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Common fixed points for weak commutative mappings on a multiplicative metric space

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Abstract

In this paper, we discuss the unique common fixed point of two pairs of weak commutative mappings on a complete multiplicative metric space. They satisfy the following inequality: $d(Sx,Ty) \leq \{\max\{d(Ax,By),d(Ax,Sx),d(By,Ty),d(Sx,By),d(Ax,Ty)\}\}^{\lambda}$, where A and S are weak commutative, B and T also are weak commutative. Our results substantially generalize and extend the results of Özavsar and Cevikel (Fixed point of multiplicative contraction mappings on multiplicative metric space).

MSC: Primary 46B20; 47A12

Keywords: weak commutative mappings; multiplicative metric space; common fixed points

1 Introduction

The study of fixed points of mappings satisfying certain contraction conditions has many applications and has been at the center of various research activities. Özavsar gave the concept of multiplicative contraction mappings and proved some fixed point theorems of such mappings on a complete multiplicative metric space in [1]. Gu proved the common fixed point theorems of weak commutative mappings on a complete metric space in [2]. A common fixed point theorem for different mappings was obtained on a 2-metric space in [3, 4]. Agarwal proved some fixed point results for monotone operators in a metric space endowed with a partial order using a weak generalized contraction-type mapping in [5]. Dhage proved fixed point theorems for a pair of coincidentally commuting mappings in a D-metric space in [6]. Mustafa discussed several fixed point theorems for a class of mappings on a complete G-metric space. In this paper, we discuss the common fixed points of two pairs of weak commutative mappings on a complete multiplicative metric space.

2 Some basic properties

Definition 2.1 [7] Let X be a nonempty set. A multiplicative metric is a mapping $d: X \times X \to R^+$ satisfying the following conditions:

(i) $d(x,y) \geq 1$ for all $x,y \in X$ and $d(x,y) = 1$ if and only if $x = y$;
(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$;
(iii) $d(x,y) \leq d(x,z) \cdot d(z,y)$ for all $x,y,z \in X$ (multiplicative triangle inequality).

Example 2.2 [1] Let R^n_+ be the collection of all n-tuples of positive real numbers. Let $d: R^n_+ \times R^n_+ \to R$ be defined as follows:

$$d(x,y) = \left|\frac{x_1}{y_1}\right| \cdot \left|\frac{x_2}{y_2}\right| \cdots \left|\frac{x_n}{y_n}\right|,$$

where $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in R_+^n$ and $|\cdot| : R_+ \to R_+$ is defined as follows:

$$|a| = \begin{cases} a & \text{if } a \geq 1; \\ \frac{1}{a} & \text{if } a < 1. \end{cases}$$

Then it is obvious that all conditions of a multiplicative metric are satisfied.

Definition 2.3 [1] Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. If for every multiplicative open ball $B_\varepsilon(x) = \{y \mid d(x, y) < \varepsilon\}$, $\varepsilon > 1$, there exists a natural number N such that $n \geq N$, then $x_n \in B_\varepsilon(x)$. The sequence $\{x_n\}$ is said to be multiplicative converging to x, denoted by $x_n \to x$ $(n \to \infty)$.

Definition 2.4 [1] Let (X, d) be a multiplicative metric space and $\{x_n\}$ be a sequence in X. The sequence is called a multiplicative Cauchy sequence if it holds that for all $\varepsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \varepsilon$ for all $m, n > N$.

Definition 2.5 [1] We call a multiplicative metric space complete if every multiplicative Cauchy sequence in it is multiplicative convergence to $x \in X$.

Definition 2.6 [2] Suppose that S, T are two self-mappings of a multiplicative metric space (X, d); S, T are called commutative mappings if it holds that for all $x \in X$, $STx = TSx$.

Definition 2.7 [2] Suppose that S, T are two self-mappings of a multiplicative metric space (X, d); S, T are called weak commutative mappings if it holds that for all $x \in X$, $d(STx, TSx) \leq d(Sx, Tx)$.

Remark Commutative mappings must be weak commutative mappings, but the converse is not true.

Definition 2.8 [1] Let (X, d) be a multiplicative metric space. A mapping $f : X \to X$ is called a multiplicative contraction if there exists a real constant $\lambda \in [0, 1)$ such that $d(f(x_1), f(x_2)) \leq d(x_1, x_2)^\lambda$ for all $x, y \in X$.

Theorem 2.9 [1] *Let (X, d) be a multiplicative metric space, and let $f : X \to X$ be a multiplicative contraction. If (X, d) is complete, then f has a unique fixed point.*

In reference [1], the authors proved that the mapping f had a unique fixed point when f was a multiplicative contraction. In the main section of this paper, we extend the only mapping to two pairs of weak commutative mappings and obtain the fixed point under certain contractive conditions.

3 Main results

Theorem 3.1 *Let S, T, A and B be self-mappings of a complete multiplicative metric space X; they satisfy the following conditions*:

(i) $SX \subset BX$, $TX \subset AX$;

(ii) *A and S are weak commutative, B and T also are weak commutative*;

(iii) *One of S, T, A and B is continuous*;

(iv) $d(Sx,Ty) \le \{\max\{d(Ax,By),d(Ax,Sx),d(By,Ty),d(Sx,By),d(Ax,Ty)\}\}^{\lambda}$, $\lambda \in (0,\frac{1}{2})$, $\forall x,y \in X$.

Then S, T, A and B have a unique common fixed point.

Proof Since $SX \subset BX$, consider a point $x_0 \in X$, $\exists x_1 \in X$ such that $Sx_0 = Bx_1 = y_0$;

$\exists x_2 \in X$ such that $Tx_1 = Ax_2 = y_1; \ldots;$

$\exists x_{2n+1} \in X$ such that $Sx_{2n} = Bx_{2n+1} = y_{2n}$;

$\exists x_{2n+2} \in X$ such that $Tx_{2n+1} = Ax_{2n+2} = y_{2n+1}; \ldots.$

Now we can define a sequence $\{y_n\}$ in X, we obtain

$$\begin{aligned} d(y_{2n},y_{2n+1}) &= d(Sx_{2n},Tx_{2n+1}) \\ &\le \bigl\{\max\bigl\{d(Ax_{2n},Bx_{2n+1}),d(Ax_{2n},Sx_{2n}),d(Bx_{2n+1},Tx_{2n+1}), \\ &\quad d(Sx_{2n},Bx_{2n+1}),d(Ax_{2n},Tx_{2n+1})\bigr\}\bigr\}^{\lambda} \\ &\le \bigl\{\max\bigl\{d(y_{2n-1},y_{2n}),d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}), \\ &\quad d(y_{2n},y_{2n}),d(y_{2n-1},y_{2n+1})\bigr\}\bigr\}^{\lambda} \\ &\le \bigl\{\max\bigl\{d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}),1,d(y_{2n-1},y_{2n})\cdot d(y_{2n},y_{2n+1})\bigr\}\bigr\}^{\lambda} \\ &= d^{\lambda}(y_{2n-1},y_{2n})\cdot d^{\lambda}(y_{2n},y_{2n+1}). \end{aligned}$$

This implies that $d(y_{2n},y_{2n+1}) \le d^{\frac{\lambda}{1-\lambda}}(y_{2n-1},y_{2n})$.
Let $\frac{\lambda}{1-\lambda} = h$, then

$$d(y_{2n},y_{2n+1}) \le d^{h}(y_{2n-1},y_{2n}). \tag{1}$$

We also obtain

$$d(y_{2n+1},y_{2n+2}) \le d^{h}(y_{2n},y_{2n+1}). \tag{2}$$

From (1) and (2), we know $d(y_n,y_{n+1}) \le d^{h}(y_{n-1},y_n) \le \cdots \le d^{h^n}(y_1,y_0)$, $\forall n \ge 2$. Let $m,n \in \mathbb{N}$ such that $m \ge n$, then we get

$$\begin{aligned} d(y_m,y_n) &\le d(y_m,y_{m-1})\cdot d(y_{m-1},y_{m-2})\cdots d(y_{n+1},y_n) \\ &\le d^{h^{m-1}}(y_1,y_0)\cdot d^{h^{m-2}}(y_1,y_0)\cdots d^{h^n}(y_1,y_0) \\ &\le d^{\frac{h^n}{1-h}}(y_1,y_0). \end{aligned}$$

This implies that $d(y_m,y_n) \to 1$ $(m,n \to \infty)$. Hence $\{y_n\}$ is a multiplicative Cauchy. By the completeness of X, there exists $z \in X$ such that $y_n \to z$ $(n \to \infty)$.

Moreover, because

$$\{Sx_{2n}\} = \{Bx_{2n+1}\} = \{y_{2n}\}$$

and

$$\{Tx_{2n+1}\} = \{Ax_{2n+2}\} = \{y_{2n+1}\}$$

are subsequences of $\{y_n\}$, so we obtain

$$\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Ax_{2n+2} = z.$$

Case 1. Suppose that A is continuous, then $\lim_{n\to\infty} ASx_{2n} = \lim_{n\to\infty} A^2x_{2n} = Az$. Since A and S are weak commutative mappings, then

$$d(ASx_{2n}, SAx_{2n}) \le d(Sx_{2n}, Ax_{2n}).$$

Let $n \to \infty$, we get $\lim_{n\to\infty} d(SAx_{2n}, Az) \le d(z,z) = 1$, *i.e.*, $\lim_{n\to\infty} SAx_{2n} = Az$,

$$\begin{aligned} d(SAx_{2n}, Tx_{2n+1}) \le \bigl\{\max\bigl\{&d\bigl(A^2x_{2n}, Bx_{2n+1}\bigr), d\bigl(A^2x_{2n}, SAx_{2n}\bigr), d(Bx_{2n+1}, Tx_{2n+1}), \\ &d(SAx_{2n}, Bx_{2n+1}), d\bigl(A^2x_{2n}, Tx_{2n+1}\bigr)\bigr\}\bigr\}^{\lambda}. \end{aligned}$$

Let $n \to \infty$, we can obtain

$$\begin{aligned} d(Az,z) &\le \bigl\{\max\bigl\{d(Az,z), d(Az,Az), d(z,z), d(Az,z), d(Az,z)\bigr\}\bigr\}^{\lambda} \\ &= \bigl\{\max\bigl\{d(Az,z),1\bigr\}\bigr\}^{\lambda} \\ &= d^{\lambda}(Az,z). \end{aligned}$$

This implies $d(Az,z) = 1$, *i.e.*, $Az = z$,

$$\begin{aligned} d(Sz, Tx_{2n+1}) \le \bigl\{\max\bigl\{&d(Az, Bx_{2n+1}), d(Az, Sz), d(Bx_{2n+1}, Tx_{2n+1}), \\ &d(Sz, Bx_{2n+1}), d(Az, Tx_{2n+1})\bigr\}\bigr\}^{\lambda}. \end{aligned}$$

Let $n \to \infty$, we can obtain

$$\begin{aligned} d(Sz,z) &\le \bigl\{\max\bigl\{d(Az,z), d(z,Sz), d(z,z), d(Sz,z), d(z,z)\bigr\}\bigr\}^{\lambda} \\ &= \bigl\{\max\bigl\{d(Sz,z),1\bigr\}\bigr\}^{\lambda} \\ &= d^{\lambda}(Sz,z), \end{aligned}$$

which implies $d(Sz,z) = 1$, *i.e.*, $Sz = z$.

$z = Sz \in SX \subseteq BX$, so $\exists z^* \in X$ such that $z = Bz^*$,

$$\begin{aligned} d\bigl(z, Tz^*\bigr) &= d\bigl(Sz, Tz^*\bigr) \\ &\le \bigl\{\max\bigl\{d\bigl(Az, Bz^*\bigr), d(Az, Sz), d\bigl(Bz^*, Tz^*\bigr) d\bigl(Sz, Bz^*\bigr), d\bigl(Az, Tz^*\bigr)\bigr\}\bigr\}^{\lambda} \\ &= \bigl\{\max\bigl\{d\bigl(z, Tz^*\bigr),1\bigr\}\bigr\}^{\lambda} \\ &= d^{\lambda}\bigl(z, Tz^*\bigr), \end{aligned}$$

which implies $d(Sz,z)=1$, *i.e.*, $Tz^*=z$. Since B and T are weak commutative, then

$$d(Bz,Tz)=d\big(BTz^*,TBz^*\big)\le d\big(Bz^*,Tz^*\big)=d(z,z)=1,$$

so $Bz=Tz$,

$$\begin{aligned} d(z,Tz) &= d(Sz,Tz) \\ &\le \big\{\max\big\{d(Az,Bz),d(Az,Sz),d(Bz,Tz),d(Sz,Bz),d(Az,Tz)\big\}\big\}^{\lambda} \\ &= \big\{\max\big\{d(z,Tz),1\big\}\big\}^{\lambda} \\ &= d^{\lambda}(z,Tz), \end{aligned}$$

which implies $d(Tz,z)=1$, *i.e.*, $Tz=z$.

Case 2. Suppose that B is continuous, we can obtain the same result by the way of Case 1.

Case 3. Suppose that S is continuous, then $\lim_{n\to\infty} SAx_{2n}=\lim_{n\to\infty} S^2x_{2n}=Sz$.

Since A and S are weak commutative, then $d(ASx_{2n},SAx_{2n})\le d(Sx_{2n},Ax_{2n})$.

Let $n\to\infty$, we get $\lim_{n\to\infty} d(ASx_{2n},Sz)\le d(z,z)=1$, *i.e.*, $\lim_{n\to\infty} ASx_{2n}=Sz$,

$$\begin{aligned} d\big(S^2x_{2n},Tx_{2n+1}\big) \le \big\{\max\big\{&d(ASx_{2n},Bx_{2n+1}),d\big(ASx_{2n},S^2x_{2n}\big),d(Bx_{2n+1},Tx_{2n+1}), \\ &d\big(S^2x_{2n},Bx_{2n+1}\big),d(ASx_{2n},Tx_{2n+1})\big\}\big\}^{\lambda}. \end{aligned}$$

Let $n\to\infty$, we can obtain

$$\begin{aligned} d(Sz,z) &\le \big\{\max\big\{d(Sz,z),d(Sz,Sz),d(z,z)d(Sz,z),d(Sz,z)\big\}\big\}^{\lambda} \\ &= \big\{\max\big\{d(Sz,z),1\big\}\big\}^{\lambda} \\ &= d^{\lambda}(Sz,z). \end{aligned}$$

This implies $d(Sz,z)=1$, *i.e.*, $Sz=z$.

Since $z=Sz\in SX\subseteq BX$, so $\exists z^*\in X$ such that $z=Bz^*$,

$$\begin{aligned} d\big(S^2x_{2n},Tz^*\big) \le \big\{\max\big\{&d\big(ASx_{2n},Bz^*\big),d\big(ASx_{2n},S^2x_{2n}\big),d\big(Bz^*,Tz^*\big), \\ &d\big(S^2x_{2n},Bz^*\big),d\big(ASx_{2n},Tz^*\big)\big\}\big\}^{\lambda}. \end{aligned}$$

Let $n\to\infty$, we can obtain

$$\begin{aligned} d\big(Sz,Tz^*\big) &\le \big\{\max\big\{d(Sz,z),d(Sz,Sz),d\big(z,Tz^*\big),d(Sz,z),d\big(Sz,Tz^*\big)\big\}\big\}^{\lambda}, \\ d\big(z,Tz^*\big) &= \big\{\max\big\{d\big(z,Tz^*\big),1\big\}\big\}^{\lambda} \\ &= d^{\lambda}\big(z,Tz^*\big), \end{aligned}$$

which implies $d(z,Tz^*)=1$, *i.e.*, $Tz^*=z$.

Since T and B are weak commutative, then

$$d(Tz,Bz)=d\big(TBz^*,BTz^*\big)\le d\big(Tz^*,Bz^*\big)=d(z,z)=1,$$

so $Bz = Tz$,

$$d(Sx_{2n}, Tz) \le \{\max\{d(Ax_{2n}, Bz), d(Ax_{2n}, Sx_{2n}), d(Bz, Tz), \\ d(Sx_{2n}, Bz), d(Ax_{2n}, Tz)\}\}^{\lambda}.$$

Let $n \to \infty$, we can obtain

$$\begin{aligned} d(z, Tz) &\le \{\max\{d(z, Tz), d(z, z), d(Tz, Tz)d(z, Tz), d(z, Tz)\}\}^{\lambda}, \\ d(z, Tz) &= \{\max\{d(z, Tz), 1\}\}^{\lambda} \\ &= d^{\lambda}(z, Tz). \end{aligned}$$

This implies $d(z, Tz) = 1$, *i.e.*, $Tz = z$.

$z = Tz \in TX \subseteq AX$, so $\exists z^{**} \in X$ such that $z = Az^{**}$,

$$\begin{aligned} d(Sz^{**}, z) &= d(Sz^{**}, Tz) \\ &\le \{\max\{d(Az^{**}, Bz), d(Az^{**}, Sz^{**}), d(Bz, Tz), \\ &\quad d(Sz^{**}, Bz), d(Az^{**}, Tz)\}\}^{\lambda} \\ &= \{\max\{d(z, z), d(z, Sz^{**}), d(Bz, Bz)d(Sz^{**}, z), d(z, z)\}\}^{\lambda} \\ &= \{\max\{d(Sz^{**}, z), 1\}\}^{\lambda} \\ &= d^{\lambda}(Sz^{**}, z). \end{aligned}$$

This implies $d(Sz^{**}, z) = 1$, *i.e.*, $Sz^{**} = z$.

Since S and A are weak commutative, then

$$d(Az, Sz) = d(ASz^{**}, SAz^{**}) \le d(Az^{**}, Sz^{**}) = d(z, z) = 1,$$

so $Az = Sz$.

We obtain $Sz = Tz = Az = Bz = z$, so z is a common fixed point of S, T, A and B.

Case 4. Suppose that T is continuous, we can obtain the same result by the way of Case 3.

In addition, we prove that S, T, A and B have a unique common fixed point. Suppose that $w \in X$ is also a common fixed point of S, T, A and B, then

$$\begin{aligned} d(z, w) &= d(Sz, Tw) \\ &\le \{\max\{d(Az, Bw), d(Az, Sz), d(Bw, Bw)d(Sz, Bw), d(Az, Tw)\}\}^{\lambda} \\ &\le \{\max\{d(z, w), 1\}\}^{\lambda} \\ &= d^{\lambda}(z, w). \end{aligned}$$

This implies $d(z, w) = 1$, *i.e.*, $z = w$.

This is a contradiction. So S, T, A and B have a unique common fixed point. □

Theorem 3.2 *Let S, T, A and B be self-mappings of a complete multiplicative metric space satisfying the following conditions*:

(i) $SX \subset BX$, $TX \subset AX$;
(ii) *A and S are commutative mappings, B and T also are commutative mappings*;
(iii) *One of S, T, A and B is continuous*;
(iv) $d(S^p x, T^q y) \leq \{\max\{d(Ax, By), d(Ax, S^p x), d(By, T^q y), d(S^p x, By), d(Ax, T^q y)\}\}^\lambda$, $\lambda \in (0, \frac{1}{2})$, $\forall x, y \in X$, $\forall p, q \in Z^+$.

Then S, T, A and B have a unique common fixed point.

Proof Since

$$S^p X \subseteq S^{p-1} X \subseteq \cdots \subseteq S^2 X \subseteq SX \subseteq BX,$$
$$T^q X \subseteq T^{q-1} X \subseteq \cdots \subseteq T^2 X \subseteq TX \subseteq AX.$$

Since A and S are commutative mappings, so

$$S^p A = S^{p-1} SA = S^{p-1} AS = S^{p-2}(SA)S = S^{p-2} AS^2 = \cdots = AS^p.$$

Since B and T are commutative mappings, so

$$T^q B = T^{q-1} TB = T^{q-1} BT = T^{q-2}(TB)T = T^{q-2} BT^2 = \cdots = BT^q.$$

That is to say, $S^p A = AS^p$, $T^q B = BT^q$.

Since a commutative mapping must be a weak commutative mapping, so S^p, A and T^q, B are weak commutative mappings.

From Theorem 3.1, we can obtain that S^p, A, T^q and B have a unique common fixed point z.

In addition, we prove that S, T, A and B have a unique common fixed point,

$$\begin{aligned} d(Sz, z) &= d\bigl(S^p(Sz), T^q z\bigr) \\ &\leq \bigl\{\max\bigl\{d(ASz, Bz), d\bigl(ASz, S^p(Sz)\bigr), d\bigl(Bz, T^q z\bigr), \\ &\quad d\bigl(S^p(Sz), Bz\bigr), d\bigl(ASz, T^q z\bigr)\bigr\}\bigr\}^\lambda \\ &= \bigl\{\max\bigl\{d(Sz, z), d(Sz, Sz), d(z, z) d(Sz, z), d(Sz, z)\bigr\}\bigr\}^\lambda \\ &= \bigl\{\max\bigl\{d(Sz, z), 1\bigr\}\bigr\}^\lambda \\ &= d^\lambda(Sz, z). \end{aligned}$$

This implies $d(Sz, z) = 1$, *i.e.*, $Sz = z$,

$$\begin{aligned} d(z, Tz) &= d\bigl(S^p z, T^q(Tz)\bigr) \\ &\leq \bigl\{\max\bigl\{d(Az, BTz), d\bigl(Az, S^p z\bigr), d\bigl(BTz, T^q(Tz)\bigr), \\ &\quad d\bigl(S^p z, BTz\bigr), d\bigl(Az, T^q(Tz)\bigr)\bigr\}\bigr\}^\lambda \\ &= \bigl\{\max\bigl\{d(z, Tz), d(z, z), d(Tz, Tz) d(z, Tz), d(z, Tz)\bigr\}\bigr\}^\lambda \\ &= \bigl\{\max\bigl\{d(z, Tz), 1\bigr\}\bigr\}^\lambda \\ &= d^\lambda(z, Tz), \end{aligned}$$

which implies $d(z, Tz) = 1$, *i.e.*, $Tz = z$.

We obtain $Sz = Tz = Az = Bz = z$, so z is a common fixed point of S, T, A and B.

In addition, we prove that S, T, A and B have a unique common fixed point. Suppose that $w \in X$ is also a common fixed point of S, T, A and B, then

$$\begin{aligned} d(z,w) &= d\big(S^p z, T^q w\big) \\ &\le \big\{\max\big\{d(Az,Bw), d\big(Az,S^p z\big), d\big(Bw,T^q w\big), \\ &\quad d\big(S^p z,Bw\big), d\big(Az,T^q w\big)\big\}\big\}^\lambda \\ &= \big\{\max\big\{d(z,w), d(z,z), d(w,w) d(z,w), d(z,w)\big\}\big\}^\lambda \\ &= \big\{\max\big\{d(z,w),1\big\}\big\}^\lambda \\ &= d^\lambda(z,w). \end{aligned}$$

This implies $d(z,w) = 1$, *i.e.*, $z = w$.

This is a contradiction. So S, T, A and B have a unique common fixed point. □

Example 3.3 Let $X = R$ be a usual metric space. Define the mapping $d : X \times X \to R^+$ by $d(x,y) = e^{|x-y|}$ for all $x,y \in X$. Clearly, (X,d) is a complete multiplicative metric space. Consider the following mappings: $Sx = x$, $Tx = \frac{1}{2}x$, $Bx = 3x$, $Ax = 2x$ for all $x \in X$.

(i) $SX = TX = BX = AX = X$, so $SX \subset BX$, $TX \subset AX$;

(ii) A and S, B and T are all commutative mappings, according to Remark, they must be weak commutative mappings;

(iii) S, T, A and B are all continuous mappings;

(iv) Let $\lambda = \frac{1}{3}$, according to the inequality of Theorem 3.1:
$d(Sx,Ty) \le \{\max\{d(Ax,By), d(Ax,Sx), d(By,Ty), d(Sx,By), d(Ax,Ty)\}\}^\lambda$ and the conditions of Example 3.3, we can know

$$\begin{aligned} e^{|x-\frac{1}{2}y|} &\le \big\{\max\big\{e^{|3x-2y|}, e^{|2x|}, e^{|\frac{3}{2}y|}, e^{|2y-x|}, e^{|3x-\frac{1}{2}y|}\big\}\big\}^\lambda \\ &= \max\big\{e^{|3x-2y|\lambda}, e^{|2x|\lambda}, e^{|\frac{3}{2}y|\lambda}, e^{|2y-x|\lambda}, e^{|3x-\frac{1}{2}y|\lambda}\big\}. \end{aligned} \tag{3}$$

Because $y = \ln x$ is an increasing mapping, so

$$(3) \quad \Longleftrightarrow \quad \left|x - \frac{1}{2}y\right| \le \max\left\{|3x-2y|\lambda, |2x|\lambda, \left|\frac{3}{2}y\right|\lambda, |2y-x|\lambda, \left|3x - \frac{1}{2}y\right|\lambda\right\}.$$

There are three situations: (1) $x \ge \frac{1}{2}y \ge 0$ or $\frac{1}{2}y \ge x \ge 0$; (2) $\frac{1}{2}y < x < 0$ or $x < \frac{1}{2}y < 0$; (3) $x > 0$, $y < 0$ or $x < 0$, $y > 0$.

No matter what kind of situation, inequality (3) is true. So the inequality of Theorem 3.1 is also true. Therefore, all the conditions of Theorem 3.1 are satisfied, then we can obtain $S0 = T0 = A0 = B0 = 0$, so 0 is a common fixed point of S, T, A and B. In fact, 0 is the unique common fixed point of S, T, A and B.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The author XH mainly conceived and wrote the entire article. The author MS guided the thinking of the article, and some details of which were corrected. The author DC provided some references. All authors read and approved the final manuscript.

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