Published for SISSA by Springer
Received: June 16, 2011
Accepted: July 5, 2011
Published: August 9, 2011

Non-relativistic solutions of $\mathcal{N} = 2$ gauged supergravity

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Abstract: We find infinite families of supersymmetric solutions of four dimensional, $\mathcal{N} = 2$ gauged supergravity with Lifshitz, Schrödinger and also AdS symmetries. We focus on the canonical example of a single hypermultiplet and a single vector multiplet and find that the spectrum of solutions depends crucially on whether the gaugings are electric or magnetic but to a far milder extent on the strength of the gaugings. For purely electric or purely magnetic gaugings we generically find Lifshitz solutions, while for a mixed gauging we find Schrödinger and AdS solutions. For some of the gaugings the theory has a known lift to string/M-theory thus giving a higher dimensional embedding of our solutions.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Holography and condensed matter physics (AdS/CMT)

ArXiv ePrint: 1102.5740

 doi:10.1007/JHEP08(2011)041

JHEP08(2011)041

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1 Introduction

The AdS/CFT duality [1] relates quantum gravity on AdS spacetime to a relativistic quantum field theory on the boundary of AdS. While the canonical example of such duality is between type IIB string theory on $\mathrm{AdS}_5 \times S^5$ and four dimensional $\mathcal{N} = 4$ SYM, there are numerous generalizations in various dimensions with less symmetry and a richer spectrum. Recently, it has been proposed to apply holography to problems in non-relativistic quantum field theory with applications to condensed matter physics. Gravitational duals of non -relativistic field theories were first proposed [2–5] by studying Einstein theory coupled to massive vectors,[1] however it is of some interest to embed such solutions in a UV finite theory such as string theory or M-theory. There has been significant progress made in this direction [6–28] and, indeed, this is the motivation for our current work. We will however take a somewhat different approach and study non-relavistic solutions of $\mathcal{N} = 2$ gauged supergravity in four dimensions, which should be holographically dual to the vacua of three-dimensional, non-relativistic quantum field theories. We will study gravitational duals to two kinds of non-relativistic solution, those with Lifshitz scaling [2]

$$t \to \lambda^z t\,, \quad x^i \to \lambda x^i \tag{1.1}$$

combined with spatial rotations and those with Schrödinger scaling [3, 4]

$$x_+ \to \lambda^z x_+\,, \quad x_- \to \lambda^{2-z} x_-\,, \quad x^i \to \lambda x^i \tag{1.2}$$

combined with Galilean boosts.

It has been well established that, for many internal manifolds, ten or eleven-dimensional supergravity can be consistently truncated to an effective gauged supergravity in lower dimensions. The prototypical examples are the maximal gauged supergravities in four, five and seven dimensions [29–31], which are believed to be consistent truncations of IIB or eleven-dimensional supergravity on the appropriate dimensional sphere. For seven dimensional gauged supergravity the consistency of this reduction has been proved [32]. These truncations keep all the lightest fields of just a subset of the various Kaluza-Klein towers, and, when further truncated to a more manageable sector, have been used to extract precise results on holographic renormalization group flows [33, 34]. It should be noted that while the explicit lift of any given solution to these gauged supergravities to the higher dimensional theory is theoretically possible, it can be quite technically challenging and in some cases perhaps prohibitively so.

Another avenue by which one can construct gauged supergravity theories from string/M-theory is to use a set of fundamental forms on the internal manifold which possess a closed set of differential and algebraic relations [12, 18, 35–38]. This results in a truncation which is not so much a restriction to the lightest modes but instead to a singlet sector under a certain symmetry group. The invariant sector of a coset reduction is a prime example, but more general geometric structures such as Sasaki-Einstein spaces and nearly Kahler manifolds also lead to such reductions. If this set of fundamental forms is chosen judiciously, the spectrum is finite and typically these truncations include massive scalars and vector fields. For these consistent truncations, the lift to the higher dimensional theory is trivial since by construction the mapping is provided.

Thanks to these impressive works on consistent truncations, we have the confidence to explore the parameter space of gauged supergravity in general and, then, to separately address the question of which subset of parameters admits an embedding into string/M-theory. We are not aware of a proof that any particular gauged supergravity cannot be

[1]The original proposal of [2] involved coupling gravity to a two-form potential and a vector field, but in four dimensions the tensor can be dualized to a scalar.

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embedded into string/M-theory, nor do we have any reason to suspect this may be true. It should be noted that while consistent truncations are useful for computing supersymmetric solutions, for non-supersymmetric solutions however, one must be wary of instabilities which lie outside the consistent truncation [39] and accounting for these can re-introduce many of the complexities which had been truncated away.

In this paper, we discuss the conditions for supersymmetry for Lifshitz and Schrödinger vacua in a general $\mathcal{N}=2$ gauged supergravity with vectors and hypermultiplets and we then apply the formalism to a specific example. We will work with the canonical example of four dimensional $\mathcal{N}=2$ gauged supergravity, namely we will retain just a single vector multiplet and a single hypermultiplet. The virtue of this approach is that by fixing the field content of the gauged supergravity but allowing for a quite general gauging, we can simultaneously scan infinite families of string/M-theory compactifications.[2] We find that while the conditions for supersymmetry produce vacua which are isolated in field space, they place extremely mild constraints on the charges and in this sense the vacua we find are somewhat universal.

Specifically, our scalar moduli space is

$$\mathcal{M}_{\rm SK}\times\mathcal{M}_{Q}=\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\times\frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times\mathrm{U}(1)}\,, \tag{1.3}$$

and we have two vector fields, the graviphoton and one from the vector multiplet. We gauge two commuting isometries of the hypermultiplet moduli space allowing for both electric and magnetic charges, in the sense discussed in section two. The type of vacua we find in any given theory depends crucially on whether the scalars are gauged electrically or magnetically, but depends only very mildly on the strength of the gaugings.

We can gauge both compact and non compact isometries of the hypermultiplet scalar manifold. We have found no interesting supersymmetric non-relativistic solutions in the case with two non-compact gaugings. We discuss in details the two cases of a pair of compact gauging, or one compact and one non-compact. Interestingly our results are essentially identical in both cases, indicating there is most likely a symmetry principle at work. Specifically, when all gaugings are either electric or magnetic, we find supersymmetric Lifshitz solutions with scaling parameter $z=2$ and we find that there are no supersymmetric Schrödinger or $\mathcal{N}=2$ AdS solutions. These results indicate possible obstacles to constructing supersymmetric holographic RG-flows (along the line of [33, 34]) between AdS and Lifshitz spacetimes.[3]

On the other hand when the gaugings are a mixture of electric and magnetic, we find Schrödinger solutions and $\mathcal{N}=2$ AdS solutions but no Lifshitz solutions. We show that under mild conditions, we can associate a supersymmetric Schrödinger solution to each $\mathcal{N}=2$ AdS_4 vacuum. The value of z in the Schrödinger solution is related to the mass (m) of the massive vector field in the corresponding AdS_4 vacuum by $z(z+1)=(mR)^2$, as in the original construction in [4].

[2]Some previous works in gauged supergravity where the charges have been left arbitrary are [40–42]
[3]Such flows were found in a non-supersymmetric effective theory in [43].

For a few particular gaugings, these solutions have in fact been found before and in those cases we find precise agreement. In particular we find the SU(3) × U(1) invariant $\mathcal{N}=2$ AdS$_4$ vacuum of [44] and we also reproduce the Schrödinger solution found in [12]. The Lifshitz solutions with $z=2$ found in [17, 23] are also probably related to our class of solutions. Most if not all possible gaugings can be viewed as arising from a consistent truncation of string/M-theory, and, in those cases, by the very nature of consistent truncations, any solutions of our gauged supergravity can be claimed to be solutions of string/M-theory. But we leave a detailed analysis of this issue for further work.

This paper is organized as follows. In section two, we review some standard facts about four dimensional, $\mathcal{N}=2$ gauged supergravity, largely to establish notation. In section three we study Lifshitz solutions, deriving and solving the conditions for supersymmetry. In section four we repeat this analysis for Schrödinger solutions and $\mathcal{N}=2$ AdS solutions. In section five we discuss the lift of these gauged supergravity theories to string/M-theory.

2 Four dimensional $\mathcal{N}=2$ gauged supergravity

In the rest of the paper we will work in the framework $\mathcal{N}=2$ gauged supergravity in four dimensions. We refer to [45, 46] for a detailed description of the formalism.

The fields of $\mathcal{N}=2$ supergravity are arranged into one graviton multiplet, n_v vector multiplets and n_h hyper-multiplets. The graviton multiplet contains the metric, the graviphoton, A^0_μ and an SU(2) doublet of gravitinos of opposite chirality, $(\psi^A_\mu, \psi_{\mu\, A})$, where $A=1,2$ is an SU(2) index. The vector multiplets consist of a vector, A^I_μ,, two spin 1/2 of opposite chirality, transforming as an SU(2) doublet, $(\lambda^{i\,A}, \lambda^{\bar{i}}_A)$, and one complex scalar z^i. $A=1,2$ is the SU(2) index, while I and i run on the number of vector multiplets $I=1,\dots,n_{\rm V}$, $i=1,\dots,n_{\rm V}$. The scalar fields z^i parametrise a special Kähler manifold of complex dimension $n_{\rm V}$, $\mathcal{M}_{\rm SK}$. Finally the hypermultiplets contain two spin 1/2 fermions of opposite chirality, $(\zeta_\alpha, \zeta^\alpha)$, and four real scalar fields, q_u, where $\alpha=1,\dots 2n_{\rm H}$ and $u=1,\dots,4n_{\rm H}$. The scalars q_u parametrise a quaternionic manifold of real dimension $4n_{\rm H}$, $\mathcal{M}_{\rm Q}$.

While in the ungauged $\mathcal{N}=2$ supergravity the vector- and the hyper-multiplets are decoupled at the two-derivative level, in the gauged theory they have non trivial interactions as can be seen from the bosonic Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm bos} = & -\frac{1}{2}R + i(\bar{\mathcal{N}}_{\Lambda\Sigma}\mathcal{F}^{-\Lambda}_{\mu\nu}\mathcal{F}^{-\Sigma\mu\nu} - \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{+\Lambda}_{\mu\nu}\mathcal{F}^{+\Sigma\mu\nu}) \\ & + g_{i\bar{j}}\nabla^\mu z^i \nabla_\mu \bar{z}^{\bar{j}} + h_{uv}\nabla^\mu q^u \nabla_\mu q^v - \mathcal{V}(z,\bar{z},q)\,,\end{aligned} \tag{2.1}$$

where $\Lambda, \Sigma = 0, 1, \dots, n_{\rm V}$. The gauge field strengths are defined as

$$\mathcal{F}^{\pm\Lambda}_{\mu\nu} = \frac{1}{2}\Big(F^\Lambda_{\mu\nu} \pm \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\Lambda\rho\sigma}\Big), \tag{2.2}$$

with $F^\Lambda_{\mu\nu} = \frac{1}{2}(\partial_\mu A^\Lambda_\nu - \partial_\nu A^\Lambda_\mu)$. In this notation, A^0 is the graviphoton and A^Λ, with $\Lambda = 1,\dots,n_{\rm V}$, denote the vectors in the vector multiplets. The period matrix $\mathcal{N}_{\Lambda\Sigma}$ is a function of the vector multiplet scalars.

$g_{i\bar{j}}$ and $h_{\rm uv}$ are the metrics on the scalar manifolds $\mathcal{M}_{\rm SK}$ and $\mathcal{M}_{\rm Q}$, respectively. The covariant derivatives are defined as

$$\nabla_\mu z^i = \partial_\mu z^i + k^i{}_\Lambda A^\Lambda_\mu\,, \tag{2.3}$$

$$\nabla_\mu q^u = \partial_\mu q^u + k^u{}_\Lambda A^\Lambda_\mu\,, \tag{2.4}$$

where k^i_Λ and k^u_Λ are the Killing vectors associated to the isometries of the vector and hypermultiplet scalar manifold, respectively, that have been gauged.

In general, the gauge group can be at most a $(1+n_{\rm V})$-dimensional subgroup G of the isometry group of the scalar manifold $\mathcal{M}_{\rm SK}\times\mathcal{M}_{\rm Q}$. If the subgroup G is non-abelian, it must necessary involve gaugings of the isometries of the vector multiplet space. In this paper we will work with abelian gaugings, so the vector multiplets are neutral and G can be identified with $1+n_{\rm V}$ isometries of the quaternionic manifold. The Killing vector fields which generate these isometries admit a prepotential called the Killing prepotential. This is a set of real functions P^x_Λ, where $x=1,2,3$ is an adjoint SU(2) index, satisfying

$$\Omega^x_{\rm uv} k^u_\Lambda = -\nabla_v P^x_\Lambda\,, \tag{2.5}$$

where $\Omega^x_{\rm uv}$ and ∇_v are the curvature and covariant derivative on $\mathcal{M}_{\rm Q}$ (see appendix B for more details on the Killing prepotentials).

The scalar potential couples the hyper and vector multiplets, and is given by

$$\mathcal{V}(z,\bar{z},q) = (g_{i\bar{j}}k^i_\Lambda k^{\bar{j}}_\Sigma + 4h_{uv}k^u_\Lambda k^v_\Sigma)\bar{L}^\Lambda L^\Sigma + (f^\Lambda_i g^{i\bar{j}} f^\Sigma_{\bar{j}} - 3\bar{L}^\Lambda L^\Sigma)\mathcal{P}^x_\Lambda \mathcal{P}^x_\Sigma\,, \tag{2.6}$$

where L^Λ are the symplectic sections[4] on $\mathcal{M}_{\rm SK}$ and $f^\Lambda_i = (\partial_i + \frac{1}{2}\partial_i K)L^\Lambda$, where K is the vector multiplet Kähler potential.

The full Lagrangian is invariant under $\mathcal{N}=2$ supersymmetry, with supersymmetry variations for the fermionic fields given by

$$\delta\psi_{\mu A} = \mathcal{D}_\mu \epsilon_A + iS_{AB}\gamma_\mu \epsilon^B + 2i(\operatorname{Im}\mathcal{N})_{\Lambda\Sigma} L^\Sigma \mathcal{F}^{-\Lambda}_{\mu\nu}\gamma^\nu \epsilon_{AB}\epsilon^B\,, \tag{2.8}$$

$$\delta\lambda^{iA} = i\nabla_\mu z^i \gamma^\mu \epsilon^A - g^{i\bar{j}}\overline{f}^\Sigma_{\bar{j}}\left(\operatorname{Im}\mathcal{N}\right)_{\Sigma\Lambda}\mathcal{F}^{-\Lambda}_{\mu\nu}\gamma^{\mu\nu}\epsilon^{AB}\epsilon_B + W^{iAB}\epsilon_B\,, \tag{2.9}$$

$$\delta\zeta_\alpha = i\mathcal{U}^{B\beta}_u \nabla_\mu q^u\, \gamma^\mu \epsilon^A \epsilon_{AB}\epsilon_{\alpha\beta} + N^A_\alpha \epsilon_A\,, \tag{2.10}$$

where $\mathcal{U}^{B\beta}_u$ are the vielbeine on the quaternionic manifold and

$$\begin{aligned} S_{AB} &= \frac{\rm i}{2}(\sigma_x)_A{}^C \epsilon_{BC}\mathcal{P}^x_\Lambda L^\Lambda\,,\\ W^{iAB} &= \epsilon^{AB}\, k^i_\Lambda \bar{L}^\Lambda \,+\, {\rm i}(\sigma_x)_C{}^B \epsilon^{CA}\mathcal{P}^x_\Lambda g^{ij^\star}\, \bar{f}^\Lambda_{j^\star}\,,\\ \mathcal{N}^A_\alpha &= 2\,\mathcal{U}^A_{\alpha u}\, k^u_\Lambda\, \bar{L}^\Lambda\,. \end{aligned} \tag{2.11}$$

In particular the covariant derivative on the spinors contains a contribution from the gauge fields

$$\mathcal{D}_\mu \epsilon_A = D_\mu \epsilon_A + \frac{i}{2}(\sigma^x)_A{}^B A^\Lambda_\mu P^x_\Lambda \epsilon_B\,. \tag{2.12}$$

[4] In the paper we will use both the symplectic sections (L^Λ, M_Λ) and the holomorphic sections

$$(X^\Lambda, F_\Lambda) = e^{-K/2}(L^\Lambda, M_\Lambda)\,. \tag{2.7}$$

In some cases which we analyse later, the supersymmetry variations are not sufficient to fully determine the solutions, so we also list here the equations of motions. The Einstein equation is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}\,, \tag{2.13}$$

where the energy momentum tensor is given by

$$\begin{aligned} T_{\mu\nu} &= -g_{\mu\nu}\Big[g_{i\bar{j}}\nabla^\rho z^i \nabla_\rho \bar{z}^{\bar{j}} + h_{uv}\nabla^\rho q^u \nabla_\rho q^v + \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\rho\sigma}F^{\Sigma\rho\sigma} - \mathcal{V}(z,\bar{z},q)\Big] \\ &\quad +2\Big[g_{i\bar{j}}\nabla^\mu z^i \nabla_\nu \bar{z}^{\bar{j}} + h_{uv}\nabla_\mu q^u \nabla_\nu q^v + 2\,\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\rho}F^{\Sigma\rho}_{\nu}\Big]\,. \end{aligned} \tag{2.14}$$

The equations of motion for the gauge fields are

$$\partial_\mu(\sqrt{-g}\,\mathrm{Im}\;G_\Lambda^{-\mu\nu}) = -\frac{\sqrt{-g}}{2}(g_{i\bar{j}}k^i{}_\Lambda k^{\bar{j}}{}_\Sigma A^{\Sigma\nu} + h_{uv}k^u{}_\Lambda k^v{}_\Sigma A^{\Sigma\nu})\,, \tag{2.15}$$

with $G_\Lambda^{-\mu\nu} = \bar{\mathcal{N}}_{\Lambda\Sigma}F^{\Lambda\mu\nu}$.

2.1 The canonical model

Having set up the general machinery, we now specialize to a particularly simple example of $\mathcal{N} = 2$ gauged supergravity in four dimensions, namely that of one vector multiplet ($n_{\mathrm{V}} = 1$) and one hypermultiplet ($n_{\mathrm{H}} = 1$). In spite of its simplicity, we will see that it exhibits a rich spectrum of supersymmetric solutions. In this case we take the scalar manifold of the theory to be

$$\mathcal{M}_{\mathrm{SK}} \times \mathcal{M}_{\mathrm{Q}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times\mathrm{U}(1)}\,. \tag{2.16}$$

The vector multiplet sector contains a single complex scalar and a natural choice of coordinates on $\mathcal{M}_{\mathrm{SK}}$ is $z = \tau$, where τ parametrizes the upper-half plane. With this choice, the Kähler potential and the metric are

$$K = -3\log\left[i(\tau - \overline{\tau})\right]\,, \tag{2.17}$$

$$\mathrm{d}s^2 = \frac{3}{4}\frac{\mathrm{d}\tau\mathrm{d}\bar{\tau}}{(\mathrm{Im}\;\tau)^2}\,. \tag{2.18}$$

The four scalars of the hypermultiplet parametrise $\mathcal{M}_{\mathrm{Q}}$. There are several alternative way to choose coordinates on $\mathcal{M}_{\mathrm{Q}}$. In this paper, depending on the model we study, we will consider the following possibilities

a) two complex coordinates

$$\{q^u\} \quad \leftrightarrow \quad (\zeta_1, \zeta_2)\,. \tag{2.19}$$

With this parametrisation the metric on $\mathcal{M}_Q$ becomes

$$\mathrm{d}s^2 = \frac{\mathrm{d}\zeta_1\mathrm{d}\overline{\zeta}_1 + \mathrm{d}\zeta_2\mathrm{d}\overline{\zeta}_2}{1 - |\zeta_2|^2 - |\zeta_2|^2} + \frac{(\zeta_1\mathrm{d}\overline{\zeta}_1 + \zeta_2 d\overline{\zeta}_2)(\overline{\zeta}_1\mathrm{d}\zeta_1 + \overline{\zeta}_2\mathrm{d}\zeta_2)}{(1 - |\zeta_2|^2 - |\zeta_2|^2)^2}. \tag{2.20}$$

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b) one complex and two real coordinates[5]

$$\{q^u\} \quad \leftrightarrow \quad (\xi, \rho, \sigma)\,. \tag{2.21}$$

Then the metric on $\mathcal{M}_Q$ is

$$\mathrm{d}s^2 = \frac{1}{4\rho^2}(\mathrm{d}\rho)^2 + \frac{1}{4\rho^2}[\mathrm{d}\sigma - i(\xi\mathrm{d}\bar{\xi} - \bar{\xi}\mathrm{d}\xi)]^2 + \frac{1}{\rho}\mathrm{d}\xi\mathrm{d}\bar{\xi}\,. \tag{2.22}$$

The $\mathcal{N} = 2$ theory is completely determined only after the gaugings and the sections (X^Λ, F_Λ) have been fixed. These may or may not be compatible with a prepotential $\mathcal{F}(X^\Lambda)$ [47], in that $F_\Lambda = \partial_{X_\Lambda}\mathcal{F}$. Typically, Kaluza-Klein reductions of string or M-theory, lead to four dimensional effective actions with a prepotential that is a cubic function of the X^Λ, and both electric and magnetic gaugings. Indeed it is by now well established that internal fluxes can generate magnetic gaugings in the lower dimensional theory. One can keep both electric and magnetic gaugings by considering Lagrangians where the hypermultiplet scalars corresponding to the symmetries that are magnetically realised are dualised into tensors (see for instance [48–50]). However, it is always possible to transform a generic dyonic gauging into a purely electric one, by a symplectic transformation on the sections (X^Λ, F_Λ)

$$(X^\Lambda, F_\Lambda) \quad \mapsto \quad (\tilde{X}^\Lambda, \tilde{F}_\Lambda) = \mathcal{S}(X^\Lambda, F_\Lambda)\,, \tag{2.23}$$

where the matrix

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{2.24}$$

is an element of $\mathrm{Sp}(2+2n_\mathrm{V}, \mathbb{R})$. This transformation leaves the Kähler potential invariant, but changes the period matrix $\mathcal{N}_{\Lambda\Sigma}$ by a fractional transformation

$$\mathcal{N}_{\Lambda\Sigma}(X,F) \quad \mapsto \quad \tilde{\mathcal{N}}_{\Lambda\Sigma}(\tilde{X},\tilde{F}) = (C + D\mathcal{N}_{\Lambda\Sigma}(X,F))(A + B\mathcal{N}_{\Lambda\Sigma}(X,F))^{-1}\,. \tag{2.25}$$

Our strategy will then be to consider purely electric gaugings, allowing for sections $(\widetilde{X}^\Lambda, \widetilde{F}_\Lambda)$ which are a general symplectic rotation of those obtained from the cubic prepotential. More precisely we start by choosing a cubic prepotential

$$\mathcal{F} = -\frac{X_1^3}{X_0}\,, \tag{2.26}$$

with sections $(\Lambda = 0, 1)$

$$\begin{aligned} X^\Lambda &= (1, \tau)\,, \\ F_\Lambda &= (\tau^3, -3\tau^2)\,. \end{aligned} \tag{2.27}$$

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[5] These two co-ordinate systems are related by

$$\xi = \frac{\zeta_2}{1+\zeta_1}\,, \qquad \rho = \frac{1 - |\zeta_1|^2 - |\zeta_2|^2}{|1+\zeta_1|^2}\,, \qquad \sigma = \frac{i(\zeta_1 - \overline{\zeta}_1)}{|1+\zeta_1|^2}\,.$$

The period matrix[6] in this case takes the form

$$\mathcal{N}_{\Lambda\Sigma} = \frac{1}{2}\begin{pmatrix} -\tau^3 - 3\tau^2\bar{\tau} & 3\tau(\tau+\bar{\tau}) \\ 3\tau(\tau+\bar{\tau}) & -3(3\tau+\bar{\tau}) \end{pmatrix}. \tag{2.29}$$

This choice corresponds to electric gaugings for both the graviphoton and the vector A^1. A dyonic configuration where the graviphoton is electrically gauged and A^1 magnetically will correspond to a symplectic rotation with

$$\mathcal{S}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{2.30}$$

while the converse case, with a magnetic graviphoton and electric A^1, is obtained by setting

$$\mathcal{S}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.31}$$

Finally, purely magnetic gaugings correspond to the rotation

$$\mathcal{S}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}. \tag{2.32}$$

We will see that these four different cases lead to very different patterns of solutions. The first and fourth cases, (2.27) and (2.32), allow for families of $\mathrm{Lif}_4(z)$ vacua, while the second and third, (2.30) and (2.31), give families of AdS_4 and $\mathrm{Schr}_4(z)$ vacua.

As already mentioned, we only consider abelian gaugings of the hypermultiplet isometries. From (2.16), it is easy to see that the isometry group of the quaternionic manifold is $\mathrm{SU}(2,1)$. The full set of corresponding Killing vectors and prepotentials are presented in appendix B. In fact we will just utilize the following three Killing vectors

$$\begin{aligned} k_3 &= -\frac{i}{2}(-\zeta_1\partial_{\zeta_1} + \zeta_2\partial_{\zeta_2} - c.c.)\,, \\ k_4 &= -\frac{i}{2}(\zeta_1\partial_{\zeta_1} + \zeta_2\partial_{\zeta_2} - c.c.)\,, \\ k_6 &= \frac{i}{2}[(1+\zeta_1^2)\partial_{\zeta_1} + \zeta_1\zeta_2\partial_{\zeta_2} - c.c.]\,. \end{aligned} \tag{2.33}$$

[6] When the holomorphic sections are compatible with the existence of a holomorphic prepotential $\mathcal{F}(X)$ such that $F_\Lambda = \partial_\Lambda\mathcal{F}(X)$, the gauge kinetic matrix $\mathcal{N}_{\Lambda\Sigma}$ can be written in terms of derivatives of the prepotential

$$\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + 2i\frac{\mathrm{Im}F_{\Lambda\mathrm{M}}\,\mathrm{Im}F_{\Sigma\Upsilon}X^{\mathrm{M}}X^{\Upsilon}}{\mathrm{Im}F_{\mathrm{M}\Upsilon}X^{\mathrm{M}}X^{\Upsilon}}. \tag{2.28}$$

where $F_{\Lambda\Sigma} = \partial^2_{\Lambda\Sigma}\mathcal{F}$.

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The vector fields (k_3, k_4) generate compact isometries while k_6 is a non-compact generator.

Since the theory contains two vectors, the graviphoton and the vector in the vector multiplet, we can gauge at most two isometries. We will consider in detail the case of two compact gaugings, and also the case of one compact and one non-compact gauging:

1. **Two compact gaugings**

   This case is best described choosing complex coordinates, (2.19), for the hypermultiplet. We choose the two U(1)'s associated with the Killing vectors k_3 and k_4, which correspond to rotations of the phases of the coordinates ζ_1 and ζ_2. The two gaugings are defined as

$$k_\Lambda = a_\Lambda \frac{(-k_3 + k_4)}{2} + b_\Lambda \frac{(k_3 + k_4)}{2}\,, \tag{2.34}$$

   where a_Λ and b_Λ are the electric (magnetic) charges. The corresponding Killing prepotentials are

$$\mathcal{P}^x_\Lambda = a_\Lambda P^x_a + b_\Lambda P^x_b\,, \tag{2.35}$$

   with

$$P_a = \frac{1}{(|\zeta_1|^2 + |\zeta_2|^2)\sqrt{1 - |\zeta_1|^2 - |\zeta_2|^2}} \begin{pmatrix} -\mathrm{Im}\,(\zeta_1\zeta_2) \\ \mathrm{Re}\,(\zeta_1\zeta_2) \\ \frac{-|\zeta_2|^4 + |\zeta_2|^2 - |\zeta_1|^2(1+|\zeta_2|^2)}{2\sqrt{1-|\zeta_1|^2-|\zeta_2|^2}} \end{pmatrix}\,, \tag{2.36}$$

$$P_b = \frac{1}{(|\zeta_1|^2 + |\zeta_2|^2)\sqrt{1 - |\zeta_1|^2 - |\zeta_2|^2}} \begin{pmatrix} \mathrm{Im}\,(\zeta_1\zeta_2) \\ -\mathrm{Re}\,(\zeta_1\zeta_2) \\ \frac{-|\zeta_1|^4 + |\zeta_1|^2 - |\zeta_2|^2(1+|\zeta_1|^2)}{2\sqrt{1-|\zeta_1|^2-|\zeta_2|^2}} \end{pmatrix}\,. \tag{2.37}$$

2. **One compact and one non compact gauging**

   For this choice, the coordinates (2.21) are more suitable. The compact isometry corresponds to the sum of the two commuting compact generators k_3 and k_4

$$k_\xi = i(\xi\partial_\xi - \bar{\xi}\partial_{\bar{\xi}}) = -(k_3 + k_4) \tag{2.38}$$

   while the non compact isometry corresponds to shifts of the coordinate σ

$$k_\sigma = \partial_\sigma = -\frac{1}{2}k_3 + \frac{3}{2}k_4 - k_6\,. \tag{2.39}$$

   The corresponding Killing prepotentials are given by

$$P_\sigma = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2\rho} \end{pmatrix} \qquad\qquad P_\xi = \begin{pmatrix} \frac{\xi+\bar{\xi}}{\sqrt{\rho}} \\ \frac{\bar{\xi}-\xi}{i\sqrt{\rho}} \\ \frac{|\xi|^2}{\rho} - 1 \end{pmatrix}\,. \tag{2.40}$$

   Then we define the generic gauging as

$$k_\Lambda = a_\Lambda k_\sigma + b_\Lambda k_\xi\,, \qquad \Lambda = 0, 1\,, \tag{2.41}$$

   with Killing prepotential

$$\mathcal{P}^x_\Lambda = a_\Lambda P^x_\sigma + b_\Lambda P^x_\xi\,. \tag{2.42}$$

As already mentioned in the Introduction, one could a priori also consider gauging two non compact isometries of the hypermultiplet manifold, (2.16). These can be chosen to be the shift of the coordinate σ as defined in (2.39) and

$$k_2 = \frac{1}{2}[\partial_\xi + \partial_{\bar{\xi}} - i(\xi - \bar{\xi})\partial_\sigma]\,, \tag{2.43}$$

corresponding to the sum of the Killing vectors k_2 and k_7 in (B.1). Such gaugings appear naturally in some dimensional reduction of type IIA theory on coset and Nearly Kähler manifolds and provide examples of $\mathcal{N} = 1$ AdS vacua [37, 38, 51]. However, we have found neither interesting supersymmetric Lifshitz and Schrödinger solutions nor $\mathcal{N} = 2$ AdS vacua in the case of two non compact gaugings.

3 Supersymmetric Lifshitz solutions

In this section we compute supersymmetric $\mathrm{Lif}_4(z)$ solutions of $\mathcal{N} = 2$ gauged supergravity with one hyper-and one vector multiplet. But before reducing to this simple case, we consider some general features of Lifshitz solutions which hold for a generic number of multiplets. A four-dimensional space-time with Lifshitz symmetry of degree z

$$(t\,, x\,, y\,, r\,) \to (\lambda^z\, t\,, \lambda\, x\,, \lambda\, y\,, \lambda^{-1}\, r\,)\,, \tag{3.1}$$

is given by [2]

$$\mathrm{d}s^2 = R^2\left(r^{2z}\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{r^2} - r^2\mathrm{d}x^2 - r^2\mathrm{d}y^2\right)\,. \tag{3.2}$$

In order to preserve the scaling symmetry, all the scalar fields z^i and q^u must be constant

$$z^i = z^i_0\,, \qquad\qquad q^u = q^u_0. \tag{3.3}$$

The interesting terms in the Lagrangian (2.1) (i.e. setting all fermions and scalar derivatives to zero) are

$$-\frac{1}{2}\mathcal{R} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\mu\nu\,\Sigma} + h_{\mathrm{uv}}k^u_\Lambda k^v_\Sigma A^\Lambda A^\Sigma - \mathcal{V}(z,\bar{z},q)\,. \tag{3.4}$$

As discussed above we will consider exclusively the case of Abelian gaugings and thus only the scalars in the hypermultiplets are charged.

We look for solutions where the gauge fields have only temporal component [2, 10]

$$A^\Lambda_t = r^z A^\Lambda\,, \tag{3.5}$$

so that the only non trivial component of the gauge field strength is

$$F^\Lambda_{r\,t} = \frac{z}{2}A^\Lambda r^{z-1}\,. \tag{3.6}$$

Einstein's equations are then algebraic

$$\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}A^\Lambda A^\Sigma = -\frac{(z-1)}{z}R^2\,, \tag{3.7}$$

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$$h_{uv}k^u{}_\Lambda k^v{}_\Sigma A^\Lambda\, A^\Sigma = (z-1)\,, \tag{3.8}$$

$$V = -\frac{z^2+z+4}{2\,R^2}\,, \tag{3.9}$$

and Maxwell's equations reduce to

$$h_{uv}k^u_\Lambda k^v_\Sigma A^\Sigma = -\frac{z}{R^2}\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}A^\Sigma\,. \tag{3.10}$$

Notice that not all these equations are independent. By contracting Maxwell's equations with A^Λ we recover one component of Einstein's equation. Finally, the equations of motion for the scalar fields require

$$\partial_{z^i}\mathcal{V}_{\text{eff}} = 0\,, \qquad \partial_{q^u}\mathcal{V}_{\text{eff}} = 0\,, \tag{3.11}$$

where we have defined the effective potential

$$\mathcal{V}_{\text{eff}}(z,\bar{z},q) = \mathcal{V}(z,\bar{z},q) - \frac{z^2}{R^4}\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}A^\Lambda A^\Sigma + \frac{2}{R^2}h_{uv}k^u_\Lambda k^v_\Sigma(\zeta)A^\Lambda A^\Sigma\,. \tag{3.12}$$

Due to the nature of our ansatz, we obtain algebraic, not differential, equations for certain real constants. The number of equations precisely matches the number of unknowns: we have n_V constants A^Λ corresponding to the electric profile for each gauge field and n_V constraints from Maxwell's equations. We have $n_s = 2n_V + 4n_H$ constants $z^i_0\,, q^u_0$ from each real scalar $z^i\,, q^u$ and n_s constraints coming from the derivatives of $V_{\text{eff}} = 0$. We have three constraints from the R_{xx}, R_{00}, R_{rr} components of Einstein's equation but one of these is implied by tracing over the Maxwell equations. Now since there are two constants in the gravity theory (z,Λ), in total we have $n_V + n_s + 2$ equations for the same number of constants.

3.1 Conditions for supersymmetric Lifshitz solutions

3.1.1 The gravitino equation

With the choice of frames

$$e^0 = Rr^z\mathrm{d}t\,,\; e^1 = Rr\mathrm{d}x\,,\; e^2 = Rr\mathrm{d}y\,,\; e^3 = R\frac{\mathrm{d}r}{r}\,, \tag{3.13}$$

the 0-,1- and 3-components of the gravitino equation (2.8) are[7]

$$\gamma^0\partial_0\epsilon_A + \frac{i}{2R}(\sigma^x)_A{}^B A^\Lambda P^x_\Lambda\gamma^0\epsilon_B + \frac{z}{2R}\gamma^3\epsilon_A - \frac{iz}{2R^2}\mathcal{N}\epsilon_{AB}\gamma^{03}\epsilon^B + iS_{AB}\epsilon^B = 0\,, \tag{3.14}$$

$$\gamma^1\partial_1\epsilon_A + \frac{1}{2R}\gamma^3\epsilon_A + \frac{iz}{2R^2}\mathcal{N}\epsilon_{AB}\gamma^{03}\epsilon^B + iS_{AB}\epsilon^B = 0\,, \tag{3.15}$$

$$\gamma^3\partial_3\epsilon_A - \frac{iz}{2R^2}\mathcal{N}\epsilon_{AB}\gamma^{03}\epsilon^B + iS_{AB}\epsilon^B = 0\,, \tag{3.16}$$

where we have defined

$$\mathcal{N} \equiv \mathrm{Im}\mathcal{N}_{\Sigma\Lambda}L^\Lambda A^\Sigma\,. \tag{3.17}$$

[7] Recall that in our conventions the gauge field configurations entering the supersymmetry variations are given by (3.5) and (3.6), so that $\mathcal{F}^{\Lambda\,-}_{rt} = \frac{z}{4}A^\Lambda r^{z-1}$ and $\mathcal{F}^{\Lambda\,-}_{xy} = -\frac{iz}{4}A^\Lambda r^2$.

When $z=1$ we recover AdS$_4$-spacetime which requires a separate treatment (see section 4), here will restrict to the Lifshitz case $z>1$. We choose a radial profile for the supersymmetry parameters[8]

$$\epsilon_A = r^{\frac{a}{2}} \epsilon_A^0 \tag{3.18}$$

where ϵ_A^0 is a constant spinor. Comparing equations (3.14) and (3.16) we find

$$\frac{i}{2R} P_\Lambda^x A^\Lambda (\sigma^x)_A{}^B \gamma^0 \epsilon_B = \frac{a-z}{2R} \gamma^3 \epsilon_A \,, \tag{3.19}$$

which by compatibility immediately implies $z=a$ and the constraint

$$P_\Lambda^x A^\Lambda = 0 \,. \tag{3.20}$$

The remaining gravitino equations now give

$$\epsilon_A = \frac{4iR}{1+z} S_{AB} \gamma^3 \epsilon^B \,, \tag{3.21}$$

$$\epsilon_A = -\frac{2iz}{R(z-1)} \mathcal{N} \gamma^0 \epsilon_{AB} \epsilon^B . \tag{3.22}$$

There are also some compatibility conditions associated to these projectors. First, by squaring (3.22) we find

$$|\mathcal{N}|^2 = \frac{(z-1)^2 R^2}{4z^2} \quad \Rightarrow \quad \mathcal{N} = e^{i\theta} \frac{(z-1)R}{2z} \,. \tag{3.23}$$

The phase $e^{i\theta}$ can be reabsorbed with a Kähler transformation in the Lagrangian and we will set it to one in the following.[9]

We thus have

$$\epsilon_A = -i \epsilon_{AB} \gamma^0 \epsilon^B \,. \tag{3.24}$$

The mutual compatibility of the γ^3 and γ^0 projections gives

$$\epsilon_A = H_A{}^B \gamma^{30} \epsilon_B \,, \tag{3.25}$$

with

$$H_A{}^B = \frac{4R}{1+z} S_{AC} \epsilon^{CB} \equiv h^x (\sigma_x)_A{}^B \,, \tag{3.26}$$

where

$$h^x = \frac{2R\,i}{1+z} \mathcal{P}_\Lambda^x L^\Lambda \,. \tag{3.27}$$

[8]A phase in the spinor, $\epsilon_A \sim r^{\frac{a+if}{2}} \epsilon_A^0$, is forbidden by the simultaneous presence of ϵ^A and its conjugate ϵ_A in the supersymmetry conditions and by the fact that the scalars have no radial profile. An r-dependent phase in ϵ_A and in the phase θ, that will be shortly introduced, presumably plays a role in solutions describing renormalization group flows. We should note indeed the similarity of the Lif$_4(z)$ supersymmetry conditions with analogous ones for black holes [52].

[9]Such Kähler transformation is equivalent to a redefinition of the sections L^Λ. In the search for the most general solution the sections in (2.27) should be allowed to have an arbitrary overall phase $e^{i\theta}$. However, we have found no interesting Lifschitz solutions with $\theta \neq 0$.

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By squaring (3.21) and (3.25) we obtain

$$\frac{16R^2}{(1+z)^2} S_{AB} S^{BC} = \delta_A^C \,, \qquad \Longrightarrow \qquad \sum_{i=1}^{3} |h^x|^2 = 1 \,,$$
$$H_A{}^B H_B{}^C = \delta_A{}^C \,, \qquad \Longrightarrow \qquad \sum_{x=1}^{3} (h^x)^2 = 1 \,. \tag{3.28}$$

These conditions immediately imply $\sum_{i=1}^{3} (\mathrm{Im} h^x)^2 = 0$ and thus

$$\mathrm{Im}\; h^x = 0 \;, \tag{3.29}$$

requiring that h^x is a real three-vector of length one. Consequently $H_A{}^B$ is an hermitian matrix with eigenvalues ± 1.

The conditions of supersymmetry are now fully compatible and can be reduced to the canonical form

$$\begin{aligned} \tilde{\epsilon}_A &= (\sigma^3)_A^B \gamma^0 \gamma^3 \tilde{\epsilon}_B \,, \\ \tilde{\epsilon}_A &= -i \epsilon_{AB} \gamma^0 \tilde{\epsilon}^B \,, \end{aligned} \tag{3.30}$$

by a unitary change of basis, thus demonstrating that any solution would be $\frac{1}{4}$-BPS or, in other words, preserves two real supercharges. $\bar{\epsilon}\gamma^\mu \epsilon$ gives the Killing vector ∂/∂_t as expected, in agreement with what found in ten-dimensional solutions [17].

3.1.2 The gaugino and hyperino equations

The gaugino equations give

$$W^{i\,AB} \epsilon_B + \frac{z}{R^2} \mathcal{N}^i \gamma^0 \gamma^3 \epsilon^{AB} \epsilon_B = 0 \,, \tag{3.31}$$

where

$$\mathcal{N}^i \equiv g^{i\bar{j}} \bar{f}_{\bar{j}}^{\Sigma} \mathrm{Im} \mathcal{N}_{\Sigma\Lambda} A^\Lambda \,, \tag{3.32}$$

and the gamma matrices can be eliminated using the gravitino conditions to yield

$$\left(-\frac{z}{R^2} \mathcal{N}^i \epsilon^{AC} H_C{}^B + W^{i\,AB} \right) \epsilon_B = 0 \,. \tag{3.33}$$

The hyperino variations give

$$R \mathcal{N}_\alpha^A \epsilon_A + i \mathcal{U}_u^{B\beta} k_\Lambda^u A^\Lambda \epsilon_{AB} \epsilon_{\alpha\beta} \gamma^0 \epsilon^A = 0 \,, \tag{3.34}$$

and, once more, by eliminating the gamma matrices we obtain

$$\left(\epsilon_{\alpha\beta} \mathcal{U}_u^{A\beta} k_\Lambda^u A^\Lambda + R \mathcal{N}_\alpha^A \right) \epsilon_A = 0 \,. \tag{3.35}$$

Since the spinors (ϵ_1, ϵ_2) are independent in a $\frac{1}{4}$-BPS solution, the matrix expressions in brackets should vanish identically and the hyperino equation becomes

$$k_\Lambda^u \left(A^\Lambda + 2R\, \bar{L}^\Lambda \right) = 0 \,. \tag{3.36}$$

3.1.3 Some general properties of the supersymmetry conditions

In total, the full set of conditions for supersymmetric Lifshitz solutions is

$$\sum_{x=1}^{3}(h^x)^2 = 1\,, \tag{3.37}$$

$$\mathrm{Im}\ h^x = 0\,, \tag{3.38}$$

$$P^x_\Lambda A^\Lambda = 0\,, \tag{3.39}$$

$$\mathcal{N} = \frac{(z-1)R}{2z}\,, \tag{3.40}$$

$$W^{i\,AB} = \frac{z}{R^2}\mathcal{N}^i \epsilon^{AC} H_C{}^B\,, \tag{3.41}$$

$$k^u_\Lambda\left(A^\Lambda + 2R\,\bar{L}^\Lambda\right) = 0\,. \tag{3.42}$$

A simple way to solve equation (3.42) would be to set $A^\Lambda = -2R\bar{L}^\Lambda$. Using equation (3.40), we obtain

$$-\frac{1}{2}\mathrm{Im}\mathcal{N}_{\Sigma\Lambda}A^\Lambda A^\Sigma = \frac{z-1}{2z}R^2\,, \tag{3.43}$$

which correctly reproduces the equation of motion (3.7). However the condition

$$\mathrm{Im}\mathcal{N}_{\Sigma\Lambda}L^\Lambda \bar{L}^\Sigma \equiv -1/2\,, \tag{3.44}$$

which is valid for all $\mathcal{N}=2$ supergravities, gives the unphysical value $z=-1$. We conclude that, in order to find interesting $\mathrm{Lif}_4(z)$ solutions, we need to find loci on the hypermultiplet manifold where the Killing vectors k^u_Λ degenerate or become aligned.

In the following we deal with cases where, on the relevant scalar locus, $\mathcal{P}^x_\Lambda$ points in a particular direction in the x space, say the $x=3$ direction. Then $h^1=h^2=0$ and we need to require $h^3=1$. The full set of gravitino conditions become

$$P^3_\Lambda A^\Lambda = 0\,, \qquad 2iR\,P^3_\Lambda L^\Lambda = z+1\,, \qquad \frac{2z}{R}\mathrm{Im}\mathcal{N}_{\Sigma\Lambda}L^\Lambda A^\Sigma = z-1\,. \tag{3.45}$$

We should also impose the gaugino and hyperino conditions (3.41) and (3.42)

$$iP^3_\Lambda \bar{f}^\Lambda_{\bar{j}} + \frac{z}{R^2}\mathrm{Im}\mathcal{N}_{\Sigma\Lambda}\bar{f}^\Lambda_{\bar{j}}A^\Sigma = 0\,, \qquad k^u_\Lambda\left(A^\Lambda + 2R\,\bar{L}^\Lambda\right) = 0\,, \tag{3.46}$$

and the Maxwell equations (3.10).

3.2 Lif$_4$(z) vacua from canonical gaugings

We now restrict our analysis to the theory with only one vector and one hypermultiplet and show that there is a $\mathrm{Lif}_4(z)$ solution in the case of a cubic prepotential with purely electric gaugings or the case of the symplectic rotation (2.32), which is equivalent to purely magnetic gaugings. These solutions exist only for $z=2$ but with very mild constraints on the gauging parameters.

3.2.1 Compact gaugings

We first consider purely electric gaugings. Consider first the hyperino variation (3.42)

$$[a_0(A^0+2R\bar{L}^0)+a_1(A^1+2R\bar{L}^1)]\zeta_1=0\,, \tag{3.47}$$

$$[b_0(A^0+2R\bar{L}^0)+b_1(A^1+2R\bar{L}^1)]\zeta_2=0\,. \tag{3.48}$$

As mentioned above, a solution with $\zeta_1\neq 0$ and $\zeta_2\neq 0$ leads to unphysical values for z. Similarly the choice $\zeta_1=\zeta_2=0$ does not lead to a solution since all quantities in the previous formulae are real except for

$$L(\tau)=e^{K/2}(1,\tau)\,, \tag{3.49}$$

and Im $\tau=0$ is a singular point of the metric (2.18). We therefore conclude that either

$$(\zeta_1,b_1)=(0,0)\quad\text{or}\quad(\zeta_2,a_1)=(0,0). \tag{3.50}$$

These two choices are clearly symmetric and we choose the latter. On the locus $\zeta_2=0$ the only non zero component of the Killing prepotentials is $\mathcal{P}^3_\Lambda$.

From the gravitino conditions (3.45), the gaugino and hyperino conditions (3.46), and Maxwell's equation (3.10) we find (with $b_1<0$)

$$A_0=-\frac{R}{\sqrt{2}(\mathrm{Im}\tau)^{3/2}}=\frac{432}{R^2\,b_1^3}\,, \tag{3.51}$$

$$A_1=-\frac{2\left(216\,b_0-108\,a_0\pm\sqrt{11664\,a_0^2+R^4\,b_1^6}\right)}{R^2\,b_1^4}\,, \tag{3.52}$$

$$\mathrm{Im}\,\tau=\frac{R^2\,b_1^2}{72}\,, \tag{3.53}$$

$$\mathrm{Re}\,\tau=\frac{A_1}{A_0}\,, \tag{3.54}$$

$$|\zeta_1|^2=\frac{-a_0\,A_0+b_0\,A_0+b_1\,A_1}{b_0\,A_0+b_1\,A_1}\,, \tag{3.55}$$

$$\zeta_2=0\,. \tag{3.56}$$

The only constraint on the parameters comes from $0\leq|\zeta_1|<1$, which can be satisfied for a large choice of gauging parameters. We have checked that the second order equations of motion are all satisfied. The solution found in [17, 23] falls in this class of vacua, which as we now see exists quite generally for electric gaugings with canonical prepotential.

Similar $\mathrm{Lif}_4(z)$ solutions also exist if we perform the simultaneous symplectic rotation (2.32) on A^0 and A^1, which is equivalent to a cubic prepotential with purely magnetic gaugings. The sections are now

$$L(\tau)=e^{K/2}(\tau^3,-3\tau^2) \tag{3.57}$$

and we still find a solution for

$$(\zeta_1,b_1)=(0,0)\quad\text{or}\quad(\zeta_2,a_1)=(0,0)\,. \tag{3.58}$$

Choosing again the second option we find

$$A_0 = \frac{16}{R^2\, b_1^3}\,, \tag{3.59}$$

$$A_1 = -\frac{2\sqrt{3}}{b_1}\,, \tag{3.60}$$

$$\mathrm{Im}\,\tau = \sqrt{3}\mathrm{Re}\,\tau = \frac{6}{R^2\, b_1^2}\,, \tag{3.61}$$

$$\mathrm{Re}\,\tau = -\frac{3}{4}\frac{A_0}{A_1} = \frac{2\sqrt{3}}{R^2\, b_1^2}\,, \tag{3.62}$$

$$|\zeta_1|^2 = \frac{-a_0\, A_0 + b_0\, A_0 + b_1\, A_1}{b_0\, A_0 + b_1\, A_1}\,, \tag{3.63}$$

$$\zeta_2 = 0\,, \tag{3.64}$$

and in addition we have to impose an algebraic relation between the gaugings

$$a_0 = \frac{32\, b_0^2 - 8\,\sqrt{3}\, R^2\, b_0\, b_1^3 + R^4\, b_1^6}{32\, b_0 - 4\,\sqrt{3}\, R^2\, b_1^3}\,. \tag{3.65}$$

With similar arguments one can check that there are no $\mathrm{Lif}_4(z)$ solutions with a single symplectic rotation on just A^0, (2.31), or A^1, (2.30). In particular, since the SU(3)-invariant sector of $\mathcal{N}=8$ gauged supergravity has one electric and one magnetic gauging [39] this demonstrates that there are no $\mathrm{Lif}_4(z)$ solutions in this theory.

3.2.2 One non-compact gauging

When one isometry is non compact we obtain almost identical results and thus we will be brief. The non trivial constraints from the hyperino variation are now (3.42)

$$[a_0(A^0 + 2R\bar{L}^0) + a_1(A^1 + 2R\bar{L}^1)] = 0\,, \tag{3.66}$$

$$[b_0(A^0 + 2R\bar{L}^0) + b_1(A^1 + 2R\bar{L}^1)]\xi = 0\,. \tag{3.67}$$

In the case of electric gaugings with $L(\tau) = e^{K/2}(1,\tau)$ we can solve the hyperino equations with $\xi = 0$ and a_1=0. The other equations then require

$$A_0 = -\frac{R}{\sqrt{2}(\mathrm{Im}\tau)^{3/2}} = -\frac{54}{R^2\, b_1^3}\,, \tag{3.68}$$

$$A_1 = \frac{54\, b_0}{R^2\, b_1^4} - \frac{1}{b_1}\,, \tag{3.69}$$

$$\mathrm{Im}\,\tau = \frac{R^2\, b_1^2}{18}\,, \tag{3.70}$$

$$\mathrm{Re}\,\tau = \frac{A_1}{A_0} = -\frac{b_0}{b_1} + \frac{R^2\, b_1^2}{54}\,, \tag{3.71}$$

$$\rho = -\frac{a_0\, A_0}{2(b_0\, A_0 + b_1\, A_1)} = -\frac{27 a_0}{R^2 b_1^3}\,, \tag{3.72}$$

$$\xi = 0\,. \tag{3.73}$$

The case of a double symplectic rotation can appear when studying type IIA solutions with Roman mass. We have $L(\tau) = e^{K/2}(\tau^3, -3\tau^2)$, we are still forced to set $a_1 = 0$. The solution is

$$\begin{aligned}
A_0 &= -\frac{2}{R^2\, b_1^3}\,, && (3.74)\\
A_1 &= \frac{\sqrt{3}}{b_1}\,, && (3.75)\\
\operatorname{Im}\tau &= \sqrt{3}\operatorname{Re}\tau = \frac{3}{2\, R^2 b_1^2}\,, && (3.76)\\
\operatorname{Re}\tau &= -\frac{3}{4}\frac{A_0}{A_1} = \frac{\sqrt{3}}{2\, R^2\, b_1^2}\,, && (3.77)\\
\rho &= -\frac{a_0\, A_0}{2(b_0\, A_0 + b_1\, A_1)} = \frac{a_0}{R^2 b_1^3}\,, && (3.78)\\
\xi &= 0\,, && (3.79)
\end{aligned}$$

with the constraint

$$b_0 = \frac{\sqrt{3}-1}{2} R^2\, b_1^3\,. \tag{3.80}$$

Once again we have found no $\mathrm{Lif}_4(z)$ solutions with a single symplectic rotation. In all cases we have checked that the second order equations of motion are satisfied.

4 Supersymmetric AdS_4 and $\mathrm{Schr}_4(z)$ solutions

We discuss now the case of AdS_4 and $\mathrm{Schr}_4(z)$ solutions. We treat them simultaneously since in our formalism the supersymmetry conditions for $\mathcal{N} = 2$ AdS_4 and $\mathrm{Schr}_4(z)$ are very similar. We will not discuss $\mathcal{N} = 1$ AdS_4 solutions,[10] where the conditions for supersymmetry typically require proportionality between ϵ^1 and ϵ^2.

4.1 The Schr_4 space-time

We first recall the form of a $\mathrm{Schr}_4(z)$ solution [3, 4]

$$\mathrm{d}s^2 = R^2\left(r^{2z}\mathrm{d}x_+^2 - 2r^2\mathrm{d}x_+\mathrm{d}x_- - \frac{\mathrm{d}r^2}{r^2} - r^2\mathrm{d}x^2\right), \tag{4.1}$$

which is invariant under the scaling symmetry

$$(x_+\,, x_-\,, x\,, r\,) \to (\lambda^z\, x_+\,, \lambda^{2-z}\, x_-\,, \lambda\, x\,, \lambda^{-1}\, r\,)\,. \tag{4.2}$$

This is a solution of Einstein's equation with a cosmological constant and a massive vector. We again set all the scalar fields $z^i\,, q^u$ to be constant and we deduce the equations of motion from the Lagrangian (3.4). The gauge fields are now

$$A_+^\Lambda = A^\Lambda r^z\,. \tag{4.3}$$

[10] Examples of this class of solutions in related contexts can be found in [37–39, 51, 53].

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Einstein's equation is

$$2h_{uv}k^u_{\ \Lambda}k^v_{\ \Sigma}A^\Lambda A^\Sigma - \frac{z^2}{R^2}\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}A^\Lambda A^\Sigma = 2z^2 - z - 1\,, \tag{4.4}$$

$$V = -\frac{3}{R^2}\,, \tag{4.5}$$

and Maxwell's equation is

$$2h_{uv}k^u_{\ \Lambda}k^v_{\ \Sigma}A^\Sigma = -\frac{z(z+1)}{R^2}\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}A^\Sigma\,. \tag{4.6}$$

In contrast to the Lifshitz solutions, there is no contribution from the gauge fields to the potential since $F_{\mu\nu}F^{\mu\nu} = A_\mu A^\mu = 0$ and thus we have the scalar equations of motion

$$\partial_{z^i}V = 0\,, \qquad \partial_{q^u}V = 0\,. \tag{4.7}$$

4.2 Conditions for supersymmetric Schr(z) solutions

It is convenient to work with the null frames

$$e^+ = \frac{1}{2}Rr^z\mathrm{d}x_+\,, \quad e^- = R\left(r^z\mathrm{d}x_+ - 2\,r^{2-z}\mathrm{d}x_-\right)\,, \quad e^2 = Rr\mathrm{d}y\,, \quad e^3 = R\frac{\mathrm{d}r}{r}\,, \tag{4.8}$$

so that the metric becomes

$$\mathrm{d}s^2 = 2e^+e^- - (e^2)^2 - (e^3)^2\,. \tag{4.9}$$

Chosing a spinor that satisfies

$$\gamma^+\epsilon^A = 0\,, \tag{4.10}$$

the components of the gravitino equation (2.8) reduce to

$$\begin{aligned}
\partial_+\epsilon_A + \frac{i}{R}(\sigma^x)_A{}^B A^\Lambda P^x_\Lambda\epsilon_B + \frac{1}{2R}\gamma^{-3}\epsilon_A - \frac{iz\mathcal{N}}{R^2}\left(\gamma^3 + i\gamma^2\right)\epsilon_{AB}\epsilon^B + iS_{AB}\gamma_+\epsilon^B &= 0\,,\\
\partial_-\epsilon_A &= 0\,,\\
\partial_2\epsilon_A - \frac{1}{2R}\gamma^{23}\epsilon_A - iS_{AB}\gamma^2\epsilon^B &= 0\,,\\
\partial_3\epsilon_A + \frac{1-z}{2R}\gamma^{-+}\epsilon_A - iS_{AB}\gamma^3\epsilon^B &= 0\,.
\end{aligned} \tag{4.11}$$

We can solve these conditions with

$$\epsilon_A = r^{\frac{2-z}{2}}\epsilon^0_A \tag{4.12}$$

and

$$iS_{AB}\epsilon^B = -\frac{1}{2R}\gamma^3\epsilon_A\,, \tag{4.13}$$

$$(\sigma^x)_A{}^B A^\Lambda P^x_\Lambda\epsilon_B = \frac{2z\mathcal{N}}{R}\gamma^3\epsilon_{AB}\epsilon^B. \tag{4.14}$$

Consistency of these equations leads to

$$\left(P^x_\Lambda A^\Lambda + 2z\mathcal{N}P^x_\Lambda\bar{L}^\Lambda\right)(\sigma^x)_A{}^B\epsilon_B = 0. \tag{4.15}$$

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Since $\gamma^+\epsilon = 0$, the gauge field contribution drops out of the gaugino and hyperino equations

$$D^{i\,AB}\epsilon_B = 0\,, \tag{4.16}$$
$$\mathcal{N}_\alpha^A \epsilon_A = 0. \tag{4.17}$$

When looking for $\mathcal{N}=2$ AdS_4 and $Schr_4(z)$ vacua we will consider the spinors (ϵ_1, ϵ_2) as independent. The conditions for supersymmetry are then

$$iS_{AB}\epsilon^B = -\frac{1}{2R}\gamma^3\epsilon_A\,, \tag{4.18}$$
$$P^x_\Lambda A^\Lambda = -2z\mathcal{N} P^x_\Lambda \bar{L}^\Lambda\,, \tag{4.19}$$
$$P^x_\Lambda \bar{f}^\Lambda_{\bar{j}} = 0\,, \tag{4.20}$$
$$k^u_\Lambda \bar{L}^\Lambda = 0\,, \tag{4.21}$$

which should be supplemented by Maxwell's equation (4.6). The spinor bilinear $\bar{\epsilon}\gamma^\mu\epsilon$ gives the Killing vector ∂/∂_-, associated with the number operator, as also found in ten-dimensional solutions [16].

The AdS solutions have $z = 1$ and $A^\Lambda = 0$. Moreover the condition $\gamma^+\epsilon_A = 0$ is superfluous and we have four independent real spinors; to these Poincaré supersymmetries we need to add the four superconformal ones which depend explicitly on (x, y, t). The AdS Killing spinors satisfy indeed $D_\mu\epsilon_A = \gamma_\mu\epsilon_A$. A class of ten dimensional Schrödinger backgrounds with $z = 2$ admit additional Poincaré and also superconformal symmetries [16]; it would be interesting to see if there is a similar phenomenon in $\mathcal{N} \geq 2$ gauged supergravities.

4.2.1 Relation between AdS_4 and $Schr_4$ vacua

A close relation between AdS and $Schr_4(z)$ vacua is expected [7, 11, 14] as has been recently discussed in great detail [24]. Here we will analyse it at the level of gauged supergravity, focusing on a theory with a single vector multiplet.

Suppose that we start with an $\mathcal{N} = 2$ AdS_4 vacuum satisfying (4.18), (4.20) and (4.21). These conditions do not depend explicitly on the vector fields A^Λ and are identical for the $\mathcal{N} = 2$ AdS_4 and $Schr(z)_4$ cases. Therefore, we would expect that for every $\mathcal{N} = 2$ AdS_4 solution there exists a corresponding supersymmetric $Schr_4(z)$ one, with the same radius R and the same value for the scalar fields, provided that the Maxwell's equations (4.6), the gravitino constraint (4.19), and the equations of motion can be satisfied for a choice of A^Λ. We now show that under mild conditions, this is the case.

Multiplying Maxwell's equations (4.6) by L^Λ and using the hyperino condition (4.21) we find $z(z+1)\mathcal{N} = 0$. Excluding uninteresting solutions with $z = 0$ or $z = -1$ we reduce the gravitino constraint (4.19) to

$$P^x_\Lambda A^\Lambda = \mathcal{N} = 0\,. \tag{4.22}$$

We see from equation (4.20) that the $\bar{f}^\Lambda_{\bar{i}}$ have a common phase. Setting

$$A^\Lambda = c f^\Lambda_i \tag{4.23}$$

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where c is a complex constant to make A^Λ real, we solve all equations in (4.22). The first equation becomes equivalent to the gaugino condition (4.20) and the second one follows from the special geometry identity $\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}L^\Lambda f_i^\Sigma = 0$ [45, 46].

Multiplying Maxwell's equations (4.6) by A^Λ and using the identity $\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\bar{f}_{\bar{j}}^\Lambda f_i^\Sigma = -\frac{1}{2}g_{i\bar{j}}$ [45, 46] we find a quadratic equation for z

$$z^2 + z - 4R^2 g^{i\bar{i}} h_{\mathrm{uv}} k_\Lambda^u k_\Sigma^v f_i^\Lambda \bar{f}_{\bar{i}}^\Sigma = 0 \tag{4.24}$$

and so we find a solution with positive z whenever $k_u^\Lambda f_i^\Lambda$ is non vanishing. This fact has a simple interpretation. In the $\mathcal{N} = 2$ AdS_4 vacuum we have a massless graviphoton and a massive vector with $m^2 = 4g^{i\bar{i}} h_{\mathrm{uv}} k_\Lambda^u k_\Sigma^v f_i^\Lambda \bar{f}_{\bar{i}}^\Sigma$, as it can be easily checked by diagonalizing the kinetic term in (3.4). The equation for z can be then written as

$$z(z+1) = (mR)^2\,. \tag{4.25}$$

We see that the exponent z is related to the mass of the vector fields in the corresponding AdS_4 vacuum, as in the original construction in [4]. Finally, the Einstein's equation (4.4) will fix the normalization of A^Λ ($z \geq 1$ is required for consistency).

This demonstrates that under mild conditions, we can associate a supersymmetric Schrödinger solution to each $\mathcal{N} = 2$ AdS_4 vacuum. These results hold for a generic number of hypermultiplets.

4.3 AdS_4 and Schr_4 vacua in the canonical model

From the Lagrangians which arise from consistent truncations of M-theory compactifications on Sasaki-Einstein manifolds [12] one finds $\mathcal{N} = 2$ AdS_4 solutions at the origin of moduli space. In our language, these gauged supergravities correspond to the case of a single symplectic rotation (2.30) with a particular choices of charges. In this section we focus again on the theory with one vector and one hypermultiplet. We show that in the case of a single symplectic rotation, (2.30) or (2.31), there are $\mathcal{N} = 2$ AdS_4 and $\mathrm{Schr}_4(z)$ vacua for a large set of gauging parameters. We found no $\mathcal{N} = 2$ AdS_4 or $\mathrm{Schr}_4(z)$ vacua in the cases of purely electric gaugings and of a double symplectic rotation, where we found $\mathrm{Lif}_4(z)$ solutions. One also finds other interesting $\mathcal{N} = 2$ AdS_4 vacua in the SU(3) sector of the $\mathcal{N} = 8$ theory [39, 44] and here we show that these vacua also exist for a very general set of gaugings.

We will discuss in details the case of a symplectic rotated prepotential corresponding to an electric-magnetic duality on the vector A^1. The case where the graviphoton is rotated is completely analogous.

4.3.1 Compact gaugings

It is still useful to start with the hyperino equation which is just $k_\Lambda^u L^\Lambda = 0$, or, explicitly,

$$[a_0 L^0(\tau) + a_1 L^1(\tau)]\zeta_1 = 0\,, \tag{4.26}$$

$$[b_0 L^0(\tau) + b_1 L^1(\tau)]\zeta_2 = 0\,. \tag{4.27}$$

Now we have $L = e^{K/2}(1, -3\tau^2)$, corresponding to an electric-magnetic duality on A^1.

We first consider $\mathcal{N}=2$ AdS_4 solutions and find two different such vacua. One is at the origin of the hypermultiplets $\zeta_1=\zeta_2=0$ and the vector multiplet scalar τ is fixed by the gaugino variation to be

$$\tau = i\sqrt{\frac{a_0-b_0}{a_1-b_1}}\,. \tag{4.28}$$

The gravitino equation simply sets the scale of R,

$$\frac{1}{R^2}=\frac{1}{2}\sqrt{(a_0-b_0)(a_1-b_1)^3}\,. \tag{4.29}$$

There is another $\mathcal{N}=2$ vacuum away from the origin. If we set $\zeta_1=0$ we can still solve the hyperino conditions by choosing

$$\tau = i\sqrt{-\frac{b_0}{3b_1}}\,. \tag{4.30}$$

The gaugino condition then fixes

$$|\zeta_2|^2=\frac{3\,a_0\,b_1+a_1\,b_0-4\,b_0\,b_1}{3\,a_0\,b_1+a_1\,b_0}\,, \tag{4.31}$$

and the gravitino equation simply sets the scale of R

$$\frac{1}{R^2}=\frac{3\sqrt{3}(a_1\,b_0-a_0\,b_1)^2}{32\sqrt{-b_0^3\,b_1}}\,. \tag{4.32}$$

There is an equivalent solution with $\zeta_2=0$.

An example of the model with one magnetic and one electric gauging is the SU(3)-invariant sector of $\mathcal{N}=8$ gauged supergravity. The values of the gauging parameters in the SU(3)-invariant sector can be determined by comparison with reference [39], where the action has been written as an $\mathcal{N}=2$ gauged supergravity. They are proportional to $(a_0,a_1)=(1,0)$ and $(b_0,b_1)=(1/2,-\sqrt{3}/2)$.[11] The vacua that we found above have $(\zeta_1,\zeta_2)=(0,0)$ and $(\zeta_1,\zeta_2)=(0,1/\sqrt{3})$ and the ratio of the values of the potential in the two vacua is equal to $3\sqrt{3}/4$. These numbers precisely correspond to those for the $\mathcal{N}=8$ vacuum with SO(8) global symmetry and the IR $\mathcal{N}=2$ solution with $\mathrm{SU}(3)\times\mathrm{U}(1)$ global symmetry in the SU(3)-invariant sector of $\mathcal{N}=8$ gauged supergravity [39]. We see that the existence of a pair of $\mathcal{N}=2$ AdS_4 vacua is quite general and holds for almost arbitrary values of the gaugings.

We now consider $\mathrm{Schr}_4(z)$ solutions. It is obvious from (4.24) that the solution in the origin, with $(\zeta_1,\zeta_2)=(0,0)$, can only give solutions with $A^\Lambda\neq 0$ in the unphysical case $z=0, z=-1$. Both vectors fields are in fact massless at the origin. On the other hand, in the case with $\zeta_1=0$ and ζ_2 given in (4.31), we can find a solution; from (4.23) and (4.24) we see that

$$A_0=\frac{3b_1}{b_0}A_1 \tag{4.33}$$

[11] To compare with the notations in reference [39] we need to perform a further (purely electric) rotation on the vectors A^0 and A^1.

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and that z solves the algebraic equation

$$z^2 + z - \frac{4(3\,a_0\,b_1 + a_1\,b_0)(3\,a_0\,b_1 + a_1\,b_0 - 4b_0 b_1)}{3(a_1\,b_0 - a_0\,b_1)^2} = 0\,. \tag{4.34}$$

The equations of motion are satisfied and one of them fixes the value of A^1. For a large choice of gauging parameters we can find physical solutions. We note that the charges corresponding to the SU(3)-invariant sector yield solutions with $z = (-2.56, 1.56)$.

The case of a rotation of the graviphoton is similar and there are analogous solutions. In the case of a cubic prepotential with electric gaugings or the case of a double electric-magnetic rotation instead we found no interesting solutions.

4.3.2 One compact and one non compact gauging

The case where one of the isometry is non compact is almost identical and we will be brief. Again there are $\mathcal{N} = 2$ AdS_4 and $\mathrm{Schr}_4(z)$ solutions for one electric and one magnetic gauging. We discuss as before the case of an electric-magnetic duality on A^1.

There is an $\mathcal{N} = 2$ AdS_4 vacuum for $\xi = 0$ and

$$\tau = i\sqrt{-\frac{a_0}{3a_1}}\,, \qquad \rho = -\frac{2a_0 a_1}{3a_1 b_0 + a_0 b_1}\,, \tag{4.35}$$

with radius

$$\frac{1}{R^2} = \frac{3\sqrt{3}(a_1\,b_0 - a_0\,b_1)^2}{8\sqrt{-a_0^3\,a_1}}\,. \tag{4.36}$$

For the same values of the scalar fields there is a $\mathrm{Schr}_4(z)$ solution with

$$A_0 = \frac{3a_1}{a_0} A_1 \tag{4.37}$$

and z determined by

$$z^2 + z - \frac{4(3\,a_1\,b_0 + a_0\,b_1)^2}{3(a_1\,b_0 - a_0\,b_1)^2} = 0\,. \tag{4.38}$$

The model with one symplectically rotated vector appears in the Lagrangian corresponding to the consistent truncation of M-theory compactified on a Sasaki-Einstein manifold SE_7 [12]. The reduction naturally gives a cubic prepotential and a tensor field; the tensor field can be dualized to the scalar σ with a simultaneous dualization of A^1. With our normalizations, the gaugings are proportional to $(a_0, a_1) = (6\sqrt{2}, -2\sqrt{2})$ and $(b_0, b_1) = (-\sqrt{2}, 0)$.[12] The AdS vacuum has $\tau = i$ and $\rho = 4$ as in [12] and it corresponds to the eleven dimensional background $\mathrm{AdS}_4 \times SE_7$ with $\mathcal{N} = 2$ supersymmetry. The $\mathrm{Schr}_4(z)$ solution has $z = (-4, 3)$, where obviously only the value $z = 3$ is physical, and corresponds to the eleven dimensional solution found in [11, 15], which is discussed from the point of view of the four dimensional theory in section 4 of [12].

[12] Reference [12] uses a different symplectic rotation given in equation (2.38) of the same reference; the gauging parameters reported above have been correspondingly rotated with respect to those in [12].

5 Embeddings into string/M-theory

Having established a wide class of supersymmetric solutions in gauged supergravity, the natural next step is to embed them into string theory or M-theory. The Lifshitz solutions of section 3 require purely electric or purely magnetic gaugings. One can achieve a purely electric gauging in a simple way by first reducing IIB on a Sasaki-Einstein five-manifold (SE_5) [54–57] where one obtains $\mathcal{N} = 4$ gauged supergravity with two vector-multiplets. Then there is a further truncation [54] to an $\mathcal{N} = 2$ theory with just the universal hyper-multiplet which is gauged electrically under the graviphoton. Dimensional reduction on a circle, with a linear profile for a hyper-scalar, introduces a further electric gauging. Specifically, suppose we take a hyper-scalar q in five dimensions and then reduce on the circle

$$ds_5^2 = ds_4^2 + (d\sigma + A_1)^2 \tag{5.1}$$

$$q = k\sigma + \widetilde{q}\,, \tag{5.2}$$

where $\widetilde{q}$ only depends on the co-ordinates of the four-dimensional space-time. It is easy to see that in four dimensions we obtain a kinetic term for $\widetilde{q}$ of the form

$$\mathcal{L}_4 \sim (d\widetilde{q} - kA_1) \wedge *(d\widetilde{q} - kA_1)\,, \tag{5.3}$$

and so $\widetilde{q}$ has electric charge k under A_1. In this way one can obtain a four dimensional $\mathcal{N} = 2$ gauged supergravity theory with cubic prepotential and electric gaugings from IIB on $SE_5 \times S^1$. The $\mathrm{Lif}_4(z)$ solutions found in [17] can probably be understood in this way.

As already discussed in section 4, certain gaugings of the form (2.30) arise from Sasaki-Einstein reductions of M-theory [12] and also in the SU(3)-invariant sector of the $\mathcal{N} = 8$ theory [39, 44, 53]. This makes it clear that our $\mathrm{Schr}_4(z)$ solutions can be embedded into these theories. It would be interesting to precisely establish which solutions of [9–24] lie within our class of solutions.

From consistent truncation of type IIA on various nearly-Kähler manifolds and cosets [36–38], one can obtain $\mathcal{N} = 2$ gauged supergravity with the same scalar manifold we have considered in this work and a rich spectrum of possible electric and magnetic non-compact gaugings. In the case of purely non compact gaugings we have found no non-relativistic, supersymmetric solutions. It would be interesting to understand if this result holds in general for models with non compact isometries, since these arise naturally in string compactifications.

Acknowledgments

We wish to thank D. Cassani, G. Dall'Agata, J. Gauntlett, A. Kashani-Poor, S. Ross, H. Samtleben, H. Triendl and A. Tomasiello for interesting discussions. N. H and A. Z would like to acknowledge the hospitality of the Galileo Galilei Institute for Theoretical Physics during the course of this project. The work of N. H. is supported by the grant number ANR-07-CEXC-006 of the Agence Nationale de La Recherche. M. Petrini is partially supported by the Institut de Physique Théorique, du CEA. A. Z. is supported in part by INFN.

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A Spinor conventions

Our conventions closely follow [45, 46]. We work with in signature $(+---)$. Spinors have the following properties

$$\gamma_5\epsilon_A = \epsilon_A\,, \tag{A.1}$$

$$\gamma_5\epsilon^A = -\epsilon^A\,, \tag{A.2}$$

$$\epsilon^A = (\epsilon_A)^C\,, \tag{A.3}$$

where $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$ and conjugation is defined on a general spinor λ as

$$\lambda^C = \gamma_0 C^{-1}\lambda^*\,, \tag{A.4}$$

and

$$CC^\dagger = 1, \quad C^2 = -1, \quad C^t = -C. \tag{A.5}$$

The gamma matrices satisfy

$$\gamma_0 = \gamma_0^\dagger\,, \tag{A.6}$$

$$\gamma_i = \gamma_0\gamma_i^\dagger\gamma_0. \tag{A.7}$$

B Hypermultiplet scalar manifold

Here we summarize various facts about the hypermultiplet scalar manifold $\mathcal{M}_\mathrm{Q}$. The eight Killing vectors are given by [40, 58]

$$\begin{aligned}
k_1 &= \frac{1}{2i}\big(z_2\partial_{z_1} + z_1\partial_{z_2} - c.c.\big)\,, & k_2 &= \frac{1}{2}\big(-z_2\partial_{z_1} + z_1\partial_{z_2} + c.c.\big)\,,\\
k_3 &= \frac{1}{2i}\big(-z_1\partial_{z_1} + z_2\partial_{z_2} - c.c.\big)\,, & k_4 &= \frac{1}{2i}\big(z_1\partial_{z_1} + z_2\partial_{z_2} - c.c.\big)\,,\\
k_5 &= \frac{1}{2}\big((-1+z_1^2)\partial_{z_1} + z_1 z_2\partial_{z_2} + c.c.\big)\,, & k_6 &= \frac{i}{2}\big((1+z_1^2)\partial_{z_1} + z_1 z_2\partial_{z_2} - c.c.\big)\,,\\
k_7 &= \frac{1}{2}\big(-z_1 z_2\partial_{z_1} + (1-z_2^2)\partial_{z_2} + c.c.\big)\,, & k_8 &= \frac{i}{2}\big(z_1 z_2\partial_{z_1} + (1+z_2^2)\partial_{z_2} - c.c.\big)\,.
\end{aligned} \tag{B.1}$$

All these Killing vectors are real, (k_1, k_2, k_3, k_4) generate compact isometries while (k_5, k_6, k_7, k_8) generate non-compact isometries. With the re-definitions

$$\begin{aligned}
k_1 &= -iF_1\,, & k_2 &= -iF_2\,,\\
k_3 &= -iF_3\,, & k_4 &= \frac{i}{\sqrt{3}}F_8\,,\\
k_5 &= F_4\,, & k_6 &= F_5\,,\\
k_7 &= F_6\,, & k_8 &= F_7\,,
\end{aligned}$$

the commutation relations are $[F_i, F_j] = if_{ijk}F_k$ with

$$f_{123} = 1, \qquad f_{147} = \frac{1}{2}, \qquad f_{156} = -\frac{1}{2}, \qquad f_{246} = \frac{1}{2}, \tag{B.2}$$

$$f_{257} = \frac{1}{2}, \qquad f_{345} = \frac{1}{2}\,, \qquad f_{367} = \frac{1}{2}\,, \tag{B.3}$$

$$f_{458} = \frac{\sqrt{3}}{2}\,, \qquad f_{678} = -\frac{\sqrt{3}}{2}\,. \tag{B.4}$$

Thus we see that (F_1, F_2, F_3) generate SU(2) and F_8 generates a commuting U(1).

The Killing prepotentials can be computed from

$$\Omega^x_{\mathrm{uv}} k^u_\Lambda = -\nabla_v P^x_\Lambda\,, \tag{B.5}$$

where

$$\Omega^x = J^i_{mn} e^m \wedge e^n, \tag{B.6}$$

J^i are a triplet of complex structures, e^i are frames on $\mathcal{M}_{\mathrm{Q}}$ and ∇_v is a covariant derivative w.r.t. the SU(2)-connection on $\mathcal{M}_{\mathrm{Q}}$. The Killing prepotentials are only well defined up to a local SU(2) transformation. In a particular gauge, the Killing prepotentials associated to the compact generators are given by (using $r^2 = |\zeta_1|^2 + |\zeta_2|^2$)

$$P_1 = \frac{1}{r^2\sqrt{1-r^2}} \begin{pmatrix} \mathrm{Im}\ (\zeta_1^2 - \zeta_2^2) \\ -\mathrm{Re}\ (\zeta_1^2 - \zeta_2^2) \\ \frac{(r^2-2)\mathrm{Re}\ (\zeta_2\bar{\zeta}_1)}{\sqrt{1-r^2}} \end{pmatrix}, \tag{B.7}$$

$$P_2 = \frac{1}{r^2\sqrt{1-r^2}} \begin{pmatrix} \mathrm{Re}\ (\zeta_1^2 + \zeta_2^2) \\ \mathrm{Im}\ (\zeta_1^2 + \zeta_2^2) \\ \frac{(r^2-2)\mathrm{Im}\ (\zeta_2\bar{\zeta}_1)}{\sqrt{1-r^2}} \end{pmatrix}, \tag{B.8}$$

$$P_3 = \frac{1}{r^2\sqrt{1-r^2}} \begin{pmatrix} 2\mathrm{Im}\ (\zeta_1\zeta_2) \\ -2\mathrm{Re}\ (\zeta_1\zeta_2) \\ \frac{|\zeta_1|^2(2-|\zeta_1|^2)-|\zeta_2|^2(2-|\zeta_2|^2)}{2\sqrt{1-r^2}} \end{pmatrix}, \tag{B.9}$$

$$P_4 = -\frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ \frac{r^2}{1-r^2} \end{pmatrix}, \tag{B.10}$$

and those associated to the non-compact generators are

$$P_5 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} \mathrm{Re}\ \zeta_2 \\ \mathrm{Im}\ \zeta_2 \\ \frac{\mathrm{Im}\ \zeta_1}{\sqrt{1-r^2}} \end{pmatrix}, \qquad P_6 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} \mathrm{Im}\ \zeta_2 \\ -\mathrm{Re}\ \zeta_2 \\ \frac{\mathrm{Re}\ \zeta_1}{\sqrt{1-r^2}} \end{pmatrix}, \tag{B.11}$$

$$P_7 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} \mathrm{Re}\ \zeta_1 \\ \mathrm{Im}\ \zeta_1 \\ -\frac{\mathrm{Im}\ \zeta_2}{\sqrt{1-r^2}} \end{pmatrix}, \qquad P_8 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} -\mathrm{Im}\ \zeta_1 \\ \mathrm{Re}\ \zeta_1 \\ \frac{\mathrm{Re}\ \zeta_2}{\sqrt{1-r^2}} \end{pmatrix}. \tag{B.12}$$

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