstrates that there exists the possibility to choose the properties of the transfer payments and cost functions in such a way that producing low-

[4] Note that if $\theta^0 = \theta^1 = \theta^2$ and the service providers deliver $D_{\theta^0}^{T*}$, then $D_{\theta^0}^{T**} = D_{\theta^0}^{T*}$ and $P_{\theta^0}^{T**} = P_{\theta^0}^{T*}$.

quality composed services is a Nash equilibrium of the static game. If we choose $T_L^i(\cdot) = C_L^i(\cdot)$ in Proposition 5.1 we obtain immediately that independently of the transfer payments for high quality $T_H^i(\cdot)$, producing low-quality services is a Nash equilibrium strategy.

Proposition 5.2. *Producing **high-quality** composed services is not a Nash equilibrium.*

Proof of Proposition 5.2. If $T_H^i(\cdot) > T_L^i(\cdot)$, the intermediary has an incentive to deviate to not being truthful. For $T_H^i(\cdot) = T_L^i(\cdot)$ service provider i has an incentive to deviate in producing low quality as $C_H^i(\cdot) > C_L^i(\cdot)$. Moreover, if the service providers are not rewarded with a bonus for high quality, then each of them has an incentive to deviate to produce low quality as $C_H^i(\cdot) > C_L^i(\cdot)$. □

In contrast to Proposition 5.1, Proposition 5.2 shows that in the static game the intermediary has no possibility to end up with a high-quality composed service as there are always possibilities for improvement for either the intermediary or the service providers. In the next section we analyze if high-quality composed services can be obtained by considering the described one-shot trading game repeatedly.

5.4.3 Trading repeatedly

The previous section showed that producing high-quality composed services is not an equilibrium if the services are traded only once. Therefore, in this section we consider repeated interaction between the intermediary and the service providers assuming an infinite time horizon. Once agreed to produce either high- or low-quality composed services in the optimal quantities, the intermediary and the service providers repeatedly trade services accordingly. We analyze if the right incentives can be provided such that this agreement is sustainable in the long run even if it is supposed to be non-binding. In our setting neither demand nor cost structures vary over time.

A refinement of the Nash equilibrium concept that fits a repeated interaction structure is the subgame-perfect Nash equilibrium Selten (1965a,b). Applied to our context, in a subgame-perfect Nash equilibrium neither the OTF provider nor the service providers are willing to unilaterally change their long run strategies at any point in time, while taking the others' (equilibrium) strategies as given. We consider the long run profits of the OTF provider and the service providers and technically apply the "one-shot deviation principle" (Mailath and Samuelson, 2006, Proposition 2.2.1 p. 25) to determine subgame-perfect Nash equilibria.

A composed service with ***low quality*** may constitute a Nash equilibrium of the trading game as shown in Proposition 5.1. If this is the case, it is also a subgame-perfect Nash equilibrium of the repeated trading game. Concerning ***high-quality*** composed services, suppose that the objective of the intermediary and the service providers is to maximize their discounted profits by using a personal discount factor $\delta_i \in (0,1)$ $(i = 0,1,2)$ from trading *high-quality* services. Suppose for the rest of this section that the condition of Proposition 5.1 is satisfied and, therefore, producing low-quality composed services in the correct quantities is a Nash equilibrium of the one-shot trading game.

Within the repeated trading game we assume that the intermediary commits himself to report his observation of the quality of the composed service *truthfully* whereas the two service providers agree to deliver him the *optimal quantity* of *high quality* services: $\left(\text{truthful}, \left(H, D_H^{T*}\right), \left(H, D_H^{T*}\right)\right)$. From this agreement the intermediary can deviate in reporting his observation *not truthfully* or the service providers can deviate in delivering *low-quality* services or a *different quantity* of services. Cheating in one period is followed by an according punishment in the periods thereafter. In case, one of the service providers does not deliver high quality, the other service provider will also not be rewarded for the high quality he produced. Therefore, he is no longer willing to deliver high quality services in future. Similarly, if the intermediary does not report his observation truthfully, the punishment of the service providers is to deliver low quality services from now on. The punishment strategy profile is therefore denoted by: $\left(\text{not truthful}, \left(L, D_L^{T*}\right), \left(L, D_L^{T*}\right)\right)$. Taking this behavior into account, we obtain the following:

Proposition 5.3. *If $T_L^i\left(D_L^{T*}\right) - C_L^i\left(D_L^{T*}\right) \geq T_L^i\left(D_H^{T*}\right) - C_L^i\left(D_H^{T*}\right)$, then producing high-quality services (in the demanded quantities) and the intermediary reporting truthfully is a subgame-perfect Nash equilibrium of the repeated game if and only if the following conditions hold for the intermediary*

$$\left(P_H^{T*} D_H^{T*} - \sum_{i=1}^{2} T_H^i\left(D_H^{T*}\right)\right) \geq (1-\delta_0)\left(P_H^{T*} D_H^{T*} - \sum_{i=1}^{2} T_L^i\left(D_H^{T*}\right)\right) + \delta_0\left(P_L^{T*} D_L^{T*} - \sum_{i=1}^{2} T_L^i\left(D_L^{T*}\right)\right), \quad (5.2)$$

and for the service providers

$$\begin{aligned} & T_H^i\left(D_H^{T*}\right) - C_H^i\left(D_H^{T*}\right) \\ & \quad \geq (1-\delta_i)\max\left\{T_L^i\left(D_L^{T*}\right) - C_L^i\left(D_L^{T*}\right), T_H^i\left(D_L^{T*}\right) - C_H^i\left(D_L^{T*}\right),\right. \\ & \quad \left. T_L^i\left(D_H^{T*} - C_L^i\left(D_H^{T*}\right)\right)\right\} + \delta_i\left(T_L^i\left(D_L^{T*}\right) - C_L^i\left(D_L^{T*}\right)\right). \end{aligned} \tag{5.3}$$

Proof of Proposition 5.3. The first condition implies that the punishment strategy after a deviation is a Nash equilibrium strategy if the services are traded once as shown in Proposition 5.1. The conditions on the discount factors are derived by comparing the discounted long run payoffs (for an infinite time horizon). □

Proposition 5.3 gives us conditions on the critical discount factors needed to ensure that high-quality composed services are produced in the long run. These depend on the transfer payments, cost functions and implicitly on the client's demand function.

5.5 Contract Design

The intermediary offers long run contracts which specify the way the service providers are paid in dependence on the (reported) quality of the composed service. The optimal contracts for the intermediary crucially depend on his strategic decision on whether to sell high- or low-quality products to his clients. Therefore, we first analyze the optimal contract design for both cases separately.

5.5.1 Contracts for low-quality composed services

Proposition 5.4. *To produce low-quality composed services the intermediary optimally chooses the transfer payments to be* $T^{L*} = \left(T_L^{L*,i}(\cdot), T_H^{L*,i}(\cdot)\right)_{i=1,2} = \left(C_L^i(\cdot), C_L^i(\cdot)\right)_{i=1,2}$.

Given T^{L*} the service providers are always paid exactly their costs for producing low-quality services and therefore have zero profits. They have no interest in producing high-quality services since they are not rewarded accordingly and would thus suffer a negative profit. In contrast to the zero profits of the service providers, the intermediary is able to extract all the generated surplus and his profit is equal to $P_L^{T^{L*}*} D_L^{T^{L*}*} - \sum_{i=1}^{2} C_L^i\left(D_L^{T^{L*}*}\right)$.

5.5.2 Contracts for high-quality composed services

Using the conditions for the service providers from Proposition 5.3 we obtain:

Proposition 5.5. *To give the appropriate incentives to the service providers in order to produce high-quality composed services in the long run, the intermediary optimally chooses the transfer payments to be* $T^{H*} = \left(T_L^{H*,i}(\cdot), T_H^{H*,i}(\cdot)\right)_{i=1,2} = \left(C_L^i(\cdot), C_H^i(\cdot) + \varepsilon^i\right)_{i=1,2}$ *with small* $\varepsilon^i > 0$.

Given these transfer payments the service providers are paid exactly their costs for producing low-quality services and strictly more than their costs for producing high- quality services. Therefore, they have a strict interest in producing high-quality services in the long run. The intermediary is able to extract almost all the generated surplus and his profit is equal to $P_H^{T^{H*}*} D_H^{T^{H*}*} - \sum_{i=1}^2 C_H^i \left(D_H^{T^{H*}*}\right) - \sum_{i=1}^2 \varepsilon^i$. Hence, the parameter ε^i is chosen to be strictly positive, but as small as possible. Using Proposition 5.3 with $T_L^{H*,i}(\cdot) = C_L^i(\cdot)$ and $T_H^{H*,i}(\cdot) = C_H^i(\cdot) + \varepsilon^i$ we have:

Corollary 5.1. *If the intermediary's discount factor satisfies*

$$\delta_0 > \frac{\sum_{i=1}^2 C_H^i \left(D_H^{T^{H*}*}\right) + \sum_{i=1}^2 \varepsilon^i - \sum_{i=1}^2 C_L^i \left(D_H^{T^{H*}*}\right)}{\left(P_H^{T^{H*}*} D_H^{T^{H*}*} - \sum_{i=1}^2 C_L^i \left(D_H^{T^{H*}*}\right)\right) - \left(P_L^{T^{H*}*} D_L^{T^{H*}*} - \sum_{i=1}^2 C_L^i \left(D_L^{T^{H*}*}\right)\right)}, \quad \text{(D0)}$$

then there exist transfer payments such that the intermediary is interested in producing high-quality composed services.

Adding and subtracting the term $\sum_{i=1}^2 C_H^i \left(D_H^{T^{H*}*}\right)$ in the denominator on the lower bound for δ_0 in Corollary 5.1 gives the following interpretation: The evaluation of future payoffs of the intermediary represented by his personal discount factor δ_0 needs to be strictly greater than the difference in the transfer payments the intermediary has to make (for $D_H^{T^{H*}*}$) relative to this difference plus the advantage in his profits for high-quality composed services.

5.5.3 Long run quality choice

Using the optimal contract design, the intermediary compares the profits on the market of low-quality composed services with the profits on the market for high-quality composed services. We can distinguish three different situations on the market. If the following condition holds

$$P_H^{T^{H*}*} D_H^{T^{H*}*} - \sum_{i=1}^{2} C_H^i \left(D_H^{T^{H*}*} \right) - \sum_{i=1}^{2} \varepsilon^i > P_L^{T^{L*}*} D_L^{T^{L*}*} - \sum_{i=1}^{2} C_L^i \left(D_L^{T^{L*}*} \right) \quad \text{(HIGH)}$$

and the intermediary's discount factor δ_0 is sufficiently large, he prefers to deliver high-quality composed services to his clients and the service providers have the right incentives to produce high quality. In contrast, if condition (HIGH) holds but the intermediary's personal discount factor is not high enough, he intends to be willing to reward the delivery of high-quality services, but as soon as the service providers deliver high-quality services the intermediary does not pay the bonus as promised. This is anticipated by the service providers. Consequently, only low-quality composed services are produced. If the following condition holds

$$P_H^{T^{H*}*} D_H^{T^{H*}*} - \sum_{i=1}^{2} C_H^i \left(D_H^{T^{H*}*} \right) - \sum_{i=1}^{2} \varepsilon^i < P_L^{T^{L*}*} D_L^{T^{L*}*} - \sum_{i=1}^{2} C_L^i \left(D_L^{T^{L*}*} \right) \quad \text{(LOW)}$$

the intermediary prefers to deliver low-quality composed services.

5.6 Concluding Comments

With our model we analyzed the incentives for high and low quality to be an equilibrium outcome of the market for composed services in a cloud computing environment. Hereby, the intermediary's discount factor is crucial even if the market for high-quality composed services is advantageous in terms of expected profits. Therefore, as a first implication for the OTF market, not only the profitability of composed services is important but also the evaluation of future profits. Hence, we have shown that if the intermediary's discount factor is lower than the payments for the single services relative to the advantage of high-quality composed services, only composed services of low quality may appear on the market.

However, we made several simplifying assumptions and therefore our model can be extended in various directions. For our theoretical analysis we suppose that the quality of the composed service and the single services is either high or low. An obvious extension is to introduce a finite or infinite number of different quality levels. Such a modification enlarges the set of possible strategic choices of the services providers, has the effect that the composed services may be available in these numerous quality levels and requires that the client's demand is defined to take this into account. A different direction for future research is the extension of asymmetric information issues to the client-intermediary relation. So far each client had a demand for high- and low-quality composed services. Differently, we may assume that there are several types of clients and the intermediary faces a client type who either demands high- or low-quality composed services. Moreover, a different reasonable assumption, especially for cloud services, is that demand and cost structures vary over time. In this case the intermediary might have to renew his contracts from time to time or the contracts might have a specific durability during which services can be flexibly demanded and delivered at a certain price. Up to now the services we considered were complementary and the intermediary needed exactly one service of each type. Adding substitute services and along with this competition is another direction to extend the current model.

5.7 Appendix: Analytical Example

The client's demand typically decreases if the price of the composed service increases. As composed services of high quality are more valuable we suppose that for a given price they are demanded in greater quantities than composed services of low quality. The demand functions $D_{\theta^0}(P) = \frac{1}{\beta_{\theta^0}^2}\frac{1}{P^2}$ for $\theta^0 \in \{L, H\}$ with $\beta_L > \beta_H > 0$ have these properties. The service providers' costs are typically increasing in the quality they provide. Linear cost functions $C_{\theta^i}^i\left(D^i\right) = \gamma_{\theta^i} D^i$ for $\theta^i \in \{L, H\}$ with $\gamma_H > \gamma_L > 0$ for $i = 1, 2$ describe the service providers' costs. The profit from producing composed services of quality θ^0 from the optimal contracts of Proposition 5.4 and 5.5 is given by

$$PD_{\theta^0}(P) - \left(1 + \tilde{\varepsilon}_{\theta^0}\right)\sum_{i=1}^{2} C_{\theta^0}^i\left(D_{\theta^0}(P)\right) = \frac{P - 2\gamma_{\theta^0}\left(1 + \tilde{\varepsilon}_{\theta^0}\right)}{\beta_{\theta^0}^2 P^2} \tag{5.4}$$

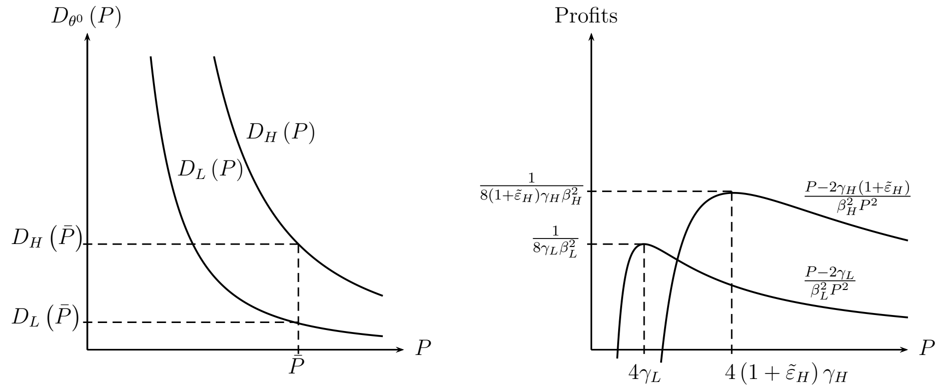
with $\tilde{\varepsilon}_L = 0$ and $\tilde{\varepsilon}_H > 0$. To simplify the calculations we use $\tilde{\varepsilon}_H$ as a multiplicative and not as an additive term (Proposition 5.5). Profit maximization yields

$$P_{L*}^{T^{L*}*} = P_{H*}^{T^{L*}*} = P_{L*}^{T^{H*}*} = 4\gamma_L \quad \text{and} \quad D_L^{T^{L*}*} = D_H^{T^{L*}*} = D_L^{T^{H*}*} = \frac{1}{16\gamma_L^2\beta_L^2}$$

and for high-quality composed services

$$P_H^{T^{H*}*} = 4\left(1 + \tilde{\varepsilon}_H\right)\gamma_H \quad \text{and} \quad D_H^{T^{H*}*} = \frac{1}{16\left(1 + \tilde{\varepsilon}_H\right)^2\gamma_H^2\beta_H^2}.$$

By the choice of the parameter $\gamma_L < \gamma_H$ we observe immediately that the price the intermediary charges for high quality is strictly greater than the price he charges for low quality, whereas the effect on the quantity remains ambiguous. Figure 5.2(a) illustrates the client's demand functions and Fig. 5.2(b) the short run profits of the OTF provider for low- and high-quality composed services.



(a) Demand functions (b) Short run profits

Figure 5.2: Example for $\beta_H = \frac{1}{2}$, $\beta_L = 1$, $\gamma_H = \frac{1}{3}$, $\gamma_L = \frac{1}{8}$, $\tilde{\varepsilon}_H = \frac{1}{100}$

Consider the long run quality choice of the intermediary. The conditions derived in Section 5.5.3 and Corollary 5.1 are for our example

$$\frac{1}{(1+\tilde{\varepsilon}_H)\gamma_H\beta_H^2} > \frac{1}{\gamma_L\beta_L^2}, \tag{Ex-HIGH}$$

$$\frac{1}{(1+\tilde{\varepsilon}_H)\gamma_H\beta_H^2} < \frac{1}{\gamma_L\beta_L^2}, \tag{Ex-LOW}$$

$$\delta_0 \geq \frac{(1+\tilde{\varepsilon}_H)\gamma_H - \gamma_L}{(1+\tilde{\varepsilon}_H)\gamma_H - \gamma_L + (1+\tilde{\varepsilon}_H)^2\gamma_H^2\beta_H^2\left(\frac{1}{(1+\tilde{\varepsilon}_H)\gamma_H\beta_H^2} - \frac{1}{\gamma_L\beta_L^2}\right)}. \tag{Ex-D0}$$

The three possible market situations are as follows: If (Ex-HIGH) and (Ex-D0) hold, high-quality composed services are produced. If (Ex-HIGH) holds and (Ex-D0) does not hold, low-quality composed services are produced even if high-quality composed services yield a higher profit for the intermediary if the services are traded once. If (Ex-LOW) holds, low-quality composed services are produced. Inserting the values from Fig. 5.2 yields that high-quality composed services are produced if $\delta_0 \geq 0.66$. This means as soon as future profits are considered to be sufficient attractive high-quality composed service will the the equilibrium outcome of the market.

Chapter 6

Outlook

Getting a broad understanding of intermediate goods markets and analyzing its structure's impact on the participants' economic behavior and interactions was the main objective of this thesis. In the previous chapters, it was shown that various challenges arise due to its two-sided structure and the accompanied interdependencies, its dynamic nature and the presence of asymmetric information with respect to inputs' and final products' quality. Furthermore, as intermediaries must not necessarily be symmetric in terms of productivity and offered product quality, the resulting competitive advantages/disadvantages have indeed an impact on all participants' decisions taken in equilibrium. All of the mentioned challenges not covered in this thesis are left for future research.

Within the analysis, methods of non-cooperative game theory were primarily applied, i.e., Nash equilibrium outcomes in static as well as repeated games determined and analyzed, optimal contracts derived and much more.

After introducing the market structure, clarifying the necessity of intermediaries and giving an extensive literature overview with respect to oligopoly theory in Chapter 1 and Chapter 2, within Chapter 3 and Chapter 4 a differentiated duopoly model was considered. We analyzed a setting in which two intermediaries procured inputs from a monopolistic supplier refined them and offered a final product to a representative customer. Differentiation in our context referred to horizontal and vertical differentiation. Thus, final products could on the one hand be substitutes, complements or mutually independent (horizontal differentiation) and on the other hand have different product qualities (vertical differentiation). Intermediaries were allowed to be asymmetric in terms of their productivity as well as their products' quality. Each of the intermediaries had the chance to foster product quality by investing in product innovation whereas productivity was exogenously given. The focus of our analysis within both chapters was put on the intermediaries' simultaneous competition for the customer's demand on the sales side and for resources on the procurement side of the market.

In contrast to Chapter 4, within Chapter 3 we assumed that intermediaries' competition for the customer's demand was conducted by either simultaneously choosing production quantities (Cournot competition) or sales prices (Bertrand competition). Our analysis considered two stages: a stage of innovation and a stage of competition.

Within the first stage of innovation we analyzed the intermediaries' incentives to take an investment that fosters their product's quality. For linear costs of investment, we found that intermediaries always choose an investment which results in either a minimum or a maximum level of product quality. For intermediaries with symmetric productivities we derived equilibrium conditions for product quality investments on the investment costs and compared the results for Cournot and Bertrand competition as well as for complements and substitutes. In this context we have seen that there exist product qualities such that for complements only symmetric equilibria are present, while the existence of asymmetric equilibria can be guaranteed for close substitutes. Thus, according to our intuition, within an environment of complementary products and less severe competition, intermediaries rather tend to coordinate their investments. However, in an environment of fierce competition with substitute products equilibria of non-coordination may also be present.

Regarding the second stage which considered competition, our focus was put on analyzing the impact of intermediaries' asymmetries on the market outcome. It turned out that the presence of a strategic input supplier may lead to an exclusion of one or both intermediaries resulting in a monopoly intermediate goods market or no trade. Our main finding in this context showed that there exist asymmetries between intermediaries such that a monopoly market is obtained if quantities are chosen where a duopoly market is present if prices are selected. This observation is due to the fact that given intermediaries compete à la Bertrand the input supplier's profit is always higher than if intermediaries face Cournot competition. Additionally, it turned out that there exist asymmetries between intermediaries such that for non-substitute products, under quantity competition equilibrium prices and profits are lower and equilibrium quantities are higher than under price competition. Thus, in a setting in which the input market is explicitly modeled and asymmetries between intermediaries are present, the results of Singh and Vives (1984) do not necessarily hold.

Differently, within Chapter 4 intermediaries competed by choosing their production quantities sequentially (Stackelberg competition). In the first stage we gave a first overview of the analysis considering intermediaries' incentives to invest in product quality. Similarly to the previous chapter, first results showed that given costs of investment are linear, in equilibrium intermediaries always choose an investment that either results in a minimum or maximum level of product quality.

In the second stage focus was put on the intermediaries' competition for the customer's demand. In this context we compared the equilibrium decisions of an intermediary when being the Stackelberg leader with his equilibrium decisions when being the Stackelberg follower. In general and analogously to the results of Chapter 3 it turned out that the presence of a strategic input supplier may lead to an exclusion of one or both intermediaries. We have shown that there exist asymmetries between intermediaries such that a less competitive Stackelberg leader is excluded whereas a less competitive Stackelberg leader is accepted in the market, ceteris paribus. Thus, being the Stackelberg follower must not necessarily be a disadvantage. Furthermore, we observed that a more competitive intermediary always offers higher quantities when being the Stackelberg leader than when being the Stackelberg follower. However, for complementary products there exist asymmetries such that a less competitive follower offers higher equilibrium quantities than a less competitive leader, showing that the classical results of Von Stackelberg (1934) do not necessarily hold in our setting. Additionally, we examined and compared equilibrium input prices for different role distributions. In this context it turned out that for equal productivities of intermediaries the input supplier demands higher equilibrium input prices and achieves higher profits if the more competitive player is the Stackelberg leader compared to the case in which he is the Stackelberg follower.

Beyond the scope of the current analysis, further investigations and extensions of the discussed settings are left for future research. For instance, within Chapter 4 the innovation stage gave a first outlook for investigating the intermediaries' optimal investment in product innovation. In a further contribution, this section may be extended by a detailed examination of Nash equilibrium investments. One focal point can be a comparison of the Stackelberg leader's and the Stackelberg follower's incentive to invest in product innovation. Furthermore, the equilibrium conditions for product quality investments on the investment costs are quite interesting and should be compared for the Stackelberg leader and Stackelberg follower as well as for complements and substitutes. Besides this, a connection to the outcome of Chapter 3 may be drawn.

From previous contributions we know that competition in which decisions are taken sequentially is not restricted to choosing production quantities, but can also be carried out by choosing sales prices. Hence, within future work Chapter 4 could be extended by sequential price competition and the resulting equilibrium outcomes be analyzed. The first- and second-mover advantages should be determined and a comparison to the already existing results established in Chapter 4 be drawn.

Our analysis within Chapter 3 and Chapter 4 was limited to discussing a monopolistic and strategically acting input supplier who was only able to charge a unique price to both of the intermediaries. In a further extension of our models, one can think of the supplier being able to discriminate in prices, i.e., charge different input prices to the intermediaries. In this context the impact of intermediaries' asymmetries on the input supplier's price choice needs to be analyzed. Moreover, the supplier's incentives to exclude one of the intermediaries when being able to discriminate in prices could be a main focus. In a further step a comparison of the results with the outcome already determined within this work may be conducted.

As a previously identified challenge but not discussed within this work, the dynamic aspect of intermediate goods markets is another interesting field and left for future research. Specifically, on the procurement side of the market it can be allowed for a large number of input suppliers providing complementary, substitute as well as mutually independent products. In such a setting suppliers may compete by either simultaneously or sequentially choosing prices or production quantities. The dynamic aspect could be considered by modeling a repeated trade game in which new input suppliers may enter while established suppliers may leave the market. Thus, when selecting the input supplies for combining a final product, an intermediary always needs to reconsider his profit maximization problem period for period. Similarly, the dynamic nature may also play a key role on the market of intermediaries, where a high fluctuation of market participants is conceivable. Hence, when determining the equilibrium production quantities intermediaries must always be aware of a constantly changing competitive environment. Furthermore a customer demanding final products in multiple time periods always needs to reconsider his problem of choosing the intermediary such that overall utility is maximized.

An intermediate goods market, slightly different than the previously discussed models was considered in the final Chapter 5. The focus was put on the contract design problem of one intermediary and two strategically interacting service providers. Specifically, we analyzed a market in which one intermediary procured two complementary goods from two input suppliers, combined them and offered the resulting final product to a representative customer. Both service providers were able to provide their products in either high or low quality. A key issue was given by the fact that the intermediary was not able to directly observe the input goods', but only the final products' quality which resulted in a problem of moral hazard. We found that in a one shot as well as a repeated game a low-quality final product can be obtained in equilibrium. A high-quality final product, however, can only be achieved in the long run. Moreover it turned out that even if providing a high-quality final product is desirable for the intermediary, the existence of high-quality products crucially depends on the intermediary's time preferences. Besides the analysis of equilibrium outcomes, we derived optimal contracts for implementing high- or low-quality composed services.

Within our model we conducted several simplifying assumptions that suggest further prospective extensions. For instance, for our theoretical analysis we supposed the quality level of composed as well as single services to either be high or low. Introducing a finite or infinite number of different product quality levels is thus an obvious modification. This kind of extension enlarges the set of possible strategic choices of the service providers and has the effect of composed services being available in more than two quality levels. In addition, a higher range of quality levels requires a consideration when defining the customer's demand.

Another branch of future research goes in the direction of assigning a higher impact to the customer's role. The customer in our model did rather take a passive role, i.e., was only considered as the source of market demand for high- and low-quality products. We assumed the customer being able to perfectly observe the final product's quality and having a demand accordingly. In a next step it can be supposed that the final product's quality is not directly observable to the customer. Instead, one can assume his evaluation to be depending on the quality which is reported by the intermediary to the input suppliers. Thus, when choosing a quality level to report, the intermediary needs to consider the procurement as well as the sales side. In this context a determination of Nash equilibria and comparing results with the outcomes of our recent model is a consequent next step.

Within the current work we have seen that although examining a uniform market structure, models' focal points and specifications can go in various directions. In Chapter 3 and Chapter 4 we focused on the intermediaries' competition and determined and analyzed the according Nash equilibrium outcomes. We assumed that intermediaries only needed to procure one input product which was refined and afterwards offered to a customer. However, we did not consider a key aspect of Chapter 5, i.e., the fact that a number of input goods needed to be combined to establish a final product. Hence, in a further step combining the central ideas of both models should be strived for. Within such a setting intermediaries procure goods from a number of input suppliers, combine them and offer the resulting final product to their clients. Competition between intermediaries and suppliers may be taken out simultaneously or sequentially where either prices or quantities are chosen. Final products can again be horizontally as well as vertically differentiated.

In conclusion, this thesis focused on two-sided markets in which intermediaries built a link between input suppliers and customers. We covered various challenges including simultaneous competition for clients and resources, asymmetries between intermediaries, asymmetric information with respect to product quality and so forth. Within our analyses we established results that gave us a broad understanding of two-sided markets and build a solid basis for further research. Extensions of the above models as well as challenges that were not in scope of this work should be considered in future literature.

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