On BPS bounds in $D = 4$ $N = 2$ gauged supergravity

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ABSTRACT: We determine the BPS bounds in minimal gauged supergravity in four space-time dimensions. We concentrate on asymptotically anti-de Sitter (AdS) spacetimes, and find that there exist two disconnected BPS ground states of the theory, depending on the presence of magnetic charge. Each of these ground states comes with a different superalgebra and a different BPS bound, which we derive. As a byproduct, we also demonstrate how the supersymmetry algebra has a built-in holographic renormalization method to define finite conserved charges.

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Contents

1 Introduction and results

The main motivation for our analysis comes from the following paradox. Similarly to BPS states in asymptotically flat spacetimes, the authors of [1] provided a BPS bound in asymptotically anti-de Sitter spacetime that in the static case reduces to:

$$M \geq \sqrt{Q_e^2 + Q_m^2}. \tag{1.1}$$

Supersymmetric configurations would have to saturate the bound with $M^2 = Q_e^2 + Q_m^2$, for a given mass M, electric charge Q_e, and magnetic charge Q_m in appropriate units. However, in $N = 2$ minimal gauged supergravity, Romans [2] found two supersymmetric solutions, one of which does not saturate the BPS bound, and therefore an apparent paradox arises. The main aim of this paper is to resolve this conflict. The resolution of the paradox will lie in understanding the BPS ground states of gauged supergravity, the associated superalgebras, and in a proper definition of the mass in asymptotically AdS$_4$ spacetimes, as we will explain in the main body of the paper.

Minimally gauged supergravity has only two bosonic fields, the metric and the graviphoton field A_μ. The most general static and spherically symmetric solution of Einstein's equation with a negative cosmological constant and an electromagnetic field is given by (in the conventions of [2]),

$$ds^2 = U^2(r)\,dt^2 - U^{-2}(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)\,, \tag{1.2}$$

with

$$U^2(r) = 1 - \frac{2M}{r} + \frac{Q_e^2 + Q_m^2}{r^2} + g^2 r^2\,, \tag{1.3}$$

and with nonvanishing components of the graviphoton

$$A_t = \frac{Q_e}{r}\,, \qquad A_\phi = -Q_m \cos\theta\,. \tag{1.4}$$

In this class, there are two solutions that preserve some fraction of supersymmetry [2]. The first one is the so-called AdS "electric Reissner-Nordström (RN)" solution, for which the magnetic charge Q_m is set to zero and $M = Q_e$ so that the factor U has the form:

$$U^2 = \left(1 - \frac{Q_e}{r}\right)^2 + g^2 r^2\,, \qquad Q_m = 0\,. \tag{1.5}$$

This solution preserves one half of the supersymmetries (it is 1/2 BPS). Clearly, it saturates the BPS bound (1.1). Notice that the function U(r) has no zeros. Therefore, there is no horizon and the point $r = 0$ is a naked singularity.[1] Asymptotically, for $r \to \infty$, the solution is that of pure AdS$_4$, with cosmological constant $\Lambda = -3g^2$ in standard conventions.

The second supersymmetric solution is the so-called "cosmic dyon", having zero mass M but nonzero fixed magnetic charge $Q_m = \pm 1/(2g)$. Such a solution will never satisfy the BPS bound (1.1). Moreover, the electric charge Q_e can take an arbitrary value:

$$M = 0\,, \quad Q_m = \pm 1/(2g)\,, \quad U^2 = \left(gr + \frac{1}{2gr}\right)^2 + \frac{Q_e^2}{r^2}\,. \tag{1.6}$$

Again, there is a naked singularity at $r = 0$. However, asymptotically, when $r \to \infty$, the solution does not approach pure AdS$_4$, due to the presence of the magnetic charge. Instead, the solution defines another vacuum, since $M = 0$, but this vacuum is topologically distinct from AdS$_4$ in which $M = Q_m = 0$. For this reason,[2] we call this vacuum magnetic anti-de Sitter, or mAdS$_4$.

The cosmic dyon solution is 1/4 BPS, i.e. it preserves two out of eight supercharges. For both the electric RN-AdS and the cosmic dyon, the Killing spinors were explicitly

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[1]In AdS spacetimes, supersymmetry does not seem to provide a cosmic censorship, contrary to most cases in asymptotically flat spacetimes [3]. Whether cosmic censorship in AdS$_4$ can be violated is still an open problem, see e.g. [4]. This issue however has nothing to do with the paradox or contradiction mentioned above.

[2]One may argue that ground states should not have naked singularities. Clearly, this discussion is related to cosmic censorship in AdS, which we mentioned in the previous footnote. It is important to disentangle this discussion from the derivation of the BPS bounds. In fact, we expect that in matter coupled supergravity, there is a magnetic ground state without naked singularities [5–7].

constructed in [2]. The fact that the BPS bound (1.1) is not satisfied for the cosmic dyon leads to an apparent contradiction since states that admit a Killing spinor should saturate the BPS bound.

In this paper we show that the cosmic dyon in fact satisfies a different BPS bound that follows from a superalgebra different from the usual AdS$_4$ superalgebra. We will determine the new BPS bound starting from the explicit calculation of the supercharges and computing the anticommutator.[3] In summary, to state the main result of this paper, for stationary configurations, the new BPS bounds are:

- For asymptotically AdS$_4$ solutions with vanishing magnetic charge, $Q_m = 0$, the BPS bound is

$$M \geq |Q_e| + g|\vec{J}|\,, \tag{1.7}$$

where $\vec{J}$ is the angular momentum.

- For asymptotically magnetic AdS$_4$ solutions with $Q_m = \pm 1/(2g)$, the BPS bound is simply

$$M \geq 0\,, \tag{1.8}$$

with unconstrained electric charge Q_e and angular momentum $\vec{J}$.

Other values for the magnetic charges are not considered. The quantization condition requires it to be an integer multiple of the minimal unit, $Q_m = n/(2g); n \in \mathbb{Z}$, but it is not known if any other supersymmetric vacua can exist with $n \neq 0, 1$.

The meaning of the BPS bound is not that all solutions to the equations of motion must automatically satisfy (1.7) or (1.8). Rather, one constructs a physical configuration space consisting of solutions that satisfy a BPS bound like (1.7) or (1.8).

Our procedure also provides a new way of defining asymptotic charges in AdS$_4$ backgrounds with automatically built-in holographic renormalization, somewhat different than the procedure developed in [9, 10]. The same technique can also be applied to non-minimal gauged supergravity (see [11] for a classification of all fully BPS vacua in $N = 2$ gauged supergravities). In this case, there exist magnetically charged BPS black holes with spherical horizons [5–7]. One can also extend this to include the black brane solutions in gauged $N = 2$ supergravity that were recently constructed in [12].[4]

Additional motivation to study more closely the magnetic AdS$_4$ case and its superalgebra is provided by the AdS/CFT correspondence. There are suggestions in the literature [14] that excitations of the dual theory are relevant for condensed matter physics in the presence of external magnetic field, e.g. quantum Hall effect and Landau level splitting at strong coupling. A better understanding of the mAdS superalgebra could then provide us with more insights about the dual field theory.

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[3]An alternative approach based on the Witten-Nester energy was proposed in [8], leading to similar conclusions.

[4]Another possible extension is to spaces with different asymptotics, e.g. quotients of AdS where one can have horizons of higher genus [13]. Here, however, we will concentrate only at spacetimes that strictly asymptote to AdS$_4$.

2 Supercurrents and charges from the Noether theorem

In this section, we define and determine the Noether currents in minimally gauged super-gravity. The currents define conserved supercharges, and the Poisson (or Dirac) brackets between these charges produce a superalgebra. In the next section, we derive BPS bounds from the superalgebra. We start this section by reviewing some (well-known) facts about Noether currents for local gauge symmetries.

2.1 Generalities

Given a Lagrangian $\mathcal{L}(\phi, \partial_\mu \phi)$, depending on fields collectively denoted by ϕ, we have that under general field variations

$$\delta \mathcal{L} = \sum_\phi \mathcal{E}_\phi \delta \phi + \partial_\mu N^\mu \ , \tag{2.1}$$

where $\mathcal{E}_\phi$ vanishes upon using the equation of motion of ϕ and

$$N^\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \delta \phi \ . \tag{2.2}$$

Under a symmetry variation, parametrized by ϵ, the Lagrangian must transform into a total derivative, such that the action is invariant for appropriate boundary conditions,

$$\delta_\epsilon \mathcal{L} = \partial_\mu K_\epsilon^\mu \ . \tag{2.3}$$

Combining this with (2.1) for symmetry variations, we obtain

$$\sum_\phi \mathcal{E}_\phi \delta_\epsilon \phi = \partial_\mu (K_\epsilon^\mu - N_\epsilon^\mu) \,. \tag{2.4}$$

From the previous expression we see that the quantity

$$J_\epsilon^\mu \equiv K_\epsilon^\mu - N_\epsilon^\mu, \tag{2.5}$$

is the (on-shell) conserved current associated with symmetry transformations. For the case of supersymmetry, we call J_ϵ^μ the supercurrent. It depends on the (arbitrary) parameter ϵ and is defined up to improvement terms of the form $\partial_\nu I^{\mu\nu}$ where I is an antisymmetric tensor, as usual for conserved currents. The associated conserved supercharge is then

$$\mathcal{Q} \equiv \int \mathrm{d}^3 x \, J_\epsilon^0(x) \,. \tag{2.6}$$

This supercharge should also generate the supersymmetry transformations of the fields,

$$\delta_\epsilon \phi = \{\mathcal{Q}, \phi\}, \tag{2.7}$$

via the classical Poisson (or Dirac in case of constraints) brackets. Since the supercurrent and correspondingly the supercharge are defined up to improvement terms and surface terms respectively, it is not directly obvious that the Noether procedure will lead to the correct supersymmetry variations using (2.7). In practice one always has the information of the supersymmetry variations together with the supergravity Lagrangian, so it is possible to cross check the answers and thus derive uniquely the correct expression of the supercharge. We now illustrate this in detail for the case of minimally gauged supergravity.

2.2 Supercharges of minimal gauged supergravity

First we compute the supercurrent from the Lagrangian of minimal $D = 4$ $N = 2$ gauged supergravity following the conventions of [15] (which is written in 1.5-formalism):

$$S = \int \mathrm{d}^4x\, e\left[R(e,\omega) + 6g^2 + \frac{2}{e}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu\gamma_5\gamma_\nu(\hat{\mathcal{D}}_\rho + igA_\rho\sigma^2)\psi_\sigma - \mathcal{F}^2 \right.$$
$$\left. - \frac{1}{2e}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\rho\gamma_5\sigma^2\psi_\sigma\left(i\overline{\psi}_\mu\sigma^2\psi_\nu - \frac{1}{2e}\epsilon_{\mu\nu}{}^{\tau\lambda}\overline{\psi}_\tau\gamma_5\sigma^2\psi_\lambda\right)\right], \tag{2.8}$$

where $\overline{\psi} = i\psi^\dagger\gamma_0, e = \sqrt{\det g_{\mu\nu}}$,

$$\hat{\mathcal{D}}_\rho = \partial_\rho - \frac{1}{4}\omega_\rho^{ab}\gamma_{ab} - \frac{i}{2}g\gamma_\rho, \tag{2.9}$$

and

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\overline{\psi}_\mu\sigma^2\psi_\nu - \frac{1}{2e}\epsilon_{\mu\nu}{}^{\rho\sigma}\overline{\psi}_\rho\gamma_5\sigma^2\psi_\sigma =$$
$$= F_{\mu\nu} + i\overline{\psi}_\mu\sigma^2\psi_\nu - \frac{1}{2e}\epsilon_{\mu\nu}{}^{\rho\sigma}\overline{\psi}_\rho\gamma_5\sigma^2\psi_\sigma. \tag{2.10}$$

The spin connection satisfies

$$\mathrm{d}e^a - \omega^a{}_b \wedge e^b = 0 \tag{2.11}$$

for a given vielbein $e^a = e_\mu^a\mathrm{d}x^\mu$. The g^2-term in the Lagrangian is related to the presence of a negative cosmological constant $\Lambda = -3g^2$.

In most of our calculations, such as in the supercurrents and supercharges, we only work to lowest order in fermions since higher order terms vanish in the expression of the (on shell) supersymmetry algebra, where we set all fermion fields to zero. The supersymmetry variations are:

$$\delta_\epsilon\psi_\mu = \widetilde{\mathcal{D}}_\mu\epsilon = \left(\partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{i}{2}g\gamma_\mu + igA_\mu\sigma^2 + \frac{1}{4}F_{\lambda\tau}\gamma^{\lambda\tau}\gamma_\mu\sigma^2\right)\epsilon, \tag{2.12}$$

$$\delta_\epsilon e_\mu^a = -i\overline{\epsilon}\gamma^a\psi_\mu, \tag{2.13}$$

$$\delta_\epsilon A_\mu = -i\overline{\epsilon}\sigma^2\psi_\mu. \tag{2.14}$$

U(1) gauge transformations act on the gauge potential and on the spinors in this way:

$$A_\mu' = A_\mu + \partial_\mu\alpha, \tag{2.15}$$

$$\psi_\mu' = e^{-ig\alpha\sigma^2}\psi_\mu. \tag{2.16}$$

We use the conventions in which all the spinors are real Majorana ones,[5] and the gamma matrix conventions and identities of appendix A.

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[5]In our conventions, the two real gravitini in the gravity multiplet are packaged together in the notation:

$$\psi_\mu = \begin{pmatrix} \psi_\mu{}^1 \\ \psi_\mu{}^2 \end{pmatrix}, \tag{2.17}$$

where each gravitino is itself a 4-component Majorana spinor. Similar conventions are used for the super-symmetry parameters. In other words, the $SU(2)_R$ indices are completely suppressed in our notation.

The quantities N^μ and K^μ for this theory are:

$$N^\mu = \frac{\partial \mathcal{L}}{\partial_\mu \omega} \delta\omega + 2\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\nu \gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\sigma \epsilon + 4ieF^{\mu\nu}\overline{\epsilon}\sigma^2 \psi_\nu \ . \tag{2.18}$$

$$K^\mu = \frac{\partial \mathcal{L}}{\partial_\mu \omega} \delta\omega - 2\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\nu \gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\sigma \epsilon + 4ieF^{\mu\nu}\overline{\epsilon}\sigma^2 \psi_\nu \ . \tag{2.19}$$

Hence the supercurrent has the form:

$$J^\mu = -4\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\nu \gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\sigma \epsilon \ . \tag{2.20}$$

This expression is gauge invariant due to the cancelation between the variation of the gravitino, the vector field and the supersymmetry parameter. Furthermore we can show that the supercurrent is conserved

$$\partial_\mu J^\mu = \partial_\mu(-4\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\nu \gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\sigma \epsilon) = 0 \tag{2.21}$$

if we enforce the equation of motion for $\overline{\psi}_\nu$ and use the antisymmetry of the Levi-Civita symbol.

The Dirac brackets defined for the given theory read (we only need those containing gravitinos):

$$\{\psi_\mu(x), \psi_\sigma(x')\}_{t=t'} = 0 \ , \tag{2.22}$$

$$\{\overline{\psi}_\mu(x), \overline{\psi}_\sigma(x')\}_{t=t'} = 0 \ , \tag{2.23}$$

$$\{\psi_\mu(x), 2\epsilon^{0\nu\rho\sigma}\overline{\psi}_\rho(x')\gamma_5\gamma_\sigma\}_{t=t'} = \delta_\mu{}^\nu \delta^3(\vec{x} - \vec{x'}) \ . \tag{2.24}$$

We can now check if (2.7) holds with the above form of the supercurrent. It turns out that, up to overall normalization, we indeed have the right expression without any ambiguity of improvement terms. We only need to rescale, since the factor of 4 in (2.20) does not appear in the supersymmetry variations (2.12)–(2.14). The supercharge is then defined as the volume integral[6]

$$\mathcal{Q} \equiv 2 \int_V \mathrm{d}\Sigma_\mu \epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\sigma \gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\nu \epsilon \ \overset{e.o.m.}{=} \ 2 \oint_{\partial V} \mathrm{d}\Sigma_{\mu\nu} \epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\sigma \gamma_5 \gamma_\rho \epsilon \ , \tag{2.26}$$

where the second equality follows from the Gauss theorem via the equations of motion (in what follows we will always deal with classical solutions of the theory). The Dirac bracket of two supersymmetry charges is then straightforwardly derived as the supersymmetry variation of (2.26):

$$\{\mathcal{Q}, \mathcal{Q}\} = 2 \oint_{\partial V} \mathrm{d}\Sigma_{\mu\nu}(\epsilon^{\mu\nu\rho\sigma}\overline{\epsilon}\gamma_5 \gamma_\rho \widetilde{\mathcal{D}}_\sigma \epsilon) \ , \tag{2.27}$$

which is again a boundary integral.

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[6]For volume and surface integrals, we use the notation that

$$\mathrm{d}\Sigma_\mu = \frac{1}{6e^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{d}x^\nu \wedge \mathrm{d}x^\rho \wedge \mathrm{d}x^\sigma \ , \qquad \mathrm{d}\Sigma_{\mu\nu} = \frac{1}{2e^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{d}x^\rho \wedge \mathrm{d}x^\sigma \ . \tag{2.25}$$

The above formula is reminiscent of the expression for the Witten-Nester energy [16, 17], which has already been implicitly assumed to generalize for supergravity applications [8, 18, 19] (see also [20]). Thus the correspondence between BPS bounds and positivity of Witten-Nester energy is confirmed also in the case of minimal gauged $N = 2$ supergravity by our explicit calculation of the supercharge anticommutator.

3   Two different BPS bounds

In this section, we derive two BPS bounds based on the two BPS sectors that we consider. What is relevant for the BPS bound are the properties of the asymptotic geometries and corresponding Killing spinors. The Killing spinors of AdS$_4$ and magnetic AdS$_4$ ("cosmic monopole") are given in appendix B, see also [2, 8]. Since only the asymptotics are important, we can set $Q_e = 0$ in the cosmic dyon solution. The AdS$_4$ solution is characterized by $M = Q_e = Q_m = 0$, while mAdS$_4$ by $M = 0$, $Q_e = 0$, $Q_m = \pm 1/(2g)$. The corresponding Killing spinors take a very different form:

$$\epsilon_{\text{AdS}} = e^{\frac{i}{2}\operatorname{arcsinh}(gr)\gamma_1} e^{\frac{i}{2}gt\gamma_0} e^{-\frac{1}{2}\theta\gamma_{12}} e^{-\frac{1}{2}\varphi\gamma_{23}}\epsilon_0 \,, \tag{3.1}$$

$$\epsilon_{\text{mAdS}} = \frac{1}{4}\sqrt{gr + \frac{1}{2gr}}(1 + i\gamma_1)(1 \mp i\gamma_{23}\sigma^2)\epsilon_0 \,, \tag{3.2}$$

where ϵ_0 is a doublet of constant Majorana spinors, carrying 8 arbitrary parameters. From here we can see that AdS$_4$ is fully supersymmetric and its Killing spinors show dependence on all the four coordinates. mAdS$_4$ on the other hand is only 1/4 BPS: its Killing spinors satisfiy a double projection that reduces the independent components to 1/4 and there is no angular or time dependence. We will come back to this remarkable fact in section 4.

The form of the Killing spinors is important because the bracket of two supercharges is a surface integral at infinity (2.27). Writing out the covariant derivative in (2.27), one obtains

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\oint_{\partial V} \mathrm{d}\Sigma_{\mu\nu} \left[\epsilon^{\mu\nu\rho\sigma}\overline{\epsilon}\gamma_5\gamma_\rho(\partial_\sigma - \frac{1}{4}\omega_\sigma^{ab}\gamma_{ab} - \frac{i}{2}g\gamma_\sigma + igA_\sigma\sigma^2 + \frac{1}{4}F_{\lambda\tau}\gamma^{\lambda\tau}\gamma_\sigma\sigma^2)\epsilon\right] \,, \tag{3.3}$$

and it depends on the asymptotic value of the Killing spinors of the solution taken into consideration. Therefore the superalgebra will be different in the two cases and there will be two different BPS bounds.

The procedure to compute the BPS bound is the following. From (2.26) we have a definition of the supercharges $\mathcal{Q}_{\text{AdS}}(\epsilon_{\text{AdS}})$ and $\mathcal{Q}_{\text{mAdS}}(\epsilon_{\text{mAdS}})$. We will then make use of the following definition for the fermionic supercharges $Q_{\text{AdS}}, Q_{\text{mAdS}}$:

$$\mathcal{Q}_{\text{AdS}} \equiv Q_{\text{AdS}}^T\epsilon_0 = \epsilon_0^T Q_{\text{AdS}} \,, \qquad \mathcal{Q}_{\text{mAdS}} \equiv Q_{\text{mAdS}}^T\epsilon_0 = \epsilon_0^T Q_{\text{mAdS}} \,, \tag{3.4}$$

i.e. any spacetime and gamma matrix dependence of the bosonic supercharges $\mathcal{Q}$ is left into the corresponding fermionic Q. We are thus able to strip off the arbitrary constant ϵ_0 in any explicit calculations and convert the Dirac brackets for $\mathcal{Q}$ into an anticommutator for the spinorial supercharges Q that is standardly used to define the superalgebra. Therefore

now we compute the surface integrals (2.27) for the Killing spinors of AdS_4 and $mAdS_4$ respectively. After stripping off the ϵ_0's, we find the anticommutator of fermionic supercharges given explicitly in terms of the other conserved charges in the respective vacua. The BPS bound is then derived in the standard way by requiring the supersymmetry anticommutator to be positive definite, see e.g. [21] for details.

3.1 Asymptotically AdS_4 states

We now derive the resulting supersymmetry algebra from the asymptotic spinors of AdS_4. For this we use the general expression (3.3) for the Dirac brackets of the supersymmetry charges, together with the asymptotic form of the Killing spinors, (3.1). Inserting the Killing spinors ϵ_{AdS} of (3.1) in (3.3), we recover something that can be written in the following form:

$$\{\mathcal{Q}, \mathcal{Q}\} = -i\,\overline{\epsilon_0}(A + B_a\gamma^a + C\gamma^5 + D_{ij}\gamma^{ij} + E_i\gamma^{0i} + F_a\gamma^{a5})\epsilon_0 \,, \tag{3.5}$$

where the charges $A, B, \ldots$ can be written down explicitly from the surface integral (3.3). They will define the electric charge (A), momentum (B), angular momentum (D, with $i, j = 1, 2, 3$ spatial indices), and boost charges (E). The charge C would correspond to a magnetic charge, which we assumed to vanish by construction. Without the charge F_a, the above bracket will fit in the $Osp(2|4)$ superalgebra (see more below). We will therefore take as definition of asymptotically AdS solutions the ones for which F_a vanishes. This choice of fall-off conditions is similar to the case of $N = 1$ supergravity, where the asymptotic charges are required to generate the $Osp(1|4)$ superalgebra [22]. Extensions of the $N = 2$ superalgebra where the charges C and F_a are non-zero have been discussed in [23].

From the previous expression we see that conserved charges like Q_e, M et cetera will arise as surface integrals of the five terms (or their combinations) appearing in the supercovariant derivative. We are going to see how this works analyzing each term in the supercovariant derivative, explicitly in terms of the ansatz of the metric (1.2) and vector fields (1.4). This will provide us with a new definition of the asymptotic charges in AdS_4 with no need to use the holographic renormalization procedure anywhere. As an explicit example one can directly read off the definition of mass $M \equiv B_0/(8\pi)$ from the explicit form of the asymptotic Killing spinors. In the stationary case,

$$M = \frac{1}{8\pi}\lim_{r\to\infty}\oint \mathrm{d}\Sigma_{tr}\{e^t_{[0}e^r_1e^\theta_{2]} + \sin\theta e^t_{[0}e^r_1e^\varphi_{3]} \tag{3.6}$$
$$+ g^2re^t_{[0}e^r_{1]} - \sqrt{g^2r^2+1}(\omega^{ab}_\theta e^t_{[0}e^r_ae^\theta_{b]} + \omega^{ab}_\varphi e^t_{[0}e^r_ae^\varphi_{b]})\} \,.$$

We are going to take into consideration both static and rotating solutions, but we will carry out our procedure and explain the calculation in full detail only the case of the electric RN-AdS black hole and comment more briefly on the rotating generalizations.

- Electric RN-AdS.

  Here we take into consideration solutions of the form (1.2)–(1.4) with zero magnetic charge, $Q_m = 0$. We now evaluate the various terms in (3.3).

To begin, it is easy to determine the piece concerning the field strength, namely

$$2 \oint \mathrm{d}\Sigma_{\mu\nu} \left[\epsilon^{\mu\nu\rho\sigma} \overline{\epsilon}(t,r,\theta,\varphi) \gamma_5 \gamma_\rho \frac{1}{4} F_{\lambda\tau} \gamma^{\lambda\tau} \gamma_\sigma \sigma^2 \epsilon(t,r,\theta,\varphi) \right] . \tag{3.7}$$

Inserting the Killing spinors for AdS$_4$ described in (3.1), and exploiting the Clifford algebra relations we get

$$2 \oint \mathrm{d}\Sigma_{tr} \, \overline{\epsilon}_{AdS}(t,r,\theta,\varphi) e \, F^{tr} \sigma^2 \epsilon_{AdS}(t,r,\theta,\varphi) \ =$$

$$2\oint \mathrm{d}\Sigma_{tr} \epsilon_0^T e^{\frac{1}{2}\varphi\gamma_{23}} e^{\frac{1}{2}\theta\gamma_{12}} e^{-igt\gamma_0} e^{\frac{i}{2}arcsinh(gr)\gamma_1} \gamma_0 e \, F^{tr} \sigma^2 e^{\frac{i}{2}arcsinh(gr)\gamma_1} e^{igt\gamma_0} e^{-\frac{1}{2}\theta\gamma_{12}} e^{-\frac{1}{2}\varphi\gamma_{23}} \epsilon_0 =$$

$$= 8i\pi \overline{\epsilon_0} Q_e \sigma^2 \epsilon_0 , \quad (3.8)$$

with the definition of the electric charge

$$Q_e = \frac{1}{4\pi} \oint_{S^2} F^{tr} r^2 \sin\theta \, \mathrm{d}\theta \mathrm{d}\phi . \tag{3.9}$$

From here we can identify the term $A = -8\pi Q_e \sigma^2$ appearing in (3.5).

Next we consider the term containing the "bare" gauge field A_μ. This gives a possible contribution to the charge F_a in (3.5). As we mentioned above, we assumed this contribution to vanish for asymptotically AdS solutions. One can explicitly check this for the class of electric RN-AdS solutions given in (1.2), since the only nonzero component of the vector field is A_t (see (1.4)), hence

$$\oint \mathrm{d}\Sigma_{tr} \left[\epsilon^{tr\rho\sigma} \overline{\epsilon}_{AdS}(t,r,\theta,\varphi) \gamma_5 \gamma_\rho i g A_\sigma \sigma^2 \epsilon_{AdS}(t,r,\theta,\varphi) \right] = 0 . \tag{3.10}$$

The term with the partial derivative ∂_σ in (3.3) in the supercovariant derivative gives nonvanishing contributions for $\sigma = \theta, \varphi$ and it amounts to the integral:

$$2 \oint \mathrm{d}\Sigma_{tr} \left[\epsilon^{tr\rho\sigma} \overline{\epsilon}_{AdS}(t,r,\theta,\varphi) \gamma_5 \gamma_\rho \partial_\sigma \epsilon_{AdS}(t,r,\theta,\varphi) \right] = -2i \oint r \overline{\epsilon_0} \gamma_0 \epsilon_0 \sin\theta \, \mathrm{d}\theta \mathrm{d}\varphi . \tag{3.11}$$

Clearly, this term will contribute, together with other terms, to the mass.

The integral containing the spin connection is:

$$-\frac{2}{4} \oint \mathrm{d}\Sigma_{tr} \epsilon^{tr\rho\sigma} \overline{\epsilon}_{AdS}(t,r,\theta,\varphi) \gamma_5 \gamma_\rho \, \omega_\sigma^{ab} \, \gamma_{ab} \, \epsilon_{AdS}(t,r,\theta,\varphi) \ =$$

$$= 2i \oint \overline{\epsilon_0} \gamma^0 r \sqrt{1+g^2r^2} \sqrt{1+g^2r^2 - \frac{2M}{r} + \frac{Q_e^2}{r^2}} \, \epsilon_0 \sin\theta \, \mathrm{d}\theta \mathrm{d}\varphi , \quad (3.12)$$

where we have used

$$e^{i \, arcsinh(gr)\gamma_1} = \sqrt{1+g^2r^2} + ig\gamma_1 r , \tag{3.13}$$

and the value of the spin connection:

$$\omega_t^{01} = U\partial_r U, \quad \omega_\theta^{12} = -U, \quad \omega_\varphi^{13} = -U\sin\theta, \quad \omega_\varphi^{23} = -\cos\theta . \tag{3.14}$$

Also (3.12) will contribute to B^0, and therefore to the mass.

The last contribution of the supercovariant derivative, the term proportional to $g\gamma_\sigma$, yields

$$-2\oint d\Sigma_{tr}\epsilon^{tr\rho\sigma}\overline{\epsilon}_{\text{AdS}}(t,r,\theta,\varphi)\frac{ig}{2}\gamma_5\gamma_\rho\gamma_\sigma\,\epsilon_{\text{AdS}}(t,r,\theta,\varphi) = -2i\oint\overline{\epsilon_0}\gamma^0 r^3 g^2 \epsilon_0\,\sin\theta\,d\theta d\varphi\,. \tag{3.15}$$

In deriving this we have used the formula $\gamma^{tr\mu}\gamma_\mu = 2\gamma^{tr}$. Again, this term contributes to the mass formula.

Collecting all the terms that contribute to the mass (the derivative term (3.11), the sum of the spin connection term (3.12), and the gamma term (3.15)) gives rise to:

$$2i\oint\overline{\epsilon_0}\gamma^0\left[r\sqrt{1+g^2r^2}\sqrt{1+g^2r^2-\frac{2M}{r}+\frac{Q_e^2}{r^2}}-r^3g^2-r\right]\epsilon_0\,\sin\theta\,d\theta d\varphi\,. \tag{3.16}$$

The integral has to be performed on a sphere with $r\to\infty$. Taking this limit one can see that in this expression all the positive powers of r are canceled. Hence all possible divergences cancel out, and we are left with a finite contribution. In this sense, our method provides a holographic renormalization of the mass. In the cases we can compare, our method agrees with previously known results.

Performing the integral on the remaining finite part we find:

$$-8i\pi\overline{\epsilon_0}M\gamma^0\epsilon_0 = -i\overline{\epsilon_0}\gamma^0 B_0\,\epsilon_0\,. \tag{3.17}$$

To sum up, for the electric RN-AdS solution, the brackets between supercharges read:

$$\{\mathcal{Q},\mathcal{Q}\} = -8\pi i\overline{\epsilon_0}(M\gamma^0 - Q_e\sigma^2)\epsilon_0$$
$$\Rightarrow \{\epsilon_0^T Q, Q^T\epsilon_0\} = 8\pi\epsilon_0^T(M - Q_e\gamma^0\sigma^2)\,\epsilon_0\,. \tag{3.18}$$

Now we can strip off the constant linearly independent doublet of spinors ϵ_0 on both sides of the above formula to restore the original SO(2) and spinor indices:

$$\{Q^{A\alpha}, Q^{B\beta}\} = 8\pi\left(M\delta^{AB}\delta^{\alpha\beta} - i\,Q_e\epsilon^{AB}(\gamma^0)^{\alpha\beta}\right)\,. \tag{3.19}$$

This expression coincides with the one expected from the algebra $Osp(2|4)$ (see (4.1) in the next section) if we identify $M_{-10} = 8\pi M$ and $T^{12} = 8\pi Q_e$.

The BPS bound for the electric RN-AdS solution is then:[7]

$$M \geq |Q_e|\,. \tag{3.20}$$

The state that saturates this bound, for which $M = |Q_e|$, preserves half of the supersymmetries, i.e. it is half-BPS. It is the ground state allowed by (3.20) and represents a naked singularity. All the excited states have higher mass and are either naked singularities or genuine black holes.

---

[7]See e.g. [21] for details on the general procedure of deriving of BPS bounds from the superalgebra.

It is interesting to look at the case of extremal black holes, in which inner and outer horizon coincide. This yields a relation between the mass and charge, which can be derived from the solution given in (1.2) and (1.3). Explicit calculation gives the following result [13]:

$$M_{\text{extr}} = \frac{1}{3\sqrt{6}g}(\sqrt{1 + 12g^2Q_e^2} + 2)(\sqrt{1 + 12g^2Q_e^2} - 1)^{1/2}\,. \qquad (3.21)$$

This lies above the BPS bound unless $Q_e = 0$, in which case we recover the fully supersymmetric AdS$_4$ space. Thus,

$$M_{\text{extr}} > M_{\text{BPS}}\,. \qquad (3.22)$$

- Kerr-AdS.

  The Kerr-AdS black hole is an example of a stationary spacetime without charges but with non-vanishing angular momentum. It is most standardly written in Boyer-Lindquist-type coordinates and we refer to [13] for more details. More details on how to calculate the angular momenta from the anticommutator of the supercharges can be found in appendix C. The BPS bound is straightforward to find also in this case, leading to

$$M \geq g|\vec{J}|\,, \qquad (3.23)$$

  where the BPS state satisfies $M = g|\vec{J}|$ and in fact corresponds to a singular limit of the Kerr-AdS black hole because the AdS boundary needs to rotate as fast as the speed of light [24]. Note that in general for the Kerr black hole we have $|\vec{J}| = aM$, where a is the rotation parameter appearing in the Kerr solution in standard notation. Thus $M = g|\vec{J}|$ implies $a = 1/g$, which is exactly the singular case. All the excited states given by $a < 1/g$ are however proper physical states, corresponding to all the regular Kerr-AdS black holes, including the extremal one. Thus the BPS bound is always satisfied but never saturated by any physical solution of the Kerr-AdS type,

$$M_{\text{extr}} > M_{\text{BPS}}\,, \qquad (3.24)$$

  as is well-known.

- KN-AdS.

  The BPS bound for Kerr-Newman-AdS (KN-AdS) black holes[8] is a bit more involved due to the presence of both electric charge and angular momentum. We will not elaborate on the details of the calculation which is straightforward. The resulting BPS bound is

$$M \geq |Q_e| + g|\vec{J}| = |Q_e| + agM\,, \qquad (3.25)$$

  and the ground (BPS) state is in fact quarter-supersymmetric. This BPS bound is also well-known and is described in [21]. The BPS bound does in general not coincide

---

[8]See again [13] for more detailed description of the KN-AdS black holes.

with the extremality bound, which in the case of the KN-AdS black holes is a rather complicated expression that can be found in [13, 24]. Interestingly, the BPS bound and the extremality bound coincide at a finite non-zero value for the mass and charge (with $ag < 1$),

$$|Q_{e,\text{crit}}| \equiv \sqrt{\frac{a}{g}} \frac{1}{1 - ag} \, . \tag{3.26}$$

Now we have two distinct possibilities for the relation between the BPS state and the extremal KN-AdS black hole depending on the actual value for the electric charge (there is exactly one BPS state and exactly one extremal black hole for any value of charge Q_e):

$$M_{\text{extr}} > M_{\text{BPS}}, \qquad |Q_e| \neq |Q_{e,\text{crit}}|,$$
$$M_{\text{extr}} = M_{\text{BPS}}, \qquad |Q_e| = |Q_{e,\text{crit}}| \, . \tag{3.27}$$

So for small or large enough electric charge the BPS solution will be a naked singularity and the extremal black hole will satisfy but not saturate the BPS bound, while for the critical value of the charge the extremal black hole is supersymmetric and all non-extremal solutions with regular horizon will satisfy the BPS bound.

3.2 Magnetic AdS$_4$

Unlike the standard AdS$_4$ case above, the Killing spinors of magnetic AdS$_4$ already break $3/4$ of the supersymmetry, cf. (B.9). The projection that they obey is,

$$\epsilon_{\text{mAdS}} = P\epsilon_{\text{mAdS}}, \qquad P \equiv \frac{1}{4}(1 + i\gamma_1)(1 \mp i\gamma_{23}\sigma^2), \tag{3.28}$$

for either the upper or lower sign, depending on the sign of the magnetic charge. Furthermore, one has the following properties of the projection operators,

$$P^\dagger P = P^\dagger i\gamma_1 P = \pm P^\dagger i\gamma_{23}\sigma^2 P = \pm P^\dagger(-i\gamma_0\gamma_5\sigma^2)P = P, \tag{3.29}$$

and all remaining quantities of the form $P^\dagger \Gamma P$ vanish, where Γ stands for any of the other twelve basis matrices generated by the Clifford algebra.

These identities allow us to derive, from (3.3), the bracket

$$\{\mathcal{Q}, \mathcal{Q}\} = \overline{P\epsilon_0}\gamma_0(-i\,8\pi)MP\epsilon_0 \qquad \Rightarrow \qquad \{\epsilon_0^T PQ, (PQ)^T\epsilon_0\} = \epsilon_0^T(8\pi M)P\epsilon_0 \, . \tag{3.30}$$

provided that the mass is given by

$$M = \frac{1}{8\pi} \lim_{r\to\infty} \oint \mathrm{d}\Sigma_{tr} \left(gr + \frac{1}{2gr} \right) \left(2g(A_\theta e^t_{[0} e^r_2 e^\theta_{3]} + A_\varphi e^t_{[0} e^r_2 e^\varphi_{3]}) \right. \tag{3.31}$$

$$\left. + \frac{\sin\theta}{g} e^t_{[0} e^r_1 e^\theta_2 e^\varphi_{3]} + g e^t_{[0} e^r_{1]} - (\omega^{ab}_\theta e^t_{[0} e^r_a e^\theta_{b]} + \omega^{ab}_\varphi e^t_{[0} e^r_a e^\varphi_{b]}) \right) \, . $$

This expression simplifies further if we choose to put the vielbein matrix in an upper triangular form, such that we have nonvanishing $e_t^{0,1,2,3}, e_r^{1,2,3}, e_\theta^{2,3}, e_\varphi^3$, and the inverse vielbein has only components $e_0^{t,r,\theta,\varphi}, e_1^{r,\theta,\varphi}, e_2^{\theta,\varphi}, e_3^\varphi$. The mass is then

$$M = \frac{1}{8\pi} \lim_{r\to\infty} \oint d\Sigma_{tr} \left(gr + \frac{1}{2gr} \right) \left(\frac{\sin\theta}{g} e_0^t e_1^r e_2^\theta e_3^\varphi + g e_0^t e_1^r - (\omega_\theta^{12} e_0^t e_1^r e_2^\theta + \omega_\varphi^{13} e_0^t e_1^r e_2^\varphi) \right). \tag{3.32}$$

Notice that this mass formula is different from the one for asymptotically AdS_4 spacetimes.

Stripping off the parameters ϵ_0 in (3.30), leaves us with a matrix equation in spinor space. Due to the projection operators, one is effectively reducing the number of supercharges to two instead of eight. These two supercharges are scalars, since the Killing spinors are invariant under rotation as they don't depend on the angular coordinates (see also in the next section). Denoting them by Q^1 and Q^2, the anticommutator then becomes

$$\{Q^I, Q^J\} = 8\pi M \delta^{IJ} , \qquad I, J = 1, 2 . \tag{3.33}$$

Hence the BPS bound is just

$$M \geq 0 . \tag{3.34}$$

Saturating the bound leads to a quarter-BPS solution. None of the other conserved charges, i.e. the electric charge and (angular) momentum, influences the BPS bound due to the projection relation (3.29). Thus Q_e and $\vec{J}$ can be completely arbitrary.

In particular, for the case of of the Reissner-Nordström solution (1.2) with fixed magnetic charge $Q_m = \pm 1/(2g)$ and Q_e arbitrary, the mass integral (3.31) yields

$$\frac{1}{8\pi} \lim_{r\to\infty} \oint \left(gr + \frac{1}{2gr} \right) \left(r\sqrt{1 + g^2r^2 - \frac{2M}{r} + \frac{Q_e^2 + 1/(4g^2)}{r^2}} - g\,r^2 - \frac{1}{2g} \right) \sin\theta\, d\theta d\varphi =$$

$$= \frac{1}{8\pi} \oint \left(-g^2r^3 - \frac{r}{2} + M - \frac{r}{2} + g^2r^3 + r \right) \sin\theta\, d\theta d\varphi = M . \tag{3.35}$$

This is exactly the mass parameter M appearing in (1.3). The supersymmetric solutions found by Romans (the so-called cosmic monopole/dyons) have vanishing mass parameter hence indeed saturate the BPS bound (3.34).

Of course in the context of a rotating black hole vanishing mass results in vanishing angular momentum due to the proportionality between the two, i.e. an asymptotically mAdS Kerr-Newman with non-zero angular momentum spacetime can never saturate the BPS bound (3.34). Nevertheless, excitations over the magnetic AdS_4 include all Reissner-Nordström and Kerr-Newman AdS black holes that have fixed magnetic charge $2gQ_m = \pm 1$ and arbitrary (positive) mass, angular momentum and electric charge. All these solutions satisfy the magnetic AdS_4 BPS bound.

4 Superalgebras

4.1 AdS$_4$

The procedure we used to find the BPS bound determines also the superalgebras of AdS$_4$ and mAdS$_4$, which are found to be different. In fact, given the Killing spinors and Killing vectors, there is a general algorithm to determine the superalgebra, see [25] and chapter 13 in [26].

For what concerns the pure AdS$_4$, in $N = 2$ gauged supergravity the superalgebra is $Osp(2|4)$, which contains as bosonic subgroup SO(2, 3) × SO(2): the first group is the isometry group of AdS$_4$ and the second one corresponds to the gauged R-symmetry group that acts by rotating the two gravitinos. The algebra contains the generators of the SO(2, 3) group M_{MN} ($M, N = -1, 0, 1, 2, 3$), and $T^{AB} = -T^{BA} = T\epsilon^{AB}, A, B = 1, 2$, the generator of SO(2). Furthermore we have supercharges $Q^{A\alpha}$ with $A = 1, 2$ that are Majorana spinors. The non-vanishing (anti-)commutators of the $Osp(2|4)$ superalgebra are:

$$
\begin{aligned}
[Q^{A\alpha}, T] &= \epsilon^{AB}Q^{B\alpha} \\
[M_{MN}, M_{PQ}] &= -\eta_{MP}M_{NQ} - \eta_{NQ}M_{MP} + \eta_{MQ}M_{NP} + \eta_{NP}M_{MQ} \\
[Q^{A\alpha}, M_{MN}] &= \frac{1}{2}(\hat{\gamma}_{MN})^{\alpha}{}_{\beta}Q^{A\beta} \\
\{Q^{A\alpha}, Q^{B\beta}\} &= \delta^{AB}(\hat{\gamma}^{MN}C^{-1})^{\alpha\beta}M_{MN} - (C^{-1})^{\alpha\beta}T\epsilon^{AB},
\end{aligned}
\tag{4.1}
$$

where $\eta_{MN} = \text{diag}(1, 1, -1, -1, -1)$, the gamma matrices are $\hat{\gamma}_M \equiv \{\gamma_5, i\gamma_\mu\gamma_5\}$, and $\hat{\gamma}_{MN} = \frac{1}{2}[\hat{\gamma}_M, \hat{\gamma}_N]$. Further details can be found in [26]. T does not have the role of a central charge, as it doesn't commute with the supercharges. Nevertheless it is associated to the electric charge.[9] The isometry group of AdS$_4$ is SO(2, 3), isomorphic to the conformal group in three dimensions, whose generators are 3 translations, 3 rotations, 3 special conformal transformations (conformal boosts) and the dilatation.

4.2 mAdS$_4$

In the case of mAdS$_4$, the symmetry group is reduced due to the presence of the magnetic charge. Spatial translations and boosts are broken, because of the presence of a magnetic monopole. There are 4 Killing vectors related to the invariance under time translations and rotations. The isometry group of this spacetime is then $\mathbb{R}$ × SO(3). Furthermore, we have also gauge invariance. The projector (3.28) reduces the independent components of the Killing spinors to 1/4, consequently the number of fermionic symmetries of the theory is reduced, as we explained in section 3.2. We have denoted the remaining two real supercharges with Q^I ($I = 1, 2$). To sum up, the symmetry generators of mAdS$_4$ are:

- the angular momentum J_i, $i = 1, 2, 3$,

- the Hamiltonian H,

---

[9]If we perform a Wigner-Inönü contraction of the algebra, T^{AB} gives rise to a central charge in the Poincaré superalgebra. See [26] for further details.

- the gauge transformation generator T,

- the two supercharges Q^I where $I = 1, 2$.

From (3.33) the anticommutator between two supercharges is

$$\{Q^I, Q^J\} = H\delta^{IJ} \ . \tag{4.2}$$

Since mAdS$_4$ is static and spherically symmetric, we have the commutation relations

$$[H, J_i] = 0 \ , \qquad [J_i, J_j] = \epsilon_{ijk} J_k \ . \tag{4.3}$$

The following commutators are then determined by imposing the Jacobi identities:

$$[Q^I, J_i] = [Q^I, H] = 0 \ . \tag{4.4}$$

Next, we add the gauge generator T to the algebra. Because of gauge invariance, we have the commutators

$$[T, J_i] = [T, H] = 0 \ . \tag{4.5}$$

From the Jacobi identities one now derives that

$$[Q^I, T] = \epsilon^{IJ} Q^J \ , \tag{4.6}$$

with a fixed normalization of T. This commutator also follows from the observation that gauge transformations act on the supersymmetry parameters in gauged supergravity, together with the fact that T commutes with the projection operator P defined in the previous section.

The first commutator in (4.4) implies that the supercharges Q^I are singlet under rotations. This is a consequence of the fact that the mAdS$_4$ Killing spinors have no angular dependence [2]. Group theoretically, this follows from the fact that the group of rotations entangles with the $SU(2)_R$ symmetry, as explained in [27].

5 Outlook

The procedure outlined in this paper to compute the BPS bound is completely general. It can be used also in other settings, for example for gauged supergravity with matter couplings. This would give new insights about the BPS ground states of the gauged supergravities. For instance, it would be interesting to investigate the BPS bounds in the presence of scalar fields that belong to vector- or hypermultiplets.

One disadvantage of our approach is that our renormalized mass formulas (3.6) and (3.31) are written in specific coordinates and therefore not manifestly diffeomorphism invariant. It would be an improvement if a coordinate independent formulation can be given, and comparison can be made to other proposals in the literature, such as [9, 10, 22, 28, 29].

Furthermore, it would be worth understanding the implications related to the AdS/CFT correspondence, especially in the presence of a magnetic charge. For instance, the fact that the supercharges carry no spin might have important consequences for the dual field theory. We leave this for further study.

Acknowledgments

A   Gamma matrix conventions

The Dirac gamma-matrices in four dimensions satisfy

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}\,, \tag{A.1}$$

and we define

$$[\gamma_a, \gamma_b] \equiv 2\gamma_{ab}\,, \qquad \gamma_5 \equiv -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{A.2}$$

In addition, they can be chosen such that

$$\gamma_0^\dagger = \gamma_0\,, \quad \gamma_0\gamma_i^\dagger\gamma_0 = \gamma_i\,, \quad \gamma_5^\dagger = \gamma_5\,, \quad \gamma_a^* = -\gamma_a\,. \tag{A.3}$$

An explicit realization of such gamma matrices is the Majorana basis, given by

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, \tag{A.4}$$

where the σ^i $(i = 1, 2, 3)$ are the Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{A.5}$$

For the charge conjugation matrix, we choose

$$C = i\gamma^0\,, \tag{A.6}$$

hence Majorana spinors have real components.

We also make use of the following identities, with curved indices:

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\rho = ie\gamma^{\mu\nu\sigma}\,, \tag{A.7}$$

$$\gamma_\mu\gamma_{\rho\sigma} = -\gamma_\rho g_{\mu\sigma} + \gamma_\sigma g_{\mu\rho} + \frac{i}{e}\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\nu\,, \tag{A.8}$$

$$\gamma_\mu\gamma_{\nu\rho}\gamma_\sigma - \gamma_\sigma\gamma_{\nu\rho}\gamma_\mu = 2g_{\mu\nu}g_{\rho\sigma} - 2g_{\mu\rho}g_{\nu\sigma} + 2\frac{i}{e}\epsilon_{\mu\nu\rho\sigma}\gamma_5\,. \tag{A.9}$$

Antisymmetrizations are taken with weight one half, and the totally antisymmetric Levi Civita symbol is defined by

$$\epsilon^{0123} = 1 = -\epsilon_{0123} . \tag{A.10}$$

With curved indices,

$$\epsilon^{\mu\nu\rho\sigma} \equiv e \, e_a^\mu e_b^\nu e_c^\rho e_d^\sigma \, \epsilon^{abcd} , \tag{A.11}$$

is a tensor-density.

Another important property that ensures the super-Jacobi identities of Osp(2|4) hold is

$$(\hat\gamma^{MN} C^{-1})^{\alpha\beta} (\hat\gamma_{MN} C^{-1})^{\gamma\delta} = (C^{-1})^{\alpha\gamma}(C^{-1})^{\beta\delta} + (C^{-1})^{\alpha\delta}(C^{-1})^{\beta\gamma} , \tag{A.12}$$

where $\hat\gamma_{MN}$ are defined in section 4.1.

B Asymptotic Killing spinors

B.1 AdS$_4$

Here we give details about the Killing spinors for AdS$_4$. We consider the metric in spherical coordinates

$$ds^2 = (1 + g^2 r^2) \, dt^2 - (1 + g^2 r^2)^{-1} \, dr^2 - r^2 \, (d\theta^2 + \sin^2\theta d\varphi^2) , \tag{B.1}$$

and corresponding vielbein

$$e_\mu^a = \text{diag}\left(\sqrt{1 + g^2 r^2}, \sqrt{1 + g^2 r^2}^{-1}, r, r\sin\theta \right) . \tag{B.2}$$

The non-vanishing components of the spin connection turn out to be:

$$\omega_t^{01} = g^2 r, \quad \omega_\theta^{12} = -\sqrt{1 + g^2 r^2}, \quad \omega_\varphi^{13} = -\sqrt{1 + g^2 r^2}\sin\theta, \quad \omega_\varphi^{23} = -\cos\theta . \tag{B.3}$$

For the AdS$_4$ solution, the field strength vanishes,

$$F_{\mu\nu} = 0 . \tag{B.4}$$

To find the Killing spinors corresponding to this spacetime we need to solve $\widetilde{\mathcal{D}}_\mu \epsilon = 0$. This equation has already been solved in [8] and we have explicitly checked that the resulting Killing spinors are given by

$$\epsilon_{\text{AdS}} = e^{\frac{i}{2}\text{arcsinh}(gr)\gamma_1} e^{\frac{i}{2}gt\gamma_0} e^{-\frac{1}{2}\theta\gamma_{12}} e^{-\frac{1}{2}\varphi\gamma_{23}} \epsilon_0 , \tag{B.5}$$

where ϵ_0 is a doublet of arbitrary constant Majorana spinors, representing the eight preserved supersymmetries of the configuration.

It is important to note that the asymptotic solution of the Killing spinor equations as $r \to \infty$ (given the same asymptotic metric) cannot change unless $A_\varphi \neq 0$. This is easy to see from the form of the supercovariant derivative (2.9) since any other term would necessarily vanish in the asymptotic limit. More precisely, any gauge field carrying an electric charge that appears in the derivative vanishes asymptotically, the only constant contribution can come when a magnetic charge is present. In other words, any spacetime with vanishing magnetic charge and asymptotic metric (B.1) has asymptotic Killing spinors given by (B.5).

B.2 Magnetic AdS$_4$

Now we will show that the asymptotic Killing spinors take a very different form when magnetic charge is present. In this case the metric is

$$ds^2 = \left(1 + g^2 r^2 + \frac{Q_m^2}{r^2}\right) dt^2 - \left(1 + g^2 r^2 + \frac{Q_m^2}{r^2}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad \text{(B.6)}$$

with corresponding vielbein:

$$e_\mu^a = \text{diag}\left(\sqrt{1 + g^2 r^2 + \frac{Q_m^2}{r^2}}, \sqrt{1 + g^2 r^2 + \frac{Q_m^2}{r^2}}^{-1}, r, r\sin\theta\right). \quad \text{(B.7)}$$

The non-vanishing components of the spin connection turn out to be:

$$\omega_t^{01} = g^2 r - \frac{Q_m^2}{r^3}, \qquad\qquad \omega_\theta^{12} = -\sqrt{1 + g^2 r^2 + \frac{Q_m^2}{r^2}},$$

$$\omega_\varphi^{13} = -\sqrt{1 + g^2 r^2 + \frac{Q_m^2}{r^2}}\sin\theta, \qquad \omega_\varphi^{23} = -\cos\theta. \quad \text{(B.8)}$$

As opposed to the previous section, now we have a non-vanishing gauge field component $A_\varphi = -Q_m\cos\theta$, resulting in $F_{\theta\varphi} = Q_m\sin\theta$. If we require $\widetilde{\mathcal{D}}_\mu \epsilon = 0$ and insist that $Q_m \neq 0$, we get a solution described by Romans in [2] as a "cosmic monopole" (which we call magnetic AdS$_4$). The magnetic charge satisfies $2gQ_m = \pm 1$, such that the metric function is an exact square $(gr + \frac{1}{2gr})^2$. The Killing spinors corresponding to solutions with $Q_m = \pm 1/(2g)$ in our conventions are given by

$$\epsilon_{\text{mAdS}} = \frac{1}{4}\sqrt{gr + \frac{1}{2gr}}(1 + i\gamma_1)(1 \mp i\gamma_{23}\sigma^2)\epsilon_0, \quad \text{(B.9)}$$

preserving two of the original eight supersymmetries. Note that in the limit $r \to \infty$ the Killing spinor projections continue to hold. Furthermore, the functional dependence is manifestly different in the expressions (B.5) and (B.9) for the Killing spinors of ordinary AdS$_4$ and its magnetic version. This leads to the conclusion that these two vacua and their corresponding excited states belong to two separate classes, i.e. they lead to two independent superalgebras and BPS bounds. Note that one can also add an arbitrary electric charge Q_e to the above solution, preserving the same amount of supersymmetry (the "cosmic dyon" of [2]). The corresponding Killing spinors [2] have the asymptotic form of (B.9), i.e. the cosmic dyons are asymptotically magnetic AdS$_4$.

C Rotations in AdS$_4$

Here we focus on stationary spacetimes with rotations. From thesupersymmetry Dirac brackets in asymptotic AdS$_4$ spaces,

$$\{\mathcal{Q}, \mathcal{Q}\} = -8\pi i\bar\epsilon_0 \left(\ldots + gJ_{ij}\gamma^{ij} + \ldots\right)\epsilon_0, \quad \text{(C.1)}$$

we can derive a definition of the conserved angular momenta. The explicit expressions are somewhat lengthy and assume a much simpler form once we choose the vielbein matrix

e_μ^a in an upper triangular form, such that its inverse e_a^μ is also upper triangular. More explicitly, in spherical coordinates we choose nonvanishing $e_t^{0,1,2,3}, e_r^{1,2,3}, e_\theta^{2,3}, e_\varphi^3$, such that the inverse vielbein has only non-vanishing components $e_0^{t,r,\theta,\varphi}, e_1^{r,\theta,\varphi}, e_2^{\theta,\varphi}, e_3^\varphi$. The resulting expressions for the angular momenta in this case become:

$$J_{12} = \frac{1}{8\pi} \lim_{r\to\infty} \int\limits_0^{2\pi} \mathrm{d}\varphi \int\limits_0^\pi \mathrm{d}\theta \left((e_0^t e_1^r e_\varphi^3 \omega_\theta^{01} + e_0^t e_1^r e_\theta^2 e_\varphi^3 e_2^\varphi \omega_\varphi^{01}) r\cos\varphi + (e_0^t e_1^r e_\theta^2 \omega_\varphi^{01}) r\cos\theta\sin\varphi \right) ,$$

$$J_{13} = \frac{1}{8\pi} \lim_{r\to\infty} \int\limits_0^{2\pi} \mathrm{d}\varphi \int\limits_0^\pi \mathrm{d}\theta \left((e_0^t e_1^r e_\varphi^3 \omega_\theta^{01} + e_0^t e_1^r e_\theta^2 e_\varphi^3 e_2^\varphi \omega_\varphi^{01}) r\sin\varphi + (e_0^t e_1^r e_\theta^2 \omega_\varphi^{01}) r\cos\theta\cos\varphi \right) ,$$

$$J_{23} = \frac{1}{8\pi} \lim_{r\to\infty} \int\limits_0^{2\pi} \mathrm{d}\varphi \int\limits_0^\pi \mathrm{d}\theta \left(e_0^t e_1^r e_\theta^2 \omega_\varphi^{01} r\sin\theta \right) . \tag{C.2}$$

It is easy to see that in case of axisymmetric solutions around φ, such as the Kerr and Kerr-Newman metrics in AdS, the angular momenta J_{12} and J_{13} automatically vanish due to $\int_0^{2\pi} \mathrm{d}\varphi \sin\varphi = \int_0^{2\pi} \mathrm{d}\varphi \cos\varphi = 0$.

One can then use the formula for J_{23} to derive the value of the angular momentum for the Kerr black hole. This is still somewhat non-trivial because one needs to change the coordinates from Boyer-Lindquist-type to spherical. The leading terms at large r were found in appendix B of [22] and are enough for the calculation of the angular momentum since subleading terms vanish when the limit is taken in (C.2). The calculation of the relevant component of the spin connection leads to

$$\omega_\varphi^{01} = -\frac{3am\sin^2\theta(1 - g^2 a^2 \sin^2\theta)^{-5/2}}{r^2} + \mathcal{O}(r^{-3}) \tag{C.3}$$

and gives the exact same result as in (B.8) of [22],

$$J_{23} = \frac{am}{(1 - g^2 a^2)^2} . \tag{C.4}$$

This expression has also been derived from different considerations in [30], thus confirming the consistency of our results.

One can also verify the result for the asymptotic mass of the Kerr and Kerr-Newman spacetimes using (3.6) and the metric in appendix B of [22]. After a somewhat lengthy but straightforward calculation one finds

$$M = \frac{m}{(1 - g^2 a^2)^2} , \tag{C.5}$$

as expected from previous studies (see, e.g., [13, 30]).

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