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A Subspace Approach to Blind Multiuser Detection for Ultra-Wideband Communication Systems

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Impulse radio-based ultra-wideband (UWB) communication systems allow multiple users to access channels simultaneously by assigning unique time-hopping codes to individual users, while each user's information stream is modulated by pulse-position modulation (PPM). However, transmitted signals undergo fading from a number of propagation paths in a dense multipath environment and meanwhile suffer from multiuser interference (MUI). Although RAKE receiver can be employed to maximally exploit path diversity, it is a single-user receiver. Multiuser receiver can significantly improve detection performance. Each of these receivers requires channel parameters. Existing maximum likelihood channel estimators treat MUI as Gaussian noise. In this paper, we derive a blind subspace channel estimator first and then design linear receivers. Following a channel input/output model that transforms a PPM signal into a sum of seemingly pulse-amplitude modulated signals, a structure similar to a code-division multiple-access (CDMA) system is observed. Code matrices for each user are identified. After considering unique statistical properties of new inputs such as mean and covariance, the model is further transformed to ensure that all signature waveforms lie in the signal subspace and are orthogonal to the noise subspace. Consequently, a subspace technique is applicable to estimate each channel. Then minimum mean square error receivers of two different versions are designed, suitable for both uplink and downlink. Asymptotic performance of both the channel estimator and receivers is studied. Closed-form bit error rate is also derived.

Keywords and phrases: ultra-wideband, subspace decomposition, multiuser detection, asymptotic performance.

1. INTRODUCTION

Research in impulse radio (IR) ultra-wideband (UWB) systems has lasted for several decades whose interest remains growing. Beginning with a focus on radar applications in military networks [1], the topic has spanned over a wide range of spectrum, such as those in commercial and other government applications [2, 3, 4]. With recent release of spectral mask from the Federal Communications Commission (FCC) [5], communication society has witnessed an increasing interest in recent years [6]. A conventional IR system transmits trains of time-hopping (TH) short-duration pulses with a low duty cycle and uses pulse-position modulation (PPM). Therefore, multipath down-to-path delay differentials in nanosecond is resolvable at the receiver, significantly mitigating multipath distortion and providing path diversity [7]. With new spectrum allocation and newly arising imperative demands for high data rates and transmission range [8], correspondingly advanced techniques need to be developed to meet specific requirements. Particular attention has to be paid to signal detection and receiver implementation. Concurrent challenges exist in complexity reduction and performance improvement.

In a UWB system, typically a RAKE receiver is employed to detect information symbols. It consists of multiple waveform correlators [4]. Compared with the optimal receiver, a RAKE receiver sacrifices performance for low complexity [9]. To fully capture signal energy spread over multiple paths, the receiver needs to know channel parameters when the correlation is performed. In a dense multipath wireless environment, channel information is not known a priori. Channel parameters can be either measured or estimated.

However, field test is sensitive to location and time, and not feasible for an unknown environment in general. Although maximum likelihood (ML) channel estimation methods [7, 10] provide blind channel estimators, they approximate multiuser interference (MUI) as a Gaussian process which may lead to degraded performance. Therefore, blind channel estimators with explicit consideration of MUI are more desirable. They are also required by either existing RAKE receivers or other advanced detectors such as linear multiuser receivers [11].

In this paper, we first focus on multiple access (MA) channel estimation based on up to the second-order statistics (SOS) of the received signal in order to construct linear receivers. Both first-order statistics and SOS can be easily estimated from data with low complexity and fast convergence. SOS have been employed in acquisition of the arrival time of the first path of UWB channels [12], linear detection of input symbols when channels are given [11, 13]. First, a UWB system is shown to follow a similar model as a direct-sequence (DS) code-division multiple-access (CDMA) system [11]. Multiple (M corresponding to the modulation level) inputs originated from the same user information can be regarded as a rate-M user in a multirate system. Code matrices can be clearly defined for each user from its unique TH sequence, like code matrices constructed from spreading codes in a multirate CDMA system [14]. But they consist of only zeros and ones, indicating existence of path contributions to the received signal from a multipath channel. Locations of zeros and ones vary with users.

However, under previous modeling, received signal shows nonzero mean due to PPM, different from a typical CDMA system where zero-mean inputs yield zero-mean channel output in general. Therefore, for convenience, zero-mean data is obtained after subtracting the estimated mean from directly received data. This results in two benefits: (a) easy application of a subspace concept [14, 15]; and (b) improvement of linear detectors' performance by significantly reducing amount of MUI. It is shown that newly defined input signals are correlated because they stem from the same modulation delay. To successfully apply the subspace technique, further transformation is performed on those signals in order to properly identify signal subspace. Thereafter, aided by unique code matrices and following standard procedures, channel parameters for the desired user can be estimated by minimizing projection of the signature waveform onto the noise subspace of data covariance matrix. Then minimum mean square error (MMSE) receivers can be built. It may take two different forms: direct matrix inversion (DMI) or subspace receiver [15]. The DMI-MMSE receiver is based on inversion of the data covariance matrix which includes the signal subspace and noise subspace components. The noise subspace components may amplify noise in a practical communication environment. Instead, subspace MMSE receiver utilizes only the signal subspace components. It shows better performance, in general, in moderate to high signal-to-noise ratio (SNR). For either of them, bit error rate (BER) is derived when M-ary PPM is adopted by the system. Channel estimation performance is evaluated when covariance is estimated from finite data samples. Meanwhile, signal-to-interference-plus-noise ratio (SINR) of each receiver is also studied jointly with channel estimation. Those results can be used to predict detection performance for given operational conditions.

Some notations following common practice are adopted throughout the paper. We denote Kronecker product [16] by $\otimes$, complex conjugate $(^*)$ transpose $(^{\mathrm{T}})$ by $(^{\mathrm{H}})$, inverse by $(^{-1})$, pseudo-inverse by $(^{\dagger})$, trace of a matrix by $\mathrm{tr}(\cdot)$, determinant by $\det(\cdot)$. $\mathrm{Re}\{\cdot\}$ represents real part, $E\{\cdot\}$ expectation, $\mathbf{I}_a$ an identity matrix of degree a whose ith column is denoted by $\mathbf{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal to one. An estimate of a quantity (scalar, vector, or matrix) is denoted by putting a hat "$\hat{}$" over it, and correspondingly, the estimation error by preceding the quantity with a δ, such as $\hat{\mathbf{X}}$ and $\delta\mathbf{X}$ for matrix $\mathbf{X}$, respectively. Meanwhile without confusing, we also use $\delta(\cdot)$ to represent a discrete-time unit impulse function. A Q function $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$ will be used in analyzing detection performance.

This paper is organized as follows. In Section 2, a discrete-time UWB system model is first described and then converted to a linear form similar to a multirate DS/CDMA model. Subspace-based channel estimation method and implementation of the MMSE receiver are proposed in Section 3. Performance of channel estimator and receivers in terms of channel mean square error (MSE), receivers' SINR, and BER are discussed in Section 4. Finally, various simulation examples are provided in Section 5 and conclusions are drawn in Section 6.

2. DISCRETE-TIME UWB SYSTEM

Assume there are K users simultaneously sharing the spectrum in an MA-TH-UWB system. The transmitted baseband UWB signal from user k can be described by [11]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k}\sum_{i=-\infty}^{\infty} w\left(t - iT_f - c_k(i)T_c - \tau_{I_k(\lfloor i/N_f\rfloor)}\right), \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(i) \in [0, N_c - 1]$ is a periodic hopping sequence with the period equal to one symbol period. Each chip has duration T_c. $I_k(\lfloor i/N_f\rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the ith frame, $\tau_{I_k(\lfloor i/N_f\rfloor)} = I_k(\lfloor i/N_f\rfloor)\sigma$ is the corresponding modulation delay in a multiple of σ seconds. Assume $T_f = N_cT_c$ and $T_c = M\sigma$. If we define $w_m(t) \triangleq w(t - m\sigma)$, where $m = 0, \dots, M-1$ and $s_{k,m}(\lfloor i/N_f\rfloor) = \delta(I_k(\lfloor i/N_f\rfloor) - m)$, then (1) may be expressed by linear modulation in a chip rate as [11]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k}\sum_{i=-\infty}^{\infty}\sum_{m=0}^{M-1} u_{k,m}(i)w_m(t - iT_c), \quad (2)$$

where the chip index has replaced the frame index in (1),

$$u_{k,m}(i) = s_{k,m}\left(\left\lfloor \frac{i}{N_c N_f} \right\rfloor\right)\tilde{c}_k(i),$$
$$\tilde{c}_k(i) = \delta\left(\left\lfloor \frac{i}{N_c} \right\rfloor N_c + c_k\left(\left\lfloor \frac{i}{N_c} \right\rfloor\right) - i\right). \tag{3}$$

It is clear according to (2) that input $u_{k,m}(i)$ is modulated by waveform $w_m(t)$ at a chip rate. The transmitted signal $\alpha_k(t)$ propagates through a linear channel with impulse response $\bar{g}_k(t)$. At the receiver, the channel output is first passed through a matched filter matched to the monopulse $w(t)$. We can define a front-end effective channel including effects from modulated pulse at the transmitter, and propagation channel and matched filter at the receiver by $g_{k,m}(t) = w_m(t) \star \bar{g}_k(t) \star w(-t)$, where $\star$ denotes convolution. Considering additive white Gaussian noise (AWGN) $v(t)$ and propagation delay d_k for user k, the output of the matched filter becomes

$$y(t) = \sum_{k,i_1,m} \sqrt{\mathcal{P}_k} u_{k,m}(i_1) g_{k,m}(t - i_1 T_c - d_k) + v(t). \tag{4}$$

Assume each effective channel has length $q\sigma$. Then $y(t)$ is sampled every σ seconds to yield a discrete-time output $y(n) = y(t)|_{t=n\sigma}$. Using the discrete-time version of the effective channel and invoking $T_c = M\sigma$, we obtain a pulse-rate model

$$y(n) = \sum_{k,m} \sum_{i_2=0}^{q} \sqrt{\mathcal{P}_k} u_{k,m}\left(\frac{n - i_2}{M}\right) g_{k,m}(i_2) + v(n). \tag{5}$$

Consider P symbol intervals of data samples with corresponding time instants $nMN_cN_f + p$ for $p = 1, \ldots, MPN_cN_f$ and collect them in a big vector $\mathbf{y}_n$ of length $\nu = MPN_cN_f$. After noticing our definition of $u_{k,m}(i)$, a vector form data model follows:

$$\mathbf{y}_n = \sum_{k,m,l} \mathbf{C}_{k,l}\mathbf{T}_m\mathbf{g}_k s_{k,m}(n + l) + \mathbf{v}_n, \tag{6}$$

where the symbol index l takes all integers $-\lceil q/(MN_cN_f)\rceil, \ldots, P - 1$, $\mathbf{g}_k$ is an unknown channel vector for user k which contains channel coefficients at the pulse rate and power factor $\sqrt{\mathcal{P}_k}$, $\mathbf{T}_m = [\mathbf{0}, \mathbf{I}_q, \mathbf{0}]^{\mathrm{T}}$ is a tall selection matrix in order to obtain the mth subchannel from $\mathbf{g}_k$ (delayed in $m\sigma$ seconds or, equivalently, downshifted by m elements), and $\mathbf{C}_{k,l}$ is a matrix constructed from corresponding $\tilde{c}_k(i)$ and is uniquely determined by the TH sequence. It consists of only zeros and ones, and repeats from symbol to symbol because the TH sequence has period equal to one symbol interval. This model can be compactly expressed in another form

$$\begin{aligned}\mathbf{y}_n &= \sum_{k,l} \mathbf{H}_{k,l}\mathbf{s}_{k,n,l} + \mathbf{v}_n \\ &= \mathbf{H}\mathbf{s}_n + \mathbf{v}_n\end{aligned} \tag{7}$$

after collecting M inputs in a vector

$$\mathbf{s}_{k,n,l} = \left[s_{k,0}(n + l), \ldots, s_{k,M-1}(n + l)\right]^{\mathrm{T}}, \tag{8}$$

defining a corresponding effective channel matrix

$$\mathbf{H}_{k,l} = \left[\mathbf{C}_{k,l}\mathbf{T}_0\mathbf{g}_k, \ldots, \mathbf{C}_{k,l}\mathbf{T}_{M-1}\mathbf{g}_k\right], \tag{9}$$

and successively stacking such matrices (or vectors) in $\mathbf{H}$ (or $\mathbf{s}_n$). By employing either structure of (6) or this structure, all channels can be estimated based on a multirate subspace concept [14].

3. SUBSPACE CHANNEL ESTIMATION AND SYMBOL DETECTION

3.1. Zero-mean data

We denote the mean of $\mathbf{y}_n$ as $\bar{\mathbf{y}}$ which can be easily found from our definition of $\mathbf{s}_{k,n,l}$. Since noise has zero mean even after the matched filter, we have

$$\bar{\mathbf{y}} = \frac{1}{M} \sum_{k,m,l} \mathbf{C}_{k,l}\mathbf{T}_m\mathbf{g}_k = \sum_{k} \mathcal{C}_k\mathbf{g}_k = \mathcal{C}\mathbf{g}, \tag{10}$$

where all channel vectors are stacked in a big vector $\mathbf{g}$. Due to nonzero mean, the autocorrelation of $\mathbf{y}_n$ has cross-terms $\mathbf{g}_{k_1}\mathbf{g}_{k_2}^{\mathrm{H}}$ of users k_1 and k_2 and is not convenient for channel estimation. Thus covariance is considered. Define a new zero-mean data vector from $\mathbf{y}_n$ as

$$\mathbf{z}_n = \mathbf{y}_n - \bar{\mathbf{y}} = \sum_{k,l} \mathbf{H}_{k,l}\mathbf{a}_{k,n,l} + \mathbf{v}_n = \mathbf{H}\mathbf{a}_n + \mathbf{v}_n, \tag{11}$$

where $\mathbf{a}_{k,n,l} = \mathbf{s}_{k,n,l} - (1/M)\mathbf{1}_M$ all of which are stacked in a big vector $\mathbf{a}_n$ with corresponding effective channel matrix defined as $\mathbf{H}$. For shorter notations, we denote the information symbol in $\mathbf{s}_{k,n,l}$ simply by I after ignoring its time and user dependence. It takes values $0, \ldots, M - 1$ with equal probability $1/M$. Then

$$\mathbf{a}_{k,n,l} = \left[\delta(I), \ldots, \delta(I - (M - 1))\right]^{\mathrm{T}} - \frac{1}{M}\mathbf{1}_M^{\mathrm{T}}. \tag{12}$$

Denote the covariance of $\mathbf{a}_{k,n,l}$ by $\mathbf{A} = E\{\mathbf{a}_{k,n,l}\mathbf{a}_{k,n,l}^{\mathrm{T}}\}$. According to the distribution of I, it can be found that

$$\mathbf{A} = \frac{1}{M} \sum_{i=1}^{M} \tilde{\mathbf{e}}_{M,i}\tilde{\mathbf{e}}_{M,i}^{\mathrm{T}}, \quad \tilde{\mathbf{e}}_{M,i} = \mathbf{e}_{M,i} - \frac{1}{M}\mathbf{1}_M. \tag{13}$$

After simplification, it becomes $\mathbf{A} = (1/M)(\mathbf{I}_M - (1/M)\mathbf{1}_M\mathbf{1}_M^{\mathrm{T}})$ which is easily shown to have rank $M - 1$ since $(1/\sqrt{M})\mathbf{1}_M$ is a unitary vector. Thus its eigenvalue decomposition (EVD) has a form $\mathbf{A} = \mathbf{B}_a\mathbf{\Lambda}_a^2\mathbf{B}_a^{\mathrm{H}}$, where $\mathbf{B}_a$ is of $M \times (M - 1)$ and $\mathbf{\Lambda}_a$ is an $(M - 1) \times (M - 1)$ diagonal matrix with all positive entries.

3.2. Channel estimator

The ideal covariance of $\mathbf{z}_n$ is then derived to be

$$\mathbf{R} = E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} = \sum_{k,l} \mathbf{H}_{k,l}\mathbf{A}\mathbf{H}_{k,l}^{\mathrm{H}} + \sigma_v^2\mathbf{I}_\nu. \quad (14)$$

Meanwhile, $\mathbf{a}_{k,n,l}$ can be whitened and correspondingly (11) becomes

$$\mathbf{z}_n = \sum_{k,l} \mathbf{H}_{k,l}\mathbf{B}_a\mathbf{\Lambda}_a\tilde{\mathbf{a}}_{k,n,l} + \mathbf{v}_n, \quad (15)$$

where $\tilde{\mathbf{a}}_{k,n,l}$ has identity covariance. Assume $\mathbf{R}$ is decomposed by EVD as

$$\mathbf{R} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^{\mathrm{H}} \\ \mathbf{U}_n^{\mathrm{H}} \end{bmatrix}, \quad (16)$$

where $\mathbf{\Lambda}_s = \mathrm{diag}\{\lambda_1^2, \dots, \lambda_\xi^2\}$, $\mathbf{\Lambda}_n = \sigma_v^2\mathbf{I}_{\nu-\xi}$, and $\mathbf{U}_s$ and $\mathbf{U}_n$ represent the signal and noise subspaces, respectively. Based on orthogonality principle, $\mathbf{U}_n^{\mathrm{H}}\mathbf{H}_{k,l}\mathbf{B}_a\mathbf{\Lambda}_a = \mathbf{0}$ or, equivalently, $\mathbf{U}_n^{\mathrm{H}}\mathbf{H}_{k,l}\mathbf{B}_a = \mathbf{0}$ for all possible k and l. Denoting the (i, j)th element of $\mathbf{B}_a$ by $b_{i,j}$, then we have

$$\mathbf{U}_n^{\mathrm{H}}[\mathbf{C}_{k,l}\mathbf{T}_0\mathbf{g}_k, \dots, \mathbf{C}_{k,l}\mathbf{T}_{M-1}\mathbf{g}_k]\mathbf{B}_a = \mathbf{0} \quad (17)$$

which can be expanded column by column as

$$\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{k,l,j}\mathbf{g}_k = \mathbf{0}, \quad j = 1, \dots, M-1, \quad (18)$$

where $\mathbf{D}_{k,l,j} = \sum_{i=1}^{M} b_{i,j}\mathbf{C}_{k,l}\mathbf{T}_{i-1}$. Therefore, we can design the following channel estimation criterion for user k by minimizing total projection error

$$\hat{\mathbf{g}}_k = \min \sum_{l,j} \left|\left|\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{k,l,j}\mathbf{g}_k\right|\right|^2. \quad (19)$$

After defining $\mathbf{O}_k = \sum_{l,j} \mathbf{D}_{k,l,j}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{k,l,j}$, (19) becomes

$$\hat{\mathbf{g}}_k = \min \sum_{l,j} \mathbf{g}_k^{\mathrm{H}}\mathbf{O}_k\mathbf{g}_k. \quad (20)$$

$\hat{\mathbf{g}}_k$ is the minimum eigenvector of $\mathbf{O}_k$.

3.3. Linear receivers

In order to detect input symbol in $\mathbf{a}_{k,n,l}$ which has only one maximum while all others are smaller, we need to design M receivers $\mathbf{f}_i$ $(i = 1, \dots, M)$ with each one corresponding to each element in $\mathbf{a}_{k,n,l}$. Then outputs of M receivers are compared and the index of the maximum element is determined. Considering I takes values $0, \dots, M-1$, our symbol detection criterion can be described as follows:

$$I = \arg \max_{i \in \{1,\dots,M\}} \mathrm{Re}\left\{\mathbf{f}_i^{\mathrm{H}}\mathbf{z}_n\right\} - 1. \quad (21)$$

The receiver takes different forms for different types. We are only interested in the MMSE receiver which is applicable to both uplink and downlink in a multiuser environment and has good performance in general. Consider the current symbol $(l = 0)$ and collect all M receivers in a matrix $\mathbf{F}_k$ for user k. Based on (11), the DMI-MMSE receivers can be found as follows after noticing that the covariance of $\mathbf{a}_{k,n,l}$ is $\mathbf{A}$:

$$\mathbf{F}_{k,\mathrm{DMI}} = \mathbf{R}^{-1}\mathbf{H}_{k,0}\mathbf{A}. \quad (22)$$

The subspace MMSE receivers can also be easily derived [15]. Since $\mathbf{U}_n^{\mathrm{H}}\mathbf{H}_{k,0}\mathbf{A} = \mathbf{0}$, according to (16), we obtain

$$\mathbf{F}_{k,\mathrm{sub}} = \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{H}_{k,0}\mathbf{A}. \quad (23)$$

Performance of the subspace channel estimator and receivers will be studied next.

4. PERFORMANCE ANALYSIS

It is found that both channel estimator and receivers depend on the data covariance matrix $\mathbf{R}$. In practical conditions, together with the mean of received data vector, it is often estimated from N data vectors as follows:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}(\mathbf{y}_n - \hat{\bar{\mathbf{y}}})(\mathbf{y}_n - \hat{\bar{\mathbf{y}}})^{\mathrm{H}}, \quad \hat{\bar{\mathbf{y}}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}_n. \quad (24)$$

The sample size N will determine the accuracy of the subspace estimate, thus affect the performance of the estimator. An estimation error occurs $\delta\mathbf{R} = \hat{\mathbf{R}} - \mathbf{R}$. For large N, it can be regarded as a small perturbation, making the perturbation technique applicable [17].

4.1. Channel estimation performance

For notational convenience, let $\mathbf{Z}$ be the noise-free data covariance matrix $\mathbf{Z} = \mathbf{R} - \sigma_v^2\mathbf{I}_\nu$. Then perturbation of $\mathbf{U}_s$ has the following form [17]:

$$\delta\mathbf{U}_n \approx -\mathbf{Z}^\dagger\delta\mathbf{R}\mathbf{U}_n. \quad (25)$$

Because channel estimate is the minimum eigenvector of $\mathbf{O}_k$, $\delta\mathbf{U}_n$ causes an error $\delta\mathbf{O}_k$. Then $\delta\mathbf{g}_k$ has the following form [14, 17]:

$$\delta\mathbf{g}_k \approx -\mathbf{O}_k^\dagger\delta\mathbf{O}_k\mathbf{g}_k. \quad (26)$$

According to our definition of $\mathbf{O}_k$, $\delta\mathbf{O}_k$ is given by

$$\delta\mathbf{O}_k \approx \sum_{l,j} \mathbf{D}_{k,l,j}^{\mathrm{H}}[\delta\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}} + \mathbf{U}_n\delta\mathbf{U}_n^{\mathrm{H}}]\mathbf{D}_{k,l,j}. \quad (27)$$

Substituting (25) in (27) then (27) in (26), and noticing (18), $\delta\mathbf{g}_k$ is related to random matrix $\delta\mathbf{R}$ by

$$\delta\mathbf{g}_k \approx \sum_{l,j} \mathbf{O}_k^\dagger\mathbf{D}_{k,l,j}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\delta\mathbf{R}\mathbf{Z}^\dagger\mathbf{D}_{k,l,j}\mathbf{g}_k. \quad (28)$$

The covariance becomes

$$\begin{aligned} \mathrm{COV}(\delta\mathbf{g}_k) &\approx \sum_{l_1,l_2,j_1,j_2} \mathbf{M}_{l_1,j_1}E\{\delta\mathbf{R}\mathbf{Y}\delta\mathbf{R}\}\mathbf{M}_{l_2,j_2}^{\mathrm{H}}, \\ \mathbf{M}_{l,j} &= \mathbf{O}_k^\dagger\mathbf{D}_{k,l,j}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}, \\ \mathbf{Y} &= \mathbf{Z}^\dagger\mathbf{D}_{k,l_1,j_1}\mathbf{g}_k\mathbf{g}_k^{\mathrm{H}}\mathbf{D}_{k,l_2,j_2}^{\mathrm{H}}\mathbf{Z}^\dagger. \end{aligned} \quad (29)$$

The channel MSE is then equal to $\mathrm{tr}(\mathrm{COV}(\delta\mathbf{g}_k))$. To evaluate either of them, it suffices to determine a general-form quantity

$$\mathbf{\Psi}(\mathbf{\Theta}) = E\{\delta\mathbf{R}\mathbf{\Theta}\delta\mathbf{R}\} \tag{30}$$

for an arbitrary weighing matrix $\mathbf{\Theta}$. Although results for a system with white inputs have been derived in [18], unfortunately our current inputs do not satisfy that condition. Therefore, new results need to be derived by following procedures therein. For convenience, we partition matrix $\mathbf{H}$ in (11) into L subblocks as $\mathbf{H} = [\mathbf{H}_1, \dots, \mathbf{H}_L]$ where each subblock corresponds to one symbol irrespective of user. Then $L = K(P + \lceil q/(MN_cN_f)\rceil)$. SOS of $\delta\mathbf{R}$ with arbitrary weighing matrix $\mathbf{\Theta}$ are given in the following proposition.

Proposition 1. *If the channel model follows* (7) *and the data covariance is estimated from N independent data vectors as* (24), *then for a real system (all quantities are real),*

$$\begin{aligned}\mathbf{\Psi}(\mathbf{\Theta}) = {} & \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\tilde{\mathbf{h}}_{l,j}^{\mathrm{T}}\mathbf{\Theta}\tilde{\mathbf{h}}_{l,j})\tilde{\mathbf{h}}_{l,j}\tilde{\mathbf{h}}_{l,j}^{\mathrm{T}} \\ & - \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\mathrm{tr}\,(\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\mathbf{\Theta})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}} \\ & - \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}(\mathbf{\Theta}+\mathbf{\Theta}^{\mathrm{T}})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}} \\ & + \frac{N-1}{N^2}\left[\mathrm{tr}(\mathbf{R}\mathbf{\Theta})\mathbf{R} + \mathbf{R}\mathbf{\Theta}^{\mathrm{T}}\mathbf{R}\right] + \frac{1}{N^2}\mathbf{R}\mathbf{\Theta}\mathbf{R}\end{aligned} \tag{31}$$

while for complex channel and noise,

$$\begin{aligned}\mathbf{\Psi}(\mathbf{\Theta}) = {} & \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}}\mathbf{\Theta}\tilde{\mathbf{h}}_{l,j})\tilde{\mathbf{h}}_{l,j}\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}} \\ & - \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\mathrm{tr}\,(\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\mathbf{\Theta})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}} \\ & - \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\mathbf{\Theta}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}} \\ & - \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}(\mathbf{H}_l^{\mathrm{H}}\mathbf{\Theta}\mathbf{H}_l)^{\mathrm{T}}\mathbf{A}\mathbf{H}_l^{\mathrm{H}} \\ & + \frac{N-1}{N^2}\left[\mathrm{tr}(\mathbf{R}\mathbf{\Theta})\mathbf{R} + \mathbf{H}\mathcal{A}(\mathbf{H}^{\mathrm{H}}\mathbf{\Theta}\mathbf{H})^{\mathrm{T}}\mathcal{A}\mathbf{H}^{\mathrm{H}}\right] \\ & + \frac{1}{N^2}\mathbf{R}\mathbf{\Theta}\mathbf{R},\end{aligned} \tag{32}$$

where $\tilde{\mathbf{h}}_{l,j} = \mathbf{H}_l\tilde{\mathbf{e}}_{M,j}$, $\mathcal{A} = \mathbf{I}_L \otimes \mathbf{A}$ are defined for shorter notations.

For the proof of the proposition, see the appendix.

It can be observed that the above results are different from [18] because of different distributions of inputs. Noticing the fact that $\tilde{\mathbf{h}}_{l,j}$, $\mathbf{H}_l\mathbf{A}$, and $\mathbf{Z}$ all lie in the signal subspace, one can verify that for either real or complex case, (29) reduces to $((N-1)/N^2)\sum_{l_1,l_2,j_1,j_2}\mathbf{M}_{l_1,j_1}\,\mathrm{tr}\{\mathbf{RY}\}\mathbf{R}\mathbf{M}_{l_2,j_2}^{\mathrm{H}}$. Replacing $\mathbf{M}_{l,j}$ and noticing that $\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{R}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}} = \sigma_v^2\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}$, the covariance can be further simplified as

$$\begin{aligned}\mathrm{COV}\,(\delta\mathbf{g}_k) \approx {} & \left(\frac{1}{N} - \frac{1}{N^2}\right)\mathrm{tr}\{\mathbf{RY}\}\sigma_v^2 \\ & \times \sum_{l_1,l_2,j_1,j_2}\mathbf{O}_k^{\dagger}\mathbf{D}_{k,l_1,j_1}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{k,l_2,j_2}\mathbf{O}_k^{\dagger}.\end{aligned} \tag{33}$$

Clearly, the covariance of $\delta\mathbf{g}_k$ or its MSE is proportional to noise power and approximately inversely proportional to the data length N.

4.2. *Detection performance*

Previously presented linear MMSE receivers are directly applied to the received data to generate estimates of entries in $\mathbf{a}_{k,n,l}$. Then I is detected following the symbol detection criterion (21). We first study performance of ideal receivers when channel and data covariance are perfectly known. Then we investigate its sensitivity to sample size which causes errors in those quantities. Without loss of generality, assume user 1 is the desired user. Its information symbol I is contained in $\mathbf{a}_{1,n,0}$ whose channel matrix is given by (9) after setting $k = 1$ and $l = 0$. That channel matrix is the first subblock in matrix $\mathbf{H}$ and has been defined as $\mathbf{H}_1$ in **Proposition** 1. We only focus on a real system although we will still use H instead of T for consistency next.

4.2.1. *Ideal receivers*

We will derive BER for given receivers. We separate the desired signal from interference in $\mathbf{z}_n$:

$$\mathbf{z}_n = \mathbf{H}_1\mathbf{a}_{1,n,0} + \mathbf{u}_n, \tag{34}$$

where $\mathbf{u}_n$ includes intersymbol interference (ISI) and MAI and is approximated as a Gaussian process for convenience of analysis. Assume information $I = 0$ is transmitted. Our data vector becomes $\mathbf{z}_n = \tilde{\mathbf{h}}_{1,1} + \mathbf{u}_n$. Denote M receivers simply by $\mathbf{f}_j$ for $j = 1, \dots, M$. It can represent any linear receiver presented before. Then the event of right detection becomes

$$\{\mathbf{f}_1^{\mathrm{H}}\mathbf{z}_n > \mathbf{f}_j^{\mathrm{H}}\mathbf{z}_n,\ j = 2, \dots, M\} = \{\Delta\mathbf{f}_j^{\mathrm{H}}\mathbf{z}_n > 0\}, \tag{35}$$

where $\Delta\mathbf{f}_j = \mathbf{f}_1 - \mathbf{f}_j$. Define an $(M-1)$-dimensional random vector $\mathbf{x}_n = \Delta\mathbf{F}^{\mathrm{H}}\mathbf{z}_n$, where $\Delta\mathbf{F}$ contains all $\Delta\mathbf{f}_j$ as columns. Since $\mathbf{z}_n$ is assumed Gaussian distributed, $\mathbf{x}_n$ is also Gaussian. We can find its probability density function

$$f_{\mathbf{x}} = \frac{e^{-(1/2)(\mathbf{x}_n - \Delta\mathbf{F}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1})^{\mathrm{H}}(\mathrm{COV}(\mathbf{x}))^{-1}(\mathbf{x}_n - \Delta\mathbf{F}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1})}}{\sqrt{(2\pi)^{M-1}\det(\mathrm{COV}(\mathbf{x}))}}, \tag{36}$$

where $\mathrm{COV}(\mathbf{x}) = \Delta\mathbf{F}^{\mathrm{H}}\mathbf{R}_{\mathrm{int}}\Delta\mathbf{F}$ is the covariance of $\mathbf{x}_n$ with $\mathbf{R}_{\mathrm{int}} = \mathbf{R} - \tilde{\mathbf{h}}_{1,1}\tilde{\mathbf{h}}_{1,1}^{\mathrm{H}}$. Probability of detection error becomes $\mathrm{BER}_0 = 1 - \mathrm{Prob}\{\mathbf{x}_n > \mathbf{0}\} = 1 - \int\cdots\int_0^{\infty} f_{\mathbf{x}}\,d\mathbf{x}$. It can be numerically evaluated. Similarly, we can find the BER when other symbols $I = 1, \dots, M-1$ are transmitted, denoted as $\mathrm{BER}_1, \dots, \mathrm{BER}_{M-1}$. Then the average probability of error becomes $\mathrm{BER} = (1/M)\sum_{m=0}^{M-1}\mathrm{BER}_m$.

Due to intractability of further analysis for an arbitrary M as evidenced by M-level integrals, we examine binary modulation where $M = 2$. The BER results can be simplified as

$$\begin{aligned} \mathrm{BER}_0 &= 1 - Q\left(-\frac{\Delta\mathbf{f}_1^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}}{\sigma_1}\right) = Q\left(\frac{\Delta\mathbf{f}_1^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}}{\sigma_1}\right), \\ \mathrm{BER}_1 &= 1 - Q\left(-\frac{\Delta\mathbf{f}_2^{\mathrm{H}}\tilde{\mathbf{h}}_{1,2}}{\sigma_2}\right) = Q\left(\frac{\Delta\mathbf{f}_2^{\mathrm{H}}\tilde{\mathbf{h}}_{1,2}}{\sigma_2}\right), \end{aligned} \tag{37}$$

where $\Delta\mathbf{f}_2 = \mathbf{f}_2 - \mathbf{f}_1 = -\Delta\mathbf{f}_1$, and $\sigma_j^2 = \Delta\mathbf{f}_j^{\mathrm{H}}(\mathbf{R} - \tilde{\mathbf{h}}_{1,j}\tilde{\mathbf{h}}_{1,j}^{\mathrm{H}})\Delta\mathbf{f}_j$ for $j = 1, 2$. Since $\tilde{\mathbf{h}}_{1,1} = \mathbf{H}_1\tilde{\mathbf{e}}_{2,1}$ and $\mathbf{H}_1 = [\mathbf{C}_{1,0}\mathbf{T}_0\mathbf{g}_1, \mathbf{C}_{1,0}\mathbf{T}_1\mathbf{g}_1]$, it can be seen that

$$\tilde{\mathbf{h}}_{1,1} = \frac{1}{2}\mathbf{C}_{1,0}(\mathbf{T}_0 - \mathbf{T}_1)\mathbf{g}_1, \tag{38}$$

$\tilde{\mathbf{h}}_{1,2} = -\tilde{\mathbf{h}}_{1,1}$, and $\sigma_1^2 = \sigma_2^2$. We can conclude that $\mathrm{BER}_0 = \mathrm{BER}_1$ as expected. Then $\mathrm{BER} = (1/2)(\mathrm{BER}_0 + \mathrm{BER}_1) = \mathrm{BER}_0$. After examining BER_0, it is found that it depends on SINR of the receiver $\Delta\mathbf{f}_1$. For convenience, denote $\Delta\mathbf{f}_1$ by $\mathbf{m}$. Then

$$\mathrm{SINR} = \frac{\left|\mathbf{m}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}\right|^2}{\sigma_1^2} = \frac{\mathbf{m}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}\tilde{\mathbf{h}}_{1,1}^{\mathrm{H}}\mathbf{m}}{\mathbf{m}^{\mathrm{H}}\mathbf{R}_{\mathrm{int}}\mathbf{m}}. \tag{39}$$

If the DMI-MMSE receiver is adopted, then according to (22), it takes an explicit form

$$\mathbf{m}_1 = \mathbf{R}^{-1}(\tilde{\mathbf{h}}_{1,1} - \tilde{\mathbf{h}}_{1,2}) = 2\mathbf{R}^{-1}\tilde{\mathbf{h}}_{1,1} = \mathbf{R}^{-1}\mathbf{S}_1\mathbf{g}_1, \tag{40}$$

where $\mathbf{S}_1 = \mathbf{C}_{1,0}(\mathbf{T}_0 - \mathbf{T}_1)$. If the subspace MMSE receiver is considered as (23), it is simplified as

$$\mathbf{m}_2 = \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{S}_1\mathbf{g}_1. \tag{41}$$

If $\mathbf{R}$ is estimated from finite data by (24), then receiver $\mathbf{m}$ will deviate from its optimum by $\delta\mathbf{m}$. Detection performance will degrade due to a change of SINR in (39) to

$$\widehat{\mathrm{SINR}} \approx \frac{\mathbf{m}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}\tilde{\mathbf{h}}_{1,1}^{\mathrm{H}}\mathbf{m} + E\{\delta\mathbf{m}^{\mathrm{H}}\tilde{\mathbf{h}}_{1,1}\tilde{\mathbf{h}}_{1,1}^{\mathrm{H}}\delta\mathbf{m}\}}{\mathbf{m}^{\mathrm{H}}\mathbf{R}_{\mathrm{int}}\mathbf{m} + E\{\delta\mathbf{m}^{\mathrm{H}}\mathbf{R}_{\mathrm{int}}\delta\mathbf{m}\}}. \tag{42}$$

We can easily find $\delta\mathbf{m}$ when the estimate of $\mathbf{R}$ produces an error $\delta\mathbf{R}$, then obtain corresponding statistics as discussed next. A general form $E\{\delta\mathbf{m}^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}\}$ will be evaluated and then $\mathbf{X}$ is replaced by either $\tilde{\mathbf{h}}_{1,1}\tilde{\mathbf{h}}_{1,1}^{\mathrm{H}}$ or $\mathbf{R}_{\mathrm{int}}$.

4.2.2. Practical receivers

If $\mathbf{R}$ is replaced by $\hat{\mathbf{R}} = \mathbf{R} + \delta\mathbf{R}$ and $\mathbf{g}_1$ is estimated with error (28), then first-order (up to $\delta\mathbf{R}$) errors in MMSE receivers can be found from (40) and (41). Currently, $M = 2$ significantly simplifies channel estimation error in (28) which requires $\mathbf{D}_{1,l,j}$ and consequently $\mathbf{B}_a$. Now j takes only one value $j = 1$. $\mathbf{B}_a$ reduces to a unitary vector and can be found from (13) to be $\mathbf{B}_a = (1/\sqrt{2})[1, -1]^{\mathrm{T}}$.

We focus on the DMI-MMSE receiver first. Its error is

$$\begin{aligned} \delta\mathbf{m}_1 \approx{}& \mathbf{R}^{-1}\mathbf{S}_1\mathbf{O}_1^{\dagger}\sum_l \mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\delta\mathbf{R}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l,1}\mathbf{g}_1 \\ &- \mathbf{R}^{-1}\delta\mathbf{R}\mathbf{m}_1. \end{aligned} \tag{43}$$

Then we obtain

$$\begin{aligned} &E\{\delta\mathbf{m}_1^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}_1\} \\ &\quad\approx \mathbf{m}_1^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{R}^{-1}\mathbf{X}\mathbf{R}^{-1}\delta\mathbf{R}\}}\mathbf{m}_1 \\ &\qquad+ \sum_{l_1,l_2}\mathbf{g}_1^{\mathrm{H}}\mathbf{D}_{1,l_1,1}^{\mathrm{H}}\mathbf{Z}^{\dagger}\underline{E\{\delta\mathbf{R}\mathbf{W}_{l_1,l_2}\delta\mathbf{R}\}}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l_2,1}\mathbf{g}_1 \\ &\qquad- \sum_l \mathbf{m}_1^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{R}^{-1}\mathbf{X}\mathbf{R}^{-1}\mathbf{S}_1\mathbf{O}_1^{\dagger}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\delta\mathbf{R}\}}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l,1}\mathbf{g}_1 \\ &\qquad- \sum_l \mathbf{g}_1^{\mathrm{H}}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{Z}^{\dagger}\underline{E\{\delta\mathbf{R}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{1,l,1}\mathbf{O}_1^{\dagger}\mathbf{S}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{X}\mathbf{R}^{-1}\delta\mathbf{R}\}}\mathbf{m}_1, \end{aligned} \tag{44}$$

where

$$\mathbf{W}_{l_1,l_2} = \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{D}_{1,l_1,1}\mathbf{O}_1^{\dagger}\mathbf{S}_1^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{X}\mathbf{R}^{-1}\mathbf{S}_1\mathbf{O}_1^{\dagger}\mathbf{D}_{1,l_2,1}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}. \tag{45}$$

All underlined terms can be evaluated according to Proposition 1.

Similarly, the subspace MMSE receiver (41) gets perturbed as follows when $\mathbf{R}$ is estimated:

$$\begin{aligned} \delta\mathbf{m}_2 \approx{}& \delta\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{S}_1\mathbf{g}_1 - \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\delta\mathbf{\Lambda}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{S}_1\mathbf{g}_1 \\ &+ \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\delta\mathbf{U}_s^{\mathrm{H}}\mathbf{S}_1\mathbf{g}_1 + \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{S}_1\delta\mathbf{g}_1. \end{aligned} \tag{46}$$

Perturbation $\delta\mathbf{R}$ causes the subspace components of $\mathbf{R}$ to be perturbed. The results can be found in the following lemma.

Lemma 1 (see [17]). If $\mathbf{R}$ is perturbed by $\delta\mathbf{R}$, then its eigencomponents are perturbed by

$$\begin{aligned} \delta\mathbf{U}_s &\approx \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\delta\mathbf{R}\mathbf{U}_s\mathbf{\Omega}^{-1}, &\qquad \delta\mathbf{U}_n &\approx -\mathbf{Z}^{\dagger}\delta\mathbf{R}\mathbf{U}_n, \\ \delta\mathbf{\Lambda}_s &\approx \mathbf{U}_s^{\mathrm{H}}\delta\mathbf{R}\mathbf{U}_s, &\qquad \delta\mathbf{\Lambda}_n &\approx \mathbf{U}_n^{\mathrm{H}}\delta\mathbf{R}\mathbf{U}_n, \end{aligned} \tag{47}$$

where $\mathbf{\Omega} = \mathbf{\Lambda}_s - \sigma_v^2\mathbf{I}_{\xi}$. All approximations are valid up to the first order of $\delta\mathbf{R}$. □

Since $\mathbf{U}_n^{\mathrm{H}}\mathbf{S}_1\mathbf{g}_1 = \mathbf{0}$, substituting (47) and (28) in (46), we obtain

$$\begin{aligned} \delta\mathbf{m}_2 \approx{}& \mathbf{A}_n\delta\mathbf{R}\mathbf{A}_{\gamma}\mathbf{S}_1\mathbf{g}_1 - \mathbf{A}_s\delta\mathbf{R}\mathbf{m}_2 \\ &+ \mathbf{A}_s\mathbf{S}_1\mathbf{O}_1^{\dagger}\sum_l \mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{A}_n\delta\mathbf{R}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l,1}\mathbf{g}_1, \end{aligned} \tag{48}$$

where for convenience, we have defined

$$\begin{aligned} \mathbf{A}_n &\triangleq \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}, &\qquad \mathbf{A}_s &\triangleq \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^{\mathrm{H}}, \\ \mathbf{A}_{\omega} &\triangleq \mathbf{U}_s\mathbf{\Omega}^{-1}\mathbf{U}_s^{\mathrm{H}}, &\qquad \mathbf{A}_{\gamma} &\triangleq \mathbf{A}_s\mathbf{A}_{\omega}. \end{aligned} \tag{49}$$

Then $E\{\delta\mathbf{m}_2^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}_2\}$ will involve nine terms as follows:

$$
\begin{aligned}
&E\{\delta\mathbf{m}_2^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}_2\}\\
&\approx \mathbf{g}_1^{\mathrm{H}}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_\gamma\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{X}\mathbf{A}_n\delta\mathbf{R}\}}\mathbf{A}_\gamma\mathbf{S}_1\mathbf{g}_1\\
&\quad-\mathbf{g}_1^{\mathrm{H}}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_\gamma\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{X}\mathbf{A}_s\delta\mathbf{R}\}}\mathbf{m}_2\\
&\quad+\sum_l \mathbf{g}_1^{\mathrm{H}}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_\gamma\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{X}\mathbf{A}_s\mathbf{S}_1\mathbf{O}_1^{\dagger}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{A}_n\delta\mathbf{R}\}}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l,1}\mathbf{g}_1\\
&\quad-\mathbf{m}_2^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{A}_s\mathbf{X}\mathbf{A}_n\delta\mathbf{R}\}}\mathbf{A}_\gamma\mathbf{S}_1\mathbf{g}_1+\mathbf{m}_2^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{A}_s\mathbf{X}\mathbf{A}_s\delta\mathbf{R}\}}\mathbf{m}_2\\
&\quad-\sum_l \mathbf{m}_2^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{A}_s\mathbf{X}\mathbf{A}_s\mathbf{S}_1\mathbf{O}_1^{\dagger}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{A}_n\delta\mathbf{R}\}}\mathbf{Z}^{\dagger}\mathbf{D}_{1,l,1}\mathbf{g}_1\\
&\quad+\sum_l \mathbf{g}_1^{\mathrm{H}}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{Z}^{\dagger}\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{D}_{1,l,1}\mathbf{O}_1^{\dagger}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_s\mathbf{X}\mathbf{A}_n\delta\mathbf{R}\}}\mathbf{A}_\gamma\mathbf{S}_1\mathbf{g}_1\\
&\quad+\sum_{l_1,l_2} \mathbf{g}_1^{\mathrm{H}}\mathbf{D}_{1,l_1,1}^{\mathrm{H}}\mathbf{Z}^{\dagger}\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{D}_{1,l_1,1}\mathbf{O}_1^{\dagger}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_s\mathbf{X}\mathbf{A}_s\mathbf{S}_1\mathbf{O}_1^{\dagger}\mathbf{D}_{1,l_2,1}^{\mathrm{H}}\mathbf{A}_n\delta\mathbf{R}\}}\\
&\qquad\times\mathbf{Z}^{\dagger}\mathbf{D}_{1,l_2,1}\mathbf{g}_1\\
&\quad-\sum_l \mathbf{g}_1^{\mathrm{H}}\mathbf{D}_{1,l,1}^{\mathrm{H}}\mathbf{Z}^{\dagger}\underline{E\{\delta\mathbf{R}\mathbf{A}_n\mathbf{D}_{1,l,1}\mathbf{O}_1^{\dagger}\mathbf{S}_1^{\mathrm{H}}\mathbf{A}_s\mathbf{X}\mathbf{A}_s\delta\mathbf{R}\}}\mathbf{m}_2.
\end{aligned}
\tag{50}
$$

Each underlined term can be obtained from Proposition 1

It is found that both (44) and (50) involve several terms. However, most terms contribute little to the final results. Each term follows a general form $\mathbf{X}_1E\{\delta\mathbf{R}\boldsymbol{\Theta}\delta\mathbf{R}\}\mathbf{X}_2$. With results in Proposition 1, it can be easily checked that this form will reduce to $((N-1)/N^2)\,\mathrm{tr}\{\mathbf{R}\boldsymbol{\Theta}\}\mathbf{X}_1\mathbf{R}\mathbf{X}_2$ when $\mathbf{X}_1$ (or $\mathbf{X}_2$) is in the signal subspace and $\boldsymbol{\Theta}$ is in the noise subspace, which is smaller than that for $\boldsymbol{\Theta}$ not in the noise subspace by an order of $O(\sigma_v^2)$. If we omit those small terms, (44) and (50) reduce to

$$E\{\delta\mathbf{m}_1^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}_1\}\approx\mathbf{m}_1^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{R}^{-1}\mathbf{X}\mathbf{R}^{-1}\delta\mathbf{R}\}}\mathbf{m}_1, \tag{51}$$

$$E\{\delta\mathbf{m}_2^{\mathrm{H}}\mathbf{X}\delta\mathbf{m}_2\}\approx\mathbf{m}_2^{\mathrm{H}}\underline{E\{\delta\mathbf{R}\mathbf{A}_s\mathbf{X}\mathbf{A}_s\delta\mathbf{R}\}}\mathbf{m}_2. \tag{52}$$

Although they are less accurate, analytical SINRs computed based on these truncated expressions yield very good approximations to practical SINRs, as will be shown by simulation examples next.

5. SIMULATIONS

In this section, we show the performance of the proposed channel estimator, receivers, and also verify our analytical results by simulations. Comparison of the proposed approach with both data-aided (DA) and non-data-aided (NDA) methods described in [10] are included.

5.1. *Performance of the proposed approach*

We consider a UWB system with $N_c = 8$, $N_f = 4$, and $M = 2$. If not stated otherwise, 8 equal-powered users are assumed in the system. Gaussian channel with maximum delay spread over one frame is considered [10, 13], which is equivalent to 16-path channel after sampling according to our data model. Each user's TH codes and channel are randomly generated once and fixed for all realizations. The received signal is the second derivative of the Gaussian function with pulse width equal to 0.7 nanosecond [4]. Simulation results are based on 100 independent realizations. User 1 is assumed to be the desired user, and the receiver is assumed to be synchronized to the desired user. In the following different cases, we demonstrate the performance of the proposed method in various simulation situations involving different finite data length N, various input E_b/N_0, variable number of active users, and different interfering users' power. In Cases 3 and 4, the RAKE receiver, which is constructed based on the proposed channel estimate, is also presented for comparison.

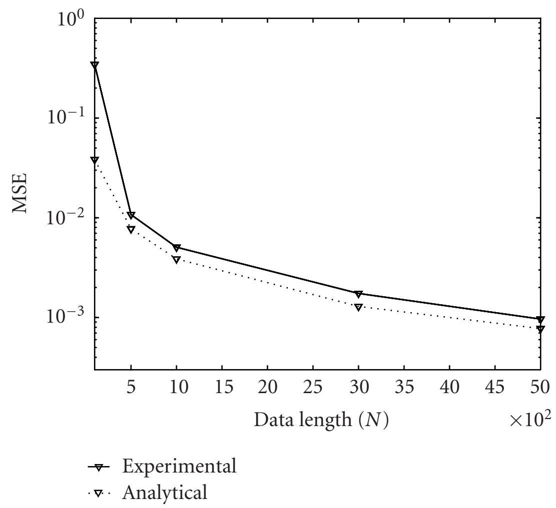

Figure 1: MSE versus N.

Case (1). Effect of N, $E_b/N_0 = 15$ dB. Effect of N on both channel estimation MSE and receivers' output SINR is investigated. Comparison between the experimental and analytical results is also presented. Figure 1 shows the channel's MSE, which decreases as N increases. Meanwhile, the experimental MSE curve is seen to converge to its analytical one (dotted line) for large N, validating our MSE analysis. The receivers' output SINR is demonstrated in Figure 2. It is observed that the output SINR of the subspace MMSE receiver converges to its analytical value very well from $N = 100$. Moreover, the truncated analytical SINR computed based on (52) is seen to be very consistent with the analytical one without truncation. On the other hand, the output SINR of the DMI-MMSE receiver converges slowly to its analytical SINR and truncated approximation, which implies that more data samples are needed for the DMI-MMSE to achieve satisfactory performance.

Case (2). Effect of E_b/N_0. Figures 3a and 3b illustrate BER performance of the subspace and DMI-MMSE receivers, respectively. Data lengths $N = 800$ and $N = 3000$ are both considered. The analytical BERs are calculated according to our previous analysis. The ideal receivers are constructed according to (22) and (23) using perfect codes and channel information of all users as well as noise power. It is observed that the BER of the subspace MMSE receiver with $N = 3000$ is very consistent with its analytical counterpart, and also very close to the BER of the ideal subspace receiver at each E_b/N_0

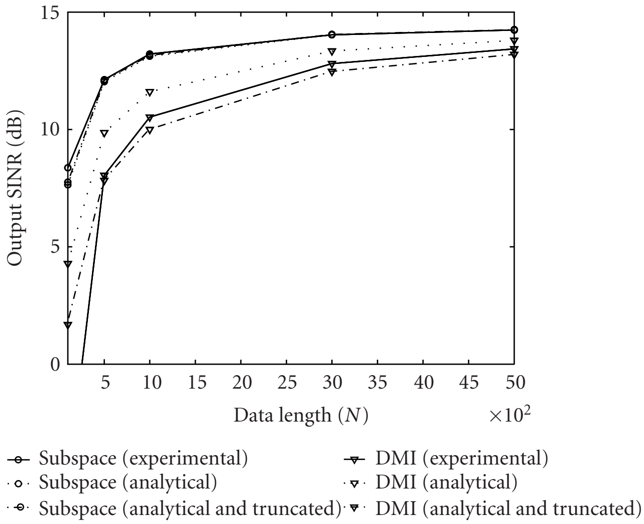

FIGURE 2: Output SINR versus N.

examined. The subspace receiver with $N = 800$ shows a little degradation. Similar results can be observed for the DMI-MMSE receiver at low E_b/N_0. However, at high E_b/N_0, the DMI receiver with either $N = 800$ or $N = 3000$ shows diverged BER from either the analytical or ideal value, due to large perturbation of the receiver incurred by the inverse of the estimated covariance matrix. Figure 3b implies that more data samples are required for the DMI-MMSE receiver to reach its analytical limit at higher E_b/N_0.

Case (3). Near-far effect, $E_b/N_0 = 15$ dB. Each interfering user is assumed to have power from 0 dB to 10 dB higher than the desired user. Corresponding BER is plotted in Figure 4. It is observed that the BER performance of all receivers degrades a little as the interfering users' power increases. However, the subspace receiver can still achieve a satisfactory performance of 2×10^{-4} even in the presence of the maximum interfering power examined.

Case (4). Effect of number of users, $E_b/N_0 = 15$ dB. The performance of the proposed method is investigated for a UWB system with different number of active users. According to Figure 5, although BER degrades for all three receivers as the number of users increases, the subspace MMSE receiver still has satisfactory performance for the cases of $K < 10$, and has an acceptable BER performance of 7×10^{-3} in the case of $K = 14$. Clearly, with the aid of the proposed multiuser detection scheme, more than N_c users can be supported by the system with satisfactory performance.

5.2. *Comparison with other approaches*

Since [10] also considers channel estimation, comparison with [10] is thus conducted and presented in this subsection. The data-aided and nondata-aided methods in [10] are termed as DA and NDA, respectively, and their RAKE receivers with one finger and three fingers are named as RAKE-1 and RAKE-3, correspondingly. For comparison, the proposed subspace MMSE receiver and RAKE receiver constructed from estimated channel vector are presented.

The system parameters are taken as $N_c = 20$, $N_f = 2$. Each three-path channel is generated by following [10] exactly. Eight hundred symbols are used for channel estimation. Instead of plotting delay and gain estimates separately as in [10], we integrate delays and gains of the desired user's channel into a channel vector by associating each of its elements with the gain of the path at a particular delay and filling zeros correspondingly if there is no path. The normalized channel MSE for the integrated channel is then plotted in Figures 6a, 6b, and 6c for the cases of $K = 1$, $K = 5$ and $K = 10$, respectively. In the case of $K = 1$, DA shows the best performance at low E_b/N_0 at the cost of using training data. In that case, DA is close to the optimal receiver due to absence of MUI and negligible ISI compared with noise power. However, at high E_b/N_0 where ISI is dominant, the proposed method, though without the aid of training data, still outperforms the training-based DA and the blind NDA methods greatly. For the case of $K = 5$ or $K = 10$, due to significant MUI, the proposed method outperforms DA significantly for most E_b/N_0 examined, and is clearly superior to NDA for all E_b/N_0 examined. The BER performance of different receivers is demonstrated in Figures 7a, 7b, and 7c for different users and E_b/N_0, respectively. In the case of one user, the proposed subspace MMSE and RAKE receivers have very similar performance to the RAKE-3 receiver of DA, while the subspace receiver shows better performance at high E_b/N_0. In the case of five users and ten users, the proposed subspace receiver shows the best performance. The proposed RAKE receiver also shows better performance than either DA or NDA method due to better channel estimation for those cases. In summary, the proposed method explicitly considers MAI, and thus achieves better performance than both DA and NDA approaches in [10].

6. CONCLUSION

In this paper, we have proposed a blind subspace channel estimator for UWB communication systems employing PPM modulation. Two MMSE receivers known as subspace and DMI-MMSE receivers are designed based on estimated channel for symbol detection. Asymptotic performance of both the channel estimator and receivers is derived based on perturbation theory. Extensive simulation results show satisfactory performance of the proposed scheme in various communication scenarios.[1]

APPENDIX

PROOF OF PROPOSITION 1

The weighted covariance $\mathbf{\Psi}(\mathbf{\Theta})$ depends on $\delta\mathbf{R} = \widehat{\mathbf{R}} - \mathbf{R}$ which in turn depends on $\widehat{\mathbf{R}}$ by (24). We thus first relate $\widehat{\mathbf{R}}$ to $\mathbf{y}_n$ and then $\mathbf{z}_n$ which shows explicit dependence of system parameters.

[1]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the US Government.

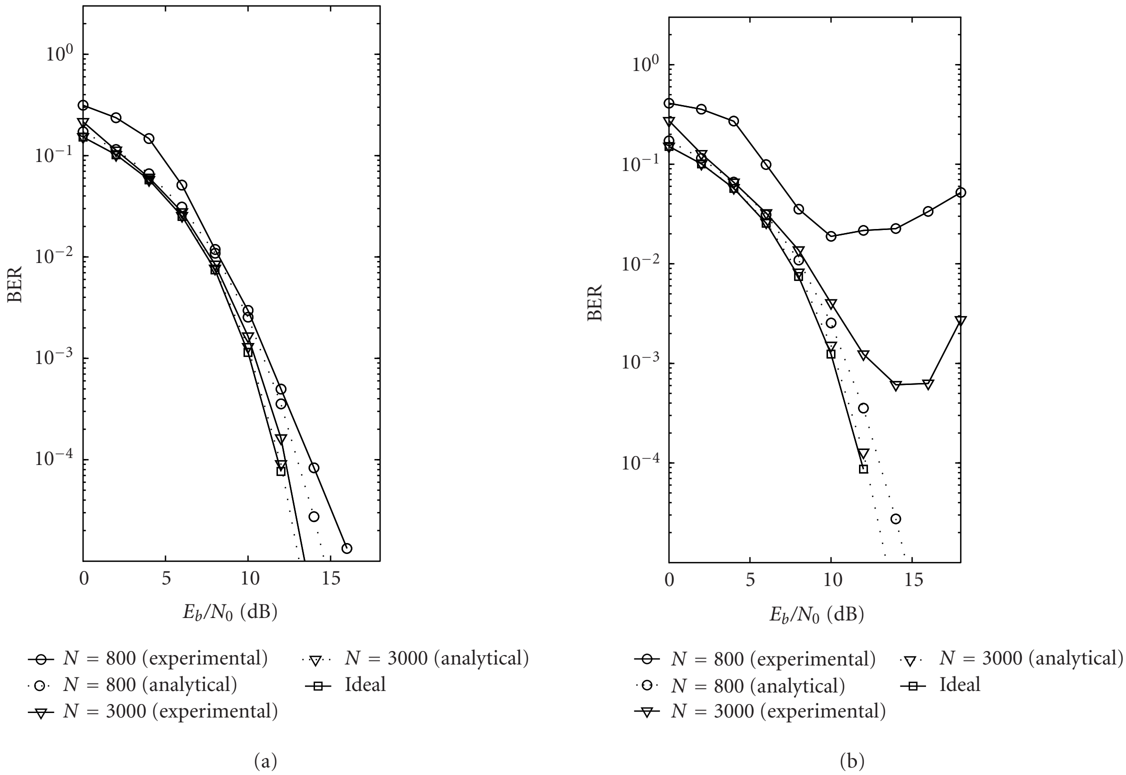

Figure 3: BER versus E_b/N_0: (a) subspace and (b) DMI.

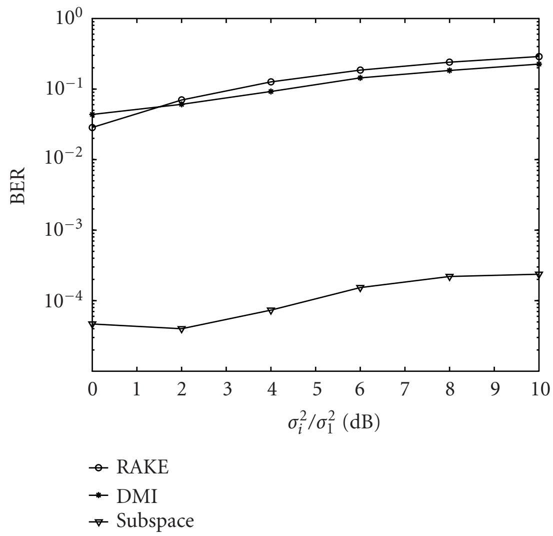

Figure 4: BER versus SIR.

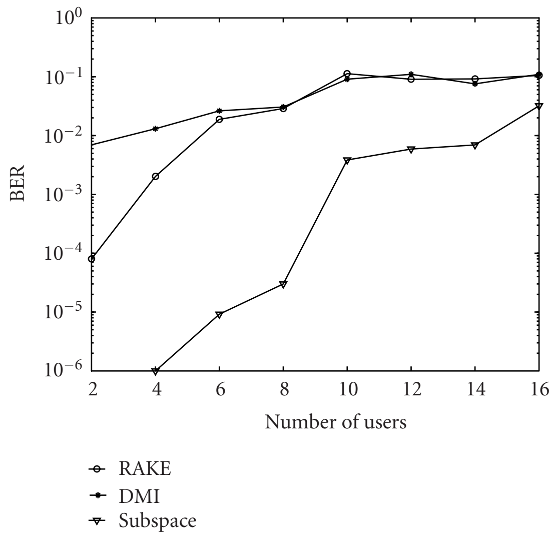

Figure 5: BER versus number of users.

After expanding summation in (24), we obtain

$$\widehat{\mathbf{R}} = \frac{1}{N}\sum_{n}\mathbf{y}_n\mathbf{y}_n^{\mathrm{H}} - \frac{1}{N^2}\sum_{n_1,n_2}\mathbf{y}_{n_1}\mathbf{y}_{n_2}^{\mathrm{H}}. \quad (\text{A.1})$$

After substituting $\mathbf{y}_n$ by $\mathbf{z}_n + \bar{\mathbf{y}}$ according to (11), (A.1) can be simplified as

$$\widehat{\mathbf{R}} = \frac{1}{N}\sum_{n}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}} - \frac{1}{N^2}\sum_{n_1,n_2}\mathbf{z}_{n_1}\mathbf{z}_{n_2}^{\mathrm{H}}. \quad (\text{A.2})$$

It can be observed that (A.2) is consistent with a typical

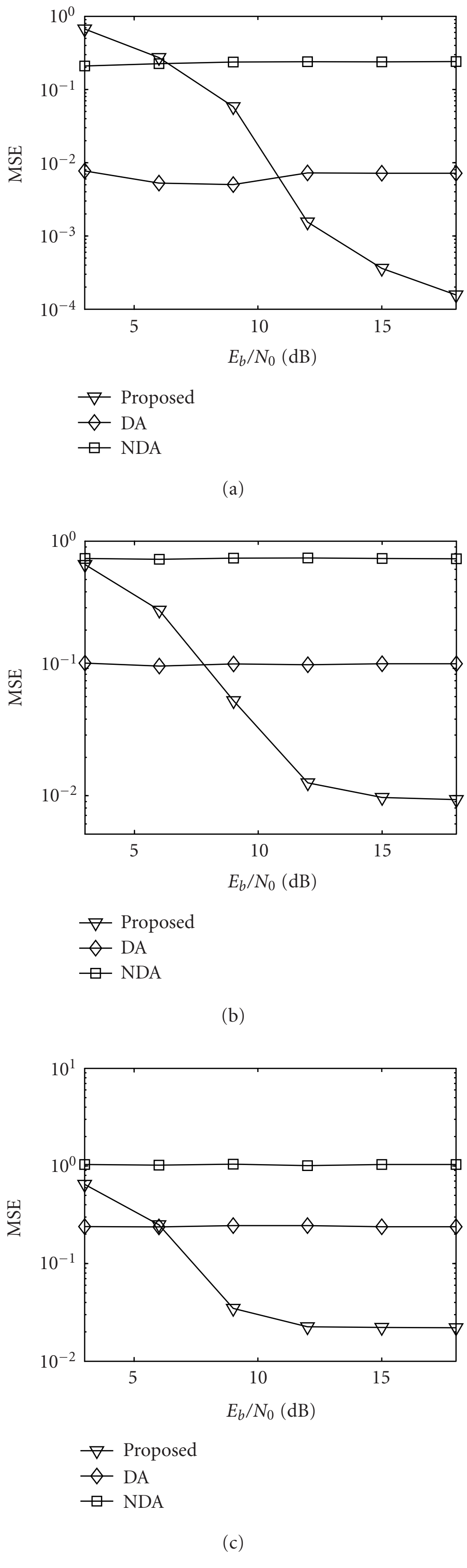

FIGURE 6: Channel MSE of different methods: (a) one user, (b) five users, and (c) ten users.

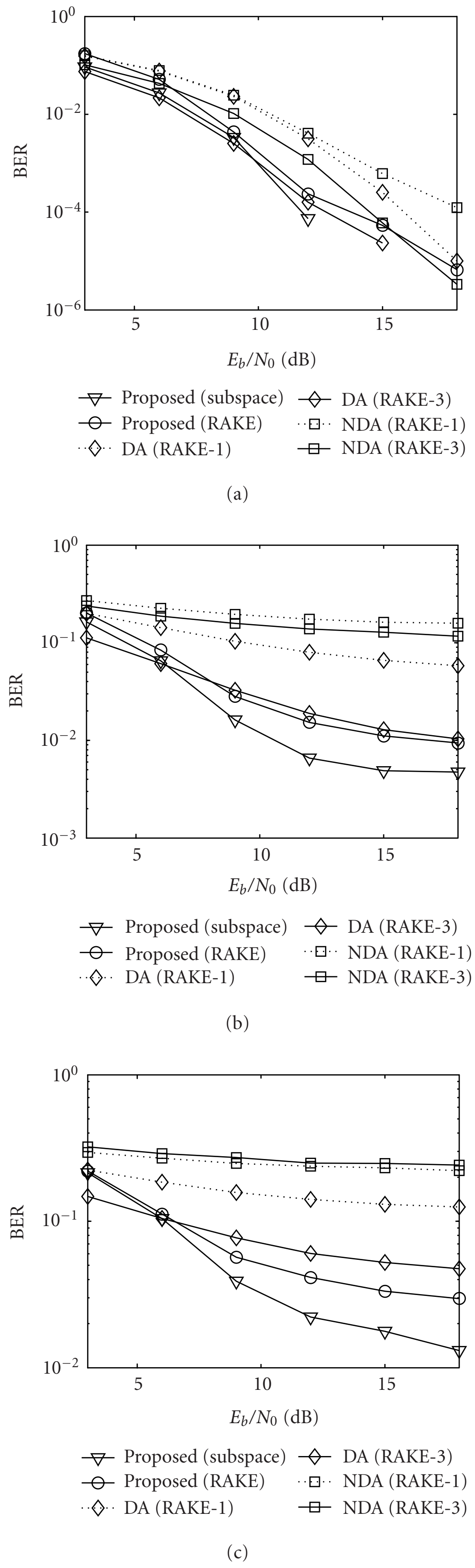

FIGURE 7: BER of different receivers: (a) one user, (b) five users, and (c) ten users.

covariance estimator

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_n (\mathbf{z}_n - \hat{\mathbf{z}})(\mathbf{z}_n - \hat{\mathbf{z}})^{\mathrm{H}}, \quad \hat{\mathbf{z}} = \frac{1}{N}\sum_n \mathbf{z}_n \tag{A.3}$$

although we estimate $\mathbf{R}$ directly from $\mathbf{y}_n$ as (24). Due to zero mean and independence assumption on $\mathbf{z}_n$ at different times, the mean of $\hat{\mathbf{R}}$ is found to be $E\{\hat{\mathbf{R}}\} = (1 - 1/N)\mathbf{R}$ from (A.2). Then $\mathbf{\Psi}(\mathbf{\Theta})$ can be expanded into

$$\begin{aligned}\mathbf{\Psi}(\mathbf{\Theta}) &= E\{(\hat{\mathbf{R}} - \mathbf{R})\mathbf{\Theta}(\hat{\mathbf{R}} - \mathbf{R})\} \\ &= E\{\hat{\mathbf{R}}\mathbf{\Theta}\hat{\mathbf{R}}\} - \left(1 - \frac{2}{N}\right)\mathbf{R}\mathbf{\Theta}\mathbf{R}.\end{aligned} \tag{A.4}$$

It thus suffices to derive $E\{\hat{\mathbf{R}}\mathbf{\Theta}\hat{\mathbf{R}}^{\mathrm{H}}\}$ for further simplification of $\mathbf{\Psi}(\mathbf{\Theta})$.

For convenience, rewrite (A.2) as

$$\hat{\mathbf{R}} = \frac{N-1}{N^2}\sum_n \mathbf{z}_n\mathbf{z}_n^{\mathrm{H}} - \frac{1}{N^2}\sum_{n_1 \neq n_2} \mathbf{z}_{n_1}\mathbf{z}_{n_2}^{\mathrm{H}}. \tag{A.5}$$

Then from (A.5) and using zero-mean property of $\mathbf{z}_n$, we obtain

$$\begin{aligned}E\{\hat{\mathbf{R}}\mathbf{\Theta}\hat{\mathbf{R}}\} = &\left(\frac{N-1}{N^2}\right)^2 \sum_{n_1,n_2} E\{\mathbf{z}_{n_1}\mathbf{z}_{n_1}^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_{n_2}\mathbf{z}_{n_2}^{\mathrm{H}}\} \\ &+ \frac{1}{N^4}\sum_{\substack{n_1 \neq n_2 \\ n_3 \neq n_4}} E\{\mathbf{z}_{n_1}\mathbf{z}_{n_2}^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_{n_3}\mathbf{z}_{n_4}^{\mathrm{H}}\}.\end{aligned} \tag{A.6}$$

The term $\sum_{n_1,n_2} E\{\mathbf{z}_{n_1}\mathbf{z}_{n_1}^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_{n_2}\mathbf{z}_{n_2}^{\mathrm{H}}\}$ becomes $NE\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} + (N^2 - N)\mathbf{R}\mathbf{\Theta}\mathbf{R}$. In the second term, there are only two different cases which give nonzero contributions because of zero mean of $\mathbf{z}_n$ and independence assumption: $n_2 = n_3$, $n_1 = n_4$ but $n_1 \neq n_2$; $n_2 = n_4$, $n_1 = n_3$ but $n_1 \neq n_2$. They correspondingly yield

$$(N^2-N)\,\mathrm{tr}(\mathbf{R}\mathbf{\Theta})\mathbf{R} + (N^2-N)E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\mathbf{\Theta}^{\mathrm{T}}\left(E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\right)^*. \tag{A.7}$$

Therefore, (A.6) becomes

$$\begin{aligned}E\{\hat{\mathbf{R}}\mathbf{\Theta}\hat{\mathbf{R}}\} = &\frac{(N-1)^2}{N^3}E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} \\ &+ \frac{(N-1)^3}{N^3}\mathbf{R}\mathbf{\Theta}\mathbf{R} + \frac{N-1}{N^3}\,\mathrm{tr}(\mathbf{R}\mathbf{\Theta})\mathbf{R} \\ &+ \frac{N-1}{N^3}E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\mathbf{\Theta}^{\mathrm{T}}\left(E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\right)^*.\end{aligned} \tag{A.8}$$

Consequently, (A.4) becomes

$$\begin{aligned}\mathbf{\Psi}(\mathbf{\Theta}) = &\frac{(N-1)^2}{N^3}\left[E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} - \mathbf{R}\mathbf{\Theta}\mathbf{R}\right] \\ &+ \frac{1}{N^2}\mathbf{R}\mathbf{\Theta}\mathbf{R} + \frac{N-1}{N^3}\,\mathrm{tr}(\mathbf{R}\mathbf{\Theta})\mathbf{R} \\ &+ \frac{N-1}{N^3}E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\mathbf{\Theta}^{\mathrm{T}}\left(E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\right)^*.\end{aligned} \tag{A.9}$$

In order to complete simplification of (A.9), $E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\}$ and $E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}$ are needed which will be derived next. We consider real and complex systems separately.

A.1. *Real system*

According to (11), we have

$$\begin{aligned}\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}} &= \mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} + \mathbf{H}\mathbf{a}_n\mathbf{v}_n^{\mathrm{T}} + \mathbf{v}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} + \mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}, \\ \mathbf{z}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{z}_n &= \mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n + \mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n + \mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n + \mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n.\end{aligned} \tag{A.10}$$

Then considering zero mean of $\mathbf{a}_n$ and $\mathbf{v}_n$, we obtain

$$\begin{aligned}&E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\} \\ &\quad= E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} + E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n\} \\ &\qquad+ E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\} + E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}^{\mathrm{T}}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\} \\ &\qquad+ E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}^{\mathrm{T}}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} \\ &\qquad+ E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\} + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\} \\ &\quad= E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} + \sigma_v^2\,\mathrm{tr}(\mathbf{\Theta})\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} \\ &\qquad+ \sigma_v^2\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta} + \sigma_v^2\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}^{\mathrm{T}} + \sigma_v^2\mathbf{\Theta}^{\mathrm{T}}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} \\ &\qquad+ \sigma_v^2\mathbf{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^2\,\mathrm{tr}\,(\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H})\mathbf{I}_\nu \\ &\qquad+ E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\}.\end{aligned} \tag{A.11}$$

Using $\mathbf{R} = \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^2\mathbf{I}_\nu$, we obtain

$$\begin{aligned}\mathbf{R}\mathbf{\Theta}\mathbf{R} = &\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^2\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta} \\ &+ \sigma_v^2\mathbf{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^4\mathbf{\Theta}.\end{aligned} \tag{A.12}$$

Then we obtain

$$\begin{aligned}&E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\} - \mathbf{R}\mathbf{\Theta}\mathbf{R} \\ &\quad= E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} + \sigma_v^2\,\mathrm{tr}(\mathbf{\Theta})\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} \\ &\qquad+ \sigma_v^2\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}^{\mathrm{T}} + \sigma_v^2\mathbf{\Theta}^{\mathrm{T}}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^2\,\mathrm{tr}\,(\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H})\mathbf{I}_\nu \\ &\qquad+ E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\mathbf{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\} - \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} - \sigma_v^4\mathbf{\Theta}.\end{aligned} \tag{A.13}$$

The first term can be simplified according to distribution of input. Express $\mathbf{H}\mathbf{a}_n$ by $\sum_{l=1}^{L}\mathbf{H}_l\mathbf{a}_{n,l}$ where $E\{\mathbf{a}_{n,l}\mathbf{a}_{n,l}^{\mathrm{T}}\} = \mathbf{A}$. Then we obtain

$$\begin{aligned}&E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\} \\ &\quad= \sum_{l_1,l_2,l_3,l_4} E\{\mathbf{H}_{l_1}\mathbf{a}_{n,l_1}\mathbf{a}_{n,l_2}^{\mathrm{T}}\mathbf{H}_{l_2}^{\mathrm{T}}(\mathbf{a}_{n,l_3}^{\mathrm{T}}\mathbf{H}_{l_3}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_{l_4}\mathbf{a}_{n,l_4})\} \\ &\quad= \sum_l E\{\mathbf{H}_l\mathbf{a}_{n,l}\mathbf{a}_{n,l}^{\mathrm{T}}\mathbf{H}_l^{\mathrm{T}}(\mathbf{a}_{n,l}^{\mathrm{T}}\mathbf{H}_l^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_l\mathbf{a}_{n,l})\} \\ &\qquad+ \sum_{l_1,l_2}\mathbf{H}_{l_1}\mathbf{A}\mathbf{H}_{l_1}^{\mathrm{T}}\,\mathrm{tr}\,(\mathbf{A}\mathbf{H}_{l_2}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_{l_2}) + \sum_{l_1,l_2}\mathbf{H}_{l_1}\mathbf{A}\mathbf{H}_{l_1}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_{l_2}\mathbf{A}\mathbf{H}_{l_2}^{\mathrm{T}} \\ &\qquad+ \sum_{l_1,l_2}\mathbf{H}_{l_1}\mathbf{A}\mathbf{H}_{l_1}^{\mathrm{T}}\mathbf{\Theta}^{\mathrm{T}}\mathbf{H}_{l_2}\mathbf{A}\mathbf{H}_{l_2}^{\mathrm{T}} \\ &\qquad- \sum_l\left[\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\,\mathrm{tr}\,(\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_l) + \mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}(\mathbf{\Theta}+\mathbf{\Theta}^{\mathrm{T}})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\right] \\ &\quad= \sum_l E\{\mathbf{H}_l\mathbf{a}_{n,l}\mathbf{a}_{n,l}^{\mathrm{T}}\mathbf{H}_l^{\mathrm{T}}(\mathbf{a}_{n,l}^{\mathrm{T}}\mathbf{H}_l^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_l\mathbf{a}_{n,l})\} \\ &\qquad- \sum_l\left[\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\,\mathrm{tr}\,(\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_l) + \mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}(\mathbf{\Theta}+\mathbf{\Theta}^{\mathrm{T}})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\right] \\ &\qquad+ \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\,\mathrm{tr}\,(\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\mathbf{\Theta}) + \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}(\mathbf{\Theta}+\mathbf{\Theta}^{\mathrm{T}})\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}.\end{aligned} \tag{A.14}$$

Also according to [18, equation (13)], the following holds for real AWGN:

$$E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{T}}\} = \sigma_v^4 \operatorname{tr}(\boldsymbol{\Theta})\mathbf{I}_\nu + \sigma_v^4(\boldsymbol{\Theta} + \boldsymbol{\Theta}^{\mathrm{T}}). \tag{A.15}$$

Substituting (A.14) and (A.15) into (A.13), using $\mathbf{R} = \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}} + \sigma_v^2\mathbf{I}_\nu$, and considering that $\mathbf{a}_{n,l}$ takes M possible values with probability $1/M$, we obtain

$$\begin{aligned} &E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\boldsymbol{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\} - \mathbf{R}\boldsymbol{\Theta}\mathbf{R} \\ &\quad = \sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}}\boldsymbol{\Theta}\tilde{\mathbf{h}}_{l,j})\tilde{\mathbf{h}}_{l,j}\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}} + \operatorname{tr}(\mathbf{R}\boldsymbol{\Theta})\mathbf{R} + \mathbf{R}\boldsymbol{\Theta}^{\mathrm{T}}\mathbf{R} \\ &\qquad - \sum_{l}\big[\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\operatorname{tr}(\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\boldsymbol{\Theta}\mathbf{H}_l) + \mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}(\boldsymbol{\Theta}+\boldsymbol{\Theta}^{\mathrm{T}})\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\big]. \end{aligned} \tag{A.16}$$

Considering $\mathbf{R} = E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}$ and substituting (A.16) into (A.9), we obtain (31).

A.2. Complex system

We will follow similar procedures as before. According to (11), we have

$$\begin{aligned} \mathbf{z}_n\mathbf{z}_n^{\mathrm{H}} &= \mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}} + \mathbf{H}\mathbf{a}_n\mathbf{v}_n^{\mathrm{H}} + \mathbf{v}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}} + \mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}, \\ \mathbf{z}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{z}_n &= \mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n + \mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n + \mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n + \mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n. \end{aligned} \tag{A.17}$$

From these two equations and since $\mathbf{v}_n$ has zero-mean circularly symmetric Gaussian entries, we obtain

$$\begin{aligned} &E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} \\ &\quad = E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\} + E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\} \\ &\qquad + E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\} + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\} \\ &\qquad + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\} + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\} \\ &\quad = E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\} + \sigma_v^2\operatorname{tr}(\boldsymbol{\Theta})\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} \\ &\qquad + \sigma_v^2\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta} + \sigma_v^2\boldsymbol{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} + \sigma_v^2\operatorname{tr}(\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H})\mathbf{I}_\nu \\ &\qquad + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\}. \end{aligned} \tag{A.18}$$

Using (A.12), we have

$$\begin{aligned} &E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} - \mathbf{R}\boldsymbol{\Theta}\mathbf{R} \\ &\quad = E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\} + \sigma_v^2\operatorname{tr}(\boldsymbol{\Theta})\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} \\ &\qquad + \sigma_v^2\operatorname{tr}(\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H})\mathbf{I}_\nu + E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\} \\ &\qquad - \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} - \sigma_v^4\boldsymbol{\Theta}. \end{aligned} \tag{A.19}$$

The first term can be similarly obtained as (A.14) after noticing that $\mathbf{a}_{n,l}$ is a real vector by

$$\begin{aligned} &E\{\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathbf{a}_n\mathbf{a}_n^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\} \\ &\quad = \sum_{l}E\{\mathbf{H}_l\mathbf{a}_{n,l}\mathbf{a}_{n,l}^{\mathrm{H}}\mathbf{H}_l^{\mathrm{H}}(\mathbf{a}_{n,l}^{\mathrm{H}}\mathbf{H}_l^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}_l\mathbf{a}_{n,l})\} \\ &\qquad - \sum_{l}\big[\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\operatorname{tr}(\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}_l) + \mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\big] \\ &\qquad - \sum_{l}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\boldsymbol{\Theta}^{\mathrm{T}}\mathbf{H}_l^{*}\mathbf{A}\mathbf{H}_l^{\mathrm{H}} + \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}}\operatorname{tr}(\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}) \\ &\qquad + \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} + \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\boldsymbol{\Theta}^{\mathrm{T}}\mathbf{H}^{*}\mathcal{A}\mathbf{H}^{\mathrm{H}}. \end{aligned} \tag{A.20}$$

According to [18, equation (20)], the following holds for complex symmetric AWGN:

$$E\{\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{v}_n\mathbf{v}_n^{\mathrm{H}}\} = \sigma_v^4\operatorname{tr}(\boldsymbol{\Theta})\mathbf{I}_\nu + \sigma_v^4\boldsymbol{\Theta}. \tag{A.21}$$

Substituting (A.20) and (A.21) into (A.19), using $\mathbf{R} = \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{H}} + \sigma_v^2\mathbf{I}_\nu$, and considering that $\mathbf{a}_{n,l}$ takes M possible values with probability $1/M$, we obtain

$$\begin{aligned} &E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{z}_n\mathbf{z}_n^{\mathrm{H}}\} - \mathbf{R}\boldsymbol{\Theta}\mathbf{R} \\ &\quad = \sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}}\boldsymbol{\Theta}\tilde{\mathbf{h}}_{l,j})\tilde{\mathbf{h}}_{l,j}\tilde{\mathbf{h}}_{l,j}^{\mathrm{H}} \\ &\qquad - \sum_{l}\big[\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\operatorname{tr}(\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}_l) + \mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\boldsymbol{\Theta}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{H}}\big] \\ &\qquad - \sum_{l}\mathbf{H}_l\mathbf{A}\mathbf{H}_l^{\mathrm{T}}\boldsymbol{\Theta}^{\mathrm{T}}\mathbf{H}_l^{*}\mathbf{A}\mathbf{H}_l^{\mathrm{H}} \\ &\qquad + \operatorname{tr}(\mathbf{R}\boldsymbol{\Theta})\mathbf{R} + \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}\boldsymbol{\Theta}^{\mathrm{T}}\mathbf{H}^{*}\mathcal{A}\mathbf{H}^{\mathrm{H}}. \end{aligned} \tag{A.22}$$

Noticing

$$E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\} = \mathbf{H}\mathcal{A}\mathbf{H}^{\mathrm{T}}, \qquad \big(E\{\mathbf{z}_n\mathbf{z}_n^{\mathrm{T}}\}\big)^{*} = \mathbf{H}^{*}\mathcal{A}\mathbf{H}^{\mathrm{H}}, \tag{A.23}$$

and substituting (A.22) into (A.9), we obtain (32).

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