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## Transition magnetic moment of Majorana neutrinos in the $\mu\nu$ SSM

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**ABSTRACT:** The nonzero vacuum expectative values of sneutrinos induce spontaneously R-parity and lepton number violation, and generate three tiny Majorana neutrino masses through the seesaw mechanism in the  $\mu\nu$ SSM, which is one of Supersymmetric extensions beyond Standard Model. Applying effective Lagrangian method, we study the transition magnetic moment of Majorana neutrinos in the model here. Under the constraints from neutrino oscillations, we consider the two possibilities on the neutrino mass spectrum with normal or inverted ordering.

**KEYWORDS:** Neutrino Physics, Supersymmetric Effective Theories, Beyond Standard Model, Supersymmetric Standard Model

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**Contents**

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The <math>\mu\nu</math>SSM</b>	<b>2</b>
<b>3</b>	<b>Neutrino magnetic moment</b>	<b>4</b>
<b>4</b>	<b>The numerical results</b>	<b>7</b>
4.1	The parameter space	7
4.2	Transition magnetic moment of Majorana neutrinos	10
<b>5</b>	<b>Conclusions</b>	<b>12</b>
<b>A</b>	<b>Mass Matrices</b>	<b>12</b>
A.1	Charged scalar mass matrix	12
A.2	Neutral fermion mass matrix	14
A.3	Charged fermion mass matrix	15
<b>B</b>	<b>Couplings</b>	<b>16</b>
B.1	Charged fermion-neutral fermion-gauge boson	16
B.2	Charged fermion-neutral fermion-charged scalar	16
<b>C</b>	<b>Form factors</b>	<b>17</b>

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**1 Introduction**

In the Standard Model (SM), neutrinos have only weak interaction and are massless. However, the observations of neutrino oscillations (see refs. [1–4]) imply that neutrinos have very small masses and are mixed. Therefore, the SM must be extended to account for the neutrino masses and mixing. In any extensions of the SM, the transition magnetic moments of neutrinos are induced through electroweak radiative corrections. In addition, the transition magnetic moments of neutrinos can generate important effects, especially in astrophysical environments, where neutrinos propagate for long distances in magnetic fields both in matter and in vacuum [5].

Applying effective Lagrangian method and on-shell scheme, here we analyze the radiative contributions from one-loop diagrams to the transition magnetic moment of Majorana neutrinos in the “ $\mu$  from  $\nu$  Supersymmetric Standard Model” ( $\mu\nu$ SSM) [6–8], where nonzero vacuum expectative values (VEVs) of sneutrinos lead to R-parity and lepton number violations. The  $\mu\nu$ SSM could generate three tiny massive Majorana neutrinos at the tree level through the mixing with the neutralinos and right-handed neutrinos [6–13]. Especially, the

$\mu$  problem [14] of the Minimal Supersymmetric Standard Model (MSSM) [15–19] had been solved in the  $\mu\nu$ SSM, through the R-parity breaking couplings  $\lambda_i \hat{\nu}_i^c \hat{H}_d^a \hat{H}_u^b$  in the superpotential. The  $\mu$  term is generated spontaneously via the nonzero VEVs of right-handed sneutrinos,  $\mu = \lambda_i \langle \hat{\nu}_i^c \rangle$ , when the electroweak symmetry is broken (EWSB).

Three flavor neutrinos  $\nu_{e,\mu,\tau}$  are mixed into the mass eigenstates  $\nu_{1,2,3}$  during their flight, and the mixing is described by the Pontecorvo-Maki-Nakagawa-Sakata unitary matrix  $U_{PMNS}$  [20, 21]. Through the several recently updated neutrino oscillation experiments T2K [22], MINOS [23], DOUBLE CHOOZ [24], RENO [25] and especially Daya Bay [26], the neutrino mixing angle  $\theta_{13}$  is now precisely known. One can give fitted value for  $\theta_{13}$  as [27]

$$\sin^2 2\theta_{13} = 0.090 \pm 0.009. \quad (1.1)$$

The other experimental observations of the neutrino oscillation parameters in  $U_{PMNS}$  show that [28]

$$\begin{aligned} \Delta m_{\odot}^2 &= 7.58_{-0.26}^{+0.22} \times 10^{-5} \text{eV}^2, & \sin^2 \theta_{12} &= 0.306_{-0.015}^{+0.018}, \\ |\Delta m_{A}^2| &= 2.35_{-0.09}^{+0.12} \times 10^{-3} \text{eV}^2, & \sin^2 \theta_{23} &= 0.42_{-0.03}^{+0.08}. \end{aligned} \quad (1.2)$$

In our numerical analysis, we use the neutrino experimental data presented in eq. (1.1) and eq. (1.2) to constrain the input parameters in the model here. Assuming neutrino mass spectrum with normal or inverted ordering, we analyse the transition magnetic moment of Majorana neutrinos in the  $\mu\nu$ SSM.

The outline of the paper is as follow. In section 2, we outline the  $\mu\nu$ SSM by introducing its superpotential and the general soft SUSY-breaking terms. In section 3, we will give formulae of the transition magnetic moment of Majorana neutrinos in the  $\mu\nu$ SSM, applying effective Lagrangian method and on-shell scheme. The results of our numerical study are given in section 4. And the conclusions are left for section 5. Finally, the relative mass matrices, couplings and form factors are collected in appendices A–C.

## 2 The $\mu\nu$ SSM

Besides the superfields of the MSSM, the  $\mu\nu$ SSM introduces three singlet right-handed neutrino superfields  $\hat{\nu}_i^c$  ( $i = 1, 2, 3$ ). The corresponding superpotential of the  $\mu\nu$ SSM is given by [6]

$$\begin{aligned} W = & \epsilon_{ab} \left( Y_{u_{ij}} \hat{H}_u^b \hat{Q}_i^a \hat{u}_j^c + Y_{d_{ij}} \hat{H}_d^a \hat{Q}_i^b \hat{d}_j^c + Y_{e_{ij}} \hat{H}_d^a \hat{L}_i^b \hat{e}_j^c + Y_{\nu_{ij}} \hat{H}_u^b \hat{L}_i^a \hat{\nu}_j^c \right) \\ & - \epsilon_{ab} \lambda_i \hat{\nu}_i^c \hat{H}_d^a \hat{H}_u^b + \frac{1}{3} \kappa_{ijk} \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c, \end{aligned} \quad (2.1)$$

where  $\hat{H}_d^T = (\hat{H}_d^0, \hat{H}_d^-)$ ,  $\hat{H}_u^T = (\hat{H}_u^+, \hat{H}_u^0)$ ,  $\hat{Q}_i^T = (\hat{u}_i, \hat{d}_i)$ ,  $\hat{L}_i^T = (\hat{\nu}_i, \hat{e}_i)$  are SU(2) doublet superfields, and  $\hat{d}_i^c$ ,  $\hat{u}_i^c$  and  $\hat{e}_i^c$  represent the singlet down-type quark, up-type quark and charged lepton superfields, respectively. In addition,  $Y_{u,d,\nu,e}$ ,  $\lambda$ ,  $\kappa$  respectively are dimensionless matrices, a vector, a totally symmetric tensor. And  $a, b$  are SU(2) indices

with antisymmetric tensor  $\epsilon_{12} = -\epsilon_{21} = 1$ . The summation convention is implied on repeated indices in this paper. In the superpotential, the first three terms are the same as the MSSM. Next two terms can generate the effective bilinear terms  $\epsilon_{ab}\epsilon_i \hat{H}_u^b \hat{L}_i^a$ ,  $\epsilon_{ab}\mu \hat{H}_d^a \hat{H}_u^b$ , and  $\epsilon_i = Y_{\nu_{ij}} \langle \tilde{\nu}_j^c \rangle$ ,  $\mu = \lambda_i \langle \tilde{\nu}_i^c \rangle$ , once the electroweak symmetry is broken. The last term generates the effective Majorana masses for neutrinos at the electroweak scale. And the last two terms explicitly violate lepton number and R-parity.

In the framework of supergravity mediated supersymmetry breaking, the general soft SUSY-breaking terms in the  $\mu\nu$ SSM are given as

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}} = & m_{\tilde{Q}_{ij}}^2 \tilde{Q}_i^{a*} \tilde{Q}_j^a + m_{\tilde{u}_{ij}^c}^2 \tilde{u}_i^{c*} \tilde{u}_j^c + m_{\tilde{d}_{ij}^c}^2 \tilde{d}_i^{c*} \tilde{d}_j^c + m_{\tilde{L}_{ij}}^2 \tilde{L}_i^{a*} \tilde{L}_j^a \\
 & + m_{\tilde{e}_{ij}^c}^2 \tilde{e}_i^{c*} \tilde{e}_j^c + m_{H_d}^2 H_d^{a*} H_d^a + m_{H_u}^2 H_u^{a*} H_u^a + m_{\tilde{\nu}_{ij}^c}^2 \tilde{\nu}_i^{c*} \tilde{\nu}_j^c \\
 & + \epsilon_{ab} \left[ (A_u Y_u)_{ij} H_u^b \tilde{Q}_i^a \tilde{u}_j^c + (A_d Y_d)_{ij} H_d^a \tilde{Q}_i^b \tilde{d}_j^c + (A_e Y_e)_{ij} H_d^a \tilde{L}_i^b \tilde{e}_j^c + \text{H.c.} \right] \\
 & + \left[ \epsilon_{ab} (A_\nu Y_\nu)_{ij} H_u^b \tilde{L}_i^a \tilde{\nu}_j^c - \epsilon_{ab} (A_\lambda \lambda)_i \tilde{\nu}_i^c H_d^a H_u^b + \frac{1}{3} (A_\kappa \kappa)_{ijk} \tilde{\nu}_i^c \tilde{\nu}_j^c \tilde{\nu}_k^c + \text{H.c.} \right] \\
 & - \frac{1}{2} \left( M_3 \tilde{\lambda}_3 \tilde{\lambda}_3 + M_2 \tilde{\lambda}_2 \tilde{\lambda}_2 + M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + \text{H.c.} \right). \tag{2.2}
 \end{aligned}$$

Here, the first two lines consist of mass squared terms of squarks, sleptons and Higgses. The next two lines contain the trilinear scalar couplings. In the last lines,  $M_3$ ,  $M_2$  and  $M_1$  denote Majorana masses corresponding to SU(3), SU(2) and U(1) gauginos  $\hat{\lambda}_3$ ,  $\hat{\lambda}_2$  and  $\hat{\lambda}_1$ , respectively. In addition to the terms from  $\mathcal{L}_{\text{soft}}$ , the tree-level scalar potential receives the usual D and F term contributions [7].

Once the electroweak symmetry is spontaneously broken, the neutral scalars develop in general the following VEVs:

$$\langle H_d^0 \rangle = v_d, \quad \langle H_u^0 \rangle = v_u, \quad \langle \tilde{\nu}_i \rangle = v_{\nu_i}, \quad \langle \tilde{\nu}_i^c \rangle = v_{\nu_i^c}. \tag{2.3}$$

Thus one can define neutral scalars as usual

$$\begin{aligned}
 H_d^0 &= \frac{h_d + iP_d}{\sqrt{2}} + v_d, & \tilde{\nu}_i &= \frac{(\tilde{\nu}_i)^{\Re} + i(\tilde{\nu}_i)^{\Im}}{\sqrt{2}} + v_{\nu_i}, \\
 H_u^0 &= \frac{h_u + iP_u}{\sqrt{2}} + v_u, & \tilde{\nu}_i^c &= \frac{(\tilde{\nu}_i^c)^{\Re} + i(\tilde{\nu}_i^c)^{\Im}}{\sqrt{2}} + v_{\nu_i^c}.
 \end{aligned} \tag{2.4}$$

And one can define

$$\tan \beta = \frac{v_u}{\sqrt{v_d^2 + v_{\nu_i} v_{\nu_i}}}. \tag{2.5}$$

For simplicity, we will assume that all parameters in the potential are real in the model. After EWSB, the charge scalar mass matrix  $M_{S^\pm}^2$ , neutral fermion mass matrix  $M_n$  and charged fermion mass matrix  $M_c$  are given in appendix A. The charged scalar mass matrix  $M_{S^\pm}^2$  contains massless unphysical Goldstone bosons  $G^\pm$ , which can be written as [29–31]

$$G^\pm = \frac{1}{\sqrt{v_u^2 + v_d^2 + v_{\nu_i} v_{\nu_i}}} \left( v_d H_d^\pm - v_u H_u^\pm + v_{\nu_i} \tilde{e}_{L_i}^\pm \right). \tag{2.6}$$

In the physical gauge, the Goldstone bosons  $G^\pm$  are eaten by  $W$ -boson, and disappear from the Lagrangian. And the mass squared of  $W$ -boson is

$$m_W^2 = \frac{e^2}{2s_W^2} (v_u^2 + v_d^2 + v_{\nu_i} v_{\nu_i}), \quad (2.7)$$

where  $e$  is the electromagnetic coupling constant,  $s_W = \sin \theta_W$  (and  $c_W = \cos \theta_W$  below) with the Weinberg angle  $\theta_W$ , respectively.

In the  $\mu\nu$ SSM, left-handed neutrinos mix with the neutralinos and right-handed neutrinos. Via the seesaw mechanism [6], the effective light neutrino mass matrix is in general given as

$$m_{\text{eff}} = -m.M^{-1}.m^T, \quad (2.8)$$

where the concrete expressions for the mass matrices  $M$  and  $m$  are given in appendix A. Diagonalized the effective neutrino mass matrix  $m_{\text{eff}}$ , we can obtain three light neutrino masses.

### 3 Neutrino magnetic moment

The magnetic dipole moment (MDM) and electric dipole moment (EDM) of the Dirac fermion (including charged lepton and neutrino etc) can be actually be written as the operators

$$\begin{aligned} \mathcal{L}_{MDM} &= \frac{1}{2} \mu_{ij} \bar{\psi}_i \sigma^{\mu\nu} \psi_j F_{\mu\nu}, \\ \mathcal{L}_{EDM} &= \frac{i}{2} \epsilon_{ij} \bar{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_j F_{\mu\nu}, \end{aligned} \quad (3.1)$$

where  $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ ,  $F_{\mu\nu}$  is the electromagnetic field strength,  $\psi_{i,j}$  denote the four-component Dirac fermions which are on-shell,  $\mu_{ij}$  and  $\epsilon_{ij}$  are Dirac diagonal ( $i = j$ ) or transition ( $i \neq j$ ) MDM and EDM between states  $\psi_i$  and  $\psi_j$ , respectively.

In fact, it convenient to get the contributions from loop diagrams to fermion diagonal or transition MDM and EDM in terms of the effective Lagrangian method, if the masses  $m_V$  of internal lines are much heavier than the external fermion mass  $m_f$ . Since  $\not{p} = m_f \ll m_V$  for on-shell fermion and  $\not{k} \rightarrow 0 \ll m_V$  for photon, we can expand the amplitude of corresponding triangle diagrams according to the external momenta of fermion and photon. After matching between the effective theory and the full theory, we get all high dimension operators together with their coefficients. It is enough to retain only those dimension 6 operators in later calculations [32–34]:

$$\begin{aligned} O_1^{L,R} &= e \bar{\psi}_i (i\mathcal{D})^3 P_{L,R} \psi_j, \\ O_2^{L,R} &= e \overline{(i\mathcal{D}_\mu \psi_i)} \gamma^\mu F \cdot \sigma P_{L,R} \psi_j, \\ O_3^{L,R} &= e \bar{\psi}_i F \cdot \sigma \gamma^\mu P_{L,R} (i\mathcal{D}_\mu \psi_j), \\ O_4^{L,R} &= e \bar{\psi}_i (\partial^\mu F_{\mu\nu}) \gamma^\nu P_{L,R} \psi_j, \\ O_5^{L,R} &= e m_{\psi_i} \bar{\psi}_i (i\mathcal{D})^2 P_{L,R} \psi_j, \\ O_6^{L,R} &= e m_{\psi_i} \bar{\psi}_i F \cdot \sigma P_{L,R} \psi_j, \end{aligned} \quad (3.2)$$

where  $\mathcal{D}_\mu = \partial^\mu + ieA_\mu$ ,  $P_L = \frac{1}{2}(1 - \gamma_5)$ ,  $P_R = \frac{1}{2}(1 + \gamma_5)$  and  $m_{\psi_i}$  is the mass of fermion  $\psi_i$ .

Certainly, all dimension 6 operators in eq. (3.2) induce the effective couplings among photons and fermions. The effective vertices with one external photon are written as

$$\begin{aligned}
O_1^{L,R} &= ie\{((p+k)^2 + p^2)\gamma_\rho + (\not{p} + \not{k})\gamma_\rho\not{p}\}P_{L,R}, \\
O_2^{L,R} &= ie(\not{p} + \not{k})[\not{k}, \gamma_\rho]P_{L,R}, \\
O_3^{L,R} &= ie[\not{k}, \gamma_\rho]\not{p}P_{L,R}, \\
O_4^{L,R} &= ie(k^2\gamma_\rho - \not{k}k_\rho)P_{L,R}, \\
O_5^{L,R} &= iem_{\psi_i}\{(\not{p} + \not{k})\gamma_\rho + \gamma_\rho\not{p}\}P_{L,R}, \\
O_6^{L,R} &= iem_{\psi_i}[\not{k}, \gamma_\rho]P_{L,R}.
\end{aligned} \tag{3.3}$$

If the full theory is invariant under the combined transformation of charge conjugation, parity and time reversal (CPT), the induced effective theory preserves the symmetry after the heavy freedoms are integrated out. The fact implies the Wilson coefficients of the operators  $O_{2,3,6}^{L,R}$  satisfying the relations [32]

$$C_3^{L,R} = C_2^{R,L*}, \quad C_6^L = C_6^{R*}, \tag{3.4}$$

where  $C_I^{L,R}$  ( $I = 1 \cdots 6$ ) represent the Wilson coefficients of the corresponding operators  $O_I^{L,R}$  in the effective Lagrangian. After applying the equations of motion to the external fermions, we find that the concerned terms in the effective Lagrangian are transformed into

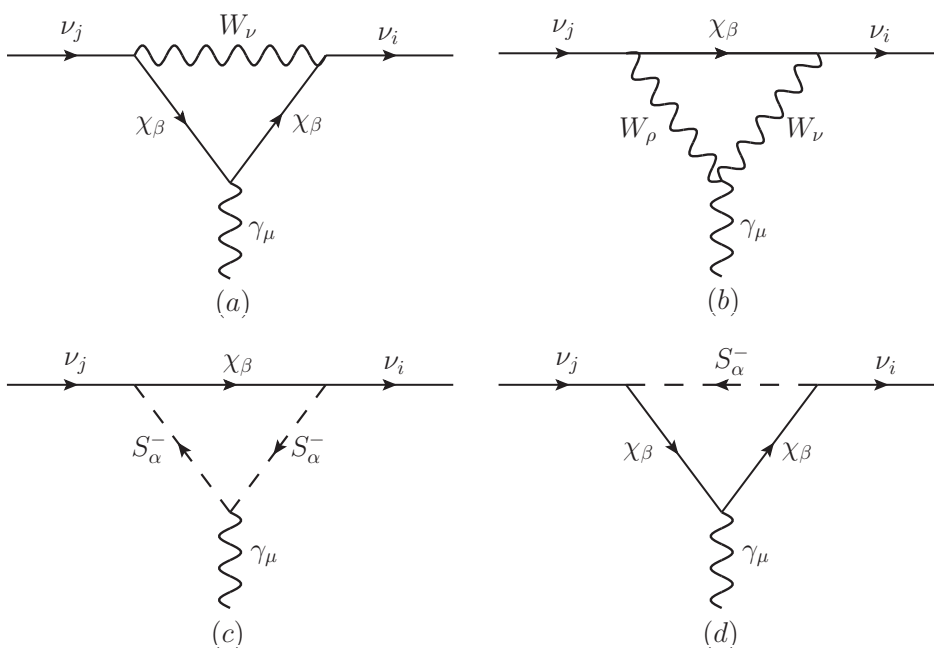
$$\begin{aligned}
&C_2^R O_2^R + C_2^L O_2^L + C_2^{L*} O_3^R + C_2^{R*} O_3^L + C_6^R O_6^R + C_6^{R*} O_6^L \\
\Rightarrow &(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) O_6^R + (C_2^{R*} + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^L + C_6^{R*}) O_6^L \\
= &em_{\psi_i} \Re(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \bar{\psi}_i \sigma^{\mu\nu} \psi_j F_{\mu\nu} \\
&+ iem_{\psi_i} \Im(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \bar{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_j F_{\mu\nu},
\end{aligned} \tag{3.5}$$

where,  $\Re(\cdots)$  and  $\Im(\cdots)$  denote the operation to take the real and imaginary part of a complex number, respectively. Matching between eq. (3.1) and eq. (3.5), we can obtain

$$\begin{aligned}
\mu_{ij} &= 4m_e m_{\psi_i} \Re(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \mu_B, \\
\epsilon_{ij} &= 4m_e m_{\psi_i} \Im(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \mu_B,
\end{aligned} \tag{3.6}$$

where  $\mu_B = e/2m_e$  and  $m_e$  is the electron mass. In other words, the MDM and EDM of the Dirac fermions are respectively proportional to real and imaginary part of the effective coupling  $C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R$ .

As a neutral fermion, the mass eigenstate of neutrino, may be not just Dirac field but also Majorana field. The Majorana neutrino coincides with its antiparticle. The four degrees of freedom of a Dirac neutrino (two helicities and two particle-antiparticle) are reduced to two (two helicities) by the Majorana constraint. The electromagnetic properties of the Majorana neutrino are possibly reduced, because the Majorana neutrino just has



**Figure 1.** One-loop diagrams contributing to transition magnetic moment of Majorana neutrinos in the  $\mu\nu$ SSM, where (a) and (b) represent the charged fermions  $\chi_\beta$  and  $W$ -boson loop contributions, (c) and (d) represent the charged fermions  $\chi_\beta$  and charged scalars  $S_\alpha^-$  loop contributions.

half the degrees of freedom of the Dirac neutrino. Through the general description of the electromagnetic form factors of Dirac and Majorana neutrinos in ref. [5], here we can get the MDM and EDM for Majorana neutrinos

$$\mu_{ij}^M = \mu_{ij}^D - \mu_{ji}^D, \quad \epsilon_{ij}^M = \epsilon_{ij}^D - \epsilon_{ji}^D, \quad (3.7)$$

with

$$\begin{aligned} \mu_{ij}^D &= 4m_\epsilon m_{\nu_i} \Re(C_2^R + \frac{m_{\nu_j}}{m_{\nu_i}} C_2^{L*} + C_6^R) \mu_B, \\ \epsilon_{ij}^D &= 4m_\epsilon m_{\nu_i} \Im(C_2^R + \frac{m_{\nu_j}}{m_{\nu_i}} C_2^{L*} + C_6^R) \mu_B, \end{aligned} \quad (3.8)$$

where  $\nu_{i,j}$  denote Majorana neutrinos. In eq. (3.7), the first terms  $\mu_{ij}^D$  and  $\epsilon_{ij}^D$  denote Dirac-neutrino-like terms for the MDM and EDM of Majorana neutrinos, the second terms  $-\mu_{ji}^D$  and  $-\epsilon_{ji}^D$  denote Dirac-antineutrino-like terms for the MDM and EDM of Majorana neutrinos, respectively. One also can find that  $\mu_{ij}^M$  and  $\epsilon_{ij}^M$  are antisymmetric. So, Majorana neutrinos don't have diagonal MDM and EDM, but can have transition MDM and EDM.

Within framework of the  $\mu\nu$ SSM, we can obtain three light massive Majorana neutrinos through the seesaw mechanism. Next we will analyse the dominant one-loop diagrams contributing to the transition magnetic moment of the Majorana neutrinos in the  $\mu\nu$ SSM, which are depicted by figure 1. Then the above Wilson coefficients can be written as a sum of four terms,

$$C_{2,6}^{L,R} = C_{2,6}^{L,R(a)} + C_{2,6}^{L,R(b)} + C_{2,6}^{L,R(c)} + C_{2,6}^{L,R(d)}. \quad (3.9)$$

The terms  $C_{2,6}^{L,R(a,b)}$  represent the charged fermions and  $W$ -boson loop contributions as

$$\begin{aligned}
 C_2^{R(a)} &= \frac{1}{2m_W^2} C_R^{W\chi_\beta \bar{\chi}_{7+i}^\circ} C_R^{W\chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_1(x_{\chi_\beta}, x_W) - I_4(x_{\chi_\beta}, x_W) \right], \\
 C_6^{R(a)} &= \frac{2m_{\chi_\beta}}{m_W^2 m_{\nu_i}} C_L^{W\chi_\beta \bar{\chi}_{7+i}^\circ} C_R^{W\chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_3(x_{\chi_\beta}, x_W) - I_1(x_{\chi_\beta}, x_W) \right], \\
 C_2^{R(b)} &= \frac{1}{2m_W^2} C_R^{W\chi_\beta \bar{\chi}_{7+i}^\circ} C_R^{W\chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_3(x_{\chi_\beta}, x_W) + I_4(x_{\chi_\beta}, x_W) \right], \\
 C_6^{R(b)} &= \frac{2m_{\chi_\beta}}{m_W^2 m_{\nu_i}} C_L^{W\chi_\beta \bar{\chi}_{7+i}^\circ} C_R^{W\chi_{7+j}^\circ \bar{\chi}_\beta} \left[ -I_3(x_{\chi_\beta}, x_W) \right], \\
 C_{2,6}^{L(a,b)} &= C_{2,6}^{R(a,b)} \Big|_{L \leftrightarrow R},
 \end{aligned} \tag{3.10}$$

where the concrete expressions for the coupling coefficients  $C_{L,R}$  and form factors  $I_k$  ( $k = 1, \dots, 4$ ) can be found in appendix B and appendix C,  $x_a = m_a^2/m_W^2$  and  $m_a$  is the mass for the corresponding particle, respectively.

Similarly, the charged fermions and charged scalars loop contributions  $C_{2,6}^{L,R(c,d)}$  are

$$\begin{aligned}
 C_2^{R(c)} &= \frac{1}{4m_W^2} C_R^{S_\alpha^- \chi_\beta \bar{\chi}_{7+i}^\circ} C_L^{S_\alpha^- \chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_4(x_{\chi_\beta}, x_{S_\alpha^-}) - I_3(x_{\chi_\beta}, x_{S_\alpha^-}) \right], \\
 C_6^{R(c)} &= \frac{m_{\chi_\beta}}{2m_W^2 m_{\nu_i}} C_R^{S_\alpha^- \chi_\beta \bar{\chi}_{7+i}^\circ} C_L^{S_\alpha^- \chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_3(x_{\chi_\beta}, x_{S_\alpha^-}) - I_1(x_{\chi_\beta}, x_{S_\alpha^-}) \right], \\
 C_2^{R(d)} &= \frac{1}{4m_W^2} C_R^{S_\alpha^- \chi_\beta \bar{\chi}_{7+i}^\circ} C_L^{S_\alpha^- \chi_{7+j}^\circ \bar{\chi}_\beta} \left[ 2I_3(x_{\chi_\beta}, x_{S_\alpha^-}) - I_1(x_{\chi_\beta}, x_{S_\alpha^-}) - I_4(x_{\chi_\beta}, x_{S_\alpha^-}) \right], \\
 C_6^{R(d)} &= \frac{m_{\chi_\beta}}{2m_W^2 m_{\nu_i}} C_R^{S_\alpha^- \chi_\beta \bar{\chi}_{7+i}^\circ} C_L^{S_\alpha^- \chi_{7+j}^\circ \bar{\chi}_\beta} \left[ I_1(x_{\chi_\beta}, x_{S_\alpha^-}) - I_2(x_{\chi_\beta}, x_{S_\alpha^-}) - I_3(x_{\chi_\beta}, x_{S_\alpha^-}) \right], \\
 C_{2,6}^{L(c,d)} &= C_{2,6}^{R(c,d)} \Big|_{L \leftrightarrow R}.
 \end{aligned} \tag{3.11}$$

## 4 The numerical results

### 4.1 The parameter space

It is well known that there are many free parameters in various supersymmetric extensions of the SM. In order to obtain a more transparent numerical results, we will do some assumptions on the concerned parameter space of the  $\mu\nu$ SSM in this paper. First, we adopt assumptions for some parameters:

$$\begin{aligned}
 \kappa_{ijk} &= \kappa \delta_{ij} \delta_{jk}, & (A_\lambda \lambda)_i &= A_\lambda \lambda, & \lambda_i &= \lambda, & \nu_{\nu_i^c} &= \nu_{\nu^c}, \\
 Y_{\nu_{ij}} &= Y_{\nu_i} \delta_{ij}, & (A_\nu Y_\nu)_{ij} &= a_{\nu_i} \delta_{ij}, & Y_{e_{ij}} &= Y_{e_i} \delta_{ij}.
 \end{aligned} \tag{4.1}$$

Restrained by the lepton masses, we have

$$Y_{e_i} = \frac{m_{l_i}}{v_d}, \tag{4.2}$$

where  $m_{l_i}$  denote the charged lepton masses.



For soft breaking slepton mass matrices  $m_{\tilde{L}, \tilde{e}^c}^2$  and trilinear coupling matrices  $(A_e Y_e)$ , we will take into account the off-diagonal terms, which are defined as [35–39]

$$m_{\tilde{L}}^2 = \begin{pmatrix} 1 & \delta_{12}^{LL} & \delta_{13}^{LL} \\ \delta_{12}^{LL} & 1 & \delta_{23}^{LL} \\ \delta_{13}^{LL} & \delta_{23}^{LL} & 1 \end{pmatrix} m_L^2, \quad (4.3)$$

$$m_{\tilde{e}^c}^2 = \begin{pmatrix} 1 & \delta_{12}^{RR} & \delta_{13}^{RR} \\ \delta_{12}^{RR} & 1 & \delta_{23}^{RR} \\ \delta_{13}^{RR} & \delta_{23}^{RR} & 1 \end{pmatrix} m_E^2, \quad (4.4)$$

$$(A_e Y_e) = \begin{pmatrix} m_{l_1} A_e & \delta_{12}^{LR} m_L m_E & \delta_{13}^{LR} m_L m_E \\ \delta_{12}^{LR} m_L m_E & m_{l_2} A_e & \delta_{23}^{LR} m_L m_E \\ \delta_{13}^{LR} m_L m_E & \delta_{23}^{LR} m_L m_E & m_{l_3} A_e \end{pmatrix} \frac{1}{v_d}. \quad (4.5)$$

Limited by the most relevant lepton flavor violating processes [39], we take the conservative choice for the off-diagonal parameters  $\delta_{12}^{LL}, \delta_{12}^{RR}, \delta_{12}^{LR} \leq 10^{-6}$ ,  $\delta_{13}^{LL}, \delta_{13}^{RR}, \delta_{13}^{LR} \leq 10^{-3}$  and  $\delta_{23}^{LL}, \delta_{23}^{RR}, \delta_{23}^{LR} \leq 10^{-2}$ . So for simplicity, we will choose the off-diagonal parameters

$$\begin{aligned} \delta_{23}^{LL} &= \delta_{23}^{RR} = \delta_{23}^{LR} \equiv \delta_{23}^X, \\ \delta_{13}^{LL} &= \delta_{13}^{RR} = \delta_{13}^{LR} \equiv 10^{-1} \delta_{23}^X, \\ \delta_{12}^{LL} &= \delta_{12}^{RR} = \delta_{12}^{LR} \equiv 10^{-4} \delta_{23}^X. \end{aligned} \quad (4.6)$$

Under above assumptions, ignoring the terms of the second order in  $Y_\nu$  and assuming  $(v_{\nu_i}^2 + v_d^2 - v_u^2) \approx (v_d^2 - v_u^2)$ , one can have the minimization conditions of the tree-level neutral scalar potential with respect to  $v_{\nu_i}$  ( $i = 1, 2, 3$ ) below:

$$m_{\tilde{L}_{ij}}^2 v_{\nu_j} + \frac{G^2}{4} (v_d^2 - v_u^2) v_{\nu_i} = \left[ \lambda v_d (v_u^2 + v_{\nu^c}^2) - \kappa v_u v_{\nu^c}^2 \right] Y_{\nu_i} - v_u v_{\nu^c} a_{\nu_i}, \quad (4.7)$$

where  $G^2 = g_1^2 + g_2^2$  and  $g_1 c_W = g_2 s_W = e$ . Solving eq. (4.7), we can gain the left-handed sneutrino VEVs

$$v_{\nu_i} = \frac{\det T_i}{\det T}, \quad (i = 1, 2, 3), \quad (4.8)$$

where

$$T = \begin{pmatrix} m_{\tilde{L}_{11}}^2 + \frac{G^2}{4} (v_d^2 - v_u^2) & m_{\tilde{L}_{12}}^2 & m_{\tilde{L}_{13}}^2 \\ m_{\tilde{L}_{21}}^2 & m_{\tilde{L}_{22}}^2 + \frac{G^2}{4} (v_d^2 - v_u^2) & m_{\tilde{L}_{23}}^2 \\ m_{\tilde{L}_{31}}^2 & m_{\tilde{L}_{32}}^2 & m_{\tilde{L}_{33}}^2 + \frac{G^2}{4} (v_d^2 - v_u^2) \end{pmatrix}, \quad (4.9)$$

and  $T_i$  can be acquired from  $T$  by replacing the  $i$ -th column with

$$\begin{pmatrix} \left[ \lambda v_d (v_u^2 + v_{\nu^c}^2) - \kappa v_u v_{\nu^c}^2 \right] Y_{\nu_1} - v_u v_{\nu^c} a_{\nu_1} \\ \left[ \lambda v_d (v_u^2 + v_{\nu^c}^2) - \kappa v_u v_{\nu^c}^2 \right] Y_{\nu_2} - v_u v_{\nu^c} a_{\nu_2} \\ \left[ \lambda v_d (v_u^2 + v_{\nu^c}^2) - \kappa v_u v_{\nu^c}^2 \right] Y_{\nu_3} - v_u v_{\nu^c} a_{\nu_3} \end{pmatrix}. \quad (4.10)$$

Assuming that the charged lepton mass matrix in the flavor basis is in the diagonal form, we parameterize the unitary matrix which diagonalizes the effective light neutrino mass matrix  $m_{\text{eff}}$  as [40–42]

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \quad (4.11)$$

where  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ , the angles  $\theta_{ij} = [0, \pi/2]$ , and CP violation phases are setting to zero, respectively. The unitary matrix  $U_\nu$  diagonalizes  $m_{\text{eff}}$  in the following way:

$$U_\nu^T m_{\text{eff}}^T m_{\text{eff}} U_\nu = \text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2), \quad (4.12)$$

In the case of 3-neutrino mixing, we have two possibilities on the neutrino mass spectrum [28]:

- (i) spectrum with normal ordering (NO):

$$\begin{aligned} m_{\nu_1} &< m_{\nu_2} < m_{\nu_3}, \\ \Delta m_{\odot}^2 &= m_{\nu_2}^2 - m_{\nu_1}^2 > 0, \quad \Delta m_A^2 = m_{\nu_3}^2 - m_{\nu_1}^2 > 0; \end{aligned} \quad (4.13)$$

- (ii) spectrum with inverted ordering (IO):

$$\begin{aligned} m_{\nu_3} &< m_{\nu_1} < m_{\nu_2}, \\ \Delta m_{\odot}^2 &= m_{\nu_2}^2 - m_{\nu_1}^2 > 0, \quad \Delta m_A^2 = m_{\nu_3}^2 - m_{\nu_2}^2 < 0. \end{aligned} \quad (4.14)$$

Limited on neutrino masses from neutrinoless double- $\beta$  decay [43] and cosmology [44], we choose the lightest neutrino mass  $m_{\nu_{\min}} = 0.1 \text{ eV}$  in the following. Through the experimental data on neutrino mass squared differences and mixing angles in eq. (1.1) and eq. (1.2), we can obtain the values of  $\theta_{ij}$  and other two neutrino masses. The effective light neutrino mass matrix  $m_{\text{eff}}$  can approximate as [9]

$$m_{\text{eff}ij} \approx \frac{2A v_{\nu^c}}{3\Delta} b_i b_j + \frac{1 - 3\delta_{ij}}{6\kappa v_{\nu^c}} a_i a_j, \quad (4.15)$$

where

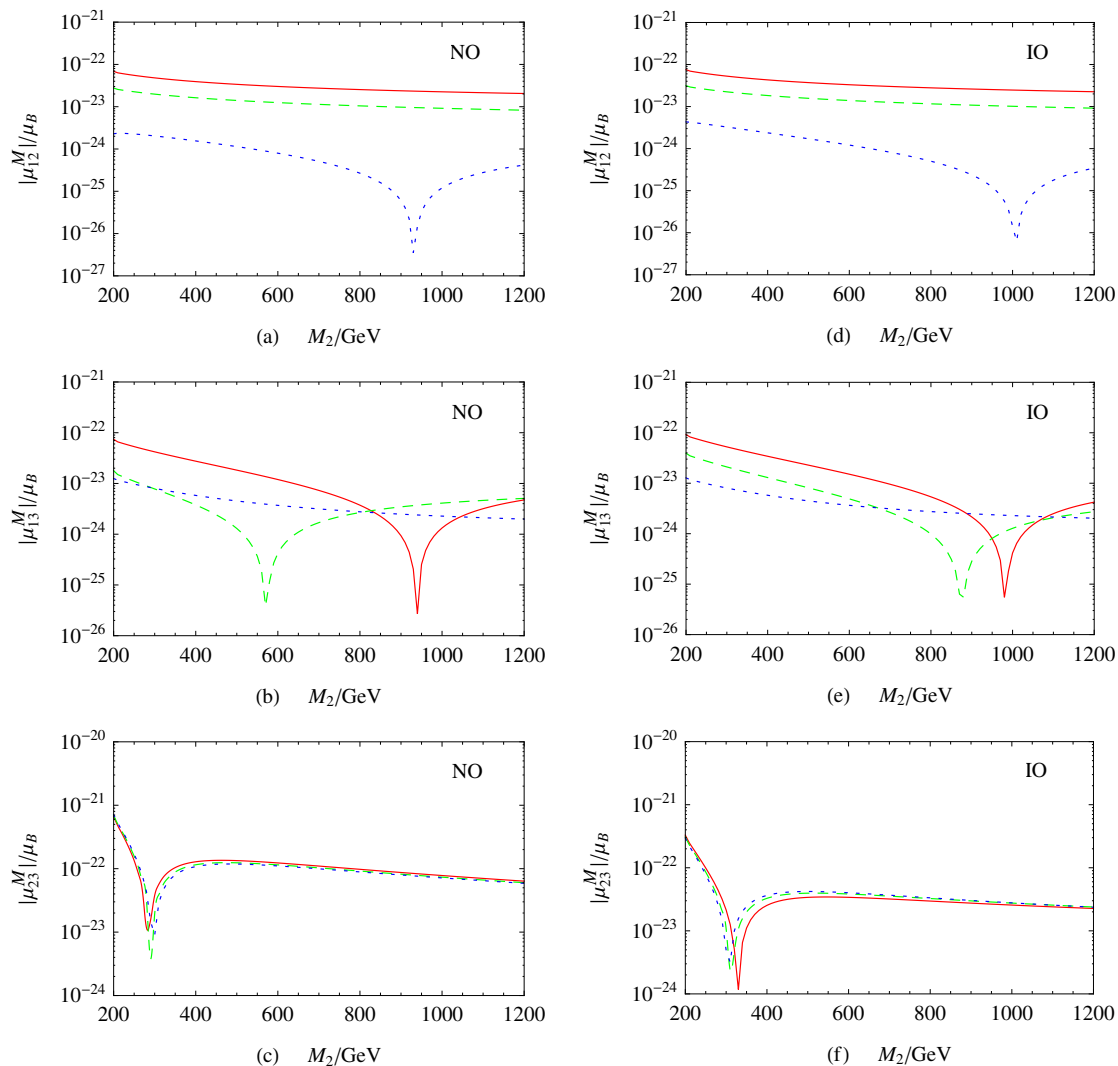
$$\begin{aligned} \Delta &= \lambda^2 (v_d^2 + v_u^2)^2 + 4\lambda\kappa v_{\nu^c} v_d v_u - 12\lambda^2 v_{\nu^c} A B, \\ A &= \kappa v_{\nu^c}^2 + \lambda v_d v_u, \quad \frac{1}{B} = \frac{e^2}{c_w^2 M_1} + \frac{e^2}{s_w^2 M_2}, \\ a_i &= Y_{\nu_i} v_u, \quad b_i = Y_{\nu_i} v_d + 3\lambda v_{\nu_i}. \end{aligned} \quad (4.16)$$

Then, we can numerically derive  $Y_{\nu_i} \sim \mathcal{O}(10^{-7})$  and  $a_{\nu_i} \sim \mathcal{O}(-10^{-4} \text{ GeV})$  from eq. (4.12). Accordingly,  $v_{\nu_i} \sim \mathcal{O}(10^{-4} \text{ GeV})$  through eq. (4.8). So  $v_{\nu_i} \ll v_{u,d}$ , then we can have

$$\tan \beta \simeq \frac{v_u}{v_d}. \quad (4.17)$$

Through the above analysis, assuming universality for the soft breaking bino and wino masses,  $M_1 = 0.5 M_2$ , the free parameters affect our next analysis are

$$\tan \beta, \kappa, \lambda, v_{\nu^c}, A_\lambda, A_e, m_L, m_E, M_2, \delta_{23}^X. \quad (4.18)$$



**Figure 2.** (Color online) Assuming neutrino mass spectrum with NO (a, b, c) or IO (d, e, f) in the  $\mu\nu$ SSM, we plot the transition magnetic moment of Majorana neutrinos varying with  $M_2$ , as  $\delta_{23}^X = 10^{-2}$  (solid line),  $\delta_{23}^X = 0.5 \times 10^{-2}$  (dashed line) and  $\delta_{23}^X = 0$  (dotted line), respectively.

## 4.2 Transition magnetic moment of Majorana neutrinos

Last year, a neutral Higgs with mass  $m_h \sim 124 - 126$  GeV was reported by ATLAS [45] and CMS [46], which contributes a strict constraint on relevant parameter space of the model. In the  $\mu\nu$ SSM, one can obtain a tree-level upper bound on the SM-like Higgs mass [7]:

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \frac{6\lambda^2 s_w^2 c_w^2}{e^2} m_Z^2 \sin^2 2\beta, \quad (4.19)$$

where  $m_Z$  is the mass of  $Z$ -boson. Compared with the MSSM, the second term of the right side in eq. (4.19) is additional contribution to the SM-like Higgs mass. So, the SM-like Higgs in the  $\mu\nu$ SSM can easily account for the mass around 125 GeV, through the main radiative corrections from the top quark and its supersymmetric partners. Also due to

this, the  $\mu\nu$ SSM favors small  $\tan\beta$  [8, 30]. Hence, we will take  $\tan\beta = 3$  for simplicity in the following.

And we choose the relevant parameters as default in our numerical calculation for convenience:

$$\begin{aligned}\kappa &= 0.01, & A_\lambda &= v_{\nu^c} = 500 \text{ GeV}, \\ \lambda &= 0.1, & A_e &= m_L = m_E = 200 \text{ GeV}.\end{aligned}\tag{4.20}$$

Here we take relatively small values of the parameters  $\kappa$  and  $\lambda$ , imposing the Landau pole condition at the high-energy scale [7, 8]. Limited on supersymmetric particle masses from the Particle Data Group [28], we choose  $A_e = m_{L,E} = 200 \text{ GeV}$  and  $M_2 \geq 200 \text{ GeV}$  in the next numerical calculation.

With the above assumptions on parameter space of the  $\mu\nu$ SSM, in figure 2 we plot the transition magnetic moment of Majorana neutrinos varying with  $M_2$  assuming neutrino mass spectrum with NO or IO, as  $\delta_{23}^X = 10^{-2}$  (solid line),  $\delta_{23}^X = 0.5 \times 10^{-2}$  (dashed line) and  $\delta_{23}^X = 0$  (dotted line), respectively. One can find that the general trend on the Majorana neutrino transition magnetic moment is decreasing, along with increasing of  $M_2$ . Here, the wino-like chargino mass is dependent on soft breaking wino mass  $M_2$ . When  $M_2$  is large, the wino-like chargino mass will be large, which causes that the wino-like chargino loop contribution is suppressed. Conversely, if  $M_2$  is enough small, we could obtain relatively large transition magnetic moment.

However, in figure 2 we also can see that transition magnetic moment of Majorana neutrinos may have a sharp decrease in some parameter space. Due to the fact that Majorana neutrino coincides with its antiparticle, the Majorana neutrino transition magnetic moment  $\mu_{ij}^M$  in eq. (3.7) contains the Dirac-neutrino-like term  $\mu_{ij}^D$  and the Dirac-antineutrino-like term  $-\mu_{ji}^D$ . Considered the supersymmetric particle loop contributions in the  $\mu\nu$ SSM, the Majorana neutrino transition magnetic moment could have a resonating absorption in some parameter space, which originates from the interference between the Dirac-neutrino-like term and the Dirac-antineutrino-like term.

In this paragraph, we will analyse the contribution from the off-diagonal parameter  $\delta_{23}^X$ . In figure 2, the transition magnetic moment  $\mu_{23}^M$  doesn't change much for different values of the off-diagonal parameter  $\delta_{23}^X$ . However, the transition magnetic moments  $\mu_{12}^M$  and  $\mu_{13}^M$  could be enhanced largely, with increasing of  $\delta_{23}^X$ . It's well known that the transition magnetic moment is flavor dependent. Taking into account the off-diagonal terms of soft breaking slepton mass matrices and trilinear coupling matrices enlarges slepton flavor mixing, which could give a contribution to the transition magnetic moment through the charged fermions and charged scalars loop diagrams.

Compared normal ordering with inverted ordering for neutrino mass spectrum, the Majorana neutrino transition magnetic moments depicted in figure 2 are somewhat alike, where the lightest neutrino mass is the same. But, we also can find some differences between normal ordering and inverted ordering. First, the value of  $M_2$  for a resonating absorption is different. And the transition magnetic moment  $\mu_{23}^M$  for normal ordering in figure 2(c) is relatively larger than that for inverted ordering in figure 2(f).

In the Standard Model augmented by nonzero Majorana neutrino masses, the Majorana neutrino transition magnetic moments  $\mu_{ij}^M \sim \mathcal{O}(10^{-24} \mu_B)$ , when the lightest neutrino mass  $m_{\nu_{\min}} = 0.1 \text{ eV}$  [5]. In figure 2, the Majorana neutrino transition magnetic moments in the  $\mu\nu\text{SSM}$  could be enhanced to  $\mathcal{O}(10^{-21,-22} \mu_B)$ , for small  $M_2$  and large  $\delta_{23}^X$ . Here the supersymmetric particle loops give the dominant contributions to the transition magnetic moments. In the oscillation of supernova neutrinos, Majorana neutrino transition magnetic moments reveal that moments as small as  $10^{-24} \mu_B$  may leave a potentially observable imprint on the energy spectra of neutrinos and antineutrinos from supernovae [47–49]. So, the relatively large transition magnetic moment of Majorana neutrinos in the  $\mu\nu\text{SSM}$  will be more easily detected than that in the Standard Model augmented by massive Majorana neutrinos.

## 5 Conclusions

The  $\mu\nu\text{SSM}$ , one supersymmetric extension of the Standard Model, can generate three tiny massive Majorana neutrinos through the seesaw mechanism. Applying effective Lagrangian method and on-shell scheme, we investigate the transition magnetic moment of Majorana neutrinos in the model. Under the present neutrino experimental constraints and some assumptions of the parameter space, we consider the two possibilities on the neutrino mass spectrum with normal or inverted ordering.

The numerical results show that the transition magnetic moment of Majorana neutrinos in the  $\mu\nu\text{SSM}$  can be enhanced to  $\mathcal{O}(10^{-21,-22} \mu_B)$ , when the supersymmetric particles are light and the off-diagonal terms of soft breaking slepton parameters are large. In the oscillation of supernova neutrinos, Majorana neutrino transition magnetic moments reveal that moments as small as  $10^{-24} \mu_B$  may leave a potentially observable from supernovae [47–49]. Therefore, the Majorana neutrino transition magnetic moments in the  $\mu\nu\text{SSM}$  have more opportunities to be detected from supernovae in the future.

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## A Mass Matrices

In this appendix, we give the relative mass matrices for this study in the  $\mu\nu\text{SSM}$ .

### A.1 Charged scalar mass matrix

The quadratic potential includes

$$V_{\text{quadratic}} = S'^T M_{S^\pm}^2 S'^+, \tag{A.1}$$

where  $S'^{\pm T} = (H_d^\pm, H_u^\pm, \tilde{e}_{L_i}^\pm, \tilde{e}_{R_i}^\pm)$  is in the unrotated basis,  $\tilde{e}_{L_i}^- \equiv \tilde{e}_i$  and  $\tilde{e}_{R_i}^+ \equiv \tilde{e}_i^c$ . The expressions for the independent coefficients of  $M_{S^\pm}^2$  are given in detail below:

$$M_{H_d^\pm H_d^\pm}^2 = m_{H_d}^2 + \frac{g_2^2}{2}(v_u^2 - v_{\nu_i} v_{\nu_i}) + \frac{G^2}{4}(v_d^2 - v_u^2 + v_{\nu_i} v_{\nu_i}) + \lambda_i \lambda_j v_{\nu_i^c} v_{\nu_j^c} + Y_{e_{ik}} Y_{e_{jk}} v_{\nu_i} v_{\nu_j}, \quad (\text{A.2})$$

$$M_{H_u^\pm H_u^\pm}^2 = m_{H_u}^2 + \frac{g_2^2}{2}(v_d^2 + v_{\nu_i} v_{\nu_i}) - \frac{G^2}{4}(v_d^2 - v_u^2 + v_{\nu_i} v_{\nu_i}) + \lambda_i \lambda_j v_{\nu_i^c} v_{\nu_j^c} + Y_{\nu_{ik}} Y_{\nu_{ij}} v_{\nu_j^c} v_{\nu_k^c}, \quad (\text{A.3})$$

$$M_{H_d^\pm H_u^\pm}^2 = (A_\lambda \lambda)_i v_{\nu_i^c} + \frac{g_2^2}{2} v_d v_u - \lambda_i \lambda_i v_d v_u + \lambda_k \kappa_{ijk} v_{\nu_i^c} v_{\nu_j^c} + Y_{\nu_{ij}} \lambda_j v_u v_{\nu_i}, \quad (\text{A.4})$$

$$M_{H_d^\pm \tilde{e}_{L_i}^\pm}^2 = \frac{g_2^2}{2} v_d v_{\nu_i} - Y_{\nu_{ij}} \lambda_k v_{\nu_k^c} v_{\nu_j^c} - Y_{e_{ij}} Y_{e_{kj}} v_d v_{\nu_k}, \quad (\text{A.5})$$

$$M_{H_u^\pm \tilde{e}_{L_i}^\pm}^2 = \frac{g_2^2}{2} v_u v_{\nu_i} - (A_\nu Y_\nu)_{ij} v_{\nu_j^c} + Y_{\nu_{ij}} \lambda_j v_d v_u - Y_{\nu_{ij}} \kappa_{ljk} v_{\nu_l^c} v_{\nu_k^c} - Y_{\nu_{ik}} Y_{\nu_{kj}} v_u v_{\nu_j}, \quad (\text{A.6})$$

$$M_{H_d^\pm \tilde{e}_{R_i}^\pm}^2 = -(A_e Y_e)_{ji} v_{\nu_j} - Y_{e_{ki}} Y_{\nu_{kj}} v_u v_{\nu_j^c}, \quad (\text{A.7})$$

$$M_{H_u^\pm \tilde{e}_{R_i}^\pm}^2 = -Y_{e_{ki}} (\lambda_j v_{\nu_j^c} v_{\nu_k} + Y_{\nu_{kj}} v_d v_{\nu_j^c}), \quad (\text{A.8})$$

$$M_{\tilde{e}_{L_i}^\pm \tilde{e}_{L_j}^\pm}^2 = m_{L_{ij}}^2 + \frac{1}{4}(g_1^2 - g_2^2)(v_d^2 - v_u^2 + v_{\nu_k} v_{\nu_k}) \delta_{ij} + \frac{g_2^2}{2} v_{\nu_i} v_{\nu_j} + Y_{\nu_{il}} Y_{\nu_{jk}} v_{\nu_l^c} v_{\nu_k^c} + Y_{e_{ik}} Y_{e_{jk}} v_d^2, \quad (\text{A.9})$$

$$M_{\tilde{e}_{L_i}^\pm \tilde{e}_{R_j}^\pm}^2 = (A_e Y_e)_{ij} v_d - Y_{e_{ij}} \lambda_k v_u v_{\nu_k^c}, \quad (\text{A.10})$$

$$M_{\tilde{e}_{R_i}^\pm \tilde{e}_{R_j}^\pm}^2 = m_{\tilde{e}_{ij}^c}^2 - \frac{1}{2} g_1^2 (v_d^2 - v_u^2 + v_{\nu_k} v_{\nu_k}) \delta_{ij} + Y_{e_{ki}} Y_{e_{kj}} v_d^2 + Y_{e_{li}} Y_{e_{kj}} v_{\nu_k} v_{\nu_l}. \quad (\text{A.11})$$

Here, imposing the minimization conditions of the tree-level neutral scalar potential with respect to  $v_d$  and  $v_u$ , one can have

$$m_{H_d}^2 = \left\{ -\frac{G^2}{4}(v_d^2 - v_u^2 + v_{\nu_i} v_{\nu_i}) v_d + (A_\lambda \lambda)_i v_u v_{\nu_i^c} + \lambda_j \kappa_{ijk} v_u v_{\nu_i^c} v_{\nu_k^c} - (\lambda_i \lambda_j v_{\nu_i^c} v_{\nu_j^c} + \lambda_i \lambda_i v_u^2) v_d + Y_{\nu_{ij}} v_{\nu_i} (\lambda_k v_{\nu_k^c} v_{\nu_j^c} + \lambda_j v_u^2) \right\} \frac{1}{v_d}, \quad (\text{A.12})$$

$$m_{H_u}^2 = \left\{ \frac{G^2}{4}(v_d^2 - v_u^2 + v_{\nu_i} v_{\nu_i}) v_u - (A_\nu Y_\nu)_{ij} v_{\nu_i} v_{\nu_j^c} + (A_\lambda \lambda)_i v_d v_{\nu_i^c} - (\lambda_i \lambda_j v_{\nu_i^c} v_{\nu_j^c} - \lambda_i \lambda_i v_u^2) v_u - Y_{\nu_{ij}} v_{\nu_i} (\kappa_{ljk} v_{\nu_l^c} v_{\nu_k^c} - 2\lambda_j v_d v_u) + \lambda_j \kappa_{ijk} v_d v_{\nu_i^c} v_{\nu_k^c} - (Y_{\nu_{ki}} Y_{\nu_{kj}} v_{\nu_i^c} v_{\nu_j^c} + Y_{\nu_{ik}} Y_{\nu_{jk}} v_{\nu_i} v_{\nu_j}) v_u \right\} \frac{1}{v_u}. \quad (\text{A.13})$$

We can use an  $8 \times 8$  unitary matrix  $R_{S^\pm}$  to diagonalize the mass matrix  $M_{S^\pm}^2$

$$R_{S^\pm}^T M_{S^\pm}^2 R_{S^\pm} = (M_{S^\pm}^{\text{diag}})^2. \quad (\text{A.14})$$

Then,  $S_\alpha^\pm$  can be rotated to the mass eigenvectors  $S_\alpha^\pm$  ( $\alpha = 1, \dots, 8$ ):

$$H_d^\pm = R_{S^\pm}^{1\alpha} S_\alpha^\pm, \quad H_u^\pm = R_{S^\pm}^{2\alpha} S_\alpha^\pm, \quad \tilde{e}_{L_i}^\pm = R_{S^\pm}^{(2+i)\alpha} S_\alpha^\pm, \quad \tilde{e}_{R_i}^\pm = R_{S^\pm}^{(5+i)\alpha} S_\alpha^\pm. \quad (\text{A.15})$$

## A.2 Neutral fermion mass matrix

Neutrinos mix with the neutralinos and in the basis  $\chi'^{oT} = (\tilde{B}^\circ, \tilde{W}^\circ, \tilde{H}_d, \tilde{H}_u, \nu_{R_i}, \nu_{L_i})$ , we can obtain the neutral fermion mass terms in the Lagrangian:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\chi'^{oT} M_n \chi'^o + \text{H.c.}, \quad (\text{A.16})$$

where

$$M_n = \begin{pmatrix} M & m^T \\ m & 0_{3 \times 3} \end{pmatrix}, \quad (\text{A.17})$$

with

$$m = \begin{pmatrix} -\frac{g_1}{\sqrt{2}}\nu_{\nu_1} & \frac{g_2}{\sqrt{2}}\nu_{\nu_1} & 0 & Y_{\nu_{1i}}\nu_{\nu_i^c} & Y_{\nu_{11}}\nu_u & Y_{\nu_{12}}\nu_u & Y_{\nu_{13}}\nu_u \\ -\frac{g_1}{\sqrt{2}}\nu_{\nu_2} & \frac{g_2}{\sqrt{2}}\nu_{\nu_2} & 0 & Y_{\nu_{2i}}\nu_{\nu_i^c} & Y_{\nu_{21}}\nu_u & Y_{\nu_{22}}\nu_u & Y_{\nu_{23}}\nu_u \\ -\frac{g_1}{\sqrt{2}}\nu_{\nu_3} & \frac{g_2}{\sqrt{2}}\nu_{\nu_3} & 0 & Y_{\nu_{3i}}\nu_{\nu_i^c} & Y_{\nu_{31}}\nu_u & Y_{\nu_{32}}\nu_u & Y_{\nu_{33}}\nu_u \end{pmatrix} \quad (\text{A.18})$$

and

$$M = \begin{pmatrix} M_1 & 0 & \frac{-g_1}{\sqrt{2}}\nu_d & \frac{g_1}{\sqrt{2}}\nu_u & 0 & 0 & 0 \\ 0 & M_2 & \frac{g_2}{\sqrt{2}}\nu_d & \frac{-g_2}{\sqrt{2}}\nu_u & 0 & 0 & 0 \\ \frac{-g_1}{\sqrt{2}}\nu_d & \frac{g_2}{\sqrt{2}}\nu_d & 0 & -\lambda_i\nu_{\nu_i^c} & -\lambda_1\nu_u & -\lambda_2\nu_u & -\lambda_3\nu_u \\ \frac{g_1}{\sqrt{2}}\nu_u & \frac{-g_2}{\sqrt{2}}\nu_u & -\lambda_i\nu_{\nu_i^c} & 0 & y_1 & y_2 & y_3 \\ 0 & 0 & -\lambda_1\nu_u & y_1 & 2\kappa_{11j}\nu_{\nu_j^c} & 2\kappa_{12j}\nu_{\nu_j^c} & 2\kappa_{13j}\nu_{\nu_j^c} \\ 0 & 0 & -\lambda_2\nu_u & y_2 & 2\kappa_{21j}\nu_{\nu_j^c} & 2\kappa_{22j}\nu_{\nu_j^c} & 2\kappa_{23j}\nu_{\nu_j^c} \\ 0 & 0 & -\lambda_3\nu_u & y_3 & 2\kappa_{31j}\nu_{\nu_j^c} & 2\kappa_{32j}\nu_{\nu_j^c} & 2\kappa_{33j}\nu_{\nu_j^c} \end{pmatrix}, \quad (\text{A.19})$$

where  $y_i = -\lambda_i\nu_d + Y_{\nu_{ji}}\nu_{\nu_j}$ . Here, the submatrix  $m$  is neutralino-neutrino mixing, and the submatrix  $M$  is neutralino mass matrix. This  $10 \times 10$  symmetric matrix  $M_n$  can be diagonalized by a  $10 \times 10$  unitary matrix  $Z_n$ :

$$Z_n^T M_n Z_n = M_{nd}, \quad (\text{A.20})$$

where  $M_{nd}$  is the diagonal neutral fermion mass matrix. Then, we have the neutral fermion mass eigenstates:

$$\chi_\alpha^\circ = \begin{pmatrix} \kappa_\alpha^\circ \\ \kappa_\alpha^\circ \end{pmatrix}, \quad \alpha = 1, \dots, 10 \quad (\text{A.21})$$

with

$$\begin{cases} \tilde{B}^\circ = Z_n^{1\alpha} \kappa_\alpha^\circ, & \tilde{H}_d = Z_n^{3\alpha} \kappa_\alpha^\circ, & \nu_{R_i} = Z_n^{(4+i)\alpha} \kappa_\alpha^\circ, \\ \tilde{W}^\circ = Z_n^{2\alpha} \kappa_\alpha^\circ, & \tilde{H}_u = Z_n^{4\alpha} \kappa_\alpha^\circ, & \nu_{L_i} = Z_n^{(7+i)\alpha} \kappa_\alpha^\circ. \end{cases} \quad (\text{A.22})$$

### A.3 Charged fermion mass matrix

Charged leptons mix with the charginos and therefore in the unrotated basis where  $\Psi^{-T} = (-i\tilde{\lambda}^-, \tilde{H}_d^-, e_{L_i}^-)$  and  $\Psi^{+T} = (-i\tilde{\lambda}^+, \tilde{H}_u^+, e_{R_i}^+)$ , one can have the charged fermion mass terms in the Lagrangian:

$$\mathcal{L}_{\text{mass}} = -\Psi^{-T} M_c \Psi^+ + \text{H.c.}, \quad (\text{A.23})$$

where

$$M_c = \begin{pmatrix} M_{\pm} & b \\ c & m_l \end{pmatrix}. \quad (\text{A.24})$$

Here, the submatrix  $M_{\pm}$  is chargino mass matrix

$$M_{\pm} = \begin{pmatrix} M_2 & g_2 v_u \\ g_2 v_d & \lambda_i v_{\nu_i^c} \end{pmatrix}. \quad (\text{A.25})$$

And the submatrices  $b$  and  $c$  give rise to chargino-charged lepton mixing. They are defined as

$$b = \begin{pmatrix} 0 & 0 & 0 \\ -Y_{e_{i1}} v_{\nu_i} & -Y_{e_{i2}} v_{\nu_i} & -Y_{e_{i3}} v_{\nu_i} \end{pmatrix}, \quad (\text{A.26})$$

$$c = \begin{pmatrix} g_2 v_{\nu_1} & -Y_{\nu_{1i}} v_{\nu_i^c} \\ g_2 v_{\nu_2} & -Y_{\nu_{2i}} v_{\nu_i^c} \\ g_2 v_{\nu_3} & -Y_{\nu_{3i}} v_{\nu_i^c} \end{pmatrix}. \quad (\text{A.27})$$

And the submatrix  $m_l$  is the charged lepton mass matrix

$$m_l = \begin{pmatrix} Y_{e_{11}} v_d & Y_{e_{12}} v_d & Y_{e_{13}} v_d \\ Y_{e_{21}} v_d & Y_{e_{22}} v_d & Y_{e_{23}} v_d \\ Y_{e_{31}} v_d & Y_{e_{32}} v_d & Y_{e_{33}} v_d \end{pmatrix}. \quad (\text{A.28})$$

This  $5 \times 5$  mass matrix  $M_c$  can be diagonalized by the  $5 \times 5$  unitary matrices  $Z_-$  and  $Z_+$ :

$$Z_-^T M_c Z_+ = M_{cd}, \quad (\text{A.29})$$

where  $M_{cd}$  is the diagonal charged fermion mass matrix. Then, one can obtain the charged fermion mass eigenstates:

$$\chi_{\alpha} = \begin{pmatrix} \kappa_{\alpha}^- \\ \kappa_{\alpha}^+ \end{pmatrix}, \quad \alpha = 1, \dots, 5 \quad (\text{A.30})$$

with

$$\begin{cases} \tilde{\lambda}^- = iZ_-^{1\alpha} \kappa_{\alpha}^-, & \tilde{H}_d^- = Z_-^{2\alpha} \kappa_{\alpha}^-, & e_{L_i} = Z_-^{(2+i)\alpha} \kappa_{\alpha}^-; \\ \tilde{\lambda}^+ = iZ_+^{1\alpha} \kappa_{\alpha}^+, & \tilde{H}_u^+ = Z_+^{2\alpha} \kappa_{\alpha}^+, & e_{R_i} = Z_+^{(2+i)\alpha} \kappa_{\alpha}^+. \end{cases} \quad (\text{A.31})$$



## B Couplings

In this appendix, we show the relevant couplings in the computation of the neutrino magnetic moment within framework of the  $\mu\nu$ SSM. And we use the indices  $i, j = 1, 2, 3, \beta = 1, \dots, 5, \alpha = 1, \dots, 8$  and  $\eta = 1, \dots, 10$ .

### B.1 Charged fermion-neutral fermion-gauge boson

The couplings of Charged fermion, neutral fermion and gauge boson are given by

$$\begin{aligned} \mathcal{L} = & eF_\mu \bar{\chi}_\beta \gamma^\mu \chi_\beta + W_\mu^+ \bar{\chi}_\eta^\circ (C_L^{W\chi_\beta \bar{\chi}_\eta^\circ} \gamma^\mu P_L + C_R^{W\chi_\beta \bar{\chi}_\eta^\circ} \gamma^\mu P_R) \chi_\beta \\ & + W_\mu^- \bar{\chi}_\beta (C_L^{W\chi_\eta^\circ \bar{\chi}_\beta} \gamma^\mu P_L + C_R^{W\chi_\eta^\circ \bar{\chi}_\beta} \gamma^\mu P_R) \chi_\eta^\circ + \dots, \end{aligned} \quad (\text{B.1})$$

where the coefficients are

$$C_L^{W\chi_\beta \bar{\chi}_\eta^\circ} = -\frac{e}{\sqrt{2}s_W} \left( \sqrt{2} Z_-^{1\beta} Z_n^{2\eta^*} + Z_-^{2\beta} Z_n^{3\eta^*} + Z_-^{(2+i)\beta} Z_n^{(7+i)\eta^*} \right), \quad (\text{B.2})$$

$$C_R^{W\chi_\beta \bar{\chi}_\eta^\circ} = -\frac{e}{\sqrt{2}s_W} \left( \sqrt{2} Z_+^{1\beta^*} Z_n^{2\eta} - Z_+^{2\beta^*} Z_n^{4\eta} \right), \quad (\text{B.3})$$

$$C_L^{W\chi_\eta^\circ \bar{\chi}_\beta} = \left( C_L^{W\chi_\beta \bar{\chi}_\eta^\circ} \right)^*, \quad C_R^{W\chi_\eta^\circ \bar{\chi}_\beta} = \left( C_R^{W\chi_\beta \bar{\chi}_\eta^\circ} \right)^*. \quad (\text{B.4})$$

### B.2 Charged fermion-neutral fermion-charged scalar

The couplings of charged fermion, neutral fermion and charged scalar are similarly written as

$$\begin{aligned} \mathcal{L} = & S_\alpha^- \bar{\chi}_\beta (C_L^{S_\alpha^- \chi_\eta^\circ \bar{\chi}_\beta} P_L + C_R^{S_\alpha^- \chi_\eta^\circ \bar{\chi}_\beta} P_R) \chi_\eta^\circ \\ & + S_\alpha^{*-} \bar{\chi}_\eta^\circ (C_L^{S_\alpha^{*-} \chi_\beta \bar{\chi}_\eta^\circ} P_L + C_R^{S_\alpha^{*-} \chi_\beta \bar{\chi}_\eta^\circ} P_R) \chi_\beta + \dots. \end{aligned} \quad (\text{B.5})$$

And the coefficients are

$$\begin{aligned} C_L^{S_\alpha^- \chi_\eta^\circ \bar{\chi}_\beta} = & \frac{-e}{\sqrt{2}s_W c_W} R_{S_\pm}^{2\alpha^*} Z_+^{2\beta} \left( c_W Z_n^{2\eta} + s_W Z_n^{1\eta} \right) - \frac{\sqrt{2}e}{s_W} R_{S_\pm}^{(5+i)\alpha^*} Z_+^{(2+i)\beta} Z_n^{1\eta} \\ & - \frac{e}{s_W} R_{S_\pm}^{2\alpha^*} Z_+^{1\beta} Z_n^{4\eta} + Y_{e_{ij}} Z_+^{(2+j)\beta} \left( R_{S_\pm}^{1\alpha} Z_n^{(7+i)\eta} - R_{S_\pm}^{(2+i)\alpha} Z_n^{3\eta} \right) \\ & + Y_{\nu_{ij}} R_{S_\pm}^{(2+i)\alpha} Z_+^{2\beta} Z_n^{(4+j)\eta} - \lambda_i R_{S_\pm}^{1\alpha} Z_+^{2\beta} Z_n^{(4+i)\eta}, \end{aligned} \quad (\text{B.6})$$

$$\begin{aligned} C_L^{S_\alpha^{*-} \chi_\beta \bar{\chi}_\eta^\circ} = & \frac{e}{\sqrt{2}s_W c_W} \left( R_{S_\pm}^{1\alpha^*} Z_-^{2\beta} + R_{S_\pm}^{(2+i)\alpha^*} Z_-^{(2+i)\beta} \right) \left( c_W Z_n^{2\eta} + s_W Z_n^{1\eta} \right) \\ & - \frac{e}{s_W} Z_-^{1\beta} \left( R_{S_\pm}^{1\alpha^*} Z_n^{3\eta} + R_{S_\pm}^{(2+i)\alpha^*} Z_n^{(7+i)\eta} \right) + Y_{\nu_{ij}} R_{S_\pm}^{2\alpha} Z_-^{(2+i)\beta} Z_n^{(4+j)\eta} \\ & + Y_{e_{ij}} R_{S_\pm}^{(5+j)\alpha} \left( Z_-^{2\beta} Z_n^{(7+i)\eta} - Z_-^{(2+i)\beta} Z_n^{3\eta} \right) - \lambda_i R_{S_\pm}^{2\alpha} Z_-^{2\beta} Z_n^{(4+i)\eta}, \end{aligned} \quad (\text{B.7})$$

$$C_R^{S_\alpha^- \chi_\eta^\circ \bar{\chi}_\beta} = \left( C_L^{S_\alpha^{*-} \chi_\beta \bar{\chi}_\eta^\circ} \right)^*, \quad C_R^{S_\alpha^{*-} \chi_\beta \bar{\chi}_\eta^\circ} = \left( C_L^{S_\alpha^- \chi_\eta^\circ \bar{\chi}_\beta} \right)^*. \quad (\text{B.8})$$

## C Form factors

Defined  $x_i = m_i^2/m_W^2$ , we can find the form factors:

$$I_1(x_1, x_2) = \frac{1}{16\pi^2} \left[ \frac{1 + \ln x_2}{(x_2 - x_1)} + \frac{x_1 \ln x_1 - x_2 \ln x_2}{(x_2 - x_1)^2} \right], \quad (\text{C.1})$$

$$I_2(x_1, x_2) = \frac{1}{16\pi^2} \left[ -\frac{1 + \ln x_1}{(x_2 - x_1)} - \frac{x_1 \ln x_1 - x_2 \ln x_2}{(x_2 - x_1)^2} \right], \quad (\text{C.2})$$

$$I_3(x_1, x_2) = \frac{1}{32\pi^2} \left[ \frac{3 + 2 \ln x_2}{(x_2 - x_1)} - \frac{2x_2 + 4x_2 \ln x_2}{(x_2 - x_1)^2} - \frac{2x_1^2 \ln x_1}{(x_2 - x_1)^3} + \frac{2x_2^2 \ln x_2}{(x_2 - x_1)^3} \right], \quad (\text{C.3})$$

$$I_4(x_1, x_2) = \frac{1}{96\pi^2} \left[ \frac{11 + 6 \ln x_2}{(x_2 - x_1)} - \frac{15x_2 + 18x_2 \ln x_2}{(x_2 - x_1)^2} + \frac{6x_2^2 + 18x_2^2 \ln x_2}{(x_2 - x_1)^3} + \frac{6x_1^3 \ln x_1 - 6x_2^3 \ln x_2}{(x_2 - x_1)^4} \right]. \quad (\text{C.4})$$

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