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REVIEW

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A survey on modeling interference and blockage in urban heterogeneous cellular networks

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Abstract

In this paper, we provide a survey on abstraction models for evaluating aggregate interference statistics in urban heterogeneous cellular networks. The two principal interference shaping factors are the path loss attenuation and the interference geometry. For both factors, our survey systematically summarizes state-of-the-art models and outlines their strengths and weaknesses. In the context of path loss attenuation, we give an overview on the basic propagation mechanisms and the various approaches for their abstraction. We specifically elaborate on random shape theory and its application for representing blockages in indoor and outdoor scenarios. In terms of interference geometry, we present techniques from stochastic geometry as well as deterministic approaches, outlining their evolution and limitations. Throughout the paper, challenges under discussion are scenarios with both indoor and outdoor environments, distance-dependent shadowing due to blockages, and correlations among node and blockage locations as well as the distinction between cell center and cell edge. Our goal is to raise awareness on not only the validity and tractability but also the limitations of state-of-the-art techniques. The presented models were chosen with regard to their adaptability for a broad range of scenarios. They are therefore expected to be adopted for describing the fifth generation of mobile networks (5G).

Keywords: Interference modeling, Aggregate interference, Blockage, Fading, Path loss, Shadowing, Stochastic geometry, Random shape theory, Point process, Urban, Heterogeneous networks, 5G

1 Review

Massive network densification and heterogeneity are two major trends heralding the fifth generation of mobile cellular networks (5). Heterogeneous networks are commonly identified as systems comprising multiple types of base stations (BS) that are distinguished by their transmit power and backhaul and radio access technology as well as the experienced propagation conditions. In such topologies, the aggregate co-channel interference from other cells (also referred to as *other-cell interference, external interference, network interference*, or simply *interference*) is one of the main performance limiting factors [1–7]. At the same time, it forms the basis for determining the signal-to-interference ratio (SIR) and the other-cell interference factor (OCIF), which constitute fundamental metrics for assessing the performance of mobile networks.

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The SIR commonly refers to the ratio between the desired signal power and the total interference power [1, 3]. In contrast, the OCIF (also termed *f-factor* or *interference factor*) is traditionally defined as the ratio of the othercell interference to the own-cell interference (also denoted as same-cell or inner-cell interference) [8–11]. Own-cell interference arises, e.g., as multiple access interference due to cross correlation of spread-spectrum signals in a code-division multiple access (CDMA) system [8]. In more recent work, the OCIF is defined as the ratio of the other-cell received power to the total inner-cell received power, encompassing both the desired signal as well as the own-cell interference [5-7, 12, 13]. This definition is still valid for mobile systems without own-cell interference, such as orthogonal frequency-division multiple access (OFDMA) [6, 14]. Therefore, the thorough statistical description of aggregate co-channel interference from other cells is essential for system analysis and design. The main goal of the interference analysis is to *capture key*



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Page 2 of 20

characteristics of the interference as a function of *relatively few parameters*. Although abstractions such as the Wyner model and the hexagonal grid first appeared two or even five decades ago [15, 16], mathematically tractable interference statistics are still the exception rather than the rule.

A frequently applied approach is the *Gaussian random process* [17, 18]. The model is reasonably accurate when aggregating a large number of interferers without a dominant term such that the central limit theorem (CLT) applies [19, 20]. In many cases, the probability density functions (PDFs) will exhibit heavier tails than those anticipated by the Gaussian approach [1, 21–25].

In general, the PDF is unknown, and aggregate interference is typically characterized by either the Laplace transform (LT), the characteristic function (CF), or the mobility generating functional (MGF), respectively [26]. In this article, the LT is considered most relevant due to its suitability for random variables (RV) with non-negative support and its *moment-generating properties*. Moreover, the CF and the MGF can directly be deduced from the LT by basic identities. Let *I* denote a RV with PDF $f_I(x)$, representing the aggregate interference. Then, its LT is given as

$$\mathcal{L}_{I}(s) = \mathbb{E}\left[e^{-sI}\right] = \int_{0}^{\infty} f_{I}(x)e^{-sx}dx.$$
 (1)

The *n*th moment of *I* is determined by

$$\mathbb{E}\left[I^n\right] = (-1)^n \left.\mathcal{L}_I^{(n)}(s)\right|_{s=0},\tag{2}$$

where $\mathcal{L}_{I}^{(n)}(s)$ refers to the *n*th derivative of $\mathcal{L}_{I}(s)$. In theory, a statistical distribution is fully characterized by specifying all of its moments, given that all moments exist and the MGF converges. Practical approaches in wireless communication engineering usually exploit only the first few of them. Application examples include *moment matching* and *deriving performance bounds by inequalities* such as the Markov inequality [27].

The two main interference shaping factors are the *path loss attenuation* and the *interference geometry* [1, 14, 28–34]. The path loss attenuation describes the difference between the transmit and receive power levels. The interference geometry condenses the transmitter locations and the channel access scheme [14, 35, 36].

1.1 Our contributions

This article provides a survey on state-of-the-art modeling and abstraction of these two factors. We particularly focus on urban environments, as they form the major field of application for heterogeneous mobile networks. In the context of signal propagation modeling, we elaborate on the basic propagation mechanisms as well as their abstraction. The main novelty of this section lies in a survey on models based on random shape theory. Those are applied for investigating the impact of blockages in indoor and outdoor scenarios. In the context of interference geometry, we outline models for abstracting the BS locations. In comparison to related surveys, we discuss both stochastic *and* deterministic models. We address strengths and limitations, and demonstrate their application by means of a case study.

This paper exclusively addresses *aggregate co-channel interference from other cells*. Other types of interference encompass *inter-carrier, inter-symbol, inter-layer, interuser*, and *own-cell* interferences. Each of these interference types has its particular characteristics and, thus, requires its own mathematical framework. Due to space limitations, these kinds of interferences are considered beyond the scope of this paper.

The majority of aggregate interference models aims at describing downlink transmissions. For this reason, we employ the terms BS and receiver, when exclusively referring to the downlink, and transmitter and receiver, when pointing out that a model is equivalently applicable for upand downlink. In this article, the term tier either refers to a ring of transmitters in a grid-based setup or the specific part of a heterogeneous network, which is associated with a certain class of transmitters, such as macro-BS and small cell BSs, respectively. The particular meaning becomes apparent from the context. Since the focus of this paper is placed upon cellular networks, we consider the field of device-to-device (D2D) communications beyond the scope of this paper. Throughout the paper, we comment on the adaptability of the presented models for abstracting (5G) topologies.

1.2 Related work

The closest related works to the contribution in this paper are [37] in the context of signal propagation modeling and [26] in the domain of interference geometry abstraction.

The authors of [37] provide a broad overview on largescale path loss modeling. They specifically elaborate on seven different types of path loss models, presenting their advantages and drawbacks. In this paper, we briefly summarize these *traditional approaches*. Compared to [37], our focus is rather placed upon heterogeneous networks in urban environments. We specifically address the abstraction of large object blockage by means of random object processes.

The authors in [26] provide a survey on stochastic geometry models for single-tier and multi-tier cognitive mobile networks. They summarize the five most prominent techniques to utilize the LT of the aggregate interference for modeling the network performance. In this paper, we briefly outline these techniques in Section 3.1.2. While the authors of [26] mainly focus on the opportunities of the stochastic geometry analysis, in this paper, we also address its limitations. Moreover, we discuss deterministic models, which, to the best of our knowledge, have not yet been surveyed.

1.3 Organization

This paper is organized as follows. In Section 2, we scrutinize signal propagation mechanisms. We review traditional models and place particular focus on statistical models for representing blockages. In Section 3, we investigate the abstraction of transmitter locations and the impact of channel access mechanisms. We elaborate on techniques from stochastic geometry and their major insights.

We also shed light on the evolution of deterministic models. We address the limitation of both approaches and compare them by means of a case study. Section 4 outlines further aspects of interference modeling. Section 5 concludes the work.

2 Signal propagation modeling

Due to the broadcast nature of the wireless medium, any signal sent from a transmitter experiences various kinds of distortion along its way to the receiver. These will depend on the environment as well as the location of the transmitter and the receiver. In this section, we discuss techniques for abstracting the mechanisms that govern the signal propagation. An overview is provided in Fig. 1.

2.1 Signal propagation mechanisms

Signal propagation is governed by four basic mechanisms [38]: *free-space loss* (distance-dependent loss along a line of sight (LOS) link), *reflections* (waves are reflected by objects that are substantially larger than the wavelength), *diffractions* (based on Huygen's principle, secondary waves form behind large impenetrable blockages), and *scattering* (energy is dispersed in various directions by objects that are small relative to the wavelength). These effects individually perturb the signal traveling from a transmitter to a receiver, thus determining the instantaneous signal strength. A formal definition of the path loss attenuation in decibel is given as

$$PL = P_t - P_r + G_t + G_r, aga{3}$$

where P_t and P_r represent the transmit and receive power levels and G_t and G_r refer to the transmit and receive antenna gains. When sectorized scenarios are considered, the antenna characteristics can be incorporated in G_{t} , including the antenna orientation and the angular dependent antenna gains, respectively. The losses caused by the four basic propagation mechanisms constitute the difference between P_t and P_r . In principle, each mechanism is well known and the resulting path loss attenuation can be exactly determined by evaluating Maxwell's equations. Such calculation requires a very accurate description of the environment. In practice, it is infeasible to solve for a single point to point link, let alone the evaluation of an entire network. Real-world propagation environments exhibit a complex structure, which leads to the necessity of abstraction. The requirement for a path loss attenuation model is to be simple enough to assure tractability while still capturing the most prominent effects of a realistic scenario.

In comparison to analytical studies, simulations enable a low degree of abstraction, i.e., they allow to incorporate a large amount of details. Path loss attenuation models may even follow a certain generation procedure, such as in the 3rd Generation Partnership Project (3GPP) spatial channel model (SCM) [39], the Wireless World Initiative New Radio (WINNER) model [40], and the 3GPP threedimensional (3D) channel model [41]. In these models, the environment is represented by statistical parameters and the exact propagation conditions are computed at



runtime. Such models are infeasible for analytical considerations, where signal propagation is commonly described by deterministic laws and RVs, as presented in the next section.

The basic propagation mechanisms are affecting the transmission in both the *below 6-GHz domain* as well as the *millimeter wave (mmWave) domain*. Therefore, most of the models that are described in the following can be adapted to represent either domain, by adjusting the influence of the individual effects accordingly. Several 5G specific references were added, in order to capture the ongoing work in this direction [42-45].

2.2 General modeling approach

A common approach for modeling path loss attenuation is expressed as

$$PL = \underbrace{L(d,f) + X_{\sigma}}_{\text{large-scale path loss}} + \underbrace{F}_{\text{small-scale path loss}}, \qquad (4)$$

where L(d, f) refers to the mean path loss, X_{σ} is the shadowing, and F denotes the small-scale fading. The term L(d, f) is mainly based on the effect of free-space path loss, which depends on the distance d between a transmitter and a receiver as well as the carrier frequency f. Note that it is independent of the node locations within the scenario. The RV X_{σ} corresponds to the shadowing caused by blockages. The RV F primarily captures the effects of the multi-path propagation. It is important to note that (4) does not model each of the four basic propagation mechanism, as presented in Section 2.1, separately. Each of the three terms rather incorporates *all* mechanisms to a certain extent.

The terms in (4) can be grouped into *large-scale path* loss, including the mean path loss and the shadowing, and *small-scale path loss* referring to *F*. This terminology is derived from the scale in space and time, where severe variations are expected to occur. The small-scale component can show large fluctuations in a short period of time as well as within few wavelengths. The corresponding models are commonly denoted as *channel models* [40, 41, 46–48]. They incorporate the effects of singleinput single-output (SISO) and multiple input multiple output (MIMO) transmissions and may include correlations over time and frequency. Modeling the influence of these effects is of interest when instantaneous transmission characteristics are investigated. In the following, we focus on the long-term average trends of the path loss, referring to the *large-scale* component in (4). A survey on MIMO channel models is provided in [49] and is considered beyond the scope of the paper.

2.3 Traditional path loss attenuation models

In literature, a substantial number of large-scale path loss models have been reported. They can be categorized into four groups: *empirical models, deterministic models, semi-deterministic models*, and *hybrid models*. The main distinctive characteristic of these models is the trade-off between accuracy and complexity. While these models aim at representing the large-scale component in (4), they do not necessarily distinguish mean path loss and shadowing.

2.3.1 Empirical models

Empirical models are typically obtained from measurement campaigns in a certain environment and describe the characteristics of the signal propagation by a deterministic law or some RV. They can be characterized by only few parameters and have found wide acceptance for analytical studies and simulations.

Examples for empirical path loss laws include the COST 231 One Slope Model and the COST 231 Hata Model [50]. The most famous example for a random abstraction of large-scale path loss is log-normal shadowing, where the effect of blockages is crammed into a log-normally distributed RV. The variance of the distribution depends on the environment and has to be determined by measurements. Thus, the model is only valid for specific scenario and requires and empirical calibration step. In real-world scenarios, the locations of large objects will be highly correlated [51]. Interference correlation in scenarios with stochastic node locations (conf. Section 3.1) is scrutinized in [52, 53]. The correlation in these papers is almost exclusively obtained by the static locations of the nodes, whereas the correlation of collocated blockages is not taken into account [54]. The authors of [54] present a correlated shadowing model by exploiting a Manhattan Poisson line process. They provide a promising method to better understand the generative processes that govern the shadowing. On the other hand, the usefulness of their approach is limited to Manhattan-type urban geometries.

Recent studies on blockage effects in urban environments indicate the dependency of shadowing on the link length [54, 55]. It follows the intuition that a longer link increases the likelihood of buildings to intersect with it. Such propagation characteristics have also been discussed recently within the 3GPP [41, 56] and cannot be reproduced by the log-normal model. As presented in Section 2.4, they can be reflected by approaches based on random shape theory.

The authors in [57] propose a multi-slope model, where the path loss law itself is a piecewise function of the distance. A related approach is to distinguish between LOS and non-line of sight (NLOS) conditions and to adapt the path loss model accordingly. In this case, it is crucial to decide whether a given link is in LOS or NLOS, depending on the link length [42]. A combination of the multi-slope model and the distinction between LOS and NLOS conditions is reported in [58].

2.3.2 Deterministic models

The goal of deterministic models is to represent the characteristics of a specific scenario with high accuracy and to include all basic propagation mechanisms. Consequently, deterministic models are characterized by the need for detailed site-specific information and large computation efforts. Two classes of deterministic models have been reported in literature. *Finite-difference time-domain models* try to replace Maxwell's differential equations with finite-difference equations, thus exhibiting a certain degree of abstraction. *Geometry models* rely on geometric rays that interact with the specified objects and are also referred to as *ray-tracing models* [59, 60]. Due to the fundamental dependency on site-specific information, it is difficult to draw general conclusions from the attained results.

2.3.3 Semi-deterministic models

Empirical and deterministic models form the two opposing ends of the accuracy-complexity trade-off. Combining both approaches leads to *semi-empirical* and *semi-deterministic* models. These models still incorporate some site-specific information while parameterizing other parts of the model by results from measurement campaigns. Some effects such as reflections may be ignored to reduce the complexity of the model. A frequently applied representative of semi-empirical models is the COST 231 Walsch-Ikegami Model [50]. A more recent, map-based approach has been proposed in [61] within the scope of the METIS 2020 project. It follows the concept that building heights are extracted from map data and are then used to estimate the path loss.

2.3.4 Hybrid models

Hybrid models combine multiple of the previously discussed propagation models. This is especially beneficial when scenarios contain sections with fundamentally different propagation conditions. A classic example is outdoor-to-indoor communication [62, 63], where the output of a 3D semi-deterministic geometry model is transformed into a 2D geometry model for describing the indoor propagation.

2.4 Stochastic blockage models

In this section, we focus on a newly emerging class of path loss models that describes attenuations due to blockages by statistical parameters. These models can expediently be used for indoor and outdoor scenarios, are mathematical tractable, and can be characterized by few parameters. Their formulation is based on concepts from *random* *shape theory,* which represents the formal framework around random objects in space [64].

While we focus on large-scale blockages such as walls and buildings, the authors of [45] show that the obstruction due to the human body can be modeled in a similar way. Body blockage is particularly distinct in the mmWave domain, where even the attenuation due to foliage affects the signal propagation, as investigated, e.g., in [65].

Let \mathcal{O} denote a set of objects on \mathbb{R}^n , which are closed and bounded, i.e., have finite area and perimeter. For instance, \mathcal{O} could be a collection of lines, circles, or rectangles on \mathbb{R}^2 (conf. Fig. 2) or a combination of cubes in \mathbb{R}^3 . For each object in \mathcal{O} , a *center point* is determined, which has to be well-defined but does not necessarily relate to the object's center of gravity. Non-symmetric objects additionally require to specify the orientation in space by a directional unit vector. In the analysis of mobile cellular networks, the objects in \mathcal{O} represent blockages such as buildings and walls.

A *random object process (ROP)* is constructed by randomly sampling objects from O and placing their corresponding center points at the points of some point process (PP). The orientation of each object is independently determined according to some probability distribution.

In general, a ROP is difficult to analyze, particularly when there are correlations between sampling, location, and orientation of the objects. For the sake of tractability, a *Boolean scheme* is commonly applied in literature [29, 43, 44, 55, 66, 67]. It satisfies the following properties: (i) the center points form a Poisson point process





(PPP); (ii) the attributes of the objects such as orientation, shape, and size are mutually independent; and (iii) for each object, sampling, location, and orientation are also independent. These assumptions of independence enable the tractability of the analysis. On the other hand, they omit correlations among blockages, as observed in practical scenarios.

Let **X** and **Y** denote the locations of the receiver and the transmitter, as indicated in Fig. 2. Further, let **XY** refer to the path between the two nodes. In a Boolean scheme, the number K of blockages crossing a link **XY** is a *Poisson RV* with mean

$$\mathbb{E}[K] = \lambda_{\mathrm{B}} \mathbb{E}[V(\mathbf{X}\mathbf{Y} \oplus B)], \qquad (5)$$

where λ_{B} denotes the density of the blockage centers and $B \in \mathcal{O}$ [55]. The operator \oplus refers to the *Minkowski* sum, which is defined as $\mathcal{A} \oplus \mathcal{B} = \bigcup_{\mathbf{x} \in \mathcal{A}, \mathbf{y} \in \mathcal{B}} (\mathbf{x} + \mathbf{y})$ for two compact sets \mathcal{A} and \mathcal{B} in \mathbb{R}^{n} , and $V(\cdot)$ is its *volume*. The expectation in Eq. 5 is calculated with respect to the objects in \mathcal{O} thus yielding the Minkowski sum with the *typical building*.

First note that $\mathbb{E}[K]$ will depend on the *length* $|\mathbf{XY}|$ of the link. Another direct consequence of the model is the probability that no blockage obstructs the link \mathbf{XY} , also referred to as *LOS probability*. It is obtained by applying the void probability of a Poisson RV: $\mathbb{P}[K = 0] = \exp(-\lambda_B \mathbb{E}[V(\mathbf{XY} \oplus B)])$. Notably, the exponential decay has been confirmed by measurement campaigns and has also been incorporated into the 3GPP standard [41].

Let γ_k denote the ratio of power loss due to the *k*th blockage. Then, the *power loss* caused by the blockages in a Boolean scheme is given by $\Gamma = \prod_{k=1}^{K} \gamma_k$, where *K* refers to the random number of blockages [55]. Assuming that γ_k are independent and identically distributed (i.i.d.) RVs on [0, 1] and *K* is a Poisson RV with means as given in Eq. 5, the distribution of Γ is in general not accessible in closed form. Recent approaches in literature therefore resort to the moments of Γ [55, 67]. The *n*th moment of Γ is obtained as $\mathbb{E}[\Gamma^n] = \exp(-\lambda_B \mathbb{E}[V(B)])(1 - \mathbb{E}[\gamma_k^n])$. Hence, on average, blockages impose an additional exponential attenuation on the mean path loss (conf. (4)). It is important to note that in this approach, reflections are ignored. They can implicitly be incorporated

by distinguishing between LOS and NLOS conditions (cf. Section 2.3.1) and adapting the path loss exponent accordingly [68].

To provide more intuition on this general result, we present an application example along the lines of [66]. In an indoor scenario, blockages are mainly constituted by walls. We represent these walls by a ROP of lines with random length and orientation. Then, the process is defined by the triple $\{X_i, L_i, \Theta_i\}$, where X_i corresponding to the PPP of wall-center positions with density λ_W , L_i is the wall length, which is distributed according to some distribution $f_L(n)$, and Θ_i denotes the wall-orientation, which is uniformly distributed in $[0, 2\pi)$. According to the introduced framework, the number K of walls blocking a link **XY** is a Poisson RV with mean

$$\mathbb{E}[K] = \frac{2\lambda_{\mathrm{W}}\mathbb{E}(L) |\mathbf{X}\mathbf{Y}|}{\pi}.$$
(6)

On the one hand, this result exhibits the dependency of $\mathbb{E}[K]$ on the link length $|\mathbf{XY}|$. On the other hand, it shows that the characteristics of a realistic environment can straightforwardly be incorporated into the model, by adapting the parameter λ_W as well as the distribution of W_i and Θ_i , respectively. This information can straightforwardly be extracted from real map data. When using convex two-dimensional (2D) objects instead of lines, the ROP is well suited to represent urban environments [55, 67].

A comparison of the discussed models is provided in Table 1. It includes necessary prior knowledge on the environment, mathematical tractability, flexibility, and accuracy. The next section elaborates on models for abstracting the interference geometry.

3 Interference geometry

When designing a mobile cellular system, its main aspects should hold across a wide range of deployment scenarios. Transmitter locations are commonly abstracted to some baseline model. For more than three decades, its most famous representative, the hexagonal grid model, has successfully withstood the test of time [16]. It has extensively been employed in both academia and industry and has found wide acceptance as a reasonably useful model

 Table 1 Comparison of discussed signal propagation models

	Prior environment knowledge necessary	Mathematical tractability	Flexibility	Accuracy
Empirical models	Low	\checkmark	Low	Low
Semi-deterministic models	Medium	\checkmark	Medium	Medium
Deterministic models	High	×	Low	High
Hybrid models	Variable	×	Variable	Variable
Stochastic models	Medium	\checkmark	High	Medium

to represent well-planned *homogeneous* BS topologies [69–72].

In the context of heterogeneous networks, small cell locations are oftentimes beyond the scope of network planning and hence exhibit a more random nature [3, 73–77]. Without preliminary information, the best statistical assumption is a uniform distribution over space, corresponding to complete spatial randomness [78]. In this case, transmitter locations can conveniently be described by some PP that further allows to leverage techniques from stochastic geometry. This powerful mathematical framework has gained momentum in recent years as the only available tool that provides a rigorous approach for modeling, design and analysis of a multi-tier network topologies [1, 4, 28-30, 33, 35, 55, 72, 79-85]. It is also considered an important approach for scrutinizing ultra dense networks (UDNs) in 5G topologies (see, e.g., [57, 58]).

Spatial randomness constitutes the philosophical opposite of a regular structure. As a results, these two extreme cases yield lower and upper performance bounds for any conceivable heterogeneous network deployment [76].

The first part of this section elaborates on the lower performance boundary, providing an overview on techniques from stochastic geometry. The second part addresses the upper bound, focusing on regular models and viewing them in the broader context of deterministic structures. In the third part of the section, a comparison in the form of a case study is carried out. In the forth part, the impact of channel access mechanisms is discussed. An overview on interference geometry models is provided in Fig. 3.

3.1 Stochastic models

The roots of stochastic geometry date back to shot noise studies of Campbell in 1909 [86, 87] and Shottky in 1918 [88]. In a planar network of nodes, which are distributed according to some PP, interference can be modeled by a generalized shot noise process [89, 90]. Key metrics such as coverage and rate had not been determined at this time. The idea of applying this framework for cellular networks appeared in the late 1990s [4, 80, 81]. Comprehensive surveys on literature related to stochastic geometry are already available, e.g., in [26, 75, 84]. For this reason, this section shall be confined to a selection of significant insights and shall outline limitations of this framework, which have found much less attention in literature.

3.1.1 Analysis of stochastic geometry

The analysis of stochastic geometry is based upon the concept of abstracting BS locations to some PP. As a result, it yields *spatial averages* over a substantial number of network realizations. When the nodes of a homogeneous BS deployment are distributed according to a PPP, i.e., they are assumed to be uniformly scattered over the infinite plane, and the fading is represented by i.i.d. non-negative RVs, the PDF of the aggregate interference yields a *skewed stable* distribution [1, 30, 91]. Yet, this is the only available case in literature that leads to known interference statistics. Still, except for a Lévy distribution, which is obtained by assuming a path loss exponent of 4, it does not result in any closed-form expressions for the aggregate interference PDF [26].



The success of stochastic geometry is rather rooted in the fact that it provides a means for systematically evaluating the Laplace transform of the aggregate interference, as defined in Eq. 1. The enabling identity is the probability generating function (PGFL): Let Φ denote an arbitrary PP. Then, its PGFL formulates as

$$\mathcal{G}[g] = \mathbb{E} \prod_{\mathsf{x} \in \Phi} g(\mathsf{x}),\tag{7}$$

where $g(x) : \mathbb{R}^d \to [0, \infty)$ is measurable.

It proves particularly useful to evaluate the LT of the sum $\sum_{\times\in\Phi}f(\times)$:

$$\mathbb{E}\left[\exp(-s\sum_{x\in\Phi}f(x)\right] = \mathbb{E}\left[\prod_{x\in\Phi}\exp(-sf(x))\right]$$
$$= \mathcal{G}[\exp(-sf(\cdot)], \qquad (8)$$

which characteristically appears in the analysis of aggregate interference with *discrete* node location models (continuous models will be explained in Section 3.2). The function $f(\cdot)$ represents the received power from an individual interferer at location \times . Consequently, $I = \sum_{x \in \Phi} f(x)$. Since I is a RV that is strictly positive, its LT always exists. It is important to note that the exact expressions for the LT, MGF, and CF are only available for *basic PPs*, encompassing PPP, binomial point process (BPP), and Poisson cluster process (PCP). For other types of PPs such as hardcore processes, only *approximations* are available.

3.1.2 Performance evaluation

Due to the *non-existence* of the aggregate interference PDF, it is generally not possible to derive exact performance metrics such as outage probability, transmission capacity, and average achievable rate. The authors in [26] summarize five techniques to go beyond moments and to model the network performance:

- #1: Resort to Rayleigh fading on desired link
 [3, 92–102]
- #2: Resort to dominant interferers by region bounds or nearest *n* interferers [85, 103]
- #3: Resort to Plancherel-Parseval theorem [104]
- #4: Directly invert the LT, CF, or MGF [22, 30, 91, 105–107]
- #5: Approximate interference by known PDF [63]

Using technique #1, the highly cited paper of Andrews et al. outlines three fundamental insights from the analysis of stochastic geometry [3]:

• In comparison to an actual BS deployment, models from stochastic geometry provide accurate lower bounds on the performance, while grid-based models yield upper bounds.

- With certain assumptions regarding path loss and fading, simple expressions for the coverage probability and the mean transmission rate can be derived.
- When the network is interference limited, i.e., the noise is considered negligible w.r.t. to the interference, the SIR statistics are independent of the BS density. Intuitively, the increasing aggregate interference is perfectly compensated by the lower average distance to the desired node.

The authors of [108] extended these results to heterogeneous cellular networks with an arbitrary number of tiers. Despite all the benefits of the stochastic approach, there are certain shortcomings one should be aware of when applying this framework. In the following, we provide a list with no claim to completeness.

3.1.3 Limitations

[Spatial averages] The analysis of stochastic geometry is based on averaging over an ensemble of spatial realizations. When the point process is ergodic, this is equivalent to averaging over a substantial number of spatial locations. Performance metrics vary from one interferer snapshot (i.e., realization of a point process) to another. Hence, the averaging only provides first-order statistics and is thus argued to hide the effect of design parameters on the uncertainties due to such variations [26]. To extend the analysis of stochastic geometry beyond spatial averages, the authors in [109] identify three sources of variability: (i) the variable distance between a node and its associated user; (ii) the variable transmission probability, which is particularly prominent in networks with contending nodes (e.g., Wireless Fidelity (Wi-Fi) and carrier sense multiple access (CSMA)); and (iii) the variability in the likelihood of successful reception. In [110] and [109], the full statistics of the SIR, also denoted as meta distribution of the SIR, and the throughput distribution are approximated.

[**Spatial correlations**] A major disadvantage of stochastic models is the difficulty to model correlations among node locations [71, 111, 112]. Those appear when reflecting topological and geographical constraints or accounting for the impact of network planning, which is not expected to loose relevance for the macro-tier in 5G networks. Therefore, it is considered imperative to investigate system models with a certain degree of regularity. In fact, the simplest and *most commonly used* PP, the PPP, assumes *completely uncorrelated* node locations. In the context of stochastic geometry, regularity can to some extent be reflected by *repulsive PPs*. Such processes impose a certain minimum acceptable distance between two BSs. When the exclusion region is fixed, the process is termed *hardcore PP*. When it is defined by a probability distribution, the process is denoted as *softcore PP*. Hard- and softcore processes significantly complicate the interference analysis due to the non-existence of the PGFL. Therefore, they require to approximate the LT of the aggregate interference or the PDF itself. Besides that, the most promising representative, the Matérn hardcore point process (HCPP), contains flaws that still have to be addressed [113–116]. It underestimates the intensity of points that can coexist for a given hardcore parameter.

[Measuring heterogeneity] Given a realistic node distribution, a particular challenge is to find a PP with the same structural properties. An objective measure for the degree of heterogeneity, also known as degree of clustering or clumping factor, should be independent of the number of nodes and the size of the area, in which the nodes are distributed as well as linear operations such as rotating and shifting [117]. Classical statistics include the J-function, the L-function [84], and Ripley's K-function [118-120]. While the J- and the L-functions are related to inter-point distances, Ripley's K-function measures second-order point location statistics. Both metrics do not allow to unambiguously identify different PPs. In [72], the authors propose to apply the coverage probability as a goodness of fit measure. Again, this measure does not allow to discriminate different models.

[Asymmetric impact of interference] Another factor that stalls the analysis of stochastic geometry is the incorporation of an asymmetric impact of the interference, as indicated by the exaggerated interference scenario in Fig. 4. With few exceptions, convenient expressions



are only achieved by assuming spatial *stationarity* and *isotropy* of the scenario, i.e., a receiver being located in the center. In [121, 122], the authors employ a *fixed cell* approach for scrutinizing eccentric receiver locations. Their method is shown to achieve accurate results only in combination with an approximation of the interference statistics by a Gamma distribution. Otherwise, notions such as *cell-center* and *cell-edge* are generally not accessible in the analysis.

3.2 Deterministic structures

The broad acceptance of models based on stochastic geometry has diverted attention away from the still ongoing improvement of deterministic structures, with their most famous representative being the hexagonal grid model. They allow to reflect the impact of the network planning, which might increasingly disappear with the emergence of self-organizing networks (SON) [123–125], and also account for more fundamental limitations such as topological and geographical constraints. In this section, we review efforts to facilitate the interference analysis using these structures, even allowing to represent multi-tier heterogeneous topologies.

Deterministic structures can broadly be categorized into *discrete* and *continuous* models [126]. Discrete models are characterized by modeling each node individually. The amount of nodes can either be finite or infinite and the arrangement of nodes follows a certain structure such as a circle or a grid, as illustrated in Fig. 5. Continuous models assume the contributions from the interferers to be uniformly distributed over a certain *n*-dimensional geometric shape, such as a circle or a ring.

3.2.1 Discrete models

Interference analysis in discrete models is based on evaluating the impact of each individual interferer on the receiver and then aggregate them. The node locations are commonly modeled along a finite or infinite regular structure, such as a square grid (also referred to as *Manhattantype model* [127–131]) or a hexagon (see. e.g., [63, 132]), as depicted in Fig. 5. It should be noted that realistic scenarios, which utilize data from network operators, such as in [133–136], also belong to this class of models. Such data may also include information about the boresight directions of the sector antennas, which is required for the calculation of the path loss (conf. Section 2.1), as well as the antenna tilting.

As explained in Section 2.1, signal propagation is usually characterized by some deterministic law and a random component accounting for the fading. Consequently, in a discrete regular structure, the aggregate interference can be viewed as a finite or infinite *sum of weighted RVs* with the weights being straightforwardly obtained from geometric considerations.



Certain fading distributions, such as Rayleigh, lognormal, or Gamma, enable to exploit a myriad of literature on the sum of weighted RVs [47, 48, 137-152]. The majority of these reports make use of variants of the CF or the MGF. When the receiver is located in the center of a symmetric grid, as indicated by the gray squares in Fig. 5, the weights are all equal or can be summarized into groups. As a result, this scenario often leads to closed-form expressions for the distribution of the aggregate interference, as demonstrated, e.g., in [153]. In the general case, when the receiver is located outside the scenario center (conf. Figs. 4 and 5), the weights are all different and the performance analysis is stalled with an inconvenient sum of RVs. To overcome this issue, two approaches are commonly applied in literature:

- Scrutinize individual link statistics to gain understanding on overall interference behavior [13, 154, 155]. The focus of these models mainly lies on link-distance statistics. While they do not lead to convenient expressions for the moments and the distribution of the aggregate interference, they allow to evaluate arbitrary receiver locations in a cell.
- Approximate aggregate interference distribution by known distribution [6, 9, 13, 14, 27, 32, 156–158]. It is well-studied that the CLT and the corresponding Gaussian model provide a very poor approximation for modeling aggregate interference statistics in large

wireless cellular networks [30, 159, 160]. Its convergence can be measured by the Berry-Esseen inequality [161] and is typically thwarted by a few strong interferers. The resulting PDF exhibits a heavier tail than what is anticipated by the Gaussian model [30]. Resorting to the approximation of the aggregate interference distribution by a known parametric distribution imposes two challenges: (i) the choice of the distribution itself and (ii) the parametrization of the selected distribution. Although there is no known criterion for choosing the optimal PDF, its tractability for further performance metrics as well as the characteristics of the spatial model, the path loss law, and the fading statistics advertise certain candidate distributions [32]. A class of continuous probability distributions that allow for positive skewness and non-negative support are normal variance-mean mixtures, in particular the normal inverse Gaussian distribution. The main penalty of such generalized distributions is the need to determine up to four parameters, which typically exhibit non-linear mappings when applying moment or cumulant matching [32]. Hence, it is beneficial to resort to special cases with only two parameters. Inverse Gamma, inverse Gaussian, log-normal, and Gamma distribution have frequently been reported to provide an accurate abstraction of the aggregate interference statistics, e.g., in [32, 160, 162], [32, 160], [151], and [32, 121, 163, 164], respectively.

It is expected that discrete models will play an important role in 5G topologies for representing the *regular* part of the network [152, 153].

3.2.2 From discrete to continuous

The discrete models discussed in the previous paragraph aim at reproducing the actual site locations. The authors in [152] propose to represent arbitrary BS topologies by nodes along circles, where the nodes do not necessarily represent physical sources. They rather correspond to mapping points of an angle-dependent power profile, as illustrated in Fig. 6. The authors provide a mapping scheme that enables to accurately preserve the interference statistics of *fully random, heterogeneous topologies* with 10,000 and more BSs by some ten nodes. It corroborates the intuition that the number of principal interferers is typically low and therefore allows to substantially reduce the amount of interfering nodes with minor loss of accuracy.

The contribution in [152] demonstrates that welldefined structures with a finite number of nodes can expediently be applied to represent massive heterogeneous BS topologies. Moreover, these models implicitly yield insights on the number of interferers that mainly determine the characteristics of the interference distribution.

3.2.3 Continuous models

A main merit of stochastic geometry is the transformation of a summation of interference terms into an integration over space by means of the PGFL (conf. Section 3.1). In the context of regular structures, the natural relationship between summation and integration has inspired the socalled *continuous-style approaches*. Those are either *line*or *area*-based.

In *line-based approaches*, the power of the interferers is uniformly distributed along a line with a certain shape. In [153], the model is constituted by a circle, while in [165], it exhibits a flower-like shape to account for the angle-dependent impact of the interferers, as illustrated in Fig. 5d. While the flower model achieves a better accuracy than the circular model, the evaluation of the results requires a numerical computation. The authors of [165] also show the extension of the flower model to a three-sector setup. The idea is to virtually shift the user position at a certain angle according to the antenna gain.

The area-based approaches incorporate power emission density (PED)-based models and *fluid* models. The common idea is that the interfering nodes are considered as a continuum of infinitesimal interferers distributed in space. In the PED-based approach in [126], each interferer is represented by an individual emitting area, as depicted in Fig. 5e. The receiver of interest is located outside these areas. When arranging the areas in a grid-based fashion, the interference analysis again entails a summation rather than simplifying to integrals. In *fluid models*, the receiver of interested is placed within a ring, which represents a round shaped network around a central cell, as illustrated in Fig. 5f and investigated in [5-7, 158, 166, 167]. The model is shown to yield good approximation for hexagonal transmitter arrangements with both omni- as well as sector antennas [5-7, 158]. The sectorization is incorporated by (i) increasing the interferer density of an omni-directional scenario in proportion to the ratio between the omni-directional-cell surfaces and the sector-cell surfaces, (ii) including the interference from the other sectors belonging to the same site, and (iii) accounting for the impact of the angle-dependent sector antenna gain on the path loss. Remarkably, the authors in [5, 7] show that the OCIF of a sectorized setup can be expressed as a linear function of the omnidirectional one. Noting that the fluid model relies on the hypothesis of a *regular* network, the authors in [167]



show its applicability for representing PPP-based models by means of a correction factor. In principle, the fact that the model allows to adjust the transmitter density makes it applicable for representing UDNs. This technique has however not been pursued in literature yet.

Similar to discrete models, a common way to obtain the aggregate interference distribution when using continuous models is to approximate it by a known distribution, as described in Section 3.2.1. Then, the model is mainly used for finding the parameters of the distribution, as demonstrated, e.g., in [5, 153, 166].

3.2.4 Limitations

This section summarizes the limitations of deterministic modeling approaches, most of which were already implicitly discussed above.

- [Finite structures] A frequently stated weakness of deterministic models, and in particular of discrete approaches, is their limitation to represent only *finite* networks with a low number of nodes [1]. Hence, they tend to underestimate the interference. An exception to this issue are fluid models, which allow to specify an infinite outer radius.
- [Approximation by known distribution] In *discrete* models, the PDF of the aggregate interference can be determined by a weighted sum of RVs. With exception to fully symmetric scenarios, in general, this approach does not lead to closed-form results. As discussed in Section 3.2.1, a commonly applied solution is to approximate the interference cPDF by a known distribution. Then, the model is mainly exploited to determine the parameters of the distribution. This technique has also been used in the domain of *continuous* models, as described above. According to Section 3.2.1, there is neither an optimal way to select the PDF nor to calculate its parameters.
- [Representing spatial randomness] Deterministic models are commonly designed to facilitate the geometric treatment of the transmitter locations by either arranging the nodes in a grid-based structure or distributing them over a convenient structure. Hence, they inherently reflect a high degree of *regularity*, with three exceptions:
 - The node locations of the discrete model represent the transmitters of a practical network. In this case, the interference statistics can only be evaluated via Monte Carlo simulations.
 - 2. The circular interference model in [152] enables to map arbitrary node locations onto circles with equivalently spaced nodes. The approach has the disadvantage that the mapping needs to be carried out individually for each snapshot of the network.

Thus, it does not allow to represent a general random topology, which is specified only by statistical parameters.

3. According to [167], the fluid model allows to accurately represent a PPP. The method requires the application of *correction factors*, which are only found empirically via simulations.

A comparison of the discussed interference geometry models is provided in Table 2. It shows whether at least one approach has a tractable interference PDF and whether the model exhibits spatial randomness and spatial correlation, respectively.

3.3 Case study

In this section, the presented transmitter location models are compared against practical outdoor BS deployments by means of a case study. The goal is to evaluate the average interference, which is experienced by a receiver at distance r away from its desired BS. The nodes are assumed to transmit with unit power and to apply universal frequency reuse. The signal propagation is abstracted by a log-distance-dependent path loss law $c^{-1} d^{-4}$, where d denotes the distance between the transmitter and the receiver. Random fading is omitted.

The practical BS deployments are obtained by extracting openly available data from Ofcom's *Sitefinder*¹ for the cities of London and Manchester. The selected area of London has an area of 4 km^2 and counts 319 BSs, yielding an average cell radius of $R_C = 63.17 \text{ m}$ and a BS density of $\mu = 79.75 \, 10^{-6} \text{ m}^2$. In the city of Manchester, 37 BSs were counted in a selected area of 1.8 km^2 , resulting in $R_C = 125.92 \text{ m}$ and $\mu = 20.56 \text{ m}^2$. The parameters are summarized in Table 3. The results are obtained by uniformly distributing users in the given area, associating them with the closest BS, and determining the aggregate interference as well as their distance to the associated BS.

Table 2 Properties of interference geometry models. Tractable interference PDF denotes the fact that at least one approach yields a closed-form solution for the interference PDF

	Tractable interference PDF	Spatial randomness	Spatial correlation
Basic point processes	\checkmark	\checkmark	x
Cluster point processes	×	\checkmark	×
Repulsive point processes	×	\checkmark	\checkmark
Grid-based model	\checkmark	×	\checkmark
Practical deployment	×	\checkmark	\checkmark
Line-based	×	×	\checkmark
Area-based	×	×	\checkmark

City	London ($A = 4 \mathrm{km}^2$)	Manchester ($A = 1.8 \mathrm{km}^2$)			
Number of BSs	319	37			
Avg. cell radius	63.17 m	125.92 m			

Table 3 BS deployment data as obtained from Ofcom's Sitefinder

 for the cities of London and Manchester

A wrap-around technique is used to avoid interference edge effect.

As a representative for a *stochastic* model (conf. Section 3.1), the hybrid approach from [121] is applied. It comprises a PPP of density μ , where the points of the process are assumed to be excluded from a *guard region* of radius $2R_{\rm C}$. In order to account for the dominant interferer, a node is randomly placed at the boundary of the guard region, i.e., a circle of radius $2R_{\rm C}$. In such scenario, the expected interference at distance *r* is found as

$$\mathbb{E}[I(r)] \approx \frac{1}{c \left(2R_{\rm C} - r\right)^{\eta}} \left(\frac{2\pi\lambda(2R_{\rm C} - r)^2}{\eta - 2} + 1\right) \qquad (9)$$

Note that without the guard region, $\mathbb{E}[I(r)]$ does not exist for any value of *r* [1].

Next to the stochastic model, a discrete and a continuous *deterministic* model (conf. Sections 3.2.1 and 3.2.3) are investigated. The discrete deterministic model is represented by the commonly used hexagonal grid model. The transmitters are spaced out by an inter-site distance of $2R_{\rm C}$ and are distributed over a circular area of radius $10R_{\rm C}$. The node at the origin is assumed to be the desired BS. The interference at a certain distance *r* away from the origin ($0 < r \le R_{\rm C}$) is obtained by averaging over all angle positions between $-\pi$ and π . The interference from the first tier of interference is exemplarily given as

$$\mathbb{E}[I(r)] = \int_{-\pi}^{\pi} \frac{1}{2\pi} \sum_{i=1}^{6} \frac{1}{c} \left((2R_{\rm C})^2 + r^2 -4R_{\rm C}r \cos\left(\frac{2\pi}{6}i - \phi\right) \right)^{-\eta/2} d\phi.$$
(10)

The continuous deterministic model is established by a fluid model, as referred from [5]. It comprises an inner radius of $2 R_{\rm C}$ and an outer radius of $10 R_{\rm C}$. According to [5], the interference at distance *r* is calculated as

$$I = \frac{2\pi\,\mu\,c}{\eta-2} \left((2R_{\rm C} - r)^{2-\eta} - (10\,R_{\rm C} - r)^{2-\eta} \right). \tag{11}$$

Note that the expressions in Eqs. 9 and 11 for the PPPbased model and the fluid approach are obtained in closed form. The hexagonal grid requires a numerical computation, while the practical models need to be simulated.

Figure 7 shows the average interference power [W] over the distance r of the desired transmitter. The results for the stochastic model as well as the fluid models are

obtained by plugging μ and $R_{\rm C}$ from the practical scenarios into Eqs. 9 and 11, respectively. The results for the hexagonal model are achieved by using the values of $R_{\rm C}$ as the inter-site distance. It is observed that the grid model yields the largest aggregate interference, while the fluid model exhibits the smallest values. According to [167], both models represent a *regular* distribution of nodes. In the hexagonal model, the dominant nodes are more concentrated at the boundary of the guard region. For this reason, it also yields a larger interference than the PPPbased model, where only a single dominant interferer is located at this boundary. Confirming results in [3], the practical model lies in between the hexagonal grid and the stochastic model, as it exhibits a certain degree of regularity.

Next to the transmitter locations, the second factor that impacts the interference geometry is the channel access scheme, as discussed in the next section.

3.4 Channel access schemes

In a wireless system, a channel access scheme, frequently referred to as medium access control (MAC), controls a transmitter's admission to use certain resources, e.g., time and frequency. It is either implemented such that each node acts autonomously or in a coordinated manner. In the presence of such scheme, a receiver experiences a thinned version of the interference situation. Moreover, a MAC generally introduces correlations among node locations and, thus, considerably complicates the analysis (conf. Section 3.1). Therefore, it is frequently neglected, amounting to the worst case, where all transmitters contribute to the aggregate interference [26, 75, 84, 127, 152, 153]. When a channel access scheme is taken into account in the interference analysis, it commonly falls into one of the following two categories:

- Exlusion region: The MAC is considered to establish an exclusion region around the transmitter-receiver pair, as indicated in Fig. 4. Such regions frequently appear in studies on cognitive radio techniques [92, 168–172].
- Independent thinning: The nodes are assumed to access the resources *randomly and independently of each other*. In the time-domain, this corresponds to the procedures of the Additive Links Online Hawaii Area (ALOHA) protocol [92, 173, 174]. This approach is particularly useful, when the node locations are modeled by a PPP. Then, by virtue of the *thinning property*, applying a random scheme again results in a PPP with different intensity [29].

In the next section, we discuss further aspects of interference analysis.



4 Further aspects

4.1 Integration

In principle, the abstractions in Section 2 and the models in Section 3 represent independent tools and can thus be combined in an arbitrary fashion. For example, one could apply the hexagonal grid model (conf. Section 3.2.1) for describing the BS locations and the ROP-based blockage model (conf. Section 2.4) together with an empirical path loss law (conf. Eq. 2.3) to characterize the signal propagation. Stochastic blockage models also enable *integrated* frameworks, as demonstrated, e.g., in [55, 67]. In such frameworks, the blockages not only affect the signal propagation but also impact the transmitter locations. As an example, consider transmitters that are restricted to be located outdoors. Then, a transmitter will be kept, if it is not covered by a blockage, and discarded otherwise.

4.2 BS cooperation

BS cooperation is viewed as a key element for interference management in mobile networks [132]. While not directly affecting the large-scale signal propagation nor the BS locations, it impacts whether a signal can be regarded as desired, interfering, or negligible [152, 175, 176].

4.3 Indoor-outdoor partitioning

It is estimated that the majority of today's mobile traffic originates and is consumed indoors [177]. Scenarios consisting of *both indoor and outdoor environments* have not received much attention in analytical studies based on stochastic geometry. With exception of [67], existing approaches mostly neglect the wall partitioning [100, 105]. This mainly stems from the fact that the walls impose inhomogeneities on the signal propagation. The authors of [67] show that the wall partitioning considerably impacts the coverage and rate performance of a typcial indoor user in an urban two-tier heterogeneous cellular network.

4.4 Random displacement

The most tractable results in stochastic geometry are obtained by assuming Rayleigh fading on the desired link. Hence, the modeling environment corresponds to a flat plane without any obstacles, as indicated in Fig. 8a. The characterization of *realistic* signal propagation environments often incorporates a separate shadowing term, which is commonly modeled by i.i.d. RVs for simplicity. This model has a very particular impact on the interference geometry. In [178], the shadowing RV are argued to act as a random displacement of the node locations. It is shown that displacing the nodes of a homogeneous PPP by i.i.d. RV again results in a homogeneous PPP with different intensity. Hence, the spatial distribution of the nodes still exhibits the same characteristics after applying the shadowing, thus not leading to further insights.

4.5 Measuring the goodness of fit

A common method to verify assumptions that simplify the interference analysis is a comparison against Monte Carlo simulations. In order to quantify the *goodness of fit* between the actual interference distribution and its approximation, *non-parametric tests*, also known as *distribution-free inferential method*, are commonly applied [152, 153]. Some of the most frequently used approaches include the *Anderson-Darling test*, the *Cramér-von Mises criterion*, and the *Kolmogorov-Smirnov test*, among others.





4.6 Exploiting the third spatial dimension

The vast majority of existing abstraction models for both interference geometry and signal propagation is based on the fundamental assumption of 2D scenarios. With the recent release of a 3D channel model for the study of elevation beamforming and full-dimension multiple-input multiple-output (MIMO) [41], the 3GPP has made a clear statement for the future of wireless network modeling. A considerable effort should be directed towards augmenting the existing models by a third dimension.

4.7 Millimeter wave transmissions

A framework for modeling blockages, as presented in Section 2.4, currently gains relevance in the study of mmWave transmissions (see, e.g., [179]), where blockages are assumed to have a substantial penetration loss and are therefore assumed to be impenetrable [55]. Consequently, they form the so-called LOS regions, as illustrated in Fig. 2. The performance of such network yields a delicate trade-off between blockage and transmitter density [43, 44, 180]. Since the stochastic modeling of transmitters and blockages follows a similar mathematical framework, there combination is a straightforward task. A combined model enables to relate parameters of the network and the environment topology, e.g., transmitter and building density. For example, in [67], this turns out to be particularly useful for evaluating indoor and outdoor coverage.

4.8 Massive MIMO

Studies on massive MIMO transmissions show that othercell interference may cease to be a major performancelimiting factor in such systems [180, 181]. They follow the intuition that a large array of antennas enables to form sharp beams such that the desired and the interfering signal establish complementary spaces. Eventually, this may lead to a paradigm shift in the performance analysis of 5G networks, where even the term *cellular* might lose its meaning [179].

5 Conclusions

In this paper, we reviewed path loss attenuation and interference geometry models to characterize aggregate interference statistics in dense urban heterogeneous cellular networks. First, we detailed the characteristics and drawbacks of traditional path loss attenuation models. Then, we focused on stochastic models that represent blockages by means of a random object process. We observed that these models can directly be parameterized by real world data and capture the dependence of blockage effects on the link length. In the context of interference geometry, we observed that discrete models, while representing the most basic method for interference evaluations are also the major obstacle for yielding convenient expressions for the aggregate interference statistics. We showed that various limitations of stochastic approaches, such as interference statistics at eccentric user locations or a certain degree of regularity among the node locations, can be resolved by deterministic structures. It is our hope that this paper provides a comprehensive view on network interference analysis that inspires researchers to develop new frameworks for evaluating the 5G.

Endnote

¹ sitefinder.ofcom.org.uk

Abbreviations

2D: Two-dimensional; 3D: Three-dimensional; 3GPP: 3rd Generation Partnership Project; 5G: Fifth generation of mobile cellular networks; ALOHA: Additive Links On-Line Hawaii Area; AMC: Adaptive modulation and coding; AWGN: Additive white Gaussian noise; BPP: Binomial point process; BS: Base station; CDF: Cumulative distribution function; CLT: Central limit theorem; CSG: Closed subscriber mode; CDMA: Code-division multiple access; CSMA: Carrier sense multiple access; CF: Characteristic function; D2D: Device-to-device; FAP: Femtocell access point; HCPP: Hardcore point process; HetNet: Heterogeneous network; i.i.d.: Independent and identically distributed; ICI: Inter carrier interference; ISI: Inter symbol interference; LOS: Line of sight; LT: Laplace transform; LTE-A: LTE-advanced; MAC: Medium access control; MAI: Multiple access interference; MCL: Minimum coupling loss; MCL [MCLs]: Minimum coupling losses; MGF: Mobility generating functional; MIMO: Multiple-input multiple-output; mmWave: Millimeter wave; NLOS: Non-line of sight; OCIF: Other-cell interference factor; OFDMA: Orthogonal frequency-division multiple access; OSG: Open subscriber mode; PCP: Poisson cluster process; PDF: Probability density function; PED: Power emission density; PGFL: Probability generating function; PP: Point process; PP [PPs]: Point processes; PPP: Poisson point process; PPP [PPPs]: Poisson point processes; ROP: Random object process; ROP [ROPs]: Random object processes; RV: Random variable; QAM: Quadrature amplitude modulation; SCM: Spatial channel model SIR: Signal-to-interference ratio; SISO: Single-input single-output; SINR: Signal-to-interference-plus-noise ratio; SON: Self-organizing network; TTI: Transmission time interval; UDN: Ultra dense network; UE: User equipment WCDMA: Wideband code division multiple access; Wi-Fi: Wireless Fidelity; WiMAX: Worldwide interoperability for microwave access; WINNER: Wireless World Initiative New Radio

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Competing interests

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