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Regularization of the light-cone gauge gluon propagator singularities using sub-gauge conditions

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Abstract: Perturbative QCD calculations in the light-cone gauge have long suffered from the ambiguity associated with the regularization of the poles in the gluon propagator. In this work we study sub-gauge conditions within the light-cone gauge corresponding to several known ways of regulating the gluon propagator. Using the functional integral calculation of the gluon propagator, we rederive the known sub-gauge conditions for the θ-function gauges and identify the sub-gauge condition for the principal value (PV) regularization of the gluon propagator's light-cone poles. The obtained sub-gauge condition for the PV case is further verified by a sample calculation of the classical Yang-Mills field of two collinear ultrarelativistic point color charges. Our method does not allow one to construct a sub-gauge condition corresponding to the well-known Mandelstam-Leibbrandt prescription for regulating the gluon propagator poles.

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Contents

1 Introduction

Consider a gluon (or photon) propagator in the

$$\eta \cdot A = A^+ = 0 \tag{1.1}$$

light-cone gauge:

$$D^{\mu\nu}(x,y) \equiv \langle 0|\mathrm{T}A^\mu(x)A^\nu(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu + k^\nu\eta^\mu}{k^+}\right]. \tag{1.2}$$

(The gluon propagator given by eq. (1.2) is diagonal in the color indices.) Our convention for four-vectors is $v^\mu = (v^+, v^-, \vec{v}_\perp)$ with $v^\pm = (v^0 \pm v^3)/\sqrt{2}$. The gauge condition (1.1) and the propagator (1.2) are defined with the help of a light-like four-vector

$$\eta^\mu \equiv (0, 1, \vec{0}_\perp), \tag{1.3}$$

such that

$$\eta^2 = 0, \quad \eta \cdot x = x^+ \ . \tag{1.4}$$

Using the gluon propagator (1.2) in practical perturbative calculations one invariably faces the need to find a suitable way of regulating the $k^+ = 0$ pole. (See [1, 2] for a retrospective of works on the subject.) Without such regularization the k^+-integral in

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eq. (1.2) is ill-defined. The singularity of eq. (1.2) at $k^+ = 0$ appears to be due to incomplete gauge fixing: the $A^+ = 0$ light-cone gauge is preserved under any x^--independent gauge transformation, given by

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x^+, \vec{x}_\perp) \tag{1.5}$$

in the Abelian case and by

$$A^\mu(x) \to S(x^+, \vec{x}_\perp) A^\mu(x) S^{-1}(x^+, \vec{x}_\perp) - \frac{i}{g} \left[\partial^\mu S(x^+, \vec{x}_\perp)\right] S^{-1}(x^+, \vec{x}_\perp) \tag{1.6}$$

in the non-Abelian case. It is usually assumed that regularization of the $k^+ = 0$ pole should follow from further gauge fixing, stemming from sub-gauge constraints imposed in addition to eq. (1.1).

The most commonly used regularization prescriptions for the $k^+ = 0$ pole of the gluon light-cone gauge propagator are as follows:

- θ-function sub-gauges [3–5]:

$$D_1^{\mu\nu}(x, y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ - i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ + i\epsilon}\right], \tag{1.7}$$

$$D_2^{\mu\nu}(x, y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ + i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ - i\epsilon}\right]. \tag{1.8}$$

  The name stems from the fact that the classical field of a point (color) charge moving along the $x^- = 0$ light cone is proportional to $A_\perp^\mu \sim \theta(-x^-)$ in the first case and $A_\perp^\mu \sim \theta(x^-)$ in the second case [4, 6–10].

- Principal value (PV) sub-gauge [11]

$$D_{PV}^{\mu\nu}(x, y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \left(k^\mu \eta^\nu + k^\nu \eta^\mu\right) \text{PV}\left\{\frac{1}{k^+}\right\}\right]. \tag{1.9}$$

- Mandelstam-Leibbrandt (ML) prescription [12, 13]

$$D_{ML}^{\mu\nu}(x, y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+ + i\epsilon k^-}\right]. \tag{1.10}$$

The goal of this work is to identify the sub-gauge conditions leading to the propagators in eqs. (1.7), (1.8), (1.9) and (1.10) and to demonstrate that these sub-gauge conditions result in the propagators listed in those formulas when implemented in Feynman functional integration. We would like to stress that the regularizations of the gluon propagator poles given in eqs. (1.7), (1.8), (1.9) and (1.10) are by no means exhaustive, and other regularizations exist which will not be considered in this work (see e.g. [14]).

The paper is structured as follows. We begin with the θ-function sub-gauges in section 2. Motivated by the $A^0 = 0$ gauge we propose the sub-gauge condition in eq. (2.1), impose this sub-gauge condition within the functional integral, and derive an expression for the gluon propagator (with the $k^+ = 0$ pole regulated) by carefully evaluating surface terms inside the functional integral. In the process we show that the sub-gauge condition (2.1) can only be imposed at $x^- = \pm\infty$. The final results for the light-cone gluon propagators are given in eqs. (2.28) and (2.29), with the corresponding sub-gauge conditions stated immediately above these propagators. The same sub-gauge conditions were employed previously in [5, 15].

We move on to the case of the PV sub-gauge in section 3. There we tackle the problem in reverse order: we search for a sub-gauge condition which yields the propagator (1.9) in the functional integral calculation similar to that in section 2. In the end we obtain

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0 \tag{1.11}$$

as the sub-gauge condition necessary to obtain the PV regularization of the light-cone gauge gluon propagator (1.9).

The same reverse strategy is applied to the Mandelstam-Leibbrandt prescription in section 4. Starting from the Mandelstam-Leibbrandt propagator (1.10) we try to reconstruct the sub-gauge condition corresponding to this propagator. Unfortunately this procedure fails to yield a valid sub-gauge condition for the ML case.

Finally, in section 5 we illustrate and test our conclusion about the proper sub-gauge fixing condition (1.11) for the PV case by constructing the classical gluon field of two ultrarelativistic color charges moving in the same direction. Problems like this arise in describing the gluon distribution of a single large nucleus in the framework of the McLerran-Venugopalan (MV) model [4, 7–10, 16]. The classical gluon field of a single nucleus was constructed in the MV model in [10, 16] by using one of the θ-function sub-gauges. In section 5, for the first time the field is obtained in the PV sub-gauge. The gluon field is constructed both by solving the classical Yang-Mills equations and by diagram summation. In particular we show that at the Abelian lowest-order in the coupling g level one may use the sub-gauge condition (see e.g. [5])

$$\vec{A}_\perp(x^- = +\infty) + \vec{A}_\perp(x^- = -\infty) = 0 \tag{1.12}$$

instead of that in eq. (1.11). However, at higher orders in g, when the non-Abelian corrections become important, it is impossible to enforce the condition (1.12) even for the classical gluon field. At the same time the condition (1.11) appears to work even at the non-Abelian level. Combined with the derivation in section 3, this result appears to put on a more solid footing the PV regularization of light-cone gluon propagator singularities, which was used in perturbative calculations in the past [11].

We conclude in section 6 by restating our main results.

For future use let us define another light-like four-vector,

$$\tilde{\eta}^\mu \equiv (1, 0, \vec{0}_\perp), \quad \tilde{\eta}^2 = 0, \quad \tilde{\eta} \cdot x = x^- \ . \tag{1.13}$$

Any four-vector can be decomposed as $k^\mu = k^+ \tilde{\eta}^\mu + k^- \eta^\mu + k_\perp^\mu$, where $k_\perp^\mu = (0, 0, k^1, k^2)$ and $a_\perp \cdot b_\perp = a^i b^i = -a_{\perp\mu} b_\perp^\mu$ with $i = 1, 2$ and $\mu = 0, \ldots, 3$. We also define $\vec{k}_\perp \equiv (k^1, k^2)$.

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2 θ-function sub-gauges

In this section we will re-derive the sub-gauge conditions and the gluon propagator for the θ-function sub-gauges of the $A^+ = 0$ light-cone gauge using the functional integral formalism. We start with a conjecture for the sub-gauge condition. Note that in the case of temporal $A^0 = 0$ gauge one has a similar situation: the gluon propagator and the prescription for regulating the singularity at $k^0 = 0$ in it are obtained by imposing a sub-gauge condition at a specific point in time: $\vec{\partial} \cdot \vec{A}(t_0, \vec{x}) = 0$ [17–21]. Motivated by the $A^0 = 0$ gauge example, we impose the following sub-gauge condition:

$$\partial_{\perp\mu} A^\mu_\perp(x^+, x^- = \sigma, \vec{x}_\perp) = 0 \; . \tag{2.1}$$

In other words, we require that the transverse divergence of the gauge field vanishes at $x^- = \sigma$ with the value of σ not specified yet. (In the $A^0 = 0$ gauge the corresponding time t_0 at which the sub-gauge condition is specified remains arbitrary.) Clearly, eq. (2.1) is not the only sub-gauge choice that can be made. For example, an alternative gauge choice is to require that the four-divergence is zero at a generic point in x^-, $\partial_\mu A^\mu(x^+, x^- = \sigma, \vec{x}_\perp) = 0$. However, as we will explain below (see e.g. appendix A), this sub-gauge choice is not supported by the functional integral calculation.

In the functional integral formalism the propagator is obtained by applying functional derivatives of the generating functional with respect to the sources,

$$\begin{aligned} \langle 0|\mathrm{T} A_\mu(x) A_\nu(y)|0\rangle &= - \left[\frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta J^\nu(y)} \, e^{-\frac{1}{2}\int d^4x' d^4y' \, J^\alpha(x') D_{\alpha\beta}(x',y') \, J^\beta(y')} \right] \Bigg|_{J=0} \\ &= - \left[\frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta J^\nu(y)} \left(\frac{Z[J]}{Z[0]} \right) \right] \Bigg|_{J=0} , \end{aligned} \tag{2.2}$$

where $D_{\mu\nu}(x,y)$ is the gluon propagator and $Z[J]$ is the generating functional. To arrive at the expression for the gluon propagator $D_{\mu\nu}(x,y)$ (with regularizations for all the poles in momentum space) using the functional integration for constructing the generating functional used in (2.2), one has to take special care of the surface terms arising from integration by parts and of the gauge conditions. In what follows we will consider the x^+ variable as time, and will define the initial and final conditions at the light-cone times x_i^+ and x_f^+ respectively. It will be implied that x_i^+ is large and negative while x_f^+ is large and positive. In addition we assume that the system is localized in space but not in time: since now x^+ is our time variable, instead of the "standard" assumption that all fields go to zero as $|\vec{x}| \to \infty$, we will assume that the fields go to zero as $|\vec{x}_\perp| \to \infty$. As will become apparent below, careful treatment will be needed of the functional integral at the boundaries in x^+ and x^- directions.

The generating functional for an Abelian gauge theory in the light-cone gauge with the sub-gauge condition (2.1) is

$$Z[J] = \lim_{\xi_1,\xi_2\to 0} \int \mathcal{D}A_i \, \mathcal{D}A_f \, \Psi_0(A_i) \Psi_0^*(A_f) \int\limits_{\substack{A(x_i^+, x^-, \vec{x}_\perp) = A_i \\ A(x_f^+, x^-, \vec{x}_\perp) = A_f}} \mathcal{D}A_\mu \exp\left\{ i \int_{x_i^+}^{x_f^+} dx^+ \int dx^- \, d^2x_\perp \left[\mathcal{L}_0(A) + \mathcal{L}_{fix}(A) + J_\mu A^\mu \right] \right\} \tag{2.3}$$

with

$$\mathcal{L}_0(A) = -\frac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu} = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2}(\partial_\mu A_\nu)(\partial^\nu A^\mu) \tag{2.4}$$

and the gauge and sub-gauge fixing terms

$$\mathcal{L}_{fix}(A) = -\frac{1}{2\xi_1}\, A_\mu\, \eta^\mu\, \eta^\nu\, A_\nu - \frac{1}{2\xi_2}\, \left(\vec{\partial}_\perp \cdot \vec{A}_\perp\right)^2\, \delta(x^- - \sigma). \tag{2.5}$$

The generating functional in eq. (2.3) can also be thought of as describing the Abelian part of a non-Abelian theory such as gluodynamics. Notice that, as discussed above, in the generating functional (2.3) we have used the light-cone coordinates with x^+ as the time direction. As is usually done, we have exponentiated the gauge conditions and the parameters ξ_1 and ξ_2 will be sent to zero at the end of the calculation.

In eq. (2.3) $\Psi_0(A)$ represents the vacuum wave function in the A_μ-representation. In the light-cone gauge it is

$$\Psi_0(A) = \exp\left\{\frac{1}{2}\int dx^- d^2x_\perp\, A^\mu \sqrt{-(\partial^+)^2} A_\mu\right\}. \tag{2.6}$$

The expression (2.6) can be obtained by starting with the vacuum wave function in the $A^0 = 0$ gauge (see eq. (7.7) in [17])

$$\Psi_0(A) = \exp\left\{-\frac{1}{2}\int d^3x\, A^i \sqrt{-\vec{\nabla}^2}\, \left[\delta^{ij} - \frac{\partial^i\,\partial^j}{\vec{\nabla}^2}\right]\, A^j\right\}, \tag{2.7}$$

(with $\vec{\nabla} = (\partial_x, \partial_y, \partial_z)$ and $i, j = 1, 2, 3$ only in this formula) and performing an ultra-boost along the $+z$ direction to change the gauge into the $A^+ = 0$ gauge and the wave function (2.7) into (2.6).

It is known that one of the advantages of using axial-type gauge conditions is the absence of ghost fields. However, now, in addition to the light-cone gauge, we have a sub-gauge condition (2.1) which introduces a non trivial determinant, leading to a ghost field $c(x)$ localized at $x^- = \sigma$:

$$\det\left[\partial^\perp_\mu\, \mathcal{D}^\mu_\perp(x^- = \sigma)\right] = \int \mathcal{D}\bar{c}\,\mathcal{D}c \exp\left\{-i\int dx^+\, d^2x_\perp\, \bar{c}\, \partial^\perp_\mu\, \mathcal{D}^\mu_\perp\, c(x^- = \sigma)\right\}, \tag{2.8}$$

where $\mathcal{D}^{ab}_\mu \equiv \partial_\mu\, \delta^{ab} + g\, f^{acb}\, A^c_\mu$ is the covariant derivative and $\bar{c}(x)$ is the complex conjugate ghost field. Just like in Feynman gauge, the ghost field is needed only in the non-Abelian case. The ghost field does not affect the gluon propagator in question. The propagator of this ghost field, along with the ghost-gluon vertices, depend only on transverse momenta, and are independent of k^-. Because of that it appears that ghost loops are zero in perturbative calculations using dimensional regularization. Therefore, in eq. (2.3) and in the subsequent analysis we omit ghost contributions arising from sub-gauge conditions.

In order to put eq. (2.3) in the same form as the first line of eq. (2.2), we will adopt the following standard procedure of "completing the square". First we perform a shift of the gauge field $A^\mu \to A^\mu + a^\mu$ and obtain

$$Z[J] = \lim_{\xi_1,\xi_2 \to 0} \int \mathcal{D}A_i\, \mathcal{D}A_f\, \Psi_0(A_i) \Psi_0^*(A_f)\, \Psi_0(a_i) \Psi_0^*(a_f) \tag{2.9}$$

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$$\times \exp\left\{\int dx^- d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2}\, a_{i\,\mu} + A_f^\mu \sqrt{-(\partial^+)^2}\, a_{f\,\mu}\right)\right\}$$

$$\times \int\limits_{\substack{A(x_i^+,x^-,\vec{x}_\perp)=A_i\\ A(x_f^+,x^-,\vec{x}_\perp)=A_f}} \mathcal{D}A_\mu \exp\Bigg\{ i \int_{x_i^+}^{x_f^+} dx^+ \int dx^-\, d^2x_\perp \Bigg[\mathcal{L}_0(A) + \mathcal{L}_{fix}(A) + \mathcal{L}_0(a) + \mathcal{L}_{fix}(a) + J^\mu A_\mu + J^\mu a_\mu +$$

$$- (\partial_\mu A_\nu)\, (\partial^\mu a^\nu) + (\partial_\mu A_\nu)\, (\partial^\nu a^\mu) - \frac{1}{\xi_1} A_\mu\, \eta^\mu\, \eta^\nu\, a_\nu - \frac{1}{\xi_2}\left(\vec{\partial}_\perp \cdot \vec{A}_\perp\right)\left(\vec{\partial}_\perp \cdot \vec{a}_\perp\right)\, \delta(x^- - \sigma)\Bigg]\Bigg\}\,.$$

In arriving at eq. (2.9) we have done an integration by parts in (parts of) the vacuum wave functions, discarding the two-dimensional boundary integral which is outside the precision of the approximation that was used in deriving eq. (2.6). We now perform integration by parts in the terms linear in a^μ in the rest of the expression to arrive at

$$Z[J] = \lim_{\xi_1,\xi_2\to 0} \int \mathcal{D}A_i\, \mathcal{D}A_f\, \Psi_0(A_i)\Psi_0^*(A_f)\, \Psi_0(a_i)\, \Psi_0^*(a_f) \tag{2.10}$$

$$\times \exp\left\{\int dx^- d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2}\, a_{i\,\mu} + A_f^\mu \sqrt{-(\partial^+)^2}\, a_{f\,\mu}\right)\right\}$$

$$\times \int\limits_{\substack{A(x_i^+,x^-,\vec{x}_\perp)=A_i\\ A(x_f^+,x^-,\vec{x}_\perp)=A_f}} \mathcal{D}A_\mu \exp\Bigg\{ i \int_{x_i^+}^{x_f^+} dx^+ \int dx^-\, d^2x_\perp \Bigg[\mathcal{L}_0(A) + \mathcal{L}_{fix}(A) + \mathcal{L}_0(a) + \mathcal{L}_{fix}(a) + J^\mu A_\mu + J^\mu a_\mu +$$

$$+ A_\nu \left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right] a_\mu - \frac{1}{\xi_1} A_\mu\, \eta^\mu\, \eta^\nu\, a_\nu + \frac{1}{\xi_2}\, A_{\perp\mu} (\partial_\perp^\mu \partial_\perp^\nu a_{\perp\nu})\, \delta(x^- - \sigma)\Bigg] - i\int d\sigma_\mu \Big[A_\nu(\partial^\mu a^\nu) - A_\nu(\partial^\nu a^\mu)\Big]\Bigg\}.$$

where $d\sigma^\mu = \pm(d^2x_\perp\, dx^+\, \tilde{\eta}^\mu + d^2x_\perp\, dx^-\, \eta^\mu + d\sigma_\perp^\mu)$ is the integration measure over the 3-dimensional surface of our four-dimensional space-time. Here $d\sigma_\perp^\mu$ is the integration measure over the surface at $x_\perp \to \infty$. The choice of a plus or minus in each of the terms depends on which boundary one is integrating over.

In order to "complete the square" we need to eliminate all the terms linear in A^μ in eq. (2.10). Starting from the 4-dimensional volume integration terms we have to choose a^μ such that

$$A_\nu \left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right] a_\mu - \frac{1}{\xi_1}\, A_\mu\, \eta^\mu\, \eta^\nu\, a_\nu + \frac{1}{\xi_2}\, A_{\perp\mu} (\partial_\perp^\mu \partial_\perp^\nu a_{\perp\nu})\, \delta(x^- - \sigma) + J_\mu A^\mu = 0 \tag{2.11}$$

for any A^μ. Solving for a^μ we get

$$a^\mu(x) = i \int d^4y\, D^{\mu\nu}(x,y)\, J_\nu(y) \tag{2.12}$$

where $D^{\mu\nu}(x,y)$ is the Green function found from

$$\left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu - \frac{1}{\xi_1}\eta^\mu\eta^\nu + \frac{1}{\xi_2}\partial_\perp^\mu \partial_\perp^\nu\, \delta(x^- - \sigma)\right] D_{\nu\rho}(x,y) = i\, \delta_\rho^\mu\, \delta^{(4)}(x-y). \tag{2.13}$$

The boundary conditions for eq. (2.13) are obtained by requiring that the 3-dimensional *surface* integration terms linear in A^μ should also vanish in the exponent of eq. (2.10),

$$\int dx^-\, d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2}\, a_{i\,\mu} + A_f^\mu \sqrt{-(\partial^+)^2}\, a_{f\,\mu}\right) - i \int d\sigma_\mu \Big[A_\nu(\partial^\mu a^\nu) - A_\nu(\partial^\nu a^\mu)\Big] = 0. \tag{2.14}$$

Note that the condition (2.14) eliminates all the boundary term dependent on a^μ from the exponent of eq. (2.10) (and not just the terms linear in A^μ). More precisely, for a^μ satisfying (2.14) one gets

$$\Psi_0(a_i)\,\Psi_0^*(a_f)\exp\left\{\int dx^- d^2x\left(A_i^\mu\sqrt{-(\partial^+)^2}a_{i\,\mu}+A_f^\mu\sqrt{-(\partial^+)^2}a_{f\,\mu}\right)\right\} \tag{2.15}$$
$$\times\exp\left\{-\frac{i}{2}\int d\sigma_\mu\Big(a_\nu(\partial^\mu a^\nu)-a_\nu(\partial^\nu a^\mu)\Big)-i\int d\sigma_\mu\Big(A_\nu(\partial^\mu a^\nu)-A_\nu(\partial^\nu a^\mu)\Big)\right\}=1.$$

With this in mind one can readily show that after using a^μ satisfying eqs. (2.12), (2.13) and (2.14) in eq. (2.10) the generating functional becomes

$$Z[J]=Z[0]\exp\left\{-\frac{1}{2}\int d^4x\,d^4y\,J_\mu(x)\,D^{\mu\nu}(x,y)\,J_\nu(y)\right\}. \tag{2.16}$$

From (2.16) we see that $D^{\mu\nu}(x,y)$ is indeed the gluon propagator, as defined in (2.2), obtained in the light-cone gauge with the sub-gauge condition (2.1). Notice that, as can be easily verified, Gauss's law is automatically satisfied in gauge theories with the generating functional (2.16), due to the self-consistency of the functional integral formalism.

We conclude that to find the gluon propagator we need to solve eq. (2.13) and verify that the solution leads to a^μ satisfying eq. (2.14).

For any $x^-\neq\sigma$ the general solution of eq. (2.13) is

$$D^{\mu\nu}(x,y)|_{x^-\neq\sigma}=\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot(x-y)}\,\frac{-i}{[k^2]}\left[g^{\mu\nu}-\frac{k^\mu\eta^\nu+k^\nu\eta^\mu}{[k^+]}\right], \tag{2.17}$$

where the regularization of the $k^2=0$ and $k^+=0$ poles is not specified on purpose, since the remaining uncertainty in this solution is solely due to the freedom to regulate these poles in various ways. For this reason we use the square brackets around the poles $k^2=0$ and $k^+=0$ (that is, $[k^2]$ and $[k^+]$) to denote that the regularization prescription has yet to be determined. Integrating eq. (2.13) over x^- in an infinitesimal interval centered at σ and assuming that $D^{\mu\nu}$ is continuous we see that for $x^-=\sigma$ (and $y^-\neq\sigma$) the solution of (2.13) has to satisfy the following condition

$$\partial_\mu^\perp\partial_\rho^\perp D^{\rho\nu}(x,y)|_{x^-=\sigma}=0\,. \tag{2.18}$$

(One also obtains continuity of $\partial_- D_{+\rho}$ at $x^-=\sigma$.) The continuity of $D^{\mu\nu}$ implies that its value at $x^-=\sigma$ is fixed by eq. (2.17), such that we can write

$$D^{\mu\nu}(x,y)=\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot(x-y)}\,\frac{-i}{[k^2]}\left[g^{\mu\nu}-\frac{k^\mu\eta^\nu+k^\nu\eta^\mu}{[k^+]}\right] \tag{2.19}$$

for all x^- with the only remaining freedom in this result being due to unspecified regularization of the $k^2=0$ and $k^+=0$ poles. In fact one may still have different regularizations (or linear combinations thereof) of the $k^2=0$ and $k^+=0$ poles for $x^->\sigma$ and $x^-<\sigma$ in eq. (2.19). (For instance one may obtain plane waves by replacing

$$\frac{1}{k^2}\to\frac{1}{2}\left[\frac{1}{k^2-i\epsilon}-\frac{1}{k^2+i\epsilon}\right]=\pi\,i\,\delta(k^2) \tag{2.20}$$

in eq. (2.19).) With the help of a direct calculation one can see that no regularization of the $k^2 = 0$ and $k^+ = 0$ poles in eq. (2.17) would lead to eq. (2.18) for an arbitrary finite value of σ and for all $x^+, \vec{x}_\perp$. This leaves $\sigma = \pm\infty$ as the only possibilities.

Let us first establish the Feynman prescription for the $k^2 = 0$ pole in eq. (2.19). Picking up the $x^+ = x_i^+$ and $x^+ = x_f^+$ surfaces in eq. (2.14) and using a^μ from eq. (2.12) with the Green function from eq. (2.19) (with $k^2 \to k^2 + i\epsilon$) while keeping in mind that $a^+ = 0$ in eq. (2.12) and $A^+ = 0$ due to $\xi_1 \to 0$ limit in eq. (2.10) yields

$$\int dx^- \, d^2x_\perp \, A_\perp^\mu(x_i^+)\Big(\sqrt{-(\partial^+)^2} + i\partial^+\Big)\, a_\mu^\perp(x_i^+) \tag{2.21a}$$
$$= \int d^4y \, dx^- \, d^2x_\perp \, A_\perp^\mu(x_i^+) \int \frac{d^4k}{(2\pi)^4} \frac{2k^+\theta(k^+)}{k^2 + i\epsilon}\left(g_\perp^{\mu\nu} - \frac{k_\perp^\mu \eta^\nu}{[k^+]}\right) e^{-i\,k^+(x^- - y^-) - i\,k^-(x_i^+ - y^+) + i\,\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{y}_\perp)} = 0$$

and

$$\int dx^- \, d^2x_\perp \, A_\perp(x_f^+)^\mu\Big(\sqrt{-(\partial^+)^2} - i\partial^+\Big)\, a_\mu^\perp(x_f^+) \tag{2.21b}$$
$$= -\int d^4y \, dx^- \, d^2x_\perp A_\perp^\mu(x_f^+) \int \frac{d^4k}{(2\pi)^4} \frac{2k^+\theta(-k^+)}{k^2 + i\epsilon}\left(g_\perp^{\mu\nu} - \frac{k_\perp^\mu \eta^\nu}{[k^+]}\right) e^{-i\,k^+(x^- - y^-) - ik^-(x_f^+ - y^+) + i\,\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{y}_\perp)} = 0.$$

To prove the validity of eqs. (2.21a) and (2.21b), it is enough to observe that the direction of the k^- -contour closure is determined by the fact that $x_i^+ - y^+ < 0$ and $x_f^+ - y^+ > 0$ for all y^+, since x_i^+ is the initial and therefore the smallest x^+ value, while x_f^+ the final and therefore the largest x^+ value in the 4-volume considered. Eqs. (2.21a) and (2.21b) are zero independent of the regularization prescription for the $k^+ = 0$ pole, and hence do not allow us to fix this prescription. Note also that other regularizations of the $k^2 = 0$ pole would not satisfy both eqs. (2.21a) and (2.21b).

We now write

$$D^{\mu\nu}(x, y) = \int \frac{d^4k}{(2\pi)^4} \, e^{-ik\cdot(x-y)} \, \frac{-i}{k^2 + i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{[k^+]}\right] \tag{2.22}$$

and directly face the need to regulate the $k^+ = 0$ pole as the only remaining ambiguity in the expression. Substituting eq. (2.22) into eq. (2.18) yields

$$\partial_\perp^\mu \int \frac{d^4k}{(2\pi)^4} \, \frac{e^{-ik^+(\sigma - y^-) - ik^-(x^+ - y^+) + i\,\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{y}_\perp)}}{k^2 + i\epsilon} \left(k_\perp^\nu + \frac{k_\perp^2 \eta^\nu}{[k^+]}\right) = 0. \tag{2.23}$$

Once again we see that for finite σ it is impossible to satisfy eq. (2.23) and hence eq. (2.18).

Since σ can not be finite, we consider $\sigma = +\infty$ first. In such case we need to close the k^+-integration contour in the lower half-plane. Before doing the calculation, it is already clear that our best chance of getting zero on the left-hand-side of eq. (2.23) is to put $[k^+] = k^+ - i\epsilon$, such that the light-cone pole would not contribute to the integral.

Using the following Fourier transform

$$\int \frac{d^4k}{(2\pi)^4} \, e^{-ik\cdot(x-y)} \, \frac{1}{k^2 + i\epsilon}\left(k_\perp^\nu + \frac{k_\perp^2 \eta^\nu}{k^+ - i\epsilon}\right) \tag{2.24}$$
$$= \frac{(x-y)_\perp^\nu}{2\pi^2[(x-y)^2 - i\epsilon]^2} + \eta^\nu\left[\frac{(x^- - y^-)}{\pi^2[(x-y)^2 - i\epsilon]^2} - i\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)\theta(y^- - x^-)\right]$$

we see that eq. (2.23) is satisfied if we use the prescription $[k^+] = k^+ - i\epsilon$ for $\sigma = +\infty$ since eq. (2.24) is zero for $x^- = +\infty$. With this result we rewrite eq. (2.22) as

$$D^{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^+-i\epsilon} - \frac{k^\nu\eta^\mu}{[k^+]}\right]. \tag{2.25}$$

It may seem that there is still an unregulated pole at $k^+ = 0$ in the last term of the square brackets in eq. (2.25). However, regularization of this last term can be fixed using the symmetry of the gluon propagator, $D^{\mu\nu}(x,y) = D^{\nu\mu}(y,x)$. This yields

$$D^{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^+-i\epsilon} - \frac{k^\nu\eta^\mu}{k^++i\epsilon}\right]. \tag{2.26}$$

The derivation is similar for the case of $\sigma = -\infty$. We employ

$$\begin{aligned}&\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+i\epsilon}\left(k_\perp^\nu + \frac{k_\perp^2\eta^\nu}{k^++i\epsilon}\right)e^{-ik\cdot(x-y)} \qquad (2.27)\\ &= \frac{(x-y)_\perp^\nu}{2\pi^2[(x-y)^2-i\epsilon]^2} + \eta^\nu\left[\frac{(x^--y^-)}{\pi^2[(x-y)^2-i\epsilon]^2} + i\delta^{(2)}(\vec{x}_\perp-\vec{y}_\perp)\delta(x^+-y^+)\theta(x^--y^-)\right]\end{aligned}$$

and observe that eq. (2.27) is zero for $x^- = -\infty$. Thus eq. (2.23) is satisfied for $[k^+] = k^+ + i\epsilon$ and $\sigma = -\infty$.

To summarize, we obtain the following two sub-gauge conditions and the corresponding gluon propagators for $\sigma = \pm\infty$ [3–5]:

- Light-cone gauge gluon propagator for the sub-gauge condition $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$

$$D_1^{\mu\nu}(x,y) \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^+-i\epsilon} - \frac{k^\nu\eta^\mu}{k^++i\epsilon}\right]; \tag{2.28}$$

- Light-cone gauge gluon propagator for the sub-gauge condition $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$

$$D_2^{\mu\nu}(x,y) \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^++i\epsilon} - \frac{k^\nu\eta^\mu}{k^+-i\epsilon}\right]. \tag{2.29}$$

As a consistency check, we now need to show that when using the propagators (2.28) or (2.29), eq. (2.14) is satisfied along the $x^- = \pm\infty$ surfaces, along with the $x_\perp = \infty$ boundary. (We have checked the $x^+ = x_i^+$ and $x^+ = x_f^+$ surfaces when deriving Feynman regularization in eqs. (2.21a) and (2.21b).) Eq. (2.14) is trivially satisfied at the $x_\perp = \infty$ boundary, since we assumed initially that the system is localized in $x_\perp$ and all fields vanish when $x_\perp \to \infty$. We are left only with the $x^- = \pm\infty$ surfaces to consider, for which eq. (2.14) reduces to

$$-i\int dx^+\, d^2x_\perp \left[A_\nu(\partial^- a^\nu) - A_\nu(\partial^\nu a^-)\right]\Big|_{x^-=-\infty}^{x^-=+\infty} = 0. \tag{2.30}$$

Let us demonstrate that eq. (2.30) is indeed valid for the case of $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$ sub-gauge. (The argument for the $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$ sub-gauge is constructed by analogy.) The a^μ-shift is (cf. eq. (2.12))

$$a_1^\mu(x) = i\int d^4y\, D_1^{\mu\nu}(x,y)\, J_\nu(y)\ . \tag{2.31}$$

We now plug eq. (2.31) into eq. (2.30) and use eq. (2.28) to integrate over k^+. Note that, just like in eqs. (2.24) and (2.27), picking up the $k^2 = 0$ pole of the k^+-integral would give us a contribution which goes to zero as $x^- \to \pm\infty$. (Those contributions are given by the first term on the right-hand side of (2.24) and (2.27) and by the first term in the square brackets of the right-hand side of (2.24) and (2.27).) Only picking the $k^+ = 0$ pole may give a term (akin to the last terms in the square brackets on the right-hand side of (2.24) and (2.27)) which may potentially violate eq. (2.30). Therefore, we substitute eq. (2.31) into eq. (2.30) and use eq. (2.28) to integrate over k^+ picking up the $k^+ = 0$ poles only. Keeping in mind the $A^+ = 0$ gauge condition we write

$$\begin{aligned}
&-i\int dx^+\, d^2x_\perp \left[A_\nu(\partial^- a_1^\nu) - A_\nu(\partial^\nu a_1^-)\right]\Big|_{x^-=-\infty}^{x^-=+\infty} \qquad (2.32)\\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y)\int\frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\,\frac{-1}{k^2+i\epsilon}\left[k^-\, A^\mu(x) + k\cdot A(x)\,\frac{1}{k^+ + i\epsilon}\,\left(k^-\,\eta^\mu + k_\perp^\mu\right)\right]\Bigg|_{x^-=-\infty}^{x^-=+\infty}\\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y)\\
&\quad\times\int\frac{d^2k_\perp\, dk^-}{(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\,\frac{i}{k_\perp^2}\,\vec{k}_\perp\cdot\vec{A}_\perp(x)\,\left(k^-\,\eta^\mu + k_\perp^\mu\right)\,\theta(x^- - y^-)\Big|_{x^-=-\infty}^{x^-=+\infty}\\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y)\int\frac{d^2k_\perp\, dk^-}{(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\,\frac{i}{k_\perp^2}\,\vec{k}_\perp\cdot\vec{A}_\perp(x^- = +\infty)\,\left(k^-\,\eta^\mu + k_\perp^\mu\right)\\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y)\int\frac{d^2k_\perp\, dk^-}{(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\,\frac{-1}{k_\perp^2}\,\vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = +\infty)\,\left(k^-\,\eta^\mu + k_\perp^\mu\right) = 0,
\end{aligned}$$

where in the final steps we replaced $\vec{k}_\perp \to -i\vec{\partial}_\perp$, integrated by parts, and employed the $\vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = +\infty) = 0$ sub-gauge condition. The details of the calculation in eq. (2.32) justifying neglecting the $k^2 = 0$ pole in eq. (2.32) along with the underlying assumptions are given in appendix B. Note that the contribution of the $k^2 = 0$ pole is independent of the regularization prescription for the $k^+ = 0$ pole: hence the conclusion of appendix B is valid for all $k^+ = 0$ pole prescriptions.

Note that a 4-divergence sub-gauge condition, $\partial_\mu A^\mu(x^- = +\infty) = 0$, would not have led to zero in eq. (2.32), and therefore does not correspond to propagator (2.28). For further reasons detailing why this is not a valid sub-gauge condition of the light-cone gauge see appendix A.

We have thus verified that a^μ from eq. (2.12) with either one of the propagators (2.28) and (2.29) satisfies eq. (2.14), while the propagators $D_1^{\mu\nu}(x, y)$ and $D_2^{\mu\nu}(x, y)$ solve eq. (2.13) with $\sigma = \pm\infty$ respectively. Therefore, eq. (2.16) is also verified, with $D_1^{\mu\nu}(x, y)$ and $D_2^{\mu\nu}(x, y)$ being valid light-cone gauge propagators satisfying corresponding sub-gauge conditions.

It is also easy to explicitly check that propagators $D_1^{\mu\nu}$ and $D_2^{\mu\nu}$ themselves respect the sub-gauge conditions

$$\begin{aligned}
\partial_\mu^\perp D_1^{\mu\nu}(x, y)\Big|_{x^-=+\infty} &= 0\,,\\
\partial_\mu^\perp D_2^{\mu\nu}(x, y)\Big|_{x^-=-\infty} &= 0\,. \qquad (2.33)
\end{aligned}$$

Propagators (2.28) and (2.29) were already obtained by different procedures in [3–5]. We observe that in ref. [5] the propagators (2.28) and (2.29) were obtained by imposing an additional sub-gauge condition, $A^-(x^- = \pm\infty) = 0$, while in the above procedure we showed that it is sufficient to assume that $\lim_{x^-\to\infty} [A^-(x^-)/x^-] = 0$ (see appendix B).

3 PV sub-gauge

In this section we will determine the sub-gauge condition that reproduces Principal Value (PV) prescription (1.9) for the k^+ pole in light-cone propagator. To this end, we will adopt the same procedure we used to arrive at propagators (2.28) and (2.29) with sub-gauge conditions $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$ and $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$ respectively, but in reverse order.

In the previous section we have assumed a sub-gauge condition (2.1), performed a shift of the field $A^\mu \to A^\mu + a^\mu$ in the generating functional, and made sure that the a^μ-dependent surface terms vanish (that is, eq. (2.14) is satisfied) for the generating functional to reduce to the form given in (2.16).

As we do not know *a priori* the sub-gauge condition that reproduces the light-cone propagator with $k^+ = 0$ pole regulated by PV prescription, we consider from the start the propagator with the PV prescription and deduce the needed sub-gauge condition in order to put the generating functional in the form (2.16). In practical terms, we have to show that eq. (15) is satisfied if we regulate the $k^+ = 0$ pole of the light-cone propagator with the PV prescription.

The gauge field propagator in the $A^+ = 0$ light-cone gauge with the PV-prescription is

$$D^{\mu\nu}_{PV}(x,y) \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon} \left[g^{\mu\nu} - \left(k^\mu \eta^\nu + k^\nu \eta^\mu\right) \text{PV}\left\{\frac{1}{k^+}\right\}\right] \tag{3.1}$$

where

$$\text{PV}\left\{\frac{1}{k^+}\right\} \equiv \frac{1}{2}\left(\frac{1}{k^+ - i\epsilon} + \frac{1}{k^+ + i\epsilon}\right). \tag{3.2}$$

Knowing the propagator means we know the shift field a^μ (cf. eq. (2.12)),

$$a^\mu_{PV} = i \int d^4y\, D^{\mu\nu}_{PV}(x,y)\, J_\nu(y). \tag{3.3}$$

Let us plug the shift field (3.3) into eq. (2.14) obtaining

$$\int dx^-\, d^2x_\perp \left(A^\mu_i \sqrt{-(\partial^+)^2}\, a^{PV}_{i\,\mu} + A^\mu_f \sqrt{-(\partial^+)^2}\, a^{PV}_{f\,\mu}\right) - i \int d\sigma_\mu \left[A_\nu(\partial^\mu a^\nu_{PV}) - A_\nu(\partial^\nu a^\mu_{PV})\right] = 0 \tag{3.4}$$

and require that the latter is satisfied everywhere along the boundary of the four-dimensional space-time volume. Eq. (3.4) is satisfied at the $x^+ = x^+_i$ and $x^+ = x^+_f$ boundaries irrespective of the regularization of the $k^+ = 0$ pole, as follows from eqs. (2.21a) and (2.21b). The boundary at $x_\perp \to \infty$ is also automatically satisfied, since we assumed from the start that all fields vanish as $x_\perp \to \infty$. We are only left with the boundary at $x^- = \pm\infty$. By analogy to eq. (2.32) we evaluate the contributions of the $x^- = \pm\infty$ boundaries by neglecting the residues of $k^2 = 0$ pole in the propagator which vanish at those boundaries (see appendix B and eqs. (2.24) and (2.27)):

$$0 = -i \int dx^+\, d^2x_\perp \left[A_\nu(\partial^- a^\nu_{PV}) - A_\nu(\partial^\nu a^-_{PV})\right]\Big|^{x^-=+\infty}_{x^-=-\infty}$$

$$
\begin{aligned}
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-1}{k^2+i\epsilon} \left[k^-\, A^\mu(x) + k\cdot A(x)\, \mathrm{PV}\left\{\frac{1}{k^+}\right\}\, (k^-\, \eta^\mu + k_\perp^\mu)\right]\Bigg|_{x^-=-\infty}^{x^-=+\infty} \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \\
&\quad \times \int \frac{d^2k_\perp\, dk^-}{(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{i}{k_\perp^2}\, \vec{k}_\perp\cdot\vec{A}_\perp(x)\, \left(k^-\, \eta^\mu + k_\perp^\mu\right)\, \frac{1}{2}\,\mathrm{Sign}(x^- - y^-)\Big|_{x^-=-\infty}^{x^-=+\infty} \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \\
&\quad \times \int \frac{d^2k_\perp\, dk^-}{2(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{i}{k_\perp^2}\, \left[\vec{k}_\perp\cdot\vec{A}_\perp(x^-=+\infty) + \vec{k}_\perp\cdot\vec{A}_\perp(x^-=-\infty)\right] \\
&\times \left(k^-\, \eta^\mu + k_\perp^\mu\right) = \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^2k_\perp\, dk^-}{2(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{-1}{k_\perp^2}\, \left(k^-\, \eta^\mu + k_\perp^\mu\right) \\
&\times \left[\vec{\partial}_\perp\cdot\vec{A}_\perp(x^-=+\infty) + \vec{\partial}_\perp\cdot\vec{A}_\perp(x^-=-\infty)\right]. \qquad (3.5)
\end{aligned}
$$

We see that for the boundary condition in eq. (3.5) to be satisfied, i.e. for the boundary term to vanish, one has to have the following sub-gauge condition:

$$
\vec{\partial}_\perp\cdot\vec{A}_\perp(x^-=+\infty) + \vec{\partial}_\perp\cdot\vec{A}_\perp(x^-=-\infty) = 0. \qquad (3.6)
$$

We have thus arrived at the sub-gauge condition which leads to the k^+ pole in the gluon propagator regulated with the PV prescription. We can check the validity of the PV-sub-gauge condition (3.6) explicitly by using the PV-propagator:

$$
\partial_\mu^\perp D_{PV}^{\mu\nu}(x,y)\Big|_{x^-=+\infty} + \partial_\mu^\perp D_{PV}^{\mu\nu}(x,y)\Big|_{x^-=-\infty} = 0. \qquad (3.7)
$$

In section 5 we will show that the PV sub-gauge condition (3.6) is consistent with reproducing the classical gluon field generated by two ultrarelativistic quarks propagating along two parallel light-cones, whereas a stronger condition

$$
\vec{A}_\perp(x^-=+\infty) + \vec{A}_\perp(x^-=-\infty) = 0, \qquad (3.8)
$$

while still satisfying eq. (3.5) does not allow one to construct the classical field of the color charges at the non-Abelian level. Therefore, it is eq. (3.6) which appears to be the correct sub-gauge condition in the PV case.

4 Mandelstam-Leibbrandt prescription

In this section we will try to obtain the sub-gauge condition that is consistent with the light-cone gauge propagator (1.10) with $k^+ = 0$ pole regulated by Mandelstam-Leibbrandt (ML) prescription [12, 13]. To this end, we will adopt the same procedure we used for the PV sub-gauge in the previous section, *i.e*, we will use the ML propagator (1.10) to construct the shift field a^μ from (2.12), and use the latter in eq. (2.14) to try to deduce the sub-gauge condition that has to be satisfied.

The light-cone propagator with Mandelstam-Leibbrandt prescription [12, 13] is

$$
D_{ML}^{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-i}{k^2+i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu\eta^\nu + k^\nu\eta^\mu}{k^+ + i\epsilon k^-}\right]. \qquad (4.1)
$$

The corresponding shift field is

$$a^{\mu}_{ML} = i \int d^4y\, D^{\mu\nu}_{ML}(x,y)\, J_\nu(y). \tag{4.2}$$

Substituting a^{μ}_{ML} into eq. (2.14) yields the following boundary condition for a^{μ}_{ML} to satisfy:

$$\int dx^-\, d^2x_\perp \left(A^\mu_i \sqrt{-(\partial^+)^2}\, a^{ML}_{i\,\mu} + A^\mu_f \sqrt{-(\partial^+)^2}\, a^{ML}_{f\,\mu} \right) - i \int d\sigma_\mu \left[A_\nu(\partial^\mu a^\nu_{ML}) - A_\nu(\partial^\nu a^\mu_{ML}) \right] = 0. \tag{4.3}$$

Again only the $x^- = \pm\infty$ boundaries need to be considered, since the other boundary conditions are automatically satisfied by the field from eq. (4.2). Discarding the contributions of the $k^2 = 0$ pole we get

$$\begin{aligned}
0 &= -i \int dx^+\, d^2x_\perp \left[A_\nu(\partial^- a^\nu_{ML}) - A_\nu(\partial^\nu a^-_{ML}) \right] \Big|^{x^-=+\infty}_{x^-=-\infty} \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{-1}{k^2+i\epsilon} \left[k^-\, A^\mu(x) + k\cdot A(x)\, \frac{1}{k^+ + i\epsilon k^-}\, (k^-\, \eta^\mu + k^\mu_\perp) \right] \Bigg|^{x^-=+\infty}_{x^-=-\infty} \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^2k_\perp\, dk^-}{(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{i}{k^2_\perp}\, \vec{k}_\perp\cdot\vec{A}_\perp(x)\, (k^-\, \eta^\mu + k^\mu_\perp) \\
&\times \frac{1}{2} \left[\theta(x^- - y^-)\, \theta(k^-) - \theta(y^- - x^-)\, \theta(-k^-) \right] \Big|^{x^-=+\infty}_{x^-=-\infty} \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^2k_\perp\, dk^-}{2(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{i}{k^2_\perp}\, (k^-\, \eta^\mu + k^\mu_\perp) \\
&\times \left[\theta(k^-)\, \vec{k}_\perp\cdot\vec{A}_\perp(x^- = +\infty) + \theta(-k^-)\, \vec{k}_\perp\cdot\vec{A}_\perp(x^- = -\infty) \right] \\
&= \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^2k_\perp\, dk^-}{2(2\pi)^3}\, e^{-ik^-(x^+-y^+)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{-1}{k^2_\perp}\, (k^-\, \eta^\mu + k^\mu_\perp) \\
&\times \left[\theta(k^-)\, \vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = +\infty) + \theta(-k^-)\, \vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = -\infty) \right].
\end{aligned} \tag{4.4}$$

It appears that to satisfy the boundary condition we need to require that the expression in the last square brackets in eq. (4.4) is zero. However, the expression in the square brackets depends on k^-: equating it to zero would result in a sub-gauge condition which would depend on the arbitrary momentum k^-, mixing up coordinate and momentum spaces. Such condition can only be satisfied if each term in the last square brackets of eq. (4.4) is zero separately.

The situation does not change if we integrate over k^- in eq. (4.4) obtaining

$$\begin{aligned}
0 = \int d^4y\, dx^+\, d^2x_\perp\, J_\mu(y) \int \frac{d^2k_\perp\, dk^-}{2(2\pi)^3}\, e^{i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{y}_\perp)}\, \frac{-1}{k^2_\perp} \Bigg[-\left(\frac{\eta^\mu}{(x^+-y^+-i\epsilon)^2} + \frac{ik^\mu_\perp}{x^+-y^+-i\epsilon} \right) \vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = +\infty) \\
+ \left(\frac{\eta^\mu}{(x^+-y^++i\epsilon)^2} + \frac{ik^\mu_\perp}{x^+-y^++i\epsilon} \right) \vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = -\infty) \Bigg].
\end{aligned} \tag{4.5}$$

The two terms in the square brackets of eq. (4.5) are multiplied by two different functions of an arbitrary variable y^+. Again the only way for these square brackets to be equal to zero is to require that

$$\begin{aligned}
&\vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = +\infty) = 0 \quad \text{and} \\
&\vec{\partial}_\perp\cdot\vec{A}_\perp(x^- = -\infty) = 0
\end{aligned} \tag{4.6}$$

at the same time. However, the sub-gauge conditions (4.6) are not satisfied by the ML-propagator (4.1). Indeed, we have

$$\partial^{\perp}_{\mu} D^{\mu\nu}_{ML}(x,y)|_{x^-=+\infty} = -\frac{1}{2\pi}\,\eta^{\nu}\,\delta^{(2)}\left(\vec{x}_{\perp}-\vec{y}_{\perp}\right)\,\frac{1}{x^+-y^+-i\epsilon} \neq 0 \tag{4.7a}$$

$$\partial^{\perp}_{\mu} D^{\mu\nu}_{ML}(x,y)|_{x^-=-\infty} = -\frac{1}{2\pi}\,\eta^{\nu}\,\delta^{(2)}\left(\vec{x}_{\perp}-\vec{y}_{\perp}\right)\,\frac{1}{x^+-y^++i\epsilon} \neq 0\,. \tag{4.7b}$$

In addition, the conditions (4.6) can not even be satisfied by the classical gluon field of a single relativistic charge, as will become apparent in section 5.

For $x^+ \neq y^+$ eq. (4.5) can be satisfied by requiring that

$$\vec{\partial}_{\perp}\cdot\vec{A}_{\perp}(x^- = +\infty) = \vec{\partial}_{\perp}\cdot\vec{A}_{\perp}(x^- = -\infty). \tag{4.8}$$

However, there is no reason here to require $x^+ \neq y^+$, since both variables are integrated over in (4.5). In addition, eq. (4.8) is not satisfied even by the field of a single ultrarelativistic charge in electrodynamics. Finally, even the ML propagators do not satisfy (4.8), as can be seen from (4.7).

We conclude that the procedure with which we successfully determined the sub-gauge condition for PV-prescription is either not the right procedure to obtain the sub-gauge condition for the light-cone gluon propagator with ML-prescription or that the ML light-cone propagator is not compatible with the functional integral formalism.

It is interesting to observe that, in [3] the ML-light-cone propagator was obtained within the functional integral formalism using complex valued fields (for a real-field gauge theory).

5 Classical Yang-Mills field

In this section we illustrate the PV sub-gauge condition (3.6) and the corresponding propagator (3.1) by an example of a classical gluon field of two color charges on parallel light cones calculated in the $A^+ = 0$ light-cone gauge. This types of problems arise in the McLerran-Venugopalan (MV) model [4, 7–10, 16] of a large nucleus, where the classical gluon field dominates over quantum corrections due to small coupling and large atomic number of the nucleus (see [22] for a detailed introduction to the subject). Classical gluon field of a single ultrarelativistic nucleus in the θ-function sub-gauges of the $A^+ = 0$ light-cone gauge was constructed in the MV model framework by solving Yang-Mills (YM) equations in [10, 16] and by summation of the corresponding tree-level diagrams in [4]. Below we will repeat both types of calculations for the PV sub-gauge of the $A^+ = 0$ light-cone gauge for a system of two color charges, which could be two valence quarks from two nucleons in a large nucleus.[1] The calculations in this section closely follows what was done in [4, 10], but in a different sub-gauge of the light-cone gauge.

[1]Note that since above we have failed to find the sub-gauge condition corresponding to the ML prescription, we can not solve classical YM equations in the ML case, since we do not know which condition to impose on the field. A diagrammatic calculation with the ML gluon propagator should lead to the field equivalent to one of the θ-function sub-gauges since $k^- < 0$ for all virtual gluon lines in this case.

Consider two ultrarelativistic quarks on two parallel light-cones. In covariant (Feynman) $\partial_\mu A^\mu = 0$ gauge their classical gluon field is known exactly [10, 16] and is

$$A^{a\,+}_{\text{cov}}(x^-, \vec{x}_\perp) = \frac{g}{2\pi}\,(t^a)_1\,\delta(x^- - b_1^-)\,\ln\Big(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\Big) + \frac{g}{2\pi}\,(t^a)_2\,\delta(x^- - b_2^-)\,\ln\Big(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\Big)\,, \tag{5.1}$$

where $(b_1^-, \vec{b}_{1\perp})$ and $(b_2^-, \vec{b}_{2\perp})$ determine the quarks' light-cone trajectories, g is the coupling constant, and $(t^a)_i$ are fundamental SU(N_c) generators in the color space of quark i.

We need to find the gauge transformation from Feynman to the light-cone gauge. It is given by

$$A_\mu^{LC} = S\,A_\mu^{\text{cov}}\,S^{-1} - \frac{i}{g}\,(\partial_\mu S)\,S^{-1}. \tag{5.2}$$

Requiring that the new gauge is the light-cone gauge, $A^+_{LC} = 0$, yields the following differential equation:

$$\partial^+ S = -i\,g\,S\,A^+_{\text{cov}}. \tag{5.3}$$

As discussed in the Introduction, eq. (5.3) does not specify S, and hence the gauge, uniquely. In the PV sub-gauge it needs to be augmented by the boundary condition (3.6).

While eq. (5.1) is the exact solution of the Yang-Mills equations for two ultrarelativistic charges, we will try to construct S by solving eq. (5.3) order-by-order in g^2, making sure the condition (3.6) is satisfied by the light-cone gauge gluon field at each order.

5.1 Abelian case

S is a unitary matrix. At the lowest non-trivial order we write

$$S = 1 + i\,\alpha(x^-, \vec{x}_\perp) + \ldots, \tag{5.4}$$

where $\alpha(x)$ is an order-g^2 correction and ellipsis represent higher-order corrections in g. Since S is unitary, $\alpha(x)$ is a hermitean matrix. Plugging eq. (5.4) into eq. (5.3) we get

$$\partial^+ \alpha = -g\,A^+_{\text{cov}}. \tag{5.5}$$

Solving this equation with A^+_{cov} given by eq. (5.1) we obtain

$$\begin{aligned}\alpha(x^-, \vec{x}_\perp) = &-\frac{g^2}{2\pi}\,t^a (t^a)_1\,\frac{1}{2}\,\text{Sign}(x^- - b_1^-)\,\ln\Big(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\Big) - \frac{g^2}{2\pi}\,t^a (t^a)_2\,\frac{1}{2}\,\text{Sign}(x^- - b_2^-)\,\ln\Big(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\Big) \\ &+ C_1(\vec{x}_\perp, b_1, b_2),\end{aligned} \tag{5.6}$$

where C_1 is the integration constant (which may be a function of all the other variables in the problem). To find C_1 we need to satisfy the boundary condition (3.6). The transverse components of the gluon field in the LC gauge are given by (note that $\partial_\perp^i = -\nabla_\perp^i$)

$$\vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) = \frac{i}{g}\,(\vec{\nabla}_\perp S)\,S^{-1} = -\frac{1}{g}\,\vec{\nabla}_\perp \alpha(x^-, \vec{x}_\perp) + \ldots. \tag{5.7}$$

Using eq. (5.6) in eq. (5.7) yields

$$\begin{aligned}\vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) = &\frac{g}{2\pi}\,t^a (t^a)_1\,\frac{1}{2}\,\text{Sign}(x^- - b_1^-)\,\frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} + \frac{g}{2\pi}\,t^a (t^a)_2\,\frac{1}{2}\,\text{Sign}(x^- - b_2^-)\,\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \\ &- \frac{1}{g}\,\vec{\nabla}_\perp C_1(\vec{x}_\perp, b_1, b_2) + \ldots.\end{aligned} \tag{5.8}$$

Clearly the gluon field from eq. (5.7) satisfies the condition (3.6) iff

$$\vec{\nabla}_\perp C_1(\vec{x}_\perp, b_1, b_2) = 0, \tag{5.9}$$

which means that $C_1 = C_1(b_1, b_2)$,

$$\vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) = \frac{g}{2\pi}\, t^a (t^a)_1\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_1^-)\, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} + \frac{g}{2\pi}\, t^a (t^a)_2\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_2^-)\, \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} + \mathcal{O}(g^3) \tag{5.10}$$

and

$$\alpha(x^-, \vec{x}_\perp) = -\frac{g^2}{2\pi}\, t^a (t^a)_1\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_1^-)\, \ln\left(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\right) - \frac{g^2}{2\pi}\, t^a (t^a)_2\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_2^-)\, \ln\left(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\right) + C_1(b_1, b_2). \tag{5.11}$$

Furthermore, since the field is Abelian at this order, the function C_1 is additive, $C_1(b_1, b_2) = \tilde{C}(b_1) + \tilde{C}(b_2)$. Applying translational invariance gives $\tilde{C}(b)$=const, while this constant we will put to zero. (The appearance of the function C_1 is related to the fact that even our sub-gauge conditions do not fix the field uniquely: an Abelian gauge transformation (1.5) with $\nabla_\perp^2 \Lambda(x^+, \vec{x}_\perp) = 0$ preserves both the light-cone gauge and the sub-gauge condition (3.6).)

Without C_1 we write

$$\alpha(x^-, \vec{x}_\perp) = -\frac{g^2}{2\pi}\, t^a (t^a)_1\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_1^-)\, \ln\left(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\right) - \frac{g^2}{2\pi}\, t^a (t^a)_2\, \frac{1}{2}\, \mathrm{Sign}(x^- - b_2^-)\, \ln\left(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\right). \tag{5.12}$$

5.2 Non-Abelian corrections

Let us find the next correction to S. Remembering that S is unitary we write

$$S = 1 + i\,\alpha - \frac{\alpha^2}{2} + i\,\alpha' + \ldots, \tag{5.13}$$

where $\alpha'(x)$ is the order-g^4 correction, which again is a hermitean matrix. Plugging (5.13) into eq. (5.3) and employing eq. (5.5) yields

$$\partial^+ \alpha' = \frac{i}{2}\, \left[\alpha, \partial^+ \alpha\right]. \tag{5.14}$$

Using eq. (5.12) in (5.14) we write

$$\begin{aligned}\partial^+ \alpha' = \frac{i}{2} \left(\frac{g^2}{2\pi}\right)^2 [t^a (t^a)_1, t^b (t^b)_2]\, \ln\left(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\right) \ln\left(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\right) \\ \times \frac{1}{2}\, \mathrm{Sign}(b_2^- - b_1^-)\, \left[\delta(x^- - b_2^-) + \delta(x^- - b_1^-)\right].\end{aligned} \tag{5.15}$$

The solution of eq. (5.15) is

$$\alpha' = \frac{i}{8} \left(\frac{g^2}{2\pi}\right)^2 [t^a (t^a)_1, t^b (t^b)_2] \ln\left(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\right) \ln\left(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\right) \mathrm{Sign}(b_2^- - b_1^-) \left[\mathrm{Sign}(x^- - b_2^-) + \mathrm{Sign}(x^- - b_1^-)\right] + C_2(\vec{x}_\perp, b_1, b_2) \tag{5.16}$$

with C_2 the integration constant.

To impose the sub-gauge condition (3.6) we need to find the transverse components of the gluon field in the light-cone gauge. We write

$$\vec{A}_\perp^{LC}(x^-,\vec{x}_\perp) = \frac{i}{g}\,(\vec{\nabla}_\perp S)\,S^{-1} = -\frac{1}{g}\,\vec{\nabla}_\perp\alpha - \frac{1}{g}\,\vec{\nabla}_\perp\alpha' - \frac{i}{2g}[\alpha,\vec{\nabla}_\perp\alpha] + \ldots. \tag{5.17}$$

Substituting eqs. (5.12) and (5.16) into eq. (5.17) gives

$$\begin{aligned}
\vec{A}_\perp^{LC}(x^-,\vec{x}_\perp) = {}& \frac{g}{4\pi}\,t^a(t^a)_1\,\text{Sign}(x^- - b_1^-)\,\frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2} + \frac{g}{4\pi}\,t^a(t^a)_2\,\text{Sign}(x^- - b_2^-)\,\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} \\
& - \frac{i}{8}\frac{g^3}{(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\text{Sign}(b_2^- - b_1^-)\,\left[\text{Sign}(x^- - b_2^-) + \text{Sign}(x^- - b_1^-)\right] \\
& \times \left[\frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{2\perp}|\,\Lambda\right) + \frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{1\perp}|\,\Lambda\right)\right] \\
& - \frac{1}{g}\,\vec{\nabla}_\perp C_2(\vec{x}_\perp,b_1,b_2) - \frac{i}{8}\frac{g^3}{(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\text{Sign}(x^- - b_1^-)\,\text{Sign}(x^- - b_2^-) \\
& \times \left[\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{1\perp}|\,\Lambda\right) - \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{2\perp}|\,\Lambda\right)\right] + \mathcal{O}(g^5).
\end{aligned} \tag{5.18}$$

The condition (3.6) is satisfied by the field in eq. (5.18) only if

$$\nabla_\perp^2 C_2(\vec{x}_\perp,b_1,b_2) = -\frac{i}{8}\frac{g^4}{2\pi}\,[t^a(t^a)_1, t^b(t^b)_2]\,\left[\delta^2\left(\vec{x}_\perp-\vec{b}_{2\perp}\right) - \delta^2\left(\vec{x}_\perp-\vec{b}_{1\perp}\right)\right]\,\ln\left(|\vec{b}_{1\perp}-\vec{b}_{2\perp}|\,\Lambda\right). \tag{5.19}$$

The solution of eq. (5.19) is

$$C_2(\vec{x}_\perp,b_1,b_2) = -\frac{i\,g^4}{8\,(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\ln\left(\frac{|\vec{x}_\perp-\vec{b}_{2\perp}|}{|\vec{x}_\perp-\vec{b}_{1\perp}|}\right)\,\ln\left(|\vec{b}_{1\perp}-\vec{b}_{2\perp}|\,\Lambda\right), \tag{5.20}$$

where we put integration constants to zero and required that C_2 is at most finite as $x_\perp \to \infty$ such that $\vec{A}_\perp^{LC} \to 0$ when $x_\perp \to \infty$, which was our assumption throughout the paper. Substituting eq. (5.20) into eq. (5.18) we obtain our final result for the gluon field in light-cone gauge,

$$\begin{aligned}
\vec{A}_\perp^{LC}(x^-,\vec{x}_\perp) = {}& \frac{g}{4\pi}\,t^a(t^a)_1\,\text{Sign}(x^- - b_1^-)\,\frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2} + \frac{g}{4\pi}\,t^a(t^a)_2\,\text{Sign}(x^- - b_2^-)\,\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} \\
& - \frac{i}{8}\frac{g^3}{(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\text{Sign}(b_2^- - b_1^-)\,\left[\text{Sign}(x^- - b_2^-) + \text{Sign}(x^- - b_1^-)\right] \\
& \times \left[\frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{2\perp}|\,\Lambda\right) + \frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{1\perp}|\,\Lambda\right)\right] \\
& + \frac{i}{8}\frac{g^3}{(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\left[\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} - \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\right]\,\ln\left(|\vec{b}_{1\perp}-\vec{b}_{2\perp}|\,\Lambda\right) \\
& - \frac{i}{8}\frac{g^3}{(2\pi)^2}\,[t^a(t^a)_1, t^b(t^b)_2]\,\text{Sign}(x^- - b_1^-)\,\text{Sign}(x^- - b_2^-) \\
& \times \left[\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{1\perp}|\,\Lambda\right) - \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\,\ln\left(|\vec{x}_\perp-\vec{b}_{2\perp}|\,\Lambda\right)\right] + \mathcal{O}(g^5).
\end{aligned} \tag{5.21}$$

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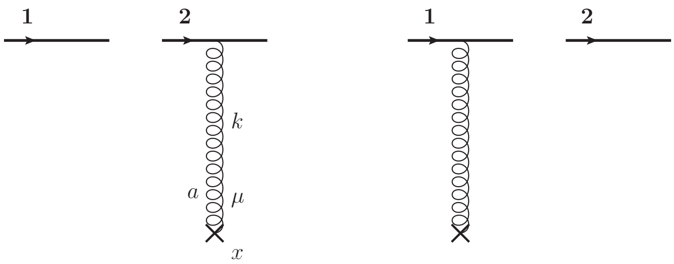

Figure 1. Diagrammatic representation of the gluon field of two quarks at the order g.

It is important to stress that imposing a stronger sub-gauge condition (3.8) onto the field of eq. (5.18) would lead to

$$\vec{\nabla}_\perp C_2(\vec{x}_\perp, b_1, b_2) = -\frac{i}{8}\frac{g^4}{(2\pi)^2}[t^a(t^a)_1, t^b(t^b)_2]\left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2}\ln\Big(|\vec{x}_\perp - \vec{b}_{1\perp}|\,\Lambda\Big) - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2}\ln\Big(|\vec{x}_\perp - \vec{b}_{2\perp}|\,\Lambda\Big)\right]. \tag{5.22}$$

However, eq. (5.22) for C_2 has no solution. The easiest way to see it is to act on both sides with $\vec{\nabla}_\perp \times$,

$$0 = \vec{\nabla}_\perp \times \vec{\nabla}_\perp C_2(\vec{x}_\perp, b_1, b_2) \neq -\frac{i}{4}\frac{g^4}{(2\pi)^2}[t^a(t^a)_1, t^b(t^b)_2]\,\frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \times \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2}, \tag{5.23}$$

obtaining a contradiction. (Here $\vec{\nabla}_\perp \times \vec{a}_\perp \equiv \partial_x a_y - \partial_y a_x$.)

We conclude that one can not *always* satisfy the condition (3.8) in a Yang-Mills theory: we have just constructed a counter-example. Therefore eq. (3.8) is not a proper sub-gauge condition of the light-cone gauge, which did not follow from our discussion in section 3. At the same time the condition (3.6) appears to have passed this non-Abelian classical field test leading to the gluon field (5.21).[2]

5.3 Diagrammatic calculation

To better understand what using the PV prescription for the propagators (3.1) entails in the actual diagrammatic calculations, let us now try to construct the gluon field of two ultrarelativistic color charges using Feynman diagrams.

We start with the order-g gluon field of two quarks in the light-cone gauge depicted in figure 1. A straightforward calculation (using PV regularization of the light-cone singularities) yields

$$\begin{aligned}\vec{A}^{LC}_\perp(x^-, \vec{x}_\perp) &= t^a \int \frac{d^2k_\perp\, dk^+}{(2\pi)^3}\, e^{-ik^+(x^- - b_1^-) + i\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{b}_{1\perp})}\, g(t^a)_1 \frac{k^\mu_\perp}{k^2_\perp}\,\text{PV}\left\{\frac{1}{k^+}\right\} + (1 \to 2) \quad (5.24)\\ &= \frac{g}{4\pi} t^a\,(t^a)_1\,\text{Sign}(x^- - b_1^-)\,\frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} + \frac{g}{4\pi} t^a\,(t^a)_2\,\text{Sign}(x^- - b_2^-)\,\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2}\end{aligned}$$

[2]One may argue that the condition (3.8) is actually two conditions, due to its (two-)vector nature, and it may over-constrain the system, whereas the condition (3.6) is only one condition, being a scalar under rotations in the transverse plane. However, presently we can not construct a proof of this conjecture in the general case.

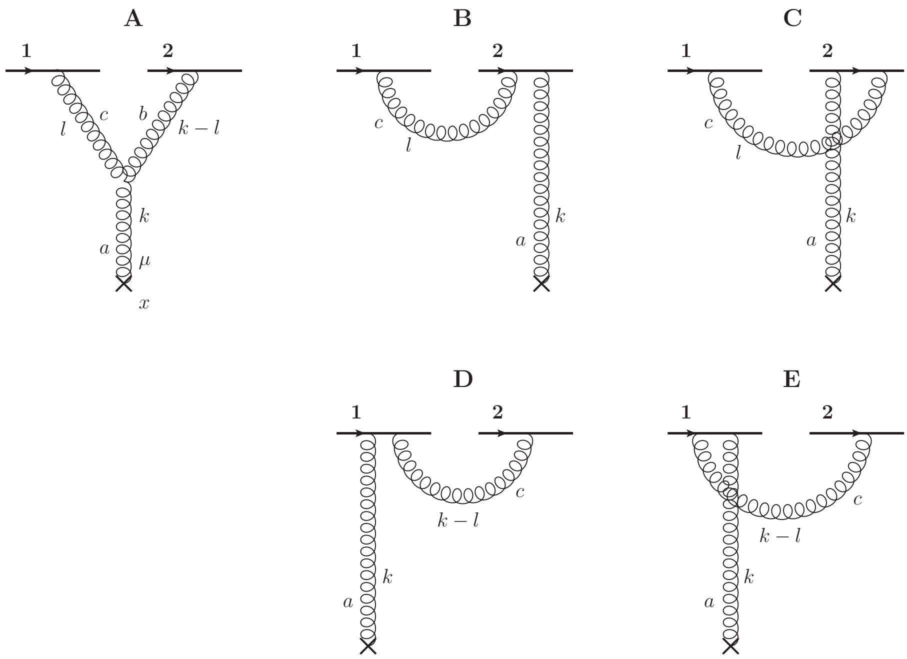

Figure 2. Diagrammatic representation of the classical gluon field of two quarks at the order g^3.

in agreement with eq. (5.10).

Now let us explore the next-to-lowest order. Diagrams contributing to the order-g^3 classical field are shown in figure 2 (cf. [4]). A straightforward but a little more tedious calculation yields (in $k^+, \vec{k}_\perp$ momentum space)

$$A = i\, g^3\, f^{abc}\, (t^b)_2\, (t^c)_1 \frac{1}{k_\perp^2\, l_\perp^2\, (\vec{k}_\perp - \vec{l}_\perp)^2} \left[\frac{-k_\perp^2\, l_\perp^\mu + \vec{k}_\perp \cdot \vec{l}_\perp\, k_\perp^\mu}{l^+\,(k^+ - l^+)} + \frac{\vec{l}_\perp \cdot (\vec{k}_\perp - \vec{l}_\perp)\, k_\perp^\mu\, (k^+ - 2l^+)}{k^+\, l^+\, (k^+ - l^+)} \right], \tag{5.25a}$$

$$B + C = i\, g^3\, f^{abc}\, (t^b)_2\, (t^c)_1 \frac{k_\perp^\mu}{k_\perp^2\, l_\perp^2}\, \frac{1}{k^+\, l^+}, \tag{5.25b}$$

$$D + E = -i\, g^3\, f^{abc}\, (t^b)_2\, (t^c)_1 \frac{k_\perp^\mu}{k_\perp^2\, (\vec{k}_\perp - \vec{l}_\perp)^2}\, \frac{1}{k^+\, (k^+ - l^+)}. \tag{5.25c}$$

The light-cone gauge gluon field due to the sum of the diagrams A through E is

$$\begin{aligned} \vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) = t^a \int \frac{d^2 k_\perp\, dk^+}{(2\pi)^3}\, \frac{d^2 l_\perp\, dl^+}{(2\pi)^3} \\ \times e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-) + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{b}_{2\perp}) + i\vec{l}_\perp \cdot (\vec{b}_{2\perp} - \vec{b}_{1\perp})}\, i\, g^3\, f^{abc}\, (t^b)_2\, (t^c)_1 \\ \times \frac{1}{k_\perp^2\, l_\perp^2\, (\vec{k}_\perp - \vec{l}_\perp)^2} \left[\frac{-k_\perp^2\, l_\perp^\mu + \vec{k}_\perp \cdot \vec{l}_\perp\, k_\perp^\mu}{l^+\,(k^+ - l^+)} + \frac{\vec{l}_\perp \cdot (\vec{k}_\perp - \vec{l}_\perp)\, k_\perp^\mu\, (k^+ - 2l^+)}{k^+\, l^+\, (k^+ - l^+)} + \frac{(\vec{k}_\perp - \vec{l}_\perp)^2\, k_\perp^\mu}{k^+\, l^+} - \frac{l_\perp^2\, k_\perp^\mu}{k^+\, (k^+ - l^+)} \right]. \end{aligned} \tag{5.26}$$

The regularization of all light-cone singularities in eq. (5.26) is (implicitly) PV. All Fourier transforms in eq. (5.26) are well-defined, except for the second term in the square brackets. There, the integral over k^+ and l^+ contains pinched poles. If we were regulating all the light-cone singularities by using the PV prescription *ad hoc*, with different $i\epsilon$'s for

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different poles, this integral would have been ill-defined, being strongly dependent on the order in which different ϵ's are sent to zero. However, since all our light-cone propagators (3.1) follow from the same generating functional (2.16), they all come with the same $i\epsilon$'s. Hence, as a result of our calculation in section 3 we have a specific prescription for the pinched-pole integral in question: use the same $i\epsilon$'s for all the light-cone poles in all the gluon propagators involved. Note that this prescription was used before in the diagrammatic calculation of next-to-leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [23–25] anomalous dimensions in [11]: here we hope to have provided a justification for this prescription.

To illustrate our prescription explicitly, let us first perform all the Fourier transforms in eq. (5.26) except for the pinched-pole integral. We get

$$\begin{aligned}
\vec{A}_{\perp}^{LC}(x^-,\vec{x}_\perp) = &- \frac{g^3}{4(2\pi)^2}\, t^a\, f^{abc}\, (t^b)_2\, (t^c)_1 \Bigg\{ \frac{1}{2} \text{Sign}(x^- - b_1^-)\, \text{Sign}(x^- - b_2^-) \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln\left(\frac{|\vec{x}_\perp - \vec{b}_{1\perp}|}{|\vec{b}_{1\perp} - \vec{b}_{2\perp}|} \right) \right. \\
&\left. - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln\left(\frac{|\vec{x}_\perp - \vec{b}_{2\perp}|}{|\vec{b}_{1\perp} - \vec{b}_{2\perp}|} \right) \right] \\
&+ \text{Sign}(b_2^- - b_1^-)\, \text{Sign}(x^- - b_2^-)\, \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln\left(|\vec{b}_{1\perp} - \vec{b}_{2\perp}|\, \Lambda \right) \\
&+ \text{Sign}(b_2^- - b_1^-)\, \text{Sign}(x^- - b_1^-)\, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln\left(|\vec{b}_{1\perp} - \vec{b}_{2\perp}|\, \Lambda \right) \\
&- 4 \int \frac{dk^+\, dl^+}{(2\pi)^2}\, e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-)}\, \frac{k^+ - 2l^+}{k^+\, l^+\, (k^+ - l^+)}\, \frac{1}{2} \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln\left(\frac{|\vec{x}_\perp - \vec{b}_{1\perp}|}{|\vec{b}_{1\perp} - \vec{b}_{2\perp}|} \right) \right. \\
&\left. + \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln\left(\frac{|\vec{x}_\perp - \vec{b}_{2\perp}|}{|\vec{b}_{1\perp} - \vec{b}_{2\perp}|} \right) \right] \Bigg\} .
\end{aligned} \tag{5.27}$$

Using the same $i\epsilon$'s to regulate all the poles in the pinched integral (similar to [11]) while using the PV prescription we get

$$\begin{aligned}
-4 \int \frac{dk^+\, dl^+}{(2\pi)^2}\, e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-)}\, \frac{k^+ - 2l^+}{k^+\, l^+\, (k^+ - l^+)} &= -4 \int \frac{dk^+\, dl^+}{(2\pi)^2}\, e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-)}\, \frac{(k^+ - l^+) - l^+}{k^+\, l^+\, (k^+ - l^+)} \\
&= \text{Sign}(b_2^- - b_1^-) \left[\text{Sign}(x^- - b_1^-) + \text{Sign}(x^- - b_2^-) \right] .
\end{aligned} \tag{5.28}$$

As one can show, using eq. (5.28) in eq. (5.27) gives eq. (5.21). (An identity

$$1 = \text{Sign}(x^- - b_1^-)\, \text{Sign}(x^- - b_2^-) + \text{Sign}(b_2^- - b_1^-) \left[\text{Sign}(x^- - b_1^-) - \text{Sign}(x^- - b_2^-) \right] \tag{5.29}$$

comes in handy.) Hence the same-$i\epsilon$'s prescription is a diagrammatic equivalent of using the sub-gauge condition (3.6) in the classical field calculations.

6 Summary

In this paper we have studied the question of whether the ambiguity associated with the regularization of the poles of the light-cone gauge gluon propagator can be eliminated by fixing the residual gauge freedom using a sub-gauge condition. We saw that this is indeed the case for the θ-function sub-gauges and for the PV sub-gauge. In the process we have elucidated the proper sub-gauge condition for the PV sub-gauge. Our main results for the

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propagators and for the sub-gauge conditions are given in (and above) eqs. (2.28) and (2.29) for the θ-function sub-gauges and by eqs. (3.1) and (3.6) for the PV sub-gauge.

We have also shown that one can construct the classical gluon field of a single ultra-relativistic nucleus in the PV sub-gauge: our perturbative calculation for the two ultra-relativistic color charges resulted in eq. (5.21) for the gluon field. Moreover, it appears that we have constructed a justification for the same-$i\epsilon$'s prescription for dealing with the light-cone gauge gluon propagator poles in the PV sub-gauge.

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A On the Lorenz-type sub-gauge condition

In section 2 we have imposed (2.1) as the sub-gauge condition requiring the transverse divergence of the gauge field to be zero at a generic point $x^- = \sigma$. An alternative sub-gauge condition is for the four-divergence to be zero at a generic point $x^- = \sigma$:

$$\partial_\mu A^\mu (x^- = \sigma) = 0 \, . \tag{A.1}$$

Here we will show that the sub-gauge choice (A.1) is not suitable for specifying the prescription of the $k^+ = 0$ pole of the light-cone gauge gluon propagator.

The propagator with sub-gauge condition (A.1) should satisfy the following differential equation (cf. eq. (2.13))

$$\left[\partial^2 g_{\mu\rho} - \partial_\mu \partial_\rho - \frac{1}{\xi_1} \eta_\mu \eta_\rho + \frac{1}{\xi_2} \partial_\mu \delta(x^- - \sigma) \partial_\rho \right] D^{\rho\nu}(x,y) = i \, \delta_\mu^\nu \, \delta^{(4)}(x-y) \, . \tag{A.2}$$

Note that ∂_μ to the left of the delta-function in eq. (A.2) acts on everything to its right.

Projecting eq. (A.2) onto $\eta^\mu \, \tilde{\eta}_\nu$ we get (cf. eq. (75) in [14])

$$\partial^+ \left[\left(1 - \frac{1}{\xi_2} \, \delta(x^- - \sigma) \right) \partial_\rho \, D^{\rho -}(x,y) \right] + \partial^2 D^{+-}(x,y) = -2i \, \delta^{(4)}(x-y). \tag{A.3}$$

This equation has no solution for finite σ. To see this one can integrate both sides over x^- in an infinitesimal interval near $x^- = \sigma$: the contribution of the δ-function term on the left-hand side of eq. (A.3) to such an integral is ill-defined, as it contains $\delta(x^- - \sigma)\Big|_{x^- = \sigma - \epsilon}^{x^- = \sigma + \epsilon}$. (If we assume that $\delta(x^- - \sigma)\Big|_{x^- = \sigma - \epsilon}^{x^- = \sigma + \epsilon} = 0$ we can simply drop the second term in eq. (A.3): however, we are not going to get a regularization of the $k^+ = 0$ poles this way.) The only way to avoid this ambiguity is to require that $\partial_\rho \, D^{\rho -}(x^- = \sigma, y) = 0$, which may only be true for $\sigma = \pm\infty$ (see a similar discussion near eq. (2.18) in the main text). This would result in the propagators (2.28) or (2.29) corresponding to $\sigma = \pm\infty$. However, for $\sigma = \pm\infty$ we saw in eq. (2.32) that the sub-gauge condition (A.1) does not give zero, and hence does not work.

To summarize, we see that for finite σ no solution of eq. (A.2) exists, while for $\sigma = \pm\infty$ the solution does not satisfy the boundary condition in eq. (2.32). From this we conclude that $\partial_\mu A^\mu(x^- = \sigma) = 0$ is not a suitable sub-gauge condition for the light-cone gauge in the functional integral formalism.

For pedagogical reasons let us arrive at the same conclusion using a slightly different technique. It is convenient to introduce the following two linearly independent tensors structures orthogonal to η^μ,

$$a^{\mu\nu} \equiv g^{\mu\nu} - \frac{\partial^\mu \eta^\nu + \partial^\nu \eta^\mu}{\eta \cdot \partial} + \frac{\partial^2 \eta^\mu \eta^\nu}{(\eta \cdot \partial)^2} - \frac{\xi_1 \partial^2 \partial^\mu \partial^\nu}{(\eta \cdot \partial)^2}, \tag{A.4a}$$

$$b^{\mu\nu} \equiv -\frac{\partial^2}{(\eta \cdot \partial)^2} \eta^\mu \eta^\nu \,, \tag{A.4b}$$

so that we can decompose $D^{\mu\nu}(x,y)$ (with the $A^+ = 0$ gauge condition imposed) as

$$D^{\mu\nu}(x,y) = a^{\mu\nu}\, a(x,y) + b^{\mu\nu}\, b(x,y) \tag{A.5}$$

with functions $a(x,y)$ and $b(x,y)$ to be determined.

Using (A.5) in (A.2) we have

$$\begin{aligned} &\left[\delta^\nu_\mu - \frac{\partial_\mu \eta^\nu}{\partial^+} + \partial^2 \frac{\eta_\mu \eta^\nu}{\partial^{+2}} - \frac{\xi_1}{\xi_2} \partial_\mu \delta(x^- - \sigma) \partial^\nu \frac{\partial^2}{\partial^{+2}}\right] \partial^2 a(x,y) \\ &+ \left[-\frac{\partial^2 \eta_\mu \eta^\nu}{\partial^{+2}} + \frac{\partial_\mu \eta^\nu}{\partial^+} - \frac{1}{\xi_2} \partial_\mu \delta(x^- - \sigma) \frac{\eta^\nu}{\partial^+}\right] \partial^2 b(x,y) = i\, \delta^{(4)}(x-y)\, \delta^\nu_\mu \,. \end{aligned} \tag{A.6}$$

Projecting eq. (A.6) onto $\eta^\mu\, \tilde{\eta}_\nu$ again we get (cf. eq. (A.3))

$$\left[1 - \frac{1}{\xi_2} \partial^+ \delta(x^- - \sigma) \frac{1}{\partial^+}\right] \partial^2 b(x,y) = i\, \delta^{(4)}(x-y). \tag{A.7}$$

(Note that we have set ξ_1 to zero because at this point the light-cone gauge has already been employed.)

Just like eq. (A.3), equation (A.7) does not provide any prescription for the $k^+ = 0$ pole for any finite σ. For $\sigma = \pm\infty$ we have already seen that sub-gauge condition (A.1) is not compatible with the path integral formalism. From this analysis we again conclude that the sub-gauge $\partial_\mu A^\mu(x^- = \sigma) = 0$ is not a suitable sub-gauge of the light-cone gauge in the functional integral formalism.

B Contribution of the Feynman pole at x^- boundary

In this appendix we provide details of the calculation carried out in eq. (2.32) (as well as those in eqs. (3.5)) and (4.4)). More specifically, in the transition from the second to the third line of eq. (2.32) we neglected the contributions of the $k^2 = 0$ Feynman pole. To justify this let us consider the Fourier transform of the terms in the square brackets of the second line of eq. (2.32). The first term is not affected by the k^+ prescription and it is zero at $x^- = \pm\infty$:

$$\int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{k^-}{k^2+i\epsilon}\, A^\mu(x)\Bigg|_{x^-=-\infty}^{x^-=+\infty} = \frac{1}{2\pi^2}\, \frac{x^- - y^-}{[(x-y)^2 - i\epsilon]^2}\, A^\mu(x)\Bigg|_{x^-=-\infty}^{x^-=+\infty} = 0. \quad \text{(B.1)}$$

In arriving at zero on the right-hand side of (B.1) we assume that $A^\mu(x)/x^- \to 0$ as $x^- \to \infty$, that is that $A^\mu(x)$ grows slower than $|x^-|$ as $x^- \to \infty$. Note that the expression in eq. (B.1) is zero at each limiting point, $x^- = +\infty$ and $x^- = -\infty$, separately.

To understand the x^--dependence of the second terms in the square brackets of the second line of eq. (2.32), note that $k \cdot A(x) = k^+ A^-(x) - \vec{k}_\perp \cdot \vec{A}_\perp(x)$ in $A^+ = 0$ light-cone gauge. (Once again we assume that the $\xi_1 \to 0$ limit is taken in eq. (2.10) enforcing the gauge condition.) The $k^+ A^-(x)$ term vanishes due to eq. (B.1) along with

$$\int \frac{d^4k}{(2\pi)^4}\, \frac{k_\perp^\mu}{k^2+i\epsilon}\, e^{-ik\cdot(x-y)} = \frac{(x-y)_\perp^\mu}{2\pi^2[(x-y)^2 - i\epsilon]^2}. \quad \text{(B.2)}$$

To find the contribution of the $\vec{k}_\perp \cdot \vec{A}_\perp(x)$ we use the following integral

$$\begin{aligned}\int \frac{d^4k}{(2\pi)^4}\, \frac{k_\perp^i\, e^{-ik\cdot(x-y)}}{(k^2+i\epsilon)(k^+ + i\epsilon)} &= -\frac{(x-y)_\perp^i}{2\pi(x-y)_\perp^2}\, \theta(x^- - y^-)\, \delta(x^+ - y^+) \\ &+ \frac{i\,(x-y)_\perp^i}{2\pi^2 (x-y)_\perp^2}\, (x^- - y^-) \left[\frac{1}{(x-y)^2 - i\epsilon} - \frac{1}{2(x^+ - y^+)(x^- - y^-) - i\epsilon}\right]. \quad \text{(B.3)}\end{aligned}$$

Note that the $k^2 = 0$ pole gives the second term on the right-hand side, which vanishes as $x^- \to \infty$. Rewriting $k^-\eta^\mu + k_\perp^\mu \to i\partial^-\eta^\mu + i\partial_\perp^\mu$ (all derivatives are with respect to x) and noticing that applying derivatives to the second term on the right-hand side of eq. (B.3) would still leave it vanishing at $x^- \to \infty$, we complete the justification of neglecting the contributions of the $k^2 = 0$ pole in going from the second to the third line of eq. (2.32). (Once again we have to assume that $A^\mu(x)/x^- \to 0$ as $x^- \to \infty$.) The first term in eq. (B.3) does not vanish for $x^- \to +\infty$: this term is due to picking up the $k^+ = 0$ pole and is the one giving us the third line of eq. (2.32).

The conclusion reached here about the $k^2 = 0$ pole contribution vanishing at $x^- \to \infty$ is independent of the regularization of the $k^+ = 0$ pole and thus applies to PV and ML sub-gauges as well.

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