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# (Dys)Zphilia or a custodial breaking Higgs at the LHC

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ABSTRACT: Electroweak precision measurements established that custodial symmetry is preserved to a good accuracy in the gauge sector after electroweak symmetry breaking. However, recent LHC results might be interpreted as pointing towards Higgs couplings that do not respect such symmetry. Motivated by this possibility, we reconsider the presence of an explicitly custodial breaking coupling in a generic Higgs parameterization. After briefly commenting on the large UV sensitivity of the T parameter to such a coupling, we perform a fit to results of Higgs searches at LHC and Tevatron, and find that the apparent enhancement of the ZZ channel with respect to WW can be accommodated. Two degenerate best-fit points are present, which we label 'Zphilic' and 'dysZphilic' depending on the sign of the hZZ coupling. Finally we highlight some measurements at future linear colliders that may remove such degeneracy.

KEYWORDS: Higgs Physics, Beyond Standard Model

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## 1 Introduction

The main goal of the LHC is to shed light on the mechanism of ElectroWeak Symmetry Breaking (EWSB). The recent excesses observed in searches for the Higgs boson at ATLAS and CMS, supplemented by some hints from the Tevatron, can be seen as the starting point in this direction. Even though they are far from being conclusive, the experimental results point to a resonance with mass around 125 GeV and, broadly speaking, Higgs-like behavior. If such hints really correspond to the first manifestation of a new degree of freedom, then the measurement and study of its properties will be crucial to unveil EWSB. This is even more true in the absence of any direct evidence of physics beyond the Standard Model (SM) so far.

The EWSB sector has been indirectly probed by the LEP precision tests, which represent a primary source of information: one of the most important outcomes of precision measurements is that the gauge sector after EWSB must approximately respect an SU(2)<sub>c</sub> custodial symmetry. Such requirement is satisfied by the SM description of EWSB. On the other hand, at the moment experimental excesses at the LHC may be interpreted as pointing to non-SM Higgs couplings, especially in the gauge sector. In fact, not only is there a trend of underproduction in the WW channel and of overproduction in the  $\gamma\gamma$ channel (for the latter, the excess is stronger in the vector boson fusion subchannel), but, even though not statistically significant, an enhancement of the ZZ signal with respect to WW is observed by ATLAS, whereas custodial symmetry implies that the two have the same strength (when normalized to their SM values). Clearly such hints could be just due to statistical fluctuations, or to issues with the modeling of complex backgrounds (for example, in the  $h \to WW$  channel). Nevertheless, it is interesting to ask what would be the implications if the current pattern of excesses were to be confirmed with more data. In this spirit, we relax the assumption of custodial invariance in the couplings of the Higgs resonance and perform a fit to the results of Higgs searches by employing a parameterization where explicit custodial breaking is allowed. Our model-independent approach is similar in spirit to other recent analyses of the Higgs experimental results, see refs.  $[1-5]^{-1}$ . We also analyze the effects on the electroweak parameter T, pointing out that if the couplings hWW and hZZ do not respect custodial symmetry then T receives quadratically divergent corrections. In a concrete model, new degrees of freedom below the cutoff must therefore conspire to make the total contribution to T compatible with electroweak precision tests (EWPT).

Not surprisingly, the fit to the results of Higgs searches points to a Higgs coupling more strongly to ZZ than to WW. Two exactly degenerate best-fit points appear, which we label 'Zphilic' and 'dysZphilic' depending on the sign of the hZZ coupling. Such sign, although unobservable in current Higgs searches, is physical in processes involving interference. We therefore discuss some future measurements at colliders that may be used to resolve the degeneracy.

We remark that many proposals for physics beyond the SM exist in the literature where the custodial symmetry is not respected: for example, models where the Higgs sector is extended with scalar triplets that get a non-vanishing vacuum expectation value, generic two Higgs doublet models, as well as theories where the Higgs arises as the pseudo-Goldstone boson of a coset G/H where H does not contain  $SO(4) \sim SU(2) \times SU(2)$ , such as  $SU(3)/(SU(2) \times U(1))$ , fall in this class.

Our paper is structured as follows: we start by introducing our parameterization and discussing the fit to LHC data in section 2, where we also briefly comment on the effect of explicit custodial breaking on the electroweak T parameter. In the light of our results, we discuss in section 3 some implications for future precision measurements of Higgs properties. Finally, we conclude in section 4.

## 2 Lagrangian, T parameter and fit to LHC data

We employ the usual parameterization of interactions of SM fields with a generic Higgs boson by considering an EW chiral Lagrangian coupled to a scalar resonance h. The Goldstone bosons corresponding to the longitudinal polarizations of the W and Z are introduced through the chiral field

$$\Sigma(x) = \exp(i\sigma^a \pi^a(x)/v) \tag{2.1}$$

with  $v \simeq 246 \,\text{GeV}$ . The Lagrangian mass terms are then

$$\mathcal{L}_{\text{mass}} = \frac{v^2}{4} \text{Tr} \left[ \left( D_{\mu} \Sigma \right)^{\dagger} \left( D^{\mu} \Sigma \right) \right] - \frac{v}{\sqrt{2}} \sum_{i,j} \left( \bar{u}_L^{(i)} d_L^{(i)} \right) \Sigma \begin{pmatrix} \lambda_{ij}^u \, u_R^{(j)} \\ \lambda_{ij}^d \, d_R^{(j)} \end{pmatrix} + \text{h.c.}$$
(2.2)

<sup>&</sup>lt;sup>1</sup>See refs. [6-12] for earlier studies on the determination of Higgs couplings, and refs. [13-15] for other recent related work.

where

$$D_{\mu}\Sigma = \partial_{\mu}\Sigma - ig\frac{\sigma^{a}}{2}\Sigma W^{a}_{\mu} + ig'\Sigma\frac{\sigma^{3}}{2}B_{\mu}. \qquad (2.3)$$

We omit for simplicity lepton masses, which could be introduced in the same way as for quarks. Notice that this Lagrangian is approximately invariant under a global  $SU(2)_L \times SU(2)_R$ , under which  $\Sigma$  transforms as

$$\Sigma \to U_L \,\Sigma \, U_R^{\dagger} \,. \tag{2.4}$$

This invariance is broken in the vacuum to the diagonal  $SU(2)_c$  (the 'custodial symmetry'), which guarantees that the  $\rho$  parameter, defined as

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1 + \hat{T} = 1 + \alpha T \,, \tag{2.5}$$

satisfies the tree level relation  $\rho = 1$ , as experimentally verified to good accuracy. In principle the Lagrangian (2.2) could contain an additional term

$$v^2 \left( \operatorname{Tr} \left[ \Sigma^{\dagger} D_{\mu} \Sigma \, \sigma^3 \right] \right)^2 \tag{2.6}$$

that is gauge invariant, but explicitly breaks  $SU(2)_L \times SU(2)_R$  and therefore the custodial symmetry. To prevent large deviations from  $\rho = 1$  and thus tensions with precision tests, its coefficient has to be very small,  $O(10^{-3})$ , so the term (2.6) is usually neglected.

As it is well known, the description (2.2) leads to amplitudes for longitudinal gauge boson scattering that grow with energy, and as a consequence to a loss of perturbative unitarity at a scale  $4\pi v \sim 3 \text{ TeV}$ . To moderate the growth of amplitudes and therefore postpone the perturbative unitarity breakdown, a scalar resonance transforming as a singlet under the custodial symmetry can be introduced. We can thus add to eq. (2.2) all possible interactions with the scalar resonance up to second order, obtaining [16] (see also refs. [17, 18] for an introduction)

$$\mathcal{L}_{h} = \frac{1}{2} \left(\partial_{\mu}h\right)^{2} - V(h) + \frac{v^{2}}{4} \operatorname{Tr}\left[\left(D_{\mu}\Sigma\right)^{\dagger}\left(D^{\mu}\Sigma\right)\right] \left(1 + 2a \frac{h}{v} + b \frac{h^{2}}{v^{2}} + \dots\right) - \frac{v}{\sqrt{2}} \sum_{i,j} \left(\bar{u}_{L}^{i} d_{L}^{i}\right) \Sigma \left(1 + c \frac{h}{v} + c_{2} \frac{h^{2}}{v^{2}} + \dots\right) \left(\lambda_{ij}^{u} u_{R}^{j}\right) + \text{h.c.}$$

$$(2.7)$$

where  $a, b, c, c_2$  are free parameters (the SM is retrieved by choosing a = b = c = 1,  $c_2 = 0$  and vanishing terms of higher order in h). We do not write explicitly the scalar self-interactions contained in V(h), as they will not be relevant in our discussion.

Since we are interested in custodial breaking effects, we add to the Lagrangian the following terms

$$\mathcal{L}_{cb} = -\frac{v^2}{8} \left( \operatorname{Tr} \left[ \Sigma^{\dagger} D_{\mu} \Sigma \, \sigma^3 \right] \right)^2 \left( t_{cb} + 2a_{cb} \frac{h}{v} + \cdots \right), \qquad (2.8)$$

where  $t_{cb}$  and  $a_{cb}$  are free parameters<sup>2</sup> and the overall normalization has been chosen for later convenience. As we already mentioned,  $t_{cb}$  contributes to T at tree level,  $\hat{T} = -t_{cb}$ .

<sup>&</sup>lt;sup>2</sup>Higher orders in the Higgs are negligible for our purposes.

On the other hand, the consequences of the coupling  $a_{cb}$  can be seen by going to the unitary gauge,  $\Sigma = 1$ : the interactions of the Higgs with vector bosons are modified as follows

$$\mathcal{L}_{hVV} = \left[a \ m_W^2 W_{\mu}^+ W_{\mu}^- + \frac{1}{2}(a + a_{cb}) \ m_Z^2 Z_{\mu} Z_{\mu}\right] \left(2\frac{h}{v}\right).$$
(2.9)

Clearly the ratio between the two couplings differs from the usual custodial-preserving value  $g_{hWW}/g_{hZZ} = \cos^2 \theta_W$ . In a SILH Lagrangian [19], where the SM gauge symmetries are linearly realized in the strong sector, we can consider the following operators

$$\mathcal{O}_H = \frac{c_H}{2f^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H), \qquad \mathcal{O}_T = \frac{c_T}{2f^2} \left( H^\dagger D_\mu H - (D_\mu H)^\dagger H \right)^2 \tag{2.10}$$

where H is the (composite) Higgs doublet emerging as a pseudo-Goldstone boson from the strong sector. We find

$$a = 1 - \frac{c_H}{2} \frac{v^2}{f^2}, \qquad a_{cb} = -2c_T \frac{v^2}{f^2}.$$
 (2.11)

However, in addition a contribution  $t_{cb} = -c_T(v^2/f^2)$  is generated, or equivalently a correction  $\hat{T} = c_T(v^2/f^2)$ . Therefore in this case the coefficients  $t_{cb}$  and  $a_{cb}$  in eq. (2.8) are of the same order. We recall that  $c_H$  is in general<sup>3</sup> positive definite [20], implying the generic expectation a < 1 in composite Higgs models. However, in the following we will not restrict ourselves to this range. For a discussion of how a > 1 could arise, see ref. [21].

## 2.1 T parameter

It is well known that when  $a \neq 1$  in eq. (2.7), a logarithmically divergent contribution to T (as well as to S) arises. Such contribution is due to the diagrams in figure 1(a), and is computable within the low-energy theory, see ref. [22]. However, in the present case we also need to consider the effects of explicit custodial breaking contained in eq. (2.8). Even if we set  $t_{cb} = 0$ , a quadratic UV sensitivity appears in T, due to the diagrams involving the Higgs shown in figure 1(b). This quadratic divergence reads

$$\hat{T}^{UV} = \Delta \epsilon_1^{UV} = \frac{1}{16\pi^2} \frac{\Lambda^2}{v^2} \left( a^2 - (a + a_{cb})^2 \right) \,, \tag{2.12}$$

where  $\Lambda$  is the cutoff: setting  $\Lambda = 4\pi v$ , we obtain a contribution of tree-level size. In a concrete model, new degrees of freedom below the cutoff will need to conspire to make the total contribution to T compatible with EW precision data. This will require in general a certain amount of tuning, which we quantify in figure 2 by showing isocontours of  $|\Delta \epsilon_1^{UV} / \epsilon_1^{exp}|^{-1}$ , where the experimental value of the  $\epsilon_1$  parameter is  $\epsilon_1^{exp} = (5.4 \pm 1.0) \times 10^{-3}$  [23]. In the same figure we also show isocontours of  $|\Delta \epsilon_1^{TL} / \epsilon_1^{exp}|^{-1}$ , where

$$\Delta \epsilon_1^{TL} = -\frac{a_{cb}}{2} \tag{2.13}$$

is the tree-level contribution that arises when the full gauge invariant operator  $\mathcal{O}_T$  is considered. We see that the level of tuning is roughly similar in the two cases. A full computation of T requires choosing a complete model, see refs. [24–27] and references therein for examples.

<sup>3</sup>The contribution to  $c_H$  arising from integrating out triplet scalars is negative. However, in models



**Figure 1.** (a) Diagrams giving a logarithmic divergence in T when  $a \neq 1$ . This is the leading correction in the custodial-preserving case. (b) Diagrams giving a quadratic divergence in T when  $a_{cb} \neq 1$ , see eq. (2.12).



**Figure 2**. Isocontours in the  $(a, a_{cb})$  plane of  $|\Delta \epsilon_1^{UV} / \epsilon_1^{exp}|^{-1}$  (solid, black) and of  $|\Delta \epsilon_1^{TL} / \epsilon_1^{exp}|^{-1}$  (red, dashed), roughly representing the amount of tuning needed to satisfy EWPT.

#### 2.2 Recent LHC results

In this section we will perform a fit to the results of experimental searches for the Higgs at LHC and at Tevatron. We are going to use the full set of data released in March by ATLAS [28, 29], CMS [30, 31] and Tevatron [32], as reported in figure 3<sup>4</sup>. Experimental

where the collective symmetry breaking mechanism is realized, such as Little Higgs theories, the total contribution to  $c_H$  is positive even in presence of scalar triplets. See ref. [20].

<sup>&</sup>lt;sup>4</sup>We have included all the channels for which a signal strength has been provided. We exclude from this set only the ATLAS results on  $h \to b\bar{b}$  and  $h \to \tau\bar{\tau}$ , which are difficult to interpret within the framework

results are given in terms of

$$\mu = \frac{(\sigma_{prod} \times BR)^{obs}}{(\sigma_{prod} \times BR)^{SM}}.$$
(2.14)

In presence of a signal a best fit for this quantity is given along with errors. Several comments are in order about the dependence of  $\mu$  on the parameters  $(a, a_{cb}, c)$  for the different channels:

• The  $pp \rightarrow hjj \rightarrow \gamma\gamma jj$  sample at CMS is assumed to be produced through Vector Boson Fusion (VBF) with a small contamination coming from gluon fusion [33], so that

$$r_{\gamma\gamma jj}(a, a_{cb}, c) = \frac{\sigma_{prod}(a, a_{cb}, c)}{\sigma_{prod}^{SM}} = \frac{0.033 \, c^2 \, \sigma_{gg} + r_{VBF}(a, a_{cb}) \, \sigma_{VBF}}{0.033 \, \sigma_{gg} + \sigma_{VBF}} \tag{2.15}$$

where

$$r_{VBF}(a, a_{cb}) = \frac{a^2 + R_{VBF}(a + a_{cb})^2}{1 + R_{VBF}}.$$
(2.16)

 $R_{VBF} \sim 1/2.93$  is the ratio between ZZ and WW fusion production in the SM (at LHC, 7 TeV) [34, 35],  $\sigma_{gg}$  is the gluon fusion production cross section and  $\sigma_{VBF}/\sigma_{gg} \approx 0.079$ .

• We include the ATLAS results from fermiophobic (FP) Higgs searches in  $pp \rightarrow hX \rightarrow \gamma\gamma X$ . Following ref. [4] we take the production to be dominated by VBF with a sizable contamination from gluon fusion:

$$r_{FP}(a, a_{cb}, c) = \frac{0.3c^2 \sigma_{gg} + r_{VBF}(a, a_{cb}) \sigma_{VBF}}{0.3\sigma_{gg} + \sigma_{VBF}} \,. \tag{2.17}$$

•  $h \to b\bar{b}$  is observed in the associated production channel  $V \to Vh \to Vb\bar{b}$ . Taking into account the possibility of V being either a W or a Z we have

$$r_{Vh}(a, a_{cb}) = \frac{a^2 + R_{Vh}(a + a_{cb})^2}{1 + R_{Vh}}$$
(2.18)

where  $R_{Vh}$  is the ratio of Zh to Wh production in the SM, equal to 0.55 at LHC and to 0.61 at Tevatron.

• All the other channels are assumed to come from inclusive production. In this case for LHC

$$r_{incl}^{LHC}(a, a_{cb}, c) = \frac{c^2 \sigma_{gg} + r_{VBF}(a, a_{cb}) \,\sigma_{VBF} + r_{Vh}(a, a_{cb}) \,\sigma_{Vh}}{\sigma_{gg} + \sigma_{VBF} + \sigma_{Vh}} \sim c^2 \tag{2.19}$$

where  $\sigma_{Vh}/\sigma_{gg} \approx 0.058$ , and the last approximate equality holds because the main production mechanism is gluon fusion. We have checked that considering inclusive WW and ZZ production as coming only from gluon fusion and VBF, as done in ref. [3], does not significantly affect our results. An equation completely analogous to (2.19) holds for inclusive production at Tevatron.

of our simple analysis.

		1	$1 \circ 0_{prod} \wedge 1$
SETTA 2Z	$\gamma \gamma$ $\gamma \gamma_{FP}$ $Z Z^*$ $W W^*$	$2^{+0.8}_{-0.7} \\ 3.3^{+1.1}_{-1.1} \\ 1.6^{+1.6}_{-1.} \\ 0.2^{+0.6}_{-0.6} \\ \end{array}$	$c^{2} \times r_{\gamma\gamma}(a,c)$ $r_{FP}(a,a_{cb},c) \times r_{\gamma\gamma}(a,c)$ $c^{2} \times (a+a_{cb})^{2}$ $c^{2} \times a^{2}$
V V V V V V V V V V V V V V V V V V V	$\begin{array}{c} \gamma \ \gamma \\ Z \ Z^{\star} \\ W \ W^{\star} \end{array}$	$ \begin{vmatrix} 2.1^{+2.}_{-1.6} \\ 0.6^{+1.}_{-0.9} \\ 2.2^{+1.5}_{-1.4} \\ 0.6^{+1.8}_{-1.7} \\ 3.6^{+2.2}_{-1.6} \\ 0.6^{+1.}_{-0.6} \\ 0.4^{+0.6}_{-0.6} \end{vmatrix} $	$c^{2} \times r_{\gamma\gamma}(a,c)$ $c^{2} \times r_{\gamma\gamma}(a,c)$ $c^{2} \times r_{\gamma\gamma}(a,c)$ $c^{2} \times r_{\gamma\gamma}(a,c)$ $r_{\gamma\gamma j j}(a, a_{cb}, c) \times r_{\gamma\gamma}(a,c)$ $c^{2} \times (a + a_{cb})^{2}$ $c^{2} \times a^{2}$
$\tau \overline{\tau}$ $\overline{\tau}$ $\overline{\tau}$ $\overline{\tau}$ $\overline{b}\overline{b}\overline{b}$ $\overline{b}\overline{b}$ 	$\begin{array}{c} b\bar{b}\\ \bar{\tau}\bar{\tau}\\ \end{array}$	$\begin{array}{c} 1.2^{+2.1}_{-1.9}\\ 0.6^{+1.1}_{-1.2}\\ 0.0^{+1.2}_{-0}\\ 2.0^{+0.8}_{-0.7}\end{array}$	$r_{Vh}(a, a_{cb}) \times c^{2}$ $c^{2} \times c^{2}$ $c^{2} \times a^{2}$ $r_{Vh}(a, a_{cb}) \times c^{2}$

Figure 3. Summary table of the experimental results that we included in our analysis. The signal strengths for all CMS and Tevatron channels, as well as for the ATLAS WW and  $\gamma\gamma_{FP}$  are taken at  $m_h = 125$  GeV. On the other hand, for the ATLAS ZZ and  $\gamma\gamma$  channels we use the peak signal strength, by such we assume that the peaks genuinely represent the signals while their appearance at different masses is due to calibration issues. We report the leading scaling with the parameters  $(a, a_{cb}, c)$  both for production cross section and partial decay width in the various channels. The predictions of the best fit points are also shown in orange.

• The partial width for  $h \to \gamma \gamma$ , which arises both from W and from heavy fermion (top, bottom and tau) loops, gets rescaled as

$$r_{\gamma\gamma}(a) = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{SM}} \simeq (1.26 \, a - 0.26 \, c)^2 \tag{2.20}$$

for  $m_h = 125 \,\mathrm{GeV}$ .

After computing production cross sections and BRs we construct a  $\chi^2$  function

$$\chi^{2}(a, a_{cb}, c) = \sum_{i} \frac{(\hat{\mu}_{i} - \mu_{i}(a, a_{cb}, c))^{2}}{\delta \mu_{i}^{2}}, \qquad (2.21)$$

where  $\hat{\mu}_i$  is the experimental central value, and  $\delta \mu_i$  is the total error. The latter is obtained by summing in quadrature the experimental error (symmetrized by means of an average in quadrature) to the theoretical error, which accounts for the rescaling of production rates. The theoretical error comes from the uncertainties on cross sections, and is relevant only



Figure 4. Left panel: best-fit region in the  $(a, a_{cb})$  plane from LHC results, as in figure 3, at 68, 95, 99% C.L. after marginalization. Dashed lines represent the analogous contours in the case c = 1. The two best fit points with (without) marginalization are shown as black dots (crosses), while the star is the SM point corresponding to  $(a, a_{cb}) = (1, 0)$ . All the observables involved are insensitive to the sign of  $a + a_{cb}$ , implying the symmetry under  $(a, a_{cb}) \rightarrow (a, -(2a + a_{cb}))$ . Right panel: isocontours of  $|\Delta \epsilon_1^{UV} / \epsilon_1^{exp}|^{-1}$  (dotted, black) and of  $|\Delta \epsilon_1^{TL} / \epsilon_1^{exp}|^{-1}$  (red, dashed), indicating the level of tuning needed to satisfy EWPT, are superimposed to the LHC best fit region.

when two or more production mechanisms are summed over. We simply propagate the errors, taking their values for the single production mechanisms from ref. [36, 37].

Since we are interested in the gauge sector, and in particular in custodial breaking effects, we treat c as a nuisance parameter. Thus a  $\chi^2$  restricted to  $(a, a_{cb})$  can be computed by marginalizing over c:

$$\chi^2(a, a_{cb}) = \min_{\{c\}} \chi^2(a, a_{cb}, c), \qquad (2.22)$$

and it can be used to perform a minimum  $\chi^2$  procedure. The result of the fit is summarized in the left panel of figure 4, where we also show for completeness the results without marginalization (fixing c = 1). The best fit points are respectively  $(a, a_{cb}) = (0.93, 0.25)$ and (0.93, -2.11), both corresponding to  $\chi^2 = 9.2$  with 13 d.o.f. As expected the best fit points are 'Zphilic' (or equivalently, Wphobic):  $\mu_{ZZ}/\mu_{WW} = (\cos^2 \theta_W g_{hZZ}/g_{hWW})^2 = (a + a_{cb})^2/a^2 \approx 1.6$ .

Notice that all the observables involved in Higgs searches are insensitive to the sign of  $a + a_{cb}$  (as such combination always appears squared), implying the symmetry of the contours under  $(a, a_{cb}) \rightarrow (a, -(2a + a_{cb}))$ . In the best-fit region where  $a + a_{cb} < 0$ , the Higgs is actually 'dysZphilic', since the sign of the hZZ coupling is opposite with respect to the standard case. We will discuss in section 3 some future measurements that may lift the degeneracy between a Zphilic and a dysZphilic Higgs. As we have already mentioned in section 2.1, new light degrees of freedom are required in order to make a sizeable  $a_{cb}$  compatible with EWPT. In the absence of a symmetry a significant tuning is generically needed, as shown in the right panel of Fig 4. In principle, such new light degrees of freedom could affect the Higgs couplings, and therefore alter the interpretation of results of Higgs searches.

An obvious consequence of  $a_{cb} \neq 0$  is that the ratio  $\mu_{ZZ}/\mu_{WW}$  differs from unity. This is shown in figure 5, where we plot for each value of a the range of  $\mu_{ZZ}/\mu_{WW}$  obtained varying  $a_{cb}$  within the 68% CL region of the LHC fit (colored region). We see that within the LHC preferred region the wide range  $0.3 \leq \mu_{ZZ}/\mu_{WW} \leq 3.5$  is obtained, with the possibility of a severe Zphilia (although Zphobia cannot be totally excluded at the moment).

Another channel that can be effectively enhanced is  $\gamma\gamma$ , due to both c and  $a_{cb}$ . For example if  $\mu_{\gamma\gamma jj}/\mu_{ZZ}$  is considered, dramatic effects are possible even within the LHC 68% C.L. region, as can be seen in figure 5.

#### 2.3 Signal strength ratios at the LHC

We have to stress that the (a, c) and  $(a, a_{cb}, c)$  parameterizations are different and in principle it is possible to distinguish between them. The best way is to look at ratios between well measured  $\mu_i$ , as most of the QCD production uncertainties are thus cancelled (especially if the production channel is the same), as well as the dependence on the total width. See refs. [3, 38] for a discussion of how to break degeneracies in similar fits by using ratios of signal strengths.

To show how it can be possible to distinguish between the different cases, we choose the ratios  $(\mu_{\gamma\gamma}/\mu_{ZZ}, \mu_{b\bar{b}}/\mu_{ZZ})$ : in figure 6 we show isocurves of such ratios in the (a, c) and  $(a, a_{cb})$  planes respectively, superimposing them to the LHC best fit regions. To simplify the comparison, in the right panel of figure 6 we have set c = 1. We see that in the (a, c) case the range allowed for the ratios is significantly smaller than it is in the custodial breaking case.

#### 3 Future implications

We are left with the issue of determining the sign of  $a_{cb}$  (or of  $a + a_{cb}$  if you prefer). Not an easy quest, as the sign is physically relevant only in the presence of interference. One readily available choice would be to look at precision tests, in particular corrections to the  $Zb\bar{b}$  vertex. However the ratio between the main 1-loop Higgs contributions and the one of interest for us goes as  $m_t^2/m_b^2$  and so we expect the latter to be negligible. Thus we have to turn our attention to other, not yet measured, processes. We are going to briefly discuss four possible experimental signatures that are, or can be in principle, sensitive to the sign of the hZZ coupling.

Before moving to a discussion of the single channels, few comments are in order. We are interested in processes where diagrams both with and without the hZZ vertex interfere, and we need such interference to be non negligible in order to distinguish between the two cases  $a + a_{cb} \ge 0$ . Let us stress that the separation has to be bigger than both experimental and theoretical uncertainties. Concerning the latter, a precise knowledge of the absolute



**Figure 5.** The colored regions show the range of  $\mu_{ZZ}/\mu_{WW}$  (left panel) and  $\mu_{\gamma\gamma jj}/\mu_{ZZ}$  (right panel) as a function of *a*, obtained varying  $a_{cb}$  within the 68% CL region of the LHC fit, whereas the full line corresponds to choosing the best-fit value of  $a_{cb}$  for the given *a*.



Figure 6. Isocurves of  $\mu_{\gamma\gamma}/\mu_{ZZ}$  (solid) and of  $\mu_{b\bar{b}}/\mu_{ZZ}$  (dashed) in the (a, c) plane (left panel) and in the  $(a, a_{cb})$  plane (right panel). In both plots the LHC best-fit regions are also shown; in the right panel, c = 1 has been set to facilitate the comparison with the custodial-preserving case.

value of the coupling constants (a and  $a + a_{cb}$  in particular) is required. Thus we are going to focus on possible scenarios at  $e^+e^-$  Linear Colliders (LC), for which it is reasonable to assume a measurement of  $g_{hZZ}$  and  $g_{hWW}$  at the level of ~ 1%, corresponding to

$$|\delta a|, |\delta (a + a_{cb})| \sim 1\%.$$
 (3.1)

Such precision is expected both at ILC [39] and CLIC [40] with reference values  $m_h = 120 \text{ GeV}, \sqrt{s} = 500 \text{ GeV}$  and with 500 fb<sup>-1</sup> of integrated luminosity. In the fol-



Figure 7. Leading order and main NLO contributions to  $h \rightarrow ZZ$ .



Figure 8. Feynman diagrams contributing to  $e^+e^- \rightarrow ht\bar{t}$ .

lowing we fix c = 1 in order to highlight the main points under study.

# 3.1 $h \rightarrow ZZ$ decay width

The first channel we investigate is the width of the decay  $h \to ZZ \to 4l$ . Here the interference occurs between tree level and higher orders, the former being sensitive to the sign flip  $a + a_{cb} \to -(a + a_{cb})$ . On the contrary we assume, in order to maximize the separation, that most of the radiative corrections arise from loops not directly involving the hZZ vertex, see the diagrams in figure 7. Then the two cases  $a + a_{cb} \ge 0$  have different relative sign between LO and NLO, and we can write the width as  $\Gamma_{ZZ}^{\pm} \approx \Gamma_{ZZ}^{0}(1 \pm \delta)$ respectively (the superscript corresponds to the sign of  $a + a_{cb}$ ), with  $\delta \approx 1\%$  for SM couplings [41]. Assuming departures from the leading approximation  $a + a_{cb} = \pm 1$  to have negligible effects, we quantify the relative separation with

$$\Delta = \left| \frac{\Gamma_Z^+ - \Gamma_Z^-}{\Gamma_Z^+ + \Gamma_Z^-} \right| = \delta \approx 1\%.$$
(3.2)

It is clear that a very high precision is required to resolve the two cases. In fact, even considering perfect knowledge of the coupling constants, the experimental uncertainties should be at least of the same size or smaller of  $\Delta$ . We conclude that the measurement under study is not realistic.

#### **3.2** $ht\bar{t}$ associated production

We now focus on a case where the interference arises between different LO contributions. In Higgs boson associated production with tops (heavy fermions in general) the process is essentially  $e^+e^- \rightarrow Z \rightarrow t\bar{t}$  with a scalar emitted either by the Z or by one of the tops (as shown in figure 8). We can write the total cross section for the two cases  $a + a_{cb} = \pm 1$  as follows

$$\sigma_{\pm} = (\sigma_t + \sigma_Z \pm \sigma_{\text{int}}), \qquad (3.3)$$



Figure 9. Representative diagrams for each of the three classes of radiative corrections to  $e^+e^- \rightarrow Zh$ , see text for details.



Figure 10. Feynman diagrams contributing to  $e^+e^- \rightarrow Zhh$ .

where the index refers to the particle the Higgs boson is emitted from. We have  $\sigma_{\rm int}/(\sigma_t + \sigma_Z) \approx 1 - 4\%$ , leading to

$$\Delta = \left| \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \right| \lesssim 4\%, \qquad (3.4)$$

that needs to be compared to the experimental resolution. It has been shown [42] that from  $e^+e^- \rightarrow t\bar{t}$  the coupling  $g_{tth}$  could be measured up to 6% precision<sup>5</sup>, which directly translates in a precision of around 10-12% on the cross section, at least 3 or 4 times larger than  $\Delta$ . So even this case seems unlikely to be able to resolve the different signs.

#### **3.3** Zh associated production

The third channel we examine is the Higgs-strahlung process  $e^+e^- \rightarrow Zh$ , see figure 9. As in the first case above, we are interested in the change in sign of NLO corrections with respect to the tree level amplitude. Following detailed analyses present in literature [43, 44] we can divide the main electroweak corrections in three different terms, as following:

- Initial State Radiation ( $\delta_{ISR}$ ): whose amplitude clearly has the same sign of the tree level one;
- Fermionic contributions  $(\delta_F)$ : they are mainly due to self energy corrections to the Z propagator. Thus, in first approximation, we expect them to have the same sign of the LO amplitude;
- Bosonic contributions  $(\delta_B)$ : they are due to box diagrams usually involving W bosons. It is reasonable to assume that most of these would not involve the hZZ vertex and so to assume that  $\delta_B$  does not present a sign flip for a disZphilic Higgs.

It is then possible to write, for  $a + a_{cb} \ge 0$ ,

$$\sigma_{\pm} = \sigma_0 (1 + \delta_{ISR} + \delta_F \pm \delta_B) \tag{3.5}$$

<sup>&</sup>lt;sup>5</sup>At ILC with  $\sqrt{s} = 800 \,\text{GeV}$  and with 1000 fb<sup>-1</sup> of integrated luminosity.

as a rough estimate of the effect. Referring to a center of mass energy of 350 GeV the expected magnitudes for such corrections<sup>6</sup> are  $\delta_{ISR} \approx 0\%$ ,  $\delta_F \approx 10\%$  and  $\delta_B \approx -10\%$ . Thus  $\sigma_+ \approx 1.\sigma_0$ ,  $\sigma_- \approx 1.2\sigma_0$  and we are able to quantify the separation between the two cases as

$$\Delta = \left| \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \right| \approx 10\%, \qquad (3.6)$$

if we consider the simple choices  $a + a_{cb} = \pm 1$ . A comparison with the expected experimental sensitivity [46], which is of ~ 3-5%, shows that this measurement would indeed be able to resolve the sign. We finally note that at the center of mass energy of 1 TeV the relative separation between  $\sigma_+$  and  $\sigma_-$  is larger (15% compared to 10%), but due to the strong reduction of the rate the sign discrimination would require more integrated luminosity than at  $\sqrt{s} = 350 \,\text{GeV}$ .

#### 3.4 Zhh production

Another process where interference is at leading order is  $e^+e^- \rightarrow Z \rightarrow Zhh$ . In this case there are three distinct constributions: the diagram with two subsequent Higgs-strahlungs, the diagram involving the hhZZ vertex, and a third one involving the Higgs self-coupling (see figure 10), the last being the only one that changes sign under  $(a + a_{cb}) \rightarrow -(a + a_{cb})$ . The cross section for  $a + a_{cb} = \pm 1$  can then be written as

$$\sigma_{\pm} = \sigma_0 \pm \sigma_{\rm int} \,, \tag{3.7}$$

and for  $\sqrt{s} = 500 \,\text{GeV}$  (which is the best choice for the process  $e^+e^- \rightarrow Zhh$ ) we find  $\sigma_+ \simeq 0.28 \,\text{fb}, \sigma_- \simeq 0.09 \,\text{fb}$ . Therefore

$$\Delta = \left| \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \right| \approx 50\%, \qquad (3.8)$$

that needs to be compared to the experimental resolution. For an integrated luminosity of  $2000 \text{ fb}^{-1}$  and SM couplings, this can be as low as 10% [47]. In the case of flipped hZZ coupling, by taking into account the reduced statistics we estimate the resolution to be still less than 20%, i.e. more than two times smaller than  $\Delta$ . So this case is promising. However, we warn the reader that in the previous discussion we have made stronger assumptions than for the other precision measurements we presented. First, when setting the Higgs self-coupling  $\lambda_{hhh}$  to its SM value, we assumed to know it to a good accuracy, even though the measurement of such coupling at the LHC would be a difficult task, and the best channel to measure the trilinear at a LC with moderate  $\sqrt{s}$  would be  $e^+e^- \rightarrow Zhh$  itself (an independent measurement of  $\lambda_{hhh}$  could come from the WW fusion process  $e^+e^- \rightarrow \nu \bar{\nu}hh$  at  $\sqrt{s} \sim 1 \text{ TeV}$ ). Second, we assumed the hhZZ coupling to have its SM value although its measurement is challenging even at a LC, and despite the fact that in a theory with  $(a, a_{cb}) \neq (1, 0)$  we should in general expect deviations from the standard values also in the couplings hhZZ and hhWW. As a consequence, one should take the estimate in eq. (3.8) with some caution.

<sup>&</sup>lt;sup>6</sup>The numerical values are extracted from ref. [45], where  $m_h = 150 \,\text{GeV}$  was assumed. However, corrections due to the lower Higgs mass we are considering should be small and nonetheless would not change our conclusions.

#### 4 Conclusions

Motivated by recent results of experimental searches for the Higgs boson, we relaxed the assumption of custodial invariance in its couplings to the W and the Z. We described custodial breaking through an additional parameter  $a_{cb}$  and we showed how it can accommodate the current pattern of observed excesses, which mildly point to a Zphilic (or Wphobic) Higgs. Should such hints be confirmed by more data, they would be evidence for custodial breaking in Higgs couplings. Such breaking implies that the electroweak T parameter receives quadratically divergent corrections. New light degrees of freedom would then be expected to play a role in mimicking the approximate custodial invariance observed in electroweak precision data, generically at the price of a sizable tuning.

We also noticed that Higgs searches are insensitive to the sign of the hZZ coupling, that is to say they do not allow to tell a Zphilic Higgs from its dysZphilic counterpart. However the sign of such coupling is physical, and processes in which interference is present can remove the degeneracy. We presented some measurements at future linear colliders that could be used for this purpose.

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# References

- D. Carmi, A. Falkowski, E. Kuflik and T. Volansky, Interpreting LHC Higgs Results from Natural New Physics Perspective, arXiv:1202.3144 [INSPIRE].
- [2] A. Azatov, R. Contino and J. Galloway, Model-Independent Bounds on a Light Higgs, JHEP 04 (2012) 127 [arXiv:1202.3415] [INSPIRE].
- [3] J. Espinosa, C. Grojean, M. Muhlleitner and M. Trott, Fingerprinting Higgs Suspects at the LHC, JHEP 05 (2012) 097 [arXiv:1202.3697] [INSPIRE].
- [4] P.P. Giardino, K. Kannike, M. Raidal and A. Strumia, Reconstructing Higgs boson properties from the LHC and Tevatron data, JHEP 06 (2012) 117 [arXiv:1203.4254] [INSPIRE].

- [5] J. Ellis and T. You, Global Analysis of Experimental Constraints on a Possible Higgs-Like Particle with Mass ~ 125 GeV, arXiv:1204.0464 [INSPIRE].
- [6] D. Zeppenfeld, R. Kinnunen, A. Nikitenko and E. Richter-Was, Measuring Higgs boson couplings at the CERN LHC, Phys. Rev. D 62 (2000) 013009 [hep-ph/0002036] [INSPIRE].
- [7] T. Plehn, D.L. Rainwater and D. Zeppenfeld, Determining the structure of Higgs couplings at the LHC, Phys. Rev. Lett. 88 (2002) 051801 [hep-ph/0105325] [INSPIRE].
- [8] M. Dührssen et al., Extracting Higgs boson couplings from CERN LHC data, Phys. Rev. D 70 (2004) 113009 [hep-ph/0406323] [INSPIRE].
- M. Dührssen, Prospects for the measurement of Higgs boson coupling parameters in the mass range from 110–190 GeV, ATLAS-PHYS-2003-030.
- [10] C. Burgess, J. Matias and M. Pospelov, A Higgs or not a Higgs? What to do if you discover a new scalar particle, Int. J. Mod. Phys. A 17 (2002) 1841 [hep-ph/9912459] [INSPIRE].
- [11] R. Lafaye, T. Plehn, M. Rauch, D. Zerwas and M. Dührssen, *Measuring the Higgs Sector*, *JHEP* 08 (2009) 009 [arXiv:0904.3866] [INSPIRE].
- S. Bock et al., Measuring Hidden Higgs and Strongly-Interacting Higgs Scenarios, Phys. Lett. B 694 (2010) 44 [arXiv:1007.2645] [INSPIRE].
- E. Cervero and J.-M. Gerard, Minimal violation of flavour and custodial symmetries in a vectophobic Two-Higgs-Doublet-Model, Phys. Lett. B 712 (2012) 255 [arXiv:1202.1973]
   [INSPIRE].
- [14] T. Li, X. Wan, Y.-k. Wang and S.-h. Zhu, Constraints on the Universal Varying Yukawa Couplings: from SM-like to Fermiophobic, arXiv:1203.5083 [INSPIRE].
- [15] M. Rauch, Determination of Higgs-boson couplings (SFitter), arXiv:1203.6826 [INSPIRE].
- [16] R. Contino, C. Grojean, M. Moretti, F. Piccinini and R. Rattazzi, Strong Double Higgs Production at the LHC, JHEP 05 (2010) 089 [arXiv:1002.1011] [INSPIRE].
- [17] R. Contino, The Higgs as a Composite Nambu-Goldstone Boson, arXiv:1005.4269
   [INSPIRE].
- [18] J. Espinosa, C. Grojean and M. Muhlleitner, Composite Higgs under LHC Experimental Scrutiny, EPJ Web Conf. 28 (2012) 08004 [arXiv:1202.1286] [INSPIRE].
- [19] G. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, The Strongly-Interacting Light Higgs, JHEP 06 (2007) 045 [hep-ph/0703164] [INSPIRE].
- [20] I. Low, R. Rattazzi and A. Vichi, Theoretical Constraints on the Higgs Effective Couplings, JHEP 04 (2010) 126 [arXiv:0907.5413] [INSPIRE].
- [21] A. Falkowski, S. Rychkov and A. Urbano, What if the Higgs couplings to W and Z bosons are larger than in the Standard Model?, JHEP 04 (2012) 073 [arXiv:1202.1532] [INSPIRE].
- [22] R. Barbieri, B. Bellazzini, V.S. Rychkov and A. Varagnolo, The Higgs boson from an extended symmetry, Phys. Rev. D 76 (2007) 115008 [arXiv:0706.0432] [INSPIRE].
- [23] ALEPH, DELPHI, L3, OPAL, SLD, LEP ELECTROWEAK WORKING GROUP, SLD ELECTROWEAK GROUP, SLD HEAVY FLAVOUR GROUP, Precision electroweak measurements on the Z resonance, Phys. Rept. 427 (2006) 257 [hep-ex/0509008] [INSPIRE].
- [24] P.H. Chankowski, S. Pokorski and J. Wagner, Z-prime and the Appelquist-Carrazzone decoupling, Eur. Phys. J. C 47 (2006) 187 [hep-ph/0601097] [INSPIRE].

- [25] M.-C. Chen, S. Dawson and T. Krupovnickas, *Higgs triplets and limits from precision measurements*, *Phys. Rev.* D 74 (2006) 035001 [hep-ph/0604102] [INSPIRE].
- [26] P.H. Chankowski, S. Pokorski and J. Wagner, (Non)decoupling of the Higgs triplet effects, Eur. Phys. J. C 50 (2007) 919 [hep-ph/0605302] [INSPIRE].
- [27] A. Carmona, E. Ponton and J. Santiago, Phenomenology of Non-Custodial Warped Models, JHEP 10 (2011) 137 [arXiv:1107.1500] [INSPIRE].
- [28] ATLAS collaboration, An update to the combined search for the Standard Model Higgs boson with the ATLAS detector at the LHC using up to 4.9 fb<sup>1</sup> of pp collision data at  $\sqrt{s} = 7$  TeV, ATLAS-CONF-2012-019 (2012).
- [29] ATLAS collaboration, Search for a fermiophobic Higgs boson in the diphoton decay channel with 4.9/fb of ATLAS data at  $\sqrt{s} = 7$  TeV, ATLAS-CONF-2012-013 (2012).
- [30] CMS collaboration, A search using multivariate techniques for a standard model Higgs boson decaying into two photons, CMS-PAS-HIG-12-008.
- [31] CMS collaboration, Combination of SM, SM4, FP Higgs boson searches, CMS-PAS-HIG-12-001.
- [32] TEVNPH (TEVATRON NEW PHENOMINA AND HIGGS WORKING GROUP), CDF COLLABORATION, D0 collaboration, Combined CDF and D0 Search for Standard Model Higgs Boson Production with up to 10.0 fb<sup>-1</sup> of Data, arXiv:1203.3774 [INSPIRE].
- [33] CMS collaboration, S. Chatrchyan et al., Search for the standard model Higgs boson decaying into two photons in pp collisions at  $\sqrt{s} = 7$  TeV, Phys. Lett. **B** 710 (2012) 403 [arXiv:1202.1487] [INSPIRE].
- [34] P. Bolzoni, F. Maltoni, S.-O. Moch and M. Zaro, Higgs production via vector-boson fusion at NNLO in QCD, Phys. Rev. Lett. 105 (2010) 011801 [arXiv:1003.4451] [INSPIRE].
- [35] Webpage, http://vbf-nnlo.phys.ucl.ac.be/vbf.html.
- [36] LHC HIGGS CROSS SECTION WORKING GROUP collaboration, S. Dittmaier et al., Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables, arXiv:1101.0593 [INSPIRE].
- [37] Webpage, https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageAt7TeV.
- [38] A. Azatov et al., Determining Higgs couplings with a model-independent analysis of  $h \rightarrow \gamma \gamma$ , JHEP 06 (2012) 134 [arXiv:1204.4817] [INSPIRE].
- [39] ILC collaboration, G. Aarons et al., International Linear Collider Reference Design Report Volume 2: Physics at the ILC, arXiv:0709.1893 [INSPIRE].
- [40] L. Linssen, A. Miyamoto, M. Stanitzki and H. Weerts, *Physics and Detectors at CLIC: CLIC Conceptual Design Report*, arXiv:1202.5940 [INSPIRE].
- [41] A. Bredenstein, A. Denner, S. Dittmaier and M. Weber, Precise predictions for the Higgs-boson decay H → WW/ZZ → 4 leptons, Phys. Rev. D 74 (2006) 013004
   [hep-ph/0604011] [INSPIRE].
- [42] A. Gay, Measurement of the top-Higgs Yukawa coupling at a Linear e<sup>+</sup>e<sup>−</sup> Collider, Eur. Phys. J. C 49 (2007) 489 [hep-ph/0604034] [INSPIRE].
- [43] B.A. Kniehl, Radiative corrections for associated ZH production at future e<sup>+</sup>e<sup>−</sup> colliders, Z. Phys. C 55 (1992) 605 [INSPIRE].

- [44] A. Denner, J. Kublbeck, R. Mertig and M. Böhm, *Electroweak radiative corrections to*  $e^+e^- \rightarrow HZ$ , Z. Phys. C 56 (1992) 261 [INSPIRE].
- [45] A. Denner, S. Dittmaier, M. Roth and M. Weber, Electroweak radiative corrections to single Higgs boson production in e<sup>+</sup>e<sup>-</sup> annihilation, Phys. Lett. B 560 (2003) 196
   [hep-ph/0301189] [INSPIRE].
- [46] P. Garcia-Abia and W. Lohmann, Measurement of the Higgs cross-section and mass with linear colliders, Eur. Phys. J. direct C 2 (2000) 2 [hep-ex/9908065] [INSPIRE].
- [47] C. Castanier, P. Gay, P. Lutz and J. Orloff, Higgs self coupling measurement in e<sup>+</sup>e<sup>-</sup> collisions at center-of-mass energy of 500 GeV, hep-ex/0101028 [INSPIRE].