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On the solutions of some nonlinear systems of difference equations

Elsayed M Elsayed[1,3*] and Hamdy El-Metwally[2,3]

*Correspondence: emelsayed@mans.edu.eg; emmelsayed@yahoo.com
[1] King Abdulaziz University, Faculty of Science, Mathematics Department, P.O. Box 80203, Jeddah 21589, Saudi Arabia
[3] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt
Full list of author information is available at the end of the article

Abstract

In this paper, we deal with the existence of solutions and the periodicity character of the following systems of rational difference equations with order three:

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(\pm 1 \pm x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(\pm 1 \pm y_n x_{n-2})},$$

where the initial conditions $x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1}$ and y_0 are nonzero real numbers.

MSC: 39A10

Keywords: difference equations; periodic solution; solution of difference equation; system of difference equations

1 Introduction

Recently, rational difference equations have attracted the attention of many researchers for various reasons. On the one hand, they provide examples of nonlinear equations which are, in some cases, treatable but their dynamics present some new features with respect to the linear case. On the other hand, rational equations frequently appear in some biological models. Hence, their study is of interest also due to their applications. A good example of both facts is Ricatti difference equations because the richness of the dynamics of Ricatti equations is very well known (see, *e.g.*, [1, 2]), and a particular case of these equations provides the classical Beverton-Holt model on the dynamics of exploited fish populations [3]. Obviously, higher-order rational difference equations and systems of rational equations have also been widely studied but still have many aspects to be investigated. The reader can find in the following books [4–6], and works cited therein, many results, applications, and open problems on higher-order equations and rational systems.

A preliminary study of planar rational systems can be found in the paper [7] by Camouzis *et al.* In the work, they give some results and provide some open questions for systems of equations of the following type:

$$x_{n+1} = \frac{\alpha_1 + \beta_1 x_n + \gamma_1 y_n}{A_1 + B_1 x_n + C_1 y_n}, \qquad y_{n+1} = \frac{\alpha_2 + \beta_2 x_n + \gamma_2 y_n}{A_2 + B_2 x_n + C_2 y_n}, \qquad n = 0, 1, 2, \ldots,$$

where the parameters are taken to be nonnegative. As shown in the cited paper, some of these systems can be reduced to some Ricatti equations or to some previously studied second-order rational equations. Furthermore, for some choices of the parameters, one obtains a system which is equivalent to the case with some other parameters. Camouzis *et al.* arrived at a list of 325 non-equivalent systems that should be focused on. They list such systems as pairs k, l, where k and l make reference to the number of the corresponding equation in their Tables 3 and 4. There are many papers that are related to the difference equations systems. For example, the periodicity of the positive solutions of the rational difference equations systems

$$x_{n+1} = \frac{m}{y_n}, \qquad y_{n+1} = \frac{py_n}{x_{n-1}y_{n-1}}$$

has been obtained by Cinar in [8].

In [9] Clark and Kulenovic investigated the global asymptotic stability of the system

$$x_{n+1} = \frac{x_n}{a + cy_n}, \qquad y_{n+1} = \frac{y_n}{b + dx_n}.$$

The behavior of the positive solutions of the following system:

$$x_{n+1} = \frac{x_{n-1}}{1 + x_{n-1}y_n}, \qquad y_{n+1} = \frac{y_{n-1}}{1 + y_{n-1}x_n}$$

has been studied by Kurbanli *et al.* [10].

Touafek *et al.* [11] investigated the periodic nature and got the form of the solutions of the following systems of rational difference equations:

$$x_{n+1} = \frac{x_{n-3}}{\pm 1 \pm x_{n-3}y_{n-1}}, \qquad y_{n+1} = \frac{y_{n-3}}{\pm 1 \pm y_{n-3}x_{n-1}}.$$

In [12] Yalçınkaya investigated the sufficient conditions for the global asymptotic stability of the following system of difference equations:

$$x_{n+1} = \frac{x_n + y_{n-1}}{x_n y_{n-1} - 1}, \qquad y_{n+1} = \frac{y_n + x_{n-1}}{y_n x_{n-1} - 1}.$$

Similarly, difference equations and nonlinear systems of the rational difference equations were investigated, see [1–39].

In this paper, we investigate the periodic nature and the form of the solutions of some nonlinear difference equations systems of order three

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(\pm 1 \pm x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(\pm 1 \pm y_n x_{n-2})},$$

where the initial conditions x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1} and y_0 are nonzero real numbers.

Definition (Periodicity) A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p} = x_n$ for all $n \geq -k$.

2 On the solution of the system $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(1+y_n x_{n-2})}$

In this section, we investigate the solutions of the two difference equations system

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(1 + x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(1 + y_n x_{n-2})}, \tag{1}$$

where $n \in \mathbb{N}_0$ and the initial conditions are arbitrary nonzero real numbers.

Theorem 1 *Assume that* $\{x_n, y_n\}$ *are solutions of system* (1). *Then for* $n = 0, 1, 2, \ldots$, *we see that all solutions of system* (1) *are given by the following formulas*:

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1} d^n} \prod_{i=0}^{n-1} \frac{(1 + (4i)cd)}{(1 + (4i + 2)af)}, \qquad x_{4n-1} = \frac{ba^n f^n}{c^n d^n} \prod_{i=0}^{n-1} \frac{(1 + (4i + 1)cd)}{(1 + (4i + 3)af)},$$

$$x_{4n} = \frac{a^{n+1} f^n}{c^n d^n} \prod_{i=0}^{n-1} \frac{(1 + (4i + 2)cd)}{(1 + (4i + 4)af)}, \qquad x_{4n+1} = \frac{a^{n+1} f^{n+1}}{ec^n d^n (1 + af)} \prod_{i=0}^{n-1} \frac{(1 + (4i + 3)cd)}{(1 + (4i + 5)af)}$$

and

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}} \prod_{i=0}^{n-1} \frac{(1 + (4i)af)}{(1 + (4i + 2)cd)}, \qquad y_{4n-1} = \frac{ec^n d^n}{a^n f^n} \prod_{i=0}^{n-1} \frac{(1 + (4i + 1)af)}{(1 + (4i + 3)cd)},$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n} \prod_{i=0}^{n-1} \frac{(1 + (4i + 2)af)}{(1 + (4i + 4)cd)}, \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1}}{ba^n f^n (1 + cd)} \prod_{i=0}^{n-1} \frac{(1 + (4i + 3)af)}{(1 + (4i + 5)cd)},$$

where $x_{-2} = c$, $x_{-1} = b$, $x_0 = a$, $y_{-2} = f$, $y_{-1} = e$ *and* $y_0 = d$.

Proof For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n - 1$, that is,

$$x_{4n-6} = \frac{a^{n-1} f^{n-1}}{c^{n-2} d^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i)cd)}{(1 + (4i + 2)af)}, \qquad x_{4n-5} = \frac{ba^{n-1} f^{n-1}}{c^{n-1} d^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 1)cd)}{(1 + (4i + 3)af)},$$

$$x_{4n-4} = \frac{a^n f^{n-1}}{c^{n-1} d^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 2)cd)}{(1 + (4i + 4)af)},$$

$$x_{4n-3} = \frac{a^n f^n}{ec^{n-1} d^{n-1} (1 + af)} \prod_{i=0}^{n-2} \frac{(1 + (4i + 3)cd)}{(1 + (4i + 5)af)},$$

$$y_{4n-6} = \frac{c^{n-1} d^{n-1}}{a^{n-1} f^{n-2}} \prod_{i=0}^{n-2} \frac{(1 + (4i)af)}{(1 + (4i + 2)cd)}, \qquad y_{4n-5} = \frac{ec^{n-1} d^{n-1}}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 1)af)}{(1 + (4i + 3)cd)},$$

$$y_{4n-4} = \frac{c^{n-1} d^n}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 2)af)}{(1 + (4i + 4)cd)},$$

$$y_{4n-3} = \frac{c^n d^n}{ba^{n-1} f^{n-1} (1 + cd)} \prod_{i=0}^{n-2} \frac{(1 + (4i + 3)af)}{(1 + (4i + 5)cd)}.$$

Now we find from Eq. (1) that

$$\begin{aligned} x_{4n-2} &= \frac{x_{4n-3} y_{4n-5}}{y_{4n-4}(1 + x_{4n-3} y_{4n-5})} \\ &= \left(\frac{a^n f^n}{ec^{n-1} d^{n-1} (1 + af)} \prod_{i=0}^{n-2} \frac{(1 + (4i + 3)cd)}{(1 + (4i + 5)af)} \right) \left(\frac{ec^{n-1} d^{n-1}}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 1)af)}{(1 + (4i + 3)cd)} \right) \\ &\quad \Bigg/ \left(\left(\frac{c^{n-1} d^n}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (4i + 2)af)}{(1 + (4i + 4)cd)} \right) \right. \end{aligned}$$

$$
\begin{aligned}
&\times \left(1+\left(\frac{a^nf^n}{ec^{n-1}d^{n-1}(1+af)}\prod_{i=0}^{n-2}\frac{(1+(4i+3)cd)}{(1+(4i+5)af)}\right)\right.\\
&\left.\times\left(\frac{ec^{n-1}d^{n-1}}{a^{n-1}f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+1)af)}{(1+(4i+3)cd)}\right)\right)\Bigg)\\
&=\frac{(\frac{af}{(1+af)}\prod_{i=0}^{n-2}\frac{(1+(4i+1)af)}{(1+(4i+5)af)})}{(\frac{c^{n-1}d^n}{a^{n-1}f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+2)af)}{(1+(4i+4)cd)})(1+\frac{af}{(1+af)}\prod_{i=0}^{n-2}\frac{(1+(4i+1)af)}{(1+(4i+5)af)})}\\
&=\frac{(\frac{af}{(1+(4n-3)af)})}{(\frac{c^{n-1}d^n}{a^{n-1}f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+2)af)}{(1+(4i+4)cd)})(1+\frac{af}{(1+(4n-3)af)})}\\
&=\frac{a^nf^n}{c^{n-1}d^n(1+(4n-2)af)}\prod_{i=0}^{n-2}\frac{(1+(4i+4)cd)}{(1+(4i+2)af)}=\frac{a^nf^n}{c^{n-1}d^n}\prod_{i=0}^{n-1}\frac{(1+(4i)cd)}{(1+(4i+2)af)},
\end{aligned}
$$

$$
\begin{aligned}
y_{4n-2}&=\frac{y_{4n-3}x_{4n-5}}{x_{4n-4}(1+y_{4n-3}x_{4n-5})}\\
&=\left(\frac{c^nd^n}{ba^{n-1}f^{n-1}(1+cd)}\prod_{i=0}^{n-2}\frac{(1+(4i+3)af)}{(1+(4i+5)cd)}\right)\left(\frac{ba^{n-1}f^{n-1}}{c^{n-1}d^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+1)cd)}{(1+(4i+3)af)}\right)\\
&\bigg/\left(\left(\frac{a^nf^{n-1}}{c^{n-1}d^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+2)cd)}{(1+(4i+4)af)}\right)\right.\\
&\times\left(1+\left(\frac{c^nd^n}{ba^{n-1}f^{n-1}(1+cd)}\prod_{i=0}^{n-2}\frac{(1+(4i+3)af)}{(1+(4i+5)cd)}\right)\right.\\
&\left.\left.\times\left(\frac{ba^{n-1}f^{n-1}}{c^{n-1}d^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+1)cd)}{(1+(4i+3)af)}\right)\right)\right)\\
&=\frac{\frac{cd}{(1+cd)}\prod_{i=0}^{n-2}\frac{(1+(4i+1)cd)}{(1+(4i+5)cd)}}{(\frac{a^nf^{n-1}}{c^{n-1}d^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+2)cd)}{(1+(4i+4)af)})(1+\frac{cd}{(1+cd)}\prod_{i=0}^{n-2}\frac{(1+(4i+1)cd)}{(1+(4i+5)cd)})}\\
&=\frac{(\frac{cd}{(1+(4n-3)cd)})}{(\frac{a^nf^{n-1}}{c^{n-1}d^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(4i+2)cd)}{(1+(4i+4)af)})(1+\frac{cd}{(1+(4n-3)cd)})}\\
&=\frac{c^nd^n}{a^nf^{n-1}(1+(4n-2)cd)}\prod_{i=0}^{n-2}\frac{(1+(4i+4)af)}{(1+(4i+2)cd)}=\frac{c^nd^n}{a^nf^{n-1}}\prod_{i=0}^{n-1}\frac{(1+(4i)af)}{(1+(4i+2)cd)}.
\end{aligned}
$$

Also, we can prove the other relations. The proof is complete. □

The following theorem can be proved similarly.

Theorem 2 *Assume that* $\{x_n, y_n\}$ *are solutions of the following system*:

$$
x_{n+1}=\frac{x_ny_{n-2}}{y_{n-1}(1+x_ny_{n-2})},\qquad y_{n+1}=\frac{y_nx_{n-2}}{x_{n-1}(1-y_nx_{n-2})}.
$$

Then for $n = 0, 1, 2, \dots,$

$$
x_{4n-2}=\frac{a^nf^n}{c^{n-1}d^n}\prod_{i=0}^{n-1}\frac{(1-(4i)cd)}{(1+(4i+2)af)},\qquad x_{4n-1}=\frac{ba^nf^n}{c^nd^n}\prod_{i=0}^{n-1}\frac{(1-(4i+1)cd)}{(1+(4i+3)af)},
$$

$$x_{4n}=\frac{a^{n+1}f^{n}}{c^{n}d^{n}}\prod_{i=0}^{n-1}\frac{(1-(4i+2)cd)}{(1+(4i+4)af)},\qquad x_{4n+1}=\frac{a^{n+1}f^{n+1}}{ec^{n}d^{n}(1+af)}\prod_{i=0}^{n-1}\frac{(1-(4i+3)cd)}{(1+(4i+5)af)},$$

$$y_{4n-2}=\frac{c^{n}d^{n}}{a^{n}f^{n-1}}\prod_{i=0}^{n-1}\frac{(1+(4i)af)}{(1-(4i+2)cd)},\qquad y_{4n-1}=\frac{ec^{n}d^{n}}{a^{n}f^{n}}\prod_{i=0}^{n-1}\frac{(1+(4i+1)af)}{(1-(4i+3)cd)},$$

$$y_{4n}=\frac{c^{n}d^{n+1}}{a^{n}f^{n}}\prod_{i=0}^{n-1}\frac{(1+(4i+2)af)}{(1-(4i+4)cd)},\qquad y_{4n+1}=\frac{c^{n+1}d^{n+1}}{ba^{n}f^{n}(1-cd)}\prod_{i=0}^{n-1}\frac{(1+(4i+3)af)}{(1-(4i+5)cd)}.$$

Example 1 For confirming the results of this section, we consider numerical example for the difference system (1) with the initial conditions $x_{-2}=3$, $x_{-1}=5$, $x_0=-1$, $y_{-2}=-0.4$, $y_{-1}=0.16$ and $y_0=7$. See Figure 1.

Example 2 We assume that the initial conditions for the difference system (1) are $x_{-2}=-0.2$, $x_{-1}=0.15$, $x_0=-0.51$, $y_{-2}=-0.23$, $y_{-1}=0.16$ and $y_0=-0.7$. See Figure 2.

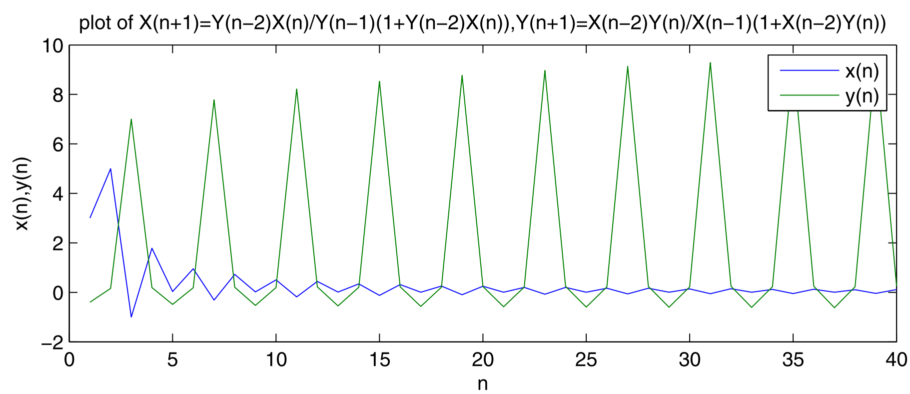

Figure 1 This figure shows the solutions of the system $x_{n+1}=\frac{x_n y_{n-2}}{y_{n-1}(1+x_n y_{n-2})}$, $y_{n+1}=\frac{y_n x_{n-2}}{x_{n-1}(1+y_n x_{n-2})}$, with the initial conditions $x_{-2}=3$, $x_{-1}=5$, $x_0=-1$, $y_{-2}=-0.4$, $y_{-1}=0.16$ and $y_0=7$.

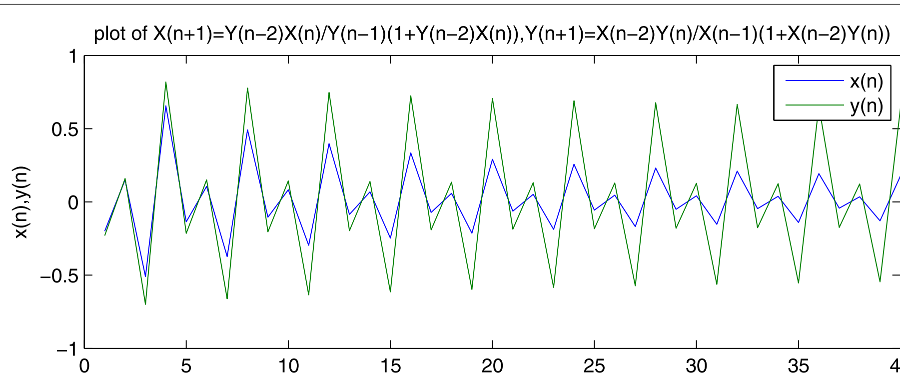

Figure 2 This figure shows the dynamics of solutions of the difference system (1) when we put the initial conditions $x_{-2}=-0.2$, $x_{-1}=0.15$, $x_0=-0.51$, $y_{-2}=-0.23$, $y_{-1}=0.16$ and $y_0=-0.7$.

3 On the solution of the system $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1+y_n x_{n-2})}$

In this section, we obtain the form of the solutions of the two difference equations system

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(1 + x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1 + y_n x_{n-2})}, \tag{2}$$

where $n \in \mathbb{N}_0$ and the initial conditions are arbitrary nonzero real numbers with $x_{-2}y_0 \neq 1$.

Theorem 3 *Let $\{x_n, y_n\}_{n=-2}^{+\infty}$ be solutions of system* (2). *Then $\{x_n\}_{n=-2}^{+\infty}$ and $\{y_n\}_{n=-2}^{+\infty}$ are given by the formula for $n = 0, 1, 2, \dots$,*

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1} d^n} \prod_{i=0}^{n-1} \frac{1}{(1 + (4i+2)af)}, \qquad x_{4n-1} = \frac{b a^n f^n}{c^n d^n} \frac{(-1 + cd)^n}{\prod_{i=0}^{n-1}(1 + (4i+3)af)},$$

$$x_{4n} = \frac{a^{n+1} f^n}{c^n d^n} \prod_{i=0}^{n-1} \frac{1}{(1 + (4i+4)af)}, \qquad x_{4n+1} = \frac{a^{n+1} f^{n+1}}{e c^n d^n (1 + af)} \frac{(-1 + cd)^n}{\prod_{i=0}^{n-1}(1 + (4i+5)af)},$$

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}} \prod_{i=0}^{n-1} \bigl(1 + (4i)af\bigr), \qquad y_{4n-1} = \frac{e c^n d^n}{a^n f^n} \frac{\prod_{i=0}^{n-1}(1 + (4i+1)af)}{(-1 + cd)^n},$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n} \prod_{i=0}^{n-1} \bigl(1 + (4i+2)af\bigr), \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1}}{b a^n f^n} \frac{\prod_{i=0}^{n-1}(1 + (4i+3)af)}{(-1 + cd)^{n+1}}.$$

Proof For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n - 1$, that is,

$$x_{4n-5} = \frac{b a^{n-1} f^{n-1}}{c^{n-1} d^{n-1}} \frac{(-1 + cd)^{n-1}}{\prod_{i=0}^{n-2}(1 + (4i+3)af)}, \qquad x_{4n-4} = \frac{a^n f^{n-1}}{c^{n-1} d^{n-1}} \prod_{i=0}^{n-2} \frac{1}{(1 + (4i+4)af)},$$

$$x_{4n-3} = \frac{a^n f^n}{e c^{n-1} d^{n-1} (1 + af)} \frac{(-1 + cd)^{n-1}}{\prod_{i=0}^{n-2}(1 + (4i+5)af)},$$

$$y_{4n-5} = \frac{e c^{n-1} d^{n-1}}{a^{n-1} f^{n-1}} \frac{\prod_{i=0}^{n-2}(1 + (4i+1)af)}{(-1 + cd)^{n-1}},$$

$$y_{4n-4} = \frac{c^{n-1} d^n}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} \bigl(1 + (4i+2)af\bigr), \qquad y_{4n-3} = \frac{c^n d^n}{b a^{n-1} f^{n-1}} \frac{\prod_{i=0}^{n-2}(1 + (4i+3)af)}{(-1 + cd)^n}.$$

Now, we obtain from Eq. (2) that

$$\begin{aligned} x_{4n-2} &= \frac{x_{4n-3} y_{4n-5}}{y_{4n-4}(1 + x_{4n-3} y_{4n-5})} \\ &= \left(\frac{a^n f^n}{e c^{n-1} d^{n-1} (1 + af)} \frac{(-1 + cd)^{n-1}}{\prod_{i=0}^{n-2}(1 + (4i+5)af)} \right) \left(\frac{e c^{n-1} d^{n-1}}{a^{n-1} f^{n-1}} \frac{\prod_{i=0}^{n-2}(1 + (4i+1)af)}{(-1 + cd)^{n-1}} \right) \\ &\quad \bigg/ \left(\left(\frac{c^{n-1} d^n}{a^{n-1} f^{n-1}} \prod_{i=0}^{n-2} (1 + (4i+2)af) \right) \right. \\ &\quad \times \left(1 + \left(\frac{a^n f^n}{e c^{n-1} d^{n-1} (1 + af)} \frac{(-1 + cd)^{n-1}}{\prod_{i=0}^{n-2}(1 + (4i+5)af)} \right) \right. \\ &\quad \left. \left. \times \left(\frac{e c^{n-1} d^{n-1}}{a^{n-1} f^{n-1}} \frac{\prod_{i=0}^{n-2}(1 + (4i+1)af)}{(-1 + cd)^{n-1}} \right) \right) \right) \end{aligned}$$

$$= \frac{(\frac{af}{(1+(4n-3)af)})}{(\frac{c^{n-1}d^n}{a^{n-1}f^{n-1}}\prod_{i=0}^{n-2}(1+(4i+2)af))(1+\frac{af}{(1+(4n-3)af)})}$$
$$= \frac{a^{n-1}f^{n-1}af}{(c^{n-1}d^n\prod_{i=0}^{n-2}(1+(4i+2)af))(1+(4n-3)af+af)}$$
$$= \frac{a^nf^n}{(c^{n-1}d^n\prod_{i=0}^{n-2}(1+(4i+2)af))(1+(4n-2)af)} = \frac{a^nf^n}{c^{n-1}d^n\prod_{i=0}^{n-1}(1+(4i+2)af)}.$$

Also, we can prove the other relations. This completes the proof. □

We consider the following systems, and the proof of the theorems is similar to the above theorem, and so it is left to the reader,

$$x_{n+1} = \frac{x_ny_{n-2}}{y_{n-1}(1+x_ny_{n-2})}, \qquad y_{n+1} = \frac{y_nx_{n-2}}{x_{n-1}(-1-y_nx_{n-2})}, \tag{3}$$
$$x_{n+1} = \frac{x_ny_{n-2}}{y_{n-1}(1-x_ny_{n-2})}, \qquad y_{n+1} = \frac{y_nx_{n-2}}{x_{n-1}(-1+y_nx_{n-2})}, \tag{4}$$
$$x_{n+1} = \frac{x_ny_{n-2}}{y_{n-1}(1-x_ny_{n-2})}, \qquad y_{n+1} = \frac{y_nx_{n-2}}{x_{n-1}(-1-y_nx_{n-2})}. \tag{5}$$

The following theorems are devoted to the expressions of the form of the solutions of systems (3), (4), (5) with $x_{-2} = c$, $x_{-1} = b$, $x_0 = a$, $y_{-2} = f$, $y_{-1} = e$ and $y_0 = d$.

Theorem 4 *Let* $\{x_n, y_n\}_{n=-2}^{+\infty}$ *be solutions of system* (3) *and* $x_{-2}y_0 \neq -1$. *Then for* $n = 0, 1, 2, \ldots,$

$$x_{4n-2} = \frac{a^nf^n}{c^{n-1}d^n}\prod_{i=0}^{n-1}\frac{1}{(1+(4i+2)af)}, \qquad x_{4n-1} = \frac{ba^nf^n}{c^nd^n}\frac{(-1-cd)^n}{\prod_{i=0}^{n-1}(1+(4i+3)af)},$$
$$x_{4n} = \frac{a^{n+1}f^n}{c^nd^n}\prod_{i=0}^{n-1}\frac{1}{(1+(4i+4)af)}, \qquad x_{4n+1} = \frac{a^{n+1}f^{n+1}}{ec^nd^n(1+af)}\frac{(-1-cd)^n}{\prod_{i=0}^{n-1}(1+(4i+5)af)},$$
$$y_{4n-2} = \frac{c^nd^n}{a^nf^{n-1}}\prod_{i=0}^{n-1}\big(1+(4i)af\big), \qquad y_{4n-1} = \frac{ec^nd^n}{a^nf^n}\frac{\prod_{i=0}^{n-1}(1+(4i+1)af)}{(-1-cd)^n},$$
$$y_{4n} = \frac{c^nd^{n+1}}{a^nf^n}\prod_{i=0}^{n-1}\big(1+(4i+2)af\big), \qquad y_{4n+1} = \frac{c^{n+1}d^{n+1}}{ba^nf^n}\frac{\prod_{i=0}^{n-1}(1+(4i+3)af)}{(-1-cd)^{n+1}}.$$

Theorem 5 *Assume that* $\{x_n, y_n\}$ *are solutions of system* (4) *with* $x_{-2}y_0 \neq 1$. *Then for* $n = 0, 1, 2, \ldots,$

$$x_{4n-2} = \frac{a^nf^n}{c^{n-1}d^n}\prod_{i=0}^{n-1}\frac{1}{(1-(4i+2)af)}, \qquad x_{4n-1} = \frac{ba^nf^n}{c^nd^n}\frac{(-1+cd)^n}{\prod_{i=0}^{n-1}(1-(4i+3)af)},$$
$$x_{4n} = \frac{a^{n+1}f^n}{c^nd^n}\prod_{i=0}^{n-1}\frac{1}{(1-(4i+4)af)}, \qquad x_{4n+1} = \frac{a^{n+1}f^{n+1}}{ec^nd^n(1+af)}\frac{(-1+cd)^n}{\prod_{i=0}^{n-1}(1-(4i+5)af)},$$
$$y_{4n-2} = \frac{c^nd^n}{a^nf^{n-1}}\prod_{i=0}^{n-1}\big(1-(4i)af\big), \qquad y_{4n-1} = \frac{ec^nd^n}{a^nf^n}\frac{\prod_{i=0}^{n-1}(1-(4i+1)af)}{(-1+cd)^n},$$

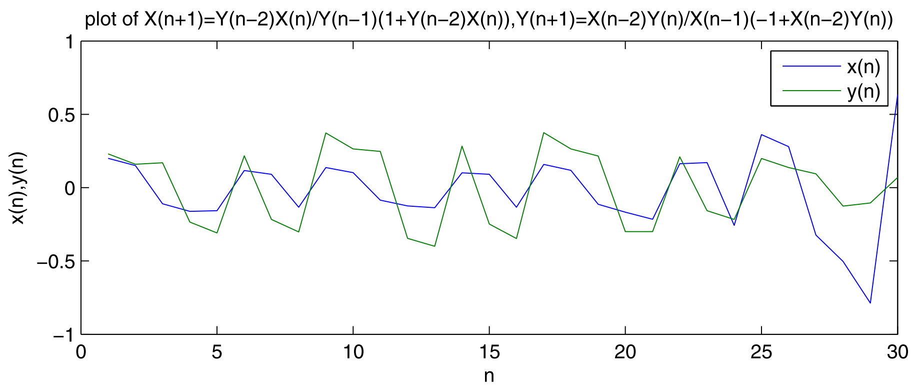

Figure 3 **This figure shows the behavior of the difference system** $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1+y_n x_{n-2})}$, **when we take the initial conditions** $x_{-2} = 0.2$, $x_{-1} = 0.15$, $x_0 = -0.11$, $y_{-2} = 0.23$, $y_{-1} = 0.16$ **and** $y_0 = 0.17$.

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n} \prod_{i=0}^{n-1} \bigl(1 - (4i+2)af\bigr), \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1}}{ba^n f^n} \frac{\prod_{i=0}^{n-1}(1 - (4i+3)af)}{(-1+cd)^{n+1}}.$$

Theorem 6 *Suppose that* $\{x_n, y_n\}$ *are solutions of system* (5) *where* $x_{-2}y_0 \neq -1$. *Then for* $n = 0, 1, 2, \ldots,$

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1} d^n} \prod_{i=0}^{n-1} \frac{1}{(1 - (4i+2)af)}, \qquad x_{4n-1} = \frac{ba^n f^n}{c^n d^n} \frac{(-1-cd)^n}{\prod_{i=0}^{n-1}(1 - (4i+3)af)},$$

$$x_{4n} = \frac{a^{n+1} f^n}{c^n d^n} \prod_{i=0}^{n-1} \frac{1}{(1 - (4i+4)af)}, \qquad x_{4n+1} = \frac{a^{n+1} f^{n+1}}{ec^n d^n (1 - af)} \frac{(-1-cd)^n}{\prod_{i=0}^{n-1}(1 - (4i+5)af)},$$

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}} \prod_{i=0}^{n-1} \bigl(1 - (4i)af\bigr), \qquad y_{4n-1} = \frac{ec^n d^n}{a^n f^n} \frac{\prod_{i=0}^{n-1}(1 - (4i+1)af)}{(-1-cd)^n},$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n} \prod_{i=0}^{n-1} \bigl(1 - (4i+2)af\bigr), \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1}}{ba^n f^n} \frac{\prod_{i=0}^{n-1}(1 - (4i+3)af)}{(-1-cd)^{n+1}}.$$

Example 3 We consider an interesting numerical example for the difference system (2) with the initial conditions $x_{-2} = 0.2$, $x_{-1} = 0.15$, $x_0 = -0.11$, $y_{-2} = 0.23$, $y_{-1} = 0.16$ and $y_0 = 0.17$. See Figure 3.

Example 4 See Figure 4, where we take system (3) with the initial conditions $x_{-2} = 0.12$, $x_{-1} = 0.15$, $x_0 = 0.11$, $y_{-2} = 0.3$, $y_{-1} = -0.6$ and $y_0 = 0.17$.

4 On the solution of the system $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(-1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1+y_n x_{n-2})}$

In this section, we get the form of the solutions of the difference equations system

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(-1 + x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1 + y_n x_{n-2})}, \tag{6}$$

where $n = 0, 1, 2, \ldots$ and the initial conditions x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1} and y_0 are arbitrary nonzero real numbers with $x_0 y_{-2}, x_{-2} y_0 \neq 1$.

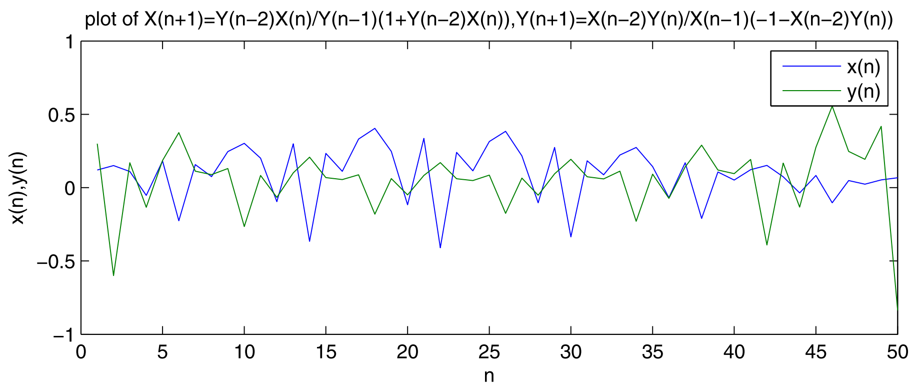

Figure 4 This figure shows the qualitative behavior of the solutions of system (3) with the initial conditions $x_{-2} = 0.12$, $x_{-1} = 0.15$, $x_0 = 0.11$, $y_{-2} = 0.3$, $y_{-1} = -0.6$ and $y_0 = 0.17$.

Theorem 7 *If* $\{x_n, y_n\}$ *are solutions of difference equation system* (6), *then for* $n = 0, 1, 2, \ldots,$

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1} d^n}, \qquad x_{4n-1} = \frac{b a^n f^n}{c^n d^n} \frac{(-1+cd)^n}{(-1+af)^n},$$

$$x_{4n} = \frac{a^{n+1} f^n}{c^n d^n}, \qquad x_{4n+1} = \frac{a^{n+1} f^{n+1} (-1+cd)^n}{e c^n d^n (-1+af)^{n+1}},$$

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}}, \qquad y_{4n-1} = \frac{e c^n d^n}{a^n f^n} \frac{(-1+af)^n}{(-1+cd)^n},$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n}, \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1} (-1+af)^n}{b a^n f^n (-1+cd)^{n+1}}.$$

Proof For $n = 0$ the result holds. Now, suppose that $n > 1$ and that our assumption holds for $n - 1$, that is,

$$x_{4n-6} = \frac{a^{n-1} f^{n-1}}{c^{n-2} d^{n-1}}, \qquad x_{4n-5} = \frac{b a^{n-1} f^{n-1} (-1+cd)^{n-1}}{c^{n-1} d^{n-1} (-1+af)^{n-1}},$$

$$x_{4n-4} = \frac{a^n f^{n-1}}{c^{n-1} d^{n-1}}, \qquad x_{4n-3} = \frac{a^n f^n (-1+cd)^{n-1}}{e c^{n-1} d^{n-1} (-1+af)^n},$$

$$y_{4n-6} = \frac{c^{n-1} d^{n-1}}{a^{n-1} f^{n-2}}, \qquad y_{4n-5} = \frac{e c^{n-1} d^{n-1} (-1+af)^{n-1}}{a^{n-1} f^{n-1} (-1+cd)^{n-1}},$$

$$y_{4n-4} = \frac{c^{n-1} d^n}{a^{n-1} f^{n-1}}, \qquad y_{4n-3} = \frac{c^n d^n (-1+af)^{n-1}}{b a^{n-1} f^{n-1} (-1+cd)^n}.$$

Now, we conclude from Eq. (6) that

$$\begin{aligned} x_{4n-2} = \frac{x_{4n-3} y_{4n-5}}{y_{4n-4}(-1 + x_{4n-3} y_{4n-5})} &= \frac{(\frac{a^n f^n (-1+cd)^{n-1}}{e c^{n-1} d^{n-1} (-1+af)^n})(\frac{e c^{n-1} d^{n-1} (-1+af)^{n-1}}{a^{n-1} f^{n-1} (-1+cd)^{n-1}})}{(\frac{c^{n-1} d^n}{a^{n-1} f^{n-1}})(-1 + (\frac{a^n f^n (-1+cd)^{n-1}}{e c^{n-1} d^{n-1} (-1+af)^n})(\frac{e c^{n-1} d^{n-1} (-1+af)^{n-1}}{a^{n-1} f^{n-1} (-1+cd)^{n-1}}))} \\ &= \frac{(\frac{af}{(-1+af)})}{(\frac{c^{n-1} d^n}{a^{n-1} f^{n-1}})(-1 + \frac{af}{(-1+af)})} = \frac{a^n f^n}{c^{n-1} d^n}, \end{aligned}$$

$$y_{4n-2} = \frac{y_{4n-3}x_{4n-5}}{x_{4n-4}(-1+y_{4n-3}x_{4n-5})} = \frac{(\frac{c^n d^n(-1+af)^{n-1}}{ba^{n-1}f^{n-1}(-1+cd)^n})(\frac{ba^{n-1}f^{n-1}(-1+cd)^{n-1}}{c^{n-1}d^{n-1}(-1+af)^{n-1}})}{(\frac{a^n f^{n-1}}{c^{n-1}d^{n-1}})(-1+(\frac{c^n d^n(-1+af)^{n-1}}{ba^{n-1}f^{n-1}(-1+cd)^n})(\frac{ba^{n-1}f^{n-1}(-1+cd)^{n-1}}{c^{n-1}d^{n-1}(-1+af)^{n-1}}))}$$

$$= \frac{(\frac{cd}{(-1+cd)})}{(\frac{a^n f^{n-1}}{c^{n-1}d^{n-1}})(-1+(\frac{cd}{(-1+cd)}))} = \frac{c^n d^n}{a^n f^{n-1}}.$$

Also, we can prove the other relations. This completes the proof. □

We consider the following system, and the proof of the theorem is similar to the above mentioned theorem and so it is left to the reader,

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(-1+x_n y_{n-2})}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1-y_n x_{n-2})}. \tag{7}$$

Theorem 8 *Let $\{x_n, y_n\}_{n=-2}^{+\infty}$ be solutions of system* (7) *and $x_0 y_{-2} \neq 1$, $x_{-2}y_0 \neq -1$. Then for $n = 0, 1, 2, \ldots,$*

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1}d^n}, \qquad x_{4n-1} = \frac{ba^n f^n}{c^n d^n}\frac{(-1-cd)^n}{(-1+af)^n},$$

$$x_{4n} = \frac{a^{n+1}f^n}{c^n d^n}, \qquad x_{4n+1} = \frac{a^{n+1}f^{n+1}(-1-cd)^n}{ec^n d^n(-1+af)^{n+1}},$$

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}}, \qquad y_{4n-1} = \frac{ec^n d^n}{a^n f^n}\frac{(-1+af)^n}{(-1-cd)^n},$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n}, \qquad y_{4n+1} = \frac{c^{n+1}d^{n+1}(-1+af)^n}{ba^n f^n(-1-cd)^{n+1}}.$$

Lemma 1 *The solution of system* (6) *is unbounded except in the following case.*

Theorem 9 *System* (6) *has a periodic solution of period four iff $cd = af = 2$ and it will take the following form*: $\{x_n\} = \{c, b, a, \frac{af}{e}, c, b, a, \ldots\}$, $\{y_n\} = \{f, e, d, \frac{cd}{b}, f, e, d, \ldots\}$.

Proof First, suppose that a prime period four solution exists

$$\{x_n\} = \left\{c, b, a, \frac{af}{e}, c, b, a, \ldots\right\}, \qquad \{y_n\} = \left\{f, e, d, \frac{cd}{b}, f, e, d, \ldots\right\}$$

of system (6). We see from the form of the solution of system (6) that

$$x_{4n-2} = c = \frac{a^n f^n}{c^{n-1}d^n}, \qquad x_{4n-1} = b = \frac{ba^n f^n}{c^n d^n}\frac{(-1+cd)^n}{(-1+af)^n},$$

$$x_{4n} = a = \frac{a^{n+1}f^n}{c^n d^n}, \qquad x_{4n+1} = \frac{af}{e} = \frac{a^{n+1}f^{n+1}(-1+cd)^n}{ec^n d^n(-1+af)^{n+1}},$$

$$y_{4n-2} = f = \frac{c^n d^n}{a^n f^{n-1}}, \qquad y_{4n-1} = e = \frac{ec^n d^n}{a^n f^n}\frac{(-1+af)^n}{(-1+cd)^n},$$

$$y_{4n} = d = \frac{c^n d^{n+1}}{a^n f^n}, \qquad y_{4n+1} = \frac{cd}{b} = \frac{c^{n+1}d^{n+1}(-1+af)^n}{ba^n f^n(-1+cd)^{n+1}}.$$

Then we get $cd = af = 2$. Second, assume that $cd = af = 2$. Then we see from the form of the solution of system (6) that

$$x_{4n-2} = \frac{a^n f^n}{c^{n-1} d^n} = c, \qquad x_{4n-1} = \frac{b a^n f^n}{c^n d^n} \frac{(-1+cd)^n}{(-1+af)^n} = b,$$

$$x_{4n} = \frac{a^{n+1} f^n}{c^n d^n} = a, \qquad x_{4n+1} = \frac{a^{n+1} f^{n+1} (-1+cd)^n}{e c^n d^n (-1+af)^{n+1}} = \frac{af}{e},$$

$$y_{4n-2} = \frac{c^n d^n}{a^n f^{n-1}} = f, \qquad y_{4n-1} = \frac{e c^n d^n}{a^n f^n} \frac{(-1+af)^n}{(-1+cd)^n} = e,$$

$$y_{4n} = \frac{c^n d^{n+1}}{a^n f^n} = c, \qquad y_{4n+1} = \frac{c^{n+1} d^{n+1} (-1+af)^n}{b a^n f^n (-1+cd)^{n+1}} = \frac{cd}{b}.$$

Thus, we have a periodic solution of period four and the proof is complete. □

Lemma 2 *The solution of system* (7) *is unbounded except in the following case.*

Theorem 10 *System* (7) *has a periodic solution of period eight iff* $cd = -2$, $af = 2$ *and it will take the following form*: $\{x_n\} = \{c, b, a, \frac{af}{e}, -c, -b, -a, -\frac{af}{e}, c, b, a, \ldots\}$, $\{y_n\} = \{f, e, d, \frac{cd}{b}, -f, -e, -d, -\frac{cd}{b}, f, e, d, \ldots\}$.

Example 5 We consider a numerical example for the difference system (6) when we put the initial conditions $x_{-2} = 0.12$, $x_{-1} = 0.5$, $x_0 = 0.31$, $y_{-2} = 0.23$, $y_{-1} = -0.6$ and $y_0 = 0.7$. See Figure 5.

Example 6 Figure 6 shows the behavior of the solution of the difference system (6) with the initial conditions $x_{-2} = 4$, $x_{-1} = -7$, $x_0 = -0.5$, $y_{-2} = -4$, $y_{-1} = 6$ and $y_0 = 0.5$.

Example 7 We consider a numerical example for the difference system (7) when we put the initial conditions $x_{-2} = 0.32$, $x_{-1} = 0.25$, $x_0 = -0.31$, $y_{-2} = 0.23$, $y_{-1} = 0.26$ and $y_0 = 0.17$. See Figure 7.

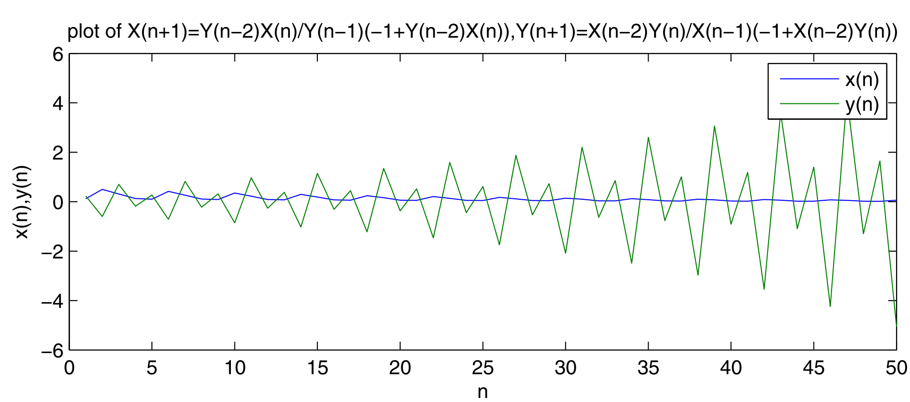

Figure 5 This figure shows the solution of the system of difference equations $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(-1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1+y_n x_{n-2})}$, when we put the initial conditions $x_{-2} = 0.12$, $x_{-1} = 0.5$, $x_0 = 0.31$, $y_{-2} = 0.23$, $y_{-1} = -0.6$ and $y_0 = 0.7$.

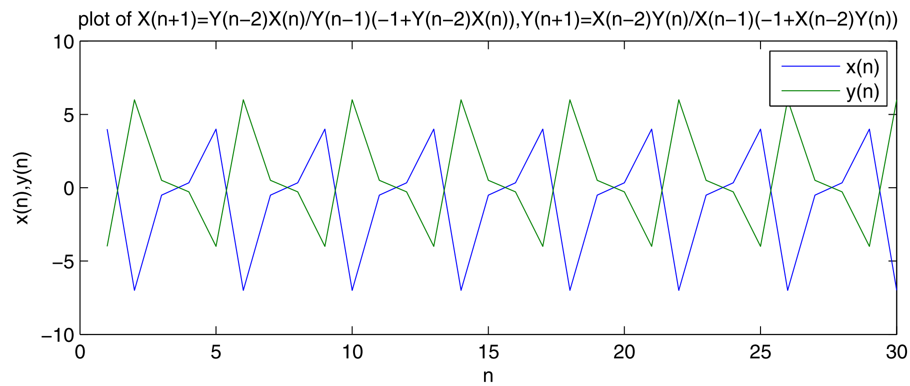

Figure 6 This figure shows the periodicity of the solutions of difference equations system (6) with initial conditions $x_{-2} = 4$, $x_{-1} = -7$, $x_0 = -0.5$, $y_{-2} = -4$, $y_{-1} = 6$ and $y_0 = 0.5$.

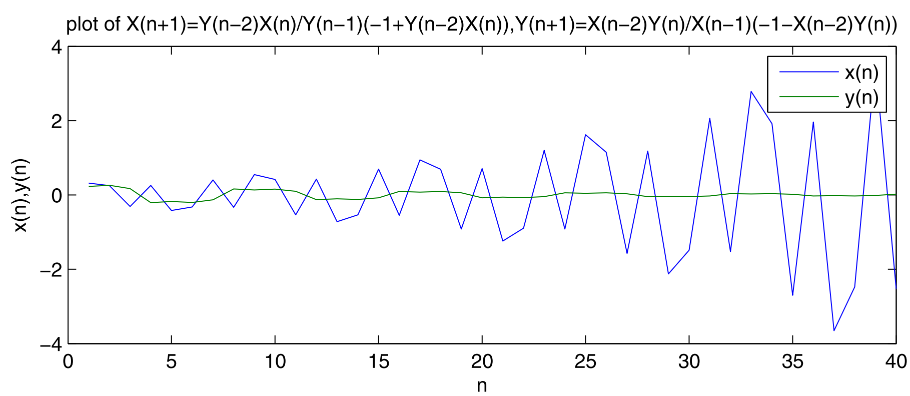

Figure 7 This figure shows the behavior of the solutions of $x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(-1+x_n y_{n-2})}$, $y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(-1-y_n x_{n-2})}$, since we consider the initial conditions equals $x_{-2} = 0.32$, $x_{-1} = 0.25$, $x_0 = -0.31$, $y_{-2} = 0.23$, $y_{-1} = 0.26$ and $y_0 = 0.17$.

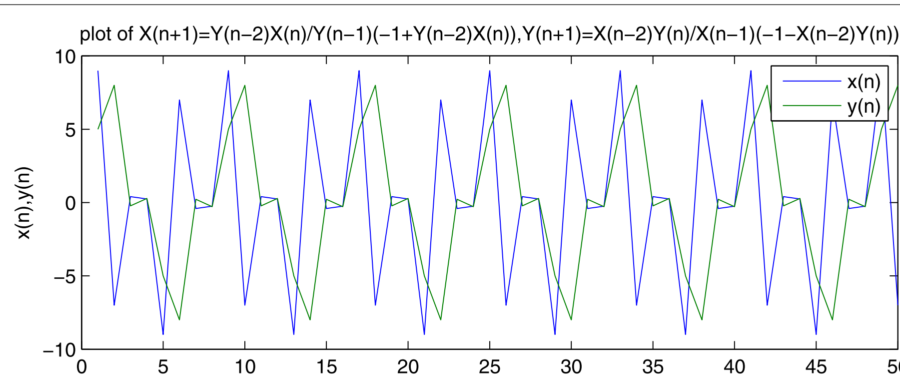

Figure 8 This figure shows the periodic nature of the solutions of system (7), where the initial conditions $x_{-2} = 9$, $x_{-1} = -7$, $x_0 = 0.4$, $y_{-2} = 5$, $y_{-1} = 8$ and $y_0 = -2/9$.

Example 8 Figure 8 shows the periodicity of the solution of the difference system (7) with the initial conditions $x_{-2} = 9$, $x_{-1} = -7$, $x_0 = 0.4$, $y_{-2} = 5$, $y_{-1} = 8$ and $y_0 = -2/9$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors declare that the study was realized in collaboration with the same responsibility. Both authors read and approved the final manuscript.

Author details
[1]King Abdulaziz University, Faculty of Science, Mathematics Department, P.O. Box 80203, Jeddah 21589, Saudi Arabia. [2]Department of Mathematics, Rabigh College of Science and Art, King Abdulaziz University, P.O. Box 344, Rabigh 21911, Saudi Arabia. [3]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt.

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