Guan and Liu *Advances in Difference Equations* (2016) 2016:52
DOI 10.1186/s13662-016-0763-0

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Finite-time H_∞ memory state feedback control for uncertain singular TS fuzzy time-delay system under actuator saturation

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Abstract

In this paper, the problem of finite-time H_∞ memory feedback control for singular T-S fuzzy systems is addressed. Conditions are obtained to guarantee that the closed-loop system is finite-time bounded with a prescribed H_∞ performance γ. The considered memory controller can be obtained by solving the LMIs. In addition, the estimation of the largest domain of attraction of the closed-loop system can be solved by solving an optimization problem. Finally, examples illustrate the feasibility of the proposed method.

Keywords: finite time; memory state feedback control; H_∞ control; singular TS fuzzy system; actuator saturation

1 Introduction

In recent years, there has been growing attention to singular systems, because of their extensive applications in many practical systems, for example, electrical circuits, power systems, networks, and other systems [1, 2]. Time delays are frequently encountered in various engineering systems such as aircraft, chemical processes, economics, networks, communication, and biological systems. It has been shown that the existence of time delays is often one of the main causes of instability and poor performance in a system. Therefore, the singular time-delay system has received considerable attention [1–4].

T-S fuzzy models have been widely studied because they can represent a wide class of nonlinear systems, especially the singular T-S fuzzy model. Many valuable stability analysis and control synthesis results for singular T-S fuzzy systems can be available, for example, memory dissipative control, memory H_∞ control, and H_∞ filters were studied in [5–7], respectively. In [8], the problem of delay-dependent dissipative control was discussed for a class of nonlinear system via a descriptor T-S fuzzy model.

In practice, actuator saturation is very ubiquitous, which is a main cause of poor performance of the closed-loop systems and sometimes it may lead to the system being unstable [9–11]. Robust H_∞ static output feedback stabilization and robust stabilization for T-S fuzzy system subject to actuator saturation were discussed in [9] and [10], respectively. Yang and Tong [11] put forward the problem of output feedback robust stabilization of switched fuzzy systems with time delay and actuator saturation. For a singular T-S fuzzy system subject to actuator saturation, the reader may refer to [5, 6]. Control of time-delay

fuzzy descriptor systems with actuator saturation was demonstrated in [12]. Furthermore, an external disturbance is often the source of instability and poor performance of systems. The H_∞ control technique is used to minimize the effects of the external disturbances, H_∞ control for fuzzy systems is addressed in [3, 4, 6, 7, 9].

In addition, a memory state feedback controller with input constraints yields less conservative sufficient conditions in terms of LMIs and allows for a wider feasible region of numerical optimization [13]. In [14], a new stabilization condition for T-S fuzzy systems with time delay was obtained by the memory state feedback controller. In [15], memory state feedback control for singular systems with multiple internal incommensurate constant point delays was demonstrated. In [16], analysis and synthesis of memory-based fuzzy sliding mode controllers were discussed.

In some practical engineering applications, the finite-time control is of practical significance. If the system state does not exceed a prescribed region during a fixed time interval, it is said to have finite-time stability (*FTB*). It is well recognized that finite-time stability is different from Lyapunov asymptotical stability [17–20]. For a singular system, there are few articles considering finite-time control. See [21], Observer-based finite-time H_∞ control for discrete singular stochastic systems was discussed. Ma *et al.* discussed the problem of finite-time H_∞ control for a class of discrete-time switched singular time-delay systems subject to actuator saturation in [22]. For switched singular linear system, Wang *et al.* used an average dwell time approach to study the problem of finite-time stabilization in [23]. However, so far, for singular T-S fuzzy time-delay system subject to actuator saturation, one has an open area for the study of finite-time control.

Motivated by the above discussion, in this paper, the problem of finite-time H_∞ memory feedback control for a singular T-S fuzzy system is demonstrated. The main contributions of this paper can be listed as follows: (1) conditions are obtained to guarantee that the closed-loop system is not only regular, impulse-free, finite-time bounded but also satisfying the presided H_∞ performance γ; (2) the considered memory controller can be obtained by solving the LMIs; (3) the estimation of the largest domain of attraction of the closed-loop system can be solved by an optimization problem; (4) examples illustrate the feasibility of the proposed method; (5) the domain of attraction is simulated in Figure 2.

Notation Throughout this paper, R^n denotes the n-dimensional Euclidean space, and $\mathrm{R}^{n\times m}$ is the set of real matrices. For $A \in \mathrm{R}^{n\times m}$, $A^{-1}$and A^{T}denote the matrix inverse and matrix transpose, respectively. $\lambda(A)$ means the eigenvalue of A. For a real symmetric matrix $A \in \mathrm{R}^{n\times n}$, $A > 0$ ($A \geq 0$) means that A is positive definite (positive semi-definite). The symbol $*$ means the symmetric term in a symmetric matrix.

2 Preliminaries

Consider the following singular TS fuzzy model:

Plant rule i: IF $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is $M_{i2} \cdots \theta_p(t)$ is M_{ip}, THEN

$$\begin{aligned} E\dot{x}(t) &= \bar{A}_i x(t) + \bar{A}_{di} x\bigl(t-d(t)\bigr) + \bar{B}_i \operatorname{sat}\bigl(u(t)\bigr) + \bar{B}_{\omega i}\omega(t), \\ z(t) &= \bar{C}_i x(t) + \bar{C}_{di} x\bigl(t-d(t)\bigr) + \bar{D}_i \operatorname{sat}\bigl(u(t)\bigr) + \bar{D}_{\omega i}\omega(t), \\ x(t) &= \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{1}$$

where $\theta(t) = [\theta_1(t)\ \theta_2(t)\ \cdots\ \theta_p(t)]^{\mathrm{T}}$ is premise variable, $i \in \Re := \{1, 2, \ldots, r\}$, r is the number of IF-THEN rules, M_{ik} $(i = 1, 2, \ldots, r,\ k = 1, 2, \ldots, p)$ is the fuzzy set. $x(t) \in \mathrm{R}^n$ is the state vector, $\omega(t) \in \mathrm{R}^q$ is the disturbance input which belongs to $L_2[0, \infty)$. $z(t) \in \mathrm{R}^p$ is the control output, $\phi(t)$ is the initial condition of the system. $d(t)$ is a time-varying continuous function that satisfies $0 \le d(t) \le d$ and $\dot{d}(t) \le h$, $h < 1$. E is a constant matrix satisfying $\operatorname{rank}(E) \le n$. $u(t) \in \mathrm{R}^l$ is the control input, and $\operatorname{sat} : \mathrm{R}^l \to \mathrm{R}^l$ is the standard saturation function defined as follows:

$$\operatorname{sat}\bigl(u(t)\bigr) = \bigl[\operatorname{sat}\bigl(u_1(t)\bigr), \ldots, \operatorname{sat}\bigl(u_l(t)\bigr)\bigr]^{\mathrm{T}},$$

without loss of generality, $\operatorname{sat}(u_i(t)) = \operatorname{sign}(u_i(t)) \min\{1, |u_i(t)|\}$. Here the notation of $\operatorname{sat}(\cdot)$ is abused to denote the scalar values and the vector valued saturation functions. For a positive scalar b and time scalar T, $\int_0^T \omega^{\mathrm{T}}(t)\omega(t) \le b$. $\bar{A}_i = A_i + \Delta A_i$, $\bar{A}_{di} = A_{di} + \Delta A_{di}$, $\bar{B}_i = B_i + \Delta B_i$, $\bar{B}_{\omega i} = B_{\omega i} + \Delta B_{\omega i}$, $\bar{C}_i = C_i + \Delta C_i$, $\bar{C}_{di} = C_{di} + \Delta C_{di}$, $\bar{D}_i = D_i + \Delta D_i$, $\bar{D}_{\omega i} = D_{\omega i} + \Delta D_{\omega i}$. A_i, A_{di}, B_i, $B_{\omega i}$, C_i, C_{di}, D_i, $D_{\omega i}$ are known real constant matrices with appropriate dimensions; ΔA_i, ΔA_{di}, ΔB_i, $\Delta B_{\omega i}$, ΔC_i, ΔC_{di}, ΔD_i, $\Delta D_{\omega i}$ are unknown matrices representing norm-bounded parametric uncertainties and are assumed to be of the form

$$\begin{bmatrix} \Delta A_i & \Delta A_{di} & \Delta B_i & \Delta B_{\omega i} \\ \Delta C_i & \Delta C_{di} & \Delta D_i & \Delta D_{\omega i} \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} \Delta [E_{1i} \quad E_{2i} \quad E_{3i} \quad E_{4i}], \tag{2}$$

where H_{1i}, H_{2i}, E_{1i}, E_{2i}, E_{3i}, E_{4i} are known real constant matrices with appropriate dimensions and Δ is for unknown real and possibly time-varying matrices satisfying $\Delta^{\mathrm{T}}\Delta \le I$.

Using a singleton fuzzifier, product inference, and a center-average defuzzifier, the global dynamics of the TS system (1) is described by the convex sum form:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r} h_i\bigl(\theta(t)\bigr)\bigl[\bar{A}_i x(t) + \bar{A}_{di} x\bigl(t - d(t)\bigr) + \bar{B}_i \operatorname{sat}\bigl(u(t)\bigr) + \bar{B}_{\omega i}\omega(t)\bigr], \\ z(t) &= \sum_{i=1}^{r} h_i\bigl(\theta(t)\bigr)\bigl[\bar{C}_i x(t) + \bar{C}_{di} x\bigl(t - d(t)\bigr) + \bar{D}_i \operatorname{sat}\bigl(u(t)\bigr) + \bar{D}_{\omega i}\omega(t)\bigr], \\ x(t) &= \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{3}$$

where $h_i(\theta(t)) = \beta_i(\theta(t))/\sum_{i=1}^{r} \beta_i(\theta(t))$, $\beta_i(\theta(t)) = \prod_{j=1}^{p} M_{ij}(\theta_j(t))$, and $M_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in M_{ij}. It is easy to see that $\beta_i(\theta(t)) \ge 0$ and $\sum_{i=1}^{r} \beta_i(\theta(t)) \ge 0$. Hence, we have $h_i(\theta(t)) \ge 0$ and $\sum_{i=1}^{r} h_i(\theta(t)) = 1$. In the sequel, for brevity we use h_i to denote $h_i(\theta(t))$.

Consider the memory state feedback fuzzy controller:

$$u(t) = \sum_{i=1}^{r} h_i\bigl(\theta(t)\bigr)\bigl[K_i x(t) + K_{di} x\bigl(t - d(t)\bigr)\bigr], \tag{4}$$

where the memoryless state feedback gain K_i and the memory state feedback gain K_{di} are matrices to be determined with appropriate dimensions.

Define the following subsets of R^n.

Let $P \in \mathrm{R}^{n\times n}$ be a symmetric matrix, ρ be a scalar. Denote

$$\varepsilon\bigl(E^{\mathrm{T}}PE,\rho\bigr) = \bigl\{x(t) \in \mathrm{R}^n : x^{\mathrm{T}}(t)E^{\mathrm{T}}PEx(t) \le \rho\bigr\}.$$

For matrices H_i, H_{di}, h_{ik}, h_{dik} are the kth row of the matrix H_i and H_{di}, respectively, we define

$$L(H_i,H_{di}) = \bigl\{x(t) \in \mathrm{R}^n : \bigl|h_{ik}x(t) + h_{dik}x\bigl(t-\tau(t)\bigr)\bigr| \le 1, k \in [1,l]\bigr\}.$$

Thus $\varepsilon(E^{\mathrm{T}}PE,\rho)$ is an ellipsoid and $L(H_i,H_{di})$ is a polyhedral consisting of states for which the saturation does not occur.

Let D be the set of $l \times l$ diagonal matrices whose diagonal elements are either 1 or 0. Suppose each element of D is labeled as E_s, $s = 1,2,\ldots,\eta = 2^l$, and denote $E_s^- = I - E_s$. Clearly, if $E_s \in D$, then $E_s^- \in D$.

Lemma 1 ([24]) *Let $F,H \in \mathrm{R}^{p\times n}$. Then for any $x(t) \in L(H)$,*

$$\mathrm{sat}\bigl(Fx(t)\bigr) \in \mathrm{co}\bigl\{E_sFx(t) + E_s^-Hx(t), s = 1,2,\ldots,\eta\bigr\};$$

or, equivalently,

$$\mathrm{sat}\bigl(Fx(t)\bigr) = \sum_{s=1}^{\eta}\alpha_s\bigl(E_sF + E_s^-H\bigr)x(t),$$

where co *stands for the convex hull, α_s for $s = 1,2,\ldots,\eta$ are some scalars which satisfy $0 \le \alpha_s \le 1$ and $\sum_{s=1}^{\eta}\alpha_s = 1$.*

Lemma 2 *For any constant matrices $N_1,N_2 \in \mathrm{R}^{n\times n}$, $L \in \mathrm{R}^{n\times p}$, positive-definite symmetric matrix $Z \in \mathrm{R}^{n\times n}$, and time-varying delay $d(t)$, we have*

$$-\int_{t-d(t)}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}ZE\dot{x}(s)\,ds \le \xi^{\mathrm{T}}(t)\bigl\{\Pi + d(t)Y^{\mathrm{T}}Z^{-1}Y\bigr\}\xi(t), \tag{5}$$

where

$$Y = [N_1 \quad N_2 \quad L], \qquad \xi^{\mathrm{T}}(t) = \bigl[x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-d(t)) \quad \omega^{\mathrm{T}}(t)\bigr],$$

$$\Pi = \begin{bmatrix} N_1^{\mathrm{T}}E + E^{\mathrm{T}}N_1 & -N_1^{\mathrm{T}}E - E^{\mathrm{T}}N_1 & E^{\mathrm{T}}L \\ * & -N_2^{\mathrm{T}}E - E^{\mathrm{T}}N_2 & -E^{\mathrm{T}}L \\ * & * & 0 \end{bmatrix}.$$

Proof Let $C = \left[\begin{smallmatrix} Z^{1/2} & Z^{1/2}Y \\ 0 & 0 \end{smallmatrix}\right]$, then $C^{\mathrm{T}}C = \left[\begin{smallmatrix} Z & Y \\ Y^{\mathrm{T}} & Y^{\mathrm{T}}Z^{-1}Y \end{smallmatrix}\right] \ge 0$. It follows that

$$\int_{t-d(t)}^{t}\begin{bmatrix} E\dot{x}(s) \\ \xi(t) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} Z & Y \\ Y^{\mathrm{T}} & Y^{\mathrm{T}}Z^{-1}Y \end{bmatrix}\begin{bmatrix} E\dot{x}(s) \\ \xi(t) \end{bmatrix}ds \ge 0. \tag{6}$$

Notice that $2\int_{t-d(t)}^{t}\xi^{\mathrm{T}}(t)Y\dot{x}(s)\,ds = 2\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}[E \ -E \ 0]\xi(t)$, rearranging (6) yield (5). □

Remark 1 Lemma 2 will play a key role in decreasing the conservatism, which can be seen from Example 1.

Lemma 3 ([25]) *Let* Υ, Γ, *and* Λ *be real matrices of appropriate dimensions with* Λ *satisfying* $\Lambda\Lambda^{\mathrm{T}} \leq I$. *Then the following inequality holds for any constant* $\varepsilon > 0$:

$$\Upsilon\Lambda\Gamma + \Gamma^{\mathrm{T}}\Lambda^{\mathrm{T}}\Upsilon^{\mathrm{T}} \leq \varepsilon\Upsilon\Upsilon^{\mathrm{T}} + \varepsilon^{-1}\Gamma^{\mathrm{T}}\Gamma.$$

Lemma 4 ([26]) *For given matrices* $E, X > 0$, Y, *if* $(E^{\mathrm{T}}X + Y\Gamma^{\mathrm{T}})$ *is nonsingular, then there exist matrices* $S > 0$, I, *such that* $ES + \mathrm{IK}^{\mathrm{T}} = (E^{\mathrm{T}}X + Y\Gamma^{\mathrm{T}})^{-1}$, *where* $X, S \in \mathrm{R}^{n\times n}$, $Y, \mathrm{I} \in R^{n\times(n-r)}$, *and* $\Gamma, \mathrm{K} \in R^{n\times(n-r)}$ *are any matrices with full column rank satisfying* $E^{\mathrm{T}}\Gamma = 0$, $E\mathrm{K} = 0$.

From Lemma 1, for any $x(t) \in L(H_j, H_{dj})$, denoting $\lambda_{sj} = E_sK_j + E_s^{-}H_j$, and $\lambda_{dsj} = E_sK_{dj} + E_s^{-}H_{dj}$, then

$$\operatorname{sat}\bigl(K_jx(t) + K_{dj}x\bigl(t - d(t)\bigr)\bigr) = \sum_{s=1}^{\eta}\alpha_s\bigl(\lambda_{sj}x(t) + \lambda_{dsj}x\bigl(t - d(t)\bigr)\bigr), \tag{7}$$

then the closed-loop system can be obtained

$$\begin{aligned} &E\dot{x}(t) = \tilde{A}x(t) + \tilde{A}_dx\bigl(t - d(t)\bigr) + \tilde{B}_\omega\omega(t), \\ &z(t) = \tilde{C}x(t) + \tilde{C}_dx\bigl(t - d(t)\bigr) + \tilde{D}_\omega\omega(t), \\ &x(t) = \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{8}$$

where

$$\begin{aligned} &\tilde{A} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{\eta}h_ih_jh_s(\bar{A}_i + \bar{B}_i\lambda_{sj}), \qquad \tilde{A}_d = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{\eta}h_ih_jh_s(\bar{A}_{di} + \bar{B}_i\lambda_{dsj}), \\ &\tilde{C} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{\eta}h_ih_jh_s(\bar{C}_i + \bar{D}_i\lambda_{sj}), \qquad \tilde{C}_d = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{\eta}h_ih_jh_s(\bar{C}_{di} + \bar{D}_i\lambda_{dsj}), \\ &\tilde{B}_\omega = \sum_{i=1}^{r}h_i\bar{B}_{\omega i}, \qquad \tilde{D}_\omega = \sum_{i=1}^{r}h_i\bar{D}_{\omega i}. \end{aligned}$$

Definition 1 ([18]) For some positive constants, c_1, b, T and symmetric positive matrix R_c, the closed-loop system (8) is finite-time bounded *FTB* subject to $(c_1\ c_2\ b\ T\ R_c)$, if there exists scalar $c_2 > c_1$, such that

$$\begin{aligned} &\sup_{-d\leq\theta\leq 0}\bigl\{x^{\mathrm{T}}(\theta)E^{\mathrm{T}}R_cEx(\theta), \dot{x}^{\mathrm{T}}(\theta)E^{\mathrm{T}}R_cE\dot{x}(\theta)\bigr\} \leq c_1 \\ &\quad\Rightarrow\quad x^{\mathrm{T}}(t)E^{\mathrm{T}}R_cEx(t) \leq c_2, \quad \forall t_0 \in [-d, 0], t \in [0, T]. \end{aligned}$$

Definition 2 ([18]) For some positive constants, c_1, b, T and symmetric positive matrix R_c, the closed-loop system (8) is finite-time bounded (FTH_∞B) subject to $(c_1\ c_2\ b\ T\ R_c)$, if

(8) is *FTB* with respect to $(c_1\ c_2\ b\ T\ R_c)$ and under the zero-initial condition such that

$$\int_0^T z^{\mathrm{T}}(t)z(t)\,dt < \gamma^2 \int_0^T \omega^{\mathrm{T}}(t)\omega(t)\,dt. \tag{9}$$

Definition 3 ([27])

(i) When $\omega(t) = 0$, the continuous-time SMJS (8) is said to be regular in time interval $[0, T]$, if the characteristic polynomial $\det(sE - \tilde{A})$ is not identically zero for all $t \in [0, T]$.

(ii) When $\omega(t) = 0$, the continuous-time SMJS (8) is said to be impulse-free in time interval $[0, T]$, if $\deg(\det(sE - \tilde{A})) = \operatorname{rank}(E)$ for all $t \in [0, T]$.

3 Main results

Theorem 1 *For positive constants c_1, b, T, δ and positive definite matrix R_c, the closed-loop system* (8) *is FTB subject to $(c_1\ c_2\ b\ T\ R_c)$ at the origin with $\varepsilon(E^{\mathrm{T}}PE, \rho)$ contained in the domain of attraction, if there exist a constant $c_2 > 0$, positive definite matrices P, Q_1, Q_2 and any matrices N_1, N_2, L with appropriate dimensions, matrix S for $i,j \in \Re$, and $\varepsilon(E^{\mathrm{T}}PE, \rho) \subset L(H_i, H_{di})$ such that*

$$\begin{bmatrix} \Theta + \Pi & \Gamma_1^{\mathrm{T}} & dY^{\mathrm{T}} \\ * & -(dQ_2)^{-1} & 0 \\ * & * & -Q_2 \end{bmatrix} < 0, \tag{10}$$

$$\left(\lambda_2 + \lambda_3 d + \lambda_4 \frac{d^2}{2}\right)c_1 + \lambda_5 b < \lambda_1 c_2 e^{-\delta T}, \tag{11}$$

where Π, Y are defined in Lemma 2, *and*

$$\Theta = \begin{bmatrix} \varpi_{11} & P\tilde{A}_d & P\tilde{B}_\omega \\ * & -(1-h)Q_1 & 0 \\ * & * & -Q \end{bmatrix},$$

$$\varpi_{11} = \hat{P}^{\mathrm{T}}\tilde{A} + \tilde{A}^{\mathrm{T}}\hat{P} - \delta E^{\mathrm{T}}\hat{P} + Q_1, \qquad \Gamma_1 = [\tilde{A} \quad \tilde{A}_d \quad \tilde{B}_\omega],$$

$$\lambda_1 = \lambda_{\min}(\bar{P}), \qquad \lambda_2 = \lambda_{\max}(\bar{P}), \qquad \lambda_3 = \lambda_{\max}(\bar{Q}_1), \qquad \lambda_4 = \lambda_{\max}(\bar{Q}_2),$$

$$\lambda_5 = \lambda_{\max}(Q), \qquad \hat{P} = \left(E^{\mathrm{T}}P + SR^{\mathrm{T}}\right)^{\mathrm{T}}, \qquad E^{\mathrm{T}}\hat{P} = E^{\mathrm{T}}R_c^{1/2}\bar{P}R_c^{1/2}E,$$

$$Q_1 = R_c^{1/2}\bar{Q}_1R_c^{1/2}, \qquad Q_2 = R_c^{1/2}\bar{Q}_2R_c^{1/2},$$

$R \in \mathrm{R}^{n\times(n-r)}$ is any matrix with full column rank satisfying $E^{\mathrm{T}}R = 0$.

Proof Firstly, we proof the system (8) with $w(t) = 0$ is regular, impulse-free.

Since $\operatorname{rank} E = r < n$, there must exist two invertible matrices G and $H \in \mathrm{R}^{n\times n}$, then R can be rewritten as $R = G^{\mathrm{T}}\left[\begin{smallmatrix} 0 \\ \Phi \end{smallmatrix}\right]$, where $\Phi \in \mathrm{R}^{(n-r)\times(n-r)}$. Denote

$$
\begin{aligned}
&GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad G\tilde{A}H = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \qquad G^{-\mathrm{T}}PG^{-1} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}, \\
&H^{\mathrm{T}}S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \qquad H_iH = [H_{i1} \quad H_{i2}], \qquad H_{di}H = [H_{di1} \quad H_{di2}], \\
&x(t) = H \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.
\end{aligned} \tag{12}
$$

Pre- and post-multiplying $\varpi_{11} < 0$ by H^{T} and H, respectively, we can get $A_4^{\mathrm{T}}\Phi S_2^{\mathrm{T}} + S_2\Phi^{\mathrm{T}}A_4 < 0$, which implies A_4 is nonsingular and thus the pair $(E, \tilde{A})$ is regular and impulse-free. From Definition 3, system (8) is regular and impulse-free.

Choose the Lyapunov function as follows:

$$
\begin{aligned}
V\bigl(x(t)\bigr) &= x^{\mathrm{T}}(t)E^{\mathrm{T}}PEx(t) + \int_{t-d(t)}^{t} e^{\delta(t-s)}x^{\mathrm{T}}(s)Q_1x(s)\,ds \\
&\quad + \int_{-d}^{0}\int_{t+\theta}^{t} e^{\delta(t-s)}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}Q_2E\dot{x}(s)\,ds\,d\theta.
\end{aligned} \tag{13}
$$

Along the trajectories of system (8), the corresponding time derivation of (13) is given by

$$
\begin{aligned}
\dot{V}\bigl(x(t)\bigr) &= 2x^{\mathrm{T}}(t)\hat{P}^{\mathrm{T}}E\dot{x}(t) + x^{\mathrm{T}}(t)Q_1x(t) \\
&\quad - \bigl(1-\dot{d}(t)\bigr)e^{\delta d(t)}x^{\mathrm{T}}\bigl(t-d(t)\bigr)Q_1x\bigl(t-d(t)\bigr) \\
&\quad + d\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}Q_2E\dot{x}(t) - d\int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}Q_2E\dot{x}(s)\,ds \\
&\le \delta V\bigl(x(t)\bigr) + 2x^{\mathrm{T}}(t)\hat{P}^{\mathrm{T}}E\dot{x}(t) + x^{\mathrm{T}}(t)Q_1x(t) \\
&\quad - (1-h)x^{\mathrm{T}}\bigl(t-d(t)\bigr)Q_1x\bigl(t-d(t)\bigr) \\
&\quad + d\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}Q_2E\dot{x}(t) - d\int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}Q_2E\dot{x}(s)\,ds - \delta x^{\mathrm{T}}(t)\hat{P}^{\mathrm{T}}Ex(t).
\end{aligned}
$$

Then, via Lemma 2,

$$
-d\int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}Q_2E\dot{x}(s)\,ds \le \xi^{\mathrm{T}}(t)\bigl\{\Pi + dY^{\mathrm{T}}Q_2^{-1}Y\bigr\}\xi(t),
$$

we have

$$
\dot{V}\bigl(x(t)\bigr) - \delta V\bigl(x(t)\bigr) - \omega^{\mathrm{T}}(t)Q\omega(t) = \xi^{\mathrm{T}}(t)\bigl(\Theta + d\Gamma_1^{\mathrm{T}}Q_2\Gamma_1 + \Pi + dY^{\mathrm{T}}Q_2^{-1}Y\bigr)\xi(t).
$$

Considering (11) and the Schur complement, it yields

$$
\dot{V}\bigl(x(t)\bigr) - \delta V\bigl(x(t)\bigr) - \omega^{\mathrm{T}}(t)Q\omega(t) < 0, \tag{14}
$$

pre- and post-multiplying (14) by $e^{-\delta t}$, and integrating it from 0 to t ($\forall t \in [0, T]$), it follows that

$$
V\bigl(x(t)\bigr) < e^{\delta t}\biggl[V\bigl(x(0)\bigr) + \int_0^t e^{-\delta s}\omega^{\mathrm{T}}(s)Q\omega(s)\,ds\biggr].
$$

From these,

$$\begin{aligned}
&V\big(x(0)\big) + \int_0^t e^{-\delta s}\omega^{\mathrm{T}}(s)Q\omega(s)\,ds \\
&\quad\le \left(\lambda_2 + \lambda_3 d + \lambda_4\frac{d^2}{2}\right)\sup_{-d\le\theta\le 0}\big\{x^{\mathrm{T}}(\theta)E^{\mathrm{T}}R_cEx(\theta), \dot{x}^{\mathrm{T}}(\theta)E^{\mathrm{T}}R_cE\dot{x}(\theta)\big\} + \lambda_5 b \\
&\quad\le \left(\lambda_2 + \lambda_3 d + \lambda_4\frac{d^2}{2}\right)c_1 + \lambda_5 b,
\end{aligned}$$

then

$$V(x_t) \le e^{\delta T}\left[\left(\lambda_2 + \lambda_3 d + \lambda_4\frac{d^2}{2}\right)c_1 + \lambda_5 b\right],$$

considering $V(x_t) \ge \lambda_1 x^{\mathrm{T}}(t)E^{\mathrm{T}}R_cEx(t)$, from condition (11), we have

$$x^{\mathrm{T}}(t)E^{\mathrm{T}}R_cEx(t) < c_2.$$

From Definition 1, the closed-loop system (8) is *FTB*. This completes the proof. □

Remark 2 In Theorem 1, sufficient conditions are obtained to guarantee that the closed-loop is finite-time bounded. Then Theorem 2 will give finite-time dissipative conditions.

Theorem 2 *For positive constants c_1, b, T, δ, positive definite matrix R_c, closed-loop system* (8) *is $FTH_\infty B$ with respect to $(c_1\ c_2\ b\ TR_c)$ at the origin with $\varepsilon(E^{\mathrm{T}}PE, \rho)$ contained in the domain of attraction, if there exist constant $c_2 > 0$, and positive definite matrices P, Q_1, Q_2 and any matrices N_1, N_2, L with appropriate dimensions, matrix S, for $i,j \in \Re$ and $\varepsilon(E^{\mathrm{T}}PE, \rho) \subset L(H_i, H_{di})$ such that the following conditions hold*:

$$\Psi = \begin{bmatrix} \Lambda + \Pi & \Gamma_1^{\mathrm{T}} & dY^{\mathrm{T}} & \Gamma_2^{\mathrm{T}} \\ * & -(dQ_2)^{-1} & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{15}$$

$$\left(\lambda_2 + \lambda_3 d + \lambda_4\frac{d^2}{2}\right)c_1 + \gamma^2 e^{-\delta T} b < \lambda_1 c_2 e^{-\delta T}, \tag{16}$$

where Π, Y, Γ_1, and ϖ_{11} are defined in Theorem 1, *and*

$$\Lambda = \begin{bmatrix} \varpi_{11} & P\tilde{A}_d & P\tilde{B}_\omega \\ * & -(1-h)Q_1 & 0 \\ * & * & -\gamma^2 e^{-\delta T} \end{bmatrix}, \qquad \Gamma_2 = [\tilde{C} \quad \tilde{C}_d \quad \tilde{D}_\omega].$$

Proof It is clear that $\Gamma_2^{\mathrm{T}}\Gamma_2 > 0$, then via the Schur complement, we can get from (15) that

$$\begin{bmatrix} \Lambda + \Pi & d\Gamma_1^{\mathrm{T}} & dY^{\mathrm{T}} \\ * & -Q_2^{-1} & 0 \\ * & * & -Q_2 \end{bmatrix} < 0. \tag{17}$$

Let $Q = -\gamma^2 e^{-\delta T} I$, by Theorem 1, combing (15) and (17), the closed-loop system (8) is *FTB* with respect to $(c_1\ c_2\ b\ T\ R_c)$.

On the other hand, select the same Lyapunov function candidate as Theorem 1 and define the following function:

$$J = \dot{V}(x_t) - \delta V(x_t) + z^{\mathrm{T}}(t)z(t) - \gamma^2 e^{-\delta T}\omega^{\mathrm{T}}(t)\omega(t),$$

using the Schur complement, it can be seen from (15) that

$$J = \xi^{\mathrm{T}}(t)\big[\Lambda + d\Gamma_1^{\mathrm{T}} Q_2 \Gamma_1 + \Pi + dY^{\mathrm{T}} Q_2^{-1} Y + \Gamma_2^{\mathrm{T}}\Gamma_2\big]\xi(t) < 0,$$

similar to the handling method in Theorem 1 and considering the zero initial condition, it is clear that

$$0 < V(x_t)e^{-\delta T} < \int_0^T e^{-\delta t}\big[\gamma^2 e^{-\delta T}\omega^{\mathrm{T}}(t)\omega(t) - z^{\mathrm{T}}(t)z(t)\big]\,dt,$$

then we have

$$\int_0^T z^{\mathrm{T}}(t)z(t)\,dt < \gamma^2 e^{-\delta T}\int_0^T \omega^{\mathrm{T}}(t)\omega(t)\,dt,$$

from Definition 2, the closed-loop system (8) is $FTH_\infty B$, and the H_∞ performance index is $\bar{\gamma} = \gamma^2 e^{-\delta T}$. This completes the proof. □

Remark 3 Theorem 2 gives the sufficient conditions for the $FTH_\infty B$ of the closed-loop system. However, the conditions (15) and (16) are nonlinear matrix inequalities, which will be transformed into LMIs in Theorem 3.

Theorem 3 *For positive constants c_1, c_2, b, T, α, positive definite matrix R_c, if there exist positive definite matrices X, $\tilde{Q}_1$, $\tilde{Q}_2$, Ψ, any matrices $\tilde{N}_1$, $\tilde{N}_2$, $\tilde{L}$, with appropriate dimensions, constants and $\mu, \eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$, $\varepsilon_{ij} > 0$, $\chi_{ij} > 0$ for $i,j \in \Re$, such that*

$$\Phi_{iis} < 0, \tag{18}$$

$$\Phi_{ijs} + \Phi_{jis} < 0, \tag{19}$$

$$\eta_1 I_n < R_c^{1/2} G^{-1}\begin{bmatrix} \left([I_r \quad 0]GEXG^{\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}\right) & 0 \\ 0 & \Psi \end{bmatrix} G^{-\mathrm{T}} R_c^{1/2} < I_n, \tag{20}$$

$$M_1 > \eta_2^{-1} R_c^{-1}, \tag{21}$$

$$M_2 > \eta_3^{-1} R_c^{-1}, \tag{22}$$

$$\begin{bmatrix} (\eta_2 d + \frac{\eta_3 d^2}{2})c_1 + (r^2 b - c_2)e^{-\delta T} & \sqrt{c_1} \\ * & -\eta_1 \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -\rho^{-1} & w_i & w_{di} \\ * & -E^{\mathrm{T}}XE & 0 \\ * & * & -E^{\mathrm{T}}XE \end{bmatrix} \le 0, \tag{24}$$

where

$$\Phi_{ijs} = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} & d\tilde{N}_1^{\mathrm{T}} & \psi_{16} & H_{1i} & \psi_{18} & X^{\mathrm{T}} & 0 & 0 \\ * & \psi_{22} & -\tilde{L} & \psi_{24} & d\tilde{N}_2^{\mathrm{T}} & \psi_{26} & 0 & \psi_{28} & 0 & X^{\mathrm{T}} & 0 \\ * & * & \psi_{33} & B_{\omega i}^{\mathrm{T}} & d\tilde{L}_1^{\mathrm{T}} & D_{\omega i}^{\mathrm{T}} & 0 & E_{4i}^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & H_{1i} & 0 & 0 & 0 & X^{\mathrm{T}} \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & H_{2i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\chi_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -M_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \frac{-M_1}{(h-1)} & 0 \\ * & * & * & * & * & * & * & * & * & * & -M_2 \end{bmatrix},$$

$$\psi_{11} = A_iX + B_i\hbar_{sj} + (A_iX + B_i\hbar_{sj})^{\mathrm{T}} - \delta X + \tilde{N}_1^{\mathrm{T}} + \tilde{N}_1,$$

$$\psi_{12} = A_{di}X + B_i\hbar_{dsj} - \tilde{N}_1^{\mathrm{T}} - \tilde{N}_2, \qquad \psi_{13} = B_{\omega i} + \tilde{L},$$

$$\psi_{14} = \hbar_{sj}^{\mathrm{T}}B_i^{\mathrm{T}} + XA_i^{\mathrm{T}}, \qquad \psi_{16} = XC_i^{\mathrm{T}} + \hbar_{sj}^{\mathrm{T}}D_i^{\mathrm{T}}, \qquad \psi_{18} = XE_{1i}^{\mathrm{T}} + \hbar_{sj}^{\mathrm{T}}E_{3i}^{\mathrm{T}},$$

$$\psi_{22} = -\tilde{N}_2^{\mathrm{T}} - \tilde{N}_2, \qquad \psi_{24} = XA_{di}^{\mathrm{T}} + \hbar_{dsj}^{\mathrm{T}}B_i^{\mathrm{T}}, \qquad \psi_{26} = XC_{di}^{\mathrm{T}} + \hbar_{dsj}^{\mathrm{T}}D_i^{\mathrm{T}},$$

$$\psi_{28} = XE_{2i}^{\mathrm{T}} + \hbar_{dsj}^{\mathrm{T}}E_{3i}^{\mathrm{T}}, \qquad \psi_{33} = -\gamma^2 e^{-\delta T}I, \qquad \psi_{44} = -M_2/d,$$

$$\psi_{55} = -X - X^{\mathrm{T}} + M_2, \qquad \hbar_{sj} = E_sY_j + E_s^-W_j, \qquad \hbar_{dsj} = E_sY_{dj} + E_s^-W_{dj},$$

G, H are nonsingular matrices that make $GEH = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]$, then closed-loop system (8) *is $FTH_\infty B$ with H_∞ performance index $\bar{\gamma} = \gamma^2 e^{-\delta T}$, and the controller feedback gains are given by*

$$K_j = Y_jX^{-1}, \qquad K_{dj} = Y_{dj}X^{-1}.$$

Proof From Theorem 2, considering (2), we can get the following relation according to matrix inequality (15):

$$\Psi = \sum_{s=1}^{\eta}\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\alpha_s(\Psi_{ijs} + \Delta\Psi_{ijs}) < 0, \tag{25}$$

where

$$\Psi_{ijs} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & N_1^{\mathrm{T}} & \varphi_{16} \\ * & \varphi_{22} & -E^{\mathrm{T}}L & \varphi_{24} & N_2^{\mathrm{T}} & \varphi_{26} \\ * & * & \varphi_{33} & B_{\omega i}^{\mathrm{T}} & L^{\mathrm{T}} & D_{\omega i}^{\mathrm{T}} \\ * & * & * & \varphi_{44} & 0 & 0 \\ * & * & * & * & -Q_2 & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\Delta\Psi_{ijs} = \begin{bmatrix} \Delta\varphi_{11} & \Delta\varphi_{12} & \Delta\varphi_{13} & \Delta\varphi_{14} & 0 & \Delta\varphi_{16} \\ * & 0 & 0 & \Delta\varphi_{24} & 0 & \Delta\varphi_{26} \\ * & * & 0 & \Delta B_{\omega i}^{\mathrm{T}} & 0 & \Delta D_{\omega i}^{\mathrm{T}} \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\varphi_{11} = \hat{P}^{\mathrm{T}}(A_i + B_i\lambda_{sj}) + (A_i + B_i\lambda_{sj})^{\mathrm{T}}\hat{P} + Q_1 - \delta\hat{P}^{\mathrm{T}}E + E^{\mathrm{T}}N_1 + N_1^{\mathrm{T}}E,$$

$$\varphi_{12} = \hat{P}^{\mathrm{T}}(A_{di} + B_i\lambda_{dsj}) - E^{\mathrm{T}}N_1 - N_2^{\mathrm{T}}E, \qquad \varphi_{13} = \hat{P}^{\mathrm{T}}B_{\omega i} + E^{\mathrm{T}}L,$$

$$\varphi_{14} = (A_i + B_i\lambda_{sj})^{\mathrm{T}}, \qquad \varphi_{22} = -(1-h)Q_1 - N_2^{\mathrm{T}}E - E^{\mathrm{T}}N_2,$$

$$\varphi_{33} = -\gamma^2 e^{-\delta T}I, \qquad \varphi_{44} = -(dQ_2)^{-1}, \qquad \varphi_{24} = (A_{di} + B_i\lambda_{dsj})^{\mathrm{T}},$$

$$\varphi_{16} = (C_i + D_i\lambda_{sj})^{\mathrm{T}}, \qquad \varphi_{26} = (C_{di} + D_i\lambda_{dsj})^{\mathrm{T}},$$

$$\Delta\varphi_{11} = \hat{P}^{\mathrm{T}}(\Delta A_i + \Delta B_i\lambda_{sj}) + (\Delta A_i + \Delta B_i\lambda_{sj})^{\mathrm{T}}\hat{P},$$

$$\Delta\varphi_{12} = \hat{P}^{\mathrm{T}}(\Delta A_{di} + \Delta B_i\lambda_{dsj}), \qquad \Delta\varphi_{13} = \hat{P}^{\mathrm{T}}\Delta B_{\omega i},$$

$$\Delta\varphi_{14} = (\Delta A_i + \Delta B_i\lambda_{sj})^{\mathrm{T}}, \qquad \Delta\varphi_{16} = (\Delta C_i + \Delta D_i\lambda_{sj})^{\mathrm{T}},$$

$$\Delta\varphi_{24} = (\Delta A_{di} + \Delta B_i\lambda_{dsj})^{\mathrm{T}}, \qquad \Delta\varphi_{26} = (\Delta C_{di} + \Delta D_i\lambda_{dsj})^{\mathrm{T}}.$$

Noticing (2) and Lemma 3, there exists a constant $\chi_{ij} > 0$, such that

$$\Delta\Psi_{ijs} = \Upsilon_1\Delta\Upsilon_2 + \Upsilon_2^{\mathrm{T}}\Delta^{\mathrm{T}}\Upsilon_1^{\mathrm{T}} \le \chi_{ij}\Upsilon_1\Upsilon_1^{\mathrm{T}} + -\chi_{ij}^{-1}\Upsilon_2^{\mathrm{T}}\Upsilon_2,$$

where

$$\Upsilon_1 = \begin{bmatrix} \left(\hat{P}^{\mathrm{T}}H_{1i}\right)^{\mathrm{T}} & 0 & 0 & H_{1i}^{\mathrm{T}} & 0 & H_{2i}^{\mathrm{T}} \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} E_{1i} + E_{3i}\lambda_{sj} & E_{2i} + E_{3i}\lambda_{dsj} & E_{4i} & 0 & 0 & 0 \end{bmatrix}.$$

Then via the Schur complement, (25) is equivalent to

$$\sum_{s=1}^{\eta}\sum_{i=1}^{r} h_i\alpha_s\Xi_{iis} + \sum_{s=1}^{\eta}\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\alpha_s(\Xi_{ijs} + \Xi_{jis}) < 0, \tag{26}$$

where

$$\Xi_{ijs} = \begin{bmatrix} \Psi_{ijs} & \Upsilon_1 & \Upsilon_2^{\mathrm{T}} \\ * & -\chi_{ij} & 0 \\ * & * & -\chi_{ij}^{\mathrm{T}} \end{bmatrix}.$$

From Theorem 1, $\varpi_{11} < 0$, we can get $\hat{P} = (E^TP + SR^{\mathrm{T}})^{\mathrm{T}}$ is nonsingular. Using Lemma 4, as the deal method in [26] there exists $X = \hat{P}^{-1} = (E\tilde{P} + \tilde{S}\tilde{R}^{\mathrm{T}})^{\mathrm{T}}$, where $\tilde{P} > 0$ and $\tilde{R} \in \mathrm{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E\tilde{R} = 0$. It is easy to see that

$$EX = X^{\mathrm{T}}E^{\mathrm{T}} = E\tilde{P}E^{\mathrm{T}} \ge 0. \tag{27}$$

Denoting $H^{-1}XG^{\mathrm{T}} = \left[\begin{smallmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{smallmatrix}\right]$, from (27), it is easy to obtain $X_{12} = 0$, and X_{11} is symmetric, then we have $H^{-1}XG^{\mathrm{T}} = \left[\begin{smallmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{smallmatrix}\right]$, so X_{11} and X_{22} are nonsingular. There, it can be concluded that $G^{-\mathrm{T}}X^{-1}H = \left[\begin{smallmatrix} X_{11}^{-1} & 0 \\ -X_{22}^{-1}X_{21}X_{11}^{-1} & X_{22}^{-1} \end{smallmatrix}\right]$ and $[I_r\ 0]GEXG^{\mathrm{T}}\left[\begin{smallmatrix} I_r \\ 0 \end{smallmatrix}\right] = X_{11}$ are nonsingular.

Then we have

$$\begin{aligned}
&H^{-\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}\left([I_r \quad 0]GEXG^{\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}\right)^{-1}[I_r \quad 0]H^{-1} \\
&\quad = H^{-\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}X_{11}^{-1}[I_r \quad 0]H^{-1} = H^{-\mathrm{T}}\begin{bmatrix} X_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}H^{-1} \\
&\quad = H^{-\mathrm{T}}\bigl(H^{\mathrm{T}}E^{\mathrm{T}}G^{\mathrm{T}}\bigr)\bigl(G^{-\mathrm{T}}X^{-1}H\bigr)H^{-1} \\
&\quad = E^{\mathrm{T}}X^{-1} = E^{\mathrm{T}}\hat{P},
\end{aligned}$$

pre- and post-multiply (18) and (19) $\operatorname{diag}\{\hat{P}^{\mathrm{T}}, \hat{P}^{\mathrm{T}}, I, I, \hat{P}^{\mathrm{T}}, I, I, I, I, I, I\}$, and denote $\hat{P}^{\mathrm{T}}\tilde{N}_1\hat{P} = N_1$, $\hat{P}^{\mathrm{T}}\tilde{N}_2\hat{P} = N_2$, $\hat{P}^{\mathrm{T}}\tilde{L} = L$, $Y_j\hat{P} = X_j$, $Y_{dj}\hat{P} = K_{dj}$, $W_j\hat{P} = H_j$, $W_{dj}\hat{P} = H_{dj}$, $\varepsilon_{ij} = \chi_{ij}^{-1}$, $M_1 = Q_1^{-1}$, $M_2 = Q_2^{-1}$, using the Schur complement, (18) and (19) are the sufficient conditions for (22) to hold.

Let

$$\bar{P} = R_c^{1/2}G^{\mathrm{T}}\begin{bmatrix} \left([I_r \quad 0]GEXG^{\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}\right)^{-1} & 0 \\ 0 & \Psi \end{bmatrix}GR_c^{1/2},$$

then we have

$$\begin{aligned}
E^{\mathrm{T}}R_c^{1/2}\bar{P}R_c^{1/2}E &= H^{-\mathrm{T}}HR_c^{1/2}G^{\mathrm{T}}\begin{bmatrix} \left([I_r \quad 0]GEXG^{\mathrm{T}}\begin{bmatrix} I_r \\ 0 \end{bmatrix}\right)^{-1} & 0 \\ 0 & \Psi^{-1} \end{bmatrix}GR_c^{1/2}HH^{-1} \\
&= H^{-\mathrm{T}}\begin{bmatrix} X_{11} & 0 \\ 0 & 0 \end{bmatrix}H^{-1} = E^{\mathrm{T}}X.
\end{aligned}$$

From (20), we have $I_n < \bar{P} < \frac{1}{\eta_1}I_n$, and then $\lambda_1 > 1$, $\lambda_2 < 1/\eta_1$. It can be seen from (21) that $\bar{Q}_1 = R_c^{-1/2}Q_1R_c^{-1/2} < \eta_2 I$, which means that $\lambda_3 < \eta_2$. Similarly, we can obtained $\lambda_4 < \eta_3$ from (21). Then using the Schur complement, we can get (16) from (23).

Then

$$\begin{aligned}
&x^{\mathrm{T}}(t)E^{\mathrm{T}}PEx(t) = x_1^{\mathrm{T}}(t)P_1x_1(t), \\
&H_ix(t) = H_{i1}x_1(t), \qquad H_{di}x(t) = H_{di1}x_1(t),
\end{aligned} \tag{28}$$

$$x^{\mathrm{T}}(t)E^{\mathrm{T}}PEx(t) \le \rho, \qquad x^{\mathrm{T}}\bigl(t-d(t)\bigr)E^{\mathrm{T}}PEx\bigl(t-d(t)\bigr) \le \rho. \tag{29}$$

So, condition $\varepsilon(E^{\mathrm{T}}PE, \rho) \subset L(H_i, H_{di})$ can be guaranteed by

$$[h_{i1k} \quad h_{\tau i1k}]\begin{bmatrix} P_1 & 0 \\ 0 & P_1 \end{bmatrix}^{-1}[h_{i1k} \quad h_{\tau i1k}]^{\mathrm{T}} \le \frac{1}{\rho}, \quad k = 1, 2, \ldots, l.$$

Via the Schur complement, it can be transformed to

$$\begin{bmatrix} -\frac{1}{\rho} & [h_{i1k} \quad h_{di1k}] \\ [h_{i1k} \quad h_{di1k}]^{\mathrm{T}} & -\begin{bmatrix} P_1 & 0 \\ 0 & P_1 \end{bmatrix} \end{bmatrix} \leq 0, \quad k = 1, 2, \ldots, l,$$

or

$$\begin{bmatrix} -\frac{1}{\rho} & [h_{i1k} \quad 0] & [h_{di1k} \quad 0] \\ * & -\mathrm{I}\tilde{P}\mathrm{I} & 0 \\ * & * & -\mathrm{I}\tilde{P}\mathrm{I} \end{bmatrix} \leq 0, \quad k = 1, 2, \ldots, l, \tag{30}$$

where $\mathrm{I}\tilde{P}\mathrm{I} = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]\left[\begin{smallmatrix} P_1 & P_2 \\ P_3 & P_4 \end{smallmatrix}\right]\left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]$, h_{i1k}, h_{di1k}, is the kth row of H_{i1} and H_{di1}, respectively.

Pre- and post-multiplying (30) by $\mathrm{diag}\{1, X^{\mathrm{T}}, X^{\mathrm{T}}\}$ and its transpose, considering $H_i X = W_i$, $H_{di} X = W_{di}$, w_{ik} is the kth row of W_i, w_{dik} is the kth row of W_{di}. Considering (12), then we can obtain (24). This completes the proof. □

Remark 4 Theorem 3 gives a LMI condition for the region $\varepsilon(E^{\mathrm{T}}PE, \rho)$ to be inside the domain of attraction for the closed-loop system (8) under the memory state feedback controller.

Remark 5 With all the ellipsoids satisfying the set invariance condition of Theorem 3, we may choose the largest one to obtain the least conservative estimate of the domain of attraction.

Let $\mathrm{X}_R \in \mathrm{R}^n$ be a prescribed bounded convex set containing the origin, which can be represented as $\mathrm{X}_R = \mathrm{co}\{x_0^1, x_0^2, \ldots, x_0^l\}$, where $x_0^1, x_0^2, \ldots, x_0^l$ are *a priori* given initial states in R^n. With Theorem 3, an exact invariant set with least degree of conservativeness can be formulated as

$$\begin{aligned} &\max \alpha \\ &\text{s.t.} \begin{cases} \text{(a)} \quad \alpha \mathrm{X}_R \subset \varepsilon(E^{\mathrm{T}}PE, \rho), \\ \text{(b)} \quad \text{inequality (18)-(24).} \end{cases} \end{aligned} \tag{31}$$

Using the Schur complement, constraint (a) is equivalent to

$$x_0^{\mathrm{T}} E^{\mathrm{T}} P E x_0 \leq \frac{\rho}{\alpha^2} \quad \Leftrightarrow \quad \begin{bmatrix} -\beta & x_0^{\mathrm{T}} E^{\mathrm{T}} \\ E x_0 & -X \end{bmatrix} \leq 0, \tag{32}$$

where $\beta = \rho/\alpha^2$.

From the above discussion, (27) can be transformed to the following LMI optimization problem:

$$\begin{aligned} &\min \beta \\ &\text{s.t. inequality (18)-(24) and (32).} \end{aligned} \tag{33}$$

Remark 6 Theorem 3 gives the sufficient conditions for designing the finite-time memory controller for TS fuzzy system with time-varying delay. It can be observed that the

conditions (18) and (19) are not strict LMIs, once we fix the parameter δ, the conditions can be turned into LMIs-based feasibility problem. Then the conditions in Theorem 3 can be turned into the following LMIs-based feasibility problem with a fixed parameter δ:

$$\begin{aligned}&\min c_2 + \gamma^2\\&X, \tilde{Q}_1, \tilde{Q}_2, \tilde{N}_1, \tilde{N}_2, \tilde{L}, b, d, \mu, \delta\\&\text{s.t. (18)-(24).}\end{aligned}$$

4 Numerical value examples

Example 1 Consider the nonlinear system with time delay [28]:

$$\begin{aligned}\dot{x}_1(t) &= -a\frac{v\bar{t}}{(L+\Delta L(t))t_0}x_1(t) - (1-a)\frac{v\bar{t}}{(L+\Delta L(t))t_0}x_1\bigl(t-\tau(t)\bigr)\\&\quad + \frac{v\bar{t}}{(l+\Delta l(t))t_0}\operatorname{sat}\bigl(u(t)\bigr),\\\dot{x}_2(t) &= a\frac{v\bar{t}}{(L+\Delta L(t))t_0}x_1(t) + (1-a)\frac{v\bar{t}}{(L+\Delta L(t))t_0}x_1\bigl(t-\tau(t)\bigr),\\\dot{x}_3(t) &= \frac{v\bar{t}}{t_0}\sin\biggl[x_2(t) + a\frac{v\bar{t}}{2(L+\Delta L(t))t_0}x_1(t)\\&\quad + (1-a)\frac{v\bar{t}}{2(L+\Delta L(t))t_0}x_1\bigl(t-\tau(t)\bigr)\biggr],\end{aligned}$$

where $x_1(t)$ is the angle difference between truck and trailer, $x_2(t)$ is the angle of trailer, $x_3(t)$ is the vertical position of rear end of trailer. The model parameters are given as $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $d = 10t_0/\pi$ and $a = 0.7$. Then the model is expressed by the following T-S fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(t)\bigl[(A_{1i}+\Delta A_{1i})x(t) + (A_{2i}+\Delta A_{2i})x\bigl(t-\tau(t)\bigr) + (B_i+\Delta B_i)\operatorname{sat}\bigl(u(t)\bigr)\bigr],$$

where

$$A_{11} = \begin{bmatrix} -av\bar{t}/(Lt_0) & 0 & 0\\ av\bar{t}/(Lt_0) & 0 & 0\\ av^2\bar{t}^2/(2Lt_0) & v\bar{t}/t_0 & 0\end{bmatrix}, \qquad A_{21} = \begin{bmatrix} -(1-a)v\bar{t}/(Lt_0) & 0 & 0\\ (1-a)v\bar{t}/(Lt_0) & 0 & 0\\ (1-a)v^2\bar{t}^2/(2Lt_0) & v\bar{t}/t_0 & 0\end{bmatrix},$$

$$B_1 = \begin{bmatrix} v\bar{t}/(lt_0)\\ 0\\ 0\end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -av\bar{t}/(Lt_0) & 0 & 0\\ av\bar{t}/(Lt_0) & 0 & 0\\ adv^2\bar{t}^2/(2Lt_0) & dv\bar{t}/t_0 & 0\end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -(1-a)v\bar{t}/(Lt_0) & 0 & 0\\ (1-a)v\bar{t}/(Lt_0) & 0 & 0\\ (1-a)dv^2\bar{t}^2/(2Lt_0) & v\bar{t}/t_0 & 0\end{bmatrix}, \qquad B_2 = \begin{bmatrix} v\bar{t}/(lt_0)\\ 0\\ 0\end{bmatrix},$$

$$\Delta A_{11} = 0.05\delta(t)\begin{bmatrix} 0.5091 & 0 & 0\\ -0.5091 & 0 & 0\\ 0.5091 & 0 & 0\end{bmatrix}, \qquad \Delta A_{21} = 0.05\delta(t)\begin{bmatrix} 0.2182 & 0 & 0\\ -0.2182 & 0 & 0\\ 0.2182 & 0 & 0\end{bmatrix},$$

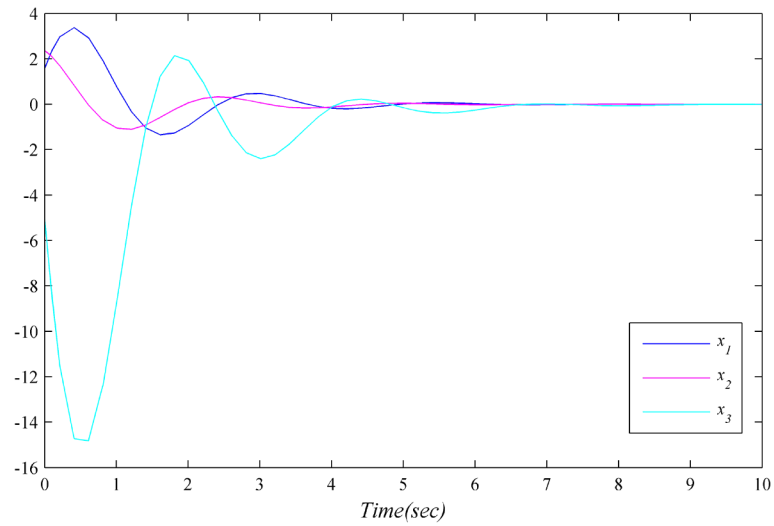

Figure 1 State response of the closed-loop system (Example 1).

$$\Delta B_1 = 0.05\delta(t)\begin{bmatrix} -0.3517 \\ 0 \\ 0 \end{bmatrix}, \qquad \Delta A_{12} = 0.05\delta(t)\begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.8107 & 0 & 0 \end{bmatrix},$$

$$\Delta A_{22} = 0.05\delta(t)\begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.3474 & 0 & 0 \end{bmatrix}, \qquad \Delta B_1 = 0.05\delta(t)\begin{bmatrix} -0.3517 \\ 0 \\ 0 \end{bmatrix},$$

where $|\delta(t)| < 1$.

On the basis of [28], the saturating constraint is ignored, and we give $\mu = 0$, $d = 0.01$, $c_1 = 1$, $c_2 = 10$, $T = 10$, $\delta = 0.01$, $R_c = I_3$. Solving the LMIs (18)-(24), we can get the memory state feedback controller gain is

$$K_{11} = [1.1380 \quad -1.5257 \quad -0.0453], \qquad K_{21} = [0.1589 \quad -0.0702 \quad -0.1001],$$

$$K_{12} = [1.2383 \quad -2.1297 \quad 0.1456], \qquad K_{22} = [0.1957 \quad -0.0052 \quad -0.0101].$$

For simulation, we choose the fuzzy weighting function to be

$$h_1(t) = 1/1 + \exp\bigl(0.5x_1(t+1)\bigr), \qquad h_2(t) = 1 - h_1(t),$$

and the initial condition $\phi^{\mathrm{T}}(t) = [0.5\pi \ 0.75\pi \ -5]^{\mathrm{T}}$, $t \in [-0.01, 0]$. Figure 1 shows the response of states of the closed-loop systems.

Example 2 Consider the TS fuzzy system subject to actuator saturation (1) with two fuzzy rules and the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$B_{\omega 1} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{\omega 1} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0.1 & 0.5 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & -0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ -0.1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.1 & 0 \end{bmatrix},$$

$$C_{d2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix}, \quad B_{\omega 2} = D_{\omega 2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$H_{ij} = \begin{bmatrix} -0.03 \\ 0.03 \\ 0.03 \end{bmatrix}, \quad E_{1i} = [0.02 \quad 00], \quad E_{2i} = [-0.35 \quad -0.450],$$

$$E_{3i} = -0.15, \quad E_{4i} = [-0.5 \quad -0.40] \quad (i, j = 1, 2).$$

For given $c_1 = 1$, $c_2 = 10$, $T = 10$, $b = 0.5$, $\delta = 0.01$, $d = 0.1$, $\mu = 0.1$, $r = 0.5$, $R_c = I_2$, the disturbance input is $\omega(t) = e^{-t}\sin(-t)$, and the membership functions are $h_1(t) = 1/[1 + \exp(0.5x_1(t+1))]$, $h_2(t) = 1 - h_1(t)$, using the Matlab toolbox, we can get

$$P = \begin{bmatrix} 5.2496 & 3.2171 & 0.1908 \\ 3.2171 & 9.8628 & -0.8331 \\ 0.1908 & -0.8331 & 20.3667 \end{bmatrix},$$

the controller gain can be obtained:

$$K_1 = [-9.6766 \quad -6.2899 \quad -1.3006], \quad K_{d1} = [6.9720 \quad 5.4183 \quad 0.1899],$$

$$K_2 = [-5.1128 \quad -2.5372 \quad -0.5392], \quad K_{d2} = [-16.6810 \quad -12.4518 \quad -0.4143].$$

For given the initial condition $x^{\mathrm{T}}(0) = [-1\ 1\ 0]^{\mathrm{T}}$, by solving the optimization problem (33), we can get $\beta^{\min} = 5.25$.

Then, using the above controller gain, Figure 2 plots the estimation of the domain of attraction and the response of the closed-loop system can be seen from Figure 3. It can be seen from Figure 3 that the closed-loop system is *FTB* subject to the memory controller. Figure 4 plots the evolution of $x^{\mathrm{T}}(t)E^{\mathrm{T}}R_cEx(t)$. It can be seen from Figure 4 that the TS fuzzy system (1) is finite-time bounded with respect to $(1, 10, I_2, 10)$ via the finite-time fuzzy memory controller.

For demonstration of the superiority of the memory state feedback controller presented in this paper, we give the memoryless controller as follows for comparison:

$$u(t) = \sum_{i=1}^{r} h_i\bigl(\theta(t)\bigr)K_i x(t). \tag{34}$$

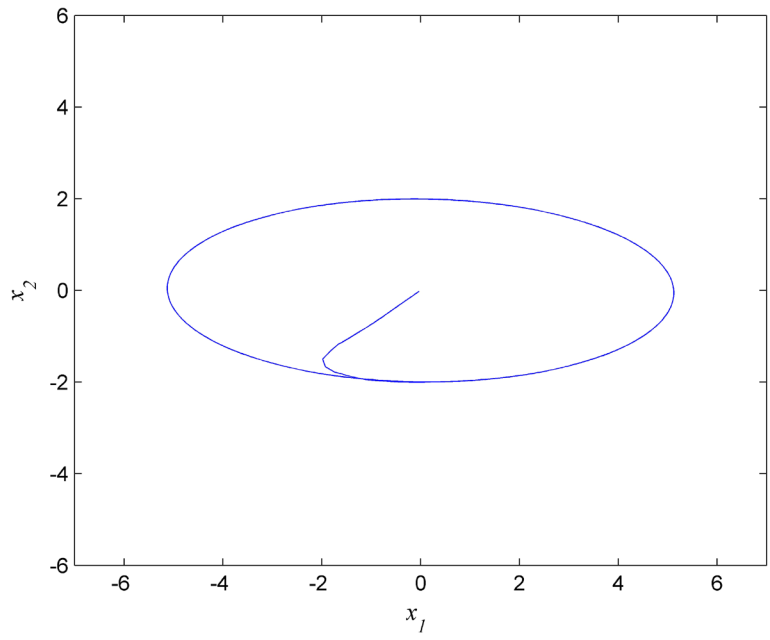

Figure 2 The estimation of the domain of attraction (Example 2).

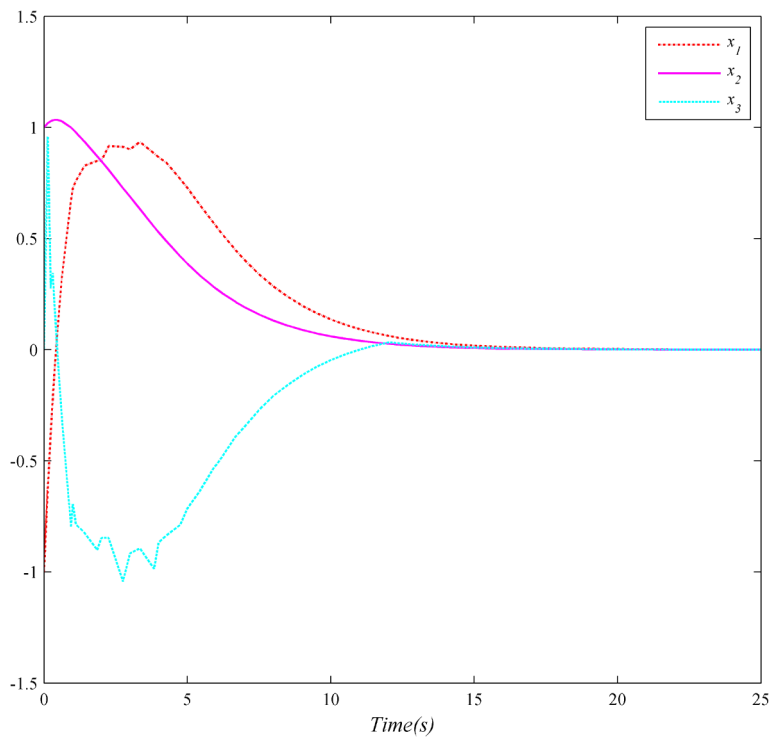

Figure 3 State response of the closed-loop system (Example 2).

We can obtain the maximum allowable d for different h in Table 1.

From the comparison in Table 1, it is obvious that the memory state feedback controller presented in this paper is less conservative.

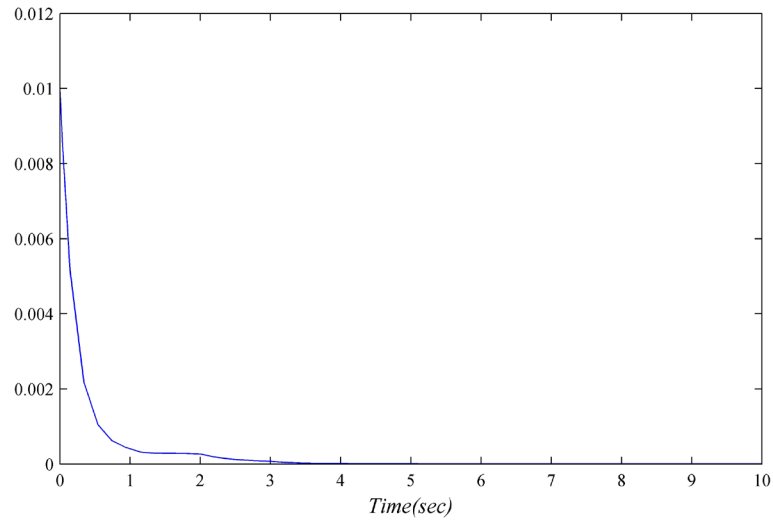

Figure 4 The evolution of $x^{\mathrm{T}}(t)E^{\mathrm{T}}R_cEx(t)$ (Example 2).

Table 1 Comparison of maximum *d* for different *h* (Example 2)

h	**0.01**	**0.02**	**0.05**	**0.07**	**0.09**
(34)	0.00264	0.00239	0.00164	0.00111	0.00053
(4)	0.00275	0.00245	0.00167	0.00113	0.00053

5 Conclusion

In this paper, the problem of finite-time H_∞ memory feedback of the singular T-S fuzzy system has been studied. Based on the finite-time stability theory, conditions were obtained, which can guarantee that the closed-loop system is finite-time H_∞ bounded with a presided H_∞ performance. The memory feedback controller problem can be solved by solving the LMIs. An optimization problem was given to deal with the largest domain of attraction of the closed-loop system. In the end, the examples were given to illustrate the feasibility of the method.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
WG carried out the main part of this paper, FL participated in the discussion and revision about the paper. All authors read and approved the final manuscript.

Acknowledgements
This work was supported by the Natural Science Foundation of Hebei province No. F2015203362.

Received: 25 October 2015 Accepted: 19 January 2016 Published online: 16 February 2016

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