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Uniformly asymptotic stability of almost periodic solutions for a delay difference system of plankton allelopathy

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Abstract

In this contribution, we investigate a delayed difference almost periodic system for the growth of two species of plankton with competition and allelopathic effects on each other. By using the methods of Lyapunov function and preliminary lemmas, sufficient conditions which guarantee the existence and uniformly asymptotic stability of a unique positive almost periodic solution of the system are established. An example together with its numerical simulations is presented to verify the validity of the proposed criteria.

Keywords: delay difference system; allelopathy; almost periodic solutions; uniformly asymptotic stability; Lyapunov function

1 Introduction

Allelopathy is a biological phenomenon by which individuals of a population release one or more biochemicals that have an effect on the growth, survival, and reproduction of the individuals of another population. As an important factor for ecosystem functioning, allelopathic interactions have occurred in various aspects: between bacteria [1], between bacteria and phytoplankton [2, 3], between phytoplankton and zooplankton [4], and also between calanoid copepods [5]. Especially, allelopathic interactions are widespread in phytoplankton communities, which deeply attract the attention of researchers. Thus, in aquatic ecology, the study of tremendous fluctuations in abundance of many phytoplankton communities is a significant theme. Recently, many workers have been aware that the increased population of one species of phytoplankton might restrain the growth of one or several other species by the production of allelopathic toxins. For detailed literature studies, we can refer to [6–15] and the references cited therein.

In [15], Qin and Liu discussed the permanence and global attractivity of the following delay difference system with plankton allelopathy:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{r_1(n) - a_{11}(n)x_1(n) - a_{12}(n)x_2(n) \\ \qquad\qquad - b_1(n)x_1(n)\sum_{p=0}^{M} k_2(p)x_2(n-p)\}, \\ x_2(n+1) = x_2(n)\exp\{r_2(n) - a_{21}(n)x_1(n) - a_{22}(n)x_2(n) \\ \qquad\qquad - b_2(n)x_2(n)\sum_{p=0}^{M} k_1(p)x_1(n-p)\}, \\ x_i(\Phi) \ge 0, \quad \Phi \in [-p, 0] \cap \mathbb{Z}; \qquad x_i(0) > 0, \quad i = 1, 2, \end{cases} \tag{1.1}$$

where $x_i(n)$ are the population densities of species x_i at the nth generation, $r_i(n)$ stand for the intrinsic growth rates of species x_i at the nth generation, $a_{ii}(n)$ are the intra-specific effects of the nth generation of species x_i on own population, and $a_{ij}(n)$ measure the inter-specific effects of the nth generation of species x_j on species x_i, $b_i(n)x_i(n)\sum_{p=0}^{M} k_j(p)x_j(n-p)$ denote the effect of toxic substances ($i,j = 1,2$; $i \neq j$), M is a positive integer.

Notice that the environment varies due to the factors such as seasonal effects and variations in weather conditions, food supplies, mating habits, harvesting *etc.* Thus it is reasonable to assume that the parameters in system (1.1) are periodic. However, if the various constituent components of the temporally nonuniform environment is with incommensurable periods (non-integral multiples), then we have to consider the environment to be almost periodic, which leads to the almost periodicity of the parameters of system (1.1). The main purpose is to establish sufficient conditions for the existence and uniformly asymptotic stability of a unique positive almost periodic solution of system (1.1). To do so, we assume that $\{r_i(n)\}$, $\{a_{ij}(n)\}$ and $\{b_i(n)\}$ for $i,j = 1,2$ are bounded nonnegative almost periodic sequences, $k_i(p)$, $i = 1,2$, is a bounded positive sequence.

Many recent works have been done on the existence and stability of almost periodic solutions for the discrete biological models without or with time delays (see [16–21]). However, to the best of our knowledge, there are few published papers concerning the above almost periodic system (1.1). For the sake of simplicity and convenience, in the following discussion, the notations below will be used

$$h^u = \sup_{n\in\mathbb{Z}^+}\bigl\{h(n)\bigr\}, \qquad h^l = \inf_{n\in\mathbb{Z}^+}\bigl\{h(n)\bigr\}, \tag{1.2}$$

where $\{h(n)\}$ is a bounded sequence defined on the set of nonnegative integers $\mathbb{Z}^+$. Meanwhile, we make a convention that $\sum_{n=a}^{b} h(n) = 0$ if $a > b$.

The rest of this paper is organized as follows. In Section 2, we introduce some notations, definitions and lemmas which are useful for our main results. Sufficient conditions for the existence and uniformly asymptotic stability of a unique positive almost periodic solution of system (1.1) are established in Section 3. In Section 4, an example and its numerical simulations are presented to illustrate the feasibility of our main results. Finally, we give some proofs of theorems in the appendices for convenience in reading.

2 Preliminaries

In this section, we give some notations, definitions and lemmas which will be useful for the later sections.

Denote by $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}$ and $\mathbb{Z}^+$ the sets of real numbers, nonnegative real numbers, integers and nonnegative integers, respectively. $\mathbb{R}^2$ and $\mathbb{R}^k$ denote the cone of a two-dimensional and k-dimensional real Euclidean space, respectively. We also set

$$[c,d]_{\mathbb{Z}} = [c,d]\cap\mathbb{Z}, \quad c,d\in\mathbb{Z}, \qquad \mathbb{K} = [-M,+\infty)_{\mathbb{Z}},$$

where M is defined in (1.1).

Definition 2.1 (see [22]) A sequence $y:\mathbb{Z}\to\mathbb{R}^k$ is called an almost periodic sequence if the ε-translation set of y

$$\mathcal{E}\{\varepsilon,y\} := \bigl\{\tau\in\mathbb{Z}: \bigl|y(n+\tau)-y(n)\bigr| < \varepsilon, \forall n\in\mathbb{Z}\bigr\}$$

is a relatively dense set in $\mathbb{Z}$ for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains an integer $\tau = \tau(\varepsilon) \in \mathcal{E}\{\varepsilon, y\}$ such that

$$\left|y(n+\tau) - y(n)\right| < \varepsilon, \quad \forall n \in \mathbb{Z}.$$

τ is called the ε-translation number of $y(n)$.

Definition 2.2 (see [22]) Let $g : \mathbb{Z} \times \mathbb{D} \to \mathbb{R}^k$, where $\mathbb{D}$ is an open set in $\mathbb{C} := \{\phi : [-\tau, 0]_{\mathbb{Z}} \to \mathbb{R}^k\}$. $g(n,\phi)$ is said to be almost periodic in n uniformly for $\phi \in \mathbb{D}$ if for any $\varepsilon > 0$ and any compact set $\mathbb{S}$ in $\mathbb{D}$, there exists a positive integer $l(\varepsilon, \mathbb{S})$ such that any interval of length $l(\varepsilon, \mathbb{S})$ contains an integer τ for which

$$\left|g(n+\tau, \phi) - g(n,\phi)\right| < \varepsilon, \quad \forall n \in \mathbb{Z}, \phi \in \mathbb{S}.$$

τ is called the ε-translation number of $g(n,\phi)$.

Lemma 2.3 (see [23]) *$\{y(n)\}$ is an almost periodic sequence if and only if for any sequence $\{p'_k\} \subset \mathbb{Z}$ there exists a subsequence $\{p_k\} \subset \{p'_k\}$ such that $y(n+p_k)$ converges uniformly on $n \in \mathbb{Z}$ as $k \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.*

Consider the following almost periodic delay difference system:

$$y(n+1) = g(n, y_n), \quad n \in \mathbb{Z}^+, \tag{2.1}$$

where

$$g : \mathbb{Z}^+ \times \mathbb{C}_B \to \mathbb{R}, \quad \mathbb{C}_B = \{\phi \in \mathbb{C} : \|\phi\| < B\}, \mathbb{C} = \{\phi : [-\tau, 0]_{\mathbb{Z}} \to \mathbb{R}\},$$

with $\|\phi\| = \sup_{s \in [-\tau,0]_{\mathbb{Z}}} |\phi(s)|$, $g(n,\phi)$ is almost periodic in n uniformly for $\phi \in \mathbb{C}_B$ and is continuous in ϕ, while $y_n \in \mathbb{C}_B$ is defined as $y_n(s) = y(n+s)$ for all $s \in [-\tau, 0]_{\mathbb{Z}}$.

The product system of (2.1) is in the form of

$$y(n+1) = g(n, y_n), \qquad z(n+1) = g(n, z_n). \tag{2.2}$$

A discrete Lyapunov function of (2.2) is a function $V : \mathbb{Z}^+ \times \mathbb{C}_B \times \mathbb{C}_B \to \mathbb{R}^+$ which is continuous in its second and third variables. Define the difference of V along the solution of system (2.2) by

$$\Delta V_{(2.2)}(n, \phi, \psi) = V\big(n+1, y_{n+1}(n,\phi), z_{n+1}(n,\psi)\big) - V(n, \phi, \psi),$$

where $(y(n,\phi), z(n,\psi))$ is a solution of system (2.2) through $(n, (\phi, \psi))$, $\phi, \psi \in \mathbb{C}_B$. And Zhang and Zheng [22] obtained the following lemma.

Lemma 2.4 (see [22]) *Suppose that there exists a Lyapunov function $V(n, \phi, \psi)$ satisfying the following conditions*:

(1) $a(|\phi(0)-\psi(0)|) \le V(n,\phi,\psi) \le b(\|\phi-\psi\|)$, *where* $a, b \in \mathcal{P}$ *with* $\mathcal{P} = \{\alpha : [0,\infty) \to [0,\infty) | \alpha(0) = 0$ *and* $\alpha(u)$ *is continuous, increasing in* $u\}$.
(2) $|V(n,\phi_1,\psi_1) - V(n,\phi_2,\psi_2)| \le L(\|\phi_1-\phi_2\| + \|\psi_1-\psi_2\|)$, *where* $L > 0$ *is a constant.*
(3) $\Delta V_{(2.2)}(n,\phi,\psi) \le -\gamma V(n,\phi,\psi)$, *where* $0 < \gamma < 1$ *is a constant.*

Moreover, if there exists a solution $y(n)$ *of system* (2.1) *such that* $\|y_n\| \le B^* < B$ *for all* $n \in \mathbb{Z}^+$, *then there exists a unique uniformly asymptotically stable almost periodic solution* $q(n)$ *of system* (2.1) *which satisfies* $|q(n)| \le B^*$ *for all* $n \in \mathbb{K}$. *In particular, if* $g(n,\phi)$ *is periodic with period* ω, *then system* (2.1) *has a unique uniformly asymptotically stable periodic solution with period* ω.

Remark 2.5 (see [19]) From the proof of [24, Theorem 6.6], it is not difficult to prove that condition (3) of Lemma 2.4 can be replaced by the following condition:

(3)′ $\Delta V_{(2.2)}(n,\phi,\psi) \le -c(|\phi(0)-\psi(0)|)$, where $c \in \{\beta : [0,\infty) \to [0,\infty) | \beta$ is continuous, $\beta(0) = 0$ and $\beta(s) > 0$ for $s > 0\}$.

Definition 2.6 (see [15]) System (1.1) is said to be permanent if there exist positive constants $\mathfrak{M}_i$ and $\mathcal{M}_i$ such that

$$\mathfrak{M}_i \le \liminf_{n\to+\infty} x_i(n) \le \limsup_{n\to+\infty} x_i(n) \le \mathcal{M}_i, \quad i = 1,2$$

for any positive solution $(x_1(n), x_2(n))$ of system (1.1).

Lemma 2.7 (see [15]) *Assume that*

$$\begin{aligned} &\min\{r_1^l - a_{12}^u M_2, r_2^l - a_{21}^u M_1\} > 0, \\ &\min\{\Delta_1 M_1, \Delta_2 M_2\} > 1. \end{aligned} \tag{2.3}$$

Then system (1.1) *is permanent. Here,* $\Delta_i = \frac{a_{ii}^u + b_i^u M_j (M+1) k_j^u}{r_i^l - a_{ij}^u M_j}$.

From the proof of [15, Lemma 2.3], we have

$$\limsup_{n\to+\infty} x_i(n) \le M_i \stackrel{\text{def}}{=} \frac{\exp(r_i^u - 1)}{a_{ii}^l} \tag{2.4}$$

and

$$\liminf_{n\to+\infty} x_i(n) \ge m_i \stackrel{\text{def}}{=} \frac{\exp[(r_i^l - a_{ij}^u M_j)(1 - \Delta_i M_i)]}{\Delta_i}, \tag{2.5}$$

where $i, j = 1, 2$, $i \ne j$.

3 Main result

According to (2.4) and (2.5), we denote by Ω the set of all solutions $(x_1(n), x_2(n))$ of system (1.1) satisfying $m_i \le x_i(n) \le M_i$, $i = 1,2$, for all $n \in \mathbb{K}$. From Lemma 2.4, we first prove that there exists a bounded solution of system (1.1) and then construct a suitable Lyapunov function for system (1.1).

Theorem 3.1 *If conditions* (2.3) *are satisfied, then* $\Omega \neq \phi$.

The proof of Theorem 3.1 is given in Appendix 1.

Theorem 3.2 *If conditions* (2.3) *and*

$$\begin{aligned} &1 - \max\left(\left|1 - a_{11}^l m_1\right|, \left|1 - a_{11}^u M_1\right|\right) - \left(b_1^u k_2^u + b_2^u k_1^u\right)(M+1)M_1M_2 - a_{21}^u M_1 > 0, \\ &1 - \max\left(\left|1 - a_{22}^l m_2\right|, \left|1 - a_{22}^u M_2\right|\right) - \left(b_1^u k_2^u + b_2^u k_1^u\right)(M+1)M_1M_2 - a_{12}^u M_2 > 0 \end{aligned} \tag{3.1}$$

are satisfied, then system (1.1) *possesses a unique almost periodic solution* $(x_1^*(n), x_2^*(n))$, *and it is uniformly asymptotically stable within* Ω.

The proof of Theorem 3.2 is given in Appendix 2.

4 Example and numerical simulations

In this section, to verify the validity of our main results, we give an example and its corresponding numerical simulations.

Example 4.1 Consider the following discrete system with a delay:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{0.85 + 0.02\sin(\sqrt{2}n\pi) - (0.80 - 0.01\sin(\sqrt{2}n\pi))x_1(n) \\ \qquad - (0.03 + 0.01\sin(\sqrt{2}n\pi))x_2(n) \\ \qquad - (0.02 - 0.01\cos(\sqrt{2}n\pi))x_1(n)[0.83x_2(n) + 0.83x_2(n-1)]\}, \\ x_2(n+1) = x_2(n)\exp\{0.80 + 0.01\cos(\sqrt{2}n\pi) \\ \qquad - (0.02 + 0.01\cos(\sqrt{2}n\pi))x_1(n) \\ \qquad - (0.65 + 0.02\sin(\sqrt{2}n\pi))x_2(n) \\ \qquad - (0.03 + 0.02\sin(\sqrt{2}n\pi))x_2(n)[0.73x_1(n) + 0.73x_1(n-1)]\}, \end{cases} \tag{4.1}$$

with the following initial conditions:

$$x_1^*(-1) = 1.06, \qquad x_1^*(0) = 1.02, \qquad x_2^*(-1) = 0.85, \qquad x_2^*(0) = 0.98. \tag{4.2}$$

By a computation, we get

$$\begin{aligned} &M_1 \approx 1.1115, \qquad M_2 \approx 1.3126, \qquad m_1 \approx 0.7305, \\ &m_2 \approx 0.8010, \qquad \Delta_1 \approx 1.1259, \qquad \Delta_2 \approx 0.9927, \\ &r_1^l - a_{12}^u M_2 \approx 0.7775 > 0, \qquad r_2^l - a_{21}^u M_1 \approx 0.7567 > 0, \\ &\Delta_1 M_1 \approx 1.2514 > 1, \qquad \Delta_2 M_2 \approx 1.3030 > 1 \end{aligned} \tag{4.3}$$

and

$$\min\left\{r_1^l - a_{12}^u M_2, r_2^l - a_{21}^u M_1\right\} > 0, \qquad \min\{\Delta_1 M_1, \Delta_2 M_2\} > 1. \tag{4.4}$$

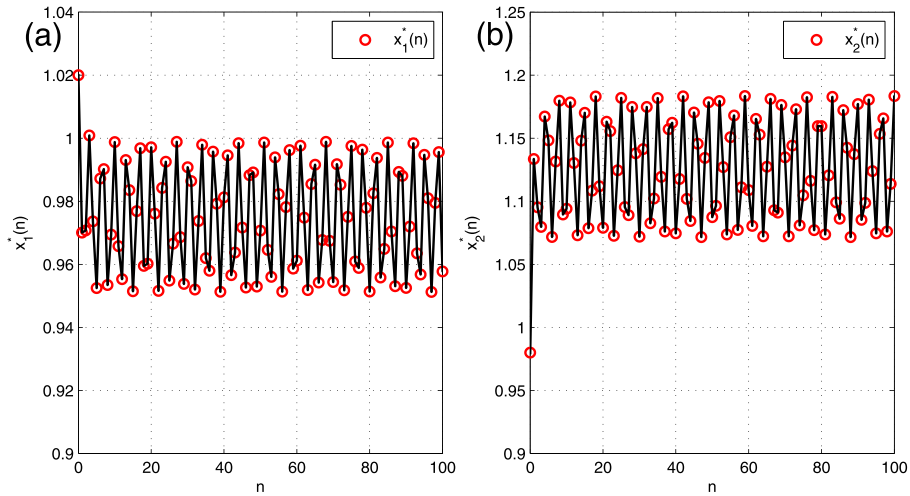

Figure 1 Positive almost periodic solution of system (4.1). (a) Time-series $x_1^*(n)$ with initial values $x_1^*(-1) = 1.06$, $x_1^*(0) = 1.02$ for $n \in [0, 100]$. **(b)** Time-series $x_2^*(n)$ with initial values $x_2^*(-1) = 0.85$, $x_2^*(0) = 0.98$ for $n \in [0, 100]$.

A further calculation shows that

$$\begin{aligned} &1 - \max\big(\big|1 - a_{11}^l m_1\big|, \big|1 - a_{11}^u M_1\big|\big) \\ &\quad - \big(b_1^u k_2^u + b_2^u k_1^u\big)(M+1)M_1 M_2 - a_{21}^u M_1 \approx 0.3646 > 0, \\ &1 - \max\big(\big|1 - a_{22}^l m_2\big|, \big|1 - a_{22}^u M_2\big|\big) \\ &\quad - \big(b_1^u k_2^u + b_2^u k_1^u\big)(M+1)M_1 M_2 - a_{12}^u M_2 \approx 0.2729 > 0. \end{aligned} \tag{4.5}$$

Clearly, the assumptions of Theorem 3.2 are satisfied, and hence system (4.1) has a unique uniformly asymptotically stable positive almost periodic solution. From Figure 1, we can see that there exists a positive almost periodic solution $(x_1^*(t), x_2^*(t))$, and the two-dimensional and three-dimensional phase portraits of almost periodic system (4.1) are revealed in Figure 2, respectively. Figure 3 shows that any positive solution $(x_1(n), x_2(n))$ tends to the almost periodic solution $(x_1^*(n), x_2^*(n))$.

Appendix 1: Proof of Theorem 3.1

By the almost periodicity of $\{r_i(n)\}$, $\{a_{ij}(n)\}$ and $\{b_i(n)\}$, $i, j = 1, 2$, any sequence $\{\tau_k\} \subset \mathbb{Z}^+$, with $\tau_k \to +\infty$ as $k \to +\infty$, is such that

$$r_i(n + \tau_k) \to r_i(n), \qquad a_{ij}(n + \tau_k) \to a_{ij}(n), \qquad b_i(n + \tau_k) \to b_i(n), \quad i, j = 1, 2, \tag{A.1}$$

as $k \to +\infty$ for $n \in \mathbb{Z}^+$. Let ε be an arbitrary small positive number. It follows from (2.4) and (2.5) that there exists a positive integer N_0 such that

$$m_i - \varepsilon \le x_i(n) \le M_i + \varepsilon \quad \text{for all } n > N_0. \tag{A.2}$$

Let $x_{ik}(n) = x_i(n + \tau_k)$ for $n \ge N_0 + M - \tau_k$, $k = 1, 2, \ldots$. For any positive integer q, we can see that there exists a sequence $\{x_{ik}(n) : k \ge q\}$ such that the sequence $\{x_{ik}(n)\}$ has a

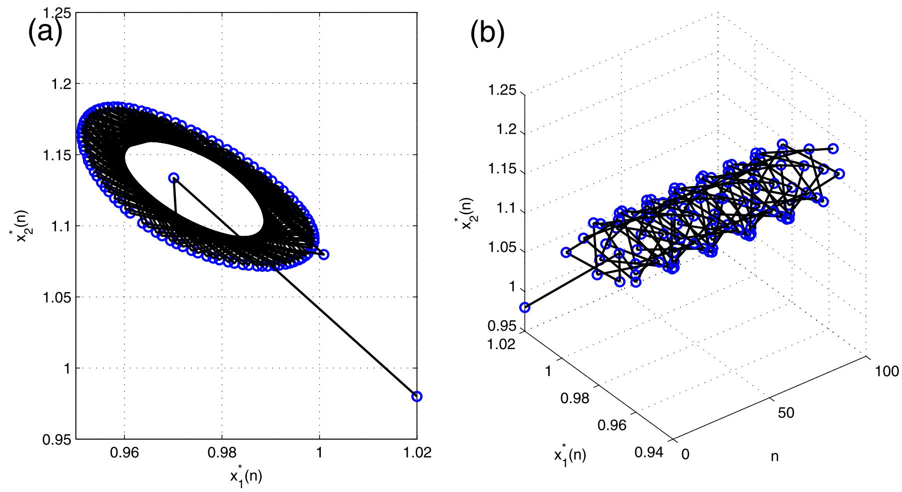

Figure 2 Phase portrait. (a) Two-dimensional phase portrait of almost periodic system (4.1). Time-series $x_1^*(n)$ and $x_2^*(n)$ with initial values $x_1^*(-1) = 1.06$, $x_1^*(0) = 1.02$, $x_2^*(-1) = 0.85$, $x_2^*(0) = 0.98$ for $n \in [0, 100]$. **(b)** Three-dimensional phase portrait of almost periodic system (4.1). Time-series $x_1^*(n)$ and $x_2^*(n)$ with the above initial values for $n \in [0, 100]$.

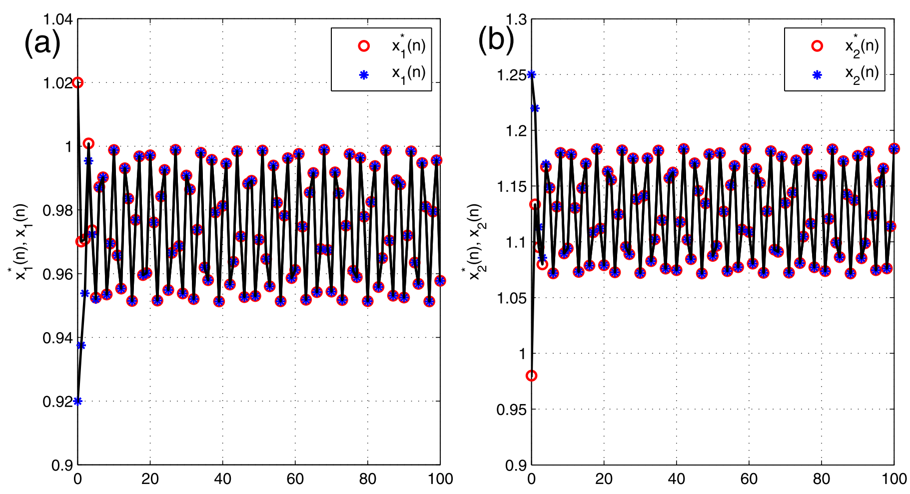

Figure 3 Uniformly asymptotic stability. (a) Time-series $x_1^*(n)$ with initial values $x_1^*(-1) = 1.06$, $x_1^*(0) = 1.02$ and $x_1(n)$ with initial values $x_1(-1) = 0.87$, $x_1(0) = 0.92$ for $n \in [0, 100]$, respectively. **(b)** Time-series $x_2^*(n)$ with initial values $x_2^*(-1) = 0.85$, $x_2^*(0) = 0.98$ and $x_2(n)$ with initial values $x_2(-1) = 1.15$, $x_2(0) = 1.25$ for $n \in [0, 100]$, respectively.

subsequence, denoted by $\{x_{ik}(n)\}$ again, converging on any finite interval of $\mathbb{K}$ as $k \to +\infty$. So, we have a sequence $\{y_i(n)\}$, $i = 1, 2$, satisfying

$$x_{ik}(n) \to y_i(n) \quad \text{for } n \in \mathbb{K} \text{ as } k \to +\infty, \tag{A.3}$$

which, together with (A.1) and

$$\begin{cases} x_{1k}(n+1) = x_{1k}(n)\exp\{r_1(n+\tau_k) - a_{11}(n+\tau_k)x_{1k}(n) - a_{12}(n+\tau_k)x_{2k}(n) \\ \qquad - b_1(n+\tau_k)x_{1k}(n)\sum_{p=0}^{M} k_2(p)x_{2k}(n-p)\}, \\ x_{2k}(n+1) = x_{2k}(n)\exp\{r_2(n+\tau_k) - a_{21}(n+\tau_k)x_{1k}(n) - a_{22}(n+\tau_k)x_{2k}(n) \\ \qquad - b_2(n+\tau_k)x_{2k}(n)\sum_{p=0}^{M} k_1(p)x_{1k}(n-p)\}, \end{cases} \tag{A.4}$$

yields

$$\begin{cases} y_1(n+1) = y_1(n)\exp\{r_1(n) - a_{11}(n)y_1(n) - a_{12}(n)y_2(n) \\ \qquad - b_1(n)y_1(n)\sum_{p=0}^{M} k_2(p)y_2(n-p)\}, \\ y_2(n+1) = y_2(n)\exp\{r_2(n) - a_{21}(n)y_1(n) - a_{22}(n)y_2(n) \\ \qquad - b_2(n)y_2(n)\sum_{p=0}^{M} k_1(p)y_1(n-p)\}. \end{cases} \tag{A.5}$$

We can easily see that $(y_1(n), y_2(n))$ is a solution of system (1.1) and $m_i - \varepsilon \le y_i(n) \le M_i + \varepsilon$ for $n \in \mathbb{K}$. Since ε is small enough, it follows that

$$m_i \le y_i(n) \le M_i, \quad i = 1, 2 \text{ for } n \in \mathbb{K}.$$

This completes the proof.

Appendix 2: Proof of Theorem 3.2

We first make the change of variables

$$p_1(n) = \ln x_1(n), \qquad p_2(n) = \ln x_2(n).$$

It follows from system (1.1) that

$$\begin{cases} p_1(n+1) = p_1(n) + r_1(n) - a_{11}(n)e^{p_1(n)} - a_{12}(n)e^{p_2(n)} \\ \qquad - b_1(n)e^{p_1(n)}\sum_{p=0}^{M} k_2(p)e^{p_2(n-p)}, \\ p_2(n+1) = p_2(n) + r_2(n) - a_{21}(n)e^{p_1(n)} - a_{22}(n)e^{p_2(n)} \\ \qquad - b_2(n)e^{p_2(n)}\sum_{p=0}^{M} k_1(p)e^{p_1(n-p)}. \end{cases} \tag{B.1}$$

From Theorem 3.1, it is easy to see that system (B.1) has a bounded solution $(p_1(n), p_2(n))$ satisfying

$$\ln m_1 \le p_1(n) \le \ln M_1, \qquad \ln m_2 \le p_2(n) \le \ln M_2 \quad \text{for all } n \in \mathbb{K}. \tag{B.2}$$

Thus $|p_1(n)| \le A_1$, $|p_2(n)| \le A_2$, where $A_i = \max\{|\ln m_i|, |\ln M_i|\}$, $i = 1, 2$. Suppose that $Y_n(s) = (p_1(n+s), p_2(n+s))$, $Z_n(s) = (q_1(n+s), q_2(n+s))$ ($n \in \mathbb{Z}^+$, $s \in [-M, 0]_{\mathbb{Z}}$) are any two solutions of system (B.1) defined on $\mathbb{S}$, where $\mathbb{S} = \{(p_1(n), p_2(n)) | \ln m_i \le p_i(n) \le \ln M_i, i = 1, 2, n \in \mathbb{K}\}$. Define the norm

$$\|Y_n(s)\| = \|(p_1(n+s), p_2(n+s))\| = \sup_{s\in[-M,0]_{\mathbb{Z}}} \{|p_1(n+s)| + |p_2(n+s)|\},$$

where $(p_1(n+s), p_2(n+s)) \in \mathbb{R}^2$, then

$$\|Y_n\| \le B, \qquad \|Z_n\| \le B,$$

where $B = A_1 + A_2$. Consider the associate product system of system (B.1)

$$\begin{cases} p_1(n+1) = p_1(n) + r_1(n) - a_{11}(n)e^{p_1(n)} - a_{12}(n)e^{p_2(n)} \\ \qquad - b_1(n)e^{p_1(n)} \sum_{p=0}^{M} k_2(p)e^{p_2(n-p)}, \\ p_2(n+1) = p_2(n) + r_2(n) - a_{21}(n)e^{p_1(n)} - a_{22}(n)e^{p_2(n)} \\ \qquad - b_2(n)e^{p_2(n)} \sum_{p=0}^{M} k_1(p)e^{p_1(n-p)}, \\ q_1(n+1) = q_1(n) + r_1(n) - a_{11}(n)e^{q_1(n)} - a_{12}(n)e^{q_2(n)} \\ \qquad - b_1(n)e^{q_1(n)} \sum_{p=0}^{M} k_2(p)e^{q_2(n-p)}, \\ q_2(n+1) = q_2(n) + r_2(n) - a_{21}(n)e^{q_1(n)} - a_{22}(n)e^{q_2(n)} \\ \qquad - b_2(n)e^{q_2(n)} \sum_{p=0}^{M} k_1(p)e^{q_1(n-p)}. \end{cases} \tag{B.3}$$

Construct a Lyapunov function $V(n) = V(n, Y_n, Z_n)$ defined on $\mathbb{Z}^+ \times \mathbb{S} \times \mathbb{S}$ as follows:

$$\begin{aligned} V(n) &= V(n, Y_n, Z_n) \\ &= |p_1(n) - q_1(n)| + |p_2(n) - q_2(n)| \\ &\quad + \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_2^u k_1^u M_1 M_2 |p_1(m) - q_1(m)| \\ &\quad + \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_1^u k_2^u M_1 M_2 |p_2(m) - q_2(m)|. \end{aligned} \tag{B.4}$$

It is easy to see that

$$\begin{aligned} |Y_n(0) - Z_n(0)| &\le V(n) \\ &\le |p_1(n) - q_1(n)| + |p_2(n) - q_2(n)| \\ &\quad + \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} D\{|p_1(m) - q_1(m)| + |p_2(m) - q_2(m)|\} \\ &\le \left[1 + \frac{(1+M)MD}{2}\right] \sup_{s\in[-M,0]_{\mathbb{Z}}} \{|p_1(n+s) - q_1(n+s)| \\ &\quad + |p_2(n+s) - q_2(n+s)|\} \\ &= \lambda \|Y_n - Z_n\|, \end{aligned} \tag{B.5}$$

where

$$\begin{aligned}\left|Y_n(0)-Z_n(0)\right| &= \sqrt{\left(p_1(n)-q_1(n)\right)^2+\left(p_2(n)-q_2(n)\right)^2} \\ &\le \left|p_1(n)-q_1(n)\right|+\left|p_2(n)-q_2(n)\right|,\end{aligned} \tag{B.6}$$

and $\lambda = 1+\frac{(1+M)MD}{2}$, $D=\max\{b_1^u k_2^u M_1M_2, b_2^u k_1^u M_1M_2\}$. Let $a,b\in C(\mathbb{R}^+,\mathbb{R}^+)$, $a(x)=x$, $b(x)=\lambda x$, so condition (1) in Lemma 2.4 is satisfied.

For $\forall y,z,\tilde{y},\tilde{z}\in\mathbb{R}$, one has

$$\begin{aligned}\big||y-z|-|\tilde{y}-\tilde{z}|\big| &= \begin{cases}|y-z|-|\tilde{y}-\tilde{z}|, & \text{if } |y-z|\ge|\tilde{y}-\tilde{z}|,\\ |\tilde{y}-\tilde{z}|-|y-z|, & \text{if } |\tilde{y}-\tilde{z}|>|y-z|\end{cases} \\ &\le \begin{cases}|(y-\tilde{y})+(\tilde{z}-z)|, & \text{if } |y-z|\ge|\tilde{y}-\tilde{z}|,\\ |(\tilde{y}-y)+(z-\tilde{z})|, & \text{if } |\tilde{y}-\tilde{z}|>|y-z|\end{cases} \\ &\le \begin{cases}|y-\tilde{y}|+|\tilde{z}-z|, & \text{if } |y-z|\ge|\tilde{y}-\tilde{z}|,\\ |\tilde{y}-y|+|z-\tilde{z}|, & \text{if } |\tilde{y}-\tilde{z}|>|y-z|\end{cases} \\ &= |y-\tilde{y}|+|z-\tilde{z}|.\end{aligned} \tag{B.7}$$

Hence, for $\forall Y_n, Z_n, \tilde{Y}_n, \tilde{Z}_n\in\mathbb{S}$, by (B.7) we have

$$\begin{aligned}&\left|V(n,Y_n,Z_n)-V(n,\tilde{Y}_n,\tilde{Z}_n)\right| \\ &\quad= \Bigg|\left|p_1(n)-q_1(n)\right|+\left|p_2(n)-q_2(n)\right|+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_2^u k_1^u M_1M_2\left|p_1(m)-q_1(m)\right| \\ &\qquad+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_1^u k_2^u M_1M_2\left|p_2(m)-q_2(m)\right| \\ &\qquad-\left|\tilde{p}_1(n)-\tilde{q}_1(n)\right|-\left|\tilde{p}_2(n)-\tilde{q}_2(n)\right|-\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_2^u k_1^u M_1M_2\left|\tilde{p}_1(m)-\tilde{q}_1(m)\right| \\ &\qquad-\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_1^u k_2^u M_1M_2\left|\tilde{p}_2(m)-\tilde{q}_2(m)\right|\Bigg| \\ &\quad\le \Big|\left|p_1(n)-q_1(n)\right|+\left|p_2(n)-q_2(n)\right|-\left|\tilde{p}_1(n)-\tilde{q}_1(n)\right|-\left|\tilde{p}_2(n)-\tilde{q}_2(n)\right|\Big| \\ &\qquad+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_2^u k_1^u M_1M_2\Big|\left|p_1(m)-q_1(m)\right|-\left|\tilde{p}_1(m)-\tilde{q}_1(m)\right|\Big| \\ &\qquad+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_1^u k_2^u M_1M_2\Big|\left|p_2(m)-q_2(m)\right|-\left|\tilde{p}_2(m)-\tilde{q}_2(m)\right|\Big| \\ &\quad\le \left\{\left|p_1(n)-\tilde{p}_1(n)\right|+\left|p_2(n)-\tilde{p}_2(n)\right|+\left|q_1(n)-\tilde{q}_1(n)\right|+\left|q_2(n)-\tilde{q}_2(n)\right|\right\} \\ &\qquad+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_2^u k_1^u M_1M_2\left\{\left|p_1(m)-\tilde{p}_1(m)\right|+\left|q_1(m)-\tilde{q}_1(m)\right|\right\}\end{aligned}$$

$$
\begin{aligned}
&+\sum_{p=0}^{M}\sum_{m=n-p}^{n-1} b_1^u k_2^u M_1 M_2\bigl\{\bigl|p_2(m)-\tilde{p}_2(m)\bigr|+\bigl|q_2(m)-\tilde{q}_2(m)\bigr|\bigr\}\\
&\le\left[1+\frac{(1+M)MD}{2}\right]\sup_{s\in[-M,0]_{\mathbb{Z}}}\bigl\{\bigl|p_1(n+s)-\tilde{p}_1(n+s)\bigr|+\bigl|p_2(n+s)-\tilde{p}_2(n+s)\bigr|\\
&\quad+\bigl|q_1(n+s)-\tilde{q}_1(n+s)\bigr|+\bigl|q_2(n+s)-\tilde{q}_2(n+s)\bigr|\bigr\}\\
&\le\lambda\bigl(\|Y_n-\tilde{Y}_n\|+\|Z_n-\tilde{Z}_n\|\bigr),
\end{aligned}
\tag{B.8}
$$

where $\lambda=1+\frac{(1+M)MD}{2}$, $D=\max\{b_1^u k_2^u M_1 M_2, b_2^u k_1^u M_1 M_2\}$. Condition (2) in Lemma 2.4 is also satisfied.

Using the mean-value theorem, we derive that

$$
e^{p_i(n)}-e^{q_i(n)}=\xi_i(n)\bigl(p_i(n)-q_i(n)\bigr),
\tag{B.9}
$$

$$
e^{p_i(n-p)}-e^{q_i(n-p)}=\eta_i(n-p)\bigl(p_i(n-p)-q_i(n-p)\bigr),
\tag{B.10}
$$

$i=1,2$, where $\xi_i(n)$ lies between $e^{p_i(n)}$ and $e^{q_i(n)}$ and $\eta_i(n-p)$ lies between $e^{p_i(n-p)}$ and $e^{q_i(n-p)}$, respectively. So, $m_i\le\xi_i(n),\eta_i(n-p)\le M_i$, $n\in\mathbb{Z}^+$.

In view of system (B.3) together with (B.9) and (B.10), we have

$$
\begin{aligned}
&\bigl|p_1(n+1)-q_1(n+1)\bigr|+\bigl|p_2(n+1)-q_2(n+1)\bigr|\\
&=\Biggl|p_1(n)-q_1(n)-a_{11}(n)\bigl(e^{p_1(n)}-e^{q_1(n)}\bigr)-a_{12}(n)\bigl(e^{p_2(n)}-e^{q_2(n)}\bigr)\\
&\quad-b_1(n)\Biggl(e^{p_1(n)}\sum_{p=0}^{M}k_2(p)e^{p_2(n-p)}-e^{q_1(n)}\sum_{p=0}^{M}k_2(p)e^{q_2(n-p)}\Biggr)\Biggr|\\
&\quad+\Biggl|p_2(n)-q_2(n)-a_{21}(n)\bigl(e^{p_1(n)}-e^{q_1(n)}\bigr)-a_{22}(n)\bigl(e^{p_2(n)}-e^{q_2(n)}\bigr)\\
&\quad-b_2(n)\Biggl(e^{p_2(n)}\sum_{p=0}^{M}k_1(p)e^{p_1(n-p)}-e^{q_2(n)}\sum_{p=0}^{M}k_1(p)e^{q_1(n-p)}\Biggr)\Biggr|\\
&=\Biggl|p_1(n)-q_1(n)-a_{11}(n)\bigl(e^{p_1(n)}-e^{q_1(n)}\bigr)-a_{12}(n)\bigl(e^{p_2(n)}-e^{q_2(n)}\bigr)\\
&\quad-b_1(n)\Biggl(e^{p_1(n)}\sum_{p=0}^{M}k_2(p)e^{p_2(n-p)}-e^{p_1(n)}\sum_{p=0}^{M}k_2(p)e^{q_2(n-p)}\\
&\quad+e^{p_1(n)}\sum_{p=0}^{M}k_2(p)e^{q_2(n-p)}-e^{q_1(n)}\sum_{p=0}^{M}k_2(p)e^{q_2(n-p)}\Biggr)\Biggr|\\
&\quad+\Biggl|p_2(n)-q_2(n)-a_{21}(n)\bigl(e^{p_1(n)}-e^{q_1(n)}\bigr)-a_{22}(n)\bigl(e^{p_2(n)}-e^{q_2(n)}\bigr)\\
&\quad-b_2(n)\Biggl(e^{p_2(n)}\sum_{p=0}^{M}k_1(p)e^{p_1(n-p)}-e^{p_2(n)}\sum_{p=0}^{M}k_1(p)e^{q_1(n-p)}
\end{aligned}
$$

$$
\begin{aligned}
&\quad + e^{p_2(n)}\sum_{p=0}^{M}k_1(p)e^{q_1(n-p)} - e^{q_2(n)}\sum_{p=0}^{M}k_1(p)e^{q_1(n-p)}\Bigg)\Bigg| \\
&= \Bigg|p_1(n) - q_1(n) - a_{11}(n)\xi_1(n)\big(p_1(n) - q_1(n)\big) - a_{12}(n)\xi_2(n)\big(p_2(n) - q_2(n)\big) \\
&\quad - b_1(n)e^{p_1(n)}\sum_{p=0}^{M}k_2(p)\big[\eta_2(n-p)\big(p_2(n-p) - q_2(n-p)\big)\big] \\
&\quad - b_1(n)\xi_1(n)\big(p_1(n) - q_1(n)\big)\sum_{p=0}^{M}k_2(p)e^{q_2(n-p)}\Bigg| \\
&\quad + \Bigg|p_2(n) - q_2(n) - a_{21}(n)\xi_1(n)\big(p_1(n) - q_1(n)\big) - a_{22}(n)\xi_2(n)\big(p_2(n) - q_2(n)\big) \\
&\quad - b_2(n)e^{p_2(n)}\sum_{p=0}^{M}k_1(p)\big[\eta_1(n-p)\big(p_1(n-p) - q_1(n-p)\big)\big] \\
&\quad - b_2(n)\xi_2(n)\big(p_2(n) - q_2(n)\big)\sum_{p=0}^{M}k_1(p)e^{q_1(n-p)}\Bigg| \\
&\le \big|1 - a_{11}(n)\xi_1(n)\big| \cdot \big|p_1(n) - q_1(n)\big| + a_{12}^{u}M_2\big|p_2(n) - q_2(n)\big| \\
&\quad + b_1^{u}k_2^{u}M_1M_2\sum_{p=0}^{M}\big|p_2(n-p) - q_2(n-p)\big| + b_1^{u}k_2^{u}M_1M_2(M+1)\big|p_1(n) - q_1(n)\big| \\
&\quad + \big|1 - a_{22}(n)\xi_2(n)\big| \cdot \big|p_2(n) - q_2(n)\big| + a_{21}^{u}M_1\big|p_1(n) - q_1(n)\big| \\
&\quad + b_2^{u}k_1^{u}M_1M_2\sum_{p=0}^{M}\big|p_1(n-p) - q_1(n-p)\big| + b_2^{u}k_1^{u}M_1M_2(M+1)\big|p_2(n) - q_2(n)\big| \\
&\le \big\{\big|1 - a_{11}(n)\xi_1(n)\big| + b_1^{u}k_2^{u}M_1M_2(M+1) + a_{21}^{u}M_1\big\} \cdot \big|p_1(n) - q_1(n)\big| \\
&\quad + b_2^{u}k_1^{u}M_1M_2\sum_{p=0}^{M}\big|p_1(n-p) - q_1(n-p)\big| \\
&\quad + \big\{\big|1 - a_{22}(n)\xi_2(n)\big| + b_2^{u}k_1^{u}M_1M_2(M+1) + a_{12}^{u}M_2\big\} \cdot \big|p_2(n) - q_2(n)\big| \\
&\quad + b_1^{u}k_2^{u}M_1M_2\sum_{p=0}^{M}\big|p_2(n-p) - q_2(n-p)\big|. \qquad \text{(B.11)}
\end{aligned}
$$

Based on (B.11), we calculate the difference of V along the solution of system (B.3)

$$
\begin{aligned}
\Delta V_{(\mathrm{B.3})}(n) &= V(n+1) - V(n) \\
&= \big|p_1(n+1) - q_1(n+1)\big| + \big|p_2(n+1) - q_2(n+1)\big| \\
&\quad + \sum_{p=0}^{M}\sum_{m=n+1-p}^{n} b_2^{u}k_1^{u}M_1M_2\big|p_1(m) - q_1(m)\big| \\
&\quad + \sum_{p=0}^{M}\sum_{m=n+1-p}^{n} b_1^{u}k_2^{u}M_1M_2\big|p_2(m) - q_2(m)\big|
\end{aligned}
$$

$$
\begin{aligned}
&- \left|p_1(n) - q_1(n)\right| - \left|p_2(n) - q_2(n)\right| - \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_2^u k_1^u M_1 M_2 \left|p_1(m) - q_1(m)\right| \\
&- \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_1^u k_2^u M_1 M_2 \left|p_2(m) - q_2(m)\right| \\
\leq{}& \left\{\left|1 - a_{11}(n)\xi_1(n)\right| + b_1^u k_2^u M_1 M_2 (M+1) + a_{21}^u M_1\right\} \cdot \left|p_1(n) - q_1(n)\right| \\
&+ \sum_{p=0}^{M} b_2^u k_1^u M_1 M_2 \left|p_1(n-p) - q_1(n-p)\right| \\
&+ \left\{\left|1 - a_{22}(n)\xi_2(n)\right| + b_2^u k_1^u M_1 M_2 (M+1) + a_{12}^u M_2\right\} \cdot \left|p_2(n) - q_2(n)\right| \\
&+ \sum_{p=0}^{M} b_1^u k_2^u M_1 M_2 \left|p_2(n-p) - q_2(n-p)\right| \\
&+ \sum_{p=0}^{M} \sum_{m=n+1-p}^{n} b_2^u k_1^u M_1 M_2 \left|p_1(m) - q_1(m)\right| \\
&+ \sum_{p=0}^{M} \sum_{m=n+1-p}^{n} b_1^u k_2^u M_1 M_2 \left|p_2(m) - q_2(m)\right| \\
&- \left|p_1(n) - q_1(n)\right| - \left|p_2(n) - q_2(n)\right| - \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_2^u k_1^u M_1 M_2 \left|p_1(m) - q_1(m)\right| \\
&- \sum_{p=0}^{M} \sum_{m=n-p}^{n-1} b_1^u k_2^u M_1 M_2 \left|p_2(m) - q_2(m)\right| \\
={}& \left\{\left|1 - a_{11}(n)\xi_1(n)\right| + \left(b_1^u k_2^u + b_2^u k_1^u\right) M_1 M_2 (M+1) \right.\\
&\left. + a_{21}^u M_1 - 1\right\} \cdot \left|p_1(n) - q_1(n)\right| \\
&+ \left\{\left|1 - a_{22}(n)\xi_2(n)\right| + \left(b_1^u k_2^u + b_2^u k_1^u\right) M_1 M_2 (M+1) \right.\\
&\left. + a_{12}^u M_2 - 1\right\} \cdot \left|p_2(n) - q_2(n)\right| \\
\leq{}& -\left\{1 - \max\left(\left|1 - a_{11}^l m_1\right|, \left|1 - a_{11}^u M_1\right|\right) - \left(b_1^u k_2^u + b_2^u k_1^u\right)(M+1) M_1 M_2 \right.\\
&\left. - a_{21}^u M_1\right\} \cdot \left|p_1(n) - q_1(n)\right| \\
&- \left\{1 - \max\left(\left|1 - a_{22}^l m_2\right|, \left|1 - a_{22}^u M_2\right|\right) - \left(b_1^u k_2^u + b_2^u k_1^u\right)(M+1) M_1 M_2 \right.\\
&\left. - a_{12}^u M_2\right\} \cdot \left|p_2(n) - q_2(n)\right| \\
\leq{}& -\theta\left(\left|p_1(n) - q_1(n)\right| + \left|p_2(n) - q_2(n)\right|\right) \\
\leq{}& -\theta \sqrt{\left(p_1(n) - q_1(n)\right)^2 + \left(p_2(n) - q_2(n)\right)^2} \\
={}& -\theta\left(\left|Y_n(0) - Z_n(0)\right|\right), \qquad \text{(B.12)}
\end{aligned}
$$

where $\theta = \min\{1 - \max(|1 - a_{11}^l m_1|, |1 - a_{11}^u M_1|) - (b_1^u k_2^u + b_2^u k_1^u)(M+1)M_1M_2 - a_{21}^u M_1, 1 - \max(|1 - a_{22}^l m_2|, |1 - a_{22}^u M_2|) - (b_1^u k_2^u + b_2^u k_1^u)(M+1)M_1M_2 - a_{12}^u M_2\}$ is a positive constant. Let $c \in C(\mathbb{R}^+, \mathbb{R}^+)$, $c(x) = \theta x$, thus condition $(3)'$ in Remark 2.5 is satisfied. Based on Lemma 2.4 and Remark 2.5, there exists a unique uniformly asymptotically stable almost periodic solution $(p_1(n), p_2(n))$ of system (B.1), that is, there is a unique uniformly asymptotically stable positive almost periodic solution $(x_1^*(n), x_2^*(n))$ of system (1.1) which satisfies $m_i \leq x_i^*(n) \leq M_i$, $i = 1, 2$, for all $n \in \mathbb{K}$. The proof is complete.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Each of the authors, QW and ZL, contributed to each part of this work equally and read and approved the final version of the manuscript.

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