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Schoen manifold with line bundles as resolved magnetized orbifolds

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Abstract: We give an alternative description of the Schoen manifold as the blow-up of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold in which one $\mathbb{Z}_2$ factor acts as a roto-translation. Since for this orbifold the fixed tori are only identified in pairs but not orbifolded, four-dimensional chirality can never be obtained in heterotic string compactifications using standard techniques alone. However, chirality is recovered when its tori become magnetized. To exemplify this, we construct an $E_8 \times E_8{}'$ heterotic SU(5) GUT on the Schoen manifold with Abelian gauge fluxes, which becomes an MSSM with three generations after an appropriate Wilson line is associated to its freely acting involution. We reproduce this model as a standard heterotic orbifold CFT of the (partially) blown down Schoen manifold with a magnetic flux. Finally, in analogy to a proposal for non-perturbative heterotic models by Aldazabal et al. we suggest modifications to the heterotic orbifold spectrum formulae in the presence of magnetized tori.

Keywords: Flux compactifications, Superstrings and Heterotic Strings, Conformal Field Models in String Theory

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1 Introduction and summary

There are two standard approaches in the literature to geometrical string compactification of the heterotic string. (Non-geometrical approaches involve e.g. free-fermionic models [1, 2] and Gepner constructions [3, 4].) Either strings are considered on singular (toroidal) orbifolds or on smooth Calabi-Yau manifolds. The main advantage of orbifolds over smooth Calabi-Yau spaces is that they are so simple that the heterotic string can be quantized on them exactly [5, 6]. Therefore, one has access to the full spectrum of the theory; not just to its zero modes. In addition, one can scan in a very systematic way through the parameter space of heterotic orbifold compactifications in order to search for interesting models for

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string phenomenology (using e.g. [7]). This has resulted, for example, in a mini-landscape of a few hundred MSSM models based on the heterotic $E_8 \times E_8{}'$ orbifold $T^6/\mathbb{Z}_{6\text{-II}}$ [8, 9].

An orbifold can be considered as a Calabi-Yau space at a singular point in its moduli space where symmetries get enhanced. To go away from the orbifold point in moduli space the orbifold singularities have to be resolved (or deformed). In this blow-up process certain (exceptional) cycles that were hidden inside the singularities acquire finite volumes. From the heterotic orbifold model perspective this corresponds to turning on Vacuum-Expectation-Values (VEVs) for twisted states, so-called blow-up modes, which are localized at the singularities of the orbifold. Unfortunately, an exact string quantization is out of reach at a generic point in moduli space and there is typically much less symmetry. For example, it turns out that in full blow-up any mini-landscape model has broken hypercharge [10, 11]. This might be interpreted in two ways: Either one does not go to the full blow-up in order to keep hypercharge unbroken and our string vacuum is very close to the orbifold point, or our string vacuum is at a generic point of the moduli space and different constructions are needed for phenomenology. As discussed in [12, 13] freely acting involutions can be used as an example for the second interpretation and an MSSM orbifold model has been constructed on the heterotic $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, which in principle can avoid hypercharge breaking in full blow-up.

Furthermore, there have been various constructions of MSSM models in the context of the heterotic string compactified on smooth Calabi-Yau manifolds. For example, a three generation MSSM has been constructed in [14] on the Schoen manifold [15] using a stable SU(5) vector bundle [16–18]. Similar constructions — yet not fully supersymmetric [19] — can be found in e.g. [20]. Even though the Schoen manifold is just one particular Calabi-Yau space, it is a typical example of a complete intersection Calabi-Yau: It can be obtained as a set of hyper surfaces within a direct product of projective spaces.

Most heterotic models built on the Schoen manifold require complicated constructions of stable SU(N) bundles. Therefore, one may wonder whether it is also possible to design MSSM-like heterotic string models on the Schoen manifolds using line bundles. As has been realized by various groups [21, 22] the analysis of line bundles on smooth Calabi-Yau spaces, described as complete intersections in toric varieties, can be performed much easier than their non-Abelian counterparts. The main reason for this is that for line bundle gauge backgrounds the stability of the bundle reduces to solving simple Donaldson-Uhlenbeck-Yau (DUY) equations [23, 24] in terms of the Kähler moduli [25, 26]. Moreover, the embedding of line bundles into the ten-dimensional heterotic gauge group ($E_8 \times E_8{}'$ or SO(32), where we focus on the $E_8 \times E_8{}'$ case, but most of our results equally apply to the SO(32) case.) can be characterized by vectors of integers [27, 28]. This makes it possible to perform computer-aided scans for potentially phenomenologically viable models.

The Schoen manifold does not only provide an interesting example of a Calabi-Yau constructed as a complete intersection. It can also be considered as a smooth limit of a certain orbifold [29]. This orbifold has some special properties: It is a $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold, where $\mathbb{Z}_{2,\text{rototrans}}$ acts as a roto-translation, i.e. as a simultaneously performed rotation and translation [30] (and [31] in the type II string context, where this kind of orbifolds are called shift orbifolds). This has far reaching consequences for the structure

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of the fixed points and tori and, in turn, modifies the breaking of higher dimensional supersymmetry to $\mathcal{N} = 1$ in four dimensions for heterotic string compactifications. As we will see, this necessarily results in vector-like spectra for this kind of orbifold geometry.

This is not a peculiar feature of this special orbifold, many more orbifolds with this property are known. Recently, there has been a classification of all six-dimensional toroidal orbifold geometries that give rise to four-dimensional $\mathcal{N} = 1$ supersymmetry [32]. These geometries can be arranged in two sets: The ones with Abelian point group and the ones with non-Abelian point group. 23 of the 138 geometries with Abelian point group share the property that they necessarily lead to non-chiral spectra for heterotic string compactifications. (These are $\mathbb{Z}_2 \times \mathbb{Z}_2$ variants, part of the classification of ref. [29].) For the non-Abelian cases these numbers are essentially unknown. However, one explicit example of a heterotic S_3 orbifold [33] also turns out to produce only vector-like spectra. We therefore expect that also a sizable portion of the non-Abelian point group orbifolds will unavoidably be non-chiral in four dimensions.

Hence, it is an important question whether there exists an unavoidable no-go theorem against four-dimensional chirality for all these heterotic orbifolds. Fortunately, we will show that it is possible to circumvent this no-go by allowing for magnetized tori on the orbifold. Concretely, we put magnetic fluxes on the tori of the Schoen orbifold and show that four-dimensional chiral spectra can be realized. More than that, we will show that it is even possible to obtain MSSM-like models in this way.

There is one technical subtlety in the construction of such orbifolds with magnetized tori: As far as we know, contrary to conventional orbifolds, it is unknown how to quantize the heterotic string exactly on them. We by-pass this obstruction in two ways: First, we consider the whole construction in blow-up, i.e. on the smooth Schoen manifold. Second, we show that one can start with a six-dimensional spectrum obtained from a standard heterotic $T^4/\mathbb{Z}_2$ orbifold, which is a subspace of the partially blown-down Schoen manifold, using conventional CFT techniques. Then, one can use field theoretical methods, discussed e.g. in [34–36], to determine the consequences of the additional (magnetic) fluxes and to obtain a chiral spectrum in four dimensions. Both approaches, i.e. the smooth approach and the hybrid approach of combining CFT and field theoretical methods, will reproduce exactly the same spectrum.

Paper overview. In section 2 we review the basics of heterotic orbifold models. In addition we introduce the DW(0-2) orbifold which is of central interest in this work. Section 3 provides an alternative description of the Schoen manifold as the resolution of this DW(0-2) orbifold. In section 4 we describe the $\mathrm{E}_8 \times \mathrm{E}_8{}'$ heterotic string with line bundles on the divisors of the Schoen manifold including magnetic fluxes on the tori of the underlying orbifold. Moreover, we identify the relevant consistency conditions for such gauge backgrounds and compute the resulting chiral spectra in both, four and six, dimensions. Then, we provide an example that mainly serves to illustrate various aspects of the general theory developed in this paper. In section 5 we construct a specific example, which is potentially phenomenologically interesting as it has the particle spectrum of the MSSM in four dimensions. We analyze this example using two approaches: First, the smooth ap-

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proach and, second, the hybrid approach of combining CFT and field theoretical methods. Finally, in section 6 we speculate on how to extend the standard heterotic CFT description of orbifolds in the presence of magnetized tori.

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2 Heterotic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

In this section we describe some basic geometrical properties of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds and explain how to determine whether such orbifolds can lead to heterotic $(\mathrm{E}_8 \times \mathrm{E}_8{}')$ string models with chiral spectra in four dimensions. We follow the classification scheme for these orbifolds developed by Donagi-Wendland [29]. (See their table 1 for details and nomenclature). In particular, we describe their DW(0-2) orbifold which can be considered as a certain singular limit of the so-called Schoen manifold. However, for comparison purposes we first recall some basic facts of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds and give some details of the more often considered DW(0-1) orbifold.

2.1 General features of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

We consider $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds defined as

$$\mathbb{R}^6/S \tag{2.1}$$

where the *space group* S specifies an equivalence relation on $\mathbb{R}^6$ as $g\,X \sim X$ for all $g \in S$ and $X \in \mathbb{R}^6$. A general space group element $g = (\vartheta, \ell)$ consists of a six-dimensional rotation matrix ϑ and a translation ℓ. It acts on $X \in \mathbb{R}^6$ as $g\,X = \vartheta\,X + \ell$. The space group is generated by two types of elements: The purely translational elements $g_i = (\mathbb{1}, e_i)$ are determined by six basis vectors e_i $(i = 1, \ldots, 6)$ that span a six-dimensional lattice and hence define a six-torus. For simplicity, we identify $\mathbb{R}^6 = \mathbb{C}^3$ and take as basis vectors

$$\begin{aligned} e_1 &= (1,0,0)\,, & e_2 &= (i,0,0)\,, & e_3 &= (0,1,0)\,, \\ e_4 &= (0,i,0)\,, & e_5 &= (0,0,1)\,, & e_6 &= (0,0,i)\,. \end{aligned} \tag{2.2}$$

Consequently, we denote the torus coordinates by $z = (z_1, z_2, z_3) \in T_1^2 \times T_2^2 \times T_3^2$ in this complex basis. The remaining two generators of the space group, g_θ and g_ω, involve $\mathbb{Z}_2 \times \mathbb{Z}_2$ rotations, denoted by θ and ω, possibly combined with some translations. When this is the

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case such elements are referred to as *roto-translations*. The phases of the rotations acting on $\mathbb{C}^3$ are

$$v_\theta = \left(0, \frac{1}{2}, -\frac{1}{2}\right) , \quad \text{and} \quad v_\omega = \left(-\frac{1}{2}, 0, \frac{1}{2}\right) , \tag{2.3}$$

respectively.

The action of the space group elements is subsequently extended to the left-moving sector of the heterotic worldsheet theory that describes the target space gauge degrees of freedom. In a bosonic formulation this sector can be described by 16 left-moving coordinates X_L^I $(I = 1, \ldots, 16)$ living on a torus $\mathbb{R}^{16}/\Lambda_{\mathrm{E}_8\times\mathrm{E}_8{}'}$ defined by the $\mathrm{E}_8 \times \mathrm{E}_8{}'$ root lattice $\Lambda_{\mathrm{E}_8\times\mathrm{E}_8{}'}$. The simplest way to extend the space group action is the *shift embedding* which acts as: $g\, X_L^I = X_L^I + 2\pi\, V_g^I$. Hence $V : g \mapsto V_g$ defines a group homomorphism of the space group S to the Abelian group $\mathbb{R}^{16}$ under addition. For a general space group element $g = g_\theta^k\, g_\omega^l\, g_1^{n_1} \cdot \ldots \cdot g_6^{n_6}$, with $k, l = 0, 1$ and $n_i \in \mathbb{Z}$, the local twist v_g and shift vector V_g can be expanded as

$$v_g = k\, v_\theta + l\, v_\omega\,, \qquad V_g = k\, V_\theta + l\, V_\omega + n_i\, W_i \tag{2.4}$$

in terms of the gauge shift vectors V_θ and V_ω and the discrete Wilson lines W_i where summation over i from 1 to 6 is understood. In order that V_g defines a proper group homomorphism, it is required that

$$2\, V_\theta \cong 2\, V_\omega \cong 2\, W_i \cong 0\,, \tag{2.5}$$

where $\cong$ means equal up to $\Lambda_{\mathrm{E}_8\times\mathrm{E}_8{}'}$ lattice vectors.

The central consistency requirement of heterotic orbifold compactifications is modular invariance. For a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold it requires for all commuting space group elements h, g that

$$V_h \cdot V_g - v_h \cdot v_g \equiv 0\,, \tag{2.6}$$

where $\equiv$ indicates that both sides are equal up to integers. Combined with equation (2.5) this leads to the following set of irreducible modular invariance conditions:

$$V_\theta^2 \equiv v_\theta^2\,, \quad V_\omega^2 \equiv v_\omega^2\,, \quad V_\theta \cdot V_\omega \equiv v_\theta \cdot v_\omega\,, \quad V_\theta \cdot W_i \equiv V_\omega \cdot W_i \equiv 0\,, \quad W_i \cdot W_j \equiv 0\,, \tag{2.7}$$

by going through all possible commuting choices of $g, h \in S$.

The spectrum of (twisted or untwisted) closed strings from the sector $g \in S$ is dictated by their left- and right-moving masses

$$M_L^2 = \frac{1}{2}\, P_{\mathrm{sh}}^2 + \widetilde{N}_g - \frac{3}{4}\,, \qquad M_R^2 = \frac{1}{2}\, p_{\mathrm{sh}}^2 - \frac{1}{4}\,, \tag{2.8}$$

in terms of the (shifted) left- and right-moving momenta

$$P_{\mathrm{sh}} = P + V_g\,, \qquad p_{\mathrm{sh}} = p + v_g\,, \tag{2.9}$$

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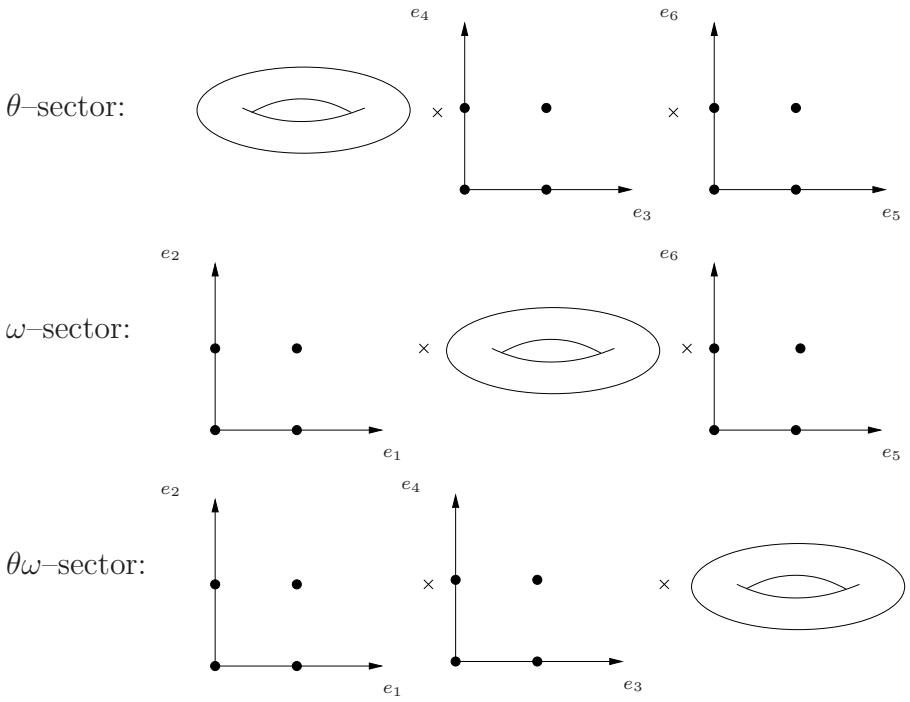

Figure 1. Fixed tori of the DW(0-1) orbifold. Every fixed torus intersects 4 + 4 other fixed tori and the intersection loci are points in six dimensions.

where $P \in \Lambda_{\mathrm{E}_8 \times \mathrm{E}_{8'}}$ and p is from the vectorial or spinorial weight lattice of SO(8). Here, the twist vector v_g is extended to a four-dimensional vector with an extra 0 as first component. Furthermore, $\widetilde{N}_g$ is a (fractional or integer) number operator counting the number of left-moving oscillators $\tilde{\alpha}_{-n}$ acting on the left-moving ground state of the g-twisted sector. The physical spectrum is subject to the level matching condition $M_L^2 = M_R^2$. The massless states in four dimensions have vanishing left- and right-moving masses, $M_L = M_R = 0$, and are subject to the projection conditions

$$V_h \cdot P_{\mathrm{sh}} - v_h \cdot \left(p_{\mathrm{sh}} + \Delta\tilde{N}_g\right) \equiv \frac{1}{2}\left(V_g \cdot V_h - v_g \cdot v_h\right) , \tag{2.10}$$

for all space group elements h that commute with g, using $\Delta\tilde{N}_g^i = \tilde{N}_g^{\bar{i}} - \tilde{N}_g^i$, $i = 0, 1, 2, 3$, where $\tilde{N}_g^{\bar{i}}$ and $\tilde{N}_g^i$ are integer oscillator numbers counting the numbers of oscillators $\tilde{\alpha}_{-n}^{\bar{i}}$ and $\tilde{\alpha}_{-n}^i$ acting on the ground state of the g-twisted sector, respectively.

2.2 The standard DW(0-1) $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Now, we consider the standard $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold which corresponds to the DW(0-1) model of ref. [29] in order to see how four-dimensional chirality arises. In this case, the elements g_θ and g_ω act only as rotations, hence the space group S is generated by the elements: $g_\theta = (\theta, 0)$, $g_\omega = (\omega, 0)$ and $g_i = (\mathbb{1}, e_i)$.

When compactifying the heterotic string on this orbifold, massless strings are attached to its fixed tori. There are 16 + 16 + 16 fixed tori associated to three twisted sectors with

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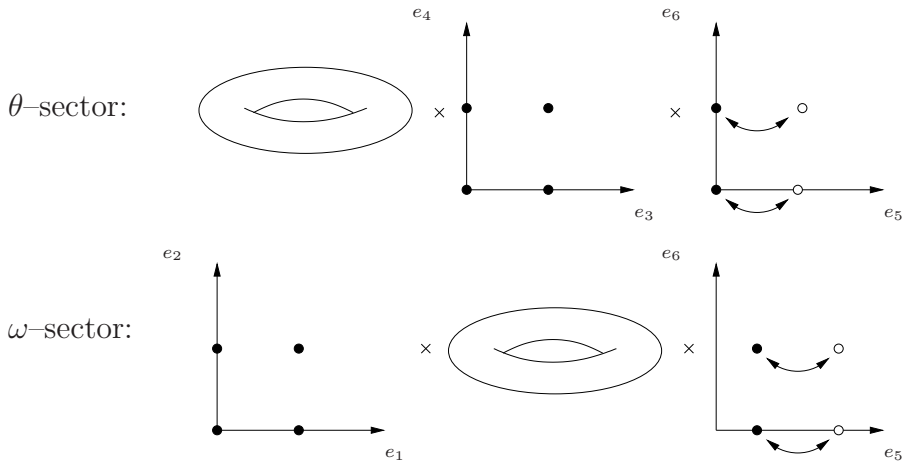

Figure 2. Fixed tori of the DW(0-2) orbifold. The fixed tori of the θ-sector never intersect the ones from the ω-sector, as they lie displaced in the third torus.

orbifold elements g_θ, g_ω and $g_\theta g_\omega$. These fixed tori are in one-to-one correspondence to the space group elements:

$$(\theta, n_i e_i) \text{ for } n_1 = n_2 = 0 \text{ and } n_3, n_4, n_5, n_6 = 0, 1\,, \tag{2.11a}$$

$$(\omega, n_i e_i) \text{ for } n_3 = n_4 = 0 \text{ and } n_1, n_2, n_5, n_6 = 0, 1\,, \tag{2.11b}$$

$$(\theta\omega, n_i e_i) \text{ for } n_5 = n_6 = 0 \text{ and } n_1, n_2, n_3, n_4 = 0, 1\,, \tag{2.11c}$$

and are displayed in figure 1. At a given fixed torus there exists a six-dimensional $\mathcal{N} = 1$ theory (i.e. $\mathcal{N} = 2$ theory in four-dimensional language) with localized hypermultiplets on it. Since every fixed torus intersects other fixed tori, six-dimensional $\mathcal{N} = 1$ supersymmetry is broken to $\mathcal{N} = 1$ in four dimensions at the intersection points. Technically, each fixed torus is orbifolded by the action of some other non-trivial elements because the orbifold generators g_θ and g_ω commute. For example, the fixed torus of $(\theta, 0)$ is orbifolded by $(\omega, 0)$ and $(\theta\omega, 0)$. Hence, the projection conditions (2.10) are active and reduce a hypermultiplet in six dimensions to a four-dimensional chiral superfield, giving chiral matter.

For example, in the orbifold standard embedding, where the twists θ and ω are embedded via the shifts $V_\theta = \left(0, \frac{1}{2}, \text{-}\frac{1}{2}, 0^5\right)\left(0^8\right)$ and $V_\omega = \left(\text{-}\frac{1}{2}, 0, \frac{1}{2}, 0^5\right)\left(0^8\right)$, we obtain a theory with 51 chiral **27**-plets (3 untwisted and $3 \cdot 16$ twisted) and 3 untwisted chiral $\overline{\mathbf{27}}$-plets of E_6 in four dimensions. This precisely corresponds to the hodge numbers of the DW(0-1) orbifold: $(h_{11}, h_{21}) = (51, 3)$.

2.3 The DW(0-2) $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Next, we turn to the orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ which will be the main focus in this work: the DW(0-2) model in the classification [29]. Its space group S is generated by the elements $g_\theta = \left(\theta, 0\right)$, $g_\omega = \left(\omega, \frac{1}{2}e_5\right)$ and $g_i = \left(\mathbb{1}, e_i\right)$. In detail, the action of g_θ, g_ω, $g_\omega g_\theta$ and $g_\theta g_\omega$

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on $\mathbb{C}^3$ is given by

$$\begin{aligned} g_\theta &: (z_1, z_2, z_3) = (z_1, -z_2, -z_3)\,, & g_\omega g_\theta &: (z_1, z_2, z_3) = \left(-z_1, -z_2, z_3 + \frac{1}{2}\right), \\ g_\omega &: (z_1, z_2, z_3) = \left(-z_1, z_2, -z_3 + \frac{1}{2}\right), & g_\theta g_\omega &: (z_1, z_2, z_3) = \left(-z_1, -z_2, z_3 - \frac{1}{2}\right). \end{aligned} \tag{2.12}$$

This shows explicitly that g_θ acts as an ordinary $\mathbb{Z}_2$ rotation, while g_ω defines a $\mathbb{Z}_2$ roto-translation, which we denote by $\mathbb{Z}_{2,\text{rototrans}}$. The remaining two elements act as an $\mathbb{Z}_2$ in the first two two-tori, but as a translation over half a lattice vector e_5 in the third two-torus.

This has various important consequences for the distribution of fixed tori within the DW(0-2) orbifold: Rather than 16+16+16 there are only 8+8 fixed tori. These are left fixed by the group elements g_θ and g_ω. Since $g_\theta g_\omega$ and $g_\omega g_\theta$ act as a pure translations in the third T^2, they do not produce any fixed tori by themselves, but rather identify the fixed tori of g_θ and g_ω in pairs. The fixed tori of g_θ and g_ω are in one-to-one correspondence to the space group elements,

$$g_r = \left(\theta, n_i e_i\right) \qquad \text{for} \quad n_1 = n_2 = 0 \text{ and } n_3, n_4, n_6 = 0, 1\,, \tag{2.13a}$$

$$g'_{r'} = \left(\omega, \frac{1}{2}e_5 + n_i e_i\right) \qquad \text{for} \quad n_3 = n_4 = 0 \text{ and } n_1, n_2, n'_6 = 0, 1\,. \tag{2.13b}$$

We will often refer to the positions $\left(z_1, \frac{1}{2}n_3 + \frac{i}{2}n_4, \frac{i}{2}n_6\right)$ and $\left(\frac{1}{2}n_1 + \frac{i}{2}n_2, z_2, \frac{1}{4} + \frac{i}{2}n'_6\right)$ of the fixed tori of g_θ and g_ω using multi-indices $r = (n_3, n_4, n_6)$ and $r' = (n_1, n_2, n'_6)$, respectively. As illustrated in figure 2 the fixed tori r and r' lie parallel to each other in the third T^2. To emphasize this important fact we use the primes on r' and n'_6 to signal that the 8 fixed points r' are shifted by $\frac{1}{4}e_5 = (0, 0, \frac{1}{4})$ in the third torus w.r.t. the fixed points r.

When compactifying the heterotic string on this orbifold, massless strings are attached to these fixed tori as for the DW(0-1) orbifold above. However, since in this case the fixed tori do not intersect, there are no projections acting locally on the six-dimensional $\mathcal{N} = 1$ theory. (Or more technically, even though θ and ω commute,the space group elements g_θ and g_ω do not, since they are defined to act on $\mathbb{R}^6$ not on T^6, see (2.12). Hence, the projection condition (2.10) is not implemented for these elements.) Nevertheless, from a four-dimensional point-of-view supersymmetry is broken to $\mathcal{N} = 1$: There are two six-dimensional theories living on the fixed tori of the g_θ and g_ω sectors which realize different six-dimensional $\mathcal{N} = 1$ supersymmetries. Hence, in the effective four-dimensional theory only $\mathcal{N} = 1$ remains. As this provides an example of non-local supersymmetry breaking, the six-dimensional hypermultiplets just branch into two chiral multiplets and consequently the resulting four-dimensional theory is necessarily non-chiral. Concretely, for the orbifold standard embedding we find for this orbifold $3 + 2 \cdot 8 = 19$ chiral **27**-plets and $3 + 2 \cdot 8 = 19$ chiral $\overline{\mathbf{27}}$-plets of E_6, i.e. the Hodge numbers are $(19, 19)$. In fact, because supersymmetry is broken non-locally, any DW(0-2) orbifold is non-chiral, independently of the choice of shifts and Wilson lines.

Because of this, it would seem that this type of orbifold can never be relevant for four-dimensional model building. One of the main messages of this paper is that one should

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not discard such orbifolds for phenomenology just yet. In fact, as shown in section 5 it is possible to construct an explicit six generation SU(5) GUT model on the resolution of this orbifold. To break the GUT to the MSSM and to reduce the number of generations by a factor two one can use a (true field-theoretical) Wilson line W_{free} associated to a free involution $\mathbb{Z}_{2,\text{free}}$ of the geometry. In terms of the complex coordinates we take this involution to act as

$$(z_1, z_2, z_3) \to \left(z_1 + \frac{i}{2}, z_2 + \frac{i}{2}, z_3 + \frac{i}{2}\right) . \tag{2.14}$$

When this involution has been modded out the resulting geometry corresponds to the DW(1-3) orbifold with Hodge numbers $(11, 11)$ in the classification [29]. In order that this is a symmetry of the full model, the discrete Wilson lines get severely restricted, i.e. $W_2 \cong W_4 \cong W_6$ and $W_{\text{free}} \cong \frac{1}{2}W_2$.

3 Schoen manifold

The *Schoen manifold* X was first introduced in [15]; here we follow the description of this manifold given e.g. in [16–18, 37]. The Schoen manifold is defined as the fiber product

$$X = B \times_{\mathbb{P}^1} B' , \tag{3.1}$$

of two (four-dimensional) rational elliptic surfaces B and B'. Hence, the manifold X is naturally equipped with two projections $\pi' : X \to B$ and $\pi : X \to B'$ that project on either factor of the fiber product. Such a *rational elliptic surface* B is defined as a two-torus fibration $\beta : B \to \mathbb{P}^1$ over the base $\mathbb{P}^1$. In terms of the fibrations β and β' the fiber product (3.1) is written as

$$X := \left\{(p, p') \in B \times B' \mid \beta(p) = \beta'(p')\right\} . \tag{3.2}$$

A two-torus can be described as an elliptic curve, i.e. via the Weierstrass mapping as the solution to an homogeneous cubic polynomial constraint $f(x) = 0$ in the homogeneous coordinates $x = (x_0, x_1, x_2) \in \mathbb{P}^2$. The complex structure of the torus is encoded in the constraint $f(x) = 0$: A homogeneous cubic polynomial is characterized by $3 + 6 + 1 = 10$ complex parameters, of which eight can be removed by complexified SU(3) rotations of x and the overall complex scale is irrelevant. Because of the fibration the complex structure in general varies over the base $\mathbb{P}^1$. Therefore, the rational elliptic surface B can be given by

$$B = \begin{matrix}\mathbb{P}^2 \\ \mathbb{P}^1\end{matrix}\begin{bmatrix}3 \\ 1\end{bmatrix} \quad : \qquad B = \left\{p = (x, t) \in (\mathbb{P}^2, \mathbb{P}^1) \mid t_0\, f_0(x) - t_1\, f_1(x) = 0\right\} . \tag{3.3}$$

The surface B can thus be considered as the blow-up of $\mathbb{P}^2$ at (generically) $3 \cdot 3 = 9$ points x where both cubic polynomials vanish simultaneously $f_0(x) = f_1(x) = 0$. Since at each of these points an exceptional cycle $\mathbb{P}^1$ is inserted, the cohomology group $H^2(B, \mathbb{Z}) = \mathbb{Z}^{10}$ is spanned by the hyper plane class ℓ of $\mathbb{P}^2$ and the exceptional classes e_ρ $(\rho = 1, \ldots, 9)$ with intersection numbers $\ell^2 = -e_\rho^2 = 1$. This surface has $c_1(B) = 3\,\ell - \sum_\rho e_\rho$ and Euler number $\chi(B) = c_2(B) = \chi(\mathbb{P}^2) + 9 \cdot \chi(\mathbb{P}^1) = 12$.

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As the description of B' is similar, the manifold X can be written as a complete intersection Calabi-Yau

$$X = \begin{matrix}\mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^1\end{matrix}\begin{bmatrix}3 & 0 \\ 0 & 3 \\ 1 & 1\end{bmatrix} \quad : \quad \begin{cases} t_0\, f_0(x) - t_1\, f_1(x) = 0 \\ t_0\, f_0'(x') - t_1\, f_1'(x') = 0 \end{cases} . \tag{3.4}$$

Clearly, the Calabi-Yau condition is satisfied as the horizontal sums of the degrees of the homogeneous polynomials are one higher than the dimension of the respective projective spaces. In this representation the number of complex structure deformations is readily counted: There are 4 cubic polynomials, $f_0(x), f_1(x), f_0'(x'), f_1'(x')$, each containing 10 complex parameters. By redefinitions of x, x' and t one can remove $2 \cdot 8 + 3$ of them and two overall complex scales are irrelevant, hence $h_{12} = 4 \cdot 10 - 2 \cdot 8 - 3 - 2 = 19$. On both B and B' there are 10 $(1,1)$-forms corresponding to the classes ℓ, e_ρ and ℓ', e'_ρ, respectively. However, since $\beta(p) = \beta'(p')$ there is one linear relation among them. Consequently, the number of Kähler deformation equals $h_{11} = 2 \cdot 10 - 1 = 19$. In summary the Hodge numbers of X are $(19, 19)$.

3.1 Singular Schoen manifold: the DW(0-2) orbifold

Putting the discussion above and the information obtained in subsection 2.3 together suggests that the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,rototrans}$ orbifold can be considered as a specific singular limit of the Schoen manifold, since their Hodge numbers agree. Indeed, by the following considerations this can be confirmed [29]: On the covering six-torus coordinates z the projects π' and π act as $\pi'(z) = (z_1, z_3)$ and $\pi(z) = (z_2, z_3)$, hence the rational elliptic surfaces B and B' are given in this singular limit as

$$B = \left\{ (z_1, z_3) \in T_1^2 \times T_3^2 \mid (z_1, z_3) \sim (z_1, -z_3) \sim \left(-z_1, \frac{1}{2} - z_3 \right) \sim \left(-z_1, z_3 + \frac{1}{2} \right) \right\}, \tag{3.5a}$$

$$B' = \left\{ (z_2', z_3') \in T_2^2 \times T_3^2 \mid (z_2', z_3') \sim \left(z_2', \frac{1}{2} - z_3' \right) \sim (-z_2', -z_3') \sim \left(-z_2', z_3' + \frac{1}{2} \right) \right\}. \tag{3.5b}$$

These spaces are isomorphic: By applying a change of coordinates $(z_2', z_3') = (z_1, z_3 + \frac{1}{4})$ the identifications in B become identical to those in B'. The $\mathbb{P}^1$ in the fiber product, which has the topology of a two-sphere, becomes a rectangular pillow

$$\mathbb{P}^1 = \left\{ (z_3) \in T_3^2 \mid (z_3) \sim (-z_3) \sim \left(\frac{1}{2} - z_3 \right) \right\}. \tag{3.6}$$

3.2 Schoen manifold as the resolution of the DW(0-2) orbifold

As has been explained e.g. in [13, 38, 39] one can construct the smooth resolutions of compact orbifolds in a systematic fashion. In particular, it is possible to determine a convenient basis of divisors and the intersection numbers of the resolution of the orbifold

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$T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$. Hence, when applying these methods to the DW(0-2) orbifold one obtains a smooth Calabi-Yau space which constitutes a different realization of the Schoen manifold. As we will use this description of the Schoen manifold in the remainder of this paper, we describe this procedure in some detail below.

Intersection ring of a basis of divisors. The *inherited divisors* R_a $(a = 1, 2, 3)$ correspond to the torus divisors $\{z_a = c_a\}$ with c_a some generic complex numbers that are made compatible with the orbifold action:

$$\begin{aligned} R_1 &:= \{z_1 = c_1\} \cup \{z_1 = -c_1\}\,, \qquad R_2 := \{z_2 = c_2\} \cup \{z_2 = -c_2\}\,, \\ R_3 &:= \{z_3 = c_3\} \cup \{z_3 = -c_3\} \cup \left\{z_3 = \tfrac{1}{2} + c_3\right\} \cup \left\{z_1 = \tfrac{1}{2} - c_1\right\}, \end{aligned} \tag{3.7}$$

Next we define the *ordinary divisors* associated to the local coordinates defined near the fixed points of the orbifold

$$\begin{aligned} D_{1,n_1 n_2} &:= \left\{z_1 = \frac{1}{2}\, n_1 + \frac{i}{2}\, n_2\right\}, \\ D_{2,n_3 n_4} &:= \left\{z_2 = \frac{1}{2}\, n_3 + \frac{i}{2}\, n_4\right\}, \\ D_{3,n_6} &:= \left\{z_3 = \frac{i}{2}\, n_6\right\} \cup \left\{z_3 = \frac{1}{2} + \frac{i}{2}\, n_6\right\}, \\ D'_{3,n'_6} &:= \left\{z_3 = \frac{1}{4} + \frac{i}{2}\, n'_6\right\} \cup \left\{z_3 = \frac{3}{4} + \frac{i}{2}\, n'_6\right\}. \end{aligned} \tag{3.8}$$

Finally, the *exceptional divisors*, denoted by $E'_{r'}$ and E_r, arise when we resolve the $\mathbb{Z}_2$ singularities using e.g. toric geometry techniques. Since the fixed tori are identified in pairs in the DW(0-2) orbifold, we can represent them in terms of the exceptional divisors in the fundamental domain of the original T^6 (before modding out this identification):

$$E_r = E_{n_3 n_4 n_6} = E_{n_3 n_4 0 n_6} \cup E_{n_3 n_4 1 n_6}\,, \qquad E'_{r'} = E'_{n_1 n_2 n'_6} = E_{n_1 n_2 0 n'_6} \cup E_{n_1 n_2 1 n'_6}\,. \tag{3.9}$$

These 8+8 exceptional divisors stem from the 8+8 fixed tori displayed in the figure 2.

Between these divisors the following linear equivalence relations hold:

$$\begin{aligned} 2\, D_{1,n_1 n_2} &= R_1 - \sum_{n'_6} E'_{n_1 n_2 n'_6}\,, \qquad 2\, D'_{3,n'_6} = R_3 - \sum_{n_1, n_2} E'_{n_1 n_2 n'_6}\,, \\ 2\, D_{2,n_3 n_4} &= R_2 - \sum_{n_6} E_{n_3 n_4 n_6}\,, \qquad 2\, D_{3,n_6} = R_3 - \sum_{n_3, n_4} E_{n_3 n_3 n_6}\,. \end{aligned} \tag{3.10}$$

These linear equivalence relations show that a basis for the $H_2(X, \mathbb{R})$ are formed by the divisors R and E. (In total we have $3 + 2 \cdot 8 = 19$ of them.) In this basis the Schoen manifold has the following non-vanishing self-intersections between these divisors:

$$R_1 R_2 R_3 = 4\,, \qquad R_2 (E'_{n_1 n_2 n'_6})^2 = R_1 (E_{n_3 n_4 n_6})^2 = -4\,. \tag{3.11}$$

Using the linear equivalence relations (3.10) (self-)intersections between any combination of R's, D's and E's are readily computed.

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Chern classes. The total Chern class $c(TX)$ of the tangent bundle of the resolution space X can be computed from the splitting principle as

$$c(TX) = \prod(1+D)\prod(1+E)\prod(1-R)^2\,, \tag{3.12}$$

where the products are taken over all divisors of the appropriate types. By expanding this out, we can determine the first and second Chern classes of X. Using the linear equivalence relations (3.10) we find that $c_1(TX) = 0$, confirming that X defines a Calabi-Yau space. For the second Chern class we obtain

$$c_2(TX) = -\frac{3}{4}\Big(\sum_{r'}(E'_{r'})^2 + \sum_{r}(E_r)^2\Big) + \ldots \tag{3.13}$$

The dots ... refer to further terms that appear in this expansion in principle, but which never contribute when integrated over any four-cycle using the intersection numbers given above.

By employing the adjunction formula, $c_2(D) = D\,c_2(X|D)$ one can determine the Euler number of the hyper surface associated to the divisor D. In particular, from

$$\chi(R_1) = c_2(R_1) = 24\,, \quad \chi(R_2) = c_2(R_2) = 24\,, \quad \text{and} \quad \chi(R_3) = c_2(R_3) = 0\,, \tag{3.14}$$

we infer that the divisors R_1 and R_2 are $K3$ surfaces and R_3 is a four-torus. Finally, one may consider $D_{1,00}$ and $D_{2,00}$ as the divisor classes associated to the rational elliptic surfaces B' and B, respectively. Indeed, the Euler number of $D_{1,00}$ equals $\chi(D_{1,00}) = 12$, and, since $D_{1,00}E_r^2 = -2$, $D_{1,00}$ contains the same fixed points of g_θ as B', see (3.5). Hence, we may identify $B' = D_{1,00}$ and similarly $B = D_{2,00}$.

4 Line bundle models on the Schoen resolution

In this section we consider Abelian gauge backgrounds on the Schoen geometry as described in the previous section. After that we determine the charged chiral spectrum in the presence of this background, showing in particular that even with line bundles it is possible to obtain chirality in four dimensions. Subsection 4.3 explains how the spectra of such line bundle backgrounds can be interpreted as heterotic orbifolds with appropriate blow-up modes switched on. The final subsection illustrates various aspects by giving an explicit line bundle model on the Schoen manifold.

4.1 Abelian gauge flux backgrounds

On the space X we consider an Abelian gauge background $\mathcal{F}$ which is embedded in the Cartan subalgebra, spanned by the generators H_I, of the $E_8 \times E_8{}'$ gauge group of the heterotic theory. In general this gauge flux is supported on both the exceptional and inherited divisors

$$\frac{\mathcal{F}}{2\pi} = \sum_a R_a\,H_{B_a} + \sum_r E_r\,H_{V_r} + \sum_{r'} E'_{r'}\,H_{V'_{r'}}\,. \tag{4.1}$$

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The *line bundle vectors* $V_r, V'_{r'}$ and the magnetic fluxes B_a on the tori are sixteen-dimensional component vectors that characterize the corresponding line bundle embedding in $\mathrm{E}_8 \times \mathrm{E}_8{}'$. To shorten the notation we have written $H_A = A_I\, H_I$, where A are referred to as bundle or flux vectors with 16 components, A_I.

In order that this gauge background is compatible with the freely acting $\mathbb{Z}_{2,\mathrm{free}}$ involution (2.14) of the orbifold, we need to require that

$$V_{n_3\, n_4\, n_6} = V_{n_3\, n_4+1\, n_6+1}\,, \qquad V'_{n_1\, n_2\, n'_6} = V'_{n_1\, n_2+1\, n'_6+1}\,, \tag{4.2}$$

Given that the indices take values $n_i = 0, 1$, the addition of indices is performed modulo 2.

Flux quantization. For a gauge flux configuration (4.1) to be physically admissible it has to be integrally quantized, i.e.

$$\int_C \frac{\mathcal{F}}{2\pi} \in \mathbb{Z}\,, \tag{4.3}$$

for all curves C that the manifold X admits. This gives a stringent set of conditions on the bundle vectors

$$2\, V'_{n_1 n_2 n'_6} \cong 2\, V_{n_3 n_4 n_6} \cong 0\,, \qquad 2\, B_1 \cong 2\, B_2 \cong 2\, B_3 \cong 0\,, \tag{4.4a}$$

$$\sum_{n_1,n_2} V'_{n_1 n_2 n'_6} + B_1 \cong \sum_{n_3,n_4} V_{n_3 n_4 n_6} + B_2 \cong 0\,, \quad \sum_{n'_6} V'_{n_1 n_2 n'_6} + \sum_{n_6} V_{n_3 n_4 n_6} + B_3 \cong 0\,. \tag{4.4b}$$

The first two conditions of (4.4a) are obtained by integrating over the curves $D_{2,n_3 n_4} E'_{n_1 n_2 n'_6}$ and $D_{1,n_1 n_2} E_{n_3 n_4 n_6}$, respectively. The first two sum conditions in (4.4b) result from integrating over $D_{2,n_3 n_4} D'_{3,n'_6}$ and $D_{1,n_1 n_2} D_{3,n_6}$, respectively. The last sum relation in (4.4b) is found by integrating over $D_{1,n_1 n_2} D_{2,n_3 n_4}$. When we combine these sum equations with the first two conditions of (4.4a), the latter three of (4.4a) are inferred. These equations are quite tricky to be solved in general. However, there are two (related) ansätze that simplify the problem considerably:

First of all, one may assume that the various bundle vectors are either equal or opposite, e.g.

$$V_{n_3 n_4 n_6} = \sum_{s=0,1} (-)^{s(n_4+n_6)}\, V_{s n_3}\,, \qquad V'_{n_1 n_2 n'_6} = \sum_{s'=0,1} (-)^{s'(n_2+n'_6)}\, V'_{s' n_1}\,. \tag{4.5}$$

This particular choice is compatible with the requirement (4.2) that the gauge fluxes admit the $\mathbb{Z}_{2,\mathrm{free}}$ action of equation (2.14). In this case, the sums of V_r and $V'_{r'}$ in the conditions (4.4b) form lattice vectors. Consequently, the magnetic fluxes of the tori B_a have to be lattice vectors themselves.

Secondly, taking inspiration from the expansion (2.4) of the local shift vectors V_g in the orbifold construction, one may write the local bundle vectors as

$$\begin{aligned} V_{n_3 n_4 n_6} &= V_\theta + n_3\, W_3 + n_4\, W_4 + n_6\, W_6 + L_{n_3 n_4 n_6}\,, \\ V'_{n_1 n_2 n'_6} &= V_\omega + n_1\, W_1 + n_2\, W_2 + n'_6\, W_6 + L'_{n_1 n_2 n'_6}\,, \end{aligned} \tag{4.6}$$

where $2V_\theta \cong 2V_\omega \cong 2W_i \cong 0$ and $L_{n_3 n_4 n_6} \cong L'_{n_1 n_2 n'_6} \cong 0$. Then, again, the sum conditions (4.4b) imply that the magnetic fluxes B_a are lattice vectors.

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Bianchi identities. The central consistency requirements for smooth compactifications are the integrated Bianchi identities. In this work we ignore the possibility of five-branes, therefore the integrated Bianchi identities have to vanish for all divisors D, i.e.

$$\int_D \left(\mathrm{tr}\mathcal{F}^2 - \mathrm{tr}\mathcal{R}^2\right) = 0 \,. \tag{4.7}$$

Using the expression for the second Chern class (3.13) of X, we find in the present case that these conditions amount to

$$B_1 \cdot V_r = 0\,, \qquad B_2 \cdot V'_{r'} = 0\,, \qquad B_1 \cdot B_2 = 0\,, \tag{4.8a}$$

$$\sum_r (V_r)^2 = 12 + 2\, B_2 \cdot B_3\,, \qquad \sum_{r'} (V'_{r'})^2 = 12 + 2\, B_1 \cdot B_3\,, \tag{4.8b}$$

by integrating over E_r, $E_{r'}$, R_3, and R_1, R_2, respectively.

The equations in (4.8a) show that the gauge fluxes B_1 and B_2 have to be perpendicular, and, that the gauge flux B_1 on R_1 is perpendicular to all the line bundle vectors V_r. Similarly, the gauge flux B_2 is perpendicular to all vectors $V'_{r'}$. However, there are no conditions on the inner products $B_1 \cdot V'_{r'}$ and $B_2 \cdot V_r$. The Bianchi identities on the second line, (4.8b), are reminiscent of the Bianchi identity on a single $\mathbb{C}^2/\mathbb{Z}_2$ resolution, i.e. $V^2 = 3/2$ (see e.g. refs. [40, 41]). For example, when B_2 or B_3 vanish and all eight V_r are equal, this condition is reproduced identically. The magnetized tori thus lead to modifications of the standard local Bianchi identity of the local $\mathbb{C}^2/\mathbb{Z}_2$ resolution.

DUY equations and the blow-down limit. Using that the volume of a divisor D is defined as $\mathrm{Vol}(D) = \int_D J^2/2$, the tree-level DUY equation $\int J^2\mathcal{F} = 0$ can be represented as

$$\sum_a \mathrm{Vol}(R_a)\, B_a + \sum_r \mathrm{Vol}(E_r)\, V_r + \sum_{r'} \mathrm{Vol}(E'_{r'})\, V'_{r'} = 0 \,. \tag{4.9}$$

These conditions can be very restrictive. They give an equal number of relations between the volumes as the number of linear independent vectors the B_a, V_r and $V'_{r'}$ can be decomposed in. However, one such relation may force many volumes to zero simultaneously, because these volumes are of course assumed to be non-negative.

As we describe the Schoen manifold as a resolution of the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\mathrm{rototrans}}$, we would like to determine the requirements under which a gauge flux configuration allows for a *regular blow-down limit* in which the underlying six torus T^6 has a finite volume. Hence, we search for solutions which allow for a full blow-down to the singular orbifold, i.e. with $\mathrm{Vol}(E_r) = \mathrm{Vol}(E'_{r'}) = 0$ and $\mathrm{Vol}(R_a) > 0$. In this case, the DUY equations simplify to

$$\mathrm{Vol}(R_1)\, B_1 + \mathrm{Vol}(R_2)\, B_2 + \mathrm{Vol}(R_3)\, B_3 = 0 \,. \tag{4.10}$$

It follows that unless the B_a are linearly dependent or all zero, at least some of the volumes $\mathrm{Vol}(R_a)$ are forced to vanish. In particular, if only one B_a is non-zero, then the corresponding volume has to be zero in the blow-down limit, and hence a regular blow-down limit does not exist. Even when B_3 is a linear combination of B_1 and B_2 but one of the coefficients is positive, two volumes are forced to zero. Hence, only if B_3 is a linear combination

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with negative coefficients of B_1 and B_2, a regular blow-down limit exists. Hence, possibly the simplest way to realize this has $B_3 = -B_1 - B_2$. We can cast this in the form of two equations

$$B_1^2 \operatorname{Vol}(R_1) + B_1 \cdot B_3 \operatorname{Vol}(R_3) = 0\,, \qquad B_2^2 \operatorname{Vol}(R_2) + B_2 \cdot B_3 \operatorname{Vol}(R_3) = 0\,, \tag{4.11}$$

since B_1 and B_2 are perpendicular, see (4.8a). Since we know from (4.4a) that $2B_a \cong 0$, we see that the ratios of the volumes of the inherited divisors, $\operatorname{Vol}(R_a)/\operatorname{Vol}(R_3)$, $a = 1, 2$, are fractional.

To allow for a full blow-up we need in addition that the fluxes located at the exceptional divisors to be chosen such that the volumes of all of them can be taken to be positive at the same time. To ensure this it is again convenient to choose that the corresponding bundle vectors are alternating, e.g. like in (4.5).

It turns out that the combination of the flux quantization, DUY equations and the Bianchi identities is extremely restrictive, hence, to obtain semi-realistic models, one is often forced to give up the requirement of a regular blow-down limit. (In addition, the one-loop correction to the DUY equation [26] can force some volumes to be non-vanishing.) When this limit does not exist, an orbifold interpretation of the model is not ruled out: Often it is possible to shrink quite a number of exceptional cycles to zero, while keeping the volumes $\operatorname{Vol}(R_a) > 0$. Hence, locally near those shrunken cycles a non-compact orbifold analysis is still possible.

4.2 Spectra computation

To determine the physical consequence of models build on such orbifold resolutions we need to be able to determine the spectrum of massless states. A convenient way of computing the spectrum on an orbifold resolution is provided by the multiplicity operator introduced e.g. in [40–42]. Using these methods we can determine both the spectra in four dimensions as well as on six-dimensional hyper surfaces.

Four-dimensional spectrum. The spectrum in four dimensions is of key interest in phenomenological studies. It can be determined by letting the operator

$$N_{4D}(X) = \int_X \left\{ \frac{1}{6} \left(\frac{\mathcal{F}}{2\pi} \right)^3 + \frac{1}{12} \operatorname{c}_2(TX) \frac{\mathcal{F}}{2\pi} \right\}, \tag{4.12}$$

act on the states contained in the ten-dimensional gaugino. This operator is normalized such that it counts the number of chiral superfields. Using the intersection numbers determined above this is computed straightforwardly:

$$N_{4D}(X) = 2\left(1 - \sum_r H_{V_r}^2\right) H_{B_1} + 2\left(1 - \sum_{r'} H_{V'_{r'}}^2\right) H_{B_2} + 4\, H_{B_1} H_{B_2} H_{B_3}\ . \tag{4.13}$$

The multiplicities of the chiral multiplets in four dimensions are then determined by evaluating this operator on the roots of $\mathrm{E}_8 \times \mathrm{E}_8{}'$.

There is some tension between solving the Bianchi identities and chirality, because of the orthogonality relations, (4.8a), among the fluxes B_a, V_r and $V'_{r'}$. However, say

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$B_1 \cdot V_r = 0$, does not imply that on all $\mathrm{E}_8 \times \mathrm{E}_8'$ roots $H_{B_1} H_{V_r}^2$ vanishes. As we show by some examples discussed in the section 5, it is indeed possible to obtain a chiral spectrum in four dimensions.

Six-dimensional spectra on divisors. In addition, we can define the multiplicity operator $N_{6D}(D)$ in six dimensions for any divisor $D \subset X$. Positive values of N_{6D} count the number of half-hyper multiplets, while negative values count (two times) the number of vector multiplets. (In six dimensions the fermions of hyper and vector multiplets have opposite chirality.) As integrals over the whole space X they read

$$N_{6D}(D) = \int_X D \left\{ \frac{1}{2} \left(\frac{\mathcal{F}}{2\pi} \right)^2 + \frac{1}{12} \mathrm{c}_2(D) \right\}, \tag{4.14}$$

where $\mathrm{c}_2(D)$ are given in (3.14). Given that the divisors R_1, R_2 may be interpreted as $K3$ surfaces and R_3 as a four-torus, the spectra on these divisors are probably the most interesting. Using the intersection numbers we readily compute this explicitly for $D = R_a$:

$$N_{6D}(R_1) = 2\left(1 - \sum_r H_{V_r}^2\right) + 4\, H_{B_2} H_{B_3}\,, \tag{4.15a}$$

$$N_{6D}(R_2) = 2\left(1 - \sum_{r'} H_{V_{r'}'}^2\right) + 4\, H_{B_1} H_{B_3}\,, \tag{4.15b}$$

$$N_{6D}(R_3) = 4\, H_{B_1} H_{B_2}\ . \tag{4.15c}$$

Relation between the six- and four-dimensional spectra. As explained in subsection 2.3 the orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\mathrm{rototrans}}$ never leads to four-dimensional chirality. The reason is basically that such models only contain hypermultiplets in six dimension, which simply branch to vector-like combinations of chiral multiplets in four dimensions. In the smooth case we have found a way to bypass this no-go. The key here are the magnetic fluxes B_a on the divisors R_a that correspond to the tori of the orbifold in the blow-down limit. Indeed, if we set all $B_a = 0$, then (4.13) simply says that $N_{4D} = 0$: no chirality. Hence, precisely by allowing for magnetized divisors R_a we can avoid this no-go and obtain chirality.

To see that this effect is expected from field theory, let us consider the case in which the flux B_2 has been switched off. The four-dimensional multiplicity operator (4.13) then leads to a relation between the six-dimensional spectrum on R_1 given by (4.15a) and the spectrum in four dimensions:

$$N_{4D}(X) = H_{B_1}\, N_{6D}(R_1)\ . \tag{4.16}$$

This equation can be interpreted as follows: When we compactify on a $K3 \times T^2$, then we can consider first the six-dimensional theory that results from the compactification on $K3$. This effective six-dimensional model is subsequently reduced on a two-torus. It is well-known that if there is no magnetic flux, this second step results in a vector-like spectrum in four dimensions. However, if there is a magnetic flux B present, a chiral spectrum arises: only the chiral fermionic states of charge q for which $Bq > 0$ survive, and their multiplicity

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is given by Bq provided that the smallest charge in the spectrum is unity [34, 36]. It is intriguing to notice that (4.16) says exactly this: $N_{6D}(R_1)$ determines the spectrum in a six-dimensional world. The operator H_{B_1} gives the charge $B_1 \cdot w$ of the six-dimensional state associated with the $\mathrm{E}_8 \times \mathrm{E}_8{}'$ root w under the magnetic flux B_1.

When all three fluxes are switched on, the relation between the four- and six-dimensional spectra apparently reads

$$N_{4D}(X) = H_{B_1}\, N_{6D}(R_1) + H_{B_2}\, N_{6D}(R_2) - H_{B_3}\, N_{6D}(R_3)\ . \tag{4.17}$$

This follows directly by identifying the six-dimensional multiplicity operators (4.15) in the four-dimensional expression (4.13). The final term corrects for over counting of states charged under B_1, B_2 and B_3 simultaneously.

4.3 Interpretation as blow-up of DW(0-2) orbifold models

So far, we have analyzed the Schoen geometry as a smooth Calabi-Yau and described line bundle backgrounds on it. The fact that the Schoen manifold is the resolution of the DW(0-2) orbifold, as discussed in section 3, has essentially been irrelevant in our investigation. Now, we would like to describe how a given line bundle model can be understood as a heterotic DW(0-2) orbifold model with a certain number of blow-up modes attaining vacuum expectation values (VEVs). We first recall how this analysis can be done in general using a $\mathcal{N} = 1$ language in four dimensions following [10, 13, 42]. After that we conclude this subsection by describing this procedure in a six-dimensional supersymmetric formulation which is more appropriate since the DW(0-2) orbifold model has $\mathcal{N} = 1$ supersymmetric sectors in six dimensions.

Four dimensional $\mathcal{N} = 1$ language. In heterotic orbifolds a fixed point gets blown up if a twisted chiral superfield $\Phi^{(r)}_{\mathrm{bm}}$, localized at that fixed point, acquires a non-vanishing VEV: $\langle\Phi^{(r)}_{\mathrm{bm}}\rangle \neq 0$. The value of this VEV determines the volume of the exceptional cycle E_r that appears in this resolution process. As we recalled in section 2.1 any twisted state is characterized by a shifted left-moving momentum P_{sh}. In refs. [10, 13, 43] it was realized that, as long as this twisted state does not involve any oscillator excitations, its shifted momentum P_{sh} precisely determines the local line bundle vector V_r associated to the exceptional divisor E_r. Some special cases might occur: Sometimes it happens that a bundle vector corresponds to a blow-up mode that has been projected out by the orbifold action in the four-dimensional theory. It is also possible that the bundle vector is associated to a massive state in the orbifold spectrum.

The spectrum of the orbifold model and the one of the blow-up theory are generically not identical, but closely related: First of all the VEVs of the twisted states $\langle\Phi^{(r)}_{\mathrm{bm}}\rangle$ lead to some gauge symmetry breaking. Furthermore, the blow-up modes are not present in the blow-up spectrum as charged states, but rather as (complexified) axions b_r. The relation between the blow-up mode and the axion reads

$$\Phi^{(r)}_{\mathrm{bm}} = e^{b_r}\,\langle\Phi^{(r)}_{\mathrm{bm}}\rangle\ . \tag{4.18}$$

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In the smooth description this axion generically gives a mass to the gauge field of the broken U(1) via the Stueckelberg mechanism. In the blow-up picture the U(1) is broken just by a standard Higgs mechanism.

As a consequence of this gauge symmetry breaking the representations of matter fields get branched. But still, this is not enough to match the orbifold and resolution spectra [10, 13]: One needs to preform field redefinitions of the other twisted matter states Φ_{orb} involving the corresponding blow-up modes to obtain an exact agreement of the spectra, i.e.

$$\Phi_{\text{orb}} = e^{\pm b_r}\, \Phi_{\text{res}} \ . \tag{4.19}$$

The signs $\pm$ have to be chosen appropriately to ensure that the weights of Φ_{res} are $\text{E}_8 \times \text{E}_8{}'$ roots, while those of Φ_{orb} belong to the shifted weight lattice defined in (2.9).

Blow-ups in six dimensions. Before we describe the blow-up procedure in a six-dimensional language, we first briefly recall some properties of $\mathcal{N} = 1$ theories in six dimensions. There are three basic irreducible representation of $\mathcal{N} = 1$ supersymmetry relevant for our discussion: i) A vector multiplet $\mathcal{V} = (\text{V}, \Phi)$ contains a vector superfield V and a chiral superfield Φ from the 4D $\mathcal{N} = 1$ perspective. ii) A hypermultiplet contains two independent chiral superfields $\mathcal{H} = (\Phi, \Phi^c)$ that live in charge-conjugate representations. This means that the gauge properties of the hypermultiplet is uniquely specified by the representation and U(1) charges of either chiral component. iii) Finally, a half-hyper multiplet is a hypermultiplet with a certain reality condition imposed. Therefore, it has only half of the number of independent components as a normal hypermultiplet. In other words, using the four-dimensional $\mathcal{N} = 1$ terminology, a half-hyper is a chiral superfield in a real or pseudo-real representation.

Now, if a twisted hypermultiplet plays the role of a blow-up mode in order to resolve a fixed torus, then only one of its chiral superfield components actually takes a VEV, while the other component only gets redefined:

$$\mathcal{H}^{(r)}_{\text{bm}} = e^{b_r}\, (\langle \Phi^{(r)}_{\text{bm}} \rangle, \Phi^{c(r)}_{\text{bm,res}}) \ . \tag{4.20}$$

Because the chiral superfield components of a hypermultiplet carry opposite U(1) charges, they have to be redefined with opposite powers of the blow-up mode:

$$\mathcal{H}_{\text{orb}} = (\Phi_{\text{orb}}, \Phi^c_{\text{orb}}) = \left(e^{\pm b_r}\, \Phi_{\text{res}}, e^{\mp b_r}\, \Phi^c_{\text{res}}\right) , \tag{4.21}$$

for appropriate choice of sign $\pm$. After these field redefinitions the chiral superfields in the blow-up mode hypermultiplet do not seem to fall into proper $\mathcal{N} = 1$ representations anymore. However, this does not signify that the blow-up breaks six-dimensional supersymmetry: The remaining chiral superfield components will be completely neutral, and therefore form half-hypermultiplets by themselves.

4.4 Sample model: an eight generation GUT

We conclude this section with a concrete example of a line bundle model which is constructed on the Schoen geometry to illustrate many aspects of the general description

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developed in this and the preceding sections. Consider the following particular line bundle model on the resolution of our $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$:

$$B_3 = -B_1\,, \quad B_2 = 0\,, \qquad V_{n_3 n_4 n_6} = (-)^{n_4+n_6}\, V_0\,, \quad V'_{n_1 n_2 n'_6} (-)^{n_2+n'_6}\, V'_{n_1}\,, \tag{4.22}$$

with

$$\begin{aligned} B_1 &= \left(1,1,1,\text{-}1,0,0^3\right)\left(0^8\right), & V'_0 &= \left(0,0,0,\frac{1}{2},\frac{1}{2},0^3\right)\left(0^8\right), \\ V_0 &= \left(0,0,\frac{1}{2},\frac{1}{2},0,0^3\right)\left(1,0^7\right), & V'_1 &= \left(\frac{1}{2},\text{-}\frac{1}{2},0,0,0,0^3\right)\left(0^8\right). \end{aligned} \tag{4.23}$$

This model exhibits the following properties:

The bundle vectors satisfy all the requirements specified in section 4.1: The quantization conditions (4.4) are fulfilled, because the alternating signs in the V_r and $V'_{r'}$ are in accordance with (4.5) and the vectors B_a are lattice vectors.

They also satisfy all Bianchi identities (4.8): B_1 is perpendicular to all vectors V_r. Since all vectors V_r square to 3/2 and $B_2 = 0$ the first condition in (4.8b) is satisfied. The second condition in (4.8b) is satisfied as well, since both sides are equal:

$$\sum_{r'} {V'_{r'}}^2 = 8 \cdot \frac{1}{2} = 4\,, \qquad 12 + 2\, B_1 B_3 = 12 - 2 \cdot 4 = 4\,. \tag{4.24}$$

Because $B_3 = -B_1$ and $B_2 = 0$ a blow-down of this model is allowed by the DUY equations (4.9) while keeping the torus radii, set by the volumes of the divisors R_a, finite. In the blow-down limit, $\text{Vol}(E_r) = \text{Vol}(E'_{r'}) = 0$, the volumes of R_1 and R_3 have to be equal, $\text{Vol}(R_3) = \text{Vol}(R_1)$. The alternating signs of V_r and $V'_{r'}$ ensure that the DUY equations also allow for a finite blow-up of all exceptional cycles.

The gauge group that is left unbroken by this Abelian gauge configuration is

$$\text{SU}(5) \times \text{SO}(14)' \times \text{U}(1)^5\,, \tag{4.25}$$

from the first and second E_8 group factor. Since for this choice of bundle vectors $B_2 = 0$ and all V_r are equal up to a sign, the 4D multiplicity operator (4.13) reduces to

$$N_{4D} = 2\, H_{B_1}\left(1 - 8\, H_{V_0}^2\right)\,. \tag{4.26}$$

The resulting spectrum,

$$8\,(\mathbf{10},\mathbf{1}) + 12\,(\overline{\mathbf{5}},\mathbf{1}) + 4\,(\mathbf{5},\mathbf{1}) + 24\,(\mathbf{1},\mathbf{1})\,, \tag{4.27}$$

is chiral w.r.t. to the five U(1) charges (which we omitted for notational simplicity). (W.r.t. the hidden gauge group at most a purely vector-like spectrum arises, which is invisible for the multiplicity operator.) Hence, the model might be considered as an eight generations SU(5) GUT toy-model with four Higgs pairs.

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5 A line bundle MSSM on the Schoen manifold

We present an MSSM-like model with three generations as a line bundle model on the resolution of $T^6/\mathbb{Z}_2\times\mathbb{Z}_{2,\text{rototrans}}$. In the first subsection we construct an SU(5) GUT model with six generations on the Schoen manifold using line bundles. In subsection 5.2 we identify a Wilson line that can be associated with a freely acting involution, which both reduces the number of generations to three and breaks the gauge group to the standard model group. In the next subsection we show that a $K3$ subspace of the Schoen manifold can be blown down to a four-dimensional orbifold $T^4/\mathbb{Z}_2$ on which the model can be quantized using standard CFT techniques. In subsection 5.4 we use this to give an alternative description of the line bundle MSSM on the Schoen manifold in terms of a blow-up of this orbifold with a magnetized torus.

5.1 Six GUT generations on the Schoen resolution

We define a line bundle model on the Schoen manifold with the flux vectors

$$B_1 = \big(3,-3,0^6\big)\big(3,3,0^6\big) \quad \text{and} \quad B_2 = B_3 = 0\,, \tag{5.1}$$

on the ordinary divisors R_a,

$$V_{(0,0,0)} = V_{(0,1,0)} = -V_{(0,0,1)} = -V_{(0,1,1)} = \left(\frac{1}{4}^8\right)\left(0,0,0,\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right), \tag{5.2a}$$

$$V_{(1,0,0)} = V_{(1,1,0)} = -V_{(1,0,1)} = -V_{(1,1,1)} = \left(0,\frac{1}{2},\frac{1}{2},0^5\right)\left(0,\frac{1}{2},0,0,0,-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right), \tag{5.2b}$$

on the exceptional divisors E_r, and finally,

$$V'_{(0,0,0)} = -V'_{(0,1,1)} = \left(0,-\frac{1}{2},-\frac{1}{2},0^5\right)\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},0,-\frac{1}{2},0,0,0\right), \tag{5.3a}$$

$$V'_{(0,1,0)} = -V'_{(0,0,1)} = \left(0,-\frac{1}{2},-\frac{1}{2},0^5\right)\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2},0,\frac{1}{2},0,0,0\right), \tag{5.3b}$$

$$V'_{(1,0,0)} = V'_{(1,1,0)} = \big(0,1,0,0^5\big)\left(-\frac{1}{2},-\frac{1}{2},0,0,0,0,0,0\right), \tag{5.3c}$$

$$V'_{(1,1,1)} = V'_{(1,0,1)} = \big(-1,0^7\big)\left(-\frac{1}{2},-\frac{1}{2},0^6\right), \tag{5.3d}$$

on the exceptional divisors $E'_{r'}$.

This choice of bundle vectors fulfills the quantization conditions (4.4) and the DUY equations (4.9) for appropriately chosen volumes. All bundle vectors V_r and $V'_{r'}$ have $V_r^2 = V_{r'}'^2 = 3/2$. This is consistent with the Bianchi identities (4.8b), which reduce to

$$\sum_r (V_r)^2 = \sum_{r'} (V'_{r'})^2 = 12\,; \tag{5.4}$$

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Superfield multiplicity	Representation $SU(5) \times SU(5)'$	U(1) charges							
		q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
6	$(\overline{\mathbf{10}}, \mathbf{1})$	0	0	0	0	1	0	-3	0
6	$(\mathbf{5}, \mathbf{1})$	0	0	0	0	0	0	-6	0
6	$(\overline{\mathbf{5}}, \mathbf{1})$	1	0	1	0	-1	0	1	0
6	$(\mathbf{5}, \mathbf{1})$	1	0	1	0	0	0	4	0
24	$(\mathbf{1}, \mathbf{1})$	2	0	0	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	-1	0	-1	0	-1	0	5	0
6	$(\mathbf{1}, \mathbf{1})$	1	0	-3	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	0	0	0	0	2	0	0	0
6	$(\mathbf{1}, \overline{\mathbf{10}})$	0	0	0	2	0	0	0	-6
24	$(\mathbf{1}, \mathbf{5})$	0	1	0	3	0	0	0	-2
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	-2	0	0	0	-8
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	1	0	7
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	-1	0	7
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	1	0	-5
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	-1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	-1	0	-5
6	$(\mathbf{1}, \mathbf{1})$	0	2	0	0	0	0	0	0

Table 1. This line bundle model on the Schoen manifold has six generations of SU(5) in both, the observable and the hidden, sectors. States in the first block are charged under the observable E_8; states in the second block are charged under the hidden group.

since there are no corrections resulting from magnetic fluxes B_a as only $B_1 \neq 0$. The unbroken gauge group in this gauge configuration reads

$$SU(5) \times SU(5)' \times U(1)^8 \ . \tag{5.5}$$

The four-dimensional multiplicity operator (4.13) is computed straightforwardly and the resulting chiral spectrum is given in table 1. In this table we have distinguished the various states, in particular the singlets, by their eight U(1) charges $(q_0, \ldots, q_7)$. Notice that, curiously, this model has six generations in both, the observable and the hidden, SU(5).

5.2 Freely acting $\mathbb{Z}_2$ and MSSM with three generations

One can define a freely acting involution $\mathbb{Z}_{2,\text{free}}$ as in equation (2.14) that reduces the number of generations by a factor 1/2. In addition, the freely acting involution can be embedded as a Wilson line that breaks SU(5) to $SU(3) \times SU(2) \times U(1)_Y$. We take this

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Wilson line,

$$W_{\text{free}} = \left(0^3, 1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right)(0^8)\,, \tag{5.6}$$

to point in the standard hypercharge direction of SU(5). This choice of W_{free} fixes the first SU(5) to define the observable sector and leads to an MSSM-like model with three generations.

Contrary to the situation in field theory, there are further requirements on this Wilson line in string theory [12, 13]: It has to satisfy $2W_{\text{free}} \cong W_2 \cong W_4 \cong W_6 \cong 0$ and it has to respect the modular invariance conditions

$$2W_{\text{free}}^2 \equiv W_{\text{free}} \cdot W_i \equiv 0\ . \tag{5.7}$$

These additional conditions were derived in context of orbifold constructions where $\mathbb{Z}_{2,\text{free}}$ is part of the space group.

5.3 Singular limits of the Schoen GUT with line bundles

Full blow down limit. Taking the magnetic flux B_1 to vanish for a moment, we can consider the full blow-down limit of the GUT model with six generations. It has an exact heterotic orbifold CFT as formulated in section 2.3 as the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold with a definite choice of gauge shifts, V_θ, V_ω, and discrete Wilson lines, W_i. As dictated by the flux vectors (5.2) and (5.3) they are given by

$$\begin{aligned}
V_\theta &= \left(\frac{1}{4}^8\right)\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, 0^3\right), \\
V_\omega &= \left(0, -\frac{1}{2}, -\frac{1}{2}, 0^5\right)\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, 0^3\right), \\
W_1 &= \left(0, \frac{1}{2}, -\frac{1}{2}, 1, 1, 0^3\right)\left(0, 1, \frac{1}{2}, 1, -\frac{1}{2}, 0^3\right), \\
W_3 &= \left(-\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}^5\right)\left(0, \frac{1}{2}, 0, \frac{1}{2}, -1, 0^3\right),
\end{aligned} \tag{5.8}$$

and the other Wilson lines vanish. This choice fulfills the conditions of modular invariance (2.7). As discussed in section 2.3, the spectrum of this orbifold can be computed using orbifold CFT techniques but is necessarily non-chiral as long as no magnetic fluxes B_a have been reintroduced.

The $T^4/\mathbb{Z}_2$ orbifold inside the Schoen manifold. Since the Schoen model defined in this section has only a single magnetic flux, B_1, switched on, see (5.1), the DUY equations (4.9) imply that in a full blow-down the volume of R_1 has to vanish as well. However, we can exploit that there also exists a partial blow-down in which all $\text{Vol}(E_r) \to 0$ while the volumes of all inherited divisors R_a and of at least some other exceptional divisors $E'_{r'}$ stay finite. Therefore, this partial blow-down leads to an intermediate $T^4/\mathbb{Z}_2$ orbifold with torus coordinates (z_2, z_3) on which the $\mathbb{Z}_2$ action acts via the twist v_θ given in (2.3) (c.f. [11]).

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For this intermediate $T^4/\mathbb{Z}_2$ orbifold an exact heterotic CFT description exists. Taking its gauge embedding as given by V_θ and W_3 from equation (5.8), its low energy limit results in a model with $\mathcal{N}=1$ supersymmetry in six dimensions with gauge group

$$\mathrm{E}_6 \times \mathrm{SU}(8)' \times \mathrm{U}(1)^3 \,. \tag{5.9}$$

The spectrum of hypermultiplets including U(1) charges of this intermediate six-dimensional orbifold theory is computed using [7] and listed in the first column of table 2.

A simple, yet non-trivial crosscheck of this spectrum is that it is free of irreducible gravitational anomalies, e.g. that the sum condition $\#(\text{hyper}) - \#(\text{vector}) = 244$ holds. Indeed, using table 2 it is straightforward to count the number of vector- and hypermultiplets:

$$\begin{aligned} \#(\text{vector}) &= 78 + 63 + 3 \cdot 1 = 144\,, \\ \#(\text{hyper}) &= 2 \cdot \left(27 + \frac{1}{2} \cdot 70 + 2\right) + 16 \cdot 2 \cdot 8 + 4 = 388\,, \end{aligned} \tag{5.10}$$

where the factor $\frac{1}{2}$ accounts for the fact that the $(\mathbf{1}, \mathbf{70})_{(0,0,0)}$ is a half-hyper. The last 4 additional hypers correspond to untwisted moduli which are not displayed in table 2.

5.4 Schoen line bundle MSSM as a blown up orbifold

The MSSM-like model of subsection 5.1 can now be reproduced as a blow-up of the $T^4/\mathbb{Z}_2$ orbifold discussed in the subsection above equipped with a magnetic flux on the torus to generate four-dimensional chirality. In short, this procedure reads:

1. Blow-up the $T^4/\mathbb{Z}_2$ orbifold to a smooth $K3$ manifold by giving VEVs to 16 blow-up modes and use field redefinitions to obtain the spectrum on $K3$.

2. Turn on the additional fluxes on the divisors $E'_{r'}$, decompose gauge group and branch the representations accordingly.

3. Generate four-dimensional chirality by switching on the magnetic flux B_1 as well.

In this process the magnetic flux B_1 does not lead to breaking of four-dimensional supersymmetry since the contribution from the fluxes on $E'_{r'}$ cancels the one from B_1 in the DUY equations (4.9). In the following we describe this procedure in detail:

Blowing up the intermediate $T^4/\mathbb{Z}_2$ orbifold. The intermediate $T^4/\mathbb{Z}_2$ orbifold gets blown up to a $K3$ surface by assigning VEVs to the blow-up modes, i.e. to 16 twisted states localized at the 16 singularities of the $T^4/\mathbb{Z}_2$ orbifold. At each singularity (labeled by the multi-index $r = (n_3, n_4, n_5, n_6)$) a blow-up mode, $\Phi^{(r)}_{\text{bm}}$ contained in a twisted hypermultiplet, is chosen such that its shifted left-moving momentum $P^{(r)}_{\text{sh}}$ agrees with the flux vector V_r localized on the divisor E_r:

$$P^{(r)}_{\text{sh}} = V_r \quad \text{for all} \quad r = (n_3, n_4, n_5, n_6)\,. \tag{5.11}$$

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All these blow-up modes are chosen to be in eight-dimensional representations of the hidden $\mathrm{SU}(8)'$ gauge group of the $T^6/\mathbb{Z}_2$ orbifold, so that, consequently, the gauge group gets broken to

$$\mathrm{E}_6 \times \mathrm{SU}(7)' \times \mathrm{U}(1)^4 \ . \tag{5.12}$$

As explained in subsection 4.3, when the blow-up mode $\Phi^{(r)}_{\mathrm{bm}}$ in a given twisted sector attains a VEV, field redefinitions, (4.20) and (4.21), have to be performed on the other states in the same twisted sector in order to ensure that all fields in the blow-up are characterized by $\mathrm{E}_8 \times \mathrm{E}_8{}'$ roots. The appropriate field redefinitions required by this blow-up procedure are listed in the second column of table 2. As the phases of the orbifold blow-up modes have been reinterpreted as axions, the remaining hypermultiplet components do not seem to form proper six-dimensional $\mathcal{N} = 1$ hypermultiplets. However, as can be verified from this table, these chiral superfield are neutral and can thus be interpreted as half-hypermultiplets.

Additional fluxes on the exceptional divisors $E'_{r'}$. Up to now, we have turned on fluxes only on the exceptional divisors E_r, which correspond to the blown-up fixed points of the $T^4/\mathbb{Z}_2$ orbifold. After the field redefinition to the blow-up field basis, the charges w.r.t. to the four U(1) factors are taken such that they correspond to the first four charges (q_0, q_1, q_2, q_3) in table 1. The U(1)'s associated to the charges (q_0, q_1, q_2) already exist at the intermediate $T^4/\mathbb{Z}_2$ orbifold, the fourth U(1) arises by symmetry breaking of the hidden $\mathrm{SU}(8)'$ in the blow-up procedure.

Turning on additional fluxes (5.3) on $E'_{r'}$ induces a further gauge symmetry breaking to

$$\mathrm{SU}(5) \times \mathrm{SU}(5)' \times \mathrm{U}(1)^8 \ . \tag{5.13}$$

Since, these fluxes are located at resolved fixed points of the other orbifold twist v_ω, they respect a different six-dimensional supersymmetry. This means that by switching on these fluxes the model becomes $\mathcal{N} = 1$ in four dimensions. However, because the divisors $E'_{r'}$ do not intersect with E_r (i.e. the fixed tori of the g_θ- and g_ω-twisted sectors do not intersect) see figure 2, this does not enforce any chiral projection on the matter spectrum (in contrast to, say, the gravitino): The matter states on $K3$ (i.e. the blow-up of the intermediate $T^4/\mathbb{Z}_2$) are simply decomposed into four-dimensional superfields and their representations are branched according to the symmetry breaking (5.13).

Generating four dimensional chirality. States from E_r feel the flux on their "fixed torus", i.e. on R_1, so that the B_1 flux induces chirality in four dimensions. Whether a state is part of the charged chiral spectrum is decided by the operator

$$\tilde{N}_{4D} = 2H_{B_1} \ . \tag{5.14}$$

In general [34, 36], if $\tilde{N}_{4D}$ is positive for a chiral superfield Φ_{res}, then $\tilde{N}_{4D}$ copies of Φ_{res} appear in the four-dimensional spectrum. While, if $\tilde{N}_{4D}$ is negative Φ_{res} is completely projected out. Thus this relation (similarly to (4.16)) shows that four dimensional chirality only arises if the flux B_1 is switched on.

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6D $\mathcal{N}=1$ super multiplet on $T^4/\mathbb{Z}_2$ ($\mathrm{E}_6\times\mathrm{SU}(8)'\times\mathrm{U}(1)^3$)	Blow-up induced redefinitions of its chiral superfield component(s) ($\mathrm{E}_6\times\mathrm{SU}(7)'\times\mathrm{U}(1)^4$)	Surviving 4D chiral superfields ($\mathrm{SU}(5)\times\mathrm{SU}(5)'\times\mathrm{U}(1)^8$)	4D multiplicity $\tilde{N}_{4D}$
untwisted gauge sector			
$(\mathbf{78},\mathbf{1})_{(0,0,0)}$ (vector)	$(\mathbf{78},\mathbf{1})_{(0,0,0)}$	$(\overline{\mathbf{10}},\mathbf{1})_{(0,0,0,0,1,0,-3,0)}$	6
		$(\mathbf{5},\mathbf{1})_{(0,0,0,0,0,0,-6,0)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,0,2,0,0,0)}$	6
$(\mathbf{1},\mathbf{63})_{(0,0,0)}$ (vector)	$(\mathbf{1},\mathbf{48})_{(0,0,0,0)}$	$(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,0,0,1,0,7)}$	6
		$(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,0,0,-1,0,7)}$	6
	$(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	—	—
	$(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,-4,0,1,0,5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,-4,0,-1,0,5)}$	6
	$(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	—	—
untwisted matter sectors: U_a, $a=2,3$			
$(\mathbf{27},\mathbf{1})_{(-1,0,-1)}$ (hyper)	$(\mathbf{27},\mathbf{1})_{(-1,0,-1,0)}$	$(\mathbf{1},\mathbf{1})_{(-1,0,-1,0,-1,0,5,0)}$	6
	$(\overline{\mathbf{27}},\mathbf{1})_{(1,0,1,0)}$	$(\mathbf{5},\mathbf{1})_{(1,0,1,0,0,0,4,0)}$	6
		$(\overline{\mathbf{5}},\mathbf{1})_{(1,0,1,0,-1,0,1,0)}$	6
$(\mathbf{1},\mathbf{70})_{(0,0,0)}$ (half-hyper)	$(\mathbf{1},\overline{\mathbf{35}})_{(0,0,0,-2)}$	$(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,-2,0,0,0,-8)}$	6
	$(\mathbf{1},\mathbf{35})_{(0,0,0,2)}$	$(\mathbf{1},\overline{\mathbf{10}})_{(0,0,0,2,0,0,0,-6)}$	6
$(\mathbf{1},\mathbf{1})_{(1,0,-3)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{(1,0,-3,0)}$	$(\mathbf{1},\mathbf{1})_{(1,0,-3,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{1})_{(-1,0,3,0)}$	—	—
$(\mathbf{1},\mathbf{1})_{(0,2,0)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{(0,2,0,0)}$	$(\mathbf{1},\mathbf{1})_{(0,2,0,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{1})_{(0,-2,0,0)}$	—	—
twisted matter sector at the fixed tori: $r=(0,n_4,n_5,0)$, $n_4,n_5=0,1$			
$(\mathbf{1},\mathbf{8})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},-\frac{7}{2}\right)}=e^{+b_r}$	blow-up mode	axion
	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\frac{7}{2}\right)}=e^{+b_r}\,(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	—	—
	$(\mathbf{1},\mathbf{7})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},-\frac{1}{2}\right)}=e^{+b_r}\,(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{1}{2}\right)}=e^{-b_r}\,(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,-1,0,-5)}$	6
$(\mathbf{1},\mathbf{8})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},\frac{7}{2}\right)}=e^{+b_r}\,(\mathbf{1},\mathbf{1})_{(1,0,3,0)}$	—	—
	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},-\frac{7}{2}\right)}=e^{-b_r}\,(\mathbf{1},\mathbf{1})_{(-1,0,-3,0)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},\frac{1}{2}\right)}=e^{+b_r}\,(\mathbf{1},\overline{\mathbf{7}})_{(0,1,0,-3)}$	$(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,-1,0,-5)}$	6
	$(\mathbf{1},\mathbf{7})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)}=e^{-b_r}\,(\mathbf{1},\mathbf{7})_{(0,-1,0,3)}$	—	—

Table 2. (Continues...)

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6D $\mathcal{N}=1$ super multiplet on $T^4/\mathbb{Z}_2$ ($\mathrm{E}_6\times\mathrm{SU}(8)'\times\mathrm{U}(1)^3$)	Blow-up induced redefinitions of its chiral superfield component(s) ($\mathrm{E}_6\times\mathrm{SU}(7)'\times\mathrm{U}(1)^4$)	Surviving 4D chiral superfields ($\mathrm{SU}(5)\times\mathrm{SU}(5)'\times\mathrm{U}(1)^8$)	4D multiplicity $\tilde{N}_{4D}$
twisted matter sector at the fixed tori: $r=(0,n_4,n_5,1)$, $n_4,n_5=0,1$			
$(\mathbf{1},\mathbf{8})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\frac{7}{2}\right)}=e^{+b_r}$	blow-up mode	axion
	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},-\frac{7}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{1}{2}\right)}=e^{+b_r}(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,-1,0,-5)}$	6
	$(\mathbf{1},\mathbf{7})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},-\frac{1}{2}\right)}=e^{-b_r}(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	—	—
$(\mathbf{1},\mathbf{8})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},-\frac{7}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{1})_{(-1,0,-3,0)}$	—	—
	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},\frac{7}{2}\right)}=e^{-b_r}(\mathbf{1},\mathbf{1})_{(1,0,3,0)}$	—	—
	$(\mathbf{1},\mathbf{7})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{7})_{(0,-1,0,3)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},\frac{1}{2}\right)}=e^{-b_r}(\mathbf{1},\overline{\mathbf{7}})_{(0,1,0,-3)}$	$(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,-1,0,-5)}$	6
twisted matter sector at the fixed tori: $r=(1,n_4,n_5,0)$, $n_4,n_5=0,1$			
$(\mathbf{1},\mathbf{8})_{\left(-1,\frac{1}{2},0\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(1,-\frac{1}{2},0,-\frac{7}{2}\right)}=e^{+b_r}$	blow-up mode	axion
	$(\mathbf{1},\mathbf{1})_{\left(-1,\frac{1}{2},0,\frac{7}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	—	—
	$(\mathbf{1},\mathbf{7})_{\left(-1,\frac{1}{2},0,-\frac{1}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(1,-\frac{1}{2},0,\frac{1}{2}\right)}=e^{-b_r}(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,-1,0,-5)}$	6
$(\mathbf{1},\mathbf{8})_{\left(1,\frac{1}{2},0\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(1,\frac{1}{2},0,\frac{7}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{1})_{(2,0,0,0)}$	$(\mathbf{1},\mathbf{1})_{(2,0,0,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{1})_{\left(-1,-\frac{1}{2},0,-\frac{7}{2}\right)}=e^{-b_r}(\mathbf{1},\mathbf{1})_{(-2,0,0,0)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(-1,-\frac{1}{2},0,\frac{1}{2}\right)}=e^{+b_r}(\mathbf{1},\overline{\mathbf{7}})_{(0,-1,0,-3)}$	—	—
	$(\mathbf{1},\mathbf{7})_{\left(1,\frac{1}{2},0,-\frac{1}{2}\right)}=e^{-b_r}(\mathbf{1},\mathbf{7})_{(0,1,0,3)}$	$(\mathbf{1},\mathbf{5})_{(0,1,0,3,0,0,0,-2)}$	6
twisted matter sector at the fixed tori: $r=(1,n_4,n_5,1)$, $n_4,n_5=0,1$			
$(\mathbf{1},\mathbf{8})_{\left(-1,\frac{1}{2},0\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(-1,\frac{1}{2},0,\frac{7}{2}\right)}=e^{+b_r}$	blow-up mode	axion
	$(\mathbf{1},\mathbf{1})_{\left(1,-\frac{1}{2},0,-\frac{7}{2}\right)}=e^{+b_r}(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	—	—
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(1,-\frac{1}{2},0,\frac{1}{2}\right)}=e^{+b_r}(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,1,0,-5)}$	6
		$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,-1,0,-5)}$	6
	$(\mathbf{1},\mathbf{7})_{\left(-1,\frac{1}{2},0,-\frac{1}{2}\right)}=e^{-b_r}(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	—	—

Table 2. (Continues...)

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6D $\mathcal{N}=1$ super multiplet on $T^4/\mathbb{Z}_2$ ($E_6 \times SU(8)' \times U(1)^3$)	Blow-up induced redefinitions of its chiral superfield component(s) ($E_6 \times SU(7)' \times U(1)^4$)	Surviving 4D chiral superfields ($SU(5) \times SU(5)' \times U(1)^8$)	4D multiplicity $\tilde{N}_{4D}$
$(\mathbf{1},\mathbf{8})_{\left(1,\frac{1}{2},0\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(-1,-\frac{1}{2},0,-\frac{7}{2}\right)} = e^{+b_r}\,(\mathbf{1},\mathbf{1})_{(-2,0,0,0)}$	—	—
	$(\mathbf{1},\mathbf{1})_{\left(1,\frac{1}{2},0,\frac{7}{2}\right)} = e^{-b_r}\,(\mathbf{1},\mathbf{1})_{(2,0,0,0)}$	$(\mathbf{1},\mathbf{1})_{(2,0,0,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{7})_{\left(1,\frac{1}{2},0,-\frac{1}{2}\right)} = e^{+b_r}\,(\mathbf{1},\mathbf{7})_{(0,1,0,3)}$	$(\mathbf{1},\mathbf{5})_{(0,1,0,3,0,0,0,-2)}$	6
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(-1,-\frac{1}{2},0,\frac{1}{2}\right)} = e^{-b_r}\,(\mathbf{1},\overline{\mathbf{7}})_{(0,-1,0,-3)}$	—	—

Table 2. The first column gives 6D $\mathcal{N}=1$ multiplets on the $T^4/\mathbb{Z}_2$ orbifold with twist g_θ and gauge embedding V_θ and W_3 from equation (5.8). The second column indicates which state is the blow-up mode and gives the field redefinitions necessary to match the orbifold and blow-up states. In the third column we only indicate the states which are part of the four-dimensional chiral spectrum, i.e. those for which $\tilde{N}_{4D}$, given in the last column, is positive.

Two important observations are in order: Chiral multiplets originating from the six-dimensional vector multiplets get an extra factor (-1) in order to account for the different chiralities of vector and hypermultiplets in six dimensions. In addition, note that the effect of the Wilson line W_1 in the presence of a flux B_1 in the same torus can be seen in a field theoretical approach as a shift in the wave-functions. Hence, concerning the spectrum of massless modes it can be neglected.

As the 16 fixed points of $T^4/\mathbb{Z}_2$ are identified pairwise by the $\mathbb{Z}_2$ action of g_ω, one has to restrict to twisted states with $n_5 = 0$. Furthermore, g_ω projects out all states from the untwisted sector U_2 as can be seen in the full orbifold model $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$. The result of the additional fluxes is listed in the third and fourth columns of table 2. The chiral part of the resulting spectrum agrees with the spectrum of the smooth model listed in table 1.

6 Towards an CFT description of orbifolds with magnetized tori

In this section we propose modifications to the standard CFT construction of heterotic orbifolds in the presence of magnetized tori. To facilitate this discussion we first recall a few standard facts of heterotic orbifolds, i.e. orbifolds without any magnetic flux supported on the two-tori, $B_a = 0$.

6.1 Standard modular invariance conditions

The conditions of modular invariance are compatible with the local Bianchi identities in the absence of B_a-fluxes in the following sense: If we choose space group elements $g = h = g_r$ or $g_{r'}$, as defined in equation (2.13) we see from (2.6) that the associated local shifts V_{g_r} and $V_{g_{r'}}$ fulfill

$$V_{g_r}^2 \equiv \frac{3}{2}\,, \qquad V_{g_{r'}}^2 \equiv \frac{3}{2}\,, \tag{6.1}$$

where r, r' label the 8+8 fixed points of the twisted sectors of θ and ω, respectively. On the other hand, in the smooth picture if we assume gauge fluxes $V_r \cong V_{g_r}$ and $V'_{r'} \cong V_{g_{r'}}$

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of length-square 3/2 at all 8+8 resolved fixed points r and r', respectively, then modular invariance corresponds (modulo integers) to 1/8th of the Bianchi identities (4.8b) with $B_a = 0$.

6.2 Heterotic description of the Schoen orbifold with magnetized tori

Inspired by the logic put forward in [44] we propose how the modular invariance conditions (2.6) are modified in the presence of magnetically charged tori, $B_a \neq 0$. Since the magnetic fluxes are constant over the tori, it is natural to assume that at a given fixed point they only contribute as one over the number of fixed points, i.e. 1/8. As can be inferred from the local Bianchi identities (4.8b) the magnetic fluxes, B_a, contribute to the energy (12 is replaced by $12+2B_a\cdot B_3$, $a=1,2$). Hence, we propose that the local modular invariance conditions (6.1) are modified to

$$V_{g_r}^2 \equiv \frac{3}{2} + \frac{1}{4}\, B_2\cdot B_3\,, \qquad V_{g_{r'}}^2 \equiv \frac{3}{2} + \frac{1}{4}\, B_1\cdot B_3\,, \tag{6.2}$$

In order to satisfy the quantization conditions (4.4) and the DUY equations in blow-down (4.10) it is convenient to expand B_3 as a linear combination of B_1 and B_2 with negative coefficients. According to equation (6.2) this reduces the lengths of the local shifts V_{g_r} and $V_{g_{r'}}$. For example, using e.g. $B_3 = -B_1 - B_2$ yields $V_{g_r}^2 \equiv \frac{3}{2} - \frac{1}{4}\, B_2^2$.

However, as we have seen in blow-up not only the consistency conditions, i.e. the Bianchi conditions, get modified in the presence of B_a-fluxes, but also the spectra. Therefore, one could imagine that the mass shell condition (2.8) on orbifolds is modified as well when $B_a \neq 0$. In analogy to the proposal in [44], we expect that the left-moving mass is modified to

$$M_L^2 = \frac{1}{2}\,(P+V_{g_r})^2 + \widetilde{N} - \frac{3}{4} - \frac{1}{8}\, B_2\cdot B_3\,, \qquad M_L^2 = \frac{1}{2}\,(P+V_{g_{r'}})^2 + \widetilde{N} - \frac{3}{4} - \frac{1}{8}\, B_1\cdot B_3\,. \tag{6.3}$$

If we follow the interpretation of the local line bundle vectors as the shifted momenta (2.9) of twisted states that generate the blow-up at r or r' these equations will contribute new twisted states as blow-up modes, which where not part of the $B_a = 0$ orbifold spectrum.

When one considers the standard heterotic orbifold, (massless) states, that survive the level matching condition, are subject to the orbifold projection conditions (2.10). Modifications of these projections are, as far as we are aware, not discussed in the literature. Moreover, since it is unknown how the heterotic string is quantized in the presence of magnetized tori, there is also not an obvious computation that would determine the appropriate corrections. However, as usual we expect that at least self-projections, i.e. taking $h = g$, should not project out any state. Hence, at least the self-projection condition should be modified to

$$V_g\cdot P_{\rm sh} - v_g\cdot\left(p_{\rm sh} + \Delta\tilde{N}_g\right) \equiv \frac{1}{2}\left(V_g^2 - v_g^2 + \frac{1}{4}\, B_a\cdot B_3\right), \tag{6.4}$$

where $a=1$ for $g=g_{r'}$ and $a=2$ for $g=g_r$.

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6.3 Sample model as blow-up of orbifold with magnetized tori

To illustrate our proposal we return to our example of an eight generation SU(5) GUT model discussed in subsection 4.4. Notice, that the bundle vectors V_r defined in (4.22) can be interpreted as the shifted left-moving momenta $P_{\rm sh}$ of twisted states without oscillator excitations of a conventional $\mathbb{Z}_2$ orbifold, since $V_r^2 = 3/2$ is interpreted as the masslessness condition $P_{\rm sh}^2 = 3/2$. The bundle vectors $V'_{r'}$ on the other hand have ${V'_{r'}}^2 = 1/2$. In a conventional orbifold model these would correspond to twisted states with oscillators. However, as discussed in section 6, we expect that the left-moving mass formula gets modified to (6.3) in the presence of magnetized tori. If correct, one still interprets the $V'_{r'}$ as shifted left-moving momenta of twisted states without oscillators. Hence, even though this model has a blow-down limit, the resulting theory in this limit is not a conventional orbifold CFT.

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