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Massive hermitian gravity

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Abstract: Einstein-Strauss Hermitian gravity was recently formulated as a gauge theory where the tangent group is taken to be the pseudo-unitary group instead of the orthogonal group. A Higgs mechanism for massive gravity was also formulated. We generalize this construction to obtain massive Hermitian gravity with the use of a complex Higgs multiplet. We show that both the graviton and antisymmetric tensor acquire the same mass. At the linearized level, the theory is ghost free around Minkowski background and describes a massive graviton with five degrees of freedom and an antisymmetric field with three degrees of of freedom. We determine the strong coupling scales for these degrees of freedom and argue that the potential nonlinear ghosts, if they exist, have to decouple from the gravitational degrees of freedom in strong coupling regime.

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Contents

1 Introduction

Hermitian gravity is based on a Hermitian metric tensor unifying gravity with an antisymmetric tensor. It was first formulated by Einstein [1] and then by Einstein and Strauss [2] in the hope of unifying gravity with electromagnetism based on a geometrical construction with a Hermitian affine connection. Schrödinger has shown that Hermitian gravity is equivalent to a theory of gravity with a non-symmetric metric tensor [3]. There exists variations of this theory depending on whether a first or second order formulation is used. A systematic study of all these models was undertaken by Damour, Deser and McCarthy [4], who have shown that these suffer either from appearance of ghost states or impose unacceptable constraints on the curvature tensor. They arrived at a no-go theorem for all these models, which however, could be evaded by adding a mass term to the antisymmetric tensor, or a cosmological constant formed from the determinant of the Hermitian metric. Adding a mass term to the antisymmetric field could not be written using the Hermitian metric only, and thus it is not geometrical. This contradicts the fundamental assumption of Hermitian gravity that all geometric invariants must be expressed in terms of the Hermitian metric.

Recently much progress was made in formulating a consistent theory of massive gravity where the graviton acquires mass through the Higgs mechanism involving four scalar fields [5–9]. The vacuum expectation values of these fields cause the excitations of three of the four scalar fields to be absorbed by the metric thus leading to a massive graviton with five degrees of freedom. The fourth scalar, a potential ghost degree of freedom, is non-dynamical in the linear approximation on the Minkowski background for the Fierz-Pauli mass term [10]. The potential nonlinear ghost [12], if exists, is in strong coupling regime above Vainshtein energy scale [11] and, hence, harmless for gravity [8]. It is then natural to consider whether the Higgs mechanism generalizes to Hermitian gravity. The aim would then be to give a mass to the antisymmetric field through spontaneous symmetry breaking mechanism with the four complex scalar fields z^A involved. In analogy with the case of real scalar fields with global $\mathrm{SO}(1,3)$ symmetry, the complex scalar fields must be

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taken to have a global $U(1,3)$ symmetry which must be imposed on the couplings to the Hermitian metric.

In this letter, we shall construct a model of massive Hermitian gravity coupled to four complex scalar fields z^A. First, we will briefly review a derivation of the Hermitian gravity action based on gauging of the $U(1,3)$ symmetry and promoting it to be the tangent group of the manifold [13, 14]. Then we will introduce four complex scalar fields and show how these fields, when acquire vacuum expectation values, generate the same masses for both the graviton and the antisymmetric tensor. Finally we determine non-covariantly the physical degrees of freedom of the antisymmetric field and find the strong coupling scales for them.

2 Hermitian gravity and the U(1,3) tangent group

The action for Hermitian gravity is most easily constructed via imposing a local $U(1,3)$ symmetry which is identified with the tangent group of the four-dimensional manifold. The main fields are then a complex vierbein e_a^μ and the connections $\omega_{\mu b}^a$, $\Gamma_{\rho\mu}^\nu(g)$, constrained by the metricity conditions [14]:

$$0 = \nabla_\mu e_a^\nu = \partial_\mu e_a^\nu + \omega_{\mu a}{}^b e_b^\nu + \Gamma_{\rho\mu}^\nu(g)\, e_a^\rho. \tag{2.1}$$

These sixty four complex conditions could be solved, perturbatively, to determine the 64 (anti) Hermitian spin-connections $\left(\omega_{\mu b}{}^c\right)^* \eta_{ca} = -\omega_{\mu a}{}^c \eta_{cb}$ and the 64 Hermitian connections $\left(\Gamma_{\mu\rho}^\nu\right)^* = \Gamma_{\rho\mu}^\nu$ in terms of e_a^μ. The curvature tensor is identified with the field strength

$$R_{\mu\nu a}{}^{b} = \partial_\mu \omega_{\nu a}{}^b - \partial_\nu \omega_{\mu a}{}^b + \omega_{\mu a}{}^c \omega_{\nu c}{}^b - \omega_{\nu a}{}^c \omega_{\mu c}{}^b, \tag{2.2}$$

which admits two possible contractions:

$$R = e_b^\mu R_{\mu\nu a}{}^{b} e^{\nu a}, \tag{2.3}$$

$$\tilde{R} = g^{\mu\nu} R_{\mu\nu a}{}^{a}, \tag{2.4}$$

where

$$g^{\mu\nu} = e_a^\mu e^{\nu a}, \quad e^{\mu a} = \left(e_b^\mu\right)^* \eta^{ab}, \quad e_a^\mu e_\nu^a = \delta_\nu^\mu \tag{2.5}$$

We have shown in [14], that using the constraint (2.1) the curvatures above can be expressed in terms of $\Gamma_{\rho\mu}^\nu$:

$$R(\omega) = \eta^{ac} e_c^{\mu *} R_{\mu\nu a}{}^{b}(\omega)\, e_b^\nu = -\eta^{ac} e_c^{\mu *} R^\nu{}_{\rho\mu\nu}(\Gamma)\, e_a^\rho = g^{\rho\mu} R^\nu{}_{\rho\nu\mu}(\Gamma) = R(\Gamma),$$

where

$$R^\sigma{}_{\rho\mu\nu}(\Gamma) = \partial_\mu \Gamma_{\rho\nu}^\sigma - \partial_\nu \Gamma_{\rho\mu}^\sigma + \Gamma_{\kappa\mu}^\sigma \Gamma_{\rho\nu}^\kappa - \Gamma_{\kappa\nu}^\sigma \Gamma_{\rho\mu}^\kappa. \tag{2.6}$$

The generalization of the Einstein action is given then by

$$S_E = -\frac{1}{2} \int d^4x \left|\det e_\mu^a\right| R, \tag{2.7}$$

and depends on metric $g_{\mu\nu}$ only (we use the units where $8\pi G = 1$)

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3 Higgs for gravity

To give mass to the graviton and antisymmetric complex part of the metric we will introduce the four complex scalar fields z^A and construct the induced "Hermitian metric"

$$H^A_{\ B} = g^{\mu\nu}\partial_\mu z^A \partial_\nu z_B = \left(H^D_{\ C}\right)^* \eta_{BD}\eta^{AC}, \tag{3.1}$$

where we have defined

$$z_A = \eta_{AB}\left(z^B\right)^*. \tag{3.2}$$

It is straightforward then to write the analogue of the Fierz-Pauli mass term in powers of $\bar{H}^A_{\ B}$, defined by

$$\bar{H}^A_{\ B} = H^A_{\ B} - \delta^A_B. \tag{3.3}$$

The action for the complex scalar fields providing us with the mass term for gravity becomes

$$S_z = \frac{m^2}{8}\int d^4x\sqrt{-g}\left(\bar{H}^2 - \bar{H}^A_{\ B}\bar{H}^B_{\ A} + O\left(\bar{H}^3\right)\right). \tag{3.4}$$

where by $O\left(\bar{H}^3\right)$ we have denoted all possible higher order extensions of the Fierz-Pauli term, which do not influence the linear propagator for massive graviton on the symmetry broken background. An elegant non-linear extension of the action for complex scalar fields (3.4) will be given in the appendix. The vacuum solution of the full action given by the sum of (2.7) and (3.4) is

$$g^{\mu\nu} = \eta^{\mu\nu}, \quad z^A = x^A. \tag{3.5}$$

Expanding z^A around this vacuum solution, we write

$$z^A = x^A + \chi^A + i\psi^A, \tag{3.6}$$

while for the metric $g^{\mu\nu}$ we have

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} + iB^{\mu\nu}, \tag{3.7}$$

with $B^{\mu\nu}$ being antisymmetric. Similarly $\bar{H}^A_{\ B}$, which is Hermitian, can be decomposed in terms of a real symmetric part and an imaginary antisymmetric part

$$\bar{H}^A_{\ B} = \bar{h}^A_{\ B} + i\bar{B}^A_{\ B}, \tag{3.8}$$

where $\bar{h}^{AB} = \bar{h}^{BA}$ and $\bar{B}^{AB} = -\bar{B}^{BA}$ and the indices are raised and lowered with the Minkowski metric η_{AB}. Substituting the expansions (3.6), (3.7) into the definition of $\bar{H}^A_{\ B}$ we find

$$\bar{h}_{AB} = h_{AB} + \partial_A\chi_B + \partial_B\chi_A + O((\partial\chi)^2, \ldots), \tag{3.9}$$

$$\bar{B}_{AB} = B_{AB} - \partial_A\psi_B + \partial_B\psi_A + O(\partial\chi\partial\psi, \ldots), \tag{3.10}$$

where we have denoted by $O((\partial\chi)^2, \ldots)$ the higher order terms in perturbations, the explicit form of which will not be needed here. Notice that $\bar{h}_{AB}$ is invariant with respect

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infinitesimal diffeomorphism transformations and the antisymmetric field $\bar{B}_{AB}$ is invariant with respect to the infinitesimal gauge transformations

$$B_{AB} \to B_{AB} + \partial_A \zeta_B - \partial_B \zeta_A, \quad \psi_A \to \psi_A + \zeta_A \, . \tag{3.11}$$

Substituting (3.8) into (3.4) we can rewrite the action for the scalar fields up to quadratic order as

$$S_z = \frac{m^2}{8} \int d^4x \sqrt{-g} \left(\bar{h}^2 - \bar{h}^{AB} \bar{h}_{AB} - \bar{B}^{AB} \bar{B}_{AB} \right) . \tag{3.12}$$

Next we expand the Einstein action (2.7) up to second order in perturbations. Using the equivalence of the expressions for curvature in terms of spin-connections $\omega_{\mu a}{}^{b}$ to that in terms of the Hermitian connections $\Gamma^{\rho}_{\mu\nu}(g)$, we can solve the equation

$$\partial_\mu g^{\nu\rho} + \Gamma^{\nu}_{\sigma\mu} g^{\sigma\rho} + \Gamma^{\rho}_{\sigma\mu} g^{\nu\sigma} = 0, \tag{3.13}$$

to determine $\Gamma^{\rho}_{\mu\nu}(g)$ perturbatively in terms of powers of $h^{\mu\nu}$ and $B^{\mu\nu}$. To the first order we have

$$\Gamma^{\rho\mu\nu(1)} = -\frac{1}{2} \left(\partial^\mu \left(h^{\nu\rho} + iB^{\nu\rho} \right) - \partial^\nu \left(h^{\rho\mu} + iB^{\rho\mu} \right) + \partial^\rho \left(h^{\mu\nu} + iB^{\mu\nu} \right) \right), \tag{3.14}$$

where the indices are raised and lowered with the Minkowski metric. This can be used back in the constraint equation to find $\Gamma^{\rho\mu\nu(2)}$ to second order. The gravitational action to second order is then given by

$$\begin{aligned} S_E =& \frac{1}{8} \int d^4x \left[\left(\partial^A h^{BC} \partial_A h_{BC} + 2\partial_B h \partial_A h^{AB} - 2\partial_A h^{AB} \partial^C h_{CB} - \partial_A h \partial^A h \right) \right. \\ & \left. + \left(\partial^A B^{BC} \partial_A B_{BC} - 2\partial^C B^{AB} \partial_A B_{CB} \right) \right] \end{aligned} \tag{3.15}$$

This action is invariant with respect to both the diffeomorphism and gauge transformations, respectively,

$$h_{AB} \to h_{AB} + \partial_A \xi_B + \partial_B \xi_A, \quad B_{AB} \to B_{AB} + \partial_A \zeta_B - \partial_B \zeta_A. \tag{3.16}$$

Hence we can replace h_{AB} and B_{AB} in the gravitational part of the action by their gauge invariant combinations with the scalar fields, $\bar{h}_{AB}$ and $\bar{B}_{AB}$, correspondingly. The field $\bar{h}_{AB}$ then satisfy the same linear equations as massive graviton with five degrees of freedom. The massive gravity and its nonlinear extensions were studied in details in [6–9] and therefore we concentrate here only on the antisymmetric massive field, the action for which becomes

$$S_{\bar{B}} = \frac{1}{8} \int d^4x \left(\partial^A \bar{B}^{BC} \partial_A \bar{B}_{BC} - 2\partial^C \bar{B}^{AB} \partial_A \bar{B}_{CB} - m^2 \bar{B}^{AB} \bar{B}_{AB} \right) . \tag{3.17}$$

The equations of motion for $\bar{B}_{AB}$ are

$$\left(\partial^2 + m^2 \right) \bar{B}_{AB} - \partial_A \partial^C \bar{B}_{CB} - \partial_B \partial^C \bar{B}_{AC} = 0. \tag{3.18}$$

They describe massive field with three degrees of freedom. Remarkably, $\bar{B}_{AB}$ is exactly the same combination of fields worked out by Kalb and Ramond in [15], where they used

Stückelberg method to introduce fake gauge invariance for the auxiliary fields (corresponding here to ψ^A) and showed that $\bar{B}_{AB}$ has two degrees of freedom from the transverse components of ψ^A plus one degree from the longitudinal part of B_{AB}.

To demonstrate this explicitly and to determine the strong coupling scales for the different degrees of freedom we will express the action (3.17) entirely in terms of physical degrees of freedom and find when they enter the strong coupling regime.

4 Physical degrees of freedom and strong coupling scales

Let us first rewrite the action (3.17) explicitly separating space and time components in $\bar{B}_{AB}$:

$$S_{\bar{B}} = \frac{1}{8}\int d^4x \left[\dot{B}_{ik}^2 + 2\dot{B}_{ik}\left(B_{i,k} - B_{k,i}\right) + \left(B_{i,k} - B_{k,i}\right)^2 \right. \\ \left. - B_{ik,j}^2 - 2B_{ik,j}B_{ji,k} + 2m^2B_i^2 - m^2B_{ik}^2\right], \tag{4.1}$$

where dot denotes the derivative with respect to time, indices $i, k, \ldots$ take the values $1, 2, 3$ and comma denotes the derivative with respect to the corresponding spatial coordinate. We have also introduced the following notations:

$$\bar{B}_{0i} \equiv B_i, \qquad \bar{B}_{ik} \equiv B_{ik}, \tag{4.2}$$

and assumed summation over repeated indices. Next we define the vector A_l dual to the antisymmetric tensor B_{ik}, so that,

$$B_{ik} = \varepsilon_{ikl}A_l, \tag{4.3}$$

and decompose the 3-vectors A_l and B_i into transverse and longitudinal parts

$$A_l = \frac{\varphi_{,l}}{\sqrt{-\Delta}} + A_l^{(T)}, \qquad B_i = \mu_{,i} + B_i^{(T)}, \tag{4.4}$$

where Δ is the Laplacian and the transverse components satisfy the conditions $A_{l,l}^{(T)} = 0, B_{i,i}^{(T)} = 0$. Substituting (4.3) and (4.4) into (4.1) the action reduces to

$$S_{\bar{B}} = \frac{1}{4}\int d^4x \left[\left(\dot{\varphi}^2 - \varphi_{,i}\varphi_{,i} - m^2\varphi^2\right) + m^2\mu_{,i}\mu_{,i} + \dot{A}_i^{(T)}\dot{A}_i^{(T)} \right. \\ \left. + 2\varepsilon_{ikl}B_{i,k}^{(T)}\dot{A}_l^{(T)} + B_{i,k}^{(T)}B_{i,k}^{(T)} - m^2A_i^{(T)}A_i^{(T)} + m^2B_i^{(T)}B_i^{(T)}\right]. \tag{4.5}$$

Variation of this action with respect to μ and $B_i^{(T)}$ give us the constraints

$$\Delta\mu = 0, \quad \varepsilon_{ikl}\dot{A}_{l,k}^{(T)} + \Delta B_i^{(T)} - m^2B_i^{(T)} = 0, \tag{4.6}$$

from which it follows, that

$$\mu = 0, \quad B_i^{(T)} = \frac{\varepsilon_{ikl}\dot{A}_{l,k}^{(T)}}{-\Delta + m^2}. \tag{4.7}$$

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Substituting these expressions into (4.5), the action becomes

$$S_{\bar{B}} = \frac{1}{4}\int d^4x \left[\left(\dot{\varphi}^2 - \varphi_{,i}\varphi_{,i} - m^2\varphi^2\right) + \left(\dot{A}_i^{(T)}\frac{m^2}{-\Delta + m^2}\dot{A}_i^{(T)} - m^2 A_i^{(T)}A_i^{(T)}\right)\right]. \qquad (4.8)$$

The three physical degrees of freedom (one pseudo-scalar φ and two independent transverse components of pseudo-vector $A_i^{(T)}$) satisfy the following equations

$$\left(\partial^2 + m^2\right)\varphi = 0, \qquad \left(\partial^2 + m^2\right)A_i^{(T)} = 0. \qquad (4.9)$$

The last term in the action is proportional to the mass and therefore when the mass m vanishes $A_i^{(T)}$ drops out from the action and in this limit the antisymmetric field $B_{\alpha\beta}$ describes a massless pseudo-scalar with only one degree of freedom. This is not surprising because as one can easily see from (4.1), three B_{0i} components of the antisymmetric metric are not dynamical and the gauge symmetry (3.16), involving only two transverse components of ζ_i removes two degrees of freedom in B_{ik}. When we couple gravity to the scalar fields ψ_B, which in the absence of $B_{\mu\nu}$ are described by the "Maxwell action"

$$-\frac{m^2}{8}\int d^4x \bar{B}^{AB}\bar{B}_{AB}, \qquad (4.10)$$

with $\bar{B}_{AB} = \partial_B\psi_A - \partial_A\psi_B$, the two physical degrees of freedom of ψ_B are absorbed by the antisymmetric metric, which thus acquires three degrees of freedom.

The scalar and vector degrees of freedom become strongly coupled at different scales. To determine these scales let us consider the plane wave with the wavelength λ. We first note that the scalar φ enters (4.8) with canonical normalization. Hence, the minimal quantum fluctuations of this field at the length-scale λ are of order $\delta\varphi_\lambda \simeq 1/\lambda$ for $\lambda \ll m^{-1}$. Because $\bar{B} \sim \varphi$ (see (4.3), (4.4)), we find that the quantum fluctuations of the antisymmetric field due to the scalar mode become of the order unity at the Planck scale, where this degree of freedom enters the strong coupling regime. For the two transverse degrees of freedom $A_i^{(T)}$ the strong coupling scale is larger than the Planck length. Actually, as follows from (4.8) the canonically normalized degrees of freedom for these modes are

$$\sqrt{\frac{m^2}{-\Delta + m^2}}A_i^{(T)} \sim m\lambda A_i^{(T)}$$

and therefore the minimal vacuum fluctuations of $m\lambda A_i^{(T)}$ decay as $1/\lambda$ for $\lambda \ll m^{-1}$.Hence the amplitude of the fluctuations of the fields $A_i^{(T)}$ itself is of order

$$\delta A_\lambda^{(T)} \simeq \frac{1}{m\lambda^2}$$

It then follows that the quantum fluctuations of $\bar{B} \sim \delta A_\lambda^{(T)}$ due to transverse degrees of freedom become of order unity at scales

$$\lambda_{strong} \simeq m^{-1/2} = \frac{1}{m}\left(\frac{m}{m_{Pl}}\right)^{1/2}$$

For masses much smaller than the Planck mass m_{Pl} this strong coupling scale is significantly smaller than the inverse mass of the field but larger than the Planck wavelength by the factor $(m_{Pl}/m)^{1/2}$. Thus the transverse modes enter strong coupling regime before the scalar mode. As a result, the scalar fields which provide mass to these transverse modes decouple and the antisymmetric field will remain with one scalar degree of freedom at energy scales above $m^{1/2}$.

In [4] it was shown that unbroken Hermitian gravity is inconsistent, due to the coupling of $B_{\mu\nu}$ to the curvature tensor. However, they also pointed out that the inconsistencies can be avoided by adding a mass term for the $B_{\mu\nu}$ field. On the other hand, adding non diffeomorphism invariant mass terms for $B_{\mu\nu}$, destroys the Hermitian symmetry and violates the diffeomorphism invariance because the antisymmetric field is also a part of the Hermitian metric. Therefore, "hard" introduction of the mass term is not acceptable as it makes the use of a Hermitian metric pointless and spoils the geometrical nature of Hermitian gravity.

In this paper we have shown that in geometrical Hermitian gravity a mass term for $B_{\mu\nu}$ can be generated only if the graviton simultaneously acquires the same mass. By adding four complex scalar fields (corresponding to 8 real fields), we demonstrated how the gravitational Higgs mechanism can be realized for Hermitian gravity. Three out of the eight fields are absorbed by the real symmetric part of the metric thus giving us massive graviton with five degrees of freedom. Two other fields are absorbed by the antisymmetric part of the metric making this field massive (with 3 degrees of freedom). The remaining three scalar fields are non-dymamical at linear level on Minkowski background. Two of them could be potential non-linear ghosts. However, these potential ghosts could certainly be dangerous for gravity only in those regions where the corresponding degrees of freedom are in the weak coupling regime and hence the perturbative analysis is trustable for them. We have shown that for a small graviton mass the strong coupling scales are much below the Planck scale and hence "the trustable nonlinear ghosts" are completely harmless even if they would exist.

A Non-linear extension for Fierz-Pauli mass terms

It is straightforward to write the analogue of the Pauli-Fierz action containing various powers of H^A_B. This can be simplified in terms of the field $\overline{H}^A_B$ defined by

$$H^A_B = \delta^A_B + \overline{H}^A_B \tag{A.1}$$

We have shown in reference [9] that in the real case there is a special simple consistent action that starts with quadratic kinetic terms for the fields z^A instead of the quartic terms normally used . To generalize this construction, we first define the auxiliary fields E^μ_A constrained so that

$$g^{\mu\nu} = E^\mu_A E^{\nu A} \tag{A.2}$$

We then define the field

$$S^A_B = E^\mu_B \partial_\mu z^A - \delta^A_B \tag{A.3}$$

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which depends on the first derivative of z^A and is constrained to be Hermitian

$$S^A_B = \left(S^B_A\right)^* \tag{A.4}$$

These constraints could be imposed through the use of Lagrange multipliers. Thus the 16 complex fields E^μ_A are subjected to 32 real constraints, and could be determined completely, in a perturbative way in terms of $g^{\mu\nu}$ and z^A. The quadratic part of the proposed action in terms of the field S^A_B is given by

$$\int d^4x\sqrt{g}\left[\frac{m^2}{2!}\delta^{AB}_{EF}S^E_A S^F_B\right] \tag{A.5}$$

This expression can be rewritten in terms of the induced metric H^A_B by using the identity

$$\begin{aligned} S'^A_C S'^C_B &= E^\mu_C \partial_\mu z^A \left(E^\nu_B \partial_\nu z^C\right) \\ &= E^\mu_C \partial_\mu z^A \left(E^\nu_C \partial_\nu z^B\right)^* \\ &= E^\mu_C E^{\nu C} \partial_\mu z^A \partial_\nu z_B \\ &= g^{\mu\nu} \partial_\mu z^A \partial_\nu z_B \\ &= H^A_B \end{aligned} \tag{A.6}$$

where

$$S'^A_B = S^A_B + \delta^A_B \tag{A.7}$$

Thus we have

$$\begin{aligned} S^A_B &= \sqrt{H^A_B} - \delta^A_B \\ &= \sqrt{\delta^A_B + \overline{H}^A_B} - \delta^A_B \\ &= \frac{1}{2}\overline{H}^A_B - \frac{1}{8}\overline{H}^A_C\overline{H}^C_B + \frac{1}{16}\overline{H}^A_C\overline{H}^C_D\overline{H}^D_B + \cdots \end{aligned} \tag{A.8}$$

There is also unique generalization of the above action that adds terms which are cubic and quartic in terms of S^A_B

$$\int d^4x\sqrt{g}\left[\frac{c_1}{3!}\delta^{ABC}_{EFG}S^E_A S^F_B S^G_C + \frac{c_2}{4!}\delta^{ABCD}_{EFGH}S^E_A S^F_B S^G_C S^H_D\right] \tag{A.9}$$

Since S^A_B is an infinite expansion in terms of $\overline{H}^A_B$ the action could be expressed in terms of $\overline{H}^A_B$. The action, up to quartic terms is given by

$$\begin{aligned} -\int d^4x\sqrt{g}\,\Bigg(&\frac{m^2}{2!}\delta^{AB}_{EF}\left(\frac{1}{4}\overline{H}^E_A\overline{H}^F_B - \frac{1}{8}\overline{H}^E_A\overline{H}^F_C\overline{H}^C_B + \frac{1}{16}\overline{H}^E_A\overline{H}^F_C\overline{H}^C_D\overline{H}^D_B + \frac{1}{64}\overline{H}^E_C\overline{H}^C_A\overline{H}^F_D\overline{H}^D_B\right) \\ &+\frac{c_1}{3!}\delta^{ABC}_{EFG}\left(\frac{1}{8}\overline{H}^E_A\overline{H}^F_B\overline{H}^G_C - \frac{3}{32}\overline{H}^E_A\overline{H}^F_B\overline{H}^G_D\overline{H}^D_C\right) + \frac{c_2}{4!}\frac{1}{16}\delta^{ABCD}_{EFGH}\overline{H}^E_A\overline{H}^F_B\overline{H}^G_C\overline{H}^H_D\Bigg) \end{aligned} \tag{A.10}$$

This is the same expression obtained in the real case that produce decoupling of ghosts up to quartic order in perturbation series (compare with equation (20) in reference [8]).

It would be very interesting to generalize the analysis carried out in this paper to non-linear terms and on non-trivial backgrounds.

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