Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2009, Article ID 394635, 9 pages
doi:10.1155/2009/394635

Research Article

On a Conjecture for a Higher-Order Rational Difference Equation

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Received 30 December 2008; Revised 11 March 2009; Accepted 14 March 2009

Recommended by Jianshe Yu

This paper studies the global asymptotic stability for positive solutions to the higher order rational difference equation $x_n = (\prod_{j=1}^{m}(x_{n-k_j}+1)+\prod_{j=1}^{m}(x_{n-k_j}-1))/(\prod_{j=1}^{m}(x_{n-k_j}+1)-\prod_{j=1}^{m}(x_{n-k_j}-1))$, $n = 0, 1, 2, \ldots$, where m is odd and $x_{-k_m}, x_{-k_m+1}, \ldots, x_{-1} \in (0, \infty)$. Our main result generalizes several others in the recent literature and confirms a conjecture by Berenhaut et al., 2007.

1. Introduction

In 2007, Berenhaut et al. [1] proved that every solution of the following rational difference equation

$$x_n = \frac{x_{n-k} + x_{n-m}}{1 + x_{n-k}x_{n-m}}, \quad n = 0, 1, 2, \ldots \tag{1.1}$$

converges to its unique equilibrium 1, where $x_{-m}, x_{-m+1}, \ldots, x_{-1} \in (0, \infty)$ and $1 \le k < m$. Based on this fact, they put forward the following two conjectures.

Conjecture 1.1. *Suppose that $1 \le k < l < m$ and that $\{x_n\}$ satisfies*

$$x_n = \frac{x_{n-k} + x_{n-l} + x_{n-m} + x_{n-k}x_{n-l}x_{n-m}}{1 + x_{n-k}x_{n-l} + x_{n-l}x_{n-m} + x_{n-m}x_{n-k}}, \quad n = 0, 1, 2, \ldots \tag{1.2}$$

with $x_{-m}, x_{-m+1}, \ldots, x_{-1} \in (0, \infty)$. Then, the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Conjecture 1.2. *Suppose that m is odd and $1 \le k_1 < k_2 < \cdots < k_m$, and define $S = \{1, 2, \ldots, m\}$. If $\{x_n\}$ satisfies*

$$x_n = \frac{f_1(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m})}{f_2(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m})}, \quad n = 0, 1, 2, \ldots \tag{1.3}$$

with $x_{-k_m}, x_{-k_m+1}, \ldots, x_{-1} \in (0, \infty)$, where

$$\begin{aligned} f_1(y_1, y_2, \ldots, y_m) &= \sum_{j\in\{1,3,\ldots,m\}} \sum_{\{t_1,t_2,\ldots,t_j\}\subset S; t_1<t_2<\cdots<t_j} y_{t_1} y_{t_2} \cdots y_{t_j}, \\ f_2(y_1, y_2, \ldots, y_m) &= 1 + \sum_{j\in\{2,4,\ldots,m-1\}} \sum_{\{t_1,t_2,\ldots,t_j\}\subset S; t_1<t_2<\cdots<t_j} y_{t_1} y_{t_2} \cdots y_{t_j}. \end{aligned} \tag{1.4}$$

Then the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Motivated by [2], Berenhaut et al. started with the investigation of the following difference equation $y_n = A + (y_{n-k}/y_{n-m})^p$ for $p > 0$ (see, [3, 4]). Among others, in [3] they used a transformation method, which has turned out to be very useful in studying (1.1) and (1.2) as well as in confirming Conjecture 1.1; see [5].

Some particular cases of (1.2) had been studied previously by Li in [6, 7], by using semicycle analysis similar to that in [8]. The problem concerning periodicity of semicycles of difference equations was solved in very general settings by Berg and Stević in [9], partially motivated also by [10].

In the meantime, it turned out that the method used in [11] by Çinar et al. can be used in confirming Conjecture 1.2 (see also [12]). More precisely [11, 12] use Corollary 3 from [13] in solving similar problems. For example, Çinar et al. has shown, in an elegant way, that the main result in [14] is a consequence of Corollary 3 in [13]. With some calculations it can be also shown that Conjecture 1.2 can be confirmed in this way (see [15]).

Some other related results can be found in [16–24].

In this paper, we will prove that Conjecture 1.2 is correct by using a new method. Obviously, our results generalize the corresponding works in [1, 5–7] and other literature.

2. Preliminaries and Notations

Observe that

$$\begin{aligned} f_1(y_1, y_2, \ldots, y_m) &= \frac{1}{2}\left[\prod_{j=1}^{m}(y_j+1) + \prod_{j=1}^{m}(y_j-1)\right], \\ f_2(y_1, y_2, \ldots, y_m) &= \frac{1}{2}\left[\prod_{j=1}^{m}(y_j+1) - \prod_{j=1}^{m}(y_j-1)\right]. \end{aligned} \tag{2.1}$$

Define function G as follows:

$$G(y_1, y_2, \ldots, y_m) = \frac{\prod_{j=1}^{m}(y_j+1) + \prod_{j=1}^{m}(y_j-1)}{\prod_{j=1}^{m}(y_j+1) - \prod_{j=1}^{m}(y_j-1)}, \quad y_1, y_2, \ldots, y_m > 0. \tag{2.2}$$

Then we can rewrite (1.3) as

$$x_n = \frac{\prod_{j=1}^{m}\left(x_{n-k_j}+1\right) + \prod_{j=1}^{m}\left(x_{n-k_j}-1\right)}{\prod_{j=1}^{m}\left(x_{n-k_j}+1\right) - \prod_{j=1}^{m}\left(x_{n-k_j}-1\right)}, \quad n = 0, 1, 2, \ldots, \tag{2.3}$$

or

$$x_n = G(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m}), \quad n = 0, 1, 2, \ldots, \tag{2.4}$$

where m is an odd integer and $x_{-k_m}, x_{-k_m+1}, \ldots, x_{-1} \in (0, \infty)$.

The following lemma can be obtained by simple calculations.

Lemma 2.1. *Let G be defined by* (2.2). *Then*

$$\frac{\partial G}{\partial y_i} = \frac{4\prod_{j=1, j\neq i}^{m}\left(y_j^2-1\right)}{\left[\prod_{j=1}^{m}(y_j+1) - \prod_{j=1}^{m}(y_j-1)\right]^2} \begin{cases} > 0, & \prod_{j=1, j\neq i}^{m}(y_j-1) > 0, \\ < 0, & \prod_{j=1, j\neq i}^{m}(y_j-1) < 0, \end{cases} \tag{2.5}$$

$i = 1, 2, \ldots, m$.

Lemma 2.2. *Assume that $0 < \alpha < 1 < \beta < +\infty$. If $\alpha \leq y_1, y_2, \ldots, y_m \leq \beta$, then*

$$\min\{A_1, A_3, \ldots, A_m\} \leq G(y_1, y_2, \ldots, y_m) \leq \max\{B_1, B_3, \ldots, B_m\}, \tag{2.6}$$

where

$$\begin{aligned} A_i &= \frac{(\alpha+1)^i(\beta+1)^{m-i} + (\alpha-1)^i(\beta-1)^{m-i}}{(\alpha+1)^i(\beta+1)^{m-i} - (\alpha-1)^i(\beta-1)^{m-i}}, \\ B_i &= \frac{(\alpha+1)^{m-i}(\beta+1)^i + (\alpha-1)^{m-i}(\beta-1)^i}{(\alpha+1)^{m-i}(\beta+1)^i - (\alpha-1)^{m-i}(\beta-1)^i}, \end{aligned} \tag{2.7}$$

$i = 1, 3, \ldots, m$.

Proof. Since $G(y_1, y_2, \ldots, y_m)$ is symmetric in $y_1, y_2, \ldots, y_m$, we can assume, without loss of generality, that $\alpha \leq y_1 \leq y_2 \leq \cdots \leq y_m \leq \beta$. Then there are $m+1$ possible cases:

(1) $\alpha \leq 1 \leq y_1 \leq y_2 \leq \cdots \leq y_m \leq \beta$;

(2) $\alpha \leq y_1 \leq 1 \leq y_2 \leq \cdots \leq y_m \leq \beta$;

(3) $\alpha \leq y_1 \leq y_2 \leq 1 \leq \cdots \leq y_m \leq \beta$;

(4) $\alpha \leq y_1 \leq y_2 \leq y_3 \leq 1 \leq \cdots \leq y_m \leq \beta$;

$\vdots$

(m+1) $\alpha \leq y_1 \leq y_2 \leq \cdots \leq y_m \leq 1 \leq \beta$.

And, for the above cases (1)–(m+1), by the monotonicity of $G(y_1, y_2, \ldots, y_m)$, in turn, we may get

(1) $1 \leq G(y_1, y_2, \ldots, y_m) \leq B_m$;

(2) $A_1 \leq G(y_1, y_2, \ldots, y_m) \leq 1$;

(3) $1 \leq G(y_1, y_2, \ldots, y_m) \leq B_{m-2}$;

(4) $A_3 \leq G(y_1, y_2, \ldots, y_m) \leq 1$;

$\vdots$

(m+1) $A_m \leq G(y_1, y_2, \ldots, y_m) \leq 1$.

From the above inequalities, it follows that (2.6) holds. The proof is complete. □

Lemma 2.3. *Assume that* $0 < \alpha < 1 < \beta < +\infty$ *.Then*

$$A_i = \frac{(\alpha+1)^i(\beta+1)^{m-i} + (\alpha-1)^i(\beta-1)^{m-i}}{(\alpha+1)^i(\beta+1)^{m-i} - (\alpha-1)^i(\beta-1)^{m-i}} \geq \alpha, \tag{2.8}$$

$$B_i = \frac{(\alpha+1)^{m-i}(\beta+1)^i + (\alpha-1)^{m-i}(\beta-1)^i}{(\alpha+1)^{m-i}(\beta+1)^i - (\alpha-1)^{m-i}(\beta-1)^i} \leq \beta, \tag{2.9}$$

$i = 1, 3, \ldots, m$.

Proof. For $i = 1, 3, \ldots, m$, it is easy to see that

$$(\alpha-1)^{i-1}(\beta-1)^{m-i} \leq (\alpha+1)^{i-1}(\beta+1)^{m-i}, \tag{2.10}$$

which yields

$$(\alpha+1)(\alpha-1)^i(\beta-1)^{m-i} \geq (\alpha-1)(\alpha+1)^i(\beta+1)^{m-i}, \tag{2.11}$$

and so

$$\alpha\left[(\alpha+1)^i(\beta+1)^{m-i} - (\alpha-1)^i(\beta-1)^{m-i}\right] \leq (\alpha+1)^i(\beta+1)^{m-i} + (\alpha-1)^i(\beta-1)^{m-i}. \tag{2.12}$$

It follows that (2.8) holds. Similarly, for $i = 1, 3, \ldots, m$, it is easy to see that

$$(\alpha - 1)^{m-i}(\beta - 1)^{i-1} \leq (\alpha + 1)^{m-i}(\beta + 1)^{i-1}, \tag{2.13}$$

which yields

$$(\beta + 1)(\alpha - 1)^{m-i}(\beta - 1)^{i} \leq (\beta - 1)(\alpha + 1)^{m-i}(\beta + 1)^{i}. \tag{2.14}$$

It follows that (2.9) holds. The proof is complete. □

Lemma 2.4. *Let*

$$\begin{aligned} \alpha_{j+1} &= \min\{A_{1j}, A_{3j}, \ldots, A_{mj}\}, \\ \beta_{j+1} &= \max\{B_{1j}, B_{3j}, \ldots, B_{mj}\}, \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} A_{ij} &= \frac{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} + (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} - (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}, \\ B_{ij} &= \frac{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} + (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} - (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}, \end{aligned} \tag{2.16}$$

$i = 1, 3, \ldots, m$; $j = 0, 1, 2, \ldots$. *Assume that* $0 < \alpha_0 < 1 < \beta_0 < +\infty$. *Then*

$$\lim_{j \to \infty} \alpha_j = \lim_{j \to \infty} \beta_j = 1. \tag{2.17}$$

Proof. By induction, we easily show that

$$0 < \alpha_j < 1 < \beta_j < +\infty, \quad j = 0, 1, 2, \ldots. \tag{2.18}$$

It follows from Lemma 2.3 that

$$\begin{aligned} A_{ij} &= \frac{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} + (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} - (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}} \geq \alpha_j, \\ B_{ij} &= \frac{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} + (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} - (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}} \leq \beta_j, \end{aligned} \tag{2.19}$$

$i = 1, 3, \ldots, m$; $j = 0, 1, 2, \ldots$. Hence, by (2.15) and (2.18), we have

$$\alpha_j \leq \alpha_{j+1} < 1 < \beta_{j+1} \leq \beta_j, \quad j = 0, 1, 2, \ldots. \tag{2.20}$$

Equation (2.20) implies that the limits $\lim_{j\to\infty}\alpha_j$ and $\lim_{j\to\infty}\beta_j$ exist, and

$$\alpha^* = \lim_{j\to\infty}\alpha_j \in [\alpha_0, 1], \qquad \beta^* = \lim_{j\to\infty}\beta_j \in [1, \beta_0]. \tag{2.21}$$

It follows from (2.16) that

$$\begin{aligned} A_i^* &:= \lim_{j\to\infty} A_{ij} = \frac{(\alpha^*+1)^i(\beta^*+1)^{m-i} + (\alpha^*-1)^i(\beta^*-1)^{m-i}}{(\alpha^*+1)^i(\beta^*+1)^{m-i} - (\alpha^*-1)^i(\beta^*-1)^{m-i}}, \\ B_i^* &:= \lim_{j\to\infty} B_{ij} = \frac{(\alpha^*+1)^{m-i}(\beta^*+1)^{i} + (\alpha^*-1)^{m-i}(\beta^*-1)^{i}}{(\alpha^*+1)^{m-i}(\beta^*+1)^{i} - (\alpha^*-1)^{m-i}(\beta^*-1)^{i}}, \end{aligned} \tag{2.22}$$

$i = 1, 3, \ldots, m$. Let $j \to \infty$ in (2.15), we have

$$\begin{aligned} \alpha^* &= \min\{A_1^*, A_3^*, \ldots, A_m^*\}, \\ \beta^* &= \max\{B_1^*, B_3^*, \ldots, B_m^*\}. \end{aligned} \tag{2.23}$$

It follows that there exist $i, j \in \{1, 3, \ldots, m\}$ such that

$$\begin{aligned} \alpha^* &= \frac{(\alpha^*+1)^i(\beta^*+1)^{m-i} + (\alpha^*-1)^i(\beta^*-1)^{m-i}}{(\alpha^*+1)^i(\beta^*+1)^{m-i} - (\alpha^*+1)^i(\beta^*+1)^{m-i}}, \\ \beta^* &= \frac{(\alpha^*+1)^{m-j}(\beta^*+1)^{j} + (\alpha^*-1)^{m-j}(\beta^*-1)^{j}}{(\alpha^*+1)^{m-j}(\beta^*+1)^{j} - (\alpha^*-1)^{m-j}(\beta^*-1)^{j}}. \end{aligned} \tag{2.24}$$

From (2.24), we have

$$\begin{aligned} &(\alpha^*-1)\left[(\alpha^*+1)^{i-1}(\beta^*+1)^{m-i} - (\alpha^*-1)^{i-1}(\beta^*-1)^{m-i}\right] = 0, \\ &(\beta^*-1)\left[(\alpha^*+1)^{m-j}(\beta^*+1)^{j-1} - (\alpha^*-1)^{m-j}(\beta^*-1)^{j-1}\right] = 0. \end{aligned} \tag{2.25}$$

Since

$$\begin{aligned} &(\alpha^*+1)^{i-1}(\beta^*+1)^{m-i} - (\alpha^*-1)^{i-1}(\beta^*-1)^{m-i} > 0, \\ &(\alpha^*+1)^{m-j}(\beta^*+1)^{j-1} - (\alpha^*-1)^{m-j}(\beta^*-1)^{j-1} > 0, \end{aligned} \tag{2.26}$$

it follows from (2.25) and (2.18) that $\alpha^* = \beta^* = 1$. The proof is complete. □

3. Proof of Conjecture 1.2

Theorem 3.1. *Suppose that* $0 < \alpha < 1 < \beta < +\infty$ *and that*

$$x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in [\alpha, \beta]. \tag{3.1}$$

Then the solution $\{x_n\}$ *of* (1.3) *satisfies*

$$x_n \in [\alpha, \beta], \quad for\ n = 0, 1, 2, \dots. \tag{3.2}$$

Theorem 3.1 is a direct corollary of Lemmas 2.2 and 2.3.

Proof of Conjecture 1.2. Let $\{x_n\}$ be a solution of (1.3) with $x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in (0, \infty)$. We need to prove that

$$\lim_{n\to\infty} x_n = 1. \tag{3.3}$$

Choose $\alpha_0 \in (0,1)$ and $\beta_0 \in (1, +\infty)$ such that

$$x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in [\alpha_0, \beta_0]. \tag{3.4}$$

In view of Theorem 3.1, we have

$$x_n \in [\alpha_0, \beta_0], \quad n = -k_m, -k_m+1, -k_m+2, \dots. \tag{3.5}$$

Let α_j, β_j, A_{ij}, and B_{ij} be defined as in Lemma 2.4. Then by (3.5) and Lemma 2.2, we have

$$\begin{aligned} \min\{A_{10}, A_{30}, \dots, A_{m0}\} &\le G(x_{n-k_1}, x_{n-k_2}, \dots, x_{n-k_m}) \\ &\le \max\{B_{10}, B_{30}, \dots, B_{m0}\}, \quad n = 0, 1, 2, \dots. \end{aligned} \tag{3.6}$$

That is

$$x_n \in [\alpha_1, \beta_1], \quad n = 0, 1, 2, \dots. \tag{3.7}$$

By (3.7) and Lemma 2.2, we obtain

$$\begin{aligned} \min\{A_{11}, A_{31}, \dots, A_{m1}\} &\le G(x_{n-k_1}, x_{n-k_2}, \dots, x_{n-k_m}) \\ &\le \max\{B_{11}, B_{31}, \dots, B_{m1}\}, \quad n = k_m, k_m+1, k_m+2, \dots. \end{aligned} \tag{3.8}$$

That is

$$x_n \in [\alpha_2, \beta_2], \quad n = k_m, k_m+1, k_m+2, \dots. \tag{3.9}$$

Repeating the above procedure, in general, we can obtain

$$x_n \in [\alpha_{j+1}, \beta_{j+1}], \quad n = jk_m, jk_m + 1, jk_m + 2, \dots, \ j = 0, 1, 2, \dots. \tag{3.10}$$

By Lemma 2.4, we have

$$\lim_{n\to\infty} x_n = \lim_{j\to\infty} \alpha_{j+1} = \lim_{j\to\infty} \beta_{j+1} = 1, \tag{3.11}$$

which implies that (3.3) holds. The proof of Conjecture 1.2 is complete. □

Acknowledgments

The authors are grateful to the referees for their careful reading of the manuscript and many valuable comments and suggestions that greatly improved the presentation of this work. This work is supported partly by NNSF of China (Grant: 10771215, 10771094), Project of Hunan Provincial Youth Key Teacher and Project of Hunan Provincial Education Department (Grant: 07C639).

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