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New $N = 5, 6$, 3D gauged supergravities and holography

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Abstract: We study $N = 5$ gauged supergravity in three dimensions with compact, non-compact and non-semisimple gauge groups. The theory under consideration is of Chern-Simons type with $\mathrm{USp}(4,k)/\mathrm{USp}(4) \times \mathrm{USp}(k)$ scalar manifold. We classify possible semisimple gauge groups of the $k = 2, 4$ cases and identify some of their critical points. A number of supersymmetric AdS_3 critical points are found, and holographic RG flows interpolating between these critical points are also investigated. As one of our main results, we consider a non-semisimple gauge group $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ for the theory with $\mathrm{USp}(4,4)/\mathrm{USp}(4) \times \mathrm{USp}(4)$ scalar manifold. The resulting theory describes $N = 5$ gauged supergravity in four dimensions reduced on $S^1/\mathbb{Z}_2$ and admits a maximally supersymmetric AdS_3 critical point with $\mathrm{Osp}(5|2,\mathbb{R}) \times \mathrm{Sp}(2,\mathbb{R})$ superconformal symmetry. We end the paper with the construction of $\mathrm{SO}(6) \ltimes \mathbf{T}^{15}$ gauged supergravity with $N = 6$ supersymmetry. The theory admits a half-supersymmetric domain wall as a vacuum solution and may be obtained from an $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions.

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Contents

1 Introduction

The duality between scalars and vectors together with the non-propagating nature of supergravity fields in three dimensions make three dimensional gauged supergravity substantially differs from its higher dimensional analogue. On one hand, only matter-coupled supergravity has propagating degrees of freedom in terms of scalars and spin-$\frac{1}{2}$ fields. Accordingly, the matter-coupled theory takes the form of a supersymmetric non-linear sigma model coupled to supergravity. On the other hand, recasting vectors to scalars, making the U-duality symmetry manifest, seems to create a trouble in any attempt to gauge the theory since the vector fields accompanying for the gauging are missing.

Special to three dimensions, vector fields can enter the gauged Lagrangian via Chern-Simons (CS) terms as opposed to the conventional Yang-Mills (YM) kinetic terms. Since CS terms do not lead to additional degrees of freedom, any number of gauge fields, or equivalently the dimension of the gauge group, can be introduced provided that the gauge group is a proper subgroup of the global symmetry group and consistent with supersymmetry. This gives rise to a very rich structure of gauged supergravity in three dimensions [1–5].

Additionally, the Chern-Simons form of gauged supergravity raises another difficulty namely the embedding of the resulting gauged theory in higher dimensions. This is due to the fact that all theories obtained from conventional dimensional reductions are of Yang-Mills form. It has been, however, shown that Yang-Mills gauged supergravity is on-shell equivalent to Chern-Simons gauged theory with a non-semisimple gauge group [6]. Up to now, there are many attempts to embed three dimensional gauged supergravity in higher dimensions and in string/M theory. These results would give rise to new string theory backgrounds with fluxes as well as new D-brane configurations [7]. However, it has been pointed out recently in [8] that there might exist supersymmetric string backgrounds which are not captured by gauged supergravities.

The rich structure and embedding in string/M theory aside, gauged supergravity proves to be a very useful tool in the AdS/CFT correspondence [9]. $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence can provide more insight not only to the AdS/CFT correspondence, including its generalizations such as the Domain Wall/Quantum Field Theory (DW/QFT) correspondence, but also to black hole physics [10, 11]. In holographic RG flows, AdS_3 vacua and domain walls interpolating between them interpreted as RG flows in the dual two dimensional field theories are of particular interest, see [12] for a thorough review. The deformations of a strongly coupled field theory can be understood in this framework. Some gauged supergravities do not admit a maximally supersymmetric AdS_3 but a half-supersymmetric domain wall as a vacuum solution. This class of gauged supergravities will be useful in the context of the DW/QFT correspondence [13–15].

In this work, we further explore the structure of gauged supergravity in three dimensions with $N = 5, 6$ supersymmetry. We begin with a study of compact and non-compact gaugings of the $N = 5$ theory with scalar manifolds $\mathrm{USp}(4, 2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ and $\mathrm{USp}(4, 4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$. We will identify some supersymmetric AdS_3 critical points and study the associated RG flow solutions. This could be useful in $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence although the embedding in higher dimensions is presently not known. The result is similar to supersymmetric RG flows studied in [16–19] and in higher dimensions such as recent solutions of new maximal gauged supergravity in four dimensions given in [20].

We then move to non-semisimple gaugings of the $N = 5$ theory containing 16 scalars encoded in $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ coset manifold with $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauge group. The gauge group is embedded in the global symmetry group $\mathrm{USp}(4,4)$. According to [6], the resulting theory is equivalent to $\mathrm{SO}(5)$ YM gauged supergravity. The latter might be obtained by a reduction of $N = 5$, $\mathrm{SO}(5)$ gauged supergravity in four dimensions on $S^1/\mathbb{Z}_2$ as pointed out in [21]. The theory may also be embedded in $N = 10$, $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauged supergravity via the embedding of the global symmetry group $\mathrm{USp}(4,4)\subset E_{6(-14)}$. The theory admits a maximally supersymmetric AdS_3 vacuum and provides another example of three dimensional gauged supergravities with known higher dimensional origin.

We finally turn to non-semisimple gauging of $N = 6$ theory with $\mathrm{SU}(4,4)/S(\mathrm{U}(4)\times\mathrm{U}(4))$ scalar manifold. The global symmetry $\mathrm{SU}(4,4)$ contains an $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ subgroup that can be consistently gauged. Similar to $N = 5$ theory, this theory is equivalent to $\mathrm{SO}(6)$ YM gauged supergravity and could be obtained by an $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions. Unlike $N = 5$ theory, the theory admits only a half-supersymmetric domain wall as a vacuum solution.

The paper is organized as follow. We give the construction of $N = 5$ theory in section 2. Relevant information and related formulae for general gauged supergravity in three dimensions are collected in appendix A. Vacua of compact and non-compact gauge groups are given in section 3 and 4, respectively. Section 5 deals with some examples of RG flows between critical points previously identified. Non-semisimple gaugings of $N = 5$ and $N = 6$ theories are constructed in sections 6 and 7, respectively. The maximally supersymmetric AdS_3 of $N = 5$ theory and a $\frac{1}{2}$-BPS domain wall of the $N = 6$ theory are explicitly given in these sections. We end the paper with some conclusions and discussions. Appendices B and C contain the explicit form of the relevant generators used in the main text as well as the scalar potential for $\mathrm{SO}(4)\times\mathrm{USp}(2)$ gauging in $N = 5$ theory.

2 $N = 5$ gauged supergravity in three dimensions

In $N = 5$ three dimensional gauged supergravity, scalar fields are described in term of $\mathrm{USp}(4,k)/\mathrm{USp}(4)\times\mathrm{USp}(k)$ coset manifold with dimensionality $4k$. The R-symmetry is given by $\mathrm{USp}(4)\sim\mathrm{SO}(5)_R$. All admissible gauge groups are embedded in the global symmetry group $\mathrm{USp}(4,k)$. In this paper, we will consider only the $k = 2$ and $k = 4$ cases.

We first introduce $\mathrm{USp}(4,k)$ generators constructed from a compact group $\mathrm{USp}(4+k)$ via the Weyl unitarity trick. In order to make contact with the $N = 6$ theory with global symmetry group $\mathrm{SU}(4,k)$ studied in section 7, we will construct the $\mathrm{USp}(4+k)$ generators by figuring out the $\mathrm{USp}(4+k)$ subgroup of $\mathrm{SU}(4+k)$, directly. The latter is generated by the well-known generalized Gell-Mann matrices given in, for example, [22]. We will denote $\mathrm{USp}(4+k)$ generators by J_i given explicitly in appendix B. The $\mathrm{SO}(5)_R$ R-symmetry generators, labeled by a pair of anti-symmetric indices $T^{IJ} = -T^{JI}$, can be identified as follow

$$\begin{aligned}
T^{12} &= \frac{1}{\sqrt{2}}(J_3 - J_6)\,, & T^{13} &= -\frac{1}{\sqrt{2}}(J_1 + J_4)\,, & T^{23} &= \frac{1}{\sqrt{2}}(J_2 - J_5)\,,\\
T^{34} &= \frac{1}{\sqrt{2}}(J_3 + J_6)\,, & T^{14} &= \frac{1}{\sqrt{2}}(J_2 + J_5)\,, & T^{24} &= \frac{1}{\sqrt{2}}(J_1 - J_4)\,,\\
T^{15} &= -J_9, & T^{25} &= -J_{10}, & T^{35} &= J_8,\\
T^{45} &= J_7\,. \qquad (2.1)
\end{aligned}$$

The non-compact generators Y^A are identified by

$$\begin{aligned}
Y^1 &= iJ_{14}, & Y^2 &= iJ_{15}, & Y^3 &= iJ_{16}, & Y^4 &= iJ_{17},\\
Y^5 &= iJ_{18}, & Y^6 &= iJ_{19}, & Y^7 &= iJ_{20}, & Y^8 &= iJ_{21},\\
Y^9 &= iJ_{25}, & Y^{10} &= iJ_{26}, & Y^{11} &= iJ_{27}, & Y^{12} &= iJ_{28},\\
Y^{13} &= iJ_{29}, & Y^{14} &= iJ_{30}, & Y^{15} &= iJ_{31}, & Y^{16} &= iJ_{32}\,. \qquad (2.2)
\end{aligned}$$

For $k = 2$ case with 8 scalars, the associated non-compact generators are given by the first 8 generators, Y^A with $A = 1, \dots, 8$.

Admissible gauge groups are completely characterized by the symmetric gauge invariant embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$, $\mathcal{M}, \mathcal{N} = 1, \dots, \dim G$. Viable gaugings are defined by the embedding tensor satisfying two constraints. The first constraint is quadratic in Θ and given by

$$\Theta_{\mathcal{P}\mathcal{L}} f^{\mathcal{K}\mathcal{L}}{}_{(\mathcal{M}}\Theta_{\mathcal{N})\mathcal{K}} = 0 \qquad (2.3)$$

ensuring that a given gauge group G_0 is a proper subgroup of G. The other constraint due to supersymmetry takes the form of a projection condition

$$\mathbb{P}_{\boxplus} T^{IJ,KL} = 0 \qquad (2.4)$$

where the T-tensor $T^{IJ,KL}$ is given by the moment map of the embedding tensor

$$T^{IJ,KL} \equiv \mathcal{V}^{\mathcal{M}\,IJ}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}\,KL}\,. \qquad (2.5)$$

The $\boxplus$ denotes the Riemann tensor-like representation of $\mathrm{SO}(N)_R$. For symmetric scalar manifolds of the form G/H, the $\mathcal{V}$ maps can be obtained from the coset representative, see appendix A, and the constraint can be written in the form

$$\mathbb{P}_{R_0}\Theta_{\mathcal{M}\mathcal{N}} = 0\,. \qquad (2.6)$$

The representation R_0 of G contains the $\boxplus$ representation of $\mathrm{SO}(N)_R$.

We are now in a position to study gaugings of $N = 5$ supergravity. We will treat compact and non-compact gauge groups separately.

3 Compact gauge groups

In this section, we explore $N = 5$ gauged supergravity with compact gauge groups. The gauge groups are subgroup of $\mathrm{USp}(4) \times \mathrm{USp}(k)$ and takes the form $\mathrm{SO}(p) \times \mathrm{SO}(5-p) \times \mathrm{USp}(k)$, $p = 5, 4, 3$.

The $\mathrm{SO}(p) \times \mathrm{SO}(5-p)$ part is embedded in $\mathrm{SO}(5)_R$ as $\mathbf{5} \to (\mathbf{p}, \mathbf{1}) + (\mathbf{1}, \mathbf{5}-\mathbf{p})$. The corresponding embedding tensor is identified in [5] and takes the form

$$\Theta_{IJ,KL} = \theta \delta^{KL}_{IJ} + \delta_{[I[K}\Xi_{L]J]} \tag{3.1}$$

where

$$\Xi_{IJ} = \begin{cases} 2\left(1 - \frac{p}{5}\right)\delta_{IJ}, & I \leq p \\ -\frac{2p}{5}\delta_{IJ}, & I > p \end{cases}, \qquad \theta = \frac{2p-5}{5}. \tag{3.2}$$

The full embedding tensor for $\mathrm{SO}(p) \times \mathrm{SO}(5-p) \times \mathrm{USp}(k)$ is given by

$$\Theta = g_1 \Theta_{\mathrm{SO}(p)\times \mathrm{SO}(5-p)} + g_2 \Theta_{\mathrm{USp}(k)} \tag{3.3}$$

with two independent coupling constants. $\Theta_{\mathrm{USp}(k)}$ is given by the Killing form of $\mathrm{USp}(k)$. Together with the explicit form of the coset representative, the scalar potential is completely determined by the embedding tensor.

3.1 The $k = 2$ case

In this case, the theory contains 8 scalars parametrized by $\mathrm{USp}(4,2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ coset space. The full 8-dimensional manifold can be conveniently parametrized by the Euler angles of $\mathrm{SO}(5) \times \mathrm{USp}(2) \sim \mathrm{USp}(4) \times \mathrm{USp}(2)$. The details of the parametrization can be found in [23], and the application to $\mathrm{SU}(n,m)/S(\mathrm{U}(n) \times \mathrm{U}(m))$ coset can be found in [19].

3.1.1 $\mathrm{SO}(5) \times \mathrm{USp}(2)$ gauging

With $\mathrm{USp}(4) \times \mathrm{USp}(2)$ Euler angles, the full $\mathrm{USp}(4,2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ coset can be parametrized by the coset representative

$$L = e^{a_1 X_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 J_7} e^{a_5 J_8} e^{a_6 J_9} e^{a_7 J_{15}} e^{b Y^7} \tag{3.4}$$

where X_i's are defined by

$$X_1 = \frac{1}{\sqrt{2}}(J_1 - J_{11}), \qquad X_2 = \frac{1}{\sqrt{2}}(J_2 - J_{12}), \qquad X_3 = \frac{1}{\sqrt{2}}(J_3 - J_{13}). \tag{3.5}$$

The resulting scalar potential is

$$\begin{aligned} V = \frac{1}{32}\Big[&64\left(g_2^2 - 12g_1^2 + 4g_1g_2\right)\cosh b - 1076g_1^2 - 180g_1g_2 - 45g_2^2 \\ &-4\left(52g_1^2 + 20g_1g_2 + 5g_2^2\right)\cosh(2b) + (2g_1 + g_2)^2\cosh(4b)\Big]. \end{aligned} \tag{3.6}$$

JHEP01(2014)159

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(5,0)$	$\mathrm{SO}(5)\times\mathrm{USp}(2)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$

Table 1. Critical points of $\mathrm{SO}(5)\times\mathrm{USp}(2)$ gauging.

Note that the scalar fields associated to the gauge generators do not appear in the potential due to gauge invariance. We find some critical points as shown in table 1. V_0 is the value of the potential at each critical point. Unbroken supersymmetry is denoted by (n_-,n_+) where n_- and n_+ correspond to the number of supersymmetry in the dual two dimensional CFT. In three dimensional language, they correspond to the numbers of negative and positive eigenvalues of A_1^{IJ} tensor. As reviewed in appendix A, these eigenvalues, $\pm\tilde{\alpha}$, satisfy $V_0=-4\tilde{\alpha}^2$. Since, in our convention, the AdS_3 radius is given by $L=\frac{1}{\sqrt{-V_0}}$, we also have a relation $L=\frac{1}{2|\tilde{\alpha}|}$.

The maximally supersymmetric critical point at $L=\mathbf{I}$ preserves the full gauge symmetry. The two non-trivial critical points preserve $\mathrm{USp}(2)\times\mathrm{USp}(2)$ symmetry. We also give the A_1 tensors at each critical point:

$$\begin{aligned}
A_1^{(\mathrm{I})} &= -4g_1\mathbf{I}_{5\times5},\\
A_1^{(\mathrm{II})} &= \mathrm{diag}\left(\alpha,\alpha,\alpha,\alpha,\frac{4g_1(g_1-g_2)}{2g_1+g_2}\right).\\
A_1^{(\mathrm{III})} &= \mathrm{diag}\left(\beta,\beta,\beta,\beta,\frac{-4g_1(3g_1+g_2)}{2g_1+g_2}\right).
\end{aligned} \tag{3.7}$$

where

$$\alpha=\frac{-4g_1(g_1+g_2)}{2g_1+g_2},\qquad \beta=\frac{-4g_1(5g_1+g_2)}{2g_1+g_2}. \tag{3.8}$$

The scalar mass spectrum at the trivial critical point is given in the table below.

m^2L^2	$\mathrm{SO}(5)\times\mathrm{USp}(2)$
$-\frac{3}{4}$	$(\mathbf{4},\mathbf{2})$

All scalars have the same mass $m^2L^2=-\frac{3}{4}$ with L being the AdS_3 radius at this critical point. The full symmetry of the background corresponds to $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$ superconformal group. Notice that in finding critical points with constant scalars we can use the gauge symmetry and the composite $\mathrm{USp}(4)\times\mathrm{USp}(k)$ symmetry to fix the scalar parametrization as, for example, in the Euler angle parametrization. In determining scalar masses, we need to compute scalar fluctuations to quadratic order. In this case, only the the composite $\mathrm{USp}(4)\times\mathrm{USp}(k)$ symmetry can be used since the vector fields are set to

JHEP01(2014)159

zero, see the discussion in [24]. The scalar masses must accordingly be computed in the so-called unitary gauge with the coset representative

$$L = \prod_{i=1}^{8} e^{a_i Y^i}\,. \tag{3.9}$$

The mass spectrum at $(4,0)$ critical point is shown below.

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$

And, scalar masses at $(1,0)$ critical point are as follow.

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
$\frac{(4g_1+g_2)(10g_1+3g_2)}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$

Notice that there are seven massless Goldstone bosons corresponding to the symmetry breaking $\mathrm{SO}(5)\times\mathrm{USp}(2)\rightarrow\mathrm{USp}(2)\times\mathrm{USp}(2)$.

3.1.2 SO(4) × USp(2) gauging

We still use the same parametrization as in the previous case. The potential in this case turns out to be much more complicated although it dose not depend on a_1, a_2 and a_3. We give its explicit form in appendix C. The trivial critical point has $N=(4,1)$ supersymmetry and preserves the full $\mathrm{SO}(4)\times\mathrm{USp}(2)$ symmetry. The A_1 tensor and scalar masses at this point are given below.

$$A_1^{(\mathrm{I})} = -4g_1 \mathrm{diag}\,(1,1,1,1,-1)\,, \tag{3.10}$$

m^2L^2	$\mathrm{SO}(4)\times\mathrm{USp}(2)\sim\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{USp}(2)$
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{2})$

The corresponding superconformal symmetry is $\mathrm{Osp}(4|2,\mathbb{R})\times\mathrm{Osp}(1|2,\mathbb{R})$.

Other critical points with $a_4=a_5=a_6=a_7=0$ are shown in table 2. Critical points II and III preserve only $\mathrm{USp}(2)_{\mathrm{diag}}\times\mathrm{USp}(2)$ subgroup of $\mathrm{SO}(4)\times\mathrm{USp}(2)$. The $\mathrm{USp}(2)_{\mathrm{diag}}$ is a diagonal subgroup of one factor in $\mathrm{USp}(2)\times\mathrm{USp}(2)\sim\mathrm{SO}(4)$ and the $\mathrm{USp}(2)$ factor in the gauge group and is generated by J_1+J_{11},J_2+J_{12} and J_3+J_{13}. Critical point II has $(4,1)$ supersymmetry with the A_1 tensor

$$A_1^{(\mathrm{II})} = -\frac{4g_1(g_1+g_2)}{2g_1+g_2}\mathrm{diag}\,(1,1,1,1,-1)\,. \tag{3.11}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(4,1)$	SO(4) × USp(2)
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,1)$	USp(2) × USp(2)
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(0,0)$	USp(2) × USp(2)

Table 2. Critical points of SO(4) × USp(2) gauging.

The scalar mass spectrum is given in the table below.

m^2L^2	USp(2) × USp(2)
0	$(\mathbf{1},\mathbf{3})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
$-\frac{g_1g_2(g_1+2g_2)}{(g_1+g_2)^2(2g_1+g_2)}$	$(\mathbf{2},\mathbf{2})$

Critical point III is non-supersymmetric with scalar masses given by

m^2L^2	USp(2) × USp(2)
0	$(\mathbf{1},\mathbf{3})$
$\frac{(4g_1+g_2)(10g_1+3g_2)}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
$-\frac{g_1(4g_1+g_2)(5g_1+2g_2)}{(2g_1+g_2)(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2})$

.

We can now check its stability by comparing the above scalar masses with the Breitenlohner-Freedman bound $m^2L^2 \geq -1$. At this critical point, the value of b is real for $g_1 > 0$ and $g_2 > -2g_1$ or $g_1 < 0$ and $g_2 < -2g_1$. For definiteness, we will consider the first possibility. The mass of the singlet scalar satisfies the BF bound for $g_1 > 0$ and $g_2 > -3g_1$ while the mass of $(\mathbf{2},\mathbf{2})$ scalars requires $g_2 > 0.21432g_1$ for $g_1 > 0$ to satisfy to BF bound. Therefore, critical point III is stable for $g_1 > 0$ and $g_2 > 0.21432g_1$.

Note that both critical points II and III contain three massless scalars which are responsible for the symmetry breaking SO(4) × USp(2) → USp(2) × USp(2).

3.1.3 SO(3) × SO(2) × USp(2) gauging

Computing the scalar potential on the full 8-dimensional manifold turns out to be very complicated even with the Euler angle parametrization (3.4). In order to make things more tractable, we employ the technique introduced in [25] and consider a submanifold of USp(4, 2)/USp(4) × USp(2) invariant under $U(1)_{\text{diag}}$ symmetry generated by $T^{12} + T^{45}$. There are four singlets under this symmetry corresponding to the non-compact generators

$$X_1 = \frac{1}{\sqrt{2}}(Y^1 + Y^6), \qquad X_2 = \frac{1}{\sqrt{2}}(Y^2 + Y^8),$$
$$X_3 = \frac{1}{\sqrt{2}}(Y^4 - Y^3), \qquad X_4 = \frac{1}{\sqrt{2}}(Y^7 - Y^5). \tag{3.12}$$

	a_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(3,2)$	$\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(2)$
II	$\frac{1}{2}\ln\left[\frac{g_2-8g_1-4\sqrt{g_1(4g_1-g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1-g_2)^2}{g_2^2}$	$(2,0)$	$\mathrm{U}(1)\times\mathrm{U}(1)$
III	$\frac{1}{2}\ln\left[\frac{g_2+8g_1-4\sqrt{g_1(4g_1+g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{g_2^2}$	$(1,2)$	$\mathrm{U}(1)\times\mathrm{U}(1)$

Table 3. Critical points of $\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(2)$ gauging.

The coset representative can be parametrized by

$$L=e^{a_1X_1}e^{a_2X_2}e^{a_3X_3}e^{a_4X_4}. \tag{3.13}$$

The resulting potential is given by

$$\begin{aligned} V\ =\ &\frac{1}{128}\left[3+\cosh a_1\cosh a_2\cosh a_3\cosh a_4\right]\big[-2\left(512g_1^2+19g_2^2\right)\\ &+\left(99g_2^2-1024g_1^2\right)\cosh a_1\cosh a_2\cosh a_3\cosh a_4+3g_2^2\cosh(2a_1)\\ &\times(\cosh a_1\cosh a_2\cosh a_3\cosh a_4)-2-12g_2^2\cosh^2 a_1\left[\cosh(2a_2)\right.\\ &\left.+2\cosh^2 a_2\left(\cosh(2a_3)+2\cosh^2 a_3\cosh(2a_4)\right)\right]+2g_2^2\cosh^3 a_1\\ &\times\cosh a_2\cosh a_3\left(3\left(\cosh(2a_2)+2\cosh^2 a_2\cosh(2a_3)\right)\cosh a_4\right.\\ &\left.+4\cosh^2 a_2\cosh^2 a_3\cosh(3a_4)\right)\big]\,. \end{aligned} \tag{3.14}$$

We find critical points as shown in table 3. We have given only the value of a_1 since, at all critical points, the four scalars are related by $a_2=a_1$ and $a_3=a_4=0$. As usual, when all scalars vanish, we have a maximally supersymmetric point with $N=(3,2)$ and $\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(2)$ symmetry. The corresponding A_1 tensor is

$$A_1^{(\mathrm{I})}=-4g_1\mathrm{diag}\,(1,1,1,-1,-1)\,. \tag{3.15}$$

This background leads to the superconformal symmetry $\mathrm{Osp}(3|2,\mathbb{R})\times\mathrm{Osp}(2|2,\mathbb{R})$. The scalar masses at this point are shown below.

m^2L^2	$\mathrm{SO}(2)\times\mathrm{SO}(3)\times\mathrm{USp}(2)$
$-\frac{3}{4}$	$(1,\mathbf{2},\mathbf{2})+(-1,\mathbf{2},\mathbf{2})$

The other two critical points preserve $\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry. The corresponding A_1 tensor at these points is given by

$$\begin{aligned} A_1^{(\mathrm{II})}&==\mathrm{diag}\,(\alpha,\alpha,\beta,-\beta,-\beta)\,,\\ A_1^{(\mathrm{III})}&=\mathrm{diag}\,(\gamma,\gamma,-\delta,\delta,\delta) \end{aligned} \tag{3.16}$$

JHEP01(2014)159

where

$$\alpha = \frac{4g_1(g_1 - g_2)}{g_2}, \qquad \beta = -\frac{4g_1(g_2 - 3g_1)}{g_2},$$
$$\gamma = -\frac{4g_1(3g_1 + g_2)}{g_2}, \qquad \delta = \frac{4g_1(g_1 + g_2)}{g_2}. \tag{3.17}$$

With some normalization of the U(1) charges, the scalar mass spectra can be computed as shown in the tables below. The original four singlets under $U(1)_{\text{diag}}$ correspond to one massless and three massive modes in the tables. The $U(1)_{\text{diag}}$ is given by a combination of the two U(1)'s in the unbroken symmetry $U(1) \times U(1)$. Therefore, the $(0, \pm 4)$ and $(\pm 4, 0)$ modes, which are singlets under one of the two U(1)'s, will not be invariant under $U(1)_{\text{diag}}$.

- $(2, 0)$ point:

m^2L^2	$U(1) \times U(1)$
0	$(0, 4) + (0, -4) + (4, 0) + (-4, 0) + (0, 0)$
$\frac{32g_1^2 - 32g_1g_2 + 6g_2^2}{(g_1 - g_2)^2}$	$(0, 0)$
$-\frac{2g_1(g_1 - 2g_2)}{(g_1 - g_2)^2}$	$(-2, -2) + (2, 2)$

- $(1, 2)$ point:

m^2L^2	$U(1) \times U(1)$
0	$(0, 4) + (0, -4) + (4, 0) + (-4, 0) + (0, 0)$
$\frac{32g_1^2 + 32g_1g_2 + 6g_2^2}{(g_1 + g_2)^2}$	$(0, 0)$
$\frac{2g_1(3g_1 + 2g_2)}{(g_1 + g_2)^2}$	$(-2, -2) + (2, 2)$

3.2 The $k = 4$ case

We now consider a bigger scalar manifold $\frac{USp(4,4)}{USp(4) \times USp(4)}$. Compact gauge groups in this case are $SO(5) \times USp(4)$, $SO(4) \times USp(4)$ and $SO(3) \times SO(2) \times USp(4)$. Analyzing the potential on the full 16-dimensional manifold would be very complicated. We then choose a particular submanifold invariant under a certain subgroup of the gauge group and study the potential on this restricted scalar manifold as in the $SO(3) \times SO(2) \times USp(2)$ gauge group of the previous case. The procedure is parallel to that of the $k = 2$ case, so we will omit some irrelevant details particularly the explicit form of the A_1 tensor at each critical point.

3.2.1 $SO(5) \times USp(4)$ gauging

We use the parametrization of a submanifold invariant under $USp(2) \subset USp(4)$. There are eight singlets under this USp(2) symmetry corresponding to non-compact generators of $USp(4, 2) \subset USp(4, 4)$. With the Euler angle parametrization, we can write the coset representative as

$$L = e^{a_1\tilde{X}_1} e^{a_2\tilde{X}_2} e^{a_3\tilde{X}_3} e^{a_4K_1} e^{a_5K_2} e^{a_6K_3} e^{a_7K_4} e^{bY^8} \tag{3.18}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(5,0)$	$\mathrm{SO}(5)\times\mathrm{USp}(4)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)^3$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)^3$

Table 4. Critical points of $\mathrm{SO}(5)\times\mathrm{USp}(4)$ gauging.

where

$$\tilde{X}_1=\frac{1}{\sqrt{2}}(J_4-J_{11}),\qquad \tilde{X}_2=\frac{1}{\sqrt{2}}(J_5-J_{12}),\qquad \tilde{X}_3=\frac{1}{\sqrt{2}}(J_6-J_{13}),$$
$$K_1=J_{31},\qquad K_2=J_{32},\qquad K_3=J_{33},\qquad K_4=J_{36}\,. \tag{3.19}$$

The scalar potential turns out to be same as in (3.6). The critical points are shown in table 4. The critical points have the same structure as in the $k=2$ case but with bigger residual symmetry. The scalar mass spectra at each critical point are given in the tables below.

- $(5,0)$ point:

m^2L^2	$\mathrm{SO}(5)\times\mathrm{USp}(4)$
$-\frac{3}{4}$	$(\mathbf{4},\mathbf{4})$

- $(4,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{4g_1^2+8g_1g_2+3g_2^2}{4(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

- $(1,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{40g_1^2+22g_1g_2+3g_2^2}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3\left(12g_1^2+8g_1g_2+g_2^2\right)}{4(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

Notice that the number of massless Goldstone bosons agrees with the corresponding symmetry breaking in each case.

JHEP01(2014)159

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(4,1)$	SO(4) × USp(4)
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,1)$	USp(2)^3
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(0,0)$	USp(2)^3

Table 5. Critical points of SO(4) × USp(4) gauging.

3.2.2 SO(4) × USp(4) gauging

With the same coset representative, we find the same potential as shown in (C.1). The critical points with different unbroken symmetry are shown in table 5. The scalar mass spectra are given below.

- $(4, 1)$ point:

m^2L^2	SO(4) × USp(2) ∼ SU(2) × SU(2) × USp(4)
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{2},\mathbf{4})$

- $(4, 1)$ point:

m^2L^2	USp(2) × USp(2) × USp(2)
0	$(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{g_1g_2(g_1+2g_2)}{(g_1+g_2)^2(2g_1+g_2)}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$
$-\frac{(2g_1+g_2)(2g_1+3g_2)}{4(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2},\mathbf{1})$

- Non-supersymmetric point:

m^2L^2	USp(2) × USp(2) × USp(2)
0	$(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{40g_1^2+22g_1g_2+3g_2^2}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3(2g_1+g_2)(6g_1+g_2)}{4(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$
$-\frac{g_1\left(20g_1^2+13g_1g_2+2g_2^2\right)}{(2g_1+g_2)(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2},\mathbf{1})$

This critical point is stable for $g_1 > 0$ and $g_2 > 0.21432g_1$.

3.2.3 SO(3) × SO(2) × USp(4) gauging

In this case, we use the parametrization of L as in (3.13). The four scalars correspond to four singlets of USp(2) × U(1)$_{\text{diag}}$. The potential is the same as (3.14) with the critical points shown in table 6. The scalar mass spectra are given in the following tables.

JHEP01(2014)159

	a_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(3,2)$	$\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(4)$
II	$\frac{1}{2}\ln\left[\frac{g_2-8g_1-4\sqrt{g_1(4g_1-g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1-g_2)^2}{g_2^2}$	$(2,0)$	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
III	$\frac{1}{2}\ln\left[\frac{g_2+8g_1-4\sqrt{g_1(4g_1+g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{g_2^2}$	$(1,2)$	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$

Table 6. Critical points of $\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(4)$ gauging.

- $(3,2)$ point:

m^2L^2	$\mathrm{SO}(3)\times\mathrm{USp}(4)$
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{4})+(\mathbf{2},\mathbf{4})$

- $(2,0)$ point:

m^2L^2	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
0	$(4,0,\mathbf{1})+(-4,0,\mathbf{1})+(0,4,\mathbf{1})+(0,-4,\mathbf{1})+(0,0,\mathbf{1})$ $+(1,-1,\mathbf{2})+(-1,1,\mathbf{2})$
$\frac{32g_1^2-32g_1g_2+6g_2^2}{(g_1-g_2)^2}$	$(0,0,\mathbf{1})$
$-\frac{2g_1(g_1-2g_2)}{(g_1-g_2)^2}$	$(-2,-2,\mathbf{1})+(2,2,\mathbf{1})$
$-\frac{4g_1^2-8g_1g_2+3g_2^2}{4(g_1-g_2)^2}$	$(-1,-1,\mathbf{2})+(1,1,\mathbf{2})$

- $(1,2)$ point:

m^2L^2	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
0	$(4,0,\mathbf{1})+(-4,0,\mathbf{1})+(0,4,\mathbf{1})+(0,-4,\mathbf{1})+(0,0,\mathbf{1})$ $+(1,-1,\mathbf{2})+(-1,1,\mathbf{2})$
$\frac{32g_1^2+32g_1g_2+6g_2^2}{(g_1+g_2)^2}$	$(0,0,\mathbf{1})$
$-\frac{2g_1(3g_1+2g_2)}{(g_1+g_2)^2}$	$(-2,-2,\mathbf{1})+(2,2,\mathbf{1})$
$-\frac{4g_1^2+8g_1g_2+3g_2^2}{4(g_1+g_2)^2}$	$(-1,-1,\mathbf{2})+(1,1,\mathbf{2})$

That critical points in the $k=4$ case are similar to those in the $k=2$ case should be related to the fact that the theory with $\mathrm{USp}(4,2)/\mathrm{USp}(4)\times\mathrm{USp}(2)$ scalar manifold can be embedded in the theory with $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ scalar manifold. We have studied the potential on scalars which are singlets under $\mathrm{USp}(2)$. These singlets are precisely parametrized by non-compact directions of $\mathrm{USp}(4,2)\subset\mathrm{USp}(4,4)$, the global symmetry group of $k = 2$ case. This might explain the fact that this particular parametrization gives rise to the same potential as in the $k = 2$ case. Turning on more scalars would give more interesting structures.

4 Non-compact gauge groups

In this section, we classify admissible non-compact gauge groups. We will consider the $k = 2$ and $k = 4$ cases separately as in the previous section.

4.1 The $k = 2$ case

In this case, there is only one non-compact subgroup of USp(4, 2) namely USp(2, 2). The USp(4, 2) itself can be gauged with the embedding tensor given by its Killing form, but the corresponding potential will become a cosmological constant. The subgroup of USp(4, 2) that can be gauged is USp(2) × USp(2, 2) ⊂ USp(4, 2). The embedding tensor reads

$$\Theta = g_1\Theta_{\mathrm{USp}(2)} + g_2\Theta_{\mathrm{USp}(2,2)} \tag{4.1}$$

where g_1 and g_2 are two independent coupling constants. $\Theta_{\mathrm{USp}(2,2)}$ and $\Theta_{\mathrm{USp}(2)}$ are given by the Killing forms of USp(2, 2) and USp(2), respectively.

Generally, scalar fields corresponding to non-compact directions in the gauge group will drop out from the potential. Therefore, we do not need to include them in the coset representative. The remaining four scalars correspond to non-compact directions of another USp(2, 2) in USp(4, 2) and can be parametrized by the coset representative of USp(2, 2)/USp(2) × USp(2). We can use Euler angles of USp(2) × USp(2) to parametrize the coset representative as

$$L = e^{a_1X_1}e^{a_2X_2}e^{a_3X_3}e^{bY^7} \tag{4.2}$$

where X_i are given in (3.5). We find the following potential

$$\begin{aligned} V = \frac{1}{16}\Big[&8(g_1-g_2+(g_1+g_2)\cosh(b))^2\sinh^2 b \\ &-(3g_1+11g_2+4(g_1-g_2)\cosh b+(g_1+g_2)\cosh(2b))^2\Big]\,. \end{aligned} \tag{4.3}$$

Some of the critical points are shown in table 7. The A_1 tensor at each supersymmetric critical point is given by

$$\begin{aligned} A_1^{(\mathrm{I})} &= (g_1+g_2)\mathrm{diag}\,(-1,-1,-1,-1,1)\,, \\ A_1^{(\mathrm{II})} &= \mathrm{diag}\left(\beta,\beta,\beta,\beta,\frac{g_2(-2g_1+g_2)}{g_1+g_2}\right), \\ A_1^{(\mathrm{III})} &= \mathrm{diag}\left(\gamma,\gamma,\gamma,\gamma,-\frac{g_2(2g_1+3g_2)}{g_1+g_2}\right) \end{aligned} \tag{4.4}$$

where

$$\beta = -\frac{g_2(2g_1+g_2)}{g_1+g_2}, \qquad \gamma = -\frac{g_2(2g_1+5g_2)}{g_1+g_2}\,. \tag{4.5}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-4(g_1+g_2)^2$	$(4,1)$	$\mathrm{USp}(2)^3$
II	$\cosh^{-1}\left(\frac{g_2-g_1}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+g_2)^2}{(g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
III	$\cosh^{-1}\left(-\frac{g_1+3g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+3g_2)^2}{(g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
IV	$\ln(2+\sqrt{3})$	$-\frac{1}{4}(27g_1^2+54g_1g_2+19g_2^2)$	$(0,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$

Table 7. Critical points of $\mathrm{USp}(2)\times\mathrm{USp}(2,2)$ gauging.

Critical point I preserves $N=(4,1)$ supersymmetry. The gauge group is broken down to its maximal compact subgroup $\mathrm{USp}(2)^3$. In this symmetry breaking, the four massless Goldstone bosons correspond to scalars associated to non-compact generators of the gauge group. The full symmetry at this point gives the superconformal symmetry $\mathrm{Osp}(4|2,\mathbb{R})\times\mathrm{Osp}(1|2,\mathbb{R})$ since the supercharges transform under $\mathrm{USp}(2)\times\mathrm{USp}(2)\subset\mathrm{SO}(5)_R$ as $(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$.

Scalar mass spectra at all critical points are given below.

- $(4,1)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{2},\mathbf{2})$
$-\frac{g_1(g_1+2g_2)}{(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

- $(4,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{4g_1(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$

- $(1,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{4(g_1+2g_2)(3g_1+5g_2)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{1})$

- Non-supersymmetric point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{12(3g_1+g_2)(3g_1+5g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{1})$

At non-trivial critical points, there are additional three massless scalars which are responsible for $USp(2) \times USp(2) \to USp(2)_{\text{diag}}$ symmetry breaking. The non-supersymmetric critical point is stable for $g_2 > \frac{3}{79}(2\sqrt{210} - 45)g_1$.

4.2 The $k = 4$ case

There are three possible non-compact subgroups of $USp(4,4)$; $USp(2,2) \times USp(2,2)$, $USp(2) \times USp(4,2)$ and $USp(2) \times USp(2) \times USp(2,2)$. Only $USp(2,2) \times USp(2,2)$ can be gauged with the following embedding tensor

$$\Theta = g_1 \Theta_{USp(2,2)} + g_2 \Theta_{USp(2,2)} \, . \tag{4.6}$$

There are two independent coupling constants g_1 and g_2, and $\Theta_{USp(2,2)}$ is given by the Killing form of $USp(2,2)$. The relevant 8 scalars can be parametrized by $\left(\frac{USp(2,2)}{USp(2)\times USp(2)}\right)^2$ coset space with the two $USp(2,2)$ factors different from those appearing in the gauge group. With the Euler angle parametrization, the coset representative reads

$$L = e^{a_1 X_1} e^{a_2 X_2} e^{a_3 X_3} e^{b_1 Y^7} e^{a_4 X_4} e^{a_5 X_5} e^{a_6 X_6} e^{b_2 Y^{16}} \tag{4.7}$$

where

$$\begin{aligned}
X_1 &= \frac{1}{\sqrt{2}}(J_1 - J_{11}), \qquad & X_2 &= \frac{1}{\sqrt{2}}(J_2 - J_{12}), \qquad & X_3 &= \frac{1}{\sqrt{2}}(J_3 - J_{13}), \\
X_4 &= \frac{1}{\sqrt{2}}(J_4 - J_{22}), \qquad & X_5 &= \frac{1}{\sqrt{2}}(J_5 - J_{23}), \qquad & X_6 &= \frac{1}{\sqrt{2}}(J_6 - J_{24}).
\end{aligned} \tag{4.8}$$

The scalar potential is given by

$$\begin{aligned}
V = \frac{1}{16} \big[&(g_1 + g_2)(6 + \cosh(2b_1)) - (4(g_1 - g_2)\cosh b_1 + 4(g_2 - g_1)\cosh b_2 \\
&+ (g_1 + g_2)\cosh(2b_2))^2 + 8(g_1 - g_2 + (g_1 + g_2)\cosh(b_1))^2 \sinh^2 b_1 \\
&+ 8(g_2 - g_1 + (g_1 + g_2)\cosh b_2)^2 \sinh^2 b_2 \big] \, .
\end{aligned} \tag{4.9}$$

We find some critical points for $b_2 = 0$ as shown in table 8. Scalar masses at all critical points are given below.

- $(4, 1)$ point:

$m^2 L^2$	$USp(2) \times USp(2) \times USp(2) \times USp(2)$
0	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) + (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2})$
$-\frac{g_2(2g_1+g_2)}{(g_1+g_2)^2}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})$
$-\frac{g_1(g_1+2g_2)}{(g_1+g_2)^2}$	$(\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1})$

	b_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-4(g_1+g_2)^2$	$(4,1)$	$\text{USp}(2)^4$
II	$\cosh^{-1}\left(\frac{-g_1+g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+g_2)^2}{(g_1+g_2)^2}$	$(4,0)$	$\text{USp}(2)^3$
III	$\cosh^{-1}\left(\frac{-g_1-3g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+3g_2)^2}{(g_1+g_2)^2}$	$(1,0)$	$\text{USp}(2)^3$
IV	$\cosh^{-1}2$	$-\frac{1}{4}(27g_1^2+54g_1g_2+19g_2^2)$	$(0,0)$	$\text{USp}(2)^3$

Table 8. Critical points of $\text{USp}(2,2)\times\text{USp}(2,2)$ gauging.

- $(4,0)$ point:

m^2L^2	$\text{USp}(2)\times\text{USp}(2)\times\text{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{4g_1(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{(g_1+g_2)(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

- $(1,0)$ point:

m^2L^2	$\text{USp}(2)\times\text{USp}(2)\times\text{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{4(3g_1^2+11g_1g_2+10g_2^2)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3(g_1^2+4g_1g_2+3g_2^2)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

- Non-supersymmetry point:

m^2L^2	$\text{USp}(2)\times\text{USp}(2)\times\text{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{12(3g_1+g_2)(3g_1+5g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{24g_2(3g_1+g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

At the trivial critical point, the $\text{SO}(5)_R$ R-symmetry is broken to $\text{SU}(2)\times\text{SU}(2)\sim\text{USp}(2)\times\text{USp}(2)$. The $N=5$ supercharges transform under this subgroup as $(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$. This gives rise to $\text{Osp}(4|2,\mathbb{R})\times\text{Osp}(1|2,\mathbb{R})$ superconformal symmetry. As in the previous case, the non-supersymmetric point is stable for $g_2>\frac{3}{79}(2\sqrt{210}-45)g_1$.

5 RG flow solutions

Given some AdS_3 critical points form the previous sections, we now consider domain wall solutions interpolating between these critical points. The solutions can be interpreted as

RG flows describing a perturbed UV CFT flowing to another CFT in the IR. Since the structure of critical points in both $k = 2$ and $k = 4$ cases is similar, we will consider only the flows in $k = 2$ case to simplify the algebra. The study of holographic RG flows is very similar to those in other gauged supergravities in three dimensions [16–19]. In this paper, we will give only examples of RG flows in compact $SO(5)\times USp(2)$ and non-compact $USp(2,2) \times USp(2)$ gauge groups.

We are interested only in supersymmetric flows connecting two supersymmetric critical points. The solution can be found by solving BPS equations arising from supersymmetry transformations of fermions $\delta\psi^I_\mu$ and $\delta\chi^{iI}$ which, for convenience, we will repeat them here from [5]

$$\begin{aligned}\delta\psi^I_\mu &= \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J,\\ \delta\chi^{iI} &= \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i{}_j \not{D}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J\end{aligned} \tag{5.1}$$

where $\mathcal{D}_\mu\epsilon^I = \left(\partial_\mu + \frac{1}{2}\omega^a_\mu\gamma_a\right)\epsilon^I$ for vanishing vector fields.

We now employ the standard domain wall ansatz for the metric

$$ds^2 = e^{2A(r)}dx^2_{1,1} + dr^2\,. \tag{5.2}$$

In order to preserve Poincare symmetry in two dimensions, all fields involving in the flow can only depend on the radial coordinate r identified with an energy scale in the dual field theory. BPS equations give rise to first order flow equations describing the dependence of active scalars on r. It can be verified that setting some of the scalars to zero satisfies their flow equations. We can then neglect all scalars that vanish at both UV and IR points.

5.1 An RG flow between (5, 0) and (4, 0) CFT's in SO(5) × USp(2) gauging

The flow involves only one active scalar parametrized by the coset representative

$$L = e^{b(r)Y^7}\,. \tag{5.3}$$

The BPS equation from $\delta\chi^{iI} = 0$ gives rise to the flow equation

$$\frac{db}{dr} = [2g_1 - g_2 + (2g_1 + g_2)\cosh b]\sinh b \tag{5.4}$$

where we have used the projection condition $\gamma_r\epsilon^I = \epsilon^I$. It is clearly seen from the above equation that there are two critical points at $b = 0$ and $b = \cosh^{-1}\frac{g_2-2g_1}{2g_1+g_2}$. This equation can be solved for r as a function of b, and the solution is given by

$$\begin{aligned}r = \frac{1}{8g_1g_2}\Big[&4g_1\ln\cosh\frac{b}{2} - (2g_1 + g_2)\ln[2g_1 - g_2 + (2g_1 + g_2)\cosh b]\\ &+2g_2\ln\sinh\frac{b}{2}\Big]\,.\end{aligned} \tag{5.5}$$

The integration constant has been neglected since we can shift the coordinate r to remove it.

The variation $\delta\psi^I_\mu = 0$ gives another equation for $A(r)$

$$\frac{dA}{dr} = \frac{1}{4}\left[4g_2\cosh b - 22g_1 - 3g_2 - 8g_1\cosh b \right. \\ \left. -2g_1\cosh(2b) - g_2\cosh(2b)\right] \tag{5.6}$$

or, in term of b,

$$\frac{dA}{db} = -\frac{[22g_1 + 3g_2 + (8g_1 - 4g_2)\cosh b + (2g_1 + g_2)\cosh(2b)]\,\text{csch}b}{8g_1 - 4g_2 + 4(2g_1 + g_2)\cosh b}\,. \tag{5.7}$$

This equation is readily solved and gives A as a function of b

$$A = \frac{1}{g_2}\left[(g_1 + g_2)\ln\left[2g_1 - g_2 + (2g_1 + g_2)\cosh b\right] - (2g_1 + g_2)\ln\cosh\frac{b}{2} \right. \\ \left. -2g_2\ln\sinh\frac{b}{2}\right]. \tag{5.8}$$

The additive integration constant can be absorbed by scaling $x^{0,1}$ coordinates. It can be verified that equation $\delta\psi^I_r = 0$ gives the Killing spinors of the unbroken supersymmetry $\epsilon^I = e^{\frac{A}{2}}\epsilon^I_0$ as usual, with constant spinors ϵ^I_0 satisfying $\gamma_r\epsilon^I_0 = \epsilon^I_0$.

Linearizing equation (5.5) near the UV point $b \approx 0$, we find

$$b(r) \sim e^{4g_1 r} = e^{-\frac{r}{2L_{\text{UV}}}}, \qquad L_{\text{UV}} = \frac{1}{8|g_1|}. \tag{5.9}$$

We have set $g_1 < 0$ to identify $r \to \infty$ as the UV point. The above behavior indicates that from a general result, see for example [12], the flow is driven by a relevant operator of dimension $\Delta = \frac{3}{2}$.

Near the IR point, we find

$$b(r) \sim e^{-\frac{8g_1 g_2 r}{2g_1 + g_2}} = e^{\frac{g_2 r}{(g_1+g_2)L_{\text{IR}}}}, \qquad L_{\text{IR}} = -\frac{2g_1 + g_2}{8g_1(g_1 + g_2)} > 0\,. \tag{5.10}$$

The reality condition for b_{IR} requires $g_2 > -2g_1$ for $g_1 < 0$. From the above equation, we find $\frac{g_2}{g_2+g_1} > 0$, so in the IR the operator becomes irrelevant with dimension $\Delta_{\text{IR}} = \frac{3g_2+2g_2}{g_1+g_2}$. This value of Δ_{IR} precisely gives the correct mass square $m^2L^2_{\text{IR}} = \frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$ given before.

The ratio of the central charges is computed to be

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{L_{\text{UV}}}{L_{\text{IR}}} = \sqrt{\frac{V_{0IR}}{V_{0UV}}} = \frac{g_1 + g_2}{2g_1 + g_2} > 1 \tag{5.11}$$

satisfying the holographic c-theorem for $g_1 < 0$ and $g_2 > -2g_1$.

5.2 An RG flow between (5,0) and (1,0) CFT's in SO(5) × USp(2) gauging

We then study another RG flow interpolating between $(5,0)$ and $(1,0)$ critical points. The coset representative is sill given by (5.3). Similar to the previous case, we obtain the following flow equations

$$\begin{aligned} \frac{db}{dr} &= [6g_1 + g_2 - (2g_1 + g_2)\cosh b]\sinh b, \\ \frac{dA}{dr} &= \frac{1}{4}\left[3g_2 - 10g_1 - 4(6g_1 + g_2)\cosh b + (2g_1 + g_2)\cosh(2b)\right]. \end{aligned} \tag{5.12}$$

The first equation gives a solution

$$r = -\frac{1}{8g_1(4g_1+g_2)}\left[4g_1 \ln\cosh\frac{b}{2} + (2g_1+g_2)\ln\left[(2g_1+g_2)\cosh b \right.\right.$$
$$\left.\left. -6g_1-g_2\right] - 2(4g_1+g_2)\ln\sinh\frac{b}{2}\right]. \qquad (5.13)$$

We can rewrite the second equation of (5.12) as

$$\frac{dA}{db} = \frac{[10g_1 - 3g_2 + 4(6g_1+g_2)\cosh b - (2g_1+g_2)\cosh(2b)]\,\text{csch} b}{4(2g_1+g_2)\cosh b - 4(6g_1+g_2)} \qquad (5.14)$$

whose solution can be found to be

$$A = \frac{1}{4g_1+g_2}\left[(3g_1+g_2)\ln\left((2g_1+g_2)\cosh b - 6g_1 - g_2\right)\right.$$
$$\left. -(2g_1+g_2)\ln\cosh\frac{b}{2} - 2(4g_1+g_2)\ln\sinh\frac{b}{2}\right]. \qquad (5.15)$$

The fluctuation around $b = 0$ behaves as

$$b(r) \sim e^{4g_1 r} = e^{-\frac{r}{2L_{\text{UV}}}}, L_{\text{UV}} = \frac{1}{8|g_1|}. \qquad (5.16)$$

As in the previous case, we have chosen $g_1 < 0$ to make the UV point corresponds to $r \to \infty$. From the above equation, the flow is again driven by a relevant operator of dimension $\Delta_{\text{UV}} = \frac{3}{2}$. Near the IR point, $b(r)$ becomes

$$b(r) \sim e^{-\frac{8g_1(4g_1+g_2)r}{2g_1+g_2}} = e^{\frac{(4g_1+g_2)r}{(3g_1+g_2)L_{\text{IR}}}}, \qquad L_{\text{IR}} = -\frac{2g_1+g_2}{8g_1(3g_1+g_2)}. \qquad (5.17)$$

We can verify that b_{IR} is real for $g_1 < 0$ and $g_2 < -2g_1$, the operator becomes irrelevant in the IR with dimension $\Delta_{\text{IR}} = \frac{10g_1+3g_2}{3g_1+g_2}$. The ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{3g_1+g_2}{2g_1+g_2} > 1, \qquad \text{for} \qquad g_1 < 0 \text{ and } g_2 < -2g_1\,. \qquad (5.18)$$

5.3 An RG flow between (4, 1) and (4, 0) CFT's in USp(2) × USp(2, 2) gauging

We next consider RG flows between critical points of non-compact USp(2) × USp(2, 2) gauge group. We will not give a non-supersymmetric flow to critical point IV in table 7 in this paper. It can be studied in the same procedure as [26] and [27]. Like in the compact case, it is consistent to truncate the full scalar manifold to a single scalar parametrized by

$$L = e^{b(r)Y^7}. \qquad (5.19)$$

The variation $\delta\chi^{iI} = 0$ gives

$$\frac{db}{dr} = (g_1 - g_2 + (g_1+g_2)\cosh b)\sinh b \qquad (5.20)$$

which is solved by the solution

$$r = \frac{1}{4g_1g_2}\left[2g_2 \ln\sinh\frac{b}{2} + 2g_1 \ln\cosh\frac{b}{2} \right.$$
$$\left. - (g_1+g_2)\ln\left[g_1 - g_2 + (g_1+g_2)\cosh b\right]\right]. \tag{5.21}$$

The equation from $\delta\psi^I_\mu = 0$ reads

$$\frac{dA}{dr} = -2\left[g_2 + g_1\cosh^4\frac{b}{2} + g_2\sinh^4\frac{b}{2}\right]. \tag{5.22}$$

The solution for A as a function of b can be found as in the previous cases. The result is given by

$$A = \frac{1}{2g_1}\left[(2g_1+g_2)\ln\left[g_1 - g_2 + (g_1+g_2)\cosh b\right] - 4g_1\ln\cosh\frac{b}{2}\right.$$
$$\left. -2(g_1+g_2)\ln\sinh\frac{b}{2}\right]. \tag{5.23}$$

Near the UV point, the b solution becomes

$$b(r) \sim e^{2g_1 r} = e^{\frac{g_1 r}{(g_1+g_2)L_{\mathrm{UV}}}}, \qquad L_{\mathrm{UV}} = \frac{1}{2(g_1+g_2)}. \tag{5.24}$$

b_{IR} is real for $g_1 < 0$ and $g_2 > -g_1$. With this range, $-\frac{g_1}{g_1+g_2} < 1$. The flow is then driven by a relevant operator of dimension $\Delta = \frac{3g_1+2g_2}{g_1+g_2} < 2$. At the IR point, we find the asymptotic behavior

$$b(r) \sim e^{-\frac{4g_1g_2 r}{g_1+g_2}} = e^{\frac{2g_2 r}{|2g_1+g_2|L_{\mathrm{IR}}}}, \qquad L_{\mathrm{IR}} = \frac{g_1+g_2}{2|g_1(2g_1+g_2)|} \tag{5.25}$$

corresponding to an irrelevant operator of dimension $\Delta = \frac{2g_2}{|2g_1+g2|} + 2$.

Finally, the ratio of the central charges is given by

$$\frac{c_{\mathrm{UV}}}{c_{\mathrm{IR}}} = \frac{|g_1(2g_1+g_2)|}{(g_1+g_2)^2}. \tag{5.26}$$

5.4 An RG flow between $(4,1)$ and $(1,0)$ CFT's in USp(2) $\times$ USp(2, 2) gauging

As a final flow solution, we quickly investigate a solution interpolating between $(4,1)$ and $(1,0)$ critical points. The flow equations are given by

$$\frac{db}{dr} = -\left[g_1 + 3g_2 + (g_1+g_2)\cosh b\right]\sinh b, \tag{5.27}$$
$$\frac{dA}{dr} = \frac{1}{4}\left[3g_1 - 5g_2 + 4(g_1+3g_2)\cosh b + (g_1+g_2)\cosh(2b)\right]. \tag{5.28}$$

The corresponding solutions take the form

$$r = -\frac{1}{4g_2(g_1+2g_2)}\Bigg[(g_1+g_2)\ln\left[g_1+3g_2+(g_1+g_2)\cosh b\right] \\ +2g_2\ln\sinh\frac{b}{2}-2(g_1+2g_2)\ln\cosh\frac{b}{2}\Bigg], \tag{5.29}$$

$$A = \frac{1}{2(g_1+2g_2)}\Bigg[(2g_1+3g_2)\ln\left[g_1+3g_2+(g_1+g_2)\cosh b\right] \\ -4(g_1+2g_2)\ln\cosh\frac{b}{2}-2(g_1+g_2)\ln\sinh\frac{b}{2}\Bigg]. \tag{5.30}$$

The fluctuations near the UV and IR points are given by

$$b(r)\sim e^{-2(g_1+2g_2)r} = e^{\frac{(g_1+2g_2)r}{(g_1+g_2)L_{\text{UV}}}}, \qquad L_{\text{UV}} = -\frac{1}{2(g_1+g_2)}, \tag{5.31}$$

$$b(r)\sim e^{-\frac{4g_2(g_1+2g_2)r}{g_1+g_2}} = e^{\frac{2g_2(g_1+2g_2)r}{|g_1(2g_1+3g2)|L_{\text{IR}}}}, \qquad L_{\text{IR}} = -\frac{(g_1+g_2)}{2|g_1(2g_1+3g_2)|}. \tag{5.32}$$

We have chosen a particular range of g_1 and g_2 namely $g_1<0$ and $-\frac{g_1}{2}<g_2<-g_1$ for which $g_1+g_2<0$. The flow is driven by a relevant operator of dimension $\Delta = \frac{3g_1+4g_2}{g_1+g_2}$. In the IR, the operator becomes irrelevant with dimension $\Delta = \frac{2g_2}{|2g_1+g2|}+2$.

The ratio of the central charges for this flow is

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{|g_1(2g_1+3g_2)|}{(g_1+g_2)^2}. \tag{5.33}$$

6 $N=5$, $SO(5)\ltimes \mathbf{T}^{10}$ gauged supergravity

In this section, we consider non-semisimple gauge groups in the form of $G_0\ltimes\mathbf{T}^{\dim G_0}$ in which G_0 is a semisimple group. $\mathbf{T}^{\dim G_0}$ constitutes a translational symmetry with $\dim G_0$ commuting generators transforming in the adjoint representation of G_0. We consider the $k=4$ case with $USp(4,4)$ global symmetry that admits a non-semisimple subgroup $SO(5)\ltimes\mathbf{T}^{10}$.

A general embedding of $G_0\ltimes\mathbf{T}^{\dim G_0}$ group is described by the embedding tensor of the form [6]

$$\Theta = g_1\Theta_{\text{ab}}+g_2\Theta_{\text{bb}}. \tag{6.1}$$

We have used the notation of [6] in denoting the semisimple and translational parts by a and b, respectively. The absence of aa coupling plays a key role in the equivalence of this theory and the Yang-Mills gauged supergravity with G_0 gauge group.

The next task is to identify $SO(5)\ltimes\mathbf{T}^{10}$ generators. The semisimple $SO(5)$ is identified with the diagonal subgroup of $SO(5)\times SO(5)\sim USp(4)\times USp(4)\subset USp(4,4)$. The corresponding generators are given by

$$J^{ij} = T^{ij}+\tilde{T}^{ij}, \qquad i,j=1,2,\ldots,5. \tag{6.2}$$

T^{ij} are the $SO(5)$ R-symmetry generators, and $\tilde{T}^{ij}$ are generators of $USp(4)$. The translational generators are constructed from a combination of $T^{ij}-\tilde{T}^{ij}$ and non-compact generators. The 16 scalars transform as $(\mathbf{4},\mathbf{4})$ under $\mathrm{SO}(5)\times\mathrm{SO}(5)$. They accordingly transform as $\mathbf{1}+\mathbf{5}+\mathbf{10}$ under $\mathrm{SO}(5)_{\mathrm{diag}}$. Scalars in the $\mathbf{10}$ representation will be part of the $\mathbf{T}^{10}$ generators which are given by

$$t^{ij}=T^{ij}-\tilde{T}^{ij}+\tilde{Y}^{ij},\qquad i,j=1,2,\ldots,5\,. \tag{6.3}$$

The explicit form of $\tilde{T}^{ij}$ and $\tilde{Y}^{ij}$ is given in appendix B.

In the present case, supersymmetry allows for any value of g_1 and g_2. Therefore, the embedding tensor contains two independent coupling constants. We begin with the scalar potential computed on the $\mathrm{SO}(5)_{\mathrm{diag}}$ singlet scalar. The above decomposition gives one singlet under this SO(5). We end up with a simple coset representative

$$L=e^{a(Y^7+Y^{16})}\,. \tag{6.4}$$

This results in the potential

$$V=-64g_1e^{-3a}\left(3e^a g_1+2g_2\right)\,. \tag{6.5}$$

The existence of a maximally supersymmetric critical point at $L=\mathbf{I}$ requires $g_2=-g_1$. This is the same as in $N=4,8$ gauged supergravities [28, 29]. With this condition and g_1 denoted by g, the potential becomes

$$V=-64g^2e^{-3a}\left(3e^a-2\right)\,. \tag{6.6}$$

Clearly, the only one critical point is given by $a=0$ with $V_0=-64g^2$ and $N=(5,0)$ supersymmetry. This critical point is a minimum of the potential as can be seen from figure 1. The vacuum is very similar to the AdS_3 vacuum found in $N=16$, $\mathrm{SO}(4)\times\mathrm{SO}(4)\ltimes(\mathbf{T}^{12},\hat{\mathbf{T}}^{34})$ gauged supergravity studied in [30]. The singlet has a positive mass square $m^2L^2=3$ as expected for a minimum point. In the dual CFT with superconformal symmetry $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$, this scalar corresponds to an irrelevant operator of dimension $\Delta=3$. The full scalar masses are given below.

m^2L^2	SO(5)
3	**1**
3	**5**
0	**10**

The ten massless scalars accompany for the symmetry breaking $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}\to\mathrm{SO}(5)$ at the vacuum.

To find other critical points, we reduce the residual symmetry of the scalar submanifold to $\mathrm{SO}(3)\subset\mathrm{SO}(5)$ under which the 16 scalars transform as $(\mathbf{2}+\mathbf{2})\times(\mathbf{2}+\mathbf{2})=4\times(\mathbf{1}+\mathbf{3})$. There are four singlets which can be parametrized by the coset representative

$$L=e^{a_1Y^4}e^{a_2Y^7}e^{a_3Y^9}e^{a_4Y^{16}}\,. \tag{6.7}$$

The resulting potential turns out to be very complicated. We, therefore, will not attempt to do the analysis of this potential in the present work.

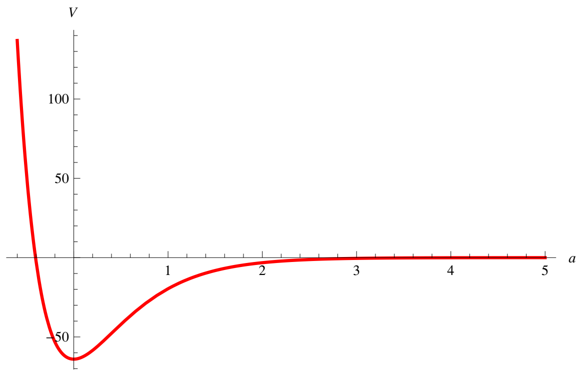

Figure 1. The scalar potential of $N = 5$, $SO(5) \ltimes \mathbf{T}^{10}$ gauged supergravity for SO(5) singlet scalar with $g = 1$.

7 $N = 6$, $SO(6) \ltimes \mathbf{T}^{15}$ gauged supergravity

In this section, we consider non-semisimple gauge groups of $N = 6$ theory. Compact and non-compact gauge groups in this theory together with their vacua and holographic RG flows have been studied in [19].

We are interested in $N = 6$ gauged supergravity with $\frac{SU(4,4)}{S(U(4)\times U(4))}$ scalar manifold. Most of our conventions here are parallel to those used in [19]. The global symmetry $SU(4,4)$ contains a non-semisimple subgroup $SO(6) \ltimes \mathbf{T}^{15}$. Similar to $N = 5$ theory, the $SO(6)$ part is given by the diagonal subgroup of $SO(6)\times SO(6) \sim SU(4)\times SU(4) \subset SU(4,4)$. The 32 scalars transform as $(\mathbf{4},\bar{\mathbf{4}}) + (\bar{\mathbf{4}},\mathbf{4})$ under $SU(4) \times SU(4)$. Under $SO(6)_{\text{diag}}$, they transform as

$$(\mathbf{4} \times \bar{\mathbf{4}}) + (\bar{\mathbf{4}} \times \mathbf{4}) = \mathbf{1} + \mathbf{15} + \mathbf{1} + \mathbf{15}. \tag{7.1}$$

The adjoint representations $\mathbf{15}$'s will be used to construct the translational generators $\mathbf{T}^{15}$. The full $SO(6) \ltimes \mathbf{T}^{15}$ generators are given in appendix B.

The embedding tensor is still given by (6.1), but in this case, the linear constraint $\mathbb{P}_{R_0}\Theta = 0$ requires $g_2 = 0$ similar to $N = 16, 10, 8$ theories [3, 21, 31]. The above decomposition gives two singlet scalars under SO(6) part of the gauge group. They correspond to non-compact generators

$$\begin{aligned} Y_{s1} &= \frac{1}{2}(Y^1 + Y^{11} + Y^{21} + Y^{31}), \\ Y_{s2} &= \frac{1}{2}(Y^2 + Y^{12} + Y^{22} + Y^{32}). \end{aligned} \tag{7.2}$$

Accordingly, the coset representative can be parametrized by

$$L = e^{\sqrt{2}b_1 Y_{s1}} e^{\sqrt{2}b_2 Y_{s2}} \tag{7.3}$$

where we have chosen a particular normalization for later convenience. The potential is, with $g = g_1$, given by

$$V = -224g^2 \left(\cosh b_1 \cosh b_2 - \sinh b_2\right)^2 . \tag{7.4}$$

The above potential does not admit any critical points, so the vacuum should be a half-supersymmetric domain wall. In the rest of this section, we will find this domain wall solution.

The supersymmetry transformations $\delta\psi^I_\mu$ and $\delta\chi^{iI}$ together with the domain wall ansatz (5.2) give rise to the following BPS equations

$$b_1' = 8g\text{sech}b_2 \sinh b_1, \tag{7.5}$$

$$b_2' = -8g\left(\cosh b_2 - \cosh b_1 \sinh b_2\right), \tag{7.6}$$

$$A' = -16g\left(\cosh b_1 \cosh b_2 - \sinh b_2\right) \tag{7.7}$$

where $'$ denotes $\frac{d}{dr}$. Equation (7.5) is readily solved by setting $b_1 = 0$. Equation (7.6) now becomes

$$b_2' = -8ge^{-b_2}. \tag{7.8}$$

The solution is given by

$$b_2 = \ln\left(-8gr + c_1\right) \tag{7.9}$$

where c_1 is an integration constant. With $b_1 = 0$ and b_2 given by (7.9), equation (7.7) becomes

$$A' = \frac{-16g}{c_1 - 8gr} \tag{7.10}$$

whose solution is easily found to be

$$A = 2\ln\left(-8gr + c_1\right) + c_2 \tag{7.11}$$

with another integration constant c_2. The two integration constants are not relevant because we can shift the coordinate r rescale $x^{0,1}$ to remove them. As in other domain wall solutions, the metric can be written in the form of a warped AdS_3 as

$$ds^2 = \frac{1}{(8g)^4\rho^2}\left(\frac{dx^2_{1,1} + d\rho^2}{\rho^2}\right) \tag{7.12}$$

where $\rho = -\frac{1}{(8g)^2 r}$.

8 Conclusions and discussions

In this paper, we have classified compact and non-compact gauge groups of $N = 5$ gauged supergravity in three dimensions with USp(4, 2)/USp(4) × USp(2) and USp(4, 4)/USp(4) × USp(4) scalar manifolds. We have also identified a number of supersymmetric AdS_3 vacua in each gauging and studied some examples of supersymmetric RG flows interpolating between these vacua in both compact and non-compact gauge groups. All of the solutions can be analytically found, and the flows describe deformations by relevant operators. They would be useful to the study of AdS_3/CFT_2 correspondence such as the computation of correlation functions in the dual field theory similar to that studied in [32].

Among our main results, we have constructed $N = 5$, $SO(5) \ltimes \mathbf{T}^{10}$ gauged supergravity. The theory is equivalent to $N = 5$ Yang-Mills gauged supergravity and could be obtained from $S^1/\mathbb{Z}_2$ reduction of $N = 5$ gauged supergravity in four dimensions as pointed out in [21]. The theory admits a maximally supersymmetric AdS_3 vacuum which should be dual to a superconformal field theory with $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$ superconformal symmetry. We have also given all of the scalar masses at this vacuum. It is interesting to further study the scalar potential of this theory in order to find other critical points as well as the associated RG flow solutions. This could give some insight to the deformations in the dual CFT.

Similar construction has then been extended to $N = 6$ gauged supergravity with $\mathrm{SU}(4,4)/S(\mathrm{U}(4)\times\mathrm{U}(4))$ scalar manifold. The resulting theory is $N = 6$ gauged supergravity with $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ gauge group. Like $N = 5$ theory, this is equivalent to SO(6) Yang-Mills gauged supergravity and should be obtained from $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions. This has also been pointed out in [21] in which the spectrum of the S^1 reduction of four dimensional $N = 6$ gauged supergravity has been given. The theory admits a half-supersymmetric domain wall vacuum rather than a maximally supersymmetric AdS_3. We have also given the domain wall solution. This solution provides another example of domain walls in three dimensional gauged supergravity similar to the solutions of [21, 31] and might be useful in the study of DW/QFT correspondence.

The above non-semisimple gaugings are of importance for embedding the theories in higher dimensions. With the full embedding at hand, any solutions in a three dimensional framework, which are usually easier to find than higher dimensional ones, can be uplifted to string/M theory in which a full geometrical interpretation can be made. Other attempts to embed Chern-Simons gauged supergravities in three dimensions can be found in [28–30, 33–35]. In many cases, the precise reduction ansatz from ten or eleven dimensions remains to be done.

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A Useful formulae

For conveniences, we collect useful formulae used throughout this paper. The detailed discussion can be found in [5]. All of our discussions involve symmetric scalar manifolds of the form G/H. The G generators are denoted by $t^{\mathcal{M}} = (T^{IJ}, T^\alpha, Y^A)$ in which T^{IJ} and T^α are $\mathrm{SO}(N)\times H'$ generators and Y^A are non-compact generators. In the present cases, we have $H' = \mathrm{USp}(k)$ for $N = 5$ and $H' = \mathrm{U}(k)$ for $N = 6$ theories, respectively. $\mathrm{SO}(N)$ is the R-symmetry.

The coset manifold, consisting of d scalars ϕ^i, $i = 1,\ldots,d = \dim(G/H)$, can be described by a coset representative L transforming by left- and right-multiplications of G

and H. Some useful relations are given by

$$L^{-1}t^{\mathcal{M}}L = \frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}T^{IJ} + \mathcal{V}^{\mathcal{M}}{}_{\alpha}T^{\alpha} + \mathcal{V}^{\mathcal{M}}{}_{A}Y^{A}, \tag{A.1}$$

$$L^{-1}\partial_i L = \frac{1}{2}Q_i^{IJ}T^{IJ} + Q_i^{\alpha}T^{\alpha} + e_i^{A}Y^{A}. \tag{A.2}$$

The first relation gives scalar matrices $\mathcal{V}$ used in defining a moment map while the second gives SO(N) $\times$ H' composite connections, Q^{IJ} and Q^{α}, and the vielbein on the manifold G/H, e_i^A. Accordingly, the metric on the scalar manifold is defined by

$$g_{ij} = e_i^A e_j^B \delta_{AB}, \qquad i, j, A, B = 1, \ldots, d. \tag{A.3}$$

The embedding tensor determines the fermionic mass-like terms and the scalar potential via the T-tensor defined by

$$T_{\mathcal{A}\mathcal{B}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}. \tag{A.4}$$

In the above equation, $\mathcal{A}$ and $\mathcal{B}$ label SO(N) $\times$ H' representations.

The A_1^{IJ} and A_{2i}^{IJ} tensors appearing in the fermionic supersymmetry transformations and the scalar potential are given in terms of linear combinations of various components of $T_{\mathcal{A}\mathcal{B}}$ by the following relations

$$\begin{aligned} A_1^{IJ} &= -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \\ A_{2j}^{IJ} &= \frac{2}{N}T^{IJ}{}_{j} + \frac{4}{N(N-2)}f^{M(Im}_{\ \ j}T^{J)M}{}_{m} + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f^{KL\ \ m}_{\ j}T^{KL}{}_{m}. \end{aligned} \tag{A.5}$$

The f_{ij}^{IJ} tensor can be constructed from SO(N) gamma matrices or from the SO(N) generators in a spinor representation. In the present case, it is given in a flat basis by

$$f_{AB}^{IJ} = -2\text{Tr}(Y^B\left[T^{IJ}, Y^A\right]). \tag{A.6}$$

The scalar potential can be computed from

$$V = -\frac{4}{N}\left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}\right). \tag{A.7}$$

We end this section by noting the condition for unbroken supersymmetry. The associated Killing spinors correspond to the eigenvectors of A_1^{IJ} with eigenvalues $\pm\sqrt{-\frac{V_0}{4}}$.

B Relevant generators

In this appendix, we give generators of various groups used throughout the paper.

B.1 $N = 5$ theory

J_i's are USp(8) generators written in terms of generalized Gell-Mann matrices λ_i generating the SU(8) group. They are explicitly given by

$$
\begin{aligned}
J_1 &= \frac{i\lambda_1}{\sqrt{2}}, & J_2 &= \frac{i\lambda_2}{\sqrt{2}}, & J_3 &= \frac{i\lambda_3}{\sqrt{2}}, \\
J_4 &= \frac{i\lambda_{13}}{\sqrt{2}}, & J_5 &= \frac{i\lambda_{14}}{\sqrt{2}}, & J_6 &= -\frac{i\lambda_8}{\sqrt{6}} + \frac{i\lambda_{15}}{\sqrt{3}}, \\
J_7 &= \frac{i\lambda_6}{2} + \frac{i\lambda_9}{2}, & J_8 &= -\frac{i\lambda_7}{2} + \frac{i\lambda_{10}}{2}, & J_9 &= \frac{i\lambda_4}{2} - \frac{i\lambda_{11}}{2}, \\
J_{10} &= -\frac{i\lambda_5}{2} - \frac{i\lambda_{12}}{2}, & J_{11} &= \frac{i\lambda_{33}}{\sqrt{2}}, & J_{12} &= \frac{i\lambda_{34}}{\sqrt{2}}, \\
J_{13} &= -\frac{i\lambda_{24}}{\sqrt{5}} + \sqrt{\frac{3}{10}}\, i\lambda_{35}, & J_{14} &= \frac{i\lambda_{18}}{2} + \frac{i\lambda_{25}}{2}, & J_{15} &= -\frac{i\lambda_{19}}{2} + \frac{i\lambda_{26}}{2}, \\
J_{16} &= \frac{i\lambda_{16}}{2} - \frac{i\lambda_{27}}{2}, & J_{17} &= \frac{i\lambda_{22}}{2} + \frac{i\lambda_{29}}{2}, & J_{18} &= -\frac{i\lambda_{23}}{2} + \frac{i\lambda_{30}}{2}, \\
J_{19} &= \frac{i\lambda_{20}}{2} - \frac{i\lambda_{31}}{2}, & J_{20} &= -\frac{i\lambda_{17}}{2} - \frac{i\lambda_{28}}{2}, & J_{21} &= -\frac{i\lambda_{21}}{2} - \frac{i\lambda_{32}}{2}, \\
J_{22} &= \frac{i\lambda_{61}}{\sqrt{2}}, & J_{23} &= \frac{i\lambda_{62}}{\sqrt{2}}, & J_{24} &= -\sqrt{\frac{3}{14}}\, i\lambda_{48} + \sqrt{\frac{2}{7}}\, i\lambda_{63}, \\
J_{25} &= \frac{i\lambda_{38}}{2} + \frac{i\lambda_{49}}{2}, & J_{26} &= -\frac{i\lambda_{39}}{2} + \frac{i\lambda_{50}}{2}, & J_{27} &= \frac{i\lambda_{36}}{2} - \frac{i\lambda_{51}}{2}, \\
J_{28} &= \frac{i\lambda_{42}}{2} + \frac{i\lambda_{53}}{2}, & J_{29} &= -\frac{i\lambda_{43}}{2} + \frac{i\lambda_{54}}{2}, & J_{30} &= \frac{i\lambda_{40}}{2} - \frac{i\lambda_{55}}{2}, \\
J_{31} &= \frac{i\lambda_{46}}{2} + \frac{i\lambda_{57}}{2}, & J_{32} &= -\frac{i\lambda_{47}}{2} + \frac{i\lambda_{58}}{2}, & J_{33} &= \frac{i\lambda_{44}}{2} - \frac{i\lambda_{59}}{2}, \\
J_{34} &= -\frac{i\lambda_{37}}{2} - \frac{i\lambda_{52}}{2}, & J_{35} &= -\frac{i\lambda_{41}}{2} - \frac{i\lambda_{56}}{2}, & J_{36} &= -\frac{i\lambda_{45}}{2} - \frac{i\lambda_{60}}{2}.
\end{aligned}
\tag{B.1}
$$

The USp(6) generators needed for constructing USp(4, 2) are given by the first 21 generators.

The $SO(5) \ltimes T^{10}$ generators are constructed as follow. The $SO(5)_{\text{diag}}$ is generated by $T^{ij} + \tilde{T}^{ij}$ in which

$$
\begin{aligned}
\tilde{T}^{12} &= \frac{1}{\sqrt{2}}\left(J_{13} - J_{24}\right), & \tilde{T}^{13} &= -\frac{1}{\sqrt{2}}\left(J_{11} + J_{22}\right), & \tilde{T}^{23} &= \frac{1}{\sqrt{2}}\left(J_{12} - J_{23}\right), \\
\tilde{T}^{34} &= \frac{1}{\sqrt{2}}\left(J_{13} + J_{24}\right), & \tilde{T}^{14} &= \frac{1}{\sqrt{2}}\left(J_{12} + J_{23}\right), & \tilde{T}^{24} &= \frac{1}{\sqrt{2}}\left(J_{11} - J_{22}\right), \\
\tilde{T}^{45} &= J_{31}, & \tilde{T}^{15} &= -J_{33}, & \tilde{T}^{25} &= -J_{36}, \\
\tilde{T}^{35} &= J_{32} \quad .
\end{aligned}
\tag{B.2}
$$

Generators $\tilde{Y}^{ij}$ in $\mathbf{T}^{10}$ are given by

$$
\begin{aligned}
\tilde{Y}^{12} &= i(J_{16} - J_{30}), & \tilde{Y}^{13} &= -i(J_{14} + J_{28}), & \tilde{Y}^{23} &= i(J_{15} + J_{29}), \\
\tilde{Y}^{34} &= i(J_{16} + J_{30}), & \tilde{Y}^{14} &= i(J_{15} + J_{29}), & \tilde{Y}^{24} &= i(J_{14} - J_{28}), \\
\tilde{Y}^{45} &= i(J_{17} + J_{25}), & \tilde{Y}^{15} &= -i(J_{19} + J_{27}), & \tilde{Y}^{25} &= i(J_{21} - J_{34}), \\
\tilde{Y}^{35} &= i(J_{18} + J_{26}).
\end{aligned}
\tag{B.3}
$$

B.2 $N = 6$ theory

For conveniences, we repeat non-compact generators of SU(4, 4) in terms of generalized Gell-Mann matrices, λ_i, $i = 1, \ldots, 63$, given in [19]

$$\bar{Y}^A = \begin{cases} \frac{1}{\sqrt{2}} c_{A+15}, & A = 1, \ldots, 8 \\ \frac{1}{\sqrt{2}} c_{A+16}, & A = 9, \ldots, 16 \\ \frac{1}{\sqrt{2}} c_{A+19}, & A = 17, \ldots, 24 \\ \frac{1}{\sqrt{2}} c_{A+24}, & A = 25, \ldots, 32 \end{cases} . \tag{B.4}$$

The $\mathrm{SO}(6)_R$ R-symmetry generators are identified to be

$$\begin{aligned}
&\bar{T}^{12} = \frac{1}{2}c_3 + \frac{1}{2\sqrt{3}}c_8 - \frac{1}{\sqrt{6}}c_{15}, && \bar{T}^{13} = -\frac{1}{2}(c_2 + c_{14}), && \bar{T}^{23} = \frac{1}{2}(c_1 - c_{13}), \\
&\bar{T}^{34} = \frac{1}{2}c_3 - \frac{1}{2\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, && \bar{T}^{14} = \frac{1}{2}(c_1 + c_{13}), && \bar{T}^{35} = -\frac{1}{2}(c_6 + c_9), \\
&\bar{T}^{56} = \frac{1}{\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, && \bar{T}^{36} = -\frac{1}{2}(c_7 + c_{10}), && \bar{T}^{24} = \frac{1}{2}(c_2 - c_{14}), \\
&\bar{T}^{45} = \frac{1}{2}(c_7 - c_{10}), && \bar{T}^{46} = \frac{1}{2}(c_9 - c_6), && \bar{T}^{15} = \frac{1}{2}(c_4 - c_{11}), \\
&\bar{T}^{16} = \frac{1}{2}(c_5 - c_{12}), && \bar{T}^{25} = \frac{1}{2}(c_5 + c_{12}), && \bar{T}^{26} = -\frac{1}{2}(c_4 + c_{11})
\end{aligned} \tag{B.5}$$

where $c_i = -i\lambda_i$.

The $\mathrm{SO}(6) \ltimes T^{15}$ generators are given by

$$\begin{aligned}
\mathrm{SO}(6):& \quad J_{\mathrm{a}}^{ij} = \bar{T}^{ij} + \tilde{\bar{T}}^{ij}, \qquad i, j = 1, \ldots, 6 \\
\mathbf{T}^{15}:& \quad J_{\mathrm{b}}^{ij} = \bar{T}^{ij} - \tilde{\bar{T}}^{ij} + \tilde{\bar{Y}}^{ij}
\end{aligned} \tag{B.6}$$

where

$$\begin{aligned}
\tilde{\bar{T}}^{12} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} - \sqrt{\frac{3}{20}}\lambda_{35} - \sqrt{\frac{3}{28}}\lambda_{48} + \frac{1}{\sqrt{7}}\lambda_{63}\right), \\
\tilde{\bar{T}}^{34} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} - \sqrt{\frac{3}{20}}\lambda_{35} + \sqrt{\frac{3}{28}}\lambda_{48} - \frac{1}{\sqrt{7}}\lambda_{63}\right), \\
\tilde{\bar{T}}^{56} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} + \frac{1}{\sqrt{15}}\lambda_{35} - \frac{2}{\sqrt{21}}\lambda_{48} - \frac{1}{\sqrt{7}}\lambda_{63}\right),
\end{aligned}$$

$$\begin{aligned}
&\tilde{\bar{T}}^{13} = \frac{i}{2}(\lambda_{34} + \lambda_{62}), && \tilde{\bar{T}}^{23} = -\frac{i}{2}(\lambda_{33} - \lambda_{61}), && \tilde{\bar{T}}^{14} = -\frac{i}{2}(\lambda_{33} + \lambda_{61}), \\
&\tilde{\bar{T}}^{24} = \frac{i}{2}(\lambda_{62} - \lambda_{34}), && \tilde{\bar{T}}^{45} = \frac{i}{2}(\lambda_{58} - \lambda_{47}), && \tilde{\bar{T}}^{15} = \frac{i}{2}(\lambda_{59} - \lambda_{44}), \\
&\tilde{\bar{T}}^{25} = -\frac{i}{2}(\lambda_{45} + \lambda_{60}), && \tilde{\bar{T}}^{35} = \frac{i}{2}(\lambda_{46} + \lambda_{57}), && \tilde{\bar{T}}^{16} = \frac{i}{2}(\lambda_{60} - \lambda_{45}), \\
&\tilde{\bar{T}}^{26} = \frac{i}{2}(\lambda_{44} + \lambda_{59}), && \tilde{\bar{T}}^{36} = \frac{i}{2}(\lambda_{47} + \lambda_{58}), && \tilde{\bar{T}}^{46} = \frac{i}{2}(\lambda_{46} - \lambda_{57})
\end{aligned} \tag{B.7}$$

and

$$\tilde{\tilde{Y}}^{12} = -\frac{1}{2}(\lambda_{27} - \lambda_{16} + \lambda_{40} - \lambda_{55}), \qquad \tilde{\tilde{Y}}^{34} = -\frac{1}{2}(\lambda_{55} - \lambda_{16} + \lambda_{27} - \lambda_{40}),$$
$$\tilde{\tilde{Y}}^{56} = -\frac{1}{2}(\lambda_{55} - \lambda_{16} - \lambda_{27} + \lambda_{40}), \qquad \tilde{\tilde{Y}}^{13} = -\frac{1}{2}(\lambda_{54} - \lambda_{19} + \lambda_{26} - \lambda_{43}),$$
$$\tilde{\tilde{Y}}^{23} = -\frac{1}{2}(\lambda_{53} - \lambda_{18} - \lambda_{25} + \lambda_{42}), \qquad \tilde{\tilde{Y}}^{14} = \frac{1}{2}(\lambda_{18} + \lambda_{25} + \lambda_{42} + \lambda_{53}),$$
$$\tilde{\tilde{Y}}^{24} = -\frac{1}{2}(\lambda_{19} - \lambda_{26} - \lambda_{43} + \lambda_{54}), \qquad \tilde{\tilde{Y}}^{45} = -\frac{1}{2}(\lambda_{50} - \lambda_{23} + \lambda_{30} - \lambda_{39}),$$
$$\tilde{\tilde{Y}}^{15} = -\frac{1}{2}(\lambda_{31} - \lambda_{20} - \lambda_{36} + \lambda_{51}), \qquad \tilde{\tilde{Y}}^{25} = -\frac{1}{2}(\lambda_{21} + \lambda_{32} - \lambda_{37} - \lambda_{52}),$$
$$\tilde{\tilde{Y}}^{35} = -\frac{1}{2}(\lambda_{22} + \lambda_{29} + \lambda_{38} + \lambda_{49}), \qquad \tilde{\tilde{Y}}^{16} = -\frac{1}{2}(\lambda_{21} - \lambda_{32} - \lambda_{37} + \lambda_{52}),$$
$$\tilde{\tilde{Y}}^{26} = -\frac{1}{2}(\lambda_{20} + \lambda_{31} + \lambda_{36} + \lambda_{51}), \qquad \tilde{\tilde{Y}}^{36} = -\frac{1}{2}(\lambda_{50} - \lambda_{23} - \lambda_{30} + \lambda_{39}),$$
$$\tilde{\tilde{Y}}^{46} = -\frac{1}{2}(\lambda_{29} - \lambda_{22} + \lambda_{38} - \lambda_{49}). \tag{B.8}$$

C Scalar potential for SO(4) × USp(2) gauging

The scalar potential for compact gauge group SO(4) × USp(2) is given by

$$\begin{aligned}
V =\ & 2g_2^2(3+\cosh b)\sinh^6\frac{b}{2} + \frac{1}{16}g_1 g_2\,[68 + 4\cos(2a_4) + 2\cos(2(a_4 - a_5)) \\
& +4\cos(2a_5) + 2\cos(2(a_4+a_5)) + 2\cos(2(a_4 - a_6)) + \cos(2(a_4 - a_5 - a_6)) \\
& +2\cos(2(a_5 - a_6)) + \cos(2(a_4 + a_5 - a_6)) + 4\cos(2a_6) + 2\cos(2(a_4 + a_6)) \\
& +\cos(2(a_4 - a_5 + a_6)) + 2\cos(2(a_5 + a_6)) + \cos(2(a_4 + a_5 + a_6)) \\
& +32\cos^2 a_4\cos^2 a_5\cos^2 a_6\cos(2a_7)]\,(3+\cosh b)\sinh^6\frac{b}{2} \\
& -4g_1^2\Big[\cos^2 a_5\cos^2 a_6\cos^2 a_7\cosh^2\frac{b}{2}(3+\cosh b)^2\sin^2(2a_4) \\
& +64\cos^2 a_4\cosh^4\frac{b}{2}\sin^2 a_4\sin^2 a_5 + 64\cos^2 a_4\cos^2 a_5\cosh^4\frac{b}{2} \\
& \times\sin^2 a_4\sin^2 a_6 + 64\cos^2 a_4\cos^2 a_5\cos^2 a_6\cosh^4\frac{b}{2}\sin^2 a_4\sin^2 a_7 \\
& +\frac{1}{16384}\Big[51 + 259\cos(2a_4) + 4(-17+63\cos(2a_4))\cosh b + (17+\cos(2a_4)) \\
& \quad\times\cosh(2b) + 16\cos^2 a_4\cos(2a_5)\sinh^4\frac{b}{2} + 32\cos^2 a_4\cos^2 a_5\cos(2a_6)\sinh^4\frac{b}{2} \\
& \quad+64\cos^2 a_4\cos^2 a_5\cos^2 a_6\cos(2a_7)\sinh^4\frac{b}{2}\Big]^2 + \frac{1}{2}\Big[-4\cos^4 a_4\cos^2 a_5\cos^2 a_6 \\
& \times\cos^2 a_7\sin^2 a_5\sinh^6\frac{b}{2} - 4\cos^4 a_4\cos^4 a_5\cos^2 a_6\cos^2 a_7\sin^2 a_6\sinh^6\frac{b}{2} \\
& -4\cos^4 a_4\cos^4 a_5\cos^4 a_6\cos^2 a_7\sin^2 a_7\sinh^6\frac{b}{2} - 4\sin^2(2a_4)\sin^2 a_5\sinh^2 b
\end{aligned}$$

JHEP01(2014)159

$$
\begin{aligned}
&-16\cos^2 a_4 \cos^2 a_5 \sin^2 a_4 \sin^2 a_6 \sinh^2 b - 16\cos^2 a_4 \cos^2 a_5 \cos^2 a_6 \sin^2 a_4 \\
&\times \sin^2 a_7 \sinh^2 b - \frac{1}{16}\cos^2 a_5 \cos^2 a_6 \cos^2 a_7 \sin^2(2a_4)\left[7\sinh\frac{b}{2} + 3\sinh\frac{3b}{2}\right]^2 \\
&-\frac{1}{4096}\Bigg[16\cos^2 a_4\left[\cos(2a_5) + 2\cos^2 a_5\left(\cos(2a_6) + 2\cos^2 a_6 \cos(2a_7)\right)\right] \\
&\quad \times \cosh\frac{b}{2}\sinh^3\frac{b}{2} + 2[63\cos(2a_4) + 17\cosh b - 17]\sinh b \\
&\quad + \cos(2a_4)\sinh(2b)\Bigg]^2\Bigg]\Bigg]. \qquad (C.1)
\end{aligned}
$$

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