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Angular distributions of the polarized photons and electron in the decays of the $^3D_3$ state of charmonium

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Abstract We calculate the combined angular-distribution functions of the polarized photons (γ_1 and γ_2) and electron (e^-) produced in the cascade process $\bar{p}p \rightarrow {}^3D_3 \rightarrow {}^3P_2 + \gamma_1 \rightarrow (\psi + \gamma_2) + \gamma_1 \rightarrow (e^+ + e^-) + \gamma_1 + \gamma_2$, when the colliding $\bar{p}$ and p are unpolarized. Our results are independent of any dynamical models and are expressed in terms of the spherical harmonics whose coefficients are functions of the angular-momentum helicity amplitudes of the individual processes. Once the joint angular distribution of (γ_1, γ_2) and that of (γ_2, e^-) with the polarization of either one of the two particles are measured, our results will enable one to determine the relative magnitudes as well as the relative phases of all the angular-momentum helicity amplitudes in the radiative decay processes $^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and $^3P_2 \rightarrow \psi + \gamma_2$.

1 Introduction

Recently there has been great interest in charmonium spectroscopy above the open charm $D\bar{D}$ threshold of 3.73 GeV [1–4]. Although the mass of the unobserved 1^3D_3 state of charmonium is expected to lie slightly above the charm threshold [2], its Zweig-allowed strong decay to $D\bar{D}$ is suppressed by the F-wave angular-momentum barrier [5,6]. The total strong width of 1^3D_3 is predicted to be just 0.5 MeV [1] and therefore other decay modes such as γ^3P_2 and $\pi\pi J/\psi$ may be observable [3]. The measurement of the angular distributions in these prominent radiative and hadronic decays of the charmonium 1^3D_3 state can provide valuable information on the true dynamics of the charmonium system above the charm threshold. In fact, the observation of the radiative decays of the charmonium states below and above the charm threshold is an important component of the planned PANDA experiments at FAIR

[7,8], which study charmonium spectroscopy in $\bar{p}p$ annihilation.

In our previous paper [9], it is shown that by measuring the combined angular distribution of the two photons (γ_1, γ_2) and that of the second photon and electron (γ_2, e^-), regardless of their polarizations, in the sequential decay process originating from unpolarized $\bar{p}p$ collisions, namely, $\bar{p}p \rightarrow {}^3D_3 \rightarrow {}^3P_2 + \gamma_1 \rightarrow (\psi + \gamma_2) + \gamma_1 \rightarrow (e^+ + e^-) + \gamma_1 + \gamma_2$, one can extract the relative magnitudes as well as the cosines of the relative phases of all the angular-momentum helicity amplitudes in the radiative decay processes $^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and $^3P_2 \rightarrow \psi + \gamma_2$. The sines of the relative phases of these helicity amplitudes, however, cannot be determined uniquely. By including the measurement of the polarization of one of the decay particles, one may also obtain unambiguously the sines of the relative phases [10,11]. So in this paper we calculate the combined angular distributions of the final particles (γ_1, γ_2 and e^-) with the determination of the polarization of one particle in the above cascade process when $\bar{p}$ and p are unpolarized.

In general, the helicity amplitudes are complex and their relative phases are nontrivial. It is important to obtain them from experiments because we can then learn about the true dynamics of the charmonium system from the decays of the charmonium states. Once the combined angular distribution of γ_1, γ_2 and e^- and the polarization of any one of the particles in unpolarized $\bar{p}p$ collisions are experimentally measured, our expressions will enable one to calculate the relative magnitudes as well as the relative phases of all the angular-momentum helicity amplitudes in the two radiative decay processes $^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and $^3P_2 \rightarrow \psi + \gamma_2$. As our calculation is based only on the general principles of quantum mechanics and symmetry, our results are independent of any dynamical models. In addition, our results on the partially integrated angular distributions where the combined angular-distribution function of γ_1, γ_2 and e^- is integrated over the direction of one of the three particles are quite interesting. They show that by measuring the two-

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particle angular distribution of (γ_1, γ_2) and that of (γ_2, e^-) with the polarization of either one of the two particles, one can also get complete information on the helicity amplitudes.

The format of the rest of this paper is as follows. In Sect. 2, we give the calculations for the combined angular distribution with polarization determination of the electron and of the two photons in the cascade process $\bar{p}p \rightarrow {}^3D_3 \rightarrow {}^3P_2 + \gamma_1 \rightarrow (\psi + \gamma_2) + \gamma_1 \rightarrow (e^+ + e^-) + \gamma_1 + \gamma_2$, when $\bar{p}$ and p are unpolarized. We then show how the measurement of this joint angular distribution of polarized γ_1, γ_2 and e^- enables us to obtain complete information on the helicity amplitudes in the two radiative decay processes ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and ${}^3P_2 \rightarrow \psi + \gamma_2$. We also present three different results for the combined angular distribution, in which the polarization of only one of the three particles, γ_1, γ_2 and e^-, is measured. In Sect. 3, we present the results for the partially integrated angular distributions in different cases where the combined angular-distribution function of the three particles is integrated over the direction of one particle. These results can all be expressed in terms of the orthogonal spherical harmonic functions. We point out how the measurement of these two-particle angular distributions will again give complete information on all the helicity amplitudes in the two radiative decay processes. Finally, in Sect. 4, we make some concluding remarks.

2 The polarized angular distributions of the photons and electron

We consider the cascade process, $\bar{p}(\lambda_1) + p(\lambda_2) \rightarrow {}^3D_3(\delta) \rightarrow {}^3P_2(\nu) + \gamma_1(\mu) \rightarrow [\psi(\sigma) + \gamma_2(\kappa)] + \gamma_1(\mu) \rightarrow [e^-(\alpha_1) + e^+(\alpha_2)] + \gamma_1(\mu) + \gamma_2(\kappa)$, in the ${}^3D_3$ rest frame or the $\bar{p}p$ c.m. frame. The Greek symbols in the brackets represent the helicities of the particles except δ, which represents the z component of the angular momentum of the stationary ${}^3D_3$ resonance. We choose the z axis to be the direction of motion of ${}^3P_2$ in the ${}^3D_3$ rest frame. The x and y axes are arbitrary and the experimentalists can choose them according to their convenience. A symbolic sketch of the cascade process is shown in Fig. 1.

Following the conventions of our previous paper [9], the probability amplitude for the cascade process can be expressed in terms of the Wigner D-functions and the angular-momentum helicity amplitudes $B_{\lambda_1\lambda_2}$, $A_{\nu\mu}$, $E_{\sigma\kappa}$ and $C_{\alpha_1\alpha_2}$ for the individual sequential processes as

$$T^{\alpha_1\alpha_2\mu\kappa}_{\lambda_1\lambda_2} = \frac{7\sqrt{15}}{16\pi^2} C_{\alpha_1\alpha_2} B_{\lambda_1\lambda_2} \sum_{\delta}^{-3\rightarrow3} \sum_{\sigma}^{-1\rightarrow1} A_{\mu+\delta,\mu} E_{\sigma\kappa}$$
$$\times D^3_{\delta\lambda}(\phi, \theta, -\phi) D^{2*}_{\mu+\delta,\sigma-\kappa}(\phi', \theta', -\phi')$$
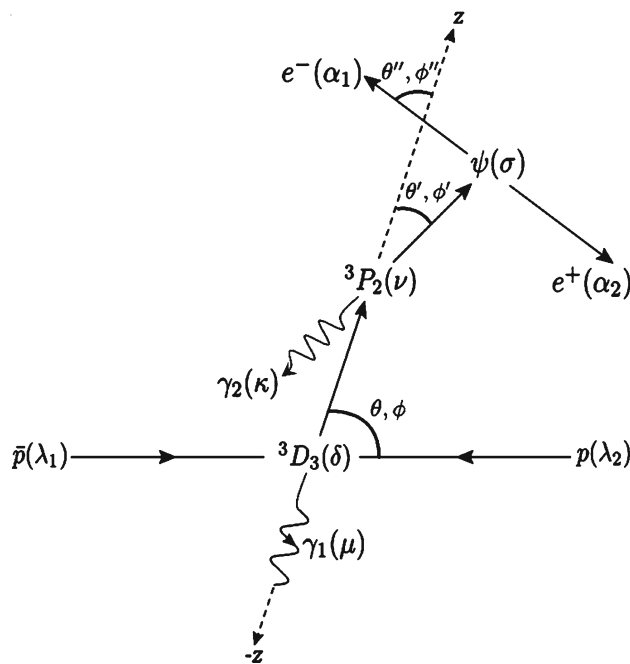$$\times D^{1*}_{\sigma\alpha}(\phi'', \theta'', -\phi''). \tag{1}$$

Fig. 1 Symbolic sketch of $\bar{p}(\lambda_1) + p(\lambda_2) \rightarrow {}^3D_3(\delta) \rightarrow {}^3P_2(\nu) + \gamma_1(\mu) \rightarrow [\psi(\sigma) + \gamma_2(\kappa)] + \gamma_1(\mu) \rightarrow [e^-(\alpha_1) + e^+(\alpha_2)] + \gamma_1(\mu) + \gamma_2(\kappa)$ showing different angles of the decay particles

In the D-functions, the angles (ϕ, θ) giving the direction of $\bar{p}$, the angles (ϕ', θ') giving the direction of ψ and the angles (ϕ'', θ'') giving the direction of e^- are measured in the ${}^3D_3$, the ${}^3P_2$ and the ψ rest frames, respectively. The angles of each decay particle observed in different rest frames can be calculated using the Lorentz transformation. The equations relating these angles are given in [12].

Because of the C and P invariances, the angular-momentum helicity amplitudes in (1) are not all independent. We have

$$B_{\lambda_1\lambda_2} \overset{\text{P}}{=} B_{-\lambda_1-\lambda_2},$$
$$B_{\lambda_1\lambda_2} \overset{\text{C}}{=} B_{\lambda_2\lambda_1},$$
$$A_{\nu\mu} \overset{\text{P}}{=} A_{-\nu-\mu},$$
$$E_{\sigma\kappa} \overset{\text{P}}{=} E_{-\sigma-\kappa}, \tag{2}$$
$$C_{\alpha_1\alpha_2} \overset{\text{P}}{=} C_{-\alpha_1-\alpha_2},$$
$$C_{\alpha_1\alpha_2} \overset{\text{C}}{=} C_{\alpha_2\alpha_1}.$$

Making use of the symmetry relations of (2), we now re-label the independent angular-momentum helicity amplitudes as follows:

$$B_0 = \sqrt{2}B_{\frac{1}{2}\frac{1}{2}}, \qquad B_1 = \sqrt{2}B_{\frac{1}{2}-\frac{1}{2}},$$
$$A_i = A_{i-2,1} = A_{2-i,-1} \quad (i = 0, 1, 2, 3, 4),$$
$$E_j = E_{j-1,1} = E_{1-j,-1} \quad (j = 0, 1, 2),$$
$$C_0 = \sqrt{2}C_{\frac{1}{2}\frac{1}{2}}, \qquad C_1 = \sqrt{2}C_{\frac{1}{2}-\frac{1}{2}}. \tag{3}$$

We will also make use of the following normalizations:

$$|B_0|^2 + |B_1|^2 = |C_0|^2 + |C_1|^2 = 1 \tag{4}$$

and

$$\sum_{i=0}^{4} |A_i|^2 = \sum_{j=0}^{2} |E_j|^2 = 1 \tag{5}$$

When $\bar{p}$ and p are unpolarized, the normalized function describing the combined angular distribution of the electron and the two photons whose polarizations are also observed can be written as

$$
W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')
$$
$$
= N \sum_{\lambda_1,\lambda_2}^{\pm\frac{1}{2}} \sum_{\alpha_2}^{\pm\frac{1}{2}} T_{\lambda_1\lambda_2}^{\alpha_1\alpha_2\mu\kappa} T_{\lambda_1\lambda_2}^{\alpha_1\alpha_2\mu\kappa*}, \tag{6}
$$

where the subscripts μ, κ and α_1 of W represent the polarizations of γ_1, γ_2 and e^-, respectively. The normalization constant N in (6) is determined by requiring that the integral of the angular-distribution function $W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$ over all the directions of γ_1, γ_2 and e^- or over all the angles, $(\theta, \phi; \theta', \phi'; \theta'', \phi'')$, is 1. In (6) we sum over the helicities α_2 since e^+ is not observed. Substituting (1) into (6) and performing the various sums will then give an expression for the angular-distribution function $W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$ in terms of the Wigner D-functions. After very long algebra, we get

$$
W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')
$$
$$
= \frac{1}{(4\pi)^3} \sum_{J_1}^{0,2,4,6} B^{J_1} \sum_{J_3=0}^{2} C_{\alpha_1}^{J_3}
$$
$$
\times \sum_{J_2=0}^{4} (-1)^{\left(\frac{2+\mu-\kappa}{2}\right)J_2 + \left(\frac{1-\kappa}{2}\right)J_3}
$$
$$
\times \sum_{d=0}^{d_m} \sum_{d'=0}^{d'_m} \left(1 - \frac{\delta_{d0}}{2}\right)\left(1 - \frac{\delta_{d'0}}{2}\right) I_{dd'}^{J_1 J_2 J_3} \tag{7}
$$

where

$$
B^{J_1} = \sqrt{7} \sum_{\lambda=0}^{1} (-1)^\lambda \langle 33; \lambda, -\lambda|J_1; 0\rangle |B_\lambda|^2, \tag{8}
$$

$$
C_{\alpha_1}^{J_3} = (-1)^{\left(\alpha_1 + \frac{1}{2}\right)J_3} \sqrt{3}
$$
$$
\times \sum_{\alpha=0}^{1} (-1)^\alpha \langle 11; \alpha, -\alpha|J_3; 0\rangle |C_\alpha|^2, \tag{9}
$$

$$
I_{dd'}^{J_1 J_2 J_3} = \beta_{d+}^{J_1 J_2} \left[\gamma_{d'+}^{J_3 J_2}(D_1 + D_1^* + D_2 + D_2^*) \right.
$$
$$
\left. + \gamma_{d'-}^{J_3 J_2}(D_1 - D_1^* + D_2 - D_2^*) \right]
$$

$$
+ \beta_{d-}^{J_1 J_2} \left[\gamma_{d'+}^{J_3 J_2}(D_1 - D_1^* - D_2 + D_2^*) \right.
$$
$$
\left. + \gamma_{d'-}^{J_3 J_2}(D_1 + D_1^* - D_2 - D_2^*) \right], , \tag{10}
$$

$$
\beta_{d\pm}^{J_1 J_2} = \frac{\sqrt{35}}{2} \sum_{s(d)} \left(A_{\frac{s+d}{2}} A_{\frac{s-d}{2}}^* \pm A_{\frac{s+d}{2}}^* A_{\frac{s-d}{2}} \right)
$$
$$
\times \left\langle 22; \frac{s+d-4}{2}, -\frac{s-d-4}{2} \bigg| J_2; d \right\rangle
$$
$$
\times \left\langle 33; \frac{s+d-6}{2}, -\frac{s-d-6}{2} \bigg| J_1; d \right\rangle, \tag{11}
$$

$$
\gamma_{d'\pm}^{J_3 J_2} = \frac{\sqrt{15}}{2} \sum_{s'(d')} \left(E_{\frac{s'+d'}{2}} E_{\frac{s'-d'}{2}}^* \pm E_{\frac{s'+d'}{2}}^* E_{\frac{s'-d'}{2}} \right)
$$
$$
\times \left\langle 22; \frac{s'+d'}{2}, -\frac{s'-d'}{2} \bigg| J_2; d' \right\rangle
$$
$$
\times \left\langle 11; \frac{s'+d'-2}{2}, -\frac{s'-d'-2}{2} \bigg| J_3; d' \right\rangle, \tag{12}
$$

$$
D_1 = D_{\mu d,0}^{J_1}(\theta, \phi, -\theta) D_{\mu d, -\kappa d'}^{J_2*}(\theta', \phi', -\theta')
$$
$$
\times D_{-\kappa d',0}^{J_3*}(\theta'', \phi'', -\theta''), \tag{13}
$$

$$
D_2 = D_{\mu d,0}^{J_1*}(\theta, \phi, -\theta) D_{\mu d, \kappa d'}^{J_2}(\theta', \phi', -\theta')
$$
$$
\times D_{\kappa d',0}^{J_3}(\theta'', \phi'', -\theta''), \tag{14}
$$

$$
s(d) = |d|, |d| + 2, \ldots, 8 - |d|, \tag{15}
$$
$$
s'(d') = |d'|, |d'| + 2, \ldots, 4 - |d'|, \tag{16}
$$

$$
d_m = \min\{4, J_1, J_2\}, \tag{17}
$$
$$
d'_m = \min\{2, J_2, J_3\}. \tag{18}
$$

The Wigner D-functions in (13) and (14) are given by [13]

$$
D_{m,m'}^{j}(\alpha, \beta, \gamma) = \langle j, m|R(\alpha, \beta, \gamma)|j, m'\rangle \tag{19}
$$

where α, β, γ are Euler angles and the rotation operator $R(\alpha, \beta, \gamma)$ can be written as

$$
R(\alpha, \beta, \gamma) = e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z}. \tag{20}
$$

The explicit expressions for all the coefficients in (7) are given in Appendix A. Making use of the orthogonal relation of the Wigner D-functions,

$$
\int_0^{2\pi} d\alpha \int_0^{2\pi} d\gamma \int_0^{\pi} D_{mm'}^{j*}(\alpha, \beta, \gamma) D_{\mu\mu'}^{j'} \sin\beta d\beta
$$
$$
= \frac{8\pi^2}{2j+1} \delta_{m\mu} \delta_{m'\mu'} \delta_{jj'}, \tag{21}
$$

we can obtain these coefficients as

$$(-1)^{\frac{J_2}{2}(2+\mu-\kappa)+\frac{J_3}{2}(1-\kappa)} B^{J_1} C_{\alpha_1}^{J_3} \left\{ \beta_{d+}^{J_1 J_2} \right.$$

$$\times \left[\gamma_{d'+}^{J_3 J_2} + \gamma_{d'-}^{J_3 J_2} (1-\delta_{d'0}) \right]$$

$$\left. + \beta_{d-}^{J_1 J_2} \left[\gamma_{d'+}^{J_3 J_2}(1-\delta_{d0}) + \gamma_{d'-}^{J_3 J_2}(1-\delta_{d0})(1-\delta_{d'0}) \right] \right\}$$

$$= (2J_1+1)(2J_2+1)(2J_3+1)$$

$$\times \int W_{\mu\kappa\alpha_1}(\theta,\phi;\theta',\phi';\theta'',\phi'')D_1^* d\Omega d\Omega' d\Omega''. \quad (22)$$

When we have sufficient experimental data for the angular-distribution function $W_{\mu\kappa\alpha}$, where the final polarizations, μ, κ and α, of all the three decay particles are measured, the integral on the right side of (22) can be determined numerically for all possible allowed values of J_1, J_2, J_3, d and d'. Thus we can extract the different coefficients B^{J_1}, $C_{\alpha_1}^{J_3}$, $\beta_{d\pm}^{J_1 J_2}$ and $\gamma_{d'\pm}^{J_3 J_2}$ on the left side of (22). From these coefficients we can determine the relative magnitudes of the A, B, C and E helicity amplitudes as well as the cosines and sines of the relative phases of the A and E helicity amplitudes in the radiative decay processes $^3D_3 \to {}^3P_2 + \gamma_1$ and $^3P_2 \to \psi + \gamma_2$, respectively. Let us illustrate more clearly how the measurements of $(J_1 J_2 J_3 dd')$ coefficients can give all the information. First, the measurement of the (00100) and (00200) coefficients yields γ_{0+}^{10} and γ_{0+}^{20}, and, with the normalization $|E_0|^2 + |E_1|^2 + |E_2|^2 = 1$, the relative magnitudes of E_j are determined. Next the measurements of (01000), (02000), (03000) and (04000) coefficients yields β_{0+}^{01}, β_{0+}^{02}, β_{0+}^{03} and β_{0+}^{04}, and with the normalization $|A_0|^2 + |A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2 = 1$, the relative magnitudes of A_i are determined. Measuring (20000) yields B^2 and with the normalization $|B_0|^2 + |B_1|^2 = 1$, the relative magnitudes of B_0 and B_1 are obtained. The measurement of (00200) yields $C_{\pm\frac{1}{2}}^2$ and with the normalization $|C_0|^2 + |C_1|^2$, the relative magnitudes of C_0 and C_1 are determined. After having obtained all the relative magnitudes, now measuring the (02101) and (02201) coefficients yields $\mathrm{Re}(E_1 E_0^*)$, $\mathrm{Re}(E_2 E_1^*)$, $\mathrm{Im}(E_1 E_0^*)$ and $\mathrm{Im}(E_2 E_1^*)$. Hence the cosines and sines of the relative phases of E_j are determined. Finally, by measuring the (22010), (24010), (42010) and (44010) coefficients, we can obtain the cosines and sines of the relative phases of A_i.

By summing over one or two helicity indices (μ, κ and α_1) of (7), we can easily obtain different combined angular-distribution functions where the polarizations of only one or two decay products (γ_1, γ_2 and e^-) are measured. Suppose we are interested in only measuring the polarization μ of γ_1, the normalized combined angular distribution of γ_1, γ_2 and e^- will then become

$$W_\mu(\theta,\phi;\theta',\phi';\theta'',\phi'')$$

$$= \frac{1}{4} \sum_{\kappa}^{\pm 1} \sum_{\alpha_1}^{\pm \frac{1}{2}} W_{\mu\kappa\alpha_1}(\theta,\phi;\theta',\phi';\theta'',\phi'')$$

$$= \frac{1}{2(4\pi)^3} \sum_{J_1}^{0,2,4,6} B^{J_1} \sum_{J_3}^{0,2} C^{J_3} \sum_{J_2=0}^{4} (-1)^{\left(\frac{1+\mu}{2}\right)J_2}$$

$$\times \sum_{d=0}^{d_m} \sum_{d'=0}^{d'_m} \left(1-\frac{\delta_{d0}}{2}\right)\left(1-\frac{\delta_{d'0}}{2}\right)\left\{ \beta_{d+}^{J_1 J_2} \right.$$

$$\times \left\{ \gamma_{d'+}^{J_3 J_2}[1+(-1)^{J_2}](D_1' + D_1'^* + D_2' + D_2'^*) \right.$$

$$\left. + \gamma_{d'-}^{J_3 J_2}[1-(-1)^{J_2}](D_1' - D_1'^* + D_2' - D_2'^*) \right\}$$

$$+ \beta_{d-}^{J_1 J_2} \left\{ \gamma_{d'+}^{J_3 J_2}[1+(-1)^{J_2}] \right.$$

$$\times (D_1' - D_1'^* - D_2' + D_2'^*)$$

$$\left. \left. + \gamma_{d'-}^{J_3 J_2}[1-(-1)^{J_2}](D_1' + D_1'^* - D_2' - D_2'^*) \right\} \right\} \quad (23)$$

where

$$D_1' = D_1(\kappa=1) = D_{\mu d,0}^{J_1} D_{\mu d,-d'}^{J_2 *} D_{-d',0}^{J_3 *} \quad (24)$$

and

$$D_2' = D_2(\kappa=1) = D_{\mu d,0}^{J_1 *} D_{\mu d,d'}^{J_2} D_{d',0}^{J_3}. \quad (25)$$

As J_3 can only take the values 0 and 2 in (23), we have defined

$$C^{J_3} = C_{\pm\frac{1}{2}}^{J_3} = \sqrt{3} \sum_{\alpha=0}^{1} (-1)^{\alpha} \langle 11; \alpha, -\alpha | J_3; 0 \rangle |C_\alpha|^2$$

$$(J_3 = 0, 2). \quad (26)$$

The coefficients of the Wigner D-functions in (23) can be obtained from

$$(-1)^{\frac{J_2}{2}(1+\mu)} B^{J_1} C^{J_3} \left\{ \beta_{d+}^{J_1 J_2} \left\{ \gamma_{d'+}^{J_3 J_2}[1+(-1)^{J_2}] \right. \right.$$

$$\left. + \gamma_{d'-}^{J_2 J_3}[1-(-1)^{J_2}](1-\delta_{d'0}) \right\}$$

$$+ \beta_{d-}^{J_1 J_2} \left\{ \gamma_{d'+}^{J_3 J_2}[1+(-1)^{J_2}](1-\delta_{d0}) \right.$$

$$\left. \left. + \gamma_{d'-}^{J_3 J_2}[1-(-1)^{J_2}](1-\delta_{d0})(1-\delta_{d'0}) \right\} \right\}$$

$$= 2(2J_1+1)(2J_2+1)(2J_3+1)$$

$$\times \int W_\mu(\theta,\phi;\theta',\phi';\theta'',\phi'')D_1'^* d\Omega d\Omega' d\Omega''. \quad (27)$$

Similarly, if we only measure the polarization κ of γ_2, the normalized combined angular distribution of γ_1, γ_2 and e^- will become

$$W_\kappa(\theta,\phi;\theta',\phi';\theta'',\phi'')$$

$$= \frac{1}{4} \sum_{\mu}^{\pm 1} \sum_{\alpha_1}^{\pm \frac{1}{2}} W_{\mu\kappa\alpha_1}(\theta,\phi;\theta',\phi';\theta'',\phi'')$$

$$= \frac{1}{2(4\pi)^3} \sum_{J_1}^{0,2,4,6} B^{J_1} \sum_{J_3}^{0,2} C^{J_3} \sum_{J_2=0}^{4} (-1)^{\left(\frac{1-\kappa}{2}\right)(J_2+J_3)}$$

$$\times \sum_{d=0}^{d_m} \sum_{d'=0}^{d'_m} \left(1 - \frac{\delta_{d0}}{2}\right) \left(1 - \frac{\delta_{d'0}}{2}\right) \Big\{ \beta_{d+}^{J_1 J_2}$$

$$\times [1 + (-1)^{J_2}] \Big[\gamma_{d'+}^{J_3 J_2} (D_1'' + D_1''^* + D_2'' + D_2''^*)$$

$$+ \gamma_{d'-}^{J_3 J_2} (D_1'' - D_1''^* + D_2'' - D_2''^*) \Big] - \beta_{d-}^{J_1 J_2}$$

$$\times [1 - (-1)^{J_2}] \Big[\gamma_{d'+}^{J_3 J_2} (D_1'' - D_1''^* - D_2'' + D_2''^*)$$

$$+ \gamma_{d'-}^{J_3 J_2} (D_1'' + D_1''^* - D_2'' - D_2''^*) \Big] \Big\} \tag{28}$$

where

$$D_1'' = D_1(\mu = 1) = D_{d,0}^{J_1} D_{d,-\kappa d'}^{J_2 *} D_{-\kappa d',0}^{J_3 *} \tag{29}$$

and

$$D_2'' = D_2(\mu = 1) = D_{d,0}^{J_1 *} D_{d,\kappa d'}^{J_2} D_{\kappa d',0}^{J_3}. \tag{30}$$

The coefficients of the angular functions in (28) can be obtained from

$$(-1)^{\left(\frac{1-\kappa}{2}\right)(J_2+J_3)} B^{J_1} C^{J_3} \Big\{ \beta_{d+}^{J_1 J_2} [1 + (-1)^{J_2}]$$

$$\times \Big[\gamma_{d'+}^{J_3 J_2} + \gamma_{d'-}^{J_3 J_2} (1 - \delta_{d'0}) \Big] - \beta_{d-}^{J_1 J_2} \Big[1 - (-1)^{J_2} \Big]$$

$$\times \Big[\gamma_{d'+}^{J_3 J_2} (1 - \delta_{d0}) + \gamma_{d'-}^{J_3 J_2} (1 - \delta_{d0})(1 - \delta_{d'0}) \Big] \Big\}$$

$$= 2(2J_1 + 1)(2J_2 + 1)(2J_3 + 1)$$

$$\times \int W_\kappa(\theta, \phi; \theta', \phi'; \theta'', \phi'') D_1''^* \mathrm{d}\Omega \mathrm{d}\Omega' \mathrm{d}\Omega'' \tag{31}$$

where again J_3 can only take the values 0 and 2.

If we are interested in only measuring the polarization α_1 of e^-, the combined angular distribution of γ_1, γ_2 and e^- will become

$$W_{\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{4} \sum_{\mu}^{\pm 1} \sum_{\kappa}^{\pm 1} W_{\mu \kappa \alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{4(4\pi)^3} \sum_{J_1}^{0,2,4,6} B^{J_1} \sum_{J_3=0}^{2} C_{\alpha_1}^{J_3} \sum_{J_2=0}^{4} \sum_{d=0}^{d_m} \sum_{d'=0}^{d'_m} \left(1 - \frac{\delta_{d0}}{2}\right)$$

$$\times \left(1 - \frac{\delta_{d'0}}{2}\right) \Big\{ \beta_{d+}^{J_1 J_2} [1 + (-1)^{J_2}] \Big\{ \gamma_{d'+}^{J_3 J_2}$$

$$\times [1 + (-1)^{J_3}](D_1''' + D_1'''^* + D_2''' + D_2'''^*) + \gamma_{d'-}^{J_3 J_2}$$

$$\times [1 - (-1)^{J_3}](D_1''' - D_1'''^* + D_2''' - D_2'''^*) \Big\}$$

$$+ \beta_{d-}^{J_1 J_2} [1 - (-1)^{J_2}] \Big\{ \gamma_{d'+}^{J_3 J_2} [(-1)^{J_3} - 1]$$

$$\times (D_1''' - D_1'''^* - D_2''' + D_2'''^*) - \gamma_{d'-}^{J_3 J_2} [1 + (-1)^{J_3}]$$

$$\times (D_1''' + D_1'''^* - D_2''' - D_2'''^*) \Big\} \Big\} \tag{32}$$

where

$$D_1''' = D_1(\mu = \kappa = 1) = D_{d,0}^{J_1} D_{d,-d'}^{J_2 *} D_{-d',0}^{J_3 *} \tag{33}$$

and

$$D_2''' = D_2(\mu = \kappa = 1) = D_{d,0}^{J_1 *} D_{d,d'}^{J_2} D_{d',0}^{J_3}. \tag{34}$$

The coefficients in (32) can be obtained from

$$B^{J_1} C_{\alpha_1}^{J_3} \Big\{ \beta_{d+}^{J_1 J_2} [1 + (-1)^{J_2}] \Big\{ \gamma_{d'+}^{J_3 J_2} [1 + (-1)^{J_3}]$$

$$+ \gamma_{d'-}^{J_3 J_2} [1 - (-1)^{J_3}](1 - \delta_{d'0}) \Big\} + \beta_{d-}^{J_1 J_2}$$

$$\times [1 - (-1)^{J_2}] \Big\{ \gamma_{d'+}^{J_3 J_2} [(-1)^{J_3} - 1](1 - \delta_{d0})$$

$$- \gamma_{d'-}^{J_3 J_2} [1 + (-1)^{J_3}](1 - \delta_{d0})(1 - \delta_{d'0}) \Big\} \Big\}$$

$$= 4(2J_1 + 1)(2J_2 + 1)(2J_3 + 1)$$

$$\times \int W_{\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'') D_1'''^* \mathrm{d}\Omega \mathrm{d}\Omega' \mathrm{d}\Omega''. \tag{35}$$

Here, J_3 can take the values 0, 1 and 2.

If we now average over the polarizations α_1 of e^- in (32) as well, we get

$$W(\theta, \phi; \theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{2} \sum_{\alpha_1}^{\pm \frac{1}{2}} W_{\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{2(4\pi)^3} \sum_{J_1}^{0,2,4,6} B^{J_1} \sum_{J_3}^{0,2} C^{J_3} \sum_{J_2=0}^{4} \sum_{d=0}^{d_m} \sum_{d'=0}^{d'_m} \left(1 - \frac{\delta_{d0}}{2}\right)$$

$$\times \left(1 - \frac{\delta_{d'0}}{2}\right) \Big\{ \beta_{d+}^{J_1 J_2} \gamma_{d'+}^{J_3 J_2} \Big[1 + (-1)^{J_2}\Big]$$

$$+ \beta_{d-}^{J_1 J_2} \gamma_{d'-}^{J_3 J_2} \Big[1 - (-1)^{J_2}\Big] \Big\}$$

$$\times \Big[(-1)^{J_2}(D_1''' + D_1'''^*) + (D_2''' + D_2'''^*)\Big]. \tag{36}$$

Using (10) and (11), we have

$$\left(1 - \frac{\delta_{d0}}{2}\right) \left(1 - \frac{\delta_{d'0}}{2}\right) \Big\{ \beta_{d+}^{J_1 J_2} \gamma_{d'+}^{J_3 J_2}$$

$$\times \Big[1 + (-1)^{J_2}\Big] + \beta_{d-}^{J_1 J_2} \gamma_{d'-}^{J_3 J_2} \Big[1 - (-1)^{J_2}\Big] \Big\}$$

$$= \frac{\beta_d^{J_1 J_2} \gamma_{d'}^{J_3 J_2}}{2} \tag{37}$$

where

$$\beta_d^{J_1 J_2} = \sqrt{35} \left(1 - \frac{\delta_{d0}}{2}\right)$$

$$\times \sum_{s(d)} \Big[A_{\frac{s+d}{2}} A_{\frac{s-d}{2}}^* + (-1)^{J_2} A_{\frac{s+d}{2}}^* A_{\frac{s-d}{2}} \Big]$$

$$\times \left\langle 22; \frac{s+d-4}{2}, -\frac{s-d-4}{2} \middle| J_2; d \right\rangle$$

$$\times \left\langle 33; \frac{s+d-6}{2}, -\frac{s-d-6}{2} \Big| J_1; d \right\rangle, \tag{38}$$

$$\gamma_{d'}^{J_3 J_2} = \sqrt{15} \left(1 - \frac{\delta_{d'0}}{2} \right)$$

$$\times \sum_{s'(d')} \left[E_{\frac{s'+d'}{2}} E^*_{\frac{s'-d'}{2}} + (-1)^{J_2} E^*_{\frac{s'+d'}{2}} E_{\frac{s'-d'}{2}} \right]$$

$$\times \left\langle 22; \frac{s'+d'}{2}, -\frac{s'-d'}{2} \Big| J_2; d' \right\rangle$$

$$\times \left\langle 11; \frac{s'+d'-2}{2}, -\frac{s'-d'-2}{2} \Big| J_3; d' \right\rangle. \tag{39}$$

By combining (37) and (36), we now recover our results in [9], where the polarizations of the decay particles are not measured.

Using (27), (31) or (35) it can be seen that once the combined angular distribution $W_\mu(\theta, \phi; \theta', \phi'; \theta'', \phi'')$, $W_\kappa(\theta, \phi; \theta', \phi'; \theta'', \phi'')$ or $W_{\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$ is measured, one can also get the same information on the helicity amplitudes as one obtained from measuring the angular distribution function $W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'')$ where the polarizations of the three particles γ_1, γ_2 and e^- are observed. In other words, by measuring the combined angular distribution of the decay particles γ_1, γ_2 and e^- and the polarization of any one particle, we can get complete information on the helicity amplitudes in the radiative decay processes $^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and $^3P_2 \rightarrow \psi + \gamma_2$. In addition, we can also get the relative magnitudes of the helicity amplitudes in the production process $\bar{p}p \rightarrow {}^3D_3$ and in the final decay process $\psi \rightarrow e^+e^-$.

3 Partially integrated angular distributions

The partially integrated angular distributions obtained from (7) will look a lot simpler and we will gain greater insight from them. There are three different cases in which the polarization and the angular distribution of only one particle (γ_1, γ_2 or e^-) are measured. We find that these results are identical to the single-particle angular-distribution functions given in [9], where the polarizations of the individual particles are not measured. So including the measurement of the polarizations in the single-particle angular distributions does not give any further information. However, we will find that the measurement of the polarizations of the decay particles can provide us more information on the helicity amplitudes when we measure the simultaneous angular distributions of two particles. We now consider three different cases of two-particle angular distributions. We will express our results in terms of the spherical harmonics by using the following relation:

$$D_{M0}^J = \sqrt{\frac{4\pi}{2J+1}} Y_{JM}^*. \tag{40}$$

Case 1 We integrate over the angles (θ'', ϕ'') or the direction of e^- and then average over the polarization α_1 of e^-. The combined angular distribution of γ_1 and γ_2, and the polarization of only one of the two particles are measured. The explicit expressions are given in the following.

Only μ is measured:

$$\widetilde{\widetilde{W}}_\mu(\theta, \phi; \theta', \phi')$$

$$= \frac{1}{4} \sum_{\kappa}^{\pm 1} \sum_{\alpha_1}^{\pm\frac{1}{2}} \int W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'') d\Omega''$$

$$= -\frac{1}{4\pi} \Bigg\{ -\frac{1}{4\pi} + \frac{1}{\sqrt{5}} B^0 \beta_{0+}^{02} \gamma_{0+}^{02} Y_{00}(\theta, \phi) Y_{20}(\theta', \phi')$$

$$+ \frac{1}{3} B^0 \beta_{0+}^{04} \gamma_{0+}^{04} Y_{00}(\theta, \phi) Y_{40}(\theta', \phi')$$

$$+ \frac{1}{\sqrt{5}} B^2 \beta_{0+}^{20} \gamma_{0+}^{00} Y_{20}(\theta, \phi) Y_{00}(\theta', \phi')$$

$$+ \frac{1}{5} B^2 \beta_{0+}^{22} \gamma_{0+}^{02} Y_{20}(\theta, \phi) Y_{20}(\theta', \phi')$$

$$+ \frac{2}{5} \Big[B^2 \beta_{1+}^{22} \gamma_{0+}^{02} \mathrm{Re}(Y_{2,\mu}(\theta, \phi) Y_{2,\mu}^*(\theta', \phi'))$$

$$- i B^2 \beta_{1-}^{22} \gamma_{0+}^{02} \mathrm{Im}(Y_{2,\mu}(\theta, \phi) Y_{2,\mu}^*(\theta', \phi'))$$

$$+ B^2 \beta_{2+}^{22} \gamma_{0+}^{02} \mathrm{Re}(Y_{2,2\mu}(\theta, \phi) Y_{2,2\mu}^*(\theta', \phi'))$$

$$- i B^2 \beta_{2-}^{22} \gamma_{0+}^{02} \mathrm{Im}(Y_{2,2\mu}(\theta, \phi) Y_{2,2\mu}^*(\theta', \phi')) \Big]$$

$$+ \frac{1}{3\sqrt{5}} B^2 \beta_{0+}^{24} \gamma_{0+}^{04} Y_{20}(\theta, \phi) Y_{40}(\theta', \phi')$$

$$+ \frac{2}{3\sqrt{5}} \Big[B^2 \beta_{1+}^{24} \gamma_{0+}^{04} \mathrm{Re}(Y_{2,\mu}(\theta, \phi) Y_{4,\mu}^*(\theta', \phi'))$$

$$- i B^2 \beta_{1-}^{24} \gamma_{0+}^{04} \mathrm{Im}(Y_{2,\mu}(\theta, \phi) Y_{4,\mu}^*(\theta', \phi'))$$

$$+ B^2 \beta_{2+}^{24} \gamma_{0+}^{04} \mathrm{Re}(Y_{2,2\mu}(\theta, \phi) Y_{4,2\mu}^*(\theta', \phi'))$$

$$- i B^2 \beta_{2-}^{24} \gamma_{0+}^{04} \mathrm{Im}(Y_{2,2\mu}(\theta, \phi) Y_{4,2\mu}^*(\theta', \phi')) \Big]$$

$$+ \frac{1}{3} B^4 \beta_{0+}^{40} \gamma_{0+}^{00} Y_{40}(\theta, \phi) Y_{00}(\theta', \phi')$$

$$+ \frac{1}{3\sqrt{5}} B^4 \beta_{0+}^{42} \gamma_{0+}^{02} Y_{40}(\theta, \phi) Y_{20}(\theta', \phi')$$

$$+ \frac{2}{3\sqrt{5}} \Big[B^4 \beta_{1+}^{42} \gamma_{0+}^{02} \mathrm{Re}(Y_{4,\mu}(\theta, \phi) Y_{2,\mu}^*(\theta', \phi'))$$

$$- i B^4 \beta_{1-}^{42} \gamma_{0+}^{02} \mathrm{Im}(Y_{4,\mu}(\theta, \phi) Y_{2,\mu}^*(\theta', \phi'))$$

$$+ B^4 \beta_{2+}^{42} \gamma_{0+}^{02} \mathrm{Re}(Y_{4,2\mu}(\theta, \phi) Y_{2,2\mu}^*(\theta', \phi'))$$

$$- i B^4 \beta_{2-}^{42} \gamma_{0+}^{02} \mathrm{Im}(Y_{4,2\mu}(\theta, \phi) Y_{2,2\mu}^*(\theta', \phi')) \Big]$$

$$+ \frac{1}{9} B^4 \beta_{0+}^{44} \gamma_{0+}^{04} Y_{40}(\theta, \phi) Y_{40}(\theta', \phi')$$

$$+ \frac{2}{9} \Big[B^4 \beta_{1+}^{44} \gamma_{0+}^{04} \mathrm{Re}(Y_{4,\mu}(\theta, \phi) Y_{4,\mu}^*(\theta', \phi'))$$

$$- i B^4 \beta_{1-}^{44} \gamma_{0+}^{04} \mathrm{Im}(Y_{4,\mu}(\theta, \phi) Y_{4,\mu}^*(\theta', \phi'))$$

$$+ B^4 \beta_{2+}^{44} \gamma_{0+}^{04} \mathrm{Re}(Y_{4,2\mu}(\theta, \phi) Y_{4,2\mu}^*(\theta', \phi'))$$

$$- i B^4 \beta^{44}_{2-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{4,2\mu}(\theta,\phi) Y^*_{4,2\mu}(\theta',\phi'))$$
$$+ B^4 \beta^{44}_{3+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{4,3\mu}(\theta,\phi) Y^*_{4,3\mu}(\theta',\phi'))$$
$$- i B^4 \beta^{44}_{3-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{4,3\mu}(\theta,\phi) Y^*_{4,3\mu}(\theta',\phi'))$$
$$+ B^4 \beta^{44}_{4+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{4,4\mu}(\theta,\phi) Y^*_{4,4\mu}(\theta',\phi'))$$
$$- i B^4 \beta^{44}_{4-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{4,4\mu}(\theta,\phi) Y^*_{4,4\mu}(\theta',\phi'))\Big]$$
$$+ \frac{1}{\sqrt{13}} B^6 \beta^{60}_{0+} \gamma^{00}_{0+} Y_{60}(\theta,\phi) Y_{00}(\theta',\phi')$$
$$+ \frac{1}{\sqrt{65}} B^6 \beta^{62}_{0+} \gamma^{02}_{0+} Y_{60}(\theta,\phi) Y_{20}(\theta',\phi')$$
$$+ \frac{2}{\sqrt{65}} \Big[B^6 \beta^{62}_{1+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{6,\mu}(\theta,\phi) Y^*_{2,\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{62}_{1-} \gamma^{02}_{0+} \, \mathrm{Im}(Y_{6,\mu}(\theta,\phi) Y^*_{2,\mu}(\theta',\phi'))$$
$$+ B^6 \beta^{62}_{2+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{6,2\mu}(\theta,\phi) Y^*_{2,2\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{62}_{2-} \gamma^{02}_{0+} \, \mathrm{Im}(Y_{6,2\mu}(\theta,\phi) Y^*_{2,2\mu}(\theta',\phi'))\Big]$$
$$+ \frac{1}{3\sqrt{13}} B^6 \beta^{64}_{0+} \gamma^{04}_{0+} Y_{60}(\theta,\phi) Y_{40}(\theta',\phi')$$
$$+ \frac{2}{3\sqrt{13}} \Big[B^6 \beta^{64}_{1+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{6,\mu}(\theta,\phi) Y^*_{4,\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{64}_{1-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{6,\mu}(\theta,\phi) Y^*_{4,\mu}(\theta',\phi'))$$
$$+ B^6 \beta^{64}_{2+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{6,2\mu}(\theta,\phi) Y^*_{4,2\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{64}_{2-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{6,2\mu}(\theta,\phi) Y^*_{4,2\mu}(\theta',\phi'))$$
$$+ B^6 \beta^{64}_{3+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{6,3\mu}(\theta,\phi) Y^*_{4,3\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{64}_{3-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{6,3\mu}(\theta,\phi) Y^*_{4,3\mu}(\theta',\phi'))$$
$$+ B^6 \beta^{64}_{4+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{6,4\mu}(\theta,\phi) Y^*_{4,4\mu}(\theta',\phi'))$$
$$- i B^6 \beta^{64}_{4-} \gamma^{04}_{0+} \, \mathrm{Im}(Y_{6,4\mu}(\theta,\phi) Y^*_{4,4\mu}(\theta',\phi'))\Big]\Big\}. \quad (41)$$

Only κ is measured:

$$\widetilde{\widetilde{W}}_\kappa(\theta,\phi;\theta',\phi')$$
$$= \frac{1}{4} \sum_{\mu}^{\pm 1} \sum_{\alpha_1}^{\pm\frac{1}{2}} \int W_{\mu\kappa\alpha_1}(\theta,\phi;\theta',\phi';\theta'',\phi'') \mathrm{d}\Omega''$$
$$= -\frac{1}{4\pi} \Big\{ -\frac{1}{4\pi} + \frac{1}{\sqrt{5}} B^0 \beta^{02}_{0+} \gamma^{02}_{0+} Y_{00}(\theta,\phi) Y_{20}(\theta',\phi')$$
$$+ \frac{1}{3} B^0 \beta^{04}_{0+} \gamma^{04}_{0+} Y_{00}(\theta,\phi) Y_{40}(\theta',\phi')$$
$$+ \frac{1}{\sqrt{5}} B^2 \beta^{20}_{0+} \gamma^{00}_{0+} Y_{20}(\theta,\phi) Y_{00}(\theta',\phi')$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{\sqrt{15}} B^2 \beta^{21}_{1-} \gamma^{01}_{0+} \, \mathrm{Im}(Y_{21}(\theta,\phi) Y^*_{11}(\theta',\phi'))$$
$$+ \frac{1}{5} B^2 \beta^{22}_{0+} \gamma^{02}_{0+} Y_{20}(\theta,\phi) Y_{20}(\theta',\phi')$$
$$+ \frac{2}{5} \Big[B^2 \beta^{22}_{1+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{21}(\theta,\phi) Y^*_{21}(\theta',\phi'))$$

$$+ B^2 \beta^{22}_{2+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{22}(\theta,\phi) Y^*_{22}(\theta',\phi'))\Big]$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{\sqrt{35}} \Big[B^2 \beta^{23}_{1-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{21}(\theta,\phi) Y^*_{31}(\theta',\phi'))$$
$$+ B^2 \beta^{23}_{2-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{22}(\theta,\phi) Y^*_{23}(\theta',\phi'))\Big]$$
$$+ \frac{1}{3\sqrt{5}} B^2 \beta^{24}_{0+} \gamma^{04}_{0+} Y_{20}(\theta,\phi) Y_{40}(\theta',\phi')$$
$$+ \frac{2}{3\sqrt{5}} \Big[B^2 \beta^{24}_{1+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{21}(\theta,\phi) Y^*_{41}(\theta',\phi'))$$
$$+ B^2 \beta^{24}_{2+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{22}(\theta,\phi) Y^*_{42}(\theta',\phi'))\Big]$$
$$+ \frac{1}{3} B^4 \beta^{40}_{0+} \gamma^{00}_{0+} Y_{40}(\theta,\phi) Y_{00}(\theta',\phi')$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{3\sqrt{3}} B^4 \beta^{41}_{1-} \gamma^{01}_{0+} \, \mathrm{Im}(Y_{41}(\theta,\phi) Y^*_{11}(\theta',\phi'))$$
$$+ \frac{1}{3\sqrt{5}} B^4 \beta^{42}_{0+} \gamma^{02}_{0+} Y_{40}(\theta,\phi) Y_{20}(\theta',\phi')$$
$$+ \frac{2}{3\sqrt{5}} \Big[B^4 \beta^{42}_{1+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{41}(\theta,\phi) Y^*_{21}(\theta',\phi'))$$
$$+ B^4 \beta^{42}_{2+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{42}(\theta,\phi) Y^*_{22}(\theta',\phi'))\Big]$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{3\sqrt{7}} \Big[B^4 \beta^{43}_{1-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{41}(\theta,\phi) Y^*_{31}(\theta',\phi'))$$
$$+ B^4 \beta^{43}_{2-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{42}(\theta,\phi) Y^*_{32}(\theta',\phi'))$$
$$+ B^4 \beta^{43}_{3-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{43}(\theta,\phi) Y^*_{33}(\theta',\phi'))\Big]$$
$$+ \frac{1}{9} B^4 \beta^{44}_{0+} \gamma^{04}_{0+} Y_{40}(\theta,\phi) Y_{40}(\theta',\phi')$$
$$+ \frac{2}{9} \Big[B^4 \beta^{44}_{1+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{41}(\theta,\phi) Y^*_{41}(\theta',\phi'))$$
$$+ B^4 \beta^{44}_{2+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{42}(\theta,\phi) Y^*_{42}(\theta',\phi'))$$
$$+ B^4 \beta^{44}_{3+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{43}(\theta,\phi) Y^*_{43}(\theta',\phi'))$$
$$+ B^4 \beta^{44}_{4+} \gamma^{04}_{0+} \, \mathrm{Re}(Y_{44}(\theta,\phi) Y^*_{44}(\theta',\phi'))\Big]$$
$$+ \frac{1}{\sqrt{13}} B^6 \beta^{60}_{0+} \gamma^{00}_{0+} Y_{60}(\theta,\phi) Y_{00}(\theta',\phi')$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{\sqrt{39}} B^6 \beta^{61}_{1-} \gamma^{01}_{0+} \, \mathrm{Im}(Y_{61}(\theta,\phi) Y^*_{11}(\theta',\phi'))$$
$$+ \frac{1}{\sqrt{65}} B^6 \beta^{62}_{0+} \gamma^{02}_{0+} Y_{60}(\theta,\phi) Y_{20}(\theta',\phi')$$
$$+ \frac{2}{\sqrt{65}} \Big[B^6 \beta^{62}_{1+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{61}(\theta,\phi) Y^*_{21}(\theta',\phi'))$$
$$+ B^6 \beta^{62}_{2+} \gamma^{02}_{0+} \, \mathrm{Re}(Y_{62}(\theta,\phi) Y^*_{22}(\theta',\phi'))\Big]$$
$$+ \frac{2i(-1)^{\frac{1}{2}(1-\kappa)}}{\sqrt{112}} \Big[B^6 \beta^{63}_{1-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{61}(\theta,\phi) Y^*_{31}(\theta',\phi'))$$
$$+ B^6 \beta^{63}_{2-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{62}(\theta,\phi) Y^*_{32}(\theta',\phi'))$$
$$+ B^6 \beta^{63}_{3-} \gamma^{03}_{0+} \, \mathrm{Im}(Y_{63}(\theta,\phi) Y^*_{33}(\theta',\phi'))\Big]$$

$$+ \frac{1}{3\sqrt{13}} B^6 \beta_{0+}^{64} \gamma_{0+}^{04} Y_{60}(\theta, \phi) Y_{40}(\theta', \phi')$$

$$+ \frac{2}{3\sqrt{13}} \Big[B^6 \beta_{1+}^{64} \gamma_{0+}^{04} \, \text{Re}(Y_{61}(\theta, \phi) Y_{41}^*(\theta', \phi'))$$

$$+ B^6 \beta_{2+}^{64} \gamma_{0+}^{04} \, \text{Re}(Y_{62}(\theta, \phi) Y_{42}^*(\theta', \phi'))$$

$$+ B^6 \beta_{3+}^{64} \gamma_{0+}^{04} \, \text{Re}(Y_{63}(\theta, \phi) Y_{43}^*(\theta', \phi'))$$

$$+ B^6 \beta_{4+}^{64} \gamma_{0+}^{04} \, \text{Re}(Y_{64}(\theta, \phi) Y_{44}^*(\theta', \phi')) \Big] \Big\}. \tag{42}$$

An inspection of (41) and (42) shows that the magnitudes of the A, B and E helicity amplitudes as well as the cosines and sines of the relative phases of the A helicity amplitudes can be extracted from the measurement of either $\widetilde{\widetilde{W}}_\mu$ or $\widetilde{\widetilde{W}}_\kappa$. It should be noted that the measurement of the polarization of one of the decay particles is essential for getting the sines of the relative phases among the A helicity amplitudes. This is not possible for the unpolarized case.

Case 2 We integrate over (θ, ϕ) or the direction of γ_1 and average over the polarization μ of γ_1. The combined angular distribution of γ_2 and e^- and the polarization of either one of them are measured. The expressions are given in the following.

Only κ is measured:

$$\widetilde{\widetilde{W}}_\kappa(\theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{4} \sum_{\mu}^{\pm 1} \sum_{\alpha_1}^{\pm \frac{1}{2}} \int W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'') d\Omega$$

$$= -\frac{1}{4\pi} \Bigg\{ -\frac{1}{4\pi} + \frac{1}{\sqrt{5}} C^2 \beta_{0+}^{00} \gamma_{0+}^{20} Y_{00}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$+ \frac{1}{\sqrt{5}} C^0 \beta_{0+}^{02} \gamma_{0+}^{02} Y_{20}(\theta', \phi') Y_{00}(\theta'', \phi'')$$

$$+ \frac{1}{5} C^2 \beta_{0+}^{02} \gamma_{0+}^{22} Y_{20}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$+ \frac{2}{5} C^2 \beta_{0+}^{02} \Big[-\gamma_{1+}^{22} \text{Re}(Y_{2,\kappa}(\theta', \phi') Y_{2,\kappa}(\theta'', \phi''))$$

$$+ i \gamma_{1-}^{22} \text{Im}(Y_{2,\kappa}(\theta', \phi') Y_{2,\kappa}(\theta'', \phi''))$$

$$+ \gamma_{2+}^{22} \text{Re}(Y_{2,2\kappa}(\theta', \phi') Y_{2,2\kappa}(\theta'', \phi''))$$

$$- i \gamma_{2-}^{22} \text{Im}(Y_{2,2\kappa}(\theta', \phi') Y_{2,2\kappa}(\theta'', \phi'')) \Big]$$

$$+ \frac{1}{3} C^0 \beta_{0+}^{04} \gamma_{0+}^{04} Y_{40}(\theta', \phi') Y_{00}(\theta'', \phi'')$$

$$+ \frac{1}{3\sqrt{5}} C^2 \beta_{0+}^{04} \gamma_{0+}^{24} Y_{40}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$+ \frac{2}{3\sqrt{5}} C^2 \beta_{0+}^{04} \Big[-\gamma_{1+}^{24} \text{Re}(Y_{4,\kappa}(\theta', \phi') Y_{2,\kappa}(\theta'', \phi''))$$

$$+ i \gamma_{1-}^{24} \text{Im}(Y_{4,\kappa}(\theta', \phi') Y_{2,\kappa}(\theta'', \phi''))$$

$$+ \gamma_{2+}^{24} \text{Re}(Y_{4,2\kappa}(\theta', \phi') Y_{2,2\kappa}(\theta'', \phi''))$$

$$- i \gamma_{2-}^{24} \text{Im}(Y_{4,2\kappa}(\theta', \phi') Y_{2,2\kappa}(\theta'', \phi'')) \Big] \Bigg\}. \tag{43}$$

Only α_1 is measured:

$$\widetilde{\widetilde{W}}_{\alpha_1}(\theta', \phi'; \theta'', \phi'')$$

$$= \frac{1}{4} \sum_{\mu}^{\pm 1} \sum_{\kappa}^{\pm 1} \int W_{\mu\kappa\alpha_1}(\theta, \phi; \theta', \phi'; \theta'', \phi'') d\Omega$$

$$= -\frac{1}{4\pi} \Bigg\{ -\frac{1}{4\pi} + \frac{1}{\sqrt{5}} C^2 \beta_{0+}^{00} \gamma_{0+}^{20} Y_{00}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$+ \frac{1}{\sqrt{5}} C^0 \beta_{0+}^{02} \gamma_{0+}^{02} Y_{20}(\theta', \phi') Y_{00}(\theta'', \phi'')$$

$$+ \frac{2i}{\sqrt{15}} C_{\alpha_1}^1 \beta_{0+}^{02} \gamma_{1-}^{12} \text{Im}(Y_{21}(\theta', \phi') Y_{11}(\theta'', \phi''))$$

$$+ \frac{1}{5} C^2 \beta_{0+}^{02} \gamma_{0+}^{22} Y_{20}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$- \frac{2}{5} C^2 \beta_{0+}^{02} \gamma_{1+}^{22} \text{Re}(Y_{21}(\theta', \phi') Y_{21}(\theta'', \phi''))$$

$$+ \frac{2}{5} C^2 \beta_{0+}^{02} \gamma_{2+}^{22} \text{Re}(Y_{22}(\theta', \phi') Y_{22}(\theta'', \phi''))$$

$$+ \frac{1}{3} C^0 \beta_{0+}^{04} \gamma_{0+}^{04} Y_{40}(\theta', \phi') Y_{00}(\theta'', \phi'')$$

$$+ \frac{2i}{3\sqrt{3}} C_{\alpha_1}^1 \beta_{0+}^{04} \gamma_{1-}^{14} \text{Im}(Y_{41}(\theta', \phi') Y_{11}(\theta'', \phi''))$$

$$+ \frac{1}{3\sqrt{5}} C^2 \beta_{0+}^{04} \gamma_{0+}^{24} Y_{40}(\theta', \phi') Y_{20}(\theta'', \phi'')$$

$$- \frac{2}{3\sqrt{5}} C^2 \beta_{0+}^{04} \gamma_{1+}^{24} \text{Re}(Y_{41}(\theta', \phi') Y_{21}(\theta'', \phi''))$$

$$+ \frac{2}{3\sqrt{5}} C^2 \beta_{0+}^{04} \gamma_{2+}^{24} \text{Re}(Y_{42}(\theta', \phi') Y_{22}(\theta'', \phi'')) \Bigg\}. \tag{44}$$

An examination of (43) and (44) shows that we can obtain the magnitudes of the E helicity amplitudes as well as both the cosines and the sines of the relative phases of the E helicity amplitudes when the simultaneous angular distribution of γ_2 and e^- with the polarization of either one particles is measured. As in case 1, the measurement of the polarization is essential for getting the sines of the relative phases of these helicity amplitudes uniquely. It is worth noting that we can now obtain all the information on the helicity amplitudes from the measurement of the joint angular distributions of only two particles.

Case 3 We integrate over (θ', ϕ') or the direction of γ_2 and then average over the polarization κ of γ_2. The combined angular distribution of γ_1 and e^- and the polarization of either one of them are measured. Since we cannot obtain any useful information from this case, we do not provide the long expressions here.

4 Concluding remarks

We have derived the model-independent expressions for the combined angular distribution of the final photons (γ_1 and γ_2)

and electron (e^-) in the cascade process, $\bar{p} + p \rightarrow {}^3D_3 \rightarrow {}^3P_2 + \gamma_1 \rightarrow (\psi + \gamma_2) + \gamma_1 \rightarrow (e^+ + e^-) + \gamma_1 + \gamma_2$, when $\bar{p}$ and p are unpolarized and the polarization of any one of the three decay particles is measured. Our expressions are based only on the general principles of quantum mechanics and the symmetry of the problem. We have also derived the partially integrated angular-distribution functions which give the two-particle angular distributions of (γ_1, γ_2) and (γ_2, e^-) with the measurement of the polarization of one particle in each cases. Once these polarized angular distributions are experimentally measured, our expressions can be used to extract the information of all the independent helicity amplitudes in the radiative decay processes ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and ${}^3P_2 \rightarrow \psi + \gamma_2$. In fact, the analysis of the angular correlations in the final decay products will serve to verify the presence of the intermediate ${}^3D_3$ state and its J^{PC} quantum numbers in the cascade process. The experimentally determined values of the helicity amplitudes can then be compared with the predictions of various dynamical models.

The great advantage of measuring the angular distributions with the polarization of one particle is that one can obtain not only the relative magnitudes of the helicity amplitudes but also both the cosines and the sines of the relative phases of the helicity amplitudes in the decay processes ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and ${}^3P_2 \rightarrow \psi + \gamma_2$. This is important because the helicity amplitudes are in general complex [14]. Therefore by measuring the combined angular distribution of γ_1, γ_2 and e^- with the polarization of any one of the three particles, we can obtain complete information on the helicity amplitudes in the two radiative decay processes. Alternatively, we can get the same information by measuring the two-particle angular distribution of γ_2 and e^- and that of γ_1 and γ_2 with the polarization of either one of the two particles.

Both the theorists and the experimentalists would like to express their results in terms of the multipole amplitudes in the radiative transitions ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and ${}^3P_2 \rightarrow \psi + \gamma_2$. The relationship between the helicity and the multipole amplitudes are given by the orthogonal transformations [15, 16]:

$$A_i = \sum_{k=1}^{5} a_k \sqrt{\frac{2k+1}{5}} \langle k, -1; 3, (i-1)|2, (i-2) \rangle \quad (45)$$
$$(i = 0, 1, 2, 3, 4),$$

and

$$E_j = \sum_{k=1}^{3} e_k \sqrt{\frac{2k+1}{5}} \langle k, 1; 1, (j-1)|2, j \rangle$$
$$(j = 0, 1, 2), \quad (46)$$

where a_k and e_k are the radiative multipole amplitudes in ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ and ${}^3P_2 \rightarrow \psi + \gamma_2$, respectively. Since the

transformations of (45) and (46) are orthogonal,

$$\sum_{i=0}^{4} |A_i|^2 = \sum_{k=1}^{5} |a_k|^2 = 1 , \quad \sum_{j=0}^{2} |E_j|^2 = \sum_{k=1}^{3} |e_k|^2 = 1 .$$
$$(47)$$

It is noteworthy that the decay process ${}^3D_3 \rightarrow {}^3P_2 + \gamma_1$ has five independent helicity amplitudes corresponding to five multipole amplitudes $E1$, $M2$, $E3$, $M4$ and $E5$. In any potential model for heavy quarkonia, the $M4$ or higher multipole amplitudes is zero to order v^2/c^2 because in this approximation there is no fourth or higher rank tensor component in the transition operator [17]. So by measuring the angular distributions, one can further test the validity of the non-relativistic potential models.

Appendix A: Expressions of coefficients

A.1 Expressions of B^{J_1}

$$B^0 = -1, \quad (48)$$
$$B^2 = \frac{2}{\sqrt{3}} \left(|B_0|^2 + \frac{3}{4}|B_1|^2 \right), \quad (49)$$
$$B^4 = -3\sqrt{\frac{2}{11}} \left(|B_0|^2 + \frac{1}{6}|B_1|^2 \right), \quad (50)$$
$$B^6 = \frac{10}{\sqrt{33}} \left(|B_0|^2 - \frac{3}{4}|B_1|^2 \right). \quad (51)$$

A.2 Expressions of $C_{\alpha_1}^{J_3}$

$$C_{\pm\frac{1}{2}}^0 = C^0 = -1, \quad (52)$$
$$C_{\pm\frac{1}{2}}^1 = \pm\sqrt{\frac{3}{2}}|C_1|^2, \quad (53)$$
$$C_{\pm\frac{1}{2}}^2 = C^2 = \sqrt{2} \left(|C_0|^2 - \frac{1}{2}|C_1|^2 \right). \quad (54)$$

A.3 Expressions of $\beta_{d\pm}^{J_1 J_2}$

$$\beta_{0+}^{00} = 1, \quad (55)$$
$$\beta_{0+}^{20} = \frac{5}{2\sqrt{3}} \left(|A_0|^2 - \frac{3}{5}|A_2|^2 - \frac{4}{5}|A_3|^2 - \frac{3}{5}|A_4|^2 \right), \quad (56)$$

$$\beta_{0+}^{40} = \frac{3}{\sqrt{22}} \left(|A_0|^2 - \frac{7}{3}|A_1|^2 \right.$$
$$\left. + \frac{1}{3}|A_2|^2 + 2|A_3|^2 + \frac{1}{3}|A_4|^2 \right), \tag{57}$$

$$\beta_{0+}^{60} = \frac{1}{2\sqrt{33}} (|A_0|^2 - 6|A_1|^2$$
$$+ 15|A_2|^2 - 20|A_3|^2 + 15|A_4|^2), \tag{58}$$

$$\beta_{0+}^{01} = -\sqrt{2} \left(|A_0|^2 + \frac{1}{2}|A_1|^2 - \frac{1}{2}|A_3|^2 - |A_4|^2 \right), \tag{59}$$

$$\beta_{0+}^{21} = -\frac{5}{\sqrt{6}} \left(|A_0|^2 + \frac{2}{5}|A_3|^2 + \frac{3}{5}|A_4|^2 \right), \tag{60}$$

$$\beta_{0+}^{41} = -\frac{3}{\sqrt{11}} \left(|A_0|^2 - \frac{7}{6}|A_1|^2 - |A_3|^2 - \frac{1}{3}|A_4|^2 \right), \tag{61}$$

$$\beta_{0+}^{61} = -\frac{1}{\sqrt{66}} (|A_0|^2 - 3|A_1|^2 + 10|A_3|^2 - 15|A_4|^2), \tag{62}$$

$$\beta_{0+}^{02} = \sqrt{\frac{10}{7}} \left(|A_0|^2 - \frac{1}{2}|A_1|^2 - |A_2|^2 - \frac{1}{2}|A_3|^2 + |A_4|^2 \right), \tag{63}$$

$$\beta_{0+}^{22} = 5\sqrt{\frac{5}{42}} \left(|A_0|^2 + \frac{3}{5}|A_2|^2 + \frac{2}{5}|A_3|^2 - \frac{3}{5}|A_4|^2 \right), \tag{64}$$

$$\beta_{0+}^{42} = 3\sqrt{\frac{5}{77}} \left(|A_0|^2 + \frac{7}{6}|A_1|^2 \right.$$
$$\left. - \frac{1}{3}|A_2|^2 - |A_3|^2 + \frac{1}{3}|A_4|^2 \right), \tag{65}$$

$$\beta_{0+}^{62} = \sqrt{\frac{5}{462}} (|A_0|^2 + 3|A_1|^2$$
$$- 15|A_2|^2 + 10|A_3|^2 + 15|A_4|^2), \tag{66}$$

$$\beta_{0+}^{03} = -\frac{1}{\sqrt{2}} (|A_0|^2 - 2|A_1|^2 + 2|A_3|^2 - |A_4|^2), \tag{67}$$

$$\beta_{0+}^{23} = -\frac{5}{2\sqrt{6}} \left(|A_0|^2 - \frac{8}{5}|A_3|^2 + \frac{3}{5}|A_4|^2 \right), \tag{68}$$

$$\beta_{0+}^{43} = -\frac{3}{2\sqrt{11}} \left(|A_0|^2 + \frac{14}{3}|A_1|^2 + 4|A_3|^2 - \frac{1}{3}|A_4|^2 \right), \tag{69}$$

$$\beta_{0+}^{63} = -\frac{1}{2\sqrt{66}} (|A_0|^2 + 12|A_1|^2 - 40|A_3|^2 - 15|A_4|^2), \tag{70}$$

$$\beta_{0+}^{04} = \frac{1}{\sqrt{14}} (|A_0|^2 - 4|A_1|^2 + 6|A_2|^2 - 4|A_3|^2 + |A_4|^2), \tag{71}$$

$$\beta_{0+}^{24} = \frac{5}{2\sqrt{42}} \left(|A_0|^2 - \frac{18}{5}|A_2|^2 + \frac{16}{5}|A_3|^2 - \frac{3}{5}|A_4|^2 \right), \tag{72}$$

$$\beta_{0+}^{44} = \frac{3}{2\sqrt{77}} \left(|A_0|^2 + \frac{28}{3}|A_1|^2 \right.$$
$$\left. + 2|A_2|^2 - 8|A_3|^2 + \frac{1}{3}|A_4|^2 \right), \tag{73}$$

$$\beta_{0+}^{64} = \frac{1}{2\sqrt{462}} (|A_0|^2 + 24|A_1|^2$$
$$+ 90|A_2|^2 + 80|A_3|^2 + 15|A_4|^2), \tag{74}$$

$$\beta_{1+}^{21} = -\frac{5}{2\sqrt{3}} \left[\mathrm{Re}(A_1 A_0^*) + \frac{3}{\sqrt{10}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. + \frac{\sqrt{3}}{5} \mathrm{Re}(A_3 A_2^*) - \frac{\sqrt{2}}{5} \mathrm{Re}(A_4 A_3^*) \right], \tag{75}$$

$$\beta_{1+}^{41} = -\sqrt{\frac{15}{11}} \left[\mathrm{Re}(A_1 A_0^*) - \frac{4}{\sqrt{10}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. - \frac{\sqrt{3}}{2} \mathrm{Re}(A_3 A_2^*) + \frac{1}{\sqrt{2}} \mathrm{Re}(A_4 A_3^*) \right], \tag{76}$$

$$\beta_{1+}^{61} = -\frac{1}{2}\sqrt{\frac{7}{33}} \left[\mathrm{Re}(A_1 A_0^*) - 6\sqrt{\frac{5}{2}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. + 5\sqrt{3} \mathrm{Re}(A_3 A_2^*) - 5\sqrt{2} \mathrm{Re}(A_4 A_3^*) \right], \tag{77}$$

$$\beta_{1+}^{22} = \frac{5}{2}\sqrt{\frac{5}{7}} \left[\mathrm{Re}(A_1 A_0^*) + \frac{1}{\sqrt{10}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. - \frac{1}{5\sqrt{3}} \mathrm{Re}(A_3 A_2^*) + \frac{\sqrt{2}}{5} \mathrm{Re}(A_4 A_3^*) \right], \tag{78}$$

$$\beta_{1+}^{42} = \frac{15}{\sqrt{77}} \left[\mathrm{Re}(A_1 A_0^*) - \frac{4}{3\sqrt{10}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. + \frac{1}{2\sqrt{3}} \mathrm{Re}(A_3 A_2^*) - \frac{1}{\sqrt{2}} \mathrm{Re}(A_4 A_3^*) \right], \tag{79}$$

$$\beta_{1+}^{62} = \frac{1}{2}\sqrt{\frac{5}{11}} \left[\mathrm{Re}(A_1 A_0^*) - \sqrt{\frac{5}{2}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. - \frac{5}{\sqrt{3}} \mathrm{Re}(A_3 A_2^*) + 5\sqrt{2} \mathrm{Re}(A_4 A_3^*) \right], \tag{80}$$

$$\beta_{1+}^{23} = -\frac{5}{2\sqrt{2}} \left[\mathrm{Re}(A_1 A_0^*) - \sqrt{\frac{2}{5}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. - \frac{2}{5\sqrt{3}} \mathrm{Re}(A_3 A_2^*) - \frac{\sqrt{2}}{5} \mathrm{Re}(A_4 A_3^*) \right], \tag{81}$$

$$\beta_{1+}^{43} = -\frac{3}{2}\sqrt{\frac{10}{11}} \left[\mathrm{Re}(A_1 A_0^*) + \frac{4}{3}\sqrt{\frac{2}{5}} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. + \frac{1}{\sqrt{3}} \mathrm{Re}(A_3 A_2^*) + \frac{1}{\sqrt{2}} \mathrm{Re}(A_4 A_3^*) \right], \tag{82}$$

$$\beta_{1+}^{63} = -\frac{1}{2}\sqrt{\frac{7}{22}} \left[\mathrm{Re}(A_1 A_0^*) + \sqrt{10} \mathrm{Re}(A_2 A_1^*) \right.$$
$$\left. - \frac{10}{\sqrt{3}} \mathrm{Re}(A_3 A_2^*) - 5\sqrt{2} \mathrm{Re}(A_4 A_3^*) \right], \tag{83}$$

$$\beta_{1+}^{24} = \frac{5}{2}\sqrt{\frac{5}{42}}\left[\mathrm{Re}(A_1A_0^*) - 3\sqrt{\frac{2}{5}}\,\mathrm{Re}(A_2A_1^*)\right.$$
$$\left. + \frac{2\sqrt{3}}{5}\,\mathrm{Re}(A_3A_2^*) + \frac{\sqrt{2}}{5}\,\mathrm{Re}(A_4A_3^*)\right], \tag{84}$$

$$\beta_{1+}^{44} = \frac{5}{2}\sqrt{\frac{6}{7}}\left[\mathrm{Re}(A_1A_0^*) + 4\sqrt{\frac{2}{5}}\,\mathrm{Re}(A_2A_1^*)\right.$$
$$\left. - \sqrt{3}\,\mathrm{Re}(A_3A_2^*) - \frac{1}{\sqrt{2}}\,\mathrm{Re}(A_4A_3^*)\right], \tag{85}$$

$$\beta_{1+}^{64} = \frac{1}{2}\sqrt{\frac{5}{66}}\left[\mathrm{Re}(A_1A_0^*) + 3\sqrt{10}\,\mathrm{Re}(A_2A_1^*)\right.$$
$$\left. + 10\sqrt{3}\,\mathrm{Re}(A_3A_2^*) + 5\sqrt{2}\,\mathrm{Re}(A_4A_3^*)\right], \tag{86}$$

$$\beta_{1-}^{21} = -\frac{5i}{2\sqrt{3}}\left[\mathrm{Im}(A_1A_0^*) + \frac{3}{\sqrt{10}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + \frac{\sqrt{3}}{5}\,\mathrm{Im}(A_3A_2^*) - \frac{\sqrt{2}}{5}\,\mathrm{Im}(A_4A_3^*)\right], \tag{87}$$

$$\beta_{1-}^{41} = -i\sqrt{\frac{15}{11}}\left[\mathrm{Im}(A_1A_0^*) - \frac{4}{\sqrt{10}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \frac{\sqrt{3}}{2}\,\mathrm{Im}(A_3A_2^*) + \frac{1}{\sqrt{2}}\,\mathrm{Im}(A_4A_3^*)\right], \tag{88}$$

$$\beta_{1-}^{61} = -\frac{i}{2}\sqrt{\frac{7}{33}}\left[\mathrm{Im}(A_1A_0^*) - 6\sqrt{\frac{5}{2}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + 5\sqrt{3}\,\mathrm{Im}(A_3A_2^*) - 5\sqrt{2}\,\mathrm{Im}(A_4A_3^*)\right], \tag{89}$$

$$\beta_{1-}^{22} = \frac{5i}{2}\sqrt{\frac{5}{7}}\left[\mathrm{Im}(A_1A_0^*) + \frac{1}{\sqrt{10}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \frac{1}{5\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) + \frac{\sqrt{2}}{5}\,\mathrm{Im}(A_4A_3^*)\right], \tag{90}$$

$$\beta_{1-}^{42} = \frac{15i}{\sqrt{77}}\left[\mathrm{Im}(A_1A_0^*) - \frac{4}{3\sqrt{10}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + \frac{1}{2\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) - \frac{1}{\sqrt{2}}\,\mathrm{Im}(A_4A_3^*)\right], \tag{91}$$

$$\beta_{1-}^{62} = \frac{i}{2}\sqrt{\frac{5}{11}}\left[\mathrm{Im}(A_1A_0^*) - \sqrt{\frac{5}{2}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \frac{5}{\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) + 5\sqrt{2}\,\mathrm{Im}(A_4A_3^*)\right], \tag{92}$$

$$\beta_{1-}^{23} = -\frac{5i}{2\sqrt{2}}\left[\mathrm{Im}(A_1A_0^*) - \sqrt{\frac{2}{5}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \frac{2}{5\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) - \frac{\sqrt{2}}{5}\,\mathrm{Im}(A_4A_3^*)\right], \tag{93}$$

$$\beta_{1-}^{43} = -\frac{3i}{2}\sqrt{\frac{10}{11}}\left[\mathrm{Im}(A_1A_0^*) + \frac{4}{3}\sqrt{\frac{2}{5}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + \frac{1}{\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) + \frac{1}{\sqrt{2}}\,\mathrm{Im}(A_4A_3^*)\right], \tag{94}$$

$$\beta_{1-}^{63} = -\frac{i}{2}\sqrt{\frac{7}{22}}\left[\mathrm{Im}(A_1A_0^*) + \sqrt{10}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \frac{10}{\sqrt{3}}\,\mathrm{Im}(A_3A_2^*) - 5\sqrt{2}\,\mathrm{Im}(A_4A_3^*)\right], \tag{95}$$

$$\beta_{1-}^{24} = \frac{5i}{2}\sqrt{\frac{5}{42}}\left[\mathrm{Im}(A_1A_0^*) - 3\sqrt{\frac{2}{5}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + \frac{2\sqrt{3}}{5}\,\mathrm{Im}(A_3A_2^*) + \frac{\sqrt{2}}{5}\,\mathrm{Im}(A_4A_3^*)\right], \tag{96}$$

$$\beta_{1-}^{44} = \frac{5i}{2}\sqrt{\frac{6}{7}}\left[\mathrm{Im}(A_1A_0^*) + 4\sqrt{\frac{2}{5}}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. - \sqrt{3}\,\mathrm{Im}(A_3A_2^*) - \frac{1}{\sqrt{2}}\,\mathrm{Im}(A_4A_3^*)\right], \tag{97}$$

$$\beta_{1-}^{64} = \frac{i}{2}\sqrt{\frac{5}{66}}\left[\mathrm{Im}(A_1A_0^*) + 3\sqrt{10}\,\mathrm{Im}(A_2A_1^*)\right.$$
$$\left. + 10\sqrt{3}\,\mathrm{Im}(A_3A_2^*) + 5\sqrt{2}\,\mathrm{Im}(A_4A_3^*)\right], \tag{98}$$

$$\beta_{2+}^{22} = \frac{5}{\sqrt{21}}\left[\mathrm{Re}(A_2A_0^*)\right.$$
$$\left. + \sqrt{3}\,\mathrm{Re}(A_3A_1^*) + 2\sqrt{\frac{3}{5}}\,\mathrm{Re}(A_4A_2^*)\right], \tag{99}$$

$$\beta_{2+}^{42} = 3\sqrt{\frac{30}{77}}\left[\mathrm{Re}(A_2A_0^*)\right.$$
$$\left. - \frac{1}{2\sqrt{3}}\,\mathrm{Re}(A_3A_1^*) - \frac{2}{3}\sqrt{\frac{5}{3}}\,\mathrm{Re}(A_4A_2^*)\right], \tag{100}$$

$$\beta_{2+}^{62} = \sqrt{\frac{10}{33}}\left[\mathrm{Re}(A_2A_0^*)\right.$$
$$\left. - 2\sqrt{3}\,\mathrm{Re}(A_3A_1^*) + \sqrt{15}\,\mathrm{Re}(A_4A_2^*)\right], \tag{101}$$

$$\beta_{2+}^{23} = -\frac{5}{2\sqrt{3}}\left[\mathrm{Re}(A_2A_0^*) - 2\sqrt{\frac{3}{5}}\,\mathrm{Re}(A_4A_2^*)\right], \tag{102}$$

$$\beta_{2+}^{43} = -\frac{3}{2}\sqrt{\frac{30}{11}}\left[\mathrm{Re}(A_2A_0^*) + \frac{2}{3}\sqrt{\frac{5}{3}}\,\mathrm{Re}(A_4A_2^*)\right], \tag{103}$$

$$\beta_{2+}^{63} = -\frac{1}{2}\sqrt{\frac{70}{33}}\left[\mathrm{Re}(A_2A_0^*) - \sqrt{15}\,\mathrm{Re}(A_4A_2^*)\right], \tag{104}$$

$$\beta_{2+}^{24} = \frac{5}{2\sqrt{7}}\Bigg[\mathrm{Re}(A_2 A_0^*)$$
$$- \frac{4}{\sqrt{3}}\mathrm{Re}(A_3 A_1^*) + 2\sqrt{\frac{3}{5}}\mathrm{Re}(A_4 A_2^*)\Bigg], \qquad (105)$$

$$\beta_{2+}^{44} = \frac{9}{2}\sqrt{\frac{10}{77}}\Bigg[\mathrm{Re}(A_2 A_0^*)$$
$$+ \frac{2\sqrt{3}}{9}\mathrm{Re}(A_3 A_1^*) - \frac{2\sqrt{15}}{9}\mathrm{Re}(A_4 A_2^*)\Bigg], \qquad (106)$$

$$\beta_{2+}^{64} = \frac{1}{2}\sqrt{\frac{10}{11}}\Bigg[\mathrm{Re}(A_2 A_0^*)$$
$$+ \frac{8}{\sqrt{3}}\mathrm{Re}(A_3 A_1^*) + \sqrt{15}\,\mathrm{Re}(A_4 A_2^*)\Bigg], \qquad (107)$$

$$\beta_{2-}^{22} = \frac{5i}{\sqrt{21}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$+ \sqrt{3}\,\mathrm{Im}(A_3 A_1^*) + 2\sqrt{\frac{3}{5}}\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (108)$$

$$\beta_{2-}^{42} = 3i\sqrt{\frac{30}{77}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$- \frac{1}{2\sqrt{3}}\mathrm{Im}(A_3 A_1^*) - \frac{2}{3}\sqrt{\frac{5}{3}}\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (109)$$

$$\beta_{2-}^{62} = i\sqrt{\frac{10}{33}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$- 2\sqrt{3}\,\mathrm{Im}(A_3 A_1^*) + \sqrt{15}\,\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (110)$$

$$\beta_{2-}^{23} = -\frac{5i}{2\sqrt{3}}\Bigg[\mathrm{Im}(A_2 A_0^*) - 2\sqrt{\frac{3}{5}}\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (111)$$

$$\beta_{2-}^{43} = -\frac{3i}{2}\sqrt{\frac{30}{11}}\Bigg[\mathrm{Im}(A_2 A_0^*) + \frac{2}{3}\sqrt{\frac{5}{3}}\mathrm{Im}(A_4 A_2^*)\Bigg], \quad (112)$$

$$\beta_{2-}^{63} = -\frac{i}{2}\sqrt{\frac{70}{33}}\Bigg[\mathrm{Im}(A_2 A_0^*) - \sqrt{15}\,\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (113)$$

$$\beta_{2-}^{24} = \frac{5i}{2\sqrt{7}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$- \frac{4}{\sqrt{3}}\mathrm{Im}(A_3 A_1^*) + 2\sqrt{\frac{3}{5}}\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (114)$$

$$\beta_{2-}^{44} = \frac{9i}{2}\sqrt{\frac{10}{77}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$+ \frac{2\sqrt{3}}{9}\mathrm{Im}(A_3 A_1^*) - \frac{2\sqrt{15}}{9}\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (115)$$

$$\beta_{2-}^{64} = \frac{i}{2}\sqrt{\frac{10}{11}}\Bigg[\mathrm{Im}(A_2 A_0^*)$$
$$+ \frac{8}{\sqrt{3}}\mathrm{Im}(A_3 A_1^*) + \sqrt{15}\,\mathrm{Im}(A_4 A_2^*)\Bigg], \qquad (116)$$

$$\beta_{3+}^{43} = -\frac{3}{2}\sqrt{\frac{35}{11}}\Bigg[\mathrm{Re}(A_3 A_0^*) + \frac{\sqrt{2}}{3}\mathrm{Re}(A_4 A_1^*)\Bigg], \quad (117)$$

$$\beta_{3+}^{63} = -\frac{1}{2}\sqrt{\frac{70}{11}}\Bigg[\mathrm{Re}(A_3 A_0^*) - \frac{3}{\sqrt{2}}\mathrm{Re}(A_4 A_1^*)\Bigg], \quad (118)$$

$$\beta_{3+}^{44} = \frac{3}{2}\sqrt{\frac{35}{11}}\Bigg[\mathrm{Re}(A_3 A_0^*) - \frac{\sqrt{2}}{3}\mathrm{Re}(A_4 A_1^*)\Bigg], \qquad (119)$$

$$\beta_{3+}^{64} = \frac{1}{2}\sqrt{\frac{70}{11}}\Bigg[\mathrm{Re}(A_3 A_0^*) + \frac{3}{\sqrt{2}}\mathrm{Re}(A_4 A_1^*)\Bigg], \qquad (120)$$

$$\beta_{3-}^{43} = -\frac{3i}{2}\sqrt{\frac{35}{11}}\Bigg[\mathrm{Im}(A_3 A_0^*) + \frac{\sqrt{2}}{3}\mathrm{Im}(A_4 A_1^*)\Bigg], \quad (121)$$

$$\beta_{3-}^{63} = -\frac{i}{2}\sqrt{\frac{70}{11}}\Bigg[\mathrm{Im}(A_3 A_0^*) - \frac{3}{\sqrt{2}}\mathrm{Im}(A_4 A_1^*)\Bigg], \quad (122)$$

$$\beta_{3-}^{44} = \frac{3i}{2}\sqrt{\frac{35}{11}}\Bigg[\mathrm{Im}(A_3 A_0^*) - \frac{\sqrt{2}}{3}\mathrm{Im}(A_4 A_1^*)\Bigg], \qquad (123)$$

$$\beta_{3-}^{64} = \frac{i}{2}\sqrt{\frac{70}{11}}\Bigg[\mathrm{Im}(A_3 A_0^*) + \frac{3}{\sqrt{2}}\mathrm{Im}(A_4 A_1^*)\Bigg], \qquad (124)$$

$$\beta_{4+}^{44} = \sqrt{\frac{105}{11}}\,\mathrm{Re}(A_4 A_0^*), \qquad (125)$$

$$\beta_{4+}^{64} = \frac{5}{2}\sqrt{\frac{14}{11}}\,\mathrm{Re}(A_4 A_0^*), \qquad (126)$$

$$\beta_{4-}^{44} = i\sqrt{\frac{105}{11}}\,\mathrm{Im}(A_4 A_0^*), \qquad (127)$$

$$\beta_{4-}^{64} = \frac{5i}{2}\sqrt{\frac{14}{11}}\,\mathrm{Im}(A_4 A_0^*). \qquad (128)$$

A.4 Expressions of $\gamma_{d'\pm}^{J_3 J_2}$

$$\gamma_{0+}^{00} = 1, \qquad (129)$$

$$\gamma_{0+}^{01} = \frac{1}{\sqrt{2}}(|E_0|^2 + 2|E_2|^2), \qquad (130)$$

$$\gamma_{0+}^{02} = -\sqrt{\frac{10}{7}}\left(|E_0|^2 + \frac{1}{2}|E_1|^2 - |E_2|^2\right), \qquad (131)$$

$$\gamma_{0+}^{03} = -\sqrt{2}\left(|E_1|^2 + \frac{1}{2}|E_2|^2\right), \qquad (132)$$

$$\gamma_{0+}^{04} = 3\sqrt{\frac{2}{7}}\left(|E_0|^2 - \frac{2}{3}|E_1|^2 + \frac{1}{6}|E_2|^2\right), \qquad (133)$$

$$\gamma_{0+}^{10} = -\sqrt{\frac{3}{2}}(|E_0|^2 - |E_2|^2), \qquad (134)$$

$$\gamma_{0+}^{11} = \sqrt{3}|E_2|^2, \qquad (135)$$

$$\gamma_{0+}^{12} = \sqrt{\frac{15}{7}}(|E_0|^2 + |E_2|^2), \qquad (136)$$

$$\gamma_{0+}^{13} = \frac{\sqrt{3}}{2}|E_2|^2, \tag{137}$$

$$\gamma_{0+}^{14} = -3\sqrt{\frac{3}{7}}(|E_0|^2 - \frac{1}{6}|E_2|^2), \tag{138}$$

$$\gamma_{0+}^{20} = \frac{1}{\sqrt{2}}(|E_0|^2 - 2|E_1|^2 + |E_2|^2), \tag{139}$$

$$\gamma_{0+}^{21} = -(|E_1|^2 - |E_2|^2), \tag{140}$$

$$\gamma_{0+}^{22} = -\sqrt{\frac{5}{7}}(|E_1|^2 - |E_1|^2 - |E_2|^2), \tag{141}$$

$$\gamma_{0+}^{23} = 2\left(|E_1|^2 + \frac{1}{4}|E_2|^2\right), \tag{142}$$

$$\gamma_{0+}^{24} = \frac{3}{\sqrt{7}}\left(|E_0|^2 + \frac{4}{3}|E_1|^2 + \frac{1}{6}|E_2|^2\right), \tag{143}$$

$$\gamma_{1+}^{11} = \frac{3}{2}\left[\mathrm{Re}(E_1 E_0^*) + \sqrt{\frac{2}{3}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{144}$$

$$\gamma_{1+}^{12} = \frac{1}{2}\sqrt{\frac{15}{7}}\left[\mathrm{Re}(E_1 E_0^*) + \sqrt{6}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{145}$$

$$\gamma_{1+}^{13} = -\sqrt{\frac{3}{2}}\left[\mathrm{Re}(E_1 E_0^*) - \sqrt{\frac{3}{2}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{146}$$

$$\gamma_{1+}^{14} = -\frac{3}{2}\sqrt{\frac{10}{7}}\left[\mathrm{Re}(E_1 E_0^*) - \frac{1}{\sqrt{6}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{147}$$

$$\gamma_{1+}^{21} = -\frac{3}{2}\left[\mathrm{Re}(E_1 E_0^*) - \sqrt{\frac{2}{3}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{148}$$

$$\gamma_{1+}^{22} = -\frac{1}{2}\sqrt{\frac{15}{7}}\left[\mathrm{Re}(E_1 E_0^*) - \sqrt{6}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{149}$$

$$\gamma_{1+}^{23} = \sqrt{\frac{3}{2}}\left[\mathrm{Re}(E_1 E_0^*) + \sqrt{\frac{3}{2}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{150}$$

$$\gamma_{1+}^{24} = \frac{3}{2}\sqrt{\frac{10}{7}}\left[\mathrm{Re}(E_1 E_0^*) + \frac{1}{\sqrt{6}}\,\mathrm{Re}(E_2 E_1^*)\right], \tag{151}$$

$$\gamma_{1-}^{11} = \frac{3i}{2}\left[\mathrm{Im}(E_1 E_0^*) + \sqrt{\frac{2}{3}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{152}$$

$$\gamma_{1-}^{12} = \frac{i}{2}\sqrt{\frac{15}{7}}\left[\mathrm{Im}(E_1 E_0^*) + \sqrt{6}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{153}$$

$$\gamma_{1-}^{13} = -i\sqrt{\frac{3}{2}}\left[\mathrm{Im}(E_1 E_0^*) - \sqrt{\frac{3}{2}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{154}$$

$$\gamma_{1-}^{14} = -\frac{3i}{2}\sqrt{\frac{10}{7}}\left[\mathrm{Im}(E_1 E_0^*) - \frac{1}{\sqrt{6}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{155}$$

$$\gamma_{1-}^{21} = -\frac{3i}{2}\left[\mathrm{Im}(E_1 E_0^*) - \sqrt{\frac{2}{3}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{156}$$

$$\gamma_{1-}^{22} = -\frac{i}{2}\sqrt{\frac{15}{7}}\left[\mathrm{Im}(E_1 E_0^*) - \sqrt{6}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{157}$$

$$\gamma_{1-}^{23} = i\sqrt{\frac{3}{2}}\left[\mathrm{Im}(E_1 E_0^*) + \sqrt{\frac{3}{2}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{158}$$

$$\gamma_{1-}^{24} = \frac{3i}{2}\sqrt{\frac{10}{7}}\left[\mathrm{Im}(E_1 E_0^*) + \frac{1}{\sqrt{6}}\,\mathrm{Im}(E_2 E_1^*)\right], \tag{159}$$

$$\gamma_{2+}^{22} = \sqrt{\frac{30}{7}}\,\mathrm{Re}(E_2 E_0^*), \tag{160}$$

$$\gamma_{2+}^{23} = \sqrt{\frac{15}{2}}\,\mathrm{Re}(E_2 E_0^*), \tag{161}$$

$$\gamma_{2+}^{24} = \frac{3}{2}\sqrt{\frac{10}{7}}\,\mathrm{Re}(E_2 E_0^*), \tag{162}$$

$$\gamma_{2-}^{22} = i\sqrt{\frac{30}{7}}\,\mathrm{Im}(E_2 E_0^*), \tag{163}$$

$$\gamma_{2-}^{23} = i\sqrt{\frac{15}{2}}\,\mathrm{Im}(E_2 E_0^*), \tag{164}$$

$$\gamma_{2-}^{24} = \frac{3i}{2}\sqrt{\frac{10}{7}}\,\mathrm{Im}(E_2 E_0^*). \tag{165}$$

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