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**ORIGINAL ARTICLE**

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# Asymmetric vortex merger: mechanism and criterion

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**Abstract** The merging of two unequal co-rotating vortices in a viscous fluid is investigated. Two-dimensional numerical simulations of initially equal sized Lamb-Oseen vortices with differing relative strengths are performed. Results show how the disparity in deformation rates between the vortices alters the interaction. Key physical mechanisms associated with vortex merging are identified. A merging criterion is formulated in terms of the relative timing of core detrainment and destruction. A critical strain parameter is defined to characterize the establishment of core detrainment. This parameter is shown to be directly related to the critical aspect ratio in the case of symmetric merger.

**Keywords** Vortex interactions · Merger · Unequal vortices

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## 1 Introduction

Vortex merger is a fundamental flow process which plays an important role in the transfer of energy and enstrophy across scales in transitional and turbulent flows. It also has practical significance, e.g., in the nearfield wake dynamics of an aircraft. Yet despite its elementary nature, the physical mechanisms of two-dimensional vortex merger have not been fully resolved [1,2,12].

In the idealized interaction of two equal size and strength (*symmetric*) co-rotating vortices, merger occurs if the aspect ratio, $a/b$ (core size/separation distance), exceeds a critical value, $(a/b)_{cr}$. The determination of $(a/b)_{cr}$ has been the focus of a number of studies, e.g., [11,10,9]. The physical mechanisms of symmetric merger have also been considered, e.g., [7,8,4,13,1,2]. In the more general interaction of two unequal size and/or strength (*asymmetric*) vortices, there is a greater range of flow behavior and the interaction may result in the destruction of the smaller/weaker vortex. Previous inviscid flow studies have identified distinct flow regimes based on the efficiency of the interaction [5,12]. *Elastic interaction* occurs when there are only small deformations and essentially no change in circulation of the vortices. *Partial* and *complete straining-out* are associated with a reduction or destruction, respectively, of the smaller vortex, with no increase in the larger vortex. *Partial* and *complete merger* are associated with increased circulation of the initially larger vortex. Flow regime maps have been presented in terms of the initial core size (or strength) ratio and initial separation distance. Trieling et al. [12] attempted to normalize the separation distance by an averaged core size, defined in terms of the second moment of vorticity [9]. However, this did not yield a universal critical aspect ratio delimiting the merging regimes. It is unclear if $(a/b)_{cr}$ can be generalized in this way. Furthermore, there has been limited consideration of viscous flows.

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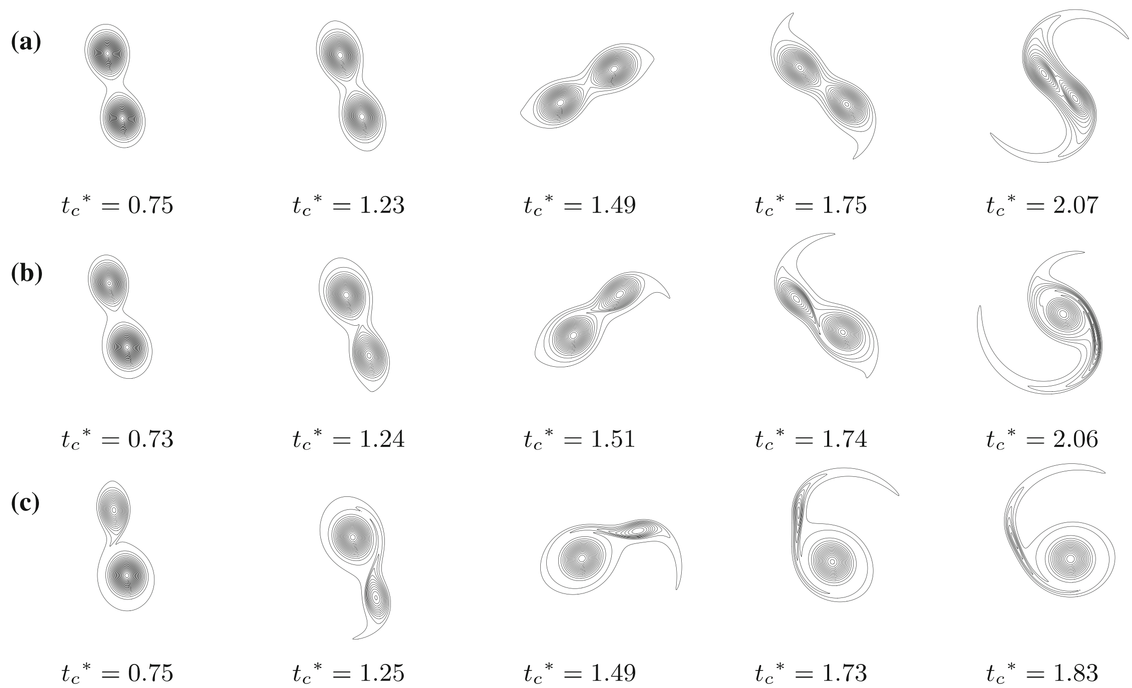


**Fig. 1** Vorticity contour plots showing time evolution of flows: **a** $\Gamma_{o,2}/\Gamma_{o,1} = 1.0$, **b** $\Gamma_{o,2}/\Gamma_{o,1} = 0.8$, **c** $\Gamma_{o,2}/\Gamma_{o,1} = 0.6$. Taken from [3]

This paper summarizes the main findings of our recent study [3] in which we investigate the interaction of two unequal co-rotating vortices in a viscous fluid. The study follows from our previous work on equal vortices [2] identifying the primary physical mechanisms associated with vortex merging and develops a more generalized description and criterion for asymmetric merger.

## 2 Numerical simulations

Numerical simulations of two-dimensional, incompressible, viscous flow are performed. The initial flow consists of two Lamb-Oseen co-rotating vortices of equal size and unequal strength. The initial aspect ratio is $a_o/b_o = 0.157$, where $a_o$ is defined based on the second moment of vorticity. The Reynolds number of the stronger vortex (vortex 1) is $Re_{\Gamma,1} = \Gamma_{o,1}/\nu = 5000$, where $\Gamma_o$ is the initial circulation and $\nu$ is the kinematic viscosity. The circulation of the weaker vortex (vortex 2) is varied in the range $0.4 \leq \Gamma_{o,2}/\Gamma_{o,1} \leq 1.0$. A convective time scale is the approximate rotational period of the system, $T = 2\pi^2 b_o{}^2/\overline{\Gamma}_o$, where $\overline{\Gamma}_o = 0.5(\Gamma_{o,1} + \Gamma_{o,2})$. Nondimensional time is $t_c{}^* = t/T$. Details are in [3].

## 3 Physical mechanisms

Figure 1 shows representative flow evolutions for several different $\Gamma_{o,2}/\Gamma_{o,1}$. Initially, the two vortices rotate about each other due to their mutually induced velocity. As in previous studies, it is useful to consider the flow structure in the co-rotating frame of reference. Figure 2 shows the instantaneous co-rotating streamlines (bold lines) indicating the inner core regions and the exchange band where fluid is advected around both vortices [7,2]. In time, all the flows result in a single vortex.

Figure 1a corresponds to symmetric vortices ($\Gamma_{o,2}/\Gamma_{o,1} = 1.0$). Our previous analysis of symmetric merger describes the merging process in terms of four phases of development [2]. In the *diffusive/deformation phase*, the separation distance remains relatively constant while the cores grow by diffusion. The cores also begin to deform. The deformation can be described in terms of the interaction of the vorticity gradient, $\nabla\omega$, and the mutually induced rate of strain, $\mathsf{S}$. As shown in Fig. 2a, each vortex exhibits a quadrapole structure of

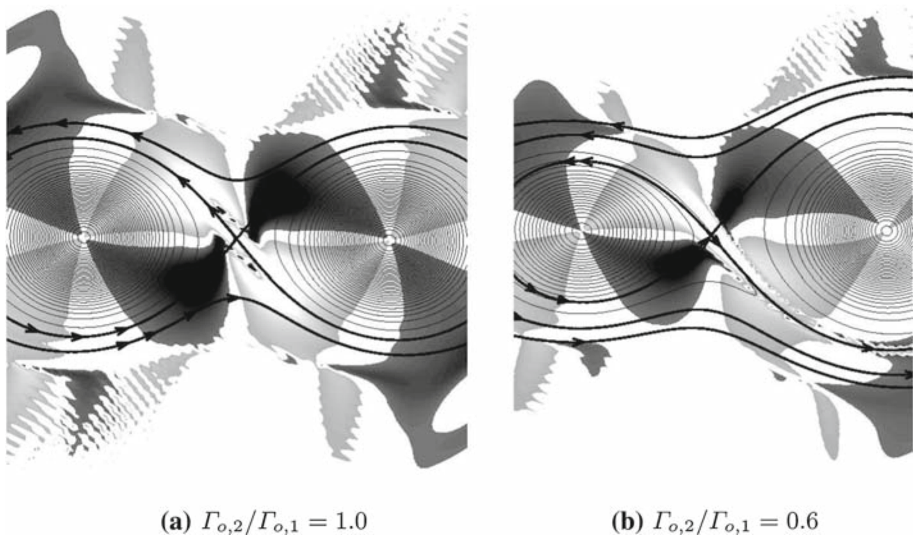


**Fig. 2** Vorticity contours (*thin lines*) with gray shading corresponding to $\nabla\omega$ production term, $P_s = -(\nabla\omega^T \mathsf{S}\nabla\omega)/|\nabla\omega|^2$ (*light gray scale*: $P_s > 0$, *dark gray scale*: $P_s < 0$), and instantaneous streamlines (*bold lines*) in the co-rotating frame at $t_c^* = 0.32$. Taken from [3]

$P_s = -(\nabla\omega^T \mathsf{S}\nabla\omega)/|\nabla\omega|^2$, which indicates alternate regions of gradient amplification/attenuation by compressive/extensional straining associated with its elliptic deformation. During this time, a distinct functional relation between $\omega$ and streamfunction exists suggesting quasi-equilibrium conditions. However, in the vicinity of the hyperbolic points, and in particular the central hyperbolic (CH) point where mutual interaction strengthens $\nabla\omega$ amplification (Fig. 2—light shading near center is $P_s > 0$), the dynamics of $\nabla\omega$ and $\mathsf{S}$ eventually produces a tilt in $\omega$ contours [1]. At the outer hyperbolic points, this initiates filamentation. During the *convective/deformation phase*, the induced flow by the filaments acts to advect the vortices towards each other and enhances the mutually induced $\mathsf{S}$ but does not drive the merger to completion. The enhanced tilting and diffusion of $\omega$ near the CH point causes $\omega$ to be detrained from the core region and enter the exchange band where it is advected away. This leads to the departure from quasi-equilibrium conditions. In the *convective/entrainment phase*, the vortex cores erode significantly. At some point, the integrity of the vortices is sufficiently diminished. The cores are then mutually entrained into the exchange band region, whose induced flow becomes dominant and transforms the flow into a single compound vortex. A critical aspect ratio, associated with the start of the convective/entrainment phase, is determined for a range of flow conditions [2]: $(a/b)_{\mathrm{cr}} = 0.235 \pm 0.006$. It is also noted that this time is comparable to the time at which the core size, $a^2(t)$, deviates from viscous (linear) growth [2]. In the final *diffusive/axisymmetrization phase*, the flow evolves towards axisymmetry by diffusion [4].

In the case of asymmetric vortex pairs (Fig. 1b, c), the difference in vortex strengths alters the flow structure and interaction. The vortices may no longer experience the flow processes simultaneously. As in the symmetric case, the vortices initially grow by diffusion. However, the deformation rates and $P_s$ (Fig. 2b) are stronger at the weaker vortex due to the difference in induced $\mathsf{S}$, and the tilt of $\omega$ contours and subsequent core detrainment occurs earlier than in the stronger vortex. However, the dominant attracting motion occurs only when, *and if*, core detrainment is established by the stronger vortex. If this occurs, then there will be some extent of mutual (reciprocal), but unequal, entrainment. This is observed in the present simulations for $0.7 \leq \Gamma_{o,2}/\Gamma_{o,1} \leq 0.9$ and is illustrated here in Fig. 1b ($\Gamma_{o,2}/\Gamma_{o,1} = 0.8$). In these cases, the stronger vortex ultimately dominates and entrains $\omega$ from the weaker vortex. Thus, the process is considered as vortex merger since the result is an *enhanced compound* vortex. If core detrainment is not established by the stronger vortex before significant erosion occurs in the weaker vortex (Fig. 1c, $\Gamma_{o,2}/\Gamma_{o,1} = 0.6$), the weaker vortex is destroyed while the stronger vortex remains relatively unaffected. In this case, convective merger does not occur.

## 4 Merging criterion

From the above description, we consider a critical state for a given vortex to be associated with the establishment of core detrainment. If both vortices reach this state, there will be some degree of mutual entrainment which

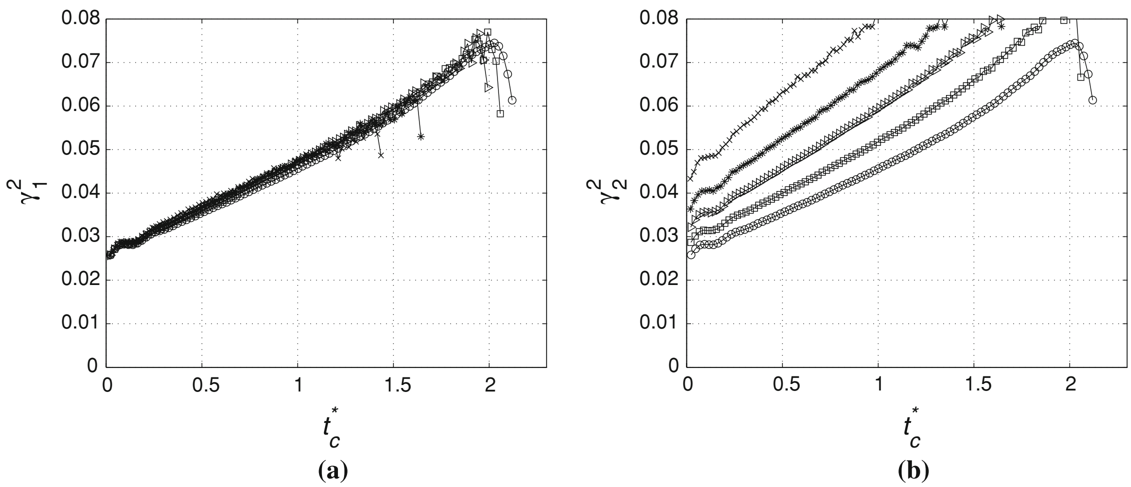


**Fig. 3** Time development of the strain parameter, $\gamma_i$ for **a** vortex 1 and **b** vortex 2. Symbols: (*circle*): $\Gamma_{o,2}/\Gamma_{o,1} = 1.0$, (*box*): $\Gamma_{o,2}/\Gamma_{o,1} = 0.9$, (*triangle*): $\Gamma_{o,2}/\Gamma_{o,1} = 0.8$, $*$: $\Gamma_{o,2}/\Gamma_{o,1} = 0.7$, $\times$: $\Gamma_{o,2}/\Gamma_{o,1} = 0.6$. Taken from [3]

results in an enhanced vortex, i.e., convective merger will occur. Based on these ideas, a merging criterion is developed.

We consider the onset of the core detrainment process to be associated with the flow achieving a sufficiently high strain rate, with respect to some characteristic $\omega$, for the process to proceed. Recall that the process is initiated by the tilting of $\boldsymbol{\omega}$ contours in the vicintiy of the CH point. We therefore consider one characteristic quantity to be the strain rate at the CH point, $S_{\text{CH}}$. A characteristic core vorticity is the maximum, $\omega_{v_i}$. In order to relate the strain rate at the CH point to the maximum vorticity of the vortex, we normalize each quantity by a characteristic local (initial) strain rate. This introduces the appropriate scaling. We define the nondimensional strain rate, $S^*_{\text{CH}} = S_{\text{CH}}/S_{\text{CH,o}}$ and nondimensional vorticity, $\omega^*_{v_i} = \omega_{v_i}/S_{v_i,o}$. A *strain parameter for vortex i* is then defined as,

$$\gamma_i(t^*) \equiv \left( \frac{S^*_{\text{CH}}(t^*)}{\frac{1}{2}\omega^*_{v_i}(t^*)} \right)^{1/2}, \tag{1}$$

which measures the relative strength of the induced strain rate at the CH point to the vortex strength. Figure 3 shows the strain parameter for the stronger and weaker vortex, $\gamma_1(t^*)$ and $\gamma_2(t^*)$, respectively, evaluated from the simulations. We consider the *critical value* of the vortex strain parameter to be the value at the critical time, $t^*_{\text{cr},i}$, when core detrainment (and entrainment into exchange band) is established, i.e., $\gamma_{\text{cr},2} = \gamma_2(t^*_{\text{cr},2})$ and $\gamma_{\text{cr},1} = \gamma_1(t^*_{\text{cr},1})$. From our simulation results, we find: $\gamma_{\text{cr},1} \approx 0.249 \pm 0.003$ and $\gamma_{\text{cr},2} \approx 0.245 \pm 0.005$ (where $t^*_{\text{cr},2}$ and $t^*_{\text{cr},1}$ are determined from the behavior of $a^2(t)$). Since the values are within the range of uncertainty, we obtain a *single value for the critical strain parameter*, $\gamma_{\text{cr},1} \approx \gamma_{\text{cr},2} \approx \gamma_{\text{cr}} = 0.247 \pm 0.007$.

In the case of a *symmetric vortex pair*, through scaling analysis [3], it is shown that,

$$\begin{aligned} \gamma_{\text{cr}} = \gamma_{\text{cr},2} &= \left( \frac{S^*_{\text{CH}}(t^*_{\text{cr}})}{\frac{1}{2}\omega^*_{v_2}(t^*_{\text{cr}})} \right)^{1/2} = \left( \frac{S_{\text{CH}}(t^*_{\text{cr}})}{4\omega_{v_i}(t^*_{\text{cr}})} \right)^{1/2} \\ &= f\, \frac{a^*_\omega(t^*_{\text{cr}})}{2d^*_{|\text{CH}-V|}(t^*_{\text{cr}})} = f\, \left( \frac{a_\omega}{b} \right)_{\text{cr}}. \end{aligned} \tag{2}$$

Thus, for symmetric merger, the critical strain parameter is directly related to the critical aspect ratio. From the simulation results, the proportionality factor is evaluated to be $f \approx 1.05 \pm 0.03$, and using the computed values of $\gamma_{\text{cr},2}$, $(a_\omega/b)_{\text{cr}} = \gamma_{\text{cr},2}/f \approx 0.233 \pm 0.005$. This compares well with the previously determined value, $(a_\omega/b)_{\text{cr}} = 0.235 \pm 0.006$ [2].

The merging criterion is formulated in terms of the timing of the key physical processes: weaker vortex core detrainment $t^*_{\text{cr},2}$, stronger vortex core detrainment $t^*_{\text{cr},1}$, and weaker vortex destruction $t^*_{\text{de},2}$. For the purposes of this study, $t^*_{\text{de},2}$ is estimated using the velocity gradient tensor [3]. We consider the classifications developed for inviscid asymmetric vortex interactions [12,5] and modify the descriptions for viscous flow. Based on our analysis, we classify the observed interactions and merging regimes as follows,

- *Complete merger* ($t^*_{\mathrm{cr},2} = t^*_{\mathrm{cr},1} < t^*_{\mathrm{de},2}$): detrainment from both vortices, mutual entrainment of the cores transforms the flow into a single vortex (present results: $\Gamma_{o,2}/\Gamma_{o,1} = 1.0$),
- *Partial merger* ($t^*_{\mathrm{cr},2} < t^*_{\mathrm{cr},1} < t^*_{\mathrm{de},2}$): detrainment from both vortices, weaker vortex is destroyed and entrained by the stronger vortex (present results: $\Gamma_{o,2}/\Gamma_{o,1} = 0.9, 0.8, 0.7$),
- *Strained-out* ($t^*_{\mathrm{cr},2} < t^*_{\mathrm{de},2} < t^*_{\mathrm{cr},1}$): detrainment from weaker vortex only, weaker vortex is destroyed (present results: $\Gamma_{o,2}/\Gamma_{o,1} \leq 0.6$).

All interactions eventually result in a single vortex. In complete merger, the circulation of the final compound vortex is greater than that of either original vortex. This increase is due to the mutual entrainment of both vortices and the transformation of the flow into a single vortex. In partial merger, the stronger vortex dominates and is enhanced by the entrained vorticity from the weaker vortex. In this case, vorticity is detrained from both vortices, however, the weaker vortex is destroyed before the stronger vortex is significantly eroded. When the weaker vortex is strained out, it is eventually destroyed and the stronger vortex remains with its circulation relatively unchanged. There is no mutual entrainment and the interaction does not yield a compound/enhanced vortex, i.e., merger does not occur.

## 5 Summary

This paper summarizes the main findings of our investigation of the interaction and merging of two unequal co-rotating vortices in a viscous fluid [3]. The key physical processes are identified and described. A merging criterion, based on the relative timing of core detrainment and core destruction, is developed. The establishment of core detrainment is characterized by a critical strain parameter, which can be directly related to the critical aspect ratio in the case of symmetric merger.



## References

1. Brandt, L.K., Nomura, K.K.: The physics of vortex merger: further insight. Phys. Fluids **18**, 1–4 (2006)
2. Brandt, L.K., Nomura, K.K.: The physics of vortex merger and the effects of ambient stable stratification. J. Fluid Mech. **592**, 413–446 (2007)
3. Brandt, L.K., Nomura, K.K.: Characterization of the interactions of two unequal co-rotating vortices. J. Fluid Mech. (Submitted)
4. Cerretelli, C., Williamson, C.H.K.: The physical mechanism for vortex merging. J. Fluid Mech. **475**, 41–77 (2003)
5. Dritschel, D.G., Waugh, D.W.: Quantification of the inelastic interaction of unequal vortices in two-dimensional vortex dynamics. Phys. Fluids **4**, 1737–1744 (1992)
6. Melander, M.V., McWilliams, J.C., Zabusky, N.J.: Asymmetric vortex merger in two dimensions: which vortex is "victorious"?. Phys. Fluids **30**, 2610–2612 (1987)
7. Melander, M.V., Zabusky, N.J., McWilliams, J.C.: Symmetric vortex merger in two dimensions: causes and conditions. J. Fluid Mech. **195**, 305–340 (1998)
8. Meunier, P.: Etude experimentale de deux tourbillons co-rotatifs. *Ph.D. Dissertation, Universite d'Aix-Marseille I, France* (2001)
9. Meunier, P., Ehrenstein, U., Leweke, T., Rossi, M.: A merging criterion for two-dimensional co-rotating vortices. Phys. Fluids **14**, 2757–2766 (2002)
10. Overman, E.A., Zabusky, N.J.: Evolution and merger of isolated vortex structures. Phys. Fluids **25**, 1297–1305 (1982)
11. Saffman, P.G., Szeto, R.: Equillibrium shapes of a pair of equal uniform vortices. Phys. Fluids **23**, 2339–2342 (1980)
12. Trieling, R.R., Velasco Fuentes, O.U., van Heijst, G.J.F.: Interaction of two unequal corotating vortices. Phys. Fluids **17**, 1–17 (2005)
13. Velasco Fuentes, O.U.: Vortex filamentation: its onset and its role on axisymmetrization and merger. Dyn. Atmos. Oceans **40**, 23–42 (2005)