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Critical solutions of topologically gauged $\mathcal{N} = 8$ CFTs in three dimensions

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Abstract: In this paper we discuss some special (critical) background solutions that arise in topological gauged $\mathcal{N} = 8$ three-dimensional CFTs with SO(N) gauge group. Depending on how many scalar fields are given a VEV the theory has background solutions for certain values of μl, where μ and l are parameters in the TMG Lagrangian. Apart from Minkowski, chiral round AdS_3 and null-warped AdS_3 (or Schrödinger($z = 2$)) we identify also a more exotic solution recently found in TMG by Ertl, Grumiller and Johansson. We also discuss the spectrum, symmetry breaking pattern and the supermultiplet structure in the various backgrounds and argue that some properties are due to their common origin in a conformal phase. Some of the scalar fields, including all higgsed ones, turn out to satisfy three-dimensional field equations similar to those of the singleton. Finally, we note that topologically gauged $\mathcal{N} = 6$ ABJ(M) theories have a similar, but more restricted, set of background solutions.

Keywords: Chern-Simons Theories, Conformal Field Models in String Theory, M-Theory

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Contents

1 Introduction

The purpose of this paper is to discuss some of the background solutions in topologically gauged CFTs in $2+1$ dimensions with $\mathcal{N} = 8$ supersymmetry and an arbitrary SO(N) gauge group [1, 2] and to point out some of their properties relevant in this context. Apart from Minkowski, and well-known geometries like round AdS_3 and null-warped AdS_3 found already in [2], we here identify a new more exotic one belonging to a different category of solutions as will be explained below. The main point of this paper is to argue that only very special solutions in topological massive gravity (TMG) will appear due to the connection to the unbroken superconformal phase of the theory.

Topological gauged CFT refers in general to superconformal Chern-Simons(CS)/matter field theories in three dimensions whose global symmetries have been gauged by coupling the theory to conformal supergravity. In three dimensions conformal supergravity is governed by gravitational CS terms [3, 4] and is therefore topological in nature. Topologically gauged CFTs of this kind were first discussed in [1] where the gauging was applied to the ordinary $\mathcal{N} = 8$ BLG theory [5–7]. For the $\mathcal{N} = 6$ ABJ(M) theories [8, 9] the same type of construction was obtained shortly afterwards in [10] where a new potential for the scalar fields was found as we will have reason to briefly discuss later. Entirely new theories with local $\mathcal{N}=8$ conformal supersymmetry, SO(N) gauge groups for any N and new scalar potential terms were subsequently discovered in [2] which also completed the task, set by the authors of [1], of gauging the BLG theory. The topological properties of the gravitational sector of the theory are important for what kind of degrees of freedom it describes. Thus, one of our goals will be to initiate an analysis of the spectrum in the different broken phases of the gauged theory with SO(N) gauge symmetry. The higgsing that turns the CS gauge fields into massive vector fields will be discussed in detail, and we will present some exact formulae for the interactions with the remaining scalar fields. We will also note that the higgsed scalars satisfy the singleton field equation.

The construction of the topological gauged BLG theory was started in [1] and completed in [2] where it was also found that if one turns off the BLG interactions it becomes possible to generalize the gauge symmetry from SO(4) to SO(N) for any N. This was shown using three different methods, one of them being the Noether method.[1] Since no details of the derivation of the potential using the Noether method were given in [2] we present some of the details in the appendix, restricting ourselves to the SO(N) theory which starts from the free matter theory. We stop the presentation at the point where we can deduce the new potential terms. The appendix also discusses the SO(N) gauge field and presents a more direct argument for the normalization of the SO(N) CS term than that given in [2]. The reader may consult [2] for the complete arguments showing the existence of these $\mathcal{N}=8$ topologically gauged SO(N) theories.

Before we turn to the theory with $\mathcal{N}=8$ let us very briefly review the situation for $\mathcal{N}=6$. The topologically gauged ABJ(M) theories were obtained in [10] and discussed further in [13] (see also [2]). Apart from the superconformal gravity sector and a standard ABJ(M) theory it contains a new $U_R(1)$ CS gauge field and a number of new interaction terms. In particular one finds a new scalar potential and the expected conformal coupling term $-\frac{1}{8}|Z|^2R$ between the curvature scalar and two scalar fields Z^A_a which are complex in this case: lower case indices are three-algebra and upper case fundamental SU(4) R-symmetry indices (for details, see [10]). The potential is then found to consist of the original (single trace (st)) term

$$V^{(st)}_{ABJ(M)}=\tfrac{2}{3}|\Upsilon^{CD}{}_{Bd}|^2,\ \Upsilon^{CD}{}_{Bd}=\lambda f^{ab}{}_{cd}Z^C_aZ^D_b\bar{Z}^c_B+\lambda f^{ab}{}_{cd}\delta^{[C}_BZ^{D]}_aZ^E_b\bar{Z}^c_E\,, \tag{1.1}$$

plus the following new terms: with one structure constant (double trace (dt))

$$V^{(dt)}_{\text{new}}=-\tfrac{1}{8}g\lambda f^{ab}{}_{cd}|Z|^2Z^C_aZ^D_b\bar{Z}^c_C\bar{Z}^d_D-\tfrac{1}{2}g\lambda f^{ab}{}_{cd}Z^B_aZ^C_b(Z^D_e\bar{Z}^e_B)\bar{Z}^c_C\bar{Z}^d_D\,, \tag{1.2}$$

and without structure constant (triple trace (tt))

$$V^{(tt)}_{\text{new}}=-g^2(\tfrac{5}{12\cdot 64}(|Z|^2)^3-\tfrac{1}{32}|Z|^2|Z|^4+\tfrac{1}{48}|Z|^6)\,, \tag{1.3}$$

where $\lambda=\frac{2\pi}{k}$ (k is the CS level) and g the gravitational coupling constant.

[1] The other two methods used in [2] are the on-shell superalgebra method and superspace. In that work the superspace method was finally successfully applied to this problem which has a number of special features that make the analysis more complicated than for Poincaré supergravity theories, see, e.g., [11, 12].

We can now break the conformal symmetries by introducing a real VEV v for one of the scalar fields Z^a_A [10] and consider the following terms in the lagrangian:[2]

$$L(v) = -\tfrac{1}{g}L_{CS(\omega)} - \frac{v^2}{8}eR - eV(v), \tag{1.4}$$

where only the triple trace terms contribute to the VEV of the potential $V(v)$. By comparing to the TMG Lagrangian discussed in the context of chiral gravity by the authors of [14] (but with an opposite sign in front of the whole Lagrangian) we find that their parameters can be expressed in terms of the ours, v and g, as follows ($\Lambda = -\frac{1}{l^2}$)

$$\mu = \tfrac{g}{\kappa^2},\ \kappa^2 = \tfrac{8}{v^2},\ \ \tfrac{1}{2l^2} = \tfrac{g^2v^4}{128}, \tag{1.5}$$

which shows that the chosen VEV produces a theory that sits exactly at the chiral point:

$$\mu l = 1. \tag{1.6}$$

Below we will repeat the above search for a critical AdS_3 solution in the $\mathcal{N}=8$ case. We will find that this does not work unless we generalize the VEV to several scalar fields, a fact first observed in [2]. This step will generate a set of solutions which will be elaborated upon in section 2. In section 3 we discuss the spectrum and supersymmetry in the various backgrounds. Here relations to AdS_3 singletons seem to appear. A few comments are collected in section 4 and some computational details of the Noether construction of the potential can be found in the appendix.

2 Field equations and background solutions

In this section we will find and discuss a number of background solutions. Two of these were briefly mentioned in [2] and are known to be in some sense (see below) critical. Here we will also identify a new solution that is unfortunately less well understood. Supersymmetry and other properties of these backgrounds will be discussed in the following section.

2.1 The bosonic part of the lagrangian with SO(N) gauge symmetry

The bosonic part of the action consists of the following terms [2]

$$L_{Bos} = -\tfrac{1}{g}L_{CS(\omega)} + \tfrac{2}{g}L_{CS(B)} - \tfrac{4}{g}L_{CS(A)} - \tfrac{e}{2}g^{\mu\nu}D_\mu X^i_a D_\nu X^i_a - \tfrac{e}{16}X^2R - eV(X), \tag{2.1}$$

where the various Chern-Simons terms are given in terms of the conventionally normalized Lagrangians $L_{CS(..)}$:

$$L_{CS(A)} = \frac{1}{2}\epsilon^{\mu\nu\rho}(A^{ab}_\mu \partial_\nu A^{ab}_\rho + \frac{2}{3}A^{ab}_\mu A^{ac}_\nu A^{cb}_\rho). \tag{2.2}$$

The conformal coupling X^2R is the $d=3$ version of the general case

$$L = -\tfrac{1}{2}(\partial_\mu\Phi)^2 - \tfrac{d-2}{8(d-1)}\Phi^2R, \tag{2.3}$$

[2]The coupling constant g was later introduced in [13] but is not really crucial for the argument.

and the new SO(N) potential, which is a special combination of triple trace terms (recall that the BLG structure constants have been set to zero in this SO(N) theory), can be written as a square as follows

$$V(X) = \frac{g^2}{2{\cdot}32{\cdot}32}(X^2 X_a^i - 4X_a^j X_b^j X_b^i)^2, \tag{2.4}$$

where the indices $a, b, ..$ and $i, j, ...$ are vector indices of the gauge group SO(N) and R-symmetry group SO(8), respectively. The covariant derivative is $D_\mu = \partial_\mu + \omega_\mu + B_\mu + A_\mu$. See the appendix for conventions and [2] for additional details.

We can now vary these terms to get the equations of motion for the bosonic fields which we will later linearize to find the spectrum, analyze stability etc. To properly analyze the issue of stability one needs, in fact, to go beyond the linear level (see, e.g., Maloney et al. [15]) but that will not be done in this paper.

Since a single scalar VEV $< X >= v$ (for one component X_1^1 of X_a^i, say) solves the Klein-Gordon equations we can just insert the VEV into the Lagrangian to analyze which geometries will satisfy the gravitational field (Cotton) equation. To this end we need the background value of the potential:

$$V(v) = \frac{9g^2v^6}{2{\cdot}32{\cdot}32}. \tag{2.5}$$

This is, however, a factor of 9 wrong if we had expected to end up at the chiral point as in the ABJ(M) case [10]! This is easily seen as follows. By considering the gravitational CS term, the X^2R term and the potential evaluated at the VEV we get

$$L_{VEV} = -\frac{1}{g}L_{CS(\omega)} - \frac{v^2e}{16}R - eV(v). \tag{2.6}$$

This may be compared to the action used by Li, Song and Strominger (LSS) [14] in their analysis of the chiral point[3]:

$$L_{LSS} = -\frac{1}{\kappa^2}\left(\frac{1}{\mu}L_{CS(\omega)} + e(R - 2\Lambda)\right). \tag{2.7}$$

Thus in this case $\mu = \frac{g}{\kappa^2}$ and $v^2 = \frac{16}{\kappa^2}$. The chiral point condition is $\mu l = 1$ where l is defined in terms of the cosmological constant as usual: $\Lambda = -\frac{1}{l^2}$. This implies that, to end up at a chiral point, the potential must satisfy

$$V(v) = -\frac{1}{e}L_{X^6}(v) = -\frac{2\Lambda}{\kappa^2} = \frac{2}{\kappa^2 l^2} = \frac{2\mu^2}{\kappa^2} = \frac{2g^2}{\kappa^6} = \frac{2g^2v^6}{16^3} = \frac{g^2v^6}{2{\cdot}32{\cdot}32}, \tag{2.8}$$

which differs from the background value above by a factor of 9. In [2] the observation was made that if two scalar fields are given the same VEV this factor of 9 disappears and one ends up at the chiral point with $\mu l = 1$. In fact, by giving three scalar fields the same VEV we find instead that $\mu l = 3$ which has a null-warped solution. Below we will elaborate on this situation and discuss the other values of μl that appear.

[3]Note that this is a TMG [16] type Lagrangian with signs opposite to those used by LSS in [14]: the signs used in our paper are dictated by the unitarity of the scalar field sector together with supersymmetry and can not be changed. However, even supersymmetric phases may have unitarity problems (appearing here only at the boundary) as indicated by the results of [14] and [17].

The reason we expect the chiral point value $\mu l = 1$, or other special values of μl, to play a role here is that we want to avoid massive propagating gravity modes in the bulk [15] which are not there in the conformal phase [1]. Introducing a similar kind of VEV in the ABJ(M) case [10] leads, in fact, directly to the chiral point as was reviewed in the previous section. That special "critical" values of μl are relevant for the broken phases also in $\mathcal{N} = 8$ theories will be a working hypothesis adopted in the following. This will be crucial also for what kind of conformal field theories that can arise at the boundary of the AdS or the null Killing vector backgrounds that we will find later. Note that Minkowski does also arise as a solution which may have a rather special "boundary CFT" (see [19, 20] and references therein). We will not discuss boundary theories in any detail in this paper but we should mention here that one case that appears as a solution is the null-warped AdS_3 with its Schrödinger symmetries at the boundary discussed, e.g., in the context of cold atoms [21–23].

As just mentioned, one important aspect of the critical point of Li, Song and Strominger [14] is that there are no massive gravity modes present. The degeneration that occurs in the spectrum when tuning the non-critical TMG theory to its critical value may result in log-modes which would be problematic from a unitarity point of view[4] (see, e.g., the recent review [24]). However, as explained in [15] by choosing the boundary conditions one can consistently truncate the theory to a chiral subsector. A similar phenomenon may be at work also in the null-warped case as argued in [25]. The behavior of scalar fields in this context has been discussed for instance in [26]. Other general properties stemming from the fact that the theory comes from a conformal phase may be extra symmetries as found for the null-warped metric (see below).[5]

2.2 Bosonic field equations and background solutions

We here summarize the bosonic field equations found in [2]. The Cotton equation reads

$$\begin{aligned} \tfrac{1}{g}C_{\mu\nu} - \tfrac{eX^2}{16}(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) + \tfrac{e}{2}g_{\mu\nu}V(X) \\ -\tfrac{e}{2}(D_\mu X^i_a D_\nu X^i_a - \tfrac{1}{2}g_{\mu\nu}D^\sigma X^i_a D_\sigma X^i_a) - \tfrac{e}{16}g_{\mu\nu}\Box X^2 + \tfrac{e}{16}D_\mu D_\nu X^2 = 0. \end{aligned} \tag{2.9}$$

Turning to the matter sector we first give the scalar field equation. Discarding the fermions it becomes $\Box X^i_a - \frac{1}{8}X^i_a R - \partial_{X^i_a} V(X) = 0$ which can be seen to be consistent with the trace of the Cotton equation. Using the expression for the potential the Klein-Gordon equation becomes

$$\begin{aligned} \Box X^i_a - \tfrac{1}{8}X^i_a R = \\ \tfrac{g^2}{32\cdot 32}(3X^i_a(X^2)^2 - 8X^i_a(X^j_b X^k_b)(X^j_c X^k_c) - 16X^2 X^k_a X^k_b X^i_b + 48X^j_a(X^j_b X^k_b)(X^k_c X^i_c)) \quad . \end{aligned} \tag{2.10}$$

Finally, for the R-symmetry gauge field we have the following field equation

$$\epsilon^{\mu\nu\rho}\, G^{ij}_{\nu\rho} + g\, e g^{\mu\nu} X^{[i}_a D_\nu X^{j]}_a = 0, \tag{2.11}$$

[4] See, however, the previous footnote.

[5] Extra symmetries have, in fact, also been found at the chiral point [27].

while for the SO(N) gauge field A_μ^{ab} we get

$$-2\epsilon^{\mu\nu\rho}F_{\nu\rho ab}+g\,eg^{\mu\nu}X^i_{[a}D_\nu X^i_{b]}=0. \tag{2.12}$$

The field equations for the two vector fields are trivially satisfied in the backgrounds we use here. Thus we can concentrate our efforts on the Cotton and Klein-Gordon equations.

We now demonstrate that these last equations allow for a number of different background solutions two of which were briefly mentioned but not analysed in [2]. The first step will be to solve the Klein-Gordon equation. To do this we introduce a VEV $p\times p$ unit matrix $v\mathbf{1_{p\times p}}$ by setting[6]

$$X^i{}_a=<X^i{}_a>+x^i{}_a=v\delta^I_A+x^i{}_a, \tag{2.13}$$

where the VEV term proportional to δ^I_A ($I=1,2,..,p,\ A=1,2,...,p\leq 8$ or $p\leq N$ if $N<8$) means that the scalar fields that are given the same VEV v are the first p ones along the diagonal starting from the upper left-hand corner of the rectangular matrix $X^i{}_a$ having 8 rows and N columns. Recall that the indices take the values $i=1,2,...,8$ and $a=1,2,...,N$ where N can be any positive integer. The capital indices $A,B,..$ and $I,J,..$ are thus of the same kind as far as their transformation properties are concerned and we will not distinguish between them from now on. $x^i{}_a$ are the fluctuations relative these VEVs. We thus have, e.g., $X^2=X^i{}_aX^i{}_a=pv^2+2vz+x^2$, where the trace $x^I{}_I=z$ and $x^2=x^i{}_ax^i{}_a$.

For the index choice $i=I,a=A$ the scalar field equation in the background of the matrix VEV becomes

$$\bar{R}=6\Lambda=-\tfrac{6}{l^2}=-\tfrac{6}{16\cdot 16}g^2v^4(p-4)^2, \tag{2.14}$$

where $\bar{R}$ refers to the background value of the curvature scalar. This equation will be a constraint valid in all considerations to be made in the rest of the paper whether the background is maximally symmetric or not. In order to discuss the other scalar equations we split the indices as follows:

$$i=(I,\hat{i}),\quad a=(A,\hat{a}). \tag{2.15}$$

We then note that using $i=I,a=\hat{a}$ etc, the remaining scalar field equations are trivially satisfied since there are no VEVs connecting the two indices in these cases.

What remains to be solved is the Cotton equation. To do this for general values of p we consider first the Lagrangian with the background put in for all fields except the metric:

$$L_{VEV}=-\tfrac{1}{g}L_{CS(\omega)}-\frac{epv^2}{16}R-\tfrac{ev^6g^2}{2\cdot 32\cdot 32}p(p-4)^2. \tag{2.16}$$

Comparing this to the Lagrangian used in the analysis of LSS [14]

$$L=-\tfrac{1}{\kappa^2}(\tfrac{1}{\mu}L_{CS(\omega)}+e\,(R+\tfrac{2}{l^2})), \tag{2.17}$$

[6]There may be other ways to introduce scalar VEVs. Only some simple modifications of the VEV used here have been checked and seen to give nothing new.

we can read off its parameters expressed in terms of our variables v, g:

$$\kappa^2\mu = g, \quad \kappa^2 = \frac{16}{pv^2}, \quad \frac{2}{\kappa^2 l^2} = p(p-4)^2 \frac{v^6 g^2}{2\cdot 32\cdot 32}, \tag{2.18}$$

where κ^2 and l have dimension L^1 and μ dimension L^{-1} since g is dimensionless. Recall that the field $X^i{}_a$ and thus v has dimension $L^{-1/2}$. The parameter relations above can be written

$$\mu = \frac{g}{\kappa^2} = \frac{gpv^2}{16}, \quad l = \frac{1}{\kappa}\frac{2\cdot 32}{|p-4|\sqrt{p}v^3 g} = \frac{16}{|p-4|gv^2}, \tag{2.19}$$

and hence

$$\mu l = |1 - \tfrac{4}{p}|^{-1}. \tag{2.20}$$

This equation gives the following values for $p = 1, 2, ..., 8$:

$$\mu l = \tfrac{1}{3}, 1, 3, \infty, 5, 3, \tfrac{7}{3}, 2. \tag{2.21}$$

The interesting cases are $p = 2$ which allows for an ordinary critical (chiral) round AdS solution together with $p = 3$ and $p = 6$ both having a null-warped AdS (see [25, 28] and references therein) as a possible solution. This latter solution has a non-zero Cotton tensor but a constant curvature scalar as we saw above is a property all solutions must satisfy. Also $p = 4$ is interesting since the potential vanishes and the solution is flat Minkowski space-time. Recent work like [19, 20] might be relevant in this case. These geometries are all very well-known and will be described briefly below. However, for $p = 5$ we get $\mu l = 5$ which is intriguing: a solution with $\mu l = 5$ was discovered only recently by Ertl, Grumiller and Johansson (EGJ) [29] and as we will see below the way this solution is obtained is very different from the other ones mentioned here.

Thus several of the μl values in the list above can be connected to solutions of TMG that are critical or in some sense special, at least this is the case for $p = 2, 3, 4, 5, 6$. It is therefore natural to wonder if the remaining values also have special solutions which, however, have not yet been found in TMG.[7] Note that non-critical solutions based on the round AdS_3 exist for any value of μl but then there are propagating massive (positive energy) gravitons. In this context we may remind the reader that the theories discussed here have a potential problem with unitarity due to negative energy black holes and boundary modes. For a discussion of this issue in bosonic TMG, see [31]. Some perhaps relevant comments concerning supersymmetric theories can be found in [17].

2.3 Some properties of the special (critical) solutions

In this subsection we discuss some of the special solutions of the Cotton equation that are possible for the values of μl that appeared for the different choices of scalar VEVs. There are several recent attempts to classify the known solutions of TMG, see for instance [32–34] and [29].[8] These papers also contain some new solutions as well as most of the original

[7] The value $\mu l = 2$ does in fact come up in the context of BTZ black holes [30]. I am grateful to H.R. Afshar for pointing this out to me. See also *Note added* at the end of the last section.

[8] A complete classification of all homogenous solutions with constant scalar invariants in TMG, NMG and GMG [35] can be found in [36].

references for the previously found solutions which appear in various guises in the literature. E.g., in [33] the Petrov-Segre classification is adapted to this situation and shown to directly account for the known solutions as belonging to a very limited set of classes. We will, however, be mostly concerned with a method discussed first by Clement [37] and later used in [29]. In the latter work the authors divide the construction of stationary axi-symmetric TMG solutions into sectors called Einstein, Schrödinger, warped and generic. After observing that all known solutions belong to the first three classes they go on to construct a new solution that belongs to the general class and which turns out to have rather special properties. The metric has $\mu l = 5$ and is non-polynomial in the radial coordinate r (see below).

It is convenient to use light-cone coordinates such that three-dimensional Minkowski space-time with signature $(-++)$ is described by the metric

$$ds^2 = d\rho^2 + 2dudv, \tag{2.22}$$

where ρ is the "radial" coordinate taking values from $-\infty$ to $+\infty$ and $2dudv = -dt^2 + dx^2$.

In the literature other closely related coordinates appear: for instance the coordinate r $(0 < r < \infty)$ related to ρ by $2\rho/l = log(r/l)$ is often used. Note, however, that in the reference [29] ρ corresponds to our radial coordinate r. Also, the commonly used coordinate z can then be introduced by $r/l = z^{-2}$.

The existence of global coordinate systems that turn the Poincaré patch into a geodecically complete space are very important in the cases below. This is one of the features that may be common to all the solutions that we call "critical" in this paper. The global coordinates for the round AdS_3 are well-known and the null-warped case is thoroughly discussed in [38] while the situation for the EGJ solution with $\mu l = 5$ is not clear.

Critical AdS_3 $(p = 2,\ \mu l = 1)$: The metric for the round AdS_3 with radius l is

$$ds^2 = d\rho^2 + 2e^{2\rho/l}dudv = l^2\frac{dr^2}{4r^2} + \frac{2r}{l}dudv = \frac{1}{z^2}(l^2dz^2 + 2dudv). \tag{2.23}$$

Criticality refers in this case to the fact that the massive bulk gravity mode disappears and a potentially chiral boundary theory becomes possible as μl is tuned to one [14, 15]. In the context of this paper with a large number of scalar fields present, the chiral limit should be reconsidered. Some relevant results in this direction may be found in [26]. Since in three dimensions the Weyl tensor vanishes, the Riemann tensor is given entirely by the traceless Ricci tensor and the curvature scalar. It then follows that being Einstein is equivalent to being maximally symmetric, and hence the above metric is the unique solution of TMG with zero Cotton tensor.[9] This corresponds to the class $\mathcal{O}$ in the Petrov-Segre classification in [33] and to the Einstein sector in [29].

In [32] the Killing spinor equation is solved and shown to have two solutions corresponding to the two components of a spinor in three dimensions. Thus this background allows for eight ordinary AdS_3 supersymmetries in the context of this paper.

Null-warped AdS_3 or $Sch_3(z = 2)$ $(p = 3$ and $6,\ \mu l = 3)$: The relation $z = \frac{\mu l+1}{2}$ is obtained by using the ansatz $ds^2 = d\rho^2 + 2e^{2\rho/l}dudv \pm e^{2z\rho/l}du^2$ to solve the Cotton equation

[9] For non-Einstein solutions with $\mu l = 1$, see [39]. See also [33].

in TMG. For the value 2 of the dynamical scaling parameter z, corresponding to $\mu l = 3$, the solution is critical in the sense that among the solutions with a null Killing vector it has no tidal forces, a global coordinate system [38] and an extra conformal generator [21, 22].[10] As discussed in [25], it may also be possible to truncate the spectrum in a chiral fashion similar to the $\mu l = 1$ case of the previous subsection. This metric can be written as follows

$$ds^2 = d\rho^2 + 2e^{2\rho/l}dudv \pm e^{4\rho/l}du^2 = l^2\frac{dr^2}{4r^2} + \frac{2r}{l}dudv \pm \frac{r^2}{l^2}du^2 = \frac{1}{z^2}(l^2dz^2 + 2dudv) \pm \frac{du^2}{z^4}, \tag{2.24}$$

where the properties of this geometry depend on the sign in front of the last term, see [28].

In this case we know from [32] that the three-dimensional geometry can only support one (component) supersymmetry due to the presence of a null Killing vector (without being the round AdS). In fact, the existence of a Killing spinor in this geometry implies that there is a null vector K_μ satisfying

$$D_\mu K_\nu = -\epsilon_{\mu\nu\rho}K^\rho. \tag{2.25}$$

Turning the argument around [32], assuming a null Killing vector (not necessarily satisfying the anti-symmetric part of the above equation), the TMG geometries are just the supersymmetric ones given above allowing, however, also for their orientation flipped versions.

The EGJ solution ($p = 5$, $\mu l = 5$): This solution was first obtained by Ertl, Grumiller and Johansson (EGJ) in [29][11] using an approach discussed originally by Clement [37]. To find all solutions of TMG that are stationary and axi-symmetric one may adopt the following ansatz for the metric:

$$ds^2 = (det\,h)^{-1}dr^2 + h_{\alpha\beta}dx^\alpha dx^\beta = (det\,h)^{-1}dr^2 + h_{++}dudu + 2h_{+-}dudv + h_{--}dvdv, \tag{2.26}$$

where the three functions in the $h_{\alpha\beta}$ part of the metric depend only on the radial coordinate r. (In this subsection we use the conventions of [29] apart from renaming their coordinate ρ as r and denoting derivatives by a prime instead of an over-dot.) Thus we denote the functions h_{++}, h_{--} and h_{+-} as X^+, X^-, Y, respectively, and note that

$$det\,h = X^+X^- - Y^2 := X^iX^j\eta_{ij}, \tag{2.27}$$

defines an auxiliary flat metric η with signature $(+,-,-)$. Setting $X^i = (X^+, X^-, Y)$, we find that (the physical) Minkowski space corresponds to $X^i = (0,0,1)$ and the maximally symmetric AdS_3 to $X^i = (0,0,r)$ while the null-warped case is obtained from $X^i = (r^2, 0, r)$. In all these cases $\mathbf{X}' \cdot \mathbf{X}'' = 0$ and $\mathbf{X}''^2 = 0$ which can be shown to imply $\mathbf{X}''' = 0$. As emphasized in [29] the first two conditions reduce the phase space to a four-dimensional hypersurface. The new solution with $\mu l = 5$ will not satisfy these conditions and therefore seems to make use of the entire six-dimensional phase space. The functions X^i then no longer satisfy $\mathbf{X}''' = 0$ and will, in fact, become non-polynomial in the radial coordinate. No closed form of the solution is yet known.

[10] This fact is important for the condensed matter applications to Fermi gas/cold atom systems. For other properties of this geometry relevant for applications, see [23, 40–42].

[11] For an earlier analysis using these methods, see [15].

The set of equations obtained by using this ansatz for the metric in the Cotton equation divides into a hamiltonian constraint, which involves fields acted upon by at most two r derivatives (see below), and three equations for the X^i containing terms that are third order in derivatives. However, one can integrate the third order equations once by employing the fact that the "angular momentum" associated to the Lorentzian symmetry of the dynamical equations containing η_{ij} and $\mathbf{X}$ is a constant of motion. In fact, acting with a derivative on

$$\mathbf{J} = \mathbf{X} \times \mathbf{X}' + \tfrac{1}{\mu}\mathbf{X} \times (\mathbf{X} \times \mathbf{X}'') - \tfrac{1}{2\mu}\mathbf{X}' \times (\mathbf{X} \times \mathbf{X}'), \tag{2.28}$$

results in a cross product of $\mathbf{X}$ and the third order equations of motion. Thus one wants to solve this last equation together with the following second order equation, which is the hamiltonian constraint in the TMG theory,

$$\tfrac{1}{2}\mathbf{X}'^2 + \frac{2}{l^2} - \frac{1}{\mu}\epsilon_{ijk}X^i X'^j X''^k = 0. \tag{2.29}$$

In fact, all the dynamical equations follow from the following TTM (topologically massive mechanics) action [37]:

$$S_{\rm TMM} = \int d\rho\, e(\tfrac{1}{2}e^{-2}\mathbf{X}'^2 - \frac{2}{l^2} - \frac{1}{2\mu}e^{-3}\epsilon_{ijk}X^i X'^j X''^k), \tag{2.30}$$

where also an einbein e has been introduced.

These equations also imply that that the curvature scalar can be expressed as

$$R = 2\mathbf{X}\cdot\mathbf{X}'' + \frac{3}{2}\mathbf{X}'^2 = -\frac{6}{l^2}, \tag{2.31}$$

which, if combined with the hamiltonian constraint, implies

$$\mu\mathbf{X}\cdot\mathbf{X}'' + \mathbf{X}\cdot\mathbf{X}'\times\mathbf{X}'' = 0. \tag{2.32}$$

Following Ertl et al. [29] for $\mu l = 5$, if we set $(s = 0, \pm 1)$

$$\mathbf{X}^{\mathbf{T}}|_0 = (1, 0, 0),\ \ \mathbf{X}'^{\mathbf{T}}|_0 = \mu(s, 0, \tfrac{2}{5}), \tag{2.33}$$

we can start solving the equations in an iterative fashion. We find

$$\mathbf{X}''^{\mathbf{T}}|_0 = (X''^{+}|_0, 0, Y''|_0), \tag{2.34}$$

where

$$Y''|_0 = \tfrac{1}{2\mu}X'''^{-}|_0,\ \ X''^{+}|_0 = \tfrac{125}{32\mu^4}(X'''^{-}|_0)^2 + \tfrac{5s}{4\mu}X'''^{-}|_0. \tag{2.35}$$

Thus, one difference between this solution and the critical ones discussed above is that the component X^- is non-zero starting at third order in r. How this affects the possibility for this geometry to support supersymmetry remains to be clarified.

Minkowski ($p = 4$, $\mu l = \infty$): Recall that we are in this paper assuming that the relevant solutions are in some sense "critical" with properties that stem for their connection to a conformal phase. In the context of Minkowski space this is a particularlry delicate issue. However, we note that there are discussions in the literature concerning the possibility to tune an AdS bulk geometry to a flat space and follow what happens to the symmetries of the CFT at the boundary, see, e.g., [19, 20]. This could be telling us to define a "critical" Minkowski solution for $p = 4$ by relating it to the BMS algebra, see, e.g., [43].[12]

3 Mode analysis and supersymmetry

To study the spectrum we should expand the Lagrangian and the field equations around the VEV v using

$$X^i{}_a =< X^i{}_a > + x^i{}_a = v\delta^I_A + x^i{}_a, \tag{3.1}$$

where the VEV matrix is proportional to the $p \times p$ unit matrix, i.e., $A, I = 1, 2, ..., p \leq 8$ (or $p \leq N$ if $N < 8$). Note that we have defined the upper index as the first one and the lower as the second one (whether indices are upper or lower will not matter from now on) and that we in the broken phases do not need to distinguish between the two sets of capital Latin indices $A, B, ...$ and $I, J, ...$ As already mentioned we define also the remaining index values $\hat{\imath}$ and $\hat{a}$ by setting $i = (I, \hat{\imath}),\ a = (A, \hat{a})$.

3.1 Symmetry breaking and massive vector fields

At this point we can insert the VEV into the Klein-Gordon term in the Lagrangian to determine the symmetry breaking pattern. The terms proportional to v^2 are

$$L(v^2) = -\tfrac{1}{2}v^2(A^{ab}_\mu \delta^I_B + B^{ij}_\mu \delta^J_A)^2 = -\tfrac{1}{2}v^2((A^{\hat{a}B}_\mu)^2 + (B^{\hat{\imath}J}_\mu)^2 + (A^{AB}_\mu - B^{AB}_\mu)^2), \tag{3.2}$$

where a square $(A_\mu{}^{AB})^2 = A^{\mu AB}A^{AB}_\mu$ etc. Note also that we have adopted the summation rule that $A^{ab}_\mu \delta^I_B := A^{aB}_\mu \delta^I_B$ etc. Thus the symmetry breaking of the bosonic gauge and R-symmetries is governed by the coset

$$G/H: \ \ G = \mathrm{SO(N)} \times \mathrm{SO(8)},\ \ \mathrm{H} = \mathrm{SO(N-p)} \times \mathrm{SO(8-p)} \times \mathrm{SO(p)_{diag}}, \tag{3.3}$$

where the factor $\mathrm{SO(p)_{diag}}$ is the diagonal part of the two SO(p) groups coming from SO(N) and SO(8) after breaking.

However, the two gauge fields involved in the SO(p) part of this system have differently normalized CS terms and the equations of motion need to be properly diagonalized to find the actual mass of the higgsed vector field. The combination of the two vector fields that remains a gauge field after breaking is determined as follows. The linearized vector equations read, for the R-symmetry gauge field, with $\delta B_\mu := b_\mu$ and $\delta A_\mu := a_\mu$,

$$2\bar{\epsilon}_\mu{}^{\nu\rho}\partial_\nu b_\rho + gv^2\bar{e}\delta^\rho_\mu(a_\rho - b_\rho) = 0, \tag{3.4}$$

and for the SO(p) gauge field

$$4\bar{\epsilon}_\mu{}^{\nu\rho}\partial_\nu a_\rho + gv^2\bar{e}\delta^\rho_\mu(a_\rho - b_\rho) = 0. \tag{3.5}$$

[12] In relation to the second of these references we note that the limit used there to get the wanted flat space CFT is similar to tuning the VEV v introduced here to zero keeping g fixed!

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As we will now see, the reduction of this system to a single vector field is similar to that used by Mukhi and Papageorgakis in [44] but will here in addition to the Yang-Mills term generate a topological mass term in a curved background. To see this we define

$$c'_\mu = 2a_\mu - b_\mu, \quad c_\mu = a_\mu + 2b_\mu, \tag{3.6}$$

which satisfy

$$\bar{\epsilon}_\mu{}^{\nu\rho}\partial_\nu c'_\rho = 0, \quad \bar{\epsilon}_\mu{}^{\nu\rho}\partial_\nu c_\rho = -\tfrac{5m}{4}\bar{e}(a_\mu - b_\mu) = -\tfrac{m}{4}\bar{e}(3c'_\mu - c_\mu), \tag{3.7}$$

where the mass $m = gv^2$. In the parity symmetric case studied in [44] the field c_μ does not appear on the right hand side of the second equation. The general non-symmetric situation with arbitrary parameters in front of the various terms is, however, discussed in [45] and contains the features seen here. Eliminating the field c'_μ we obtain in our case the following field equation for $\bar{H}^{IJ}_{\mu\nu} = \partial_\mu c_\nu - \partial_\nu c_\mu$:

$$\bar{e}\,\bar{D}^\nu \bar{H}_{\nu\mu} = \frac{m}{8}\bar{\epsilon}_\mu{}^{\nu\rho}\bar{H}_{\nu\rho}, \tag{3.8}$$

which is a topologically massive gauge theory [46, 47] in a curved background.[13] Thus the Yang-Mills coupling constant g^2_{YM} is proportional to the mass parameter $m = gv^2$.

The last task concerning the vector fields is to rewrite the covariant derivative in terms of the gauge field c_μ which will also give us a hint about the structure of the full non-abelian case. Thus, using the above expression for c'_μ, we get

$$a_\mu = \frac{1}{5}(c_\mu + 2c'_\mu) = \frac{1}{3}(c_\mu - \frac{8}{5m\bar{e}}\bar{\epsilon}_\mu{}^{\nu\rho}\bar{D}_\nu c_\rho), \tag{3.9}$$

and

$$b_\mu = \frac{1}{5}(c_\mu + 2c'_\mu) = \frac{1}{3}(c_\mu + \frac{4}{5m\bar{e}}\bar{\epsilon}_\mu{}^{\nu\rho}\bar{D}_\nu c_\rho). \tag{3.10}$$

Now we rescale c_μ to cancel the factor $\frac{1}{3}$, rename the field as C_μ and express the covariant derivative as follows

$$\begin{aligned} D_\mu X^I{}_A = \partial_\mu X^I{}_A + A_{\mu AB} X^I{}_B + B^{IJ}_\mu X^J{}_A \rightarrow \\ D_\mu X^I{}_A - \frac{4}{5me}(2(\epsilon D C^{IJ})_\mu X^J{}_A - (\epsilon D C_{AB})_\mu X^I{}_B), \end{aligned} \tag{3.11}$$

where the new covariant derivative, also denoted D_μ, is

$$D_\mu X^I{}_A = \partial_\mu X^I{}_A + C_{\mu AB} X^I{}_B + C^{IJ}_\mu X^J{}_A. \tag{3.12}$$

A more complete treatment using the non-abelian field strength $H^{IJ}_{\mu\nu}$ defined by the commutator as usual is obtained by the replacement

$$(\epsilon D C^{IJ})_\mu \rightarrow \tfrac{1}{2}\epsilon_\mu{}^{\nu\rho} H^{IJ}_{\nu\rho}. \tag{3.13}$$

[13] For a very nice discussion of the various mass terms that appear in this context and the relations between them, see [45].

To find the full non-abelian version of the above equations and to see how they can be solved also with the scalar source terms present we write the field equations schematically as

$$2\epsilon F + m(A-B) = gXD(A,B)X, \quad \epsilon G + m(A-B) = -gXD(A,B)X, \tag{3.14}$$

where all terms are gauge covariant in the broken phase (A and B are then the same gauge field up to covariant terms as we saw above). Solving for B from the first equation and inserting the answer into the second one gives, in the limit $g \to 0$ keeping $m = gv^2$ fixed,

$$\epsilon F = \tfrac{4}{m}\epsilon P(\epsilon F) + \tfrac{8}{m^2}\epsilon(\epsilon F, \epsilon F), \tag{3.15}$$

where $P = \partial + A$ and where we have used

$$\epsilon G(B) = \epsilon G(A + (B-A)) = \epsilon F(A) + 2\epsilon P(B-A) + 2\epsilon(B-A, B-A). \tag{3.16}$$

To linear order in $\frac{1}{m}$ this gives the same field equation as obtained for C_μ above. This may be compared to [45] where a similar set of equations is discussed. As seen there, choosing other combinations of the two gauge fields as the remaining one may lead to situations which require unlimited iterations of the kind we will see below when the scalar source terms are kept in the analysis.

Turning on g implies that one needs to solve the equations iteratively to eliminate B in the derivative $D = \partial + A + B$ which only appears in the expression XDX. This will produce an infinite series of terms in powers of $\frac{1}{m}X^2$. In fact, the iteration needed is just to consider the first equation in (3.14) and repeatedly eliminate B on the r.h.s. of

$$B = A + \tfrac{2}{m}\epsilon F - \tfrac{g}{m}XPX - \tfrac{g}{m}X(B-A)X. \tag{3.17}$$

Formally the solution is (for $m \neq 0$)

$$m(B-A) = \Sigma_{n=0}^{\infty}(\tfrac{X}{v})^n(2\epsilon F - gXPX)(\tfrac{X}{v})^n, \tag{3.18}$$

which gives the final answer when inserted into the second field equation in (3.14).

To summarize the situation in the gauge field sector: the vector fields corresponding to broken generators have all become massive in the higgs process and possess now one propagating mode each. The SO(p) gauge field in the final version of the theory is massive due to the appearance of both a Yang-Mills term and a CS term which is a generalized version of the higgs effect found by Mukhi and Papageorgakis [44] (see also [45]). The fields $A_\mu^{\hat{a}B}$ and $B_\mu^{\hat{i}J}$, on the other hand, both get a mass from a term involving the square of the gauge field which as we saw above gets added to their respective CS term, and there are no Yang-Mills terms involved in these cases. In the next subsection we will identify the scalar fields that get absorbed by the vector fields in the higgsing process.

3.2 Scalar mass terms

When we now turn to the scalar fields we need to divide them as follows:

$$x^i{}_a = (\hat{x}^{\hat{i}}{}_{\hat{a}}, x^{\hat{i}}{}_A, x^I{}_{\hat{a}}, x^I{}_A), \tag{3.19}$$

where, since the indices A and I are identified, the last field must be further split into

$$x^I{}_A = (z, w^{(\tilde{IA})}, y^{[IA]}). \tag{3.20}$$

Here w is symmetric and traceless and $z = x^{II} = \delta^I_A\, x^I_A$. The propagating modes absorbed by the gauge fields in order to become massive are $x^I{}_{\hat{a}}$, $x^{\hat{i}}{}_A$ and y^{IJ}, respectively, for the three mass terms in $L(v^2)$ (3.2) discussed in the previous subsection. These three scalar fields are thus eliminated by the higgsing leaving only the scalars $x^{\hat{i}}{}_{\hat{a}}, z, w^{IJ}$ in the theory.

We need to expand the expression in (2.4) whose square gives the new potential around the VEV. Using $X^i{}_a = v\delta^I_A + x^i{}_a$ we get

$$\begin{aligned}
&X^2 X^i_a - 4(X^i_b X^j_b)X^j_a = (pv^2 + 2vz + x^2)(v\delta^I_A + x^i{}_a)\\
&-4(v^2\delta^{IJ} + v(x^{iJ} + x^{jI}) + x^i{}_b x^j{}_b)(v\delta^J_A + x^j{}_a))\\
= \;&(p-4)v^3\delta^I_A + v^2(px^i{}_a + 2z\delta^I_A - 4x^{Ia} - 4(x^{iA} + x^{AI}))\\
&+v(x^2\delta^I_A + 2zx^i{}_a - 4x^i{}_b x^A{}_b - 4x^{iJ}x^{Ja} - 4x^{jI}x^{ja}) + x^2 x^{ia} - 4x^i{}_b x^j{}_b x^j{}_a.
\end{aligned} \tag{3.21}$$

The terms in the potential directly relevant for an analysis of the spectrum are of $\mathcal{O}(v^4)$. The expression that multiplies v^4 in the square of (3.21) reads

$$\begin{aligned}
&(3p^2 - 8p)x^2 + (12p - 64)z^2 - 16(p-3)x^{Ia}x^{Ia}\\
&-16(p-3)x^{iI}x^{iI} + 48x^{IJ}x^{IJ} - 16(p-6)x^{IA}x^{AI}.
\end{aligned} \tag{3.22}$$

We start by analyzing the scalar fields $x^{\hat{i}}{}_{\hat{a}}$. We find

$$L((x^{\hat{i}}{}_{\hat{a}})^2) = -\tfrac{1}{2}(D_\mu x^{\hat{i}}{}_{\hat{a}})^2 - \tfrac{1}{16}(x^{\hat{i}}{}_{\hat{a}})^2 \bar{R} - \tfrac{v^4 g^2}{2\cdot 32\cdot 32} p(3p-8)(x^{\hat{i}}{}_{\hat{a}})^2. \tag{3.23}$$

Inserting the constant background value for R, that is

$$\bar{R} = -\tfrac{6}{16\cdot 16} g^2 v^4 (p-4)^2 = 6\Lambda(p) = -\tfrac{6}{l^2}, \tag{3.24}$$

gives

$$L((x^{\hat{i}}{}_{\hat{a}})^2) = -\tfrac{1}{2}(D_\mu x^{\hat{i}}{}_{\hat{a}})^2 - \tfrac{2g^2 v^4}{16\cdot 16}(p-3)(x^{\hat{i}}{}_{\hat{a}})^2. \tag{3.25}$$

Comparing the BF bound to the $p = 2$ scalar $x^{\hat{i}}{}_{\hat{a}}$ mass value we see that they coincide:

$$\hat{m}^2(p=2) = -\tfrac{1}{64} g^2 v^4 = \Lambda(p=2). \tag{3.26}$$

Also the flat Minkowski case is consistent with unitarity since $\hat{m}^2(p=4) > 0$. One might also note that the two null-warped cases $p = 3$ and $p = 6$ with the same geometry (and perhaps without a BF-bound as argued in [48, 49]) seem nevertheless to be different since the masses are not the same for the two values of p.

We now turn to the trace $z = x^{II}$. Using $X^I{}_A = (v + \frac{z}{p})\delta^I_A + ...$, we get

$$\bar{\Box} z - \tfrac{1}{8} z\bar{R} - \tfrac{pv}{8} R^{(1)} - \tfrac{g^2 v^4}{32\cdot 32} 15(p-4)^2 z = 0, \tag{3.27}$$

where we have included the first variation of the scalar curvature in case there is a mixing between z and a gravity mode. Inserting also the expression for the background curvature scalar $\bar{R}$ quoted above it reads

$$\bar{\Box}z - \tfrac{3}{16\cdot 16}g^2v^4(p-4)^2z - \tfrac{pv}{8}R^{(1)} = 0. \tag{3.28}$$

To see if there is a mixing with gravity recall that

$$R^{(1)} := \delta R = -\bar{\Box}h + \bar{\nabla}_\mu\bar{\nabla}_\nu h^{\mu\nu} - h^{\mu\nu}\bar{R}_{\mu\nu}, \tag{3.29}$$

where $\bar{R}_{\mu\nu}$ is the background Ricci tensor which is non-trivial in all geometries except the round AdS. In our case we must allow for null-warped, and even more exotic, metrics with non-zero Cotton and traceless Ricci tensors. As we saw above, however, the scalar curvature is constant in all cases. We will continue the analysis of the field equation for z in the next subsection since we will need also the linearized Cotton equation which is the main subject of that subsection.

Next we consider the field w^{IJ} which is symmetric and traceless. We have

$$\bar{\Box}w + \tfrac{g^2v^4}{32}(p-6)w = 0, \tag{3.30}$$

corresponding to the mass

$$\hat{m}^2(w) = -\tfrac{g^2v^4}{16}(p-6). \tag{3.31}$$

Note that once again the null-warped cases $p=3$ and $p=6$ are different with even a zero mass value in the latter case (which also happens for $p=3$ in the case of $x^{\hat{i}}{}_{\hat{a}}$). This is a property that will be significant for some of the other scalar fields in the discussion of the higgs effect below.

For $x^{\hat{i}}_A$ and $x^I_{\hat{a}}$ we find the same linearized field equation:

$$\Box x - \tfrac{1}{8}\bar{R}\,x - \tfrac{3}{32\cdot 32}g^2v^4(p-4)^2x = 0. \tag{3.32}$$

Inserting the background value for the curvature scalar we find for each of these fields that the total mass term vanishes for all values of p:

$$\Box x^{\hat{i}}_A = 0,\ \ \Box x^I_{\hat{a}} = 0. \tag{3.33}$$

We may note that in three dimensions and for the round AdS_3, this happens to be the upper bound of the mass, using the standard formula also for $d=3$, where both Dirichlet and Neumann boundary conditions are allowed.

The final scalar field to analyze is the anti-symmetric part of $x^I{}_A$. Recall the definition $x^I{}_A = (z, w^{(\widetilde{IA})}, y^{[IA]})$, with w traceless and z the trace of $x^I{}_A$. One easily checks that the field $y^{IJ} = x^{[IJ]}$ behaves the same way as the last two scalar fields just discussed, namely

$$\Box y^{IJ} = 0. \tag{3.34}$$

Thus all scalar fields that are eaten by the vector fields corresponding to broken symmetries behave this way and this is so in all the backgrounds discussed here. More interesting is,

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however, the fact that for some values of p also physical scalar fields behave this way. The zero mass Klein-Gordon equation is also the equation for the singleton in AdS_3 [50], the implications of which need further study. However, it may be noted that in [50] the authors mention two different methods to realize singletons in the AdS_3 bulk theory, either as vector fields or by involving $\Box^2$ field equations. If and how any of these options is realized in the present context of the topologically gauged theories considered in this paper is not clear (see, however, the next subsection).

3.3 Linearized field equations for maximally symmetric backgrounds ($p = 2, 4$)

Due to the complications in the warped cases we will in this subsection restrict ourselves to the conformally flat cases, i.e., we assume that the background is either the maximally symmetric AdS or Minkowski obtained for $p = 2$ and $p = 4$, respectively. We will continue to use p dependent formulae when possible but we should be careful to remember that in this subsection the results are only valid for these two values of p.

For maximally symmetric backgrounds we have a zero Cotton tensor and

$$\bar{R}_{\mu\nu} = 2\Lambda g_{\mu\nu} = -\tfrac{2}{16\cdot 16} g^2 v^4 (p-4)^2 g_{\mu\nu}. \tag{3.35}$$

The first variation of the curvature scalar then becomes

$$R^{(1)} := \delta R = -\bar{\Box} h + \bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} - h^{\mu\nu}\bar{R}_{\mu\nu} = -\bar{\Box} h + \bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} - 2\Lambda h. \tag{3.36}$$

Using this expression in the Klein-Gordon equation for the field z we find

$$\bar{\Box}(\tfrac{z}{p} + \tfrac{v}{8} h) + \Lambda(3\tfrac{z}{p} + \tfrac{v}{4} h) = \tfrac{v}{8} H, \tag{3.37}$$

where $\bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} = H$ and $h = h^\mu{}_\mu$.

We thus seem to need another equation relating the fields z, h and H. This equation must come from the untraced Cotton equation since the traced one just gives back the scalar field equation for z. In fact, by decomposing the metric according to

$$h_{\mu\nu} = h^{TT}_{\mu\nu} + \bar{D}_{(\mu} V^T_{\nu)} + (\bar{D}_\mu \bar{D}_\nu - \tfrac{1}{3}\bar{g}_{\mu\nu}\Box)\phi + \tfrac{1}{3}\bar{g}_{\mu\nu} h, \tag{3.38}$$

we will obtain such an equation below. The Cotton equation is, after using the Klein-Gordon equation to eliminate some terms,

$$\begin{aligned} &\tfrac{1}{g} C_{\mu\nu} - \tfrac{eX^2}{16}(R_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R) - \tfrac{e}{4} g_{\mu\nu} V(X) \\ &-\tfrac{3e}{8} D_\mu X^i_a D_\nu X^i_a + \tfrac{e}{8} g_{\mu\nu} D^\sigma X^i_a D_\sigma X^i_a + \tfrac{e}{8} X^i_a D_\mu D_\nu X^i_a = 0, \end{aligned} \tag{3.39}$$

which now has to be linearized. This has been done in many places in the literature (usually with at most one scalar field present) and we just quote the result

$$\begin{aligned} &-\tfrac{v^2}{16}(\bar{e}\,\delta^\beta_{(\mu} - \tfrac{1}{\mu}\bar{\epsilon}_{(\mu}{}^{\alpha\beta}\bar{D}_{|\alpha})(-\tfrac{1}{2}\bar{\Box} h_{\beta|\nu)} + \tfrac{1}{2}\bar{\nabla}_{\beta|}\bar{\nabla}^\rho h_{\nu)\rho} \\ &\quad + \tfrac{1}{2}\bar{\nabla}_{\nu)}\bar{\nabla}^\rho h_{\beta\rho} - \tfrac{1}{2}\bar{\nabla}_{\beta|}\bar{\nabla}_{\nu)} h + \Lambda h_{\beta|\nu)} - \Lambda h \bar{g}_{\beta|\nu)}) \\ &+\tfrac{\bar{e}v\Lambda}{8}(\tfrac{z}{p} - \tfrac{v}{4} h)\bar{g}_{\mu\nu} + \tfrac{\bar{e}v^2}{64}(-\bar{\Box} h + H)\bar{g}_{\mu\nu} + \tfrac{v}{8}\bar{e}\bar{D}_\mu \partial_\nu \tfrac{z}{p} = 0. \end{aligned} \tag{3.40}$$

If the Cotton equation is traced we get

$$(\bar{\Box}+3\Lambda)\tfrac{z}{p}+\tfrac{v}{8}(\bar{\Box}+2\Lambda)h=\tfrac{v}{8}H \tag{3.41}$$

which as expected is identical to the equation coming from the Klein-Gordon equation for z given above.

We now need to analyze also the vector part of the Cotton equation. That is, we should keep the vector fields and get the equation for $W_\mu=\nabla^\mu h_{\mu\nu}$. Using the decomposition of the metric given above we find after some algebra

$$\tfrac{3}{2}\tfrac{\Lambda}{\mu}\epsilon_\mu{}^{\alpha\beta}\bar{D}_\alpha W_\beta=\tfrac{8}{v}\bar{D}_\mu((\bar{\Box}+3\Lambda)\tfrac{z}{p}+\tfrac{v}{8}(\bar{\Box}+2\Lambda)h-\tfrac{v}{8}H). \tag{3.42}$$

The scalar equation for z obtained above puts the expression in the r.h.s. bracket to zero and hence

$$\epsilon_\mu{}^{\alpha\beta}\bar{D}_\alpha W_\beta=0. \tag{3.43}$$

Relating V^T_μ in the metric decomposition to W_μ we get an equation whose divergence becomes

$$H=\tfrac{2}{3}\bar{\Box}(\bar{\Box}+3\Lambda)\phi+\tfrac{1}{3}\bar{\Box}h, \tag{3.44}$$

and using this in the scalar equation for z leads to the following result:

$$(\bar{\Box}+3\Lambda)(\tfrac{z}{p}+\tfrac{v}{12}(h-\bar{\Box}\phi))=0, \tag{3.45}$$

which means that there is actually only one physical index-free scalar field in the theory.

In order to choose a convenient gauge[14] we note that the equation $\epsilon_\mu{}^{\alpha\beta}\bar{D}_\alpha W_\beta=0$ suggests the gauge choice $V^T_\mu=0$. Choosing also $\bar{\Box}\phi=h$ it follows that

$$h_{\mu\nu}=h^{TT}_{\mu\nu}+(\bar{D}_\mu\bar{D}_\nu-\tfrac{1}{3}\bar{g}_{\mu\nu}\bar{\Box})\phi+\tfrac{1}{3}\bar{g}_{\mu\nu}h=h^{TT}_{\mu\nu}+\bar{D}_\mu\bar{D}_\nu\phi, \tag{3.46}$$

and

$$H=\bar{D}^\mu\bar{D}^\nu\bar{D}_\mu\bar{D}_\nu\phi=(\bar{\Box}+2\Lambda)\bar{\Box}\phi=(\bar{\Box}+2\Lambda)h. \tag{3.47}$$

As in the previous subsection, we find also here some features indicating that AdS_3 bulk singletons play a role. Writing the parameter of coordinate transformations as $\xi_\mu=\xi^T_\mu+\partial_\mu\xi$ we get a transformation of the trace of the metric involving a $\Box$ which together with the appearance of $\Box^2$ above should be compared to the discussion in [50].

Finally, the equation for the traceless transverse part of the metric $h^{TT}_{\mu\nu}$ is identical to the one obtained in pure TMG [14] namely

$$(\bar{\mathcal{D}}(\mu)\bar{\mathcal{D}}(l)\bar{\mathcal{D}}(-l))_{(\mu}{}^\rho h^{TT}_{\nu)\rho}=0, \tag{3.48}$$

where the operators $\mathcal{D}(l)$ etc are defined as

$$\bar{\mathcal{D}}(l)_\mu{}^\rho=\bar{e}\delta^\rho_\mu-\tfrac{1}{l}\bar{\epsilon}_\mu{}^{\alpha\rho}\bar{D}_\alpha. \tag{3.49}$$

An analysis with more properties of supergravity taken into account can be found in the work by Becker et al. [17]. In particular, it is found there that at the critical point (and only there) super-TMG theories with $\mathcal{N}=(1,0)$, $\mathcal{N}=(0,1)$ and $\mathcal{N}=(1,1)$ supersymmetry but without a matter sector satisfy a positive energy theorem (in the sign conventions of [14]) and are chiral in the same sense as in the bosonic case studied in [14].

[14] Fixing the gauge completely, e.g., using the physical light-cone gauge as done in [51], one finds that all non-zero components of the metric can be expressed in terms of the stress tensor for the matter fields.

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3.4 Susy rules for any p

In this subsection we will briefly discuss what the transformation rules tell us about the multiplet structure in the different backgrounds. The following formulae are valid for all values of p.

The fields that appear after the superconformal symmetry breaking will organize themselves into supermultiplets according to their number of SO(N − p) vector indices for the simple reason that the supersymmetry parameter does not have any such indices. Thus we find one multiplet with $8(N-p)$ d.o.f. for both bosons and fermions containing the following fields (the $\hat{a}$-vector multiplet)

$$x^{\hat{i}}{}_{\hat{a}},\ \psi_{\hat{a}},\ A_\mu^{A\hat{a}}(\text{massive}), \tag{3.50}$$

and one with $8p$ d.o.f. for both bosons and fermions containing (the $\hat{a}$-scalar multiplet)

$$C_\mu{}^{IJ}(\text{massive}),\ w^{IJ},\ z,\ \psi_A,\ B_\mu^{\hat{i}J}(\text{massive}). \tag{3.51}$$

The remaining vector fields $A_\mu^{\hat{a}\hat{b}}$(massless) and $B_\mu^{\hat{i}\hat{j}}$(massless) couple to both multiplets as usual for CS gauge fields carrying no degrees of freedom. These two multiplets will also couple to the gravitational field with spin 2 which is still massless and without propagating degrees of freedom. The corresponding statement for the spin 3/2 fields depends on the number of surviving supersymmetries.[15] Below we will present some properties of the transformation rules that support this picture.

The supersymmetry transformation rules are as quoted from [1, 2], with $\epsilon_m = A\epsilon_g$ and $A^2 = \frac{1}{2}$,

$$\begin{aligned}
\delta e_\mu{}^\alpha &= i\bar{\epsilon}_g\gamma^\alpha\chi_\mu, && (3.52)\\
\delta\chi_\mu &= \tilde{D}_\mu\epsilon_g, && (3.53)\\
\delta B_\mu^{ij} &= -\tfrac{i}{2e}\bar{\epsilon}_g\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{3ig}{8}\bar{\psi}_a\gamma_\mu\Gamma^{[i}\epsilon_m X_a^{j]} - \tfrac{ig}{16}\bar{\psi}_a\gamma_\mu\Gamma^{ijk}\epsilon_m X_a^k \\
&\quad -\tfrac{ig}{4}\bar{\chi}_\mu\Gamma^{k[i}\epsilon_g X_a^{j]}X_a^k - \tfrac{ig}{32}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X^2, && (3.54)\\
\delta X_a^i &= i\bar{\epsilon}_m\Gamma^i\psi_a, && (3.55)\\
\delta\psi_a &= \gamma^\mu\Gamma^i\epsilon_m(\tilde{D}_\mu X_a^i - iA\bar{\chi}_\mu\Gamma^i\psi_a) + \tfrac{g}{8}\Gamma^i\epsilon_m X_b^i X_b^j X_a^j - \tfrac{g}{32}\Gamma^i\epsilon_m X_a^i X^2, && (3.56)\\
\delta A_\mu^{ab} &= \tfrac{ig}{4}\bar{\epsilon}_m\gamma_\mu\Gamma^i\psi_{[a}X_{b]}^i + \tfrac{ig}{8}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X_a^i X_b^j, && (3.57)
\end{aligned}$$

which we want to linearize around a general background. Consider first $\delta\psi_a$ written as

$$\delta\psi_a = \gamma^\mu\Gamma^i\epsilon_m(\tilde{D}_\mu X_a^i - iA\bar{\chi}_\mu\Gamma^i\psi_a) - \tfrac{g}{32}\Gamma^i\epsilon_m(X^2X_a^i - 4X_b^iX_b^jX_a^j), \tag{3.58}$$

where we recognize in the last term the expression whose square is the potential and which has been expanded in powers of the VEV in the previous section.

Choosing first $a = A$ we get

$$\begin{aligned}
\delta\psi_A &= \gamma^\mu\Gamma^{\hat{i}}\epsilon_m D_\mu X^{\hat{i}}{}_A + \gamma^\mu\Gamma^I\epsilon_m D_\mu X^I{}_A - \tfrac{gv^3}{32}\Gamma^I\epsilon_m(p-4)\delta_A^I \\
&\quad + \tfrac{gv^2}{8}\Gamma^I\epsilon_m(x^{IA} + 2x^{(IA)}) - \tfrac{gv^2}{32}\Gamma^I\epsilon_m(p\,x^I{}_A + 2\delta_A^I z) \\
&\quad - \tfrac{gv^2}{32}\Gamma^{\hat{i}}\epsilon_m(p-4)x^{\hat{i}A} + O(x^2).
\end{aligned} \tag{3.59}$$

[15] This number will depend on p as is clear from the analysis of [32].

Note that in all non-Minkowskian backgrounds ($p \neq 4$) there are non-zero constant terms indicating a symmetry breaking of the superconformal symmetry. However, these terms can be removed by adding a special superconformal transformation

$$\delta_S \psi_a =< X^i{}_a > \Gamma^i \eta_m, \;\; \eta_m = \epsilon_m \tfrac{gv^2}{32}(p-4). \tag{3.60}$$

Thus, the Q transformations present in any of the broken phases (except the Minkowski one) are obtained by this special combination of the Q and S transformations in the unbroken conformal phase. For instance, in the round AdS case obtained for $p = 2$ this leads to the covariant derivative

$$\delta\chi_\mu = D_\mu \epsilon_g + \gamma_\mu \eta_g = (D_\mu - \tfrac{gv^2}{16}\gamma_\mu)\epsilon_g := \hat{D}_\mu \epsilon_g, \tag{3.61}$$

where we assumed that the same relation between η_m and η_g is true as for the ordinary susy parameters. Note that as expected the new term is related to the cosmological constant

$$\Lambda = -\tfrac{1}{l^2} = -\tfrac{g^2v^4}{16\cdot 16}(p-4)^2, \tag{3.62}$$

as $\frac{gv^2}{16} = \frac{1}{2l}$. Thus for $p = 2$ we find that

$$\hat{D}_\mu = D_\mu - \tfrac{1}{2l}\gamma_\mu, \tag{3.63}$$

which is the same result as found in the ABJM case in [13]. In fact, this form of the covariant derivative is valid for all values of p (with $l = \infty$ for $p = 4$).

With this understanding of the mixing of Q and S transformations we have

$$\begin{aligned}\delta\psi_A &= \gamma^\mu \Gamma^I \epsilon_m D'_\mu (w^{IA} + \tfrac{1}{p}\delta^{IA} z) - v(A^{IA}_\mu - B^{IA}_\mu)) + \gamma^\mu \Gamma^{\hat{i}} \epsilon_m v B^{\hat{i}A}_\mu \\ &\quad + \tfrac{gv^2}{8}\Gamma^I \epsilon_m (3w^{IA} + \tfrac{3}{p}\delta^{IA} z) - \tfrac{gv^2}{32}\Gamma^I \epsilon_m (p\, w^{IA} + 3\delta^I_A z)] + O(x^2),\end{aligned} \tag{3.64}$$

where $D'_\mu x^I{}_A = \partial_\mu x^I{}_A + A_{\mu AB} x^I{}_B + B^{IJ}_\mu x^J{}_A$ and where we have only kept the physical scalar fields that are not eaten in the higgs effect. At this point we should recall the discussion in the beginning of this section concerning the reduction of the two gauge fields to a single massive one and the structure of the interaction terms involving the remaining scalar fields that arose in that analysis. Using that information we will find that the above transformation rule is in fact rather non-trivial when written out in detail.

Next we consider the transformation rule for the other choice of index, i.e., $a = \hat{a}$, which after higgsing reads

$$\delta\psi_{\hat{a}} = \gamma^\mu (\Gamma^{\hat{i}} \epsilon_m D'_\mu x^{\hat{i}\hat{a}} + v\Gamma^I \epsilon_m A^{\hat{a}I}_\mu) - \tfrac{gv^2 p}{32}\Gamma^{\hat{i}} \epsilon_m \hat{x}^{\hat{i}}{}_{\hat{a}} + \mathcal{O}(x^2), \tag{3.65}$$

which also supports the multiplet structure given above. There are many features here that need further study. These will be studied elsewhere.

4 Comments

The topologically gauging [1] of free matter CFTs in three dimensions with eight supersymmetries gives rise to an $O(N)$ type model with a novel six order scalar potential [2]. This potential consists of three different triple trace terms, one of them being $(X^2)^3$ where $X^2 := X^i{}_a X^i{}_a$ with $i = 1, 2, .., 8$ and $a = 1, 2, .., N$. Neglecting the R-symmetry index this term is precisely the scalar term $(\phi^i\phi^i)^3$ that has been discussed recently, see, e.g., Aharony et al. [52], in the context of the AdS_4/CFT_3 correspondence relating $O(N)$ models in three dimensions to four-dimensional Vasiliev higher spins systems [53]. Note that when construction the other two sixth order terms appearing in the potential of this topologically gauged model the R-symmetry index play a key role. These two terms in the potential are therefore not present in the usual treatments of marginal deformations of $O(N)$ type models in three dimensions but are crucial for the critical solutions to appear in our models.

In relation to the AdS_4/CFT_3 correspondence it may also the pointed out that in these topologically gauged $O(N)$ models the Chern-Simons terms of both the vector fields and the spin connection are multiplied by the same coupling constant (denoted g). Thus if the interpolation between the A and B type HS models in [53], parametrized by the parameter θ_0, is related to the introduction of gauge interactions and a non-parity symmetric Chern-Simons term as argued in [54], and hence also to the related bosonization phenomenon, then in versions with $\mathcal{N} = 8$ supersymmetry also gravitational Chern-Simons terms will enter on the field theory side. One may speculate that such spin two terms may be related to turning on θ_2, the second coefficient among the θ_{2n} parameters defining the HS theories that interpolate between the A and B type models in Vasiliev's system in AdS_4.

Some features of topologically gauged CFTs indicate that they may have a deeper role to play in the context of AdS/CFT. The AdS_4/CFT_3 correspondence was mentioned above but also the AdS_3/CFT_2 correspondence has recently been investigated in depth in many papers using W_N algebras in two dimensions and its connection to Vasiliev's higher spin systems in three dimensions, see [55]. In view of the fact that AdS_3 arises naturally as a spontaneously broken phase of a three-dimensional topologically gauged superconformal theory as discussed in this paper, one may ask if this conformal theory could not itself be the boundary theory of an AdS_4 theory. The *sequential* AdS/CFT that is suggested by these facts was first discussed in [56]. The new information since that paper was written, namely that the topologically gauged CFT_3 with eight supersymmetries is actually a kind of $O(N)$ model, may thus be important. Also the possible role of singletons found in this paper may be pointing in the direction of such a *sequence*. In the topologically gauged ABJ(M) models first derived in [10] and developed further in [13] the situation is a bit more complicated since in that case there are more than one independent coupling constant for any choice of gauge group. The idea that several AdS/CFTs may follow one after the other has appeared previously in the literature. Based on higher spin and unfolding arguments, Vasiliev raised this possibility in [57] and made it explicit in a recent paper [58]. Speculations with the same goal based on AdS_d foliations of AdS_{d+1} can be found in [59] (see also [60] for related comments). However, the scenario of a “sequential AdS/CFT” coming from a topologically gauged CFT_3 is the first one which relies on a dynamical model and a conformal symmetry breaking mechanism interpolating between two AdS/CFTs as pointed out in [56].

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The main purpose of this paper was to elaborate on the observation that the topologically gauged $O(N)$ theory with eight supersymmetries has a number of special background solutions with interesting properties. These solutions, of which two were found in [2], depend on the number of scalar fields that are given a VEV and can be characterized by the value of μl where μ is the coupling constant of the gravitational CS term and l is related as usual to the cosmological constant. The solutions that appear correspond to the values $\mu l = \frac{1}{3}, 1, 3, \infty, 5, 3, \frac{7}{3}, 2$. Here we recognize the second one as connected to chiral gravity, the third and sixth ones to the null-warped, or Schrödingier($z = 2$), geometry while $\mu l = 5$ can be associated with a solution recently discovered in [29]. $\mu l = \infty$ corresponds to Minkowski space and requires a separate discussion.

In this paper we have tried to argue that although for each of these values there are more than one kind of solution, the ones that are relevant as broken phases of the superconformal topologically gauged theory are only the "critical" ones.[16] For $\mu l = 1$ this is based on the fact that the critical, or chiral, case has no propagating massive gravitons which should be a direct consequence of the connection to the superconformal unbroken phase which is also lacking such modes. The $\mu l = 3$ null-warped, or Schrödingier($z = 2$), case has also been argued to be chiral in [25] but is also "critical" for seemingly different reasons, see, e.g., [38]. The working hypothesis adopted here that all the above values of μl have special solutions is indeed also supported by the existence of a special solution for $\mu l = 5$ [29]. The topologically gauged ABJ(M) theory [10] have similar properties but for a smaller set of solutions.

For $p = 8$ we get $\mu l = 2$ which stands out because it is even. If there is a special solution of this kind it should contain odd powers[17] of $e^{\rho/l}$. Examples with such a dependence on ρ are known in theories containing a scalar field with a potential, see, e.g., [61]. In [62] the Fefferman-Graham expansion for NMG is discussed in detail and a generalized expansion introduced that can accommodate both novel boundary behavior in AdS as well as entirely different non-AdS boundary behavior like for the $\mu l = 3$ null-warped solution. There are also generalizations with higher values $\mu l = 5, 7, ...$ [63, 64].

The "critical" null-warped, or Schrödinger($z = 2$), solution is one of the most attractive three-dimensional geometries for condensed matter applications. This geometry (often with extra flat directions) is designed to have Schrödinger symmetries on the boundary that play a crucial role in, e.g., unitary Fermi gases (cold atoms) etc. Finally, let us return to the topologically gauged ABJ(M) case mentioned in the introduction. There we recalled the result from [10] that giving a real VEV v to one of the complex scalar fields gives rise to a background solution corresponding to a super-TMG theory at the chiral point. In the context of the topologically gauged SO(N) model investigated in this paper the VEV was generalized to a $p \times p$ diagonal VEV matrix leading to a number of interesting backgrounds.

[16] A way to make this more concrete may be to consider the unitary representations of SO(3, 2) that are involved and how they behave under the symmetry breaking. This way of looking at it could, e.g., explain why only the representations of SO(2, 2) with the correct properties appear in AdS_3.

[17] See *Note added* at the end of this section for recent developments.

Repeating this step for the ABJ(M) case we find, for $p \leq 4$,

$$\mu l = \sqrt{\frac{-3p^2}{5p^2 - 24p + 16}}. \tag{4.1}$$

The values produced by this formula are

$$\mu l = 1, 1, \sqrt{\tfrac{27}{11}}, \infty, \tag{4.2}$$

where we recognize the first two as critical round AdS and the last one as Minkowski. This analysis for ABJ(M) is valid for infinite level but one should note that if the other two sets of potential terms (i.e., the single and double trace terms) are kept they may be non-zero in some of these backgrounds. From the properties of the structure constants $f^{ab}{}_{cd}$ summarized in [56] we see that even the part of the potential linear in the structure constant may contribute: $f^{ab}{}_{ab} \to N^2N' - NN'^2$ giving $p(p-1)$ in a vector model (that is, for $N' = 1$) with $N = p$ in the background.

Note added. Since this paper appeared on the ArXiv, there has been developments relevant for the list of known solutions realizing the values of μl listed in (2.21) and discussed in section 2.3. There the value $\mu l = 2$ was not discussed since no such solution seemed to be known in the literature. However, a solution with $\mu l = 2$ was found recently in extended topologically massive gravity with $(1, 1)$ supersymmetry in [65]. That solution involves in a crucial way a topologically massive vector field. All the necessary ingredients for the $\mu l = 2$ solution used in [65] are at hand also in the topologically gauged $\mathcal{N} = 8$ theory discussed in the present paper and we thus expect this kind of $\mu l = 2$ solution to exist also here. Note that this $\mu l = 2$ solution is a null-warped one [65] of the kind that is known to appear in our case for $\mu l = 3$.

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A Cancelation of terms in δL with one or no D

Before starting the computation we give our conventions. We use a mostly plus metric and a Levi-Civita tensor defined by

$$\epsilon^{\mu\nu\rho}: \quad \epsilon^{012} = +1. \tag{A.1}$$

Then

$$\epsilon^{\mu\nu\rho}\epsilon_{\tau\nu\rho} = -2e^2\delta^\mu_\tau, \quad \epsilon^{\mu\nu\rho}\epsilon_{\alpha\beta\rho} = -2e^2\delta^{\mu\nu}_{\alpha\beta}. \tag{A.2}$$

Our gamma matrices satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \tag{A.3}$$

and are chosen such that

$$e\gamma^{\mu\nu\rho} = \epsilon^{\mu\nu\rho}, \quad e\gamma^{\mu\nu} = \epsilon^{\mu\nu\rho}\gamma_\rho, \quad 2e\gamma^\mu = -\epsilon^{\mu\nu\rho}\gamma_{\nu\rho}. \tag{A.4}$$

The lagrangian that we need in the following reads

$$\begin{aligned} L = & \tfrac{1}{g}L^{\rm conf}_{\rm sugra} + \tfrac{1}{\alpha}L_{CS(A)} - \tfrac{1}{2}eg^{\mu\nu}D_\mu X^i_a D_\nu X^i_a + \tfrac{i}{2}e\bar{\psi}_a\gamma^\mu D_\mu\psi_a \\ & +ieA\bar{\chi}_\mu\Gamma^i\gamma^\nu\gamma^\mu\psi_a(D_\nu X^i_a - \frac{i}{2}\hat{A}\bar{\chi}_\nu\Gamma^i\psi_a) \\ & -iA'\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\Gamma^{ij}\chi_\nu(D_\rho X^i_a)X^j_a \\ & +iA''\bar{f}\cdot\gamma\Gamma^i\psi X^i_a + iA_{12}\bar{f}\cdot\chi X^2 + A_{13}eRX^2 \\ & +ieA_{14}\bar{\psi}_a\psi_a X^2 + ieA'_{14}\bar{\psi}_a\psi_b X^i_a X^i_b + ieA_{15}\bar{\psi}_a\Gamma^{ij}\psi_b X^i_a X^j_b \\ & +ie\bar{\chi}\cdot\gamma\Gamma^i\psi_a(A_{16}X^i_a X^2 + A'_{16}X^j_a X^i_b X^j_b) \\ & +ie\bar{\chi}\cdot\chi(A_{17}(X^2)^2 + A'_{17}(X^i_a X^j_a)(X^i_b X^j_b)) \\ & +ie\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\gamma_\nu\chi_\rho(A_{18}(X^2)^2 + A'_{18}(X^i_a X^j_a)(X^i_b X^j_b)) \\ & +eA_{19}(X^2)^3 + eA'_{19}(X^2)(X^i_a X^j_a)(X^i_b X^j_b) + eA''_{19}(X^i_a X^j_a)(X^j_b X^k_b)(X^k_c X^i_c), \end{aligned} \tag{A.5}$$

where all the terms in the first four lines (except $\frac{1}{\alpha}L_{CS(A)}$) were determined in [1] with the following result:

$$\hat{A} = A, \quad A' = -\tfrac{1}{4}, \quad A'' = A, \quad A_{12} = \tfrac{1}{4}, \quad A_{13} = -\tfrac{1}{16}, \quad \textit{and} \quad A^2 = \tfrac{1}{2}. \tag{A.6}$$

$\frac{1}{\alpha}L_{CS(A)}$ plus the potential were found in [2] by various methods. This appendix is a continuation of the Noether computation started in [1] and supplies the missing details of the presentation in [2] where the final result was first presented. Here we also give a more direct argument leading to the normalization of $\frac{1}{\alpha}L_{CS(A)}$ than that given in [2]. The new terms in $\delta\psi$ and δB^{ij}_μ will be crucial. We therefore give them explicitly:

$$\delta\psi_a = \gamma^\mu\Gamma^i\epsilon_m(D_\mu X^i_a - iA\bar{\chi}_\mu\Gamma^i\psi_a) + B_5\Gamma^i\epsilon_m X^i_a X^2 + B_6\Gamma^i\epsilon_m X^i_b X^j_a X^j_b, \tag{A.7}$$

where $A = \pm\frac{1}{\sqrt{2}}$, and

$$\begin{aligned} \delta B^{ij}_\mu = & -\frac{i}{2e}\bar{\epsilon}_g\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{ig}{16}\bar{\psi}_a\gamma_\mu\Gamma^{ijk}\epsilon_m X^k_a - \tfrac{3ig}{8}\bar{\psi}_a\gamma_\mu\Gamma^{[i}\epsilon_m X^{j]}_a \\ & -\tfrac{ig}{4}\bar{\chi}_\mu\Gamma^{k[i}\epsilon_g X^{j]}_a X^k_a - \tfrac{ig}{32}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X^2. \end{aligned} \tag{A.8}$$

Now we add also a variation of the gauge field but without the usual three-algebra structure constant, i.e.,

$$\delta A_{\mu ab} = 2iq\bar{\epsilon}_m\gamma_\mu\Gamma^i\psi_{[a}X^i_{b]} + q'i\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X^i_a X^j_b, \tag{A.9}$$

leading to the following form of the covariant derivative

$$D_\mu X^i_a = \partial_\mu X^i_a + B^{ij}_\mu X^j_a + A_{\mu a}{}^b X^i_b. \tag{A.10}$$

The various kinds of terms with one derivative D that can appear in δL and need to be canceled are with two fermions

$$\epsilon D\psi X^3,\ \ \epsilon D\chi X^4, \tag{A.11}$$

and a D together with four fermions

$$\epsilon D\psi\chi\psi,\ \ \epsilon D\psi\chi\chi X,\ \epsilon D\chi\chi^2 X^2. \tag{A.12}$$

The D^2 and D^3 terms in δL were dealt with in [1].

A.1 Terms with one D and two fermions: $\epsilon D\psi X^3$ terms

Starting with the cancelation of $\epsilon D\psi X^3$ these terms arise from a number of places, namely $\delta L_{KG}|_{\delta B=\epsilon\psi X}$, $\delta L_{\text{Dirac}}|_{\delta\psi=\epsilon X^3}$ and $\delta L_{14(\text{Yuk})}|_{\delta\psi=\epsilon DX}$.

Adding these should give something that can be canceled by adding a term $\chi\psi X^3$ and vary χ. Note that B_5 and B_6 are obtained from the computation now to be done.

$$\begin{aligned}
&\delta L_{KG}|_{\delta B=\epsilon\psi X,\delta A=\epsilon\psi X}\\
&= -e(D^\mu X^i_a)X^j_a(-\tfrac{ig}{16}\bar{\psi}_b\gamma_\mu\Gamma^{ijk}\epsilon_m X^k_b - \tfrac{3ig}{8}\bar{\psi}_b\gamma_\mu\Gamma^{[i}\epsilon_m X^{j]}_b)\\
&\quad -iqe(D^\mu X^i_a)X^i_b\bar{\epsilon}_m\gamma_\mu\Gamma^j(\psi_a X^j_b - \psi_b X^j_a),
\end{aligned} \tag{A.13}$$

where we see that the first term needs to be canceled by the Yukawa term containing Γ^{ij} and the other can be written with the antisymmetry written out and with an index b on the spinor and i on the Γ in all terms:

$$\begin{aligned}
&\delta L_{4-KG}|_{\delta B=\epsilon\psi X,\delta A=\epsilon\psi X}\\
&= -e(D^\mu X^i_a)X^j_a(-\tfrac{ig}{16}\bar{\psi}_b\gamma_\mu\Gamma^{ijk}\epsilon_m X^k_b - \tfrac{3ig}{16}\bar{\psi}_b\gamma_\mu\Gamma^{i}\epsilon_m X^j_b + \tfrac{3ig}{16}\bar{\psi}_b\gamma_\mu\Gamma^{j}\epsilon_m X^i_b)\\
&\quad -iqe(D^\mu X^j_b)X^j_a\bar{\epsilon}_m\gamma_\mu\Gamma^i\psi_b X^i_a + iqe(D^\mu X^j_a)X^j_b\bar{\epsilon}_m\gamma_\mu\Gamma^i\psi_b X^i_a.
\end{aligned} \tag{A.14}$$

Next we derive the contribution from $\delta L_{5(\text{Dirac})}|_{\delta\psi=\epsilon X^3}$:

$$\begin{aligned}
&\delta L_{5-\text{Dirac}}|_{\delta\psi=\epsilon X^3}\\
&= ie\bar{\psi}_b\Gamma^i\gamma^\mu\tilde{D}_\mu\epsilon_m(B_5 X^i_b X^2 + B_6 X^j_b X^i_a X^j_a)\\
&\quad +ie\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m\tilde{D}_\mu(B_5 X^i_b X^2 + B_6 X^j_b X^i_a X^j_a),
\end{aligned} \tag{A.15}$$

and from $\delta L_{\text{Yuk}}|_{\delta\psi=\epsilon DX}$ we get:

$$\begin{aligned}
&\delta L_{\text{Yuk}}|_{\delta\psi=\epsilon DX}\\
&= 2ieA_{14}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_b)X^2 + 2ieA'_{14}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_a)X^j_a X^j_b\\
&\quad +2ieA_{15}\bar{\psi}_b\Gamma^{jk}\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_a)X^j_b X^k_a\\
&= 2ieA_{14}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_b)X^2 + 2ieA'_{14}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_a)X^j_a X^j_b\\
&\quad +2ieA_{15}\bar{\psi}_b\Gamma^{ijk}\gamma^\mu\epsilon_m(\tilde{D}_\mu X^i_a)X^j_b X^k_a\\
&\quad +ieA_{15}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^2)X^i_b - 2ieA_{15}\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m(\tilde{D}_\mu X^j_a)X^j_b X^i_a.
\end{aligned} \tag{A.16}$$

Since we are avoiding derivatives on ψ we must cancel terms as they are without integrations by part. Then all terms except the $D\epsilon$ must cancel directly. The first terms to cancel are the Γ^{ijk} terms giving

$$2A_{15} - \tfrac{g}{16} = 0. \tag{A.17}$$

Then from the cancelation of $\bar{\psi}...\epsilon_m X^2 \tilde{D}_\mu X^i_b$ and $\bar{\psi}...\epsilon_m X^i_b \tilde{D}_\mu X^2$ we get

$$B_5 + 2A_{14} = 0, \quad B_5 + A_{15} = 0, \tag{A.18}$$

which implies

$$B_5 = -\tfrac{g}{32}, \;\; B_5 = -2A_{14}, \;\; A_{15} = \tfrac{g}{32}. \tag{A.19}$$

Looking now at the terms $(D^\mu X^j_a) X^i_a X^j_b$ and $(D^\mu X^i_a) X^j_a X^j_b$ we find cancelation for

$$-\tfrac{3g}{16} + B_6 - 2A_{15} + q = 0, \tag{A.20}$$

and

$$\tfrac{3g}{16} + B_6 + 2A'_{14} = 0, \tag{A.21}$$

and for the last kind of such terms $(D^\mu X^j_b) X^i_a X^j_a$:

$$B_6 - q = 0, \tag{A.22}$$

giving the result

$$B_6 = q = \tfrac{g}{8}, \;\; A'_{14} = -\tfrac{5g}{32}, \;\; \textit{using} \;\; A_{15} = \tfrac{g}{32}. \tag{A.23}$$

Finally to cancel the $\tilde{D}_\mu \epsilon$ term we must add

$$ie\bar{\chi} \cdot \gamma \Gamma^i \psi_b (A_{16} X^i_b X^2 + A'_{16} X^j_b X^j_a X^i_a) \tag{A.24}$$

with

$$A_{16} = -AB_5 \;\; \textit{and} \;\; A'_{16} = -AB_6 \tag{A.25}$$

A.2 Terms with one D and two fermions: $\epsilon D\chi X^4$ terms

These come from the following variations

$$\begin{aligned}
\delta L_{4(KG)}|_{\delta B=\epsilon\chi X, \delta A=\epsilon\chi X} &= e(\tilde{D}_\mu X^i_a) X^j_a \big(\tfrac{ig}{4} \bar{\chi}^\mu \Gamma^{k[i} \epsilon_g X^{j]}_b X^k_b + \tfrac{ig}{32} \bar{\chi}^\mu \Gamma^{ij} \epsilon_g X^2\big) \\
&\quad - q' ie (\tilde{D}_\mu X^i_a) X^i_b \bar{\chi}_\mu \Gamma^{jk} \epsilon_g X^j_a X^k_b, \qquad\qquad (A.26) \\
\delta L_{9(SC)}|_{\delta\psi=\epsilon X^3} &= ieAB_5 \bar{\chi}_\nu \Gamma^i \Gamma^j \gamma^\mu \gamma^\nu \epsilon_m (\tilde{D}_\mu X^i_a) X^j_a X^2 \\
&\quad + ieAB_6 \bar{\chi}_\nu \Gamma^i \Gamma^j \gamma^\mu \gamma^\nu \epsilon_m (\tilde{D}_\mu X^i_a) X^j_b X^k_a X^k_b, \qquad (A.27) \\
\delta L_{10}|_{\delta\psi=\epsilon X^3} &= iA'' B_5 \bar{f}^\mu \gamma_\mu \Gamma^i \Gamma^j \epsilon_m X^j_a X^2 X^i_a + iA'' B_6 \bar{f}^\mu \gamma_\mu \Gamma^i \Gamma^j \epsilon_m X^j_b X^k_a X^k_b X^i_a \\
&= iA'' B_5 \bar{f}^\mu \gamma_\mu \epsilon_m (X^2)^2 + iA'' B_6 \bar{f}^\mu \gamma_\mu \epsilon_m X^i_a X^j_a X^i_b X^j_b, \qquad (A.28)
\end{aligned}$$

since the Γ^{ij} term vanishes! Next term is

$$\begin{aligned}
\delta L_{16}|_{\delta\psi=\epsilon\tilde{D}X} &= ie\bar{\chi} \cdot \gamma \Gamma^i \delta\psi_b (A_{16} X^i_b X^2 + A'_{16} X^j_b X^j_a X^i_a) \\
&= ie\bar{\chi} \cdot \gamma \Gamma^i \Gamma^k \gamma^\mu \epsilon_m (D_\mu X^k_a)(A_{16} X^i_a X^2 + A'_{16} X^j_a X^j_b X^i_b). \qquad (A.29)
\end{aligned}$$

Here there will be a nice test of the coefficients so far since all Γ^{ij} terms must cancel when summing up the expressions above. The reason is that no $\chi^2 X^4$ terms can be written down with Γ^{ij} matrices.

We now have all the contributions and can start to require cancelations from susy. First, the Γ^{ij} matrix terms give for the X^2 terms, using also the relation for B_4,

$$\tfrac{g}{32}g^{\mu\nu}\epsilon_g + AB_5\gamma^\mu\gamma^\nu\epsilon_m - A_{16}\gamma^\nu\gamma^\mu\epsilon_m = 0, \tag{A.30}$$

which means

$$\tfrac{g}{32}g^{\mu\nu}\epsilon_g + A^2B_5\gamma^\mu\gamma^\nu\epsilon_g - AA_{16}\gamma^\nu\gamma^\mu\epsilon_g = 0, \tag{A.31}$$

giving for the $\gamma^{\mu\nu}$ terms

$$\tfrac{1}{2}B_5 + AA_{16} = 0, \tag{A.32}$$

and for the $g^{\mu\nu}$ terms

$$\tfrac{g}{32} + \tfrac{1}{2}B_5 - AA_{16} = 0. \tag{A.33}$$

Adding and subtracting them give the following two equations

$$B_5 = -\tfrac{g}{32}, \quad AA_{16} = \tfrac{g}{64}. \tag{A.34}$$

Next we turn to the Γ^{ij} matrix terms give for the non-X^2 terms

$$\begin{aligned}
&\tfrac{ig}{8}e(\tilde{D}_\mu X^i_a)X^j_a(\bar{\chi}^\mu\Gamma^{ki}\epsilon_g X^j_b X^k_b - \bar{\chi}^\mu\Gamma^{kj}\epsilon_g X^i_b X^k_b)\\
&\quad -q' ie(\tilde{D}_\mu X^i_a)X^j_a\bar{\chi}_\mu\Gamma^{jk}\epsilon_g X^i_b X^k_b\\
&\quad +ieAB_6\bar{\chi}_\nu\Gamma^{ij}\gamma^\mu\gamma^\nu\epsilon_m(\tilde{D}_\mu X^i_a)X^j_b X^k_a X^k_b\\
&\quad +ieA'_{16}\bar{\chi}\cdot\gamma\Gamma^{ik}\gamma^\mu\epsilon_m(D_\mu X^k_a)X^j_a X^j_b X^i_b = 0
\end{aligned} \tag{A.35}$$

Changing indices to get the same factor of $(DX)X$ and then dropping it gives

$$\begin{aligned}
&-\tfrac{g}{8}\bar{\chi}^\mu\Gamma^{ik}\epsilon_g X^j_b X^k_b + \tfrac{g}{8}\bar{\chi}^\mu\Gamma^{jk}\epsilon_g X^i_b X^k_b - q'\bar{\chi}_\mu\Gamma^{jk}\epsilon_g X^i_b X^k_b\\
&\quad +AB_6\bar{\chi}_\nu\Gamma^{ik}\gamma^\mu\gamma^\nu\epsilon_m X^k_b X^j_b - A'_{16}\bar{\chi}\cdot\gamma\Gamma^{ik}\gamma^\mu\epsilon_m X^j_b X^k_b = 0.
\end{aligned} \tag{A.36}$$

The γ-terms must give rise to an anticommutator which means that

$$A'_{16} = -AB_6, \tag{A.37}$$

and then the whole equation becomes

$$\begin{aligned}
&-\tfrac{g}{8}\bar{\chi}^\mu\Gamma^{ik}\epsilon_g X^j_b X^k_b + \tfrac{g}{8}\bar{\chi}^\mu\Gamma^{jk}\epsilon_g X^i_b X^k_b - q'\bar{\chi}_\mu\Gamma^{jk}\epsilon_g X^i_b X^k_b\\
&\qquad\qquad +2AB_6\bar{\chi}_\nu\Gamma^{ik}\epsilon_m X^k_b X^j_b = 0.
\end{aligned} \tag{A.38}$$

Using that $\epsilon_m = A\epsilon_g$ then gives

$$-\tfrac{g}{8}\Gamma^{ik}X^j_b X^k_b + \tfrac{g}{8}\Gamma^{jk}X^i_b X^k_b - q'\Gamma^{jk}X^i_b X^k_b + 2A^2B_6\Gamma^{ik}X^k_b X^j_b = 0, \tag{A.39}$$

implying

$$q' = \tfrac{g}{8}, \;\; 2A^2B_6 = \tfrac{g}{8} \;\; \textit{or} \;\; B_6 = \tfrac{g}{8}. \tag{A.40}$$

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Now we check the remaining terms, i.e., those without Γ-matrices

$$\begin{aligned}&\tfrac{ie}{4}AB_5\bar{\chi}_\nu\gamma^\mu\gamma^\nu\epsilon_m\tilde{D}_\mu(X^2)^2+\tfrac{ie}{4}AB_6\bar{\chi}_\nu\gamma^\mu\gamma^\nu\epsilon_m\tilde{D}_\mu(X^i_aX^i_bX^j_aX^j_b)\\&+iA''B_5\bar{f}^\mu\gamma_\mu\epsilon_m(X^2)^2+iA''B_6\bar{f}^\mu\gamma_\mu\epsilon_mX^i_aX^i_bX^j_aX^j_b\\&+\tfrac{ie}{4}A_{16}\bar{\chi}_\nu\gamma^\nu\gamma^\mu\epsilon_m\tilde{D}_\mu(X^2)^2+\tfrac{ie}{4}A'_{16}\bar{\chi}_\nu\gamma^\nu\gamma^\mu\epsilon_m\tilde{D}_\mu(X^i_aX^i_bX^j_aX^j_b),\end{aligned}\tag{A.41}$$

where the last two terms come from the above variation of $\tilde{\omega}$ in the RX^2 term. Note that the very last term then cancels the second term!

Then with $e\gamma^{\mu\nu}=\epsilon^{\mu\nu\rho}\gamma_\rho$ the X^2 terms containing f become (the rest of the terms work the same way)

$$\begin{aligned}&+iA''B_5\bar{f}^\mu\gamma_\mu\epsilon_m(X^2)^2=\tfrac{i}{2}A''B_5\epsilon^{\mu\nu\rho}\tilde{D}_\nu\bar{\chi}_\rho\gamma_\mu\epsilon_m(X^2)^2\\&=-\tfrac{i}{2}A''B_5\epsilon^{\mu\nu\rho}\bar{\chi}_\rho\gamma_\mu(\tilde{D}_\nu\epsilon_m)(X^2)^2-\tfrac{i}{2}A''B_5\epsilon^{\mu\nu\rho}\bar{\chi}_\rho\gamma_\mu\epsilon_m(\tilde{D}_\nu(X^2)^2)+\text{contortion}\\&=\tfrac{ie}{2}A''B_5\bar{\chi}_\mu\gamma^{\mu\nu}(\tilde{D}_\nu\epsilon_m)(X^2)^2-\tfrac{ie}{2}A''B_5\bar{\chi}_\mu\gamma^{\mu\nu}\epsilon_m(\tilde{D}_\nu(X^2)^2)+\text{contortion},\end{aligned}\tag{A.42}$$

after an integration by parts.

We can now collect and cancel the $D(X^2)^2g^{\mu\nu}\epsilon_m$ terms:

$$AA_{16}=-\tfrac{1}{2}B_5,\tag{A.43}$$

while the antisymmetric part implies

$$-\tfrac{1}{4}AB_5+\tfrac{1}{4}A_{16}+\tfrac{1}{2}A''B_5=0,\tag{A.44}$$

which just means that the previous relation is obtained once again.

Add now term 18 in L

$$L_{18}=ieA_{18}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\gamma_\nu\chi_\rho(X^2)^2,\tag{A.45}$$

which varies into

$$\delta L_{18}=2ieA_{18}\epsilon^{\mu\nu\rho}\tilde{D}_\mu\bar{\epsilon}_g\gamma_\nu\chi_\rho(X^2)^2-4eA_{18}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\gamma_\nu\chi_\rho(X^2)X^i_a\bar{\epsilon}_m\Gamma^i\psi_a.\tag{A.46}$$

Thus if

$$2A_{18}\epsilon_g=\tfrac{1}{2}AB_5\epsilon_m,\tag{A.47}$$

the one-derivative terms cancel so (sing $2A^2=1$)

$$A_{18}=\tfrac{1}{8}B_5.\tag{A.48}$$

Since the other terms work the same way the full new term in L is

$$L_{18}=ieA_{18}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\gamma_\nu\chi_\rho(X^2)^2+ieA'_{18}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\gamma_\nu\chi_\rho X^i_aX^i_bX^j_aX^j_b,\tag{A.49}$$

with

$$A_{18}=\tfrac{1}{8}B_5\ \ and\ \ A'_{18}=\tfrac{1}{8}B_6.\tag{A.50}$$

Note that no χ^2X^2 term without the Levi-Civita tensor is not needed just as in ABJM. With the obtained values we see that

$$A_{18}=-\tfrac{g}{256}\ \ and\ \ A'_{18}=\tfrac{g}{64}\tag{A.51}$$

A.3 The normalization of the CS term for the gauge field A_μ^{ab}

After having determined the coefficients q and q' in the variation δA_μ^{ab} we must now return to the question of the corresponding CS term appearing in L and its normalization in terms of the parameter α. We will trace the places in the previous derivation of the lagrangian where the field strength $F_{\mu\nu}^{ab}$ appears simply by looking for where $G_{\mu\nu}^{ab}$ appears as a result of evaluating the commutator of two covariant derivatives acting on X_a^i. Note that this computation also arises acting on the supersymmetry parameter in some cases but then $F_{\mu\nu}^{ab}$ will not appear since the susy parameter is inert under gauge symmetry.

There are two places where $F_{\mu\nu}^{ab}$ appears: in the variation of the Dirac kinetic term giving

$$\tfrac{i}{2}\epsilon^{\mu\nu\rho}\bar{\psi}_a\gamma_\rho\Gamma^i\epsilon_m(G_{\mu\nu}^{ij}X_a^j + F_{\mu\nu}^{ab}X_b^i), \tag{A.52}$$

and from the variation of the term denoted L'

$$iA'\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\Gamma^{ik}\epsilon_g(G_{\mu\nu}^{ij}X_a^j + F_{\mu\nu}^{ab}X_b^i)X_a^k. \tag{A.53}$$

These contributions to δL must be cancelled by adding terms to the variation of the gauge fields using

$$\delta L_{CS(B,A)} = \tfrac{1}{g}\epsilon^{\mu\nu\rho}\delta B_\mu^{ij}|_{\text{new}}G_{\nu\rho}^{ij} + \tfrac{1}{2\alpha}\epsilon^{\mu\nu\rho}\delta A_\mu^{ab}|_{\text{new}}F_{\nu\rho}^{ab}. \tag{A.54}$$

For the R-symmetry terms this implies

$$B_2 = -\tfrac{g}{2},\;\; B_3 = gA', \tag{A.55}$$

as we already have seen. However, for the gauge field A_μ^{ab} the results are new and read

$$\alpha = -2q,\;\; 2\alpha A' = q', \tag{A.56}$$

which must give the same answer for α. Inserting $q = q' = \frac{g}{8}$ and $A' = -\frac{1}{4}$ we find that this is indeed the case:

$$\alpha = -\tfrac{g}{4}. \tag{A.57}$$

A.4 Cancellation of terms with no D and two fermions

Here we concentrate on the cancellations that will lead us to the form of the potential.

Start by varying the X^6 potential

$$L_{X^6} = eA_{19}(X^2)^3 + eA'_{19}(X^2)(X_a^iX_a^j)(X_b^iX_b^j) + eA''_{19}(X_a^iX_a^j)(X_b^jX_b^k)(X_c^kX_c^i). \tag{A.58}$$

We find that varying this term gives

$$\begin{aligned}\delta L_{X^6} = {} & ie(\bar{\epsilon}_g\gamma^\mu\chi_\mu)(A_{19}(X^2)^3 + A'_{19}(X^2)(X_a^iX_a^j)(X_b^iX_b^j)\\ & +A''_{19}(X_a^iX_a^j)(X_b^jX_b^k)(X_c^kX_c^i))\\ & +ieA_{19}6(X^2)^2X_a^i\bar{\epsilon}_m\Gamma^i\psi_a\\ & +ieA'_{19}(2X_c^k\bar{\epsilon}_m\Gamma^k\psi_c(X_a^iX_a^j)(X_b^iX_b^j) + 4X^2(X_a^i\bar{\epsilon}_m\Gamma^j\psi_a)(X_b^iX_b^j))\\ & +ieA''_{19}(6\bar{\epsilon}_m\Gamma^i\psi_aX_a^j)(X_b^jX_b^k)(X_c^kX_c^i).\end{aligned} \tag{A.59}$$

From the χ terms we can obtain uniquely the A_{16} coefficients in front of the $\chi\psi X^3$ terms using $\delta\psi = \epsilon X^3$. This variation reads

$$\begin{aligned} \delta L_{\chi\psi X^3}|_{\delta\psi|_{\epsilon X^3}} &= ie\bar{\chi}\cdot\gamma\Gamma^i(\delta\psi_a)|_{\epsilon X^3}(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b) \\ &= ie\bar{\chi}\cdot\gamma\Gamma^i\Gamma^k\epsilon_m(B_5X^k_aX^l_cX^l_c + B_6X^k_cX^l_aX^l_c)(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b). \end{aligned} \tag{A.60}$$

Here all Γ^{ik} terms vanish since all expressions in terms of six scalars are symmetric in two free ik indices. Thus the above becomes

$$\begin{aligned} \delta L_{\chi\psi X^3}|_{\delta\psi|_{\epsilon X^3}} &= ie\bar{\chi}\cdot\gamma\epsilon_m(B_5X^i_aX^l_cX^l_c + B_6X^i_cX^l_aX^l_c)(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b) \\ = -ie\bar{\epsilon}_m\gamma\cdot\chi(B_5A_{16}(X^2)^3 &+ (B_5A'_{16} + B_6A_{16})X^2X^{ij}X^{ij} + B_6A'_{16}X^{ij}X^{jk}X^{ki}). \end{aligned} \tag{A.61}$$

Thus the cancelation of these terms gives the relations

$$A_{19} = B_5(AA_{16}),\;\; A'_{19} = B_5(AA'_{16}) + B_6(AA_{16}),\;\; A''_{19} = B_6(AA'_{16}) \tag{A.62}$$

Now recall

$$\begin{aligned} &\delta L_{5(\mathrm{Dirac})}|_{\delta\psi=\epsilon X^3} \\ &= ie\bar{\psi}_b\Gamma^i\gamma^\mu\tilde{D}_\mu\epsilon_m(B_5X^i_bX^2 + B_6X^j_bX^i_aX^j_a) \\ &\quad + ie\bar{\psi}_b\Gamma^i\gamma^\mu\epsilon_m\tilde{D}_\mu(B_5X^i_bX^2 + B_6X^j_bX^i_aX^j_a), \end{aligned} \tag{A.63}$$

where only the $D\epsilon$ terms remain to be canceled which is done by the term

$$L_{\chi\psi X^3} = ie\bar{\chi}\cdot\gamma\Gamma^i\psi_a(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b). \tag{A.64}$$

The $\delta\chi_\mu = D_\mu\epsilon_g$ variation gives

$$\begin{aligned} \delta L_{\chi\psi X^3}|_{\delta\chi_\mu = D_\mu\epsilon_g} &= ieD_\mu\bar{\epsilon}_g\gamma_\mu\Gamma^i\psi_a(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b) \\ &= ie\bar{\psi}_a\gamma_\mu\Gamma^iD_\mu\epsilon_g(A_{16}X^i_aX^2 + A'_{16}X^j_aX^i_bX^j_b). \end{aligned} \tag{A.65}$$

Cancelation implies

$$AB_5 = -A_{16},\;\; AB_6 = -A'_{16}. \tag{A.66}$$

Hence we know the six order potential:

$$\begin{aligned} A_{19} &= -A_{16}^2 = -(AB_5)^2 = -\tfrac{g^2}{2\cdot 32\cdot 32}, \\ A'_{19} &= -2A_{16}A'_{16} = -2A^2B_5B_6 = \tfrac{g^2}{8\cdot 32}, \\ A''_{19} &= -(A'_{16})^2 = -(AB_6)^2 = -\tfrac{g^2}{2\cdot 8\cdot 8}. \end{aligned} \tag{A.67}$$

With a potential the theory should have an AdS vacuum that puts the theory at a chiral point. If we set the VEV $< X >= v$ we find that the potential gives

$$L_{X^6}(v) = (A_{19} + A'_{19} + A''_{19})v^6. \tag{A.68}$$

Adding the gravitational CS term and the X^2R term evaluated at the VEV we get

$$L_{\mathrm{AdS}} = L_{CS(\omega)} - \tfrac{v^2 e}{16}R + L_{X^6}(v) \tag{A.69}$$

We should compare this to Li, Song and Strominger (LSS) for the chiral point but with TMG signs in the lagrangian:

$$L_{\mathrm{LSS}} = \frac{1}{\kappa^2}(\frac{1}{\mu}L_{CS(\omega)} - e(R-2\Lambda)). \tag{A.70}$$

Thus $\mu = \frac{1}{\kappa^2}$ and $v^2 = \frac{16}{\kappa^2}$. The chiral point condition is $\mu l = 1$ where l is defined by means of the cosmological constant as $\Lambda = -\frac{1}{l^2}$. This implies that, to end up a chiral point, the potential must satisfy

$$\tfrac{1}{e}L_{X^6}(v) = \tfrac{2\Lambda}{\kappa^2} = -\tfrac{2}{\kappa^2 l^2} = -\tfrac{2\mu^2}{\kappa^2} = -\tfrac{2}{\kappa^6} = -\tfrac{2v^6}{16^3}. \tag{A.71}$$

Thus we see that for the theory to be at the chiral point we must require

$$A_{19} + A'_{19} + A''_{19} = -\tfrac{2}{16^3} = -\tfrac{1}{2048}, \tag{A.72}$$

(which strangely enough happens to be exactly A_{19} above!).

Next we consider the variation of the Yukawa terms that connect to the variation of the X^6 potential above

$$L_{\mathrm{Yuk}} = ieA_{14}\bar{\psi}_a\psi_a X^2 + ieA'_{14}\bar{\psi}_a\psi_b X^i_a X^i_b + ieA_{15}\bar{\psi}_a\Gamma^{ij}\psi_b X^i_a X^j_b \tag{A.73}$$

Vary this using the $\psi = \epsilon X^3$ expression

$$\delta\psi_a|_{\epsilon X^3} = B_5\Gamma^k\epsilon_m X^k_a X^l_b X^l_b + B_6\Gamma^k\epsilon_m X^k_b X^l_a X^l_b \tag{A.74}$$

We get

$$\begin{aligned}\delta L_{\mathrm{Yuk}} &= 2ieA_{14}\bar{\psi}_a\delta\psi_a X^2 + 2ieA'_{14}\bar{\psi}_a\delta\psi_b X^i_a X^i_b + 2ieA_{15}\bar{\psi}_a\Gamma^{ij}\delta\psi_b X^i_a X^j_b\\ &= 2ieA_{14}\bar{\psi}_a(B_5\Gamma^k\epsilon_m X^k_a X^2 + B_6\Gamma^k\epsilon_m X^k_b X^l_a X^l_b)X^2\\ &\quad +2ieA'_{14}\bar{\psi}_a(B_5\Gamma^k\epsilon_m X^k_b X^2 + B_6\Gamma^k\epsilon_m X^k_c X^l_b X^l_c)X^i_a X^i_b\\ &\quad +2ieA_{15}\bar{\psi}_a\Gamma^{ij}(B_5\Gamma^k\epsilon_m X^k_b X^2 + B_6\Gamma^k\epsilon_m X^k_c X^l_b X^l_c)X^i_a X^j_b\end{aligned} \tag{A.75}$$

From the conformal variation of the spin 3/2 field in the 16'th term in L we get

$$(\delta L_{16} + \delta L_{16'})|_{\chi=\gamma\epsilon X^2} = 3ieB_7\bar{\epsilon}_m\Gamma^i\psi_a(A_{16}X^i_a(X^2)^2 + A'_{16}X^j_a X^i_b X^j_b X^2) \tag{A.76}$$

Cancelation gives the relations

$$\begin{aligned}(X^2)^2X^i:&\ \ 6A_{19} = 2B_5(A_{14}+A_{15})\\ (X^{jk}X^{jk})X^i:&\ \ 2A'_{19} = 2B_6A_{15}\\ X^2X^{ji}X^i:&\ \ 4A'_{19} = 2B_5(A'_{14}-A_{15}) + 2B_6A_{14}\\ X^{kj}X^{ji}X^i:&\ \ 6A''_{19} = 2B_6(A'_{14}-A_{15}).\end{aligned} \tag{A.77}$$

Inserting the values of the various parameters on the right hand sides as derived previously we confirm the values of $A_{19}, A'_{19}, A''_{19}$ found above.

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