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Research Article

Common Fixed Point Theorems for Four Mappings on Cone Metric Type Space

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In this paper we consider the so called a cone metric type space, which is a generalization of a cone metric space. We prove some common fixed point theorems for four mappings in those spaces. Obtained results extend and generalize well-known comparable results in the literature. All results are proved in the settings of a solid cone, without the assumption of continuity of mappings.

1. Introduction

Replacing the real numbers, as the codomain of a metric, by an ordered Banach space we obtain a generalization of metric space. Such a generalized space, called a cone metric space, was introduced by Huang and Zhang in [1]. They described the convergence in cone metric space, introduced their completeness, and proved some fixed point theorems for contractive mappings on cone metric space. Cones and ordered normed spaces have some applications in optimization theory (see [2]). The initial work of Huang and Zhang [1] inspired many authors to prove fixed point theorems, as well as common fixed point theorems for two or more mappings on cone metric space, for example, [3–14].

In this paper we consider the so-called a cone metric type space, which is a generalization of a cone metric space and prove some common fixed point theorems for four mappings in those spaces. Obtained results are generalization of theorems proved in [13]. For some special choices of mappings we obtain theorems which generalize results from [1, 8, 15].

All results are proved in the settings of a solid cone, without the assumption of continuity of mappings.

The paper is organized as follows. In Section 2 we repeat some definitions and well-known results which will be needed in the sequel. In Section 3 we prove common fixed point theorems. Also, we presented some corollaries which show that our results are generalization of some existing results in the literature.

2. Definitions and Notation

Let E be a real Banach space and P a subset of E. By θ we denote zero element of E and by $\operatorname{int} P$ the interior of P. The subset P is called *a cone* if and only if

(i) P is closed, nonempty and $P \neq \{\theta\}$;

(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$, and $x, y \in P$ imply $ax + by \in P$;

(iii) $P \cap (-P) = \{\theta\}$.

For a given cone P, a partial ordering $\preceq$ with respect to P is introduced in the following way: $x \preceq y$ if and only if $y - x \in P$. One writes $x \prec y$ to indicate that $x \preceq y$, but $x \neq y$. If $y - x \in \operatorname{int} P$, one writes $x \ll y$.

If $\operatorname{int} P \neq \emptyset$, the cone P is called *solid*.

In the sequel we always suppose that E is a real Banach space, P is a solid cone in E, and $\preceq$ is partial ordering with respect to P.

Analogously with definition of metric type space, given in [16], we consider cone metric type space.

Definition 2.1. Let X be a nonempty set and E a real Banach space with cone P. A vector-valued function $d : X \times X \to E$ is said to be a cone metric type function on X with constant $K \geq 1$ if the following conditions are satisfied:

(d_1) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(d_2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(d_3) $d(x, y) \preceq K(d(x, z) + d(z, y))$ for all $x, y, z \in X$.

The pair (X, d) is called a cone metric type space (in brief CMTS).

Remark 2.2. For $K = 1$ in Definition 2.1 we obtain a cone metric space introduced in [1].

Definition 2.3. Let (X, d) be a CMTS and $\{x_n\}$ a sequence in X.

(c_1) $\{x_n\}$ converges to $x \in X$ if for every $c \in E$ with $\theta \ll c$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) \ll c$ for all $n > n_0$. We write $\lim_{n\to\infty} x_n = x$, or $x_n \to x$, $n \to \infty$.

(c_2) If for every $c \in E$ with $\theta \ll c$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_m) \ll c$ for all $n, m > n_0$, then $\{x_n\}$ is called a Cauchy sequence in X.

If every Cauchy sequence is convergent in X, then X is called a complete CMTS.

Example 2.4. Let $B = \{e_i \mid i = 1, \ldots, n\}$ be orthonormal basis of $\mathbb{R}^n$ with inner product $(\cdot,\cdot)$. Let $p > 0$, and define

$$X_p = \left\{ [x] \mid x : [0,1] \longrightarrow \mathbb{R}^n,\ \int_0^1 |(x(t), e_k)|^p \mathrm{d}t \in \mathbb{R},\ k = 1, \ldots, n \right\}, \tag{2.1}$$

where $[x]$ represents class of element x with respect to equivalence relation of functions equal almost everywhere. We choose $E = \mathbb{R}^n$ and

$$P_B = \{ y \in \mathbb{R}^n \mid (y, e_i) \geq 0,\ i = 1, \ldots, n \}. \tag{2.2}$$

We show that P_B is a solid cone. Let $y_k \in P_B$, $k \in \mathbb{N}$, with property $\lim_{k\to+\infty} y_k = y$. Since scalar product is continuous, we get $\lim_{k\to+\infty}(y_k, e_i) = (\lim_{k\to+\infty} y_k, e_i) = (y, e_i)$, $i = 1, \ldots, n$. Clearly, it must be $(y, e_i) \geq 0$, $i = 1, \ldots, n$, and, hence, $y \in P_B$, that is, P_B is closed. It is obvious that $\theta \neq e_1 \in P_B \neq \{\theta\}$, and for $a, b \geq 0$, and all $z, y \in P_B$, we have $(az+by, e_i) = a(z, e_i)+b(y, e_i) \geq 0$, $i = 1, \ldots, n$. Finally, if $z \in P_B \cap (-P_B)$ we have $(z, e_i) \geq 0$ and $(-z, e_i) \geq 0$, $i = 1, \ldots, n$, and it follows that $(z, e_i) = 0$, $i = 1, \ldots, n$, and, since B is complete, we get $z = 0$. Let us choose $z = \sum_{i=1}^n e_i$. It is obvious that $z \in \operatorname{int} P_B$, since if not, for every $\varepsilon > 0$ there exists $y \notin P_B$ such that $|1 - (y, e_i)| \leq (\sum_{i=1}^n |1 - (y, e_i)|^2)^{1/2} = \|z - y\| < \varepsilon$. If we choose $\varepsilon = 1/4$, we conclude that it must be $(y, e_i) > 1 - 1/4 > 0$, hence $y \in P_B$, which is contradiction.

Finally, define $d : X_p \times X_p \to P_B$ by

$$d(f, g) = \sum_{i=1}^n e_i \int_0^1 \left| ((f - g)(t), e_i) \right|^p \mathrm{d}t, \quad f, g \in X_p. \tag{2.3}$$

Then it is obvious that (X_p, d) is CMTS with $K = 2^{p-1}$. Let f, g, h be functions such that $(f, e_1) = 1$, $(g, e_1) = -2$, $(h, e_1) = 0$, and $(f, e_i) = (g, e_i) = (h, e_i) = 0$, $i = 2, \ldots, n$, with $p = 2$ give $d(f, g) = 9e_1$, $d(f, h) = e_1$, and $d(h, g) = 4e_1$, which proves $5e_1 = d(f, h) + d(h, g) \preceq d(f, g) = 9e_1$, but $9e_1 = d(f, g) \preceq 2(d(f, h) + d(h, g)) = 10e_1$.

The following properties are well known in the case of a cone metric space, and it is easy to see that they hold also in the case of a CMTS.

Lemma 2.5. *Let (X, d) be a CMTS over-ordered real Banach space E with a cone P. The following properties hold $(a, b, c \in E)$.*

- (p_1) *If $a \preceq b$ and $b \ll c$, then $a \ll c$.*
- (p_2) *If $\theta \preceq a \ll c$ for all $c \in \operatorname{int} P$, then $a = \theta$.*
- (p_3) *If $a \preceq \lambda a$, where $a \in P$ and $0 \leq \lambda < 1$, then $a = \theta$.*
- (p_4) *Let $x_n \to \theta$ in E and let $\theta \ll c$. Then there exists positive integer n_0 such that $x_n \ll c$ for each $n > n_0$.*

Definition 2.6 (see [17]). Let $F, G : X \to X$ be mappings of a set X. If $y = Fx = Gx$ for some $x \in X$, then x is called a coincidence point of F and G, and y is called a point of coincidence of F and G.

Definition 2.7 (see [17]). Let F and G be self-mappings of set X and $\mathcal{C}(F,G) = \{x \in X : Fx = Gx\}$. The pair $\{F,G\}$ is called weakly compatible if mappings F and G commute at all their coincidence points, that is, if $FGx = GFx$ for all $x \in \mathcal{C}(F,G)$.

Lemma 2.8 (see [5]). *Let F and G be weakly compatible self-mappings of a set X. If F and G have a unique point of coincidence $y = Fx = Gx$, then y is the unique common fixed point of F and G.*

3. Main Results

Theorem 3.1. *Let (X,d) be a CMTS with constant $1 \le K \le 2$ and P a solid cone. Suppose that self-mappings $F,G,S,T : X \to X$ are such that $SX \subset GX$, $TX \subset FX$ and that for some constant $\lambda \in (0,1/K)$ for all $x,y \in X$ there exists*

$$u(x,y) \in \left\{Kd(Fx,Gy),\ Kd(Fx,Sx),\ Kd(Gy,Ty),\ K\frac{d(Fx,Ty)+d(Gy,Sx)}{2}\right\}, \tag{3.1}$$

such that the following inequality

$$d(Sx,Ty) \preceq \frac{\lambda}{K}u(x,y), \tag{3.2}$$

holds. If one of SX, TX, FX, or GX is complete subspace of X, then $\{S,F\}$ and $\{T,G\}$ have a unique point of coincidence in X. Moreover, if $\{S,F\}$ and $\{T,G\}$ are weakly compatible pairs, then F, G, S, and T have a unique common fixed point.

Proof. Let us choose $x_0 \in X$ arbitrary. Since $SX \subset GX$, there exists $x_1 \in X$ such that $Gx_1 = Sx_0 = z_0$. Since $TX \subset FX$, there exists $x_2 \in X$ such that $Fx_2 = Tx_1 = z_1$. We continue in this manner. In general, $x_{2n+1} \in X$ is chosen such that $Gx_{2n+1} = Sx_{2n} = z_{2n}$, and $x_{2n+2} \in X$ is chosen such that $Fx_{2n+2} = Tx_{2n+1} = z_{2n+1}$.

First we prove that

$$d(z_n, z_{n+1}) \preceq \alpha d(z_{n-1}, z_n), \quad n \ge 1, \tag{3.3}$$

where $\alpha = \max\{\lambda, \lambda K/(2-\lambda K)\}$, which will lead us to the conclusion that $\{z_n\}$ is a Cauchy sequence, since $\alpha \in (0,1)$ (it is easy to see that $0 < \lambda K/(2-\lambda K) < 1$). To prove this, it is necessary to consider the cases of an odd integer n and of an even n.

For $n = 2\ell+1$, $\ell \in \mathbb{N}_0$, we have $d(z_{2\ell+1}, z_{2\ell+2}) = d(Sx_{2\ell+2}, Tx_{2\ell+1})$, and from (3.2) there exists

$$\begin{aligned} u(x_{2\ell+2}, x_{2\ell+1}) \in \Bigg\{&Kd(Fx_{2\ell+2}, Gx_{2\ell+1}), Kd(Fx_{2\ell+2}, Sx_{2\ell+2}), \\ &Kd(Gx_{2\ell+1}, Tx_{2\ell+1}), K\frac{d(Fx_{2\ell+2}, Tx_{2\ell+1}) + d(Gx_{2\ell+1}, Sx_{2\ell+2})}{2}\Bigg\} \\ = \Bigg\{&Kd(z_{2\ell+1}, z_{2\ell}), Kd(z_{2\ell+1}, z_{2\ell+2}), \frac{Kd(z_{2\ell}, z_{2\ell+2})}{2}\Bigg\}, \end{aligned} \tag{3.4}$$

such that $d(z_{2\ell+1}, z_{2\ell+2}) \preceq (\lambda/K)u(x_{2\ell+2}, x_{2\ell+1})$. Thus we have the following three cases:

(i) $d(z_{2\ell+1}, z_{2\ell+2}) \preceq \lambda d(z_{2\ell+1}, z_{2\ell})$;

(ii) $d(z_{2\ell+1}, z_{2\ell+2}) \preceq \lambda d(z_{2\ell+1}, z_{2\ell+2})$, which, because of property (p_3), implies $d(z_{2\ell+1}, z_{2\ell+2}) = \theta$;

(iii) $d(z_{2\ell+1}, z_{2\ell+2}) \preceq (\lambda/2)d(z_{2\ell}, z_{2\ell+2})$, that is, by using (d_3),

$$d(z_{2\ell+1}, z_{2\ell+2}) \preceq \frac{\lambda K}{2} d(z_{2\ell}, z_{2\ell+1}) + \frac{\lambda K}{2} d(z_{2\ell+1}, z_{2\ell+2}), \tag{3.5}$$

which implies $d(z_{2\ell+1}, z_{2\ell+2}) \preceq (\lambda K/(2-\lambda K))d(z_{2\ell}, z_{2\ell+1})$.

Thus, inequality (3.3) holds in this case.

For $n = 2\ell$, $\ell \in \mathbb{N}_0$, we have

$$d(z_{2\ell}, z_{2\ell+1}) = d(Sx_{2\ell}, Tx_{2\ell+1}) \preceq \frac{\lambda}{K} u(x_{2\ell}, x_{2\ell+1}), \tag{3.6}$$

where

$$\begin{aligned} u(x_{2\ell}, x_{2\ell+1}) \in \Big\{ & Kd(Fx_{2\ell}, Gx_{2\ell+1}), Kd(Fx_{2\ell}, Sx_{2\ell}), \\ & Kd(Gx_{2\ell+1}, Tx_{2\ell+1}), K\frac{d(Fx_{2\ell}, T_{2\ell+1}) + d(Gx_{2\ell+1}, Sx_{2\ell})}{2} \Big\} \\ = \Big\{ & Kd(z_{2\ell-1}, z_{2\ell}), Kd(z_{2\ell}, z_{2\ell+1}), \frac{Kd(z_{2\ell-1}, z_{2\ell+1})}{2} \Big\}. \end{aligned} \tag{3.7}$$

Thus we have the following three cases:

(i) $d(z_{2\ell}, z_{2\ell+1}) \preceq \lambda d(z_{2\ell-1}, z_{2\ell})$;

(ii) $d(z_{2\ell}, z_{2\ell+1}) \preceq \lambda d(z_{2\ell}, z_{2\ell+1})$, which implies $d(z_{2\ell}, z_{2\ell+1}) = \theta$;

(iii) $d(z_{2\ell}, z_{2\ell+1}) \preceq (\lambda/2)d(z_{2\ell-1}, z_{2\ell+1}) \preceq (\lambda K/2)d(z_{2\ell-1}, z_{2\ell}) + (\lambda K/2)d(z_{2\ell}, z_{2\ell+1})$, which implies $d(z_{2\ell}, z_{2\ell+1}) \preceq (\lambda K/(2-\lambda K))d(z_{2\ell}, z_{2\ell-1})$.

So, inequality (3.3) is satisfied in this case, too.

Therefore, (3.3) is satisfied for all $n \in \mathbb{N}_0$, and by iterating we get

$$d(z_n, z_{n+1}) \preceq \alpha^n d(z_0, z_1). \tag{3.8}$$

Since $K \geq 1$, for $m > n$ we have

$$\begin{aligned} d(z_n, z_m) &\preceq Kd(z_n, z_{n+1}) + K^2 d(z_{n+1}, z_{n+2}) + \cdots + K^{m-n-1} d(z_{m-1}, z_m) \\ &\preceq \left(K\alpha^n + K^2\alpha^{n+1} + \cdots + K^{m-n}\alpha^{m-1}\right) d(z_0, z_1) \\ &\preceq \frac{K\alpha^n}{1-K\alpha} d(z_0, z_1) \longrightarrow \theta, \quad \text{as } n \longrightarrow \infty. \end{aligned} \tag{3.9}$$

Now, by (p_4) and (p_1), it follows that for every $c \in \operatorname{int} P$ there exists positive integer n_0 such that $d(z_n, z_m) \ll c$ for every $m > n > n_0$, so $\{z_n\}$ is a Cauchy sequence.

Let us suppose that SX is complete subspace of X. Completeness of SX implies existence of $z \in SX$ such that $\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} Sx_{2n} = z$. Then, we have

$$\lim_{n\to\infty} Gx_{2n+1} = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Fx_{2n} = \lim_{n\to\infty} Tx_{2n+1} = z, \tag{3.10}$$

that is, for any $\theta \ll c$, for sufficiently large n we have $d(z_n, z) \ll c$. Since $z \in SX \subset GX$, there exists $y \in X$ such that $z = Gy$. Let us prove that $z = Ty$. From (d_3) and (3.2), we have

$$d(Ty, z) \preceq Kd(Ty, Sx_{2n}) + Kd(Sx_{2n}, z) \preceq \lambda u(x_{2n}, y) + Kd(z_{2n}, z), \tag{3.11}$$

where

$$\begin{aligned} u(x_{2n}, y) \in & \left\{ Kd(Fx_{2n}, Gy), Kd(Fx_{2n}, Sx_{2n}), Kd(Gy, Ty), K\frac{d(Fx_{2n}, Ty) + d(Gy, Sx_{2n})}{2} \right\} \\ & = \left\{ Kd(z_{2n-1}, z), Kd(z_{2n-1}, z_{2n}), Kd(z, Ty), K\frac{d(z_{2n-1}, Ty) + d(z, z_{2n})}{2} \right\}. \end{aligned} \tag{3.12}$$

Therefore we have the following four cases:

(i) $d(Ty, z) \preceq K\lambda d(z_{2n-1}, z) + Kd(z_{2n}, z) \ll K\lambda \cdot c/(2K\lambda) + K \cdot c/(2K) = c$, as $n \to \infty$;

(ii) $d(Ty, z) \preceq K\lambda d(z_{2n-1}, z_{2n}) + Kd(z_{2n}, z) \ll K\lambda \cdot c/(2K\lambda) + K \cdot c/(2K) = c$, as $n \to \infty$;

(iii) $d(Ty, z) \preceq K\lambda d(z, Ty) + Kd(z_{2n}, z)$, that is,

$$d(Ty, z) \preceq \frac{K}{1 - K\lambda} d(z_{2n}, z) \ll \frac{K}{1 - K\lambda} \cdot \frac{1 - K\lambda}{K} \cdot c = c, \quad \text{as } n \longrightarrow \infty; \tag{3.13}$$

(iv) $d(Ty, z) \preceq (K\lambda/2)(d(z_{2n-1}, Ty) + d(z, z_{2n})) + Kd(z_{2n}, z)$, that is, because of (d_3),

$$d(Ty, z) \preceq \frac{K\lambda}{2}\left(Kd(z_{2n-1}, z) + Kd(z, Ty) + d(z, z_{2n})\right) + Kd(z_{2n}, z), \tag{3.14}$$

which implies

$$\begin{aligned} d(Ty, z) &\preceq \frac{1}{1 - K^2\lambda/2}\left[\frac{K^2\lambda}{2} d(z_{2n-1}, z) + \left(\frac{K\lambda}{2} + K\right) d(z_{2n}, z)\right] \\ &\ll \frac{K^2\lambda}{2 - K^2\lambda} \frac{2 - K^2\lambda}{K^2\lambda} \frac{c}{2} + \frac{K(\lambda + 2)}{2 - K^2\lambda} \frac{2 - K^2\lambda}{K(\lambda + 2)} \frac{c}{2} = c, \quad \text{as } n \longrightarrow \infty, \end{aligned} \tag{3.15}$$

since from $1 \le K \le 2$ and $\lambda \in (0, 1/K)$ we have $\lambda < 1/K \le 2/K^2$, and therefore $1 - K^2\lambda/2 > 0$.

Therefore, $d(Ty,z) \ll c$ for each $c \in \operatorname{int} P$. So, by (p_2) we have $d(Ty,z) = \theta$, that is, $Ty = Gy = z$, y is a coincidence point, and z is a point of coincidence of T and G.

Since $TX \subset FX$, there exists $v \in X$ such that $z = Fv$. Let us prove that $Sv = z$. From (d_3) and (3.2), we have

$$d(Sv,z) \preceq Kd(Sv,Tx_{2n+1}) + Kd(Tx_{2n+1},z) \preceq \lambda u(v,x_{2n+1}) + Kd(z_{2n+1},z), \tag{3.16}$$

where

$$\begin{aligned} &u(v,x_{2n+1}) \\ &\quad \in \left\{Kd(Fv,Gx_{2n+1}), Kd(Fv,Sv), Kd(Gx_{2n+1},Tx_{2n+1}), K\frac{d(Fv,Tx_{2n+1}) + d(Gx_{2n+1},Sv)}{2}\right\} \\ &\quad = \left\{Kd(z,z_{2n}), Kd(z,Sv), Kd(z_{2n},z_{2n+1}), K\frac{d(z,z_{2n+1}) + d(z_{2n},Sv)}{2}\right\}. \end{aligned} \tag{3.17}$$

Therefore we have the following four cases:

(i) $d(Sv,z) \preceq K\lambda d(z,z_{2n}) + Kd(z_{2n+1},z)$;

(ii) $d(Sv,z) \preceq K\lambda d(z,Sv) + Kd(z_{2n+1},z)$;

(iii) $d(Sv,z) \preceq K\lambda d(z_{2n},z_{2n+1}) + Kd(z_{2n+1},z)$;

(iv) $d(Sv,z) \preceq (K\lambda/2)(d(z,z_{2n+1}) + d(z_{2n},Sv)) + Kd(z_{2n+1},z)$.

By the same arguments as above, we conclude that $d(Sv,z) = \theta$, that is, $Sv = Fv = z$. So, z is a point of coincidence of S and F, too.

Now we prove that z is unique point of coincidence of pairs $\{S,F\}$ and $\{T,G\}$. Suppose that there exists z^* which is also a point of coincidence of these four mappings, that is, $Fv^* = Gy^* = Sv^* = Ty^* = z^*$. From (3.2),

$$d(z,z^*) = d(Sv,Ty^*) \preceq \frac{\lambda}{K}u(v,y^*), \tag{3.18}$$

where

$$\begin{aligned} u(v,y^*) &\in \left\{Kd(Fv,Gy^*), Kd(Fv,Sv), d(Gy^*,Ty^*), K\frac{d(Fv,Ty^*) + d(Gy^*,Sv)}{2}\right\} \\ &= \{Kd(z,z^*), \theta\}. \end{aligned} \tag{3.19}$$

Using (p_3) we get $d(z,z^*) = \theta$, that is, $z = z^*$. Therefore, z is the unique point of coincidence of pairs $\{S,F\}$ and $\{T,G\}$. If these pairs are weakly compatible, then z is the unique common fixed point of S, F, T, and G, by Lemma 2.8.

Similarly, we can prove the statement in the cases when FX, GX, or TX is complete. □

We give one simple, but illustrative, example.

Example 3.2. Let $X = \mathbb{R}$, $E = \mathbb{R}$, and $P = [0, +\infty)$. Let us define $d(x, y) = |x - y|^2$ for all $x, y \in X$. Then (X, d) is a CMTS, but it is not a cone metric space since the triangle inequality is not satisfied. Starting with Minkowski inequality (see [18]) for $p = 2$, by using the inequality of arithmetic and geometric means, we get

$$|x - z|^2 \leq |x - y|^2 + |y - z|^2 + 2|x - y||x - z| \leq 2\left(|x - y|^2 + |y - z|^2\right). \tag{3.20}$$

Here, $K = 2$.

Let us define four mappings $S, F, T, G : X \to X$ as follows:

$$Sx = M(ax + b), \qquad Fx = ax + b, \qquad Tx = M(cx + d), \qquad Gx = cx + d, \tag{3.21}$$

where $x \in X$, $a \neq 0$, $c \neq 0$, and $M < 1/\sqrt{2}$. Since $SX = FX = TX = GX = X$ we have trivially $SX \subset GX$ and $TX \subset FX$. Also, X is a complete space. Further, $d(Sx, Ty) = |M(ax + b) - M(cy + d)|^2 = M^2 d(Fx, Gy)$, that is, there exists $\lambda = M^2 < 1/2 = 1/K$ such that (3.2) is satisfied.

According to Theorem 3.1, $\{S, F\}$ and $\{T, G\}$ have a unique point of coincidence in X, that is, there exists unique $z \in X$ and there exist $x, y \in X$ such that $z = Sx = Fx = Ty = Gy$. It is easy to see that $x = -b/a$, $y = -d/c$, and $z = 0$.

If $\{S, F\}$ is weakly compatible pair, we have $SFx = FSx$, which implies $Mb = b$, that is, $b = 0$. Similarly, if $\{T, G\}$ is weakly compatible pair, we have $TGy = GTy$, which implies $Md = d$, that is, $d = 0$. Then $x = y = 0$, and $z = 0$ is the unique common fixed point of these four mappings.

The following two theorems can be proved in the same way as Theorem 3.1, so we omit the proofs.

Theorem 3.3. *Let (X, d) be a CMTS with constant $K \geq 2$ and P a solid cone. Suppose that self-mappings $F, G, S, T : X \to X$ are such that $SX \subset GX$, $TX \subset FX$ and that for some constant $\lambda \in (0, 2/K^2)$ for all $x, y \in X$ there exists*

$$u(x, y) \in \left\{ Kd(Fx, Gy), Kd(Fx, Sx), Kd(Gy, Ty), K\frac{d(Fx, Ty) + d(Gy, Sx)}{2} \right\}, \tag{3.22}$$

such that the following inequality

$$d(Sx, Ty) \leq \frac{\lambda}{K} u(x, y), \tag{3.23}$$

holds. If one of SX, TX, FX, or GX is complete subspace of X, then $\{S, F\}$ and $\{T, G\}$ have a unique point of coincidence in X. Moreover, if $\{S, F\}$ and $\{T, G\}$ are weakly compatible pairs, then F, G, S, and T have a unique common fixed point.

Theorem 3.4. *Let* (X, d) *be a CMTS with constant* $K \geq 1$ *and* P *a solid cone. Suppose that self-mappings* $F, G, S, T : X \to X$ *are such that* $SX \subset GX$, $TX \subset FX$ *and that for some constant* $\lambda \in (0, 1/K)$ *for all* $x, y \in X$ *there exists*

$$u(x,y) \in \left\{ Kd(Fx,Gy), Kd(Fx,Sx), Kd(Gy,Ty), \frac{d(Fx,Ty)+d(Gy,Sx)}{2} \right\}, \tag{3.24}$$

such that the following inequality

$$d(Sx,Ty) \preceq \frac{\lambda}{K} u(x,y), \tag{3.25}$$

holds. If one of SX, TX, FX, *or* GX *is complete subspace of* X, *then* $\{S, F\}$ *and* $\{T, G\}$ *have a unique point of coincidence in* X. *Moreover, if* $\{S, F\}$ *and* $\{T, G\}$ *are weakly compatible pairs, then* F, G, S, *and* T *have a unique common fixed point.*

Theorems 3.1 and 3.4 are generalizations of [13, Theorem 2.2]. As a matter of fact, for $K = 1$, from Theorems 3.1 and 3.4, we get [13, Theorem 2.2].

If we choose $T = S$ and $G = F$, from Theorems 3.1, 3.3, and 3.4 we get the following results for two mappings on CMTS.

Corollary 3.5. *Let* (X, d) *be a CMTS with constant* $1 \leq K \leq 2$ *and* P *a solid cone. Suppose that self-mappings* $F, S : X \to X$ *are such that* $SX \subset FX$ *and that for some constant* $\lambda \in (0, 1/K)$ *for all* $x, y \in X$ *there exists*

$$u(x,y) \in \left\{ Kd(Fx,Fy), Kd(Fx,Sx), Kd(Fy,Sy), K\frac{d(Fx,Sy)+d(Fy,Sx)}{2} \right\}, \tag{3.26}$$

such that the following inequality

$$d(Sx,Sy) \preceq \frac{\lambda}{K} u(x,y), \tag{3.27}$$

holds. If FX *or* SX *is complete subspace of* X, *then* F *and* S *have a unique point of coincidence in* X. *Moreover, if* $\{F, S\}$ *is a weakly compatible pair, then* F *and* S *have a unique common fixed point.*

Corollary 3.6. *Let* (X, d) *be a CMTS with constant* $K \geq 2$ *and* P *a solid cone. Suppose that self-mappings* $F, S : X \to X$ *are such that* $SX \subset FX$ *and that for some constant* $\lambda \in (0, 2/K^2)$ *for all* $x, y \in X$ *there exists*

$$u(x,y) \in \left\{ Kd(Fx,Fy), Kd(Fx,Sx), Kd(Fy,Sy), K\frac{d(Fx,Sy)+d(Fy,Sx)}{2} \right\}, \tag{3.28}$$

such that the following inequality

$$d(Sx,Sy) \preceq \frac{\lambda}{K} u(x,y), \tag{3.29}$$

holds. If FX or SX is complete subspace of X, then F and S have a unique point of coincidence in X. Moreover, if $\{F, S\}$ is a weakly compatible pair, then F and S have a unique common fixed point.

Corollary 3.7. *Let (X, d) be a CMTS with constant $K \geq 1$ and P a solid cone. Suppose that self-mappings $F, S : X \to X$ are such that $SX \subset FX$ and that for some constant $\lambda \in (0, 1/K)$ for all $x, y \in X$ there exists*

$$u(x,y) \in \left\{ Kd(Fx,Fy), Kd(Fx,Sx), Kd(Fy,Sy), \frac{d(Fx,Sy)+d(Fy,Sx)}{2} \right\}, \tag{3.30}$$

such that the following inequality

$$d(Sx,Sy) \preceq \frac{\lambda}{K} u(x,y), \tag{3.31}$$

holds. If FX or SX is complete subspace of X, then F and S have a unique point of coincidence in X. Moreover, if $\{F, S\}$ is a weakly compatible pair, then F and S have a unique common fixed point.

Theorem 3.8. *Let (X, d) be a CMTS with constant $K \geq 1$ and P a solid cone. Suppose that self-mappings $F, G, S, T : X \to X$ are such that $SX \subset GX$, $TX \subset FX$ and that there exist nonnegative constants a_i, $i = 1, \ldots, 5$, satisfying*

$$a_1 + a_2 + a_3 + 2K \max\{a_4, a_5\} < 1, \qquad a_3K + a_4K^2 < 1, \qquad a_2K + a_5K^2 < 1, \tag{3.32}$$

such that for all $x, y \in X$ inequality

$$d(Sx,Ty) \preceq a_1 d(Fx,Gy) + a_2 d(Fx,Sx) + a_3 d(Gy,Ty) + a_4 d(Fx,Ty) + a_5 d(Gy,Sx), \tag{3.33}$$

holds. If one of SX, TX, FX, or GX is complete subspace of X, then $\{S, F\}$ and $\{T, G\}$ have a unique point of coincidence in X. Moreover, if $\{S, F\}$ and $\{T, G\}$ are weakly compatible pairs, then F, G, S, and T have a unique common fixed point.

Proof. We define sequences $\{x_n\}$ and $\{z_n\}$ as in the proof of Theorem 3.1. First we prove that

$$d(z_n, z_{n+1}) \preceq \alpha d(z_{n-1}, z_n), \quad n \geq 1, \tag{3.34}$$

where

$$\alpha = \max\left\{ \frac{a_1 + a_3 + a_5K}{1 - a_2 - a_5K}, \frac{a_1 + a_2 + a_4K}{1 - a_3 - a_4K} \right\}, \tag{3.35}$$

which implies that $\{z_n\}$ is a Cauchy sequence, since, because of (3.32), it is easy to check that $\alpha \in [0, 1)$. To prove this, it is necessary to consider the cases of an odd and of an even integer n.

For $n = 2\ell + 1$, $\ell \in \mathbb{N}_0$, we have $d(z_{2\ell+1}, z_{2\ell+2}) = d(Sx_{2\ell+2}, Tx_{2\ell+1})$, and from (3.33) we have

$$\begin{aligned} d(Sx_{2\ell+2}, Tx_{2\ell+1}) \preceq\ & a_1 d(Fx_{2\ell+2}, Gx_{2\ell+1}) + a_2 d(Fx_{2\ell+2}, Sx_{2\ell+2}) \\ & + a_3 d(Gx_{2\ell+1}, Tx_{2\ell+1}) + a_4 d(Fx_{2\ell+2}, Tx_{2\ell+1}) + a_5 d(Gx_{2\ell+1}, Sx_{2\ell+2}), \end{aligned} \tag{3.36}$$

that is,

$$\begin{aligned} d(z_{2\ell+1}, z_{2\ell+2}) \preceq\ & a_1 d(z_{2\ell+1}, z_{2\ell}) + a_2 d(z_{2\ell+1}, z_{2\ell+2}) + a_3 d(z_{2\ell}, z_{2\ell+1}) \\ & + a_4 d(z_{2\ell+1}, z_{2\ell+1}) + a_5 d(z_{2\ell}, z_{2\ell+2}) \\ =\ & (a_1 + a_3) d(z_{2\ell}, z_{2\ell+1}) + a_2 d(z_{2\ell+1}, z_{2\ell+2}) + a_5 d(z_{2\ell}, z_{2\ell+2}) \\ \preceq\ & (a_1 + a_3) d(z_{2\ell}, z_{2\ell+1}) + a_2 d(z_{2\ell+1}, z_{2\ell+2}) + a_5 K d(z_{2\ell}, z_{2\ell+1}) \\ & + a_5 K d(z_{2\ell+1}, z_{2\ell+2}) \\ =\ & (a_1 + a_3 + a_5 K) d(z_{2\ell}, z_{2\ell+1}) + (a_2 + a_5 K) d(z_{2\ell+1}, z_{2\ell+2}). \end{aligned} \tag{3.37}$$

Therefore,

$$d(z_{2\ell+1}, z_{2\ell+2}) \preceq \frac{a_1 + a_3 + a_5 K}{1 - a_2 - a_5 K} d(z_{2\ell}, z_{2\ell+1}), \tag{3.38}$$

that is, inequality (3.34) holds in this case.

Similarly, for $n = 2\ell$, $\ell \in \mathbb{N}_0$, we have $d(z_{2\ell}, z_{2\ell+1}) = d(Sx_{2\ell}, Tx_{2\ell+1})$, and from (3.33) we have

$$\begin{aligned} d(Sx_{2\ell}, Tx_{2\ell+1}) \preceq\ & a_1 d(Fx_{2\ell}, Gx_{2\ell+1}) + a_2 d(Fx_{2\ell}, Sx_{2\ell}) \\ & + a_3 d(Gx_{2\ell+1}, Tx_{2\ell+1}) + a_4 d(Fx_{2\ell}, Tx_{2\ell+1}) \\ & + a_5 d(Gx_{2\ell+1}, Sx_{2\ell}), \end{aligned} \tag{3.39}$$

that is,

$$\begin{aligned} d(z_{2\ell}, z_{2\ell+1}) \preceq\ & a_1 d(z_{2\ell-1}, z_{2\ell}) + a_2 d(z_{2\ell-1}, z_{2\ell}) + a_3 d(z_{2\ell}, z_{2\ell+1}) \\ & + a_4 d(z_{2\ell-1}, z_{2\ell+1}) + a_5 d(z_{2\ell}, z_{2\ell}) \\ =\ & (a_1 + a_2) d(z_{2\ell-1}, z_{2\ell}) + a_3 d(z_{2\ell}, z_{2\ell+1}) + a_4 d(z_{2\ell-1}, z_{2\ell+1}) \\ \preceq\ & (a_1 + a_2) d(z_{2\ell-1}, z_{2\ell}) + a_3 d(z_{2\ell}, z_{2\ell+1}) + a_4 K d(z_{2\ell-1}, z_{2\ell}) + a_4 K d(z_{2\ell}, z_{2\ell+1}) \\ =\ & (a_1 + a_2 + a_4 K) d(z_{2\ell-1}, z_{2\ell}) + (a_3 + a_4 K) d(z_{2\ell}, z_{2\ell+1}). \end{aligned} \tag{3.40}$$

Thus,

$$d(z_{2\ell}, z_{2\ell+1}) \preceq \frac{a_1 + a_2 + a_4 K}{1 - a_3 - a_4 K} d(z_{2\ell-1}, z_{2\ell}), \tag{3.41}$$

and inequality (3.34) holds in this case, too.

By the same arguments as in Theorem 3.1 we conclude that $\{z_n\}$ is a Cauchy sequence.

Let us suppose that SX is complete subspace of X. Completeness of SX implies existence of $z \in SX$ such that $\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} Sx_{2n} = z$. Then, we have

$$\lim_{n\to\infty} Gx_{2n+1} = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Fx_{2n} = \lim_{n\to\infty} Tx_{2n+1} = z, \tag{3.42}$$

that is, for any $\theta \ll c$, for sufficiently large n we have $d(z_n, z) \ll c$. Since $z \in SX \subset GX$, there exists $y \in X$ such that $z = Gy$. Let us prove that $z = Ty$. From (d_3) and (3.33), we have

$$\begin{aligned}
d(Ty, z) &\preceq Kd(Ty, Sx_{2n}) + Kd(Sx_{2n}, z) \\
&\preceq a_1 Kd(Fx_{2n}, Gy) + a_2 Kd(Fx_{2n}, Sx_{2n}) + a_3 Kd(Gy, Ty) \\
&\quad + a_4 Kd(Fx_{2n}, Ty) + a_5 Kd(Gy, Sx_{2n}) + Kd(Sx_{2n}, z) \\
&= a_1 Kd(z_{2n-1}, z) + a_2 Kd(z_{2n-1}, z_{2n}) + a_3 Kd(z, Ty) \\
&\quad + a_4 Kd(z_{2n-1}, Ty) + a_5 Kd(z, z_{2n}) + Kd(z_{2n}, z) \\
&\preceq a_1 Kd(z_{2n-1}, z) + a_2 Kd(z_{2n-1}, z_{2n}) + a_3 Kd(z, Ty) \\
&\quad + a_4 K^2 d(z_{2n-1}, z) + a_4 K^2 d(z, Ty) + a_5 Kd(z, z_{2n}) + Kd(z_{2n}, z).
\end{aligned} \tag{3.43}$$

The sequence $\{z_n\}$ converges to z, so for each $c \in \operatorname{int} P$ there exists $n_0 \in \mathbb{N}$ such that for every $n > n_0$

$$\begin{aligned}
d(Ty, z) \preceq{}& \frac{1}{1 - a_3 K - a_4 K^2} \\
&\left(a_1 Kd(z_{2n-1}, z) + a_2 Kd(z_{2n-1}, z_{2n}) + a_4 K^2 d(z_{2n-1}, z) + a_5 Kd(z, z_{2n}) + Kd(z_{2n}, z)\right) \\
\ll{}& \frac{a_1 K}{1 - a_3 K - a_4 K^2} \cdot \frac{1 - a_3 K - a_4 K^2}{a_1 K} \cdot \frac{c}{5} + \frac{a_2 K}{1 - a_3 K - a_4 K^2} \cdot \frac{1 - a_3 K - a_4 K^2}{a_2 K} \cdot \frac{c}{5} \\
&+ \frac{a_4 K^2}{1 - a_3 K - a_4 K^2} \cdot \frac{1 - a_3 K - a_4 K^2}{a_4 K^2} \cdot \frac{c}{5} + \frac{a_5 K}{1 - a_3 K - a_4 K^2} \cdot \frac{1 - a_3 K - a_4 K^2}{a_5 K} \cdot \frac{c}{5} \\
&+ \frac{K}{1 - a_3 K - a_4 K^2} \cdot \frac{1 - a_3 K - a_4 K^2}{K} \cdot \frac{c}{5} \\
={}& c,
\end{aligned} \tag{3.44}$$

because of (3.32). Now, by (p_2) it follows that $d(Ty, z) = \theta$, that is, $Ty = z$. So, we have $Ty = Gy = z$, that is, y is a coincidence point, and z is a point of coincidence of mappings T and G.

Since $TX \subset FX$, there exists $v \in X$ such that $z = Fv$. Let us prove that $Sv = z$, too. From (d_3) and (3.33), we have

$$\begin{aligned}
d(Sv, z) &\preceq Kd(Sv, Tx_{2n+1}) + Kd(Tx_{2n+1}, z) \\
&\preceq a_1 Kd(Fv, Gx_{2n+1}) + a_2 Kd(Fv, Sv) + a_3 Kd(Gx_{2n+1}, Tx_{2n+1}) \\
&\quad + a_4 Kd(Fv, Tx_{2n+1}) + a_5 Kd(Gx_{2n+1}, Sv) + Kd(Tx_{2n+1}, z) \\
&= a_1 Kd(z, z_{2n}) + a_2 Kd(z, Sv) + a_3 Kd(z_{2n}, z_{2n+1}) \\
&\quad + a_4 Kd(z, z_{2n+1}) + a_5 Kd(z_{2n}, Sv) + Kd(Tx_{2n+1}, z) \\
&\preceq a_1 Kd(z, z_{2n}) + a_2 Kd(z, Sv) + a_3 Kd(z_{2n}, z_{2n+1}) \\
&\quad + a_4 Kd(z, z_{2n+1}) + a_5 K^2 d(z_{2n}, z) + a_5 K^2 d(Sv, z) + Kd(Tx_{2n+1}, z),
\end{aligned} \tag{3.45}$$

and by the same arguments as above, we conclude that $d(Sv, z) = \theta$, that is, $Sv = Fv = z$. Thus, z is a point of coincidence of mappings S and F, too.

Suppose that there exists z^* which is also a point of coincidence of these four mappings, that is, $Fv^* = Gy^* = Sv^* = Ty^* = z^*$. From (3.33) we have

$$\begin{aligned}
d(z, z^*) &= d(Sv, Ty^*) \\
&\preceq a_1 Kd(Fv, Gy^*) + a_2 Kd(Fv, Sv) + a_3 Kd(Gy^*, Ty^*) \\
&\quad + a_4 Kd(Fv, Ty^*) + a_4 Kd(Gy^*, Sv) \\
&= a_1 Kd(z, z^*) + a_2 Kd(z, z) + a_3 Kd(z^*, z^*) + a_4 Kd(z, z^*) + a_5 Kd(z^*, z) \\
&= (a_1 + a_4 + a_5) Kd(z, z^*),
\end{aligned} \tag{3.46}$$

and (because of (p_3)) it follows that $z = z^*$. Therefore, z is the unique point of coincidence of pairs $\{S, F\}$ and $\{T, G\}$, and we have $z = Sv = Fv = Gy = Ty$. If $\{S, F\}$ and $\{T, G\}$ are weakly compatible pairs, then z is the unique common fixed point of S, F, T, and G, by Lemma 2.8.

The proofs for the cases in which FX, GX, or TX is complete are similar. □

Theorem 3.8 is a generalization of [13, Theorem 2.8]. Choosing $K = 1$ from Theorem 3.8 we get the following corollary.

Corollary 3.9. *Let (X, d) be cone metric space and P a solid cone. Suppose that self-mappings $F, G, S, T : X \to X$ are such that $SX \subset GX$, $TX \subset FX$ and that there exist nonnegative constants a_i, $i = 1, \ldots, 5$, satisfying $a_1 + a_2 + a_3 + 2\max\{a_4, a_5\} < 1$, such that for all $x, y \in X$ inequality*

$$d(Sx, Ty) \preceq a_1 d(Fx, Gy) + a_2 d(Fx, Sx) + a_3 d(Gy, Ty) + a_4 d(Fx, Ty) + a_5 d(Gy, Sx), \tag{3.47}$$

holds. If one of SX, TX, FX, *or* GX *is complete subspace of* X, *then* $\{S, F\}$ *and* $\{T, G\}$ *have a unique point of coincidence in* X. *Moreover, if* $\{S, F\}$ *and* $\{T, G\}$ *are weakly compatible pairs, then* F, G, S, *and* T *have a unique common fixed point.*

If we choose $T = S$ and $G = F$, from Theorem 3.8, we get the following result for two mappings on CMTS.

Corollary 3.10. *Let* (X, d) *be a CMTS with constant* $K \geq 1$ *and* P *a solid cone. Suppose that self-mappings* $F, S : X \to X$ *are such that* $SX \subset FX$ *and that there exist nonnegative constants* a_i, $i = 1, \ldots, 5$, *satisfying*

$$a_1 + a_2 + a_3 + 2K \max\{a_4, a_5\} < 1, \qquad a_3 K + a_4 K^2 < 1, \qquad a_2 K + a_5 K^2 < 1, \tag{3.48}$$

such that for all $x, y \in X$ *inequality*

$$d(Sx, Sy) \preceq a_1 d(Fx, Fy) + a_2 d(Fx, Sx) + a_3 d(Fy, Sy) + a_4 d(Fx, Sy) + a_5 d(Fy, Sx), \tag{3.49}$$

holds. If one of SX *or* FX *is complete subspace of* X, *then* S *and* F *have a unique point of coincidence in* X. *Moreover, if* $\{F, S\}$ *is a weakly compatible pair, then* F *and* S *have a unique common fixed point.*

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