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Petrov type I condition and Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity

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Abstract: Recently the Petrov type I condition is introduced to reduce the degrees of freedom of the extrinsic curvature of a timelike hypersurface to the degrees of freedom in the dual Rindler fluid in Einstein gravity. In this paper we show that the Petrov type I condition holds for the solutions of vacuum Einstein-Gauss-Bonnet gravity up to the second order in the relativistic hydrodynamic expansion. On the other hand, if imposing the Petrov type I condition and Hamiltonian constraint on a finite cutoff hypersurface, the stress tensor of the relativistic Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity can be recovered with correct first order and second order transport coefficients. The case in the non-relativistic hydrodynamic expansion is also discussed.

Keywords: Gauge-gravity correspondence, Classical Theories of Gravity, Holography and condensed matter physics (AdS/CMT)

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Contents

1 Introduction

There has been increasing interest in the holographic duality between fluid dynamics and gravity in the past few years, while the suggestion of such a connection can be dated back to the 1970s suggested by Damour [1–3]. Later, the approach was developed into the membrane paradigm [4], which relates the evolution and diffusion of a black hole to those in hydrodynamics [5–9]. In recent years, along with the progress in the anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [10–13], the dual fluid has been generalized to the conformal fluid living on the boundary of AdS spacetime, which describes the long wavelength and low frequency limit of conformal field theory [14–16]. In particular, a systematic method to study the duality was proposed in the fluid/gravity correspondence [17], which translates problems in fluid dynamics into problems in general relativity. It was then further expanded to the case in an arbitrary dimension in [18–20] and to the non-relativistic hydrodynamics case in [21].

To build up a connection between the fluid/gravity correspondence and membrane paradigm, a timelike hypersurface outside a horizon of spacetime is introduced to study the universality of the hydrodynamic limit in the AdS/CFT correspondence and membrane paradigm [22–24]. Significantly, the authors in [24] consider the fluid living on the finite cutoff hypersurface outside the horizon from the viewpoint of Wilsonian renormalization, and impose the Dirichlet boundary condition on the hypersurface and the regularity on the horizon. Then the fluid/gravity correspondence on the cutoff hypersurface can be generalized to the asymptotically flat [25, 26] or de Sitter spacetime [27], and it has been further studied in [28–36]. More general discussions in the fluid/gravity correspondence can also be found in [37–41], as well as in the frame of the AdS/Ricci-flat correspondence [42, 43].

Recently, one of the most important developments in the gravity/fluid duality is the so-called Rindler hydrodynamics [25, 44–49], where the dual fluid lives on a constant acceleration hypersurface with a flat induced metric. While Bredberg et al. showed in [25] that for the four-dimensional case the geometry for the vacuum Einstein gravity is, at least at leading nontrivial order for the non-relativisic hydrodynamic expansion, of an algebraically special variety known as restricted Petrov type I , it was found in [50] that in the near-horizon limit, instead of the regularity condition on the horizon, imposing the Petrov type I condition on the hypersurface can reduce the vacuum Einstein constraint equations on the hypersurface to the incompressible Navier-Stokes equations in one lower dimensional flat spacetime. It is mathematically much simpler than solving gravitational field equations. Further study based on this framework can be found in [51–56]. From the point of view of degrees of freedom, the Petrov type I condition gives $(p+2)(p-1)/2$ constraints on the extrinsic curvature of a $p+1$ dimensional timelike hypersurface, or equivalently on the dual Brown-York stress tensor. Then the degrees of freedom of the stress tensor are reduced to be $p+2$, which can be interpreted as energy density, pressure and velocity field of dual fluid [50]. Furthermore the momentum constraint of Einstein gravity turns out to be the equation of motion of the dual fluid, while the Hamiltonian constraint of the gravity can be interpreted as the equation of state for the dual fluid.

Very recently, it has been shown in [57, 58] that, the Petrov type I condition can be used to recover the stress tensor of the dual fluid on the hypersurface order by order under appropriate gauge choice. Without solving the gravitational field equations, the Rindler fluid dual to vacuum Einstein gravity can be recovered at least up to the second order in the relativistic hydrodynamic expansion [58]. Note that the stress tensor of Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity is found to be modified by the Gauss-Bonnet term with coupling coefficient α in [45, 48]. It is then quite interesting to ask whether the Petrov type I condition holds or not in vacuum Einstein-Gauss-Bonnet gravity and whether it can be used to recover the dual stress tensor. In this paper, we find that the Petrov type I condition for the solution of vacuum Einstein-Gauss-Bonnet equations still holds up to the second order in the relativistic hydrodynamic expansion, and that turn the logic around, imposing the Petrov type I condition and Hamiltonian constraint, the stress tensor of the relativistic Rindler fluid can be recovered with correct first order and second order transport coefficients including the Gauss-Bonnet term corrections.

This paper is organized as follows. In section 2, we first review the Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity, and show that the spacetime is at least Petrov type I up to the second order in the relativistic hydrodynamic expansion. In section 3, we give a detailed derivation of the Petrov type I condition on a cutoff hypersurface in the vacuum Einstein-Gauss-Bonnet gravity. In section 4, we turn the logic around and obtain the stress tensor of the dual fluid without using the details of the solution, but assuming the Hamiltonian constraint and Petrov type I conditiont on a finite cutoff hypersurface. We further study the Petrov type I condition in non-relativistic hydrodynamic expansion in section 5, and make the conclusion in section 6.

2 Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity

To study the fluid dual to the vacuum Einstein-Gauss-Bonnet gravity, we begin with the Einstein-Hilbert action on a $(p+2)$ dimensional Lorentz manifold $\mathcal{M}$, with the Gauss-Bonnet term $\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$ and appropriate surface term [59]

$$S = \frac{1}{16\pi G_{p+2}} \int d^{p+2}x\sqrt{-g}(R - 2\Lambda + \alpha\mathcal{L}_{GB}) + S_{\partial\mathcal{M}}. \tag{2.1}$$

where α is the Gauss-Bonnet coefficient. Varying this action with respect to the metric $g_{\mu\nu}$ yields the vacuum Einstein-Gauss-Bonnet field equations,

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = 0, \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \quad \mu,\nu = 0,1,\dots,p+1, \tag{2.2}$$

$$H_{\mu\nu} \equiv RR_{\mu\nu} - 2R_{\mu\lambda}R^{\lambda}{}_{\nu} - 2R^{\sigma\lambda}R_{\mu\sigma\nu\lambda} + R_{\mu}{}^{\sigma\lambda\rho}R_{\nu\sigma\lambda\rho} - \frac{1}{4}g_{\mu\nu}\mathcal{L}_{GB}. \tag{2.3}$$

The $p+2$ dimensional Rindler metric

$$\mathrm{d}s^2_{p+2} = -r\mathrm{d}\tau^2 + 2\mathrm{d}\tau\mathrm{d}r + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \quad i,j = 1,\dots,p, \tag{2.4}$$

is an exact solution of the field equations (2.2). On a timelike hypersurface Σ_c with $r = r_c$, the induced metric is intrinsic flat,

$$\mathrm{d}s^2_{p+1} = \gamma_{ab}\mathrm{d}x^a\mathrm{d}x^b = -r_c\mathrm{d}\tau^2 + \mathrm{d}x_i\mathrm{d}x^i, \quad a,b = 0,1,\dots,p. \tag{2.5}$$

And after setting $16\pi G_{p+2} = 1$, the Brown-York stress tensor of Einstein-Gauss-Bonnet gravity on the cutoff surface Σ_c can be written as [28, 60],

$$T^{(GB)}_{ab} = -2\left(K_{ab} - K\gamma_{ab}\right) - 4\alpha\left(3J_{ab} - J\gamma_{ab}\right), \qquad J \equiv \gamma^{ab}J_{ab}, \tag{2.6}$$

$$J_{ab} \equiv \frac{1}{3}\left(2KK_{ac}K^{c}{}_{b} + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}\right). \tag{2.7}$$

Here K_{ab} is the extrinsic curvature of the hypersurface Σ_c.

2.1 Rindler fluid in relativistic hydrodynamic expansion

In order to study the dual fluid on the hypersurface Σ_c, one introduces the $(p+1)$ independent parameters $u^a = \gamma_v(1,\, v^i)$ and $\mathbb{p}$, which are slowly varying functions of $x^a = (\tau, x^i)$. Here γ_v is fixed through $\gamma_{ab}u^au^b = -1$. Keep the induced metric on a timelike hypersurface Σ_c flat and impose the regularity on the future horizon, the solution of vacuum Einstein-Gauss-Bonnet field equations (2.2) up to the second order in the derivative expansion is given by [48],

$$\mathrm{d}s^2_{p+2} = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -2\mathbb{p}u_a\mathrm{d}x^a\mathrm{d}r + g_{ab}\mathrm{d}x^a\mathrm{d}x^b, \tag{2.8}$$

$$g_{ab} = g^{(0)}_{ab} + g^{(1)}_{ab} + g^{(2)}_{ab} + O(\partial^3). \tag{2.9}$$

The superscript indices $(0),(1),(2),\ldots$ on the metric components g_{ab} denote the order of the derivative of the velocity u^a and pressure $\mathbb{p}$ with respect to the transverse space-time coordinates x^a: $\partial \sim \varepsilon$, where we have introduced the small parameter $\varepsilon \ll 1$. The leading order term of g_{ab} in the derivative expansion is given by

$$g_{ab}^{(0)} = [1-\mathbb{p}^2(r-r_c)]u_a u_b + h_{ab}, \tag{2.10}$$

where the projection tensor $h_{ab} \equiv \gamma_{ab} + u_a u_b$. We can read off the horizon position through $r_h = r_c - 1/\mathbb{p}^2$ with $g_{ab}^{(0)}$ in the case of equilibrium state. The first order term in g_{ab} in the derivative expansion is

$$g_{ab}^{(1)} = 2\mathbb{p}(r-r_c)\left[(D\ln\mathbb{p})u_a u_b + 2a_{(a}u_{b)}\right], \tag{2.11}$$

where $D \equiv u^c\partial_c$ and the acceleration $a^a \equiv u^b\partial_b u^a$. At the second order in the derivative expansion, the Gauss-Bonnet corrections appear in the metric,

$$\begin{aligned} u^c u^d g_{cd}^{(2)} &= +2(r-r_c)\mathcal{K}_{cd}\mathcal{K}^{cd} + \frac{1}{2}\mathbb{p}^2(r-r_c)^2\left(\mathcal{K}_{cd}\mathcal{K}^{cd} + 2a_c a^c\right) + \frac{1}{2}\mathbb{p}^4(r-r_c)^3\Omega_{cd}\Omega^{cd} \\ &\quad + 2\alpha\mathbb{p}^2(r-r_c)\left(\mathcal{K}_{cd}\mathcal{K}^{cd} - \frac{6}{p}\Omega_{cd}\Omega^{cd}\right) + 3\alpha\mathbb{p}^4(r-r_c)^2\frac{p-2}{p}\Omega_{cd}\Omega^{cd}, \end{aligned} \tag{2.12}$$

$$h_a^c u^d g_{cd}^{(2)} = -2(r-r_c)h_a^c\partial_d\mathcal{K}^d_c + \mathbb{p}^2(r-r_c)^2\left[h_a^b\partial_c\mathcal{K}^c_b - (\mathcal{K}_{ad}+\Omega_{ad})a^d\right], \tag{2.13}$$

$$\begin{aligned} h_a^c h_b^d g_{cd}^{(2)} &= +2(r-r_c)\left(-\mathcal{K}_a{}^c\mathcal{K}_{cb} + 2\mathcal{K}_{c(a}\Omega^c{}_{b)} - 2h_a^c h_b^d D\mathcal{K}_{cd}\right) - \mathbb{p}^2(r-r_c)^2\Omega_{ac}\Omega^c{}_b \\ &\quad + 12\alpha\mathbb{p}^2(r-r_c)\left[\Omega_{ac}\Omega^c{}_b + \frac{1}{p}\left(\Omega_{cd}\Omega^{cd}\right)h_{ab}\right]. \end{aligned} \tag{2.14}$$

Here the fluid shear and vorticity are defined as

$$\mathcal{K}_{ab} \equiv h_a^c h_b^d \partial_{(c}u_{d)}, \quad \Omega_{ab} \equiv h_a^c h_b^d \partial_{[c}u_{d]}. \tag{2.15}$$

The components of inverse metric up to the second order in the derivative expansion are

$$\begin{aligned} g^{rr} &= \mathbb{p}^{-2}\left[1 + \mathbb{p}^2(r-r_c) - \left(g_{cd}^{(1)} + g_{cd}^{(2)} - h^{ab}g_{ac}^{(1)}g_{bd}^{(1)}\right)u^c u^d\right], \\ g^{ra} &= \mathbb{p}^{-1}\left(u^a + h^{ab}g_{bc}^{(1)}u^c + h^{ab}g_{bc}^{(2)}u^c\right), \\ g^{ab} &= h^{ab} - h^{ac}h^{bd}g_{cd}^{(2)}. \end{aligned} \tag{2.16}$$

One also needs to consider the following constraints

$$\begin{aligned} \partial_a u^a &= 2\mathbb{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab} + O(\partial^3), \\ a_a + D_a^\perp \ln\mathbb{p} &= 2\mathbb{p}^{-1}h_a^c\partial_b\mathcal{K}^b_c + O(\partial^3), \end{aligned} \tag{2.17}$$

with $D_a^\perp \equiv h_a^c\partial_c$, so that the metric (2.8) solves the vacuum Einstein-Gauss-Bonnet field equations (2.2) up to the second order in the derivative expansion.

With the metric (2.8) and the gauge choice where the fluid velocity u^a is defined such that the momentum density vanishes in the local rest frame of the fluid, and the pressure $\mathbb{p}$ is defined by imposing the isotropy gauge so that there do not contain terms proportional to h_{ab} at higher derivative orders [46], the dual stress tensor $T_{ab}^{(GB)}$ for the vacuum Einstein-Gauss-Bonnet gravity on the finite cutoff surface Σ_c in (2.6) has been obtained in [48],

$$\begin{aligned} T_{ab}^{(GB)} &= +\mathbb{p} h_{ab} - 2\mathcal{K}_{ab} - 2\mathbb{p}^{-1}\left(\mathcal{K}_{cd}\mathcal{K}^{cd}\right)u_a u_b \\ &\quad + \mathbb{p}^{-1}\Big[-2\left(1+2\alpha\mathbb{p}^2\right)\mathcal{K}_{ac}\mathcal{K}^c{}_b - 4\mathcal{K}_{c(a}\Omega^c{}_{b)} - 4\left(1+3\alpha\mathbb{p}^2\right)\Omega_{ac}\Omega^c{}_b \\ &\qquad -4h_a^c h_b^d\partial_c\partial_d\ln\mathbb{p} - 4\mathcal{K}_{ab}D\ln\mathbb{p} + 4(D_a^\perp\ln\mathbb{p})(D_b^\perp\ln\mathbb{p})\Big]. \end{aligned} \tag{2.18}$$

On the other hand, a general stress tensor $T_{ab}^{(R)}$ for $(p+1)$-dimensional relativistic fluid with vanishing equilibrium energy density is constructed in [46] as

$$\begin{aligned} T_{ab}^{(R)} &= +\mathbb{p} h_{ab} - 2\eta\mathcal{K}_{ab} + \zeta'(D\ln\mathbb{p})u_a u_b \\ &\quad + \mathbb{p}^{-1}\left[d_1\mathcal{K}_{cd}\mathcal{K}^{cd} + d_2\Omega_{cd}\Omega^{cd} + d_3(D\ln\mathbb{p})^2 + d_4 DD\ln\mathbb{p} + d_5(D_\perp\ln\mathbb{p})^2\right]u_a u_b \\ &\quad + \mathbb{p}^{-1}\Big[c_1\mathcal{K}_{ac}\mathcal{K}^c{}_b + c_2\mathcal{K}_{c(a}\Omega^c{}_{b)} + c_3\Omega_{ac}\Omega^c{}_b + c_4 h_a^c h_b^d\partial_c\partial_d\ln\mathbb{p} + c_5\mathcal{K}_{ab}\,D\ln\mathbb{p} \\ &\qquad + c_6 D_a^\perp\ln\mathbb{p}\,D_b^\perp\ln\mathbb{p}\Big]. \end{aligned} \tag{2.19}$$

Compare $T_{ab}^{(GB)}$ in (2.18) with $T_{ab}^{(R)}$ in (2.19), one can read off the holographic transport coefficients of Rindler fluid dual to the vacuum Einstein-Gauss-Bonnet gravity as

$$\begin{aligned} &\zeta' = 0, \qquad \eta = 1, \qquad d_1 = -2, \qquad\qquad d_2 = d_3 = d_4 = d_5 = 0, \\ &c_1 = -2(1+2\alpha\mathbb{p}^2), \qquad c_3 = -4(1+3\alpha\mathbb{p}^2), \qquad c_2 = c_4 = c_5 = -c_6 = -4. \end{aligned} \tag{2.20}$$

It turns out that there are no Gauss-Bonne corrections to the shear viscosity η and the parameter ζ', the latter measures variations of the energy density. The Gauss-Bonnet corrections appear in the second order transport coefficients c_1 and c_3. Considering the equilibrium entropy density $s = 4\pi$, one obtains the universal ratio of shear viscosity over entropy density as $\eta/s = 1/4\pi$ [45]. There is no Gauss-Bonnet term correction to the ratio or to the shear viscosity itself here, which is different from the result in the context of the AdS/CFT correspondence [61–63]. To understand the difference, as pointed out in [28], one can start with the $(p+2)$ dimensional Einstein-Gauss-Bonnet gravity in AdS spacetime with AdS radius L and a negative cosmological constant $\Lambda = -p(p+1)/2L^2$. Then the ratio of shear viscosity over entropy density in the dual hydrodynamics is $\eta/s = \left[1-2(p-2)(p+1)\alpha/L^2\right]/4\pi$. Taking the large AdS radius limit $L\to\infty$ or the zero cosmological constant limit $\Lambda\to 0$, the asymptotically AdS spacetime becomes an asymptotically flat one, meanwhile, the Gauss-Bonnet term correction in the ratio $-2(p-2)(p+1)\alpha/L^2$ disappears. This implies that the shear viscosity in the Rindler fluid is protected against quantum corrections or other deformations [45].

2.2 The solution is Petrov type I

The Petrov type classification of Weyl tensor in higher dimensions is summarized in appendix A. Choose $(p+2)$ Newman-Penrose-like vector fields, which include two null vectors $\boldsymbol{\ell}^2 = \boldsymbol{k}^2 = 0$, and p orthonormal spacelike vectors $\boldsymbol{m}_i$. The null vectors obey $\boldsymbol{\ell}_\mu\boldsymbol{k}^\mu = 1$ and all other products with $\boldsymbol{m}_i (i = 1, \dots p)$ vanish, such that the metric can be decomposed as $g_{\mu\nu} = 2\boldsymbol{\ell}_{(\mu}\boldsymbol{k}_{\nu)} + \delta_{ij}\boldsymbol{m}^i{}_\mu \boldsymbol{m}^j{}_\nu$. Define

$$\mathbb{P}^{(r)}_{ij} \equiv 2C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j} \equiv 2\boldsymbol{\ell}^\mu \boldsymbol{m}_i{}^\nu \boldsymbol{\ell}^\alpha \boldsymbol{m}_j{}^\beta C_{\mu\nu\alpha\beta}. \tag{2.21}$$

Then the Weyl tensor $C_{\mu\nu\alpha\beta}$ is at least Petrov type I if there exists a frame $\boldsymbol{\ell}, \boldsymbol{k}, \boldsymbol{m}_i$ such that $\mathbb{P}^{(r)}_{ij} = 0$. In this subsection, we will show that the Weyl tensors $C_{\mu\nu\alpha\beta}$ of the metric $g_{\mu\nu}$ in (2.8) is at least Petrov type I .

A special kind of frame has been chosen in [58]. If we denote $\boldsymbol{n} = \boldsymbol{n}^\mu \partial_\mu$ as the spacelike unit normal vector of a constant r hypersurface, $\boldsymbol{u} = \boldsymbol{u}^\mu \partial_\mu$ is the normalized $(p+2)$ velocity along with the hypersurface, and $\boldsymbol{m}_i = \boldsymbol{m}_i{}^\mu \partial_\mu$ being the remaining orthonormal spatial vectors, then the inverse of the metric (2.8) can be decomposed as

$$g^{\mu\nu} = \boldsymbol{n}^\mu \boldsymbol{n}^\nu - \boldsymbol{u}^\mu \boldsymbol{u}^\nu + \delta^{ij}\boldsymbol{m}_i{}^\mu \boldsymbol{m}_j{}^\nu, \tag{2.22}$$

where $\boldsymbol{n}^\mu$ and $\boldsymbol{u}^\mu$ associated with a constant r hypersurface have been taken as

$$\begin{aligned} \boldsymbol{n}^r &= (g^{rr})^{1/2}, \quad & \boldsymbol{n}^a &= (g^{rr})^{-1/2} g^{ra}, \\ \boldsymbol{u}^r &= 0, & \boldsymbol{u}^a &= \boldsymbol{n}^a. \end{aligned} \tag{2.23}$$

Considering the fact that $m_i{}^a m^i{}_b = h^a_b = \delta^a_b + u^a u_b$, where

$$\begin{aligned} m_i{}^a &= \delta_i{}^a + r_c^{-1/2} u_i \delta^a_\tau + (1 + r_c^{1/2}\gamma_v)^{-1} u_i u^j \delta^a_j, \\ m^i{}_a &= \delta^i{}_a - r_c^{+1/2} u^i \delta^\tau_a + (1 + r_c^{1/2}\gamma_v)^{-1} u^i u_j \delta^j_a, \end{aligned} \tag{2.24}$$

we can fix the freedom of $\boldsymbol{m}_i{}^\mu$ through choosing them as

$$\boldsymbol{m}_i{}^r = 0, \quad \boldsymbol{m}_i{}^a = m_i{}^a - \frac{1}{2} m_i{}^b g^{(2)}_{bc} h^{ca}, \tag{2.25}$$

which satisfy $g_{\mu\nu}\boldsymbol{m}^\mu_i \boldsymbol{m}^\nu_j = \delta_{ij}$ up to the order ∂^2.

Further one can construct the two null vector fields as the combinations of $\boldsymbol{n}$ and $\boldsymbol{u}$ as

$$\sqrt{2}\boldsymbol{\ell} = -\boldsymbol{n} + \boldsymbol{u}, \quad \sqrt{2}\boldsymbol{k} = -\boldsymbol{n} - \boldsymbol{u}. \tag{2.26}$$

Then the metric (2.8) as well as its inverse (2.16) can be decomposed as

$$g_{\mu\nu} = 2\boldsymbol{\ell}_{(\mu}\boldsymbol{k}_{\nu)} + \delta_{ij}\boldsymbol{m}^i{}_\mu \boldsymbol{m}^j{}_\nu, \quad g^{\mu\nu} = 2\boldsymbol{\ell}^{(\mu}\boldsymbol{k}^{\nu)} + \delta^{ij}\boldsymbol{m}_i{}^\mu \boldsymbol{m}_j{}^\nu. \tag{2.27}$$

Concretely, the components of the frame with superscript index are given as follows.

$$\begin{aligned} \sqrt{2}\boldsymbol{\ell}^\mu &= -\boldsymbol{n}^r \delta^\mu_r - (\boldsymbol{n}^a - \boldsymbol{u}^a)\delta^\mu_a = -(g^{rr})^{1/2}\delta^\mu_r, \\ \sqrt{2}\boldsymbol{k}^\mu &= -\boldsymbol{n}^r \delta^\mu_r - (\boldsymbol{n}^a + \boldsymbol{u}^a)\delta^\mu_a = -(g^{rr})^{1/2}\delta^\mu_r - 2(g^{rr})^{-1/2} g^{ra}\delta^\mu_a, \\ \boldsymbol{m}_i{}^\mu &= \boldsymbol{m}_i{}^a \delta^\mu_a = \left(m_i{}^a - \frac{1}{2} m_i{}^b g^{(2)}_{bc} h^{ca} \right)\delta^\mu_a. \end{aligned} \tag{2.28}$$

And the components with subscript index are given by

$$\sqrt{2}\boldsymbol{\ell}_\mu = -(\boldsymbol{n}_r - \boldsymbol{u}_r)\delta^r_\mu + \boldsymbol{u}_a \delta^a_\mu = (g^{rr})^{1/2} \mathrm{p} u_a \delta^a_\mu,$$

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$$\sqrt{2}\boldsymbol{k}_\mu = -(\boldsymbol{n}_r + \boldsymbol{u}_r)\delta^r_\mu - \boldsymbol{u}_a\delta^a_\mu = -2(g^{rr})^{-1/2}\delta^r_\mu - (g^{rr})^{1/2}\mathrm{p}u_a\delta^a_\mu,$$
$$\boldsymbol{m}_\mu{}^i = \left[m_a{}^i + u_a u^b(g^{(1)}_{bc} + g^{(2)}_{bc})h^{cd}m_d{}^i - \frac{1}{2}h^b_a g^{(2)}_{bc}h^{cd}m_d{}^i\right]\delta^a_\mu. \tag{2.29}$$

To check the Petrov type I condition $\mathbb{P}^{(r)}_{ij} = 0$ of the Weyl tensor of the solution we introduce another covariant formula $\mathbb{P}^{(r)}_{ab}$, which is defined as

$$\mathbb{P}^{(r)}_{ab} \equiv 2h^c_a h^d_b C_{(\boldsymbol{\ell})c(\boldsymbol{\ell})d} = \boldsymbol{n}^r h^c_a \boldsymbol{n}^r h^d_b C_{rcrd}, \quad \mathbb{P}^{(r)}_{ij} = \boldsymbol{m}_i{}^a \boldsymbol{m}_j{}^b \mathbb{P}^{(r)}_{ab}. \tag{2.30}$$

Then after a straightforward calculation of the Weyl tensors with metric (2.8), we find

$$\mathbb{P}^{(r)}_{ab} = -g^{rr}\left(\frac{1}{2}h^c_a h^d_b \partial^2_r g^{(2)}_{cd} + \mathrm{p}^2\Omega_{ac}\Omega^c{}_b\right) + O(\partial^3). \tag{2.31}$$

Considering $g^{(2)}_{cd}$ with Gauss-Bonnet corrections in (2.14), we can conclude that $\mathbb{P}^{(r)}_{ab} = O(\partial^3)$ at arbitrary r, which also indicates $\mathbb{P}^{(r)}_{ij} = O(\partial^3)$ at every spacetime point in (2.8). As a result, we have shown that the Weyl tensor or the spacetime with metric (2.8) is at least Petrov type I up to ∂^2, even when the Gauss-Bonnet term is included.

3 Petrov type I condition on the hypersurface Σ_c

The Petrov type I condition is introduced to reduce the degrees of freedom in the extrinsic curvature of the hypersurface Σ_c to the degrees of freedom in the dual fluid on Σ_c in [50]. On a hypersurface with intrinsic metric γ_{ab}, the covariant Petrov type I condition is defined as [58],

$$\mathbb{P}_{ab} \equiv \mathbb{P}^{(r_c)}_{ab} = 2h^c_a h^d_b C_{(\boldsymbol{\ell})c(\boldsymbol{\ell})d}|_{\Sigma_c} = 0, \tag{3.1}$$

where $h^a_b \equiv \gamma^a_b + u^a u_b$ and $u^a \equiv \boldsymbol{u}^a|_{r=r_c}$. With (2.26), we have

$$2C_{(\boldsymbol{\ell})c(\boldsymbol{\ell})d} = C_{(\boldsymbol{u})c(\boldsymbol{u})d} - C_{(\boldsymbol{u})c(\boldsymbol{n})d} - C_{(\boldsymbol{u})d(\boldsymbol{n})c} + C_{(\boldsymbol{n})c(\boldsymbol{n})d}, \tag{3.2}$$

where the subscript indexes $(\boldsymbol{u})$ and $(\boldsymbol{n})$ denote contractions with the vectors $\boldsymbol{u}^\mu$ and $\boldsymbol{n}^\mu$ introduced in (2.23), respectively. We need to rewrite the Weyl tensor in terms of the extrinsic curvature K_{ab}, through using the Gauss-Codazzi equations on the intrinsic flat hypersurface Σ_c. Thus, we firstly define the following hypersurface quantities

$$M_{abcd} \equiv \gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c\gamma^\delta_d R_{\alpha\beta\gamma\delta} = K_{ad}K_{bc} - K_{ac}K_{bd},$$
$$N_{abc} \equiv \gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c n^\delta R_{\alpha\beta\gamma\delta} = \partial_a K_{bc} - \partial_b K_{ac},$$
$$Y_{ab} \equiv \gamma^\alpha_a n^\beta\gamma^\gamma_b n^\delta R_{\alpha\beta\gamma\delta} = KK_{ab} - K_{ac}K^c{}_b + \gamma^\alpha_a\gamma^\beta_b R_{\alpha\beta}, \tag{3.3}$$

where $\alpha, \beta, \ldots$ are the bulk indexes and $a, b, \ldots$ are hypersurface indexes. Associated with the hypersurface Σ_c, $n^\alpha \equiv \boldsymbol{n}^\alpha|_{r=r_c}$ is the unit normal vector, and γ^α_a are the remaining projection vectors. In our coordinate system, $\gamma^\alpha_a \equiv \frac{\partial x^\alpha}{\partial x^a} = \delta^\alpha_a$ are identical to the $a-$components of the projection tensor $\gamma^\alpha_\beta \equiv \delta^\alpha_\beta - n^\alpha n_\beta$. The flat induced metric γ_{ab} is related to the bulk metric through $\gamma_{ab} \equiv g_{\alpha\beta}\gamma^\alpha_a\gamma^\beta_b|_{r=r_c}$. Thus we can obtain

$$M_{ac} \equiv \gamma^{bd}M_{abcd} = K_{ab}K^b{}_c - KK_{ac}, \qquad N_b \equiv \gamma^{ac}N_{abc} = \partial_a\left(K^a{}_b - K\delta^a{}_b\right),$$
$$M \equiv \gamma^{ac}M_{ac} = K_{ab}K^{ab} - K^2, \qquad Y \equiv \gamma^{ac}Y_{ac} = -M + \gamma^{\alpha\beta}R_{\alpha\beta}. \tag{3.4}$$

Then using the equations of motion (2.2) which lead to

$$R_{\mu\nu} = -\frac{2}{p}\alpha H g_{\mu\nu} - 2\alpha H_{\mu\nu}, \quad R = \frac{4}{p}\alpha H, \quad H \equiv H_{\mu\nu}g^{\mu\nu}, \tag{3.5}$$

we can obtain the projections of the Weyl tensor on the hypersurface Σ_c,

$$\begin{aligned}
\gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c\gamma^\delta_d C_{\alpha\beta\gamma\delta} &= M_{abcd} - \frac{8\alpha H}{p(p+1)}\gamma_{a[c}\gamma_{d]b} + \alpha\frac{4}{p}\gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c\gamma^\delta_d(g_{\alpha[\gamma}H_{\delta]\beta} - g_{\beta[\gamma}H_{\delta]\alpha}),\\
\gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c n^\delta C_{\alpha\beta\gamma\delta} &= N_{abc} + \alpha\frac{4}{p}\gamma^\alpha_a\gamma^\beta_b\gamma^\gamma_c n^\delta(g_{\alpha[\gamma}H_{\delta]\beta} - g_{\beta[\gamma}H_{\delta]\alpha}),\\
\gamma^\alpha_a n^\beta\gamma^\gamma_c n^\delta C_{\alpha\beta\gamma\delta} &= Y_{ac} - \frac{4\alpha H}{p(p+1)}\gamma_{ac} + \alpha\frac{4}{p}\gamma^\alpha_a n^\beta\gamma^\gamma_c n^\delta\left(g_{\alpha[\gamma}H_{\delta]\beta} - g_{\beta[\gamma}H_{\delta]\alpha}\right).
\end{aligned} \tag{3.6}$$

This is similar to the derivation in [53] for the case of Einstein gravity with matter. Then put (3.6) into (3.1) and consider (3.2), we obtain $\mathbb{P}_{ab} = \mathbb{P}^{(\alpha)}_{ab} + \delta\mathbb{P}^{(H)}_{ab}$, where

$$\mathbb{P}^{(\alpha)}_{ab} \equiv M^\perp_{(u)a(u)b} + 2N^\perp_{(u)(ab)} - M^\perp_{ab}, \tag{3.7}$$

$$\delta\mathbb{P}^{(H)}_{ab} \equiv -2\alpha H^\perp_{ab} + 2\alpha p^{-1}\left[H_{(n)(n)} - 2H_{(n)(u)} + H_{(u)(u)} + H\right]h_{ab}. \tag{3.8}$$

For convenience, we here have defined

$$M^\perp_{(u)a(u)b} = h^m_a h^n_b M_{cmdn}u^c u^d, \quad N^\perp_{(u)(ab)} = h^m_{(a}h^n_{b)}N_{cmn}u^c, \quad M^\perp_{ab} = h^m_a h^n_b M_{mn}, \tag{3.9}$$

as well as

$$\begin{aligned}
H^\perp_{ab} &\equiv H_{\mu\nu}\gamma^\mu_c\gamma^\nu_d h^c_a h^d_b, & H_{(n)(n)} &\equiv H_{\mu\nu}n^\mu n^\nu,\\
H_{(u)(u)} &\equiv H_{\mu\nu}\gamma^\mu_a\gamma^\nu_b u^a u^b, & H_{(n)(u)} &\equiv H_{\mu\nu}n^\mu\gamma^\nu_b u^b.
\end{aligned} \tag{3.10}$$

On the other hand, the Hamiltonian constraint for vacuum Einstein-Gauss-Bonnet field equations (2.2) is

$$\mathbb{H} \equiv -2(G_{\mu\nu} + 2\alpha H_{\mu\nu})n^\mu n^\nu = 0. \tag{3.11}$$

With the decomposition of the Riemann tensor in appendix B, we obtain $\mathbb{H} = \mathbb{H}^{(\alpha)} + \delta\mathbb{H}^{(H)}$, where [64]

$$\mathbb{H}^{(\alpha)} \equiv M, \quad \delta\mathbb{H}^{(H)} \equiv \alpha\left(M^2 - 4M_{ab}M^{ab} + M_{abcd}M^{abcd}\right), \tag{3.12}$$

while the momentum constraint for the equations of motion (2.2) turns out to be

$$\partial^a T^{(GB)}_{ab} \equiv -2(E_{\mu\nu} + 2\alpha H_{\mu\nu})n^\mu\gamma^\nu_b = 0, \tag{3.13}$$

where $T^{(GB)}_{ab}$ is the one given in (2.6).

Notice that $\mathbb{P}^{(\alpha)}_{ab}$ in (3.7) is a hypersurface function of extrinsic curvature K_{ab}, but it is not true for $\delta\mathbb{P}^{(H)}_{ab}$ in (3.8). For example, we can see from [64] that the term

$$Y_{ab} = -M_{ab} + \gamma^\mu_a\gamma^\nu_b R_{\mu\nu} = -\mathscr{L}_n K_{ab} + K_{ac}K^c{}_b \tag{3.14}$$

appears in $2\alpha H^\perp_{ab}$, thus Y_{ab} can not be obtained only from the extrinsic curvature K_{ab} and other intrinsic quantities, because additional information for the bulk such as $R_{\mu\nu}$, or the analytic continuation of K_{ab} out of the hypersurface along n is needed. Thus unlike the case in the Einstein gravity, the goal that Petrov type I condition gives constraints to the extrinsic curvature can not be realized in the case with the Gauss-Bonnet term. However, if we consider the linear approximation of Gauss-Bonnet parameter α, and take the Petrov type I condition up to the linear order of α, the above difficulty can be solved.

To see this, we first define all the quantities with bars have the same expressions as those without bars when $\alpha = 0$. Then put (3.5) into (3.14) and (2.3), we obtain $\bar{Y}_{ab} = -\bar{M}_{ab}$, as well as

$$H_{\mu\nu} = \bar{H}_{\mu\nu} + O(\alpha), \quad \bar{H}_{\mu\nu} \equiv \bar{R}_{\mu}{}^{\sigma\lambda\rho}\bar{R}_{\nu\sigma\lambda\rho} - \frac{1}{4}\left(\bar{R}^{\kappa\sigma\lambda\rho}\bar{R}_{\kappa\sigma\lambda\rho}\right)\bar{g}_{\mu\nu}. \tag{3.15}$$

With the calculations in appendix B, the equation (3.8) becomes $\delta\mathbb{P}^{(H)}_{ab} = \delta\bar{\mathbb{P}}^{(H)}_{ab} + O(\alpha^2)$, where $\delta\bar{\mathbb{P}}^{(H)}_{ab}$ is the linear order term of α as

$$\begin{aligned}\delta\bar{\mathbb{P}}^{(H)}_{ab} &\equiv -2\alpha\bar{H}^{\perp}_{ab} + 2\alpha p^{-1}h_{ab}\left[\bar{H}_{(n)(n)} - 2\bar{H}_{(n)(u)} + \bar{H}_{(u)(u)} + \bar{H}\right] &(3.16)\\ &= -2\alpha h^m_a h^n_b\left(\bar{M}_m{}^{cde}\bar{M}_{ncde} + 2\bar{N}_m{}^{cd}\bar{N}_{ncd} + \bar{N}^{cd}{}_m\bar{N}_{cdn} + 2\bar{M}_m{}^d\bar{M}_{nd}\right)\\ &\quad + \alpha p^{-1}h_{ab}\Big[2\left(\bar{M}_{(u)}{}^{cde}\bar{M}_{(u)cde} + 2\bar{N}_{(u)}{}^{cd}\bar{N}_{(u)cd} + \bar{N}^{cd}{}_{(u)}\bar{N}_{cd(u)} + 2\bar{M}_{(u)}{}^d\bar{M}_{(u)d}\right)\\ &\quad + 4(\bar{M}_{(u)cde}\bar{N}^{dec} - 2\bar{M}^{cd}\bar{N}_{(u)cd}) + \left(\bar{M}^{cdef}\bar{M}_{cdef} + 6\bar{N}^{cde}\bar{N}_{cde} + 8\bar{M}^{cd}\bar{M}_{cd}\right)\Big]. &(3.17)\end{aligned}$$

Now we can say that $\delta\bar{\mathbb{P}}^{(H)}_{ab}$ is a function of K_{ab}, γ_{ab} as well as u_a. On the other hand, notice that the extrinsic curvature K_{ab} can be decomposed as

$$K_{ab} = \bar{K}_{ab} + \delta K^{(\alpha)}_{ab} + O(\alpha^2), \tag{3.18}$$

where $\bar{K}_{ab}$ is the contribution from vacuum Einstein gravity, and $\delta K^{(\alpha)}_{ab}$ includes the linear order terms from the Gauss-Bonnet parameter α. Then from (3.7) we have $\mathbb{P}^{(\alpha)}_{ab} = \bar{\mathbb{P}}_{ab} + \delta\mathbb{P}^{(\alpha)}_{ab} + O(\alpha^2)$, where

$$\bar{\mathbb{P}}_{ab} \equiv \bar{M}^{\perp}_{(u)a(u)b} + 2\bar{N}^{\perp}_{(u)(ab)} - \bar{M}^{\perp}_{ab}, \tag{3.19}$$

$$\delta\mathbb{P}^{(\alpha)}_{ab} = \delta M^{\perp(\alpha)}_{(u)a(u)b} + 2\delta N^{\perp(\alpha)}_{(u)(ab)} - \delta M^{\perp(\alpha)}_{ab}. \tag{3.20}$$

Finally, the covariant Petrov type I condition (3.1) up to the linear order terms of α becomes

$$\mathbb{P}_{ab} \equiv \bar{\mathbb{P}}_{ab} + \delta\mathbb{P}^{(\alpha)}_{ab} + \delta\bar{\mathbb{P}}^{(H)}_{ab} = 0. \tag{3.21}$$

Similarly, the Hamiltonian constraint (3.11) up to the linear order terms of α becomes,

$$\mathbb{H} = \bar{\mathbb{H}} + \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0, \tag{3.22}$$

where

$$\bar{\mathbb{H}} \equiv \bar{M}, \qquad \delta\mathbb{H}^{(\alpha)} \equiv \delta M^{(\alpha)}, \tag{3.23}$$

$$\delta\bar{\mathbb{H}}^{(H)} \equiv \alpha\left(\bar{M}^2 - 4\bar{M}_{ab}\bar{M}^{ab} + \bar{M}_{abcd}\bar{M}^{abcd}\right). \tag{3.24}$$

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With the expansion of K_{ab} in (3.18), the Brown-York stress tensor (2.6) can also be expanded as

$$T_{ab}^{(GB)} \equiv \bar{T}_{ab} + \delta T_{ab} + O(\alpha^2), \tag{3.25}$$

$$\bar{T}_{ab} \equiv -2(\bar{K}_{ab} - \bar{K}\gamma_{ab}), \qquad \delta T_{ab} = \delta T_{ab}^{(\alpha)} + \delta \bar{T}_{ab}^{(J)}, \tag{3.26}$$

where $\bar{T}_{ab}$ is just the Brown-York stress tensor of Einstein gravity, and δT_{ab} comes from the linear order Gauss-Bonnet terms,

$$\delta T_{ab}^{(\alpha)} \equiv -2\left(\delta K_{ab}^{(\alpha)} - \delta K^{(\alpha)}\gamma_{ab}\right), \quad \delta \bar{T}_{ab}^{(J)} \equiv -4\alpha(3\bar{J}_{ab} - \bar{J}\gamma_{ab}). \tag{3.27}$$

In the following section, with the Petrov type I condition (3.21) and Hamiltonian constraint (3.22), as well as the stress tensor (3.25), we will directly recover the stress tensor (2.18) of Rindler fluid in the vacuum Einstein-Gauss-Bonnet gravity.

Notice that in the Einstein gravity, $\bar{K}_{ab}$ can be expressed in terms of its Brown-York stress tensor through $\bar{T}_{ab} = 2(\bar{K}\gamma_{ab} - \bar{K}_{ab})$. But if we consider the Gauss-Bonnet corrections in (2.6), as the cube terms of K_{ab} appear in J_{ab}, one cannot obtain the extrinsic curvature K_{ab} in terms of the stress tensor $T_{ab}^{(GB)}$ in (3.25) at finite α. But, up to the linear order terms of α, we can have from (3.25) that

$$2\bar{K}_{ab} = -\bar{T}_{ab} + p^{-1}\bar{T}\gamma_{ab}, \tag{3.28}$$

$$2\delta K_{ab}^{(\alpha)} = -\delta T_{ab} + p^{-1}\delta T\gamma_{ab} - 4\alpha\left(3\bar{J}_{ab} - 2p^{-1}\bar{J}\gamma_{ab}\right), \tag{3.29}$$

such that the Petrov type I condition on the hypersurface can also be expressed in terms of the Brown-York stress tensor in Einstein-Gauss-Bonnet gravity $T_{ab}^{(GB)} = \bar{T}_{ab} + \delta T_{ab}$. The spirit for the formulas (3.28) and (3.29) in terms of the stress tensor is in accord with the original goal of the Petrov type I condition introduced in [50]. We will also use this strategy to study the Petrov type I condition in the non-relativistic hydrodynamic expansion in section 5.

Let us stress here that in this section, so far we have considered the expansion of extrinsic curvature and other geometric quantities in terms of the Gauss-Bonnet coefficient up to its linear order, and have not introduced any hydrodynamic expansion parameters yet. In the following sections, we will meet the parameter $\partial \sim \varepsilon$ ($v \sim \epsilon$) for the relativistic (non-relativistic) hydrodynamic expansions. The hydrodynamic expansion is independent with the α expansion. Namely, in what follows, we will study the hydrodynamic expansion based on the α expansion.

4 From Petrov type I condition to Rindler fluid

In this section, we will show how to obtain the stress tensor (2.18) by use of the Petrov type I condition and intrinsic flatness on a timelike accelerated hypersurface in the vacuum Einstein-Gauss-Bonnet gravity without using the solution given in (2.8). We also derive the momentum constraint for the equations of motion (2.2). To be clear, we first consider the case with $\alpha = 0$ and obtain the stress tensor for the Rindler fluid in the vacuum Einstein gravity from Petrov type I condition and Hamiltonian constraint, and then obtain the linear order Gauss-Bonnet corrections to the stress tensor in the vacuum Einstein-Gauss-Bonnet gravity case.

4.1 Recover the Rindler fluid in vacuum Einstein gravity

Firstly, setting $\alpha = 0$ in (3.21), we have the Petrov type I condition $\bar{\mathbb{P}}_{ab} = 0$ on the finite cutoff hypersurface Σ_c in the vacuum Einstein gravity, where

$$\bar{\mathbb{P}}_{ab} = \bar{M}^{\perp}_{(u)a(u)b} + 2\bar{N}^{\perp}_{(u)(ab)} - \bar{M}^{\perp}_{ab}. \tag{4.1}$$

Similar to (3.9), we have defined

$$\begin{aligned} \bar{M}^{\perp}_{(u)a(u)b} &= h^m_a h^n_b \left(\bar{K}_{cm}\bar{K}_{dn} - \bar{K}_{cd}\bar{K}_{mn}\right) u^c u^d, \\ \bar{N}^{\perp}_{(u)(ab)} &= h^m_{(a} h^n_{b)} \left(u^c \partial_c \bar{K}_{mn} - u^c \partial_m \bar{K}_{nc}\right), \\ \bar{M}^{\perp}_{ab} &= -h^m_a h^n_b \left(\bar{K}\bar{K}_{mn} - \bar{K}_{mc}\bar{K}^c{}_n\right). \end{aligned} \tag{4.2}$$

On the other hand, from (3.28), we have

$$2\bar{K}_{ab} = -\bar{T}_{ab} + p^{-1}\bar{T}\gamma_{ab}, \qquad 2\bar{K} = p^{-1}\bar{T}. \tag{4.3}$$

Then $\bar{\mathbb{P}}_{ab}$ in (4.1) can be expressed as [58]

$$\begin{aligned} 4\bar{\mathbb{P}}_{ab} = h^m_a h^n_b \big[\left(\bar{T}_{mc}\bar{T}_{nd} - \bar{T}_{mn}\bar{T}_{cd}\right) u^c u^d - \bar{T}_{mc}\bar{T}^c{}_n - 4u^c\partial_c\bar{T}_{mn} + 4u^c\partial_{(m}\bar{T}_{n)c}\big] \\ + p^{-2}\big[\bar{T}(\bar{T} + p\,\bar{T}_{cd}u^c u^d) + 4p\,u^c\partial_c\bar{T}\big] h_{ab}. \end{aligned} \tag{4.4}$$

Now we decompose an arbitrary stress tensor $\bar{T}_{ab}$ associated with a $(p+1)$-velocity u_a as

$$\bar{T}_{ab} = \mathbb{e} u_a u_b + 2\mathbb{j}_{(a}u_{b)} + \Pi_{ab}, \quad \bar{T} = -\mathbb{e} + \Pi, \tag{4.5}$$

where we have defined

$$\mathbb{e} \equiv \bar{T}_{ab}u^a u^b, \quad \mathbb{j}_a \equiv -h^c_a \bar{T}_{cd}u^d, \quad \Pi_{ab} \equiv h^c_a h^d_b \bar{T}_{cd}, \quad \Pi \equiv \Pi_{ab}h^{ab}. \tag{4.6}$$

Substituting (4.5) into (4.4), the Petrov type I condition $\bar{\mathbb{P}}_{ab} = 0$ reads

$$\begin{aligned} 4\bar{\mathbb{P}}_{ab} \equiv -\mathbb{e}\Pi_{ab} + 2\mathbb{j}_a\mathbb{j}_b - \Pi_{ac}\Pi^c{}_b - 8a_{(a}\mathbb{j}_{b)} - 4h^c_a h^d_b D\Pi_{cd} - 4\mathbb{e}\mathcal{K}_{ab} - 4D^{\perp}_{(a}\mathbb{j}_{b)} - 4\Pi_{(a}{}^c D^{\perp}_{b)}u_c \\ + p^{-2}\left[\Pi^2 + (p-2)\mathbb{e}\Pi - (p-1)\mathbb{e}^2 + 4p\,D(\Pi - \mathbb{e})\right] h_{ab}. \end{aligned} \tag{4.7}$$

In the case with $\alpha = 0$, the Hamiltonian constraint $\bar{\mathbb{H}} = 0$ in (3.22) becomes

$$4\bar{\mathbb{H}} \equiv p\bar{T}_{ab}\bar{T}^{ab} - \bar{T}^2 = 2\mathbb{e}\Pi + (p-1)\mathbb{e}^2 - 2p\mathbb{j}_a\mathbb{j}_b h^{ab} + p\Pi_{ab}\Pi^{ab} - \Pi^2. \tag{4.8}$$

Expanding the undetermined stress tensor $\bar{T}_{ab}$ in (4.5) in terms of the derivative expansion $\partial \sim \varepsilon$ as

$$\begin{aligned} \mathbb{e} &= \mathbb{e}^{(0)} + \mathbb{e}^{(1)} + \mathbb{e}^{(2)} + O(\partial^3), \\ \mathbb{j}_a &= \mathbb{j}^{(0)}_a + \mathbb{j}^{(1)}_a + \mathbb{j}^{(2)}_a + O(\partial^3), \end{aligned}$$

$$\Pi_{ab} = \Pi_{ab}^{(0)} + \Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} + O(\partial^3),$$
$$\Pi = \Pi^{(0)} + \Pi^{(1)} + \Pi^{(2)} + O(\partial^3), \tag{4.9}$$

and assuming that the stress tensor at the zeroth order has the same form as that in the Rindler fluid (2.19) with

$$\mathbb{e}^{(0)} = 0, \qquad \mathbb{j}_a^{(0)} = 0, \qquad \Pi_{ab}^{(0)} = \mathbb{p} h_{ab}, \qquad \Pi^{(0)} = p\mathbb{p}, \tag{4.10}$$

which gives the stress tensor for the Rindler fluid dual to the Rindler solution (2.4), one can recover the first and second order terms of total stress tensor (2.18) with $\alpha = 0$, by imposing the Hamiltonian constraint (4.8) and Petrov type I condition (4.7). We fix the fluid frame by defining the relativistic fluid velocity u^a such that $\mathbb{j}_a = u^c \bar{T}_{cd} h_a^d \equiv 0$ at arbitrary orders, and choose the isotropy gauge where there are no higher order corrections to the term proportional to h_{ab}, that is, only $\mathbb{p} h_{ab}$ appears in the stress tensor [46]. Concretely, we go as follows step by step.

i) First order.

We put (4.9) and (4.10) into the Hamiltonian constraint (4.8) and Petrov type I condition (4.7), and then expand them in terms of the derivative expansion. Assuming $\mathbb{j}_a^{(1)} = 0$, at the first order, we have

$$\bar{\mathbb{H}}^{(1)} = 0 \Rightarrow \mathbb{e}^{(1)} = 0, \tag{4.11}$$
$$\bar{\mathbb{P}}_{ab}^{(1)} = 0 \Rightarrow \Pi_{ab}^{(1)} = -2\mathcal{K}_{ab} + p^{-1}\left(\Pi^{(1)} - \mathbb{e}^{(1)}\right) h_{ab}. \tag{4.12}$$

Choosing the isotropy gauge such that $\Pi^{(1)} = \mathbb{e}^{(1)} = 0$, we reach $\Pi_{ab}^{(1)} = -2\mathcal{K}_{ab}$.

ii) Second order.

With the results in the first order, assuming $\mathbb{j}_a^{(2)} = 0$, we can obtain the second order terms through

$$\bar{\mathbb{H}}^{(2)} = 0 \Rightarrow \mathbb{e}^{(2)} = -2\mathbb{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab}, \tag{4.13}$$
$$\bar{\mathbb{P}}_{ab}^{(2)} = 0 \Rightarrow \Pi_{ab}^{(2)} = \mathbb{p}^{-1}\left[2\mathcal{K}_{ac}\mathcal{K}^c{}_b - 4\mathcal{K}_{c(a}\Omega^c{}_{b)} + 4h_a^c h_b^d D\mathcal{K}_{cd}\right] + p^{-1}\left(\Pi^{(2)} - \mathbb{e}^{(2)}\right) h_{ab}. \tag{4.14}$$

Choosing the isotropy gauge such that $\Pi^{(2)} = \mathbb{e}^{(2)} = -2\mathbb{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab}$, and employing the derivatives of momentum constraint equation (2.17) which leads to the identities,

$$h_a^c h_b^d D\mathcal{K}_{cd} = -h_a^c h_b^d \partial_c \partial_d \ln\mathbb{p} - \mathcal{K}_{ab} D\ln\mathbb{p} + D_a^\perp \ln\mathbb{p} D_b^\perp \ln\mathbb{p} - \mathcal{K}_a{}^c\mathcal{K}_{cb} - \Omega_a{}^c\Omega_{cb} + O(\partial^3),$$
$$h^{cd} D\mathcal{K}_{cd} = D\mathcal{K} = O(\partial^3), \tag{4.15}$$

we finally reach the stress tensor up to the second order in the derivative expansion,

$$\bar{T}_{ab} = +\mathbb{p} h_{ab} + \left(\mathbb{e}^{(1)} + \mathbb{e}^{(2)}\right) u_a u_b + \Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} \tag{4.16}$$
$$= +\mathbb{p} h_{ab} - 2\mathcal{K}_{ab} - 2\mathbb{p}^{-1}\left(\mathcal{K}_{cd}\mathcal{K}^{cd}\right) u_a u_b + \mathbb{p}^{-1}\Big[-2\mathcal{K}_{ac}\mathcal{K}^c{}_b - 4\mathcal{K}_{c(a}\Omega^c{}_{b)}$$
$$-4\Omega_{ac}\Omega^c{}_b - 4h_a^c h_b^d \partial_c\partial_d \ln\mathbb{p} - 4\mathcal{K}_{ab} D\ln\mathbb{p} + 4(D_a^\perp \ln\mathbb{p})(D_b^\perp \ln\mathbb{p})\Big]. \tag{4.17}$$

Comparing the above stress tensor $\bar{T}_{ab}$ with the general stress tensor $T^{(R)}_{ab}$ in (2.19), one can read off exactly the same coefficients in (2.20) when $\alpha = 0$. Thus, taking the Hamiltonian constraint and Petrov type I condition, we recover the Brown-York stress tensor (2.18) dual to the bulk metric in (2.8) in the case of Einstein gravity.

4.2 Recover the Rindler fluid in Einstein-Gauss-Bonnet gravity

In this subsection, we will recover the stress tensor of the Rindler fluid dual to the vacuum Einstein-Gauss-Bonnet gravity. Because $\bar{\mathbb{H}} \equiv 0$, we can write the Hamiltonian constraint (3.22) as

$$\mathbb{H} = \mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0, \tag{4.18}$$

where $\mathbb{H}^{(\alpha)}$ and $\delta\bar{\mathbb{H}}^{(H)}$ can be found in (3.12) and (3.24), respectively. Since $\bar{\mathbb{P}}_{ab} \equiv 0$, the Petrov type I condition in (3.21) becomes

$$\mathbb{P}_{ab} = \mathbb{P}^{(\alpha)}_{ab} + \delta\bar{\mathbb{P}}^{(H)}_{ab} = \delta\mathbb{P}^{(\alpha)}_{ab} + \delta\bar{\mathbb{P}}^{(H)}_{ab} = 0, \tag{4.19}$$

where $\mathbb{P}^{(\alpha)}_{ab}$ and $\delta\bar{\mathbb{P}}^{(H)}_{ab}$ can be found in (3.7) and (3.17), respectively. On the other hand, with the results in (4.16), one has from (3.28) that

$$2\bar{K}_{ab} = -(\mathbb{p} + \mathbb{e}^{(2)})u_a u_b - \Pi^{(1)}_{ab} - \Pi^{(2)}_{ab} + O(\partial^3). \tag{4.20}$$

We then assume the following decomposition of the extrinsic curvature

$$K_{ab} = \varrho\, u_a u_b + \pi_{ab}, \qquad \varrho \equiv K_{ab}u^a u^b, \qquad \pi_{ab} \equiv h^c_a h^d_b K_{cd}, \tag{4.21}$$

$$\delta K^{(\alpha)}_{ab} = \delta\varrho^{(\alpha)} u_a u_b + \delta\pi^{(\alpha)}_{ab}, \qquad \delta\varrho^{(\alpha)} \equiv \delta K^{(\alpha)}_{ab} u^a u^b, \qquad \delta\pi^{(\alpha)}_{ab} \equiv h^c_a h^d_b \delta K^{(\alpha)}_{cd}. \tag{4.22}$$

From (3.18), we conclude

$$2\varrho = -\mathbb{p} - \mathbb{e}^{(2)} + 2\delta\varrho^{(\alpha)} + O(\partial^3), \tag{4.23}$$

$$2\pi_{ab} = -\Pi^{(1)}_{ab} - \Pi^{(2)}_{ab} + 2\delta\pi^{(\alpha)}_{ab} + O(\partial^3). \tag{4.24}$$

Putting (4.20) into (3.24) and (3.7), one has

$$\delta\bar{\mathbb{H}}^{(H)} = O(\partial^3), \quad \delta\bar{\mathbb{P}}^{(H)}_{ab} = -6\alpha\mathbb{p}^2 \left[\Omega_{ac}\Omega^c{}_b + p^{-1} h_{ab}\Omega_{cd}\Omega^{cd}\right] + O(\partial^3). \tag{4.25}$$

As the Gauss-Bonnet corrections to Hamiltonian constraint and Petrov type I condition appear at the second order in the derivative expansion, we only need to consider the second order corrections with $\delta\varrho^{(\alpha)} \sim \delta\pi^{(\alpha)}_{ab} \sim O(\partial^2)$. Then put (4.21) into (3.12) and (3.7), we have

$$\mathbb{H}^{(\alpha)} = (2\varrho - \pi)\pi + \pi_{ab}\pi^{ab}, \tag{4.26}$$

$$\mathbb{P}^{(\alpha)}_{ab} = (\pi - 2\varrho)\pi_{ab} - \pi_{ac}\pi^c{}_b + 2\varrho\mathcal{K}_{ab} + 2\mathcal{K}_{(a}{}^c\pi_{b)c} + 2\Omega_{(a}{}^c\pi_{b)c} + 2h^c_a h^d_b D\pi_{cd}. \tag{4.27}$$

Taking into account of (4.23) and (4.24) , up to the linear order terms of α, we obtain

$$\delta\mathbb{H}^{(\alpha)} = \mathbb{H}^{(\alpha)} = -\mathbb{p}\delta\pi^{(\alpha)}, \quad \delta\mathbb{P}^{(\alpha)}_{ab} = \mathbb{P}^{(\alpha)}_{ab} = \mathbb{p}\delta\pi^{(\alpha)}_{ab}. \tag{4.28}$$

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With (4.25) and (4.28), at the second order in the derivative expansion, the Hamiltonian constraint leads to

$$\mathbb{H}^{(2)} = \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0 \;\Rightarrow\; \delta\pi^{(\alpha)} = 0. \tag{4.29}$$

And the Petrov type I condition leads to

$$\mathbb{P}^{(2)}_{ab} = \delta\mathbb{P}^{(\alpha)}_{ab} + \delta\bar{\mathbb{P}}^{(H)}_{ab} = 0 \Rightarrow \delta\pi^{(\alpha)}_{ab} = 6\alpha\mathrm{p}\left[\Omega_{ac}\Omega^{c}{}_{b} + p^{-1}h_{ab}\Omega_{cd}\Omega^{cd}\right]. \tag{4.30}$$

We can see that there is no constraint on $\varrho^{(\alpha)}$ at this order, and it will be determined by the gauge choice of the stress tensor. Then from (3.27), we obtain

$$\delta T^{(\alpha)}_{ab} = -2\delta\pi^{(\alpha)}u_a u_b + 2(\delta\pi^{(\alpha)} - \delta\varrho^{(\alpha)})h_{ab} - 2\delta\pi^{(\alpha)}_{ab}. \tag{4.31}$$

On the other hand, a straightforward calculation from (3.27) and (4.20) gives

$$\delta\bar{T}^{(J)}_{ab} = \alpha\mathrm{p}\left[-\Pi^{(1)}_{ac}\Pi^{c(1)}_{\ \ b} + \frac{1}{2}\left(\Pi^{(1)}_{cd}\Pi^{cd}_{(1)}\right)h_{ab}\right], \tag{4.32}$$

where $\Pi^{(1)}_{ab}$ has been obtained in (4.12). Put them together, we obtain

$$\begin{aligned}\delta T_{ab} = \delta T^{(\alpha)}_{ab} + \delta\bar{T}^{(J)}_{ab} &= -4\alpha\mathrm{p}\left(\mathcal{K}_{ac}\mathcal{K}^{c}{}_{b} + 3p^{-1}\Omega_{ac}\Omega^{c}{}_{b}\right)\\ &\quad + \left[-2\delta\varrho^{(2)} + 2\alpha\mathrm{p}\left(\mathcal{K}_{cd}\mathcal{K}^{cd} - 6p^{-1}\Omega_{cd}\Omega^{cd}\right)\right]h_{ab}.\end{aligned} \tag{4.33}$$

The isotropic gauge of the pressure leads to $\delta\varrho^{(2)} = \alpha\mathrm{p}\left(\mathcal{K}_{cd}\mathcal{K}^{cd} - 6p^{-1}\Omega_{cd}\Omega^{cd}\right)$. Then the stress tensor from Petrov type I condition turns out to be $\bar{T}_{ab} + \delta T_{ab}$ with (4.17) and (4.33), which matches exactly with $T^{(GB)}_{ab}$ in (2.18) from the fluid/gravity duality calculation. Meanwhile, we can see from (3.13) that the conservation of the stress tensor $\partial^a T^{(GB)}_{ab}$ results in the momentum constraint of the vacuum Einstein-Gauss-Bonnet gravity.

5 The non-relativistic hydrodynamic expansion

The Rindler fluid in the vacuum Einstein-Gauss-Bonnet gravity has been studied in [44, 45] with the following non-relativistic hydrodynamic expansion

$$v_i \sim \epsilon, \qquad P \sim \epsilon^2, \qquad \partial_i \sim \epsilon, \qquad \partial_\tau \sim \epsilon^2. \tag{5.1}$$

The dual stress tensor turns out to be $T^{(GB)}_{ab} = \bar{T}_{ab} + \delta T_{ab}$, where $\bar{T}_{ab}$ come from the Einstein sector, which are given by [44],

$$\begin{aligned}\bar{T}^{\tau}{}_{i} &= +r_c^{-3/2}v_i + r_c^{-5/2}\left[v_i(v^2+P) - 2r_c\sigma_{ij}v^j\right] + O(\epsilon^5),\\ \bar{T}^{\tau}{}_{\tau} &= -r_c^{-3/2}v^2 - r_c^{-5/2}\left[v^2(v^2+P) - 2r_c\sigma_{ij}v^iv^j - 2r_c^2\sigma_{ij}\sigma^{ij}\right] + O(\epsilon^6),\\ \bar{T}_{ij} &= +r_c^{-1/2}\delta_{ij} + r_c^{-3/2}\left[P\delta_{ij} + v_iv_j - 2r_c\sigma_{ij}\right]\\ &\quad + r_c^{-5/2}\left[v_iv_j(v^2+P) - r_c\sigma_{ij}v^2 + 2r_cv_{(i}\partial_{j)}P - r_cv_{(i}\partial_{j)}v^2 - 2r_c^2v_{(i}\partial^2v_{j)}\right.\\ &\quad \left. - 2r_c^2\sigma_{ik}\sigma^{k}{}_{j} - 4r_c^2\sigma_{k(i}\omega^{k}{}_{j)} - 4r_c^2\omega_{ik}\omega^{k}{}_{j} - 4r_c^2\partial_i\partial_jP + 3r_c^3\partial^2\sigma_{ij}\right] + O(\epsilon^6),\\ \bar{T} &= \bar{T}^{\tau}{}_{\tau} + \bar{T}^{i}{}_{i} = pr_c^{-1/2} + pr_c^{-3/2}P + O(\epsilon^6).\end{aligned} \tag{5.2}$$

Here the fluid shear $\sigma_{ij} = \partial_{(i}v_{j)}$ and vorticity $\omega_{ij} = \partial_{[i}v_{j]}$. And δT_{ab} come from the Gauss-Bonnet term, with the non-vanishing components [45, 48],

$$\delta T_{ij} = -4\alpha r_c^{-3/2}\left(\sigma_{ik}\sigma^k{}_j + 3\,\omega_{ik}\omega^k{}_j\right) + O(\epsilon^6), \tag{5.3}$$

$$\delta T = \delta^{ij}\delta T_{ij} = -4\alpha r_c^{-3/2}\left(\sigma_{ij}\sigma^{ij} - 3\,\omega_{ij}\omega^{ij}\right) + O(\epsilon^6). \tag{5.4}$$

We can see that the contributions from the Gauss-Bonnet term only appear at order ϵ^4. This comes from the fact that the first non-zero components of the Riemann tensor appear at order ϵ^2 [45]. And notice that the situation for the case of Einstein gravity has been studied in [57]. Thus here we only focus on the linear order Gauss-Bonnet corrections to the Petrov type I condition and Hamiltonian constraint at ϵ^4. Since the nontrivial terms on the expansion of α only appear at the highest order ϵ^4 terms which we are interested in, it is enough to consider the linear order of α in the non-relativistic hydrodynamic expansion, once again.

5.1 Petrov type I condition in non-relativistic hydrodynamic expansion

Introduce the new coordinate $x^0 = \sqrt{r_c}\tau$, the flat induced metric γ_{ab} in (2.5) becomes

$$ds^2_{p+1} = \eta_{ab}dx_a dx^b = -(dx^0)^2 + \delta_{ij}dx^i dx^j. \tag{5.5}$$

The $(p+2)$ Newman-Penrose-like vector fields are given with respect to the ingoing and outgoing pair of null vectors as [50]

$$\sqrt{2}\ell = \partial_0 - n, \quad \sqrt{2}k = -\partial_0 - n, \quad m_i = \partial_i. \tag{5.6}$$

Here n is the unit normal vector of the hypersurface Σ_c, ∂_0 and ∂_i are the tangent vectors to Σ_c. The spacetime is at least Petrov type I if

$$\mathrm{P}_{ij} \equiv 2C_{(\ell)i(\ell)j} = 0, \quad C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta}. \tag{5.7}$$

With the Guass-Codazzi equations given in (3.6), we have the Petrov type I condition up to linear order in the Gauss-Bonnet parameter α as

$$\mathrm{P}_{ij} = \bar{\mathrm{P}}_{ij} + \delta\mathrm{P}^{(\alpha)}_{ij} + \delta\bar{\mathrm{P}}^{(H)}_{ij} = 0, \tag{5.8}$$

$$\bar{\mathrm{P}}_{ij} \equiv -\bar{M}^\perp_{ij} + 2\bar{N}^\perp_{0ij} + \bar{M}^\perp_{0i0j}, \quad \delta\mathrm{P}^{(\alpha)}_{ij} \equiv -\delta M^\perp_{ij} + 2\delta N^\perp_{0ij} + \delta M^\perp_{0i0j}, \tag{5.9}$$

with

$$\delta\bar{\mathrm{P}}^{(H)}_{ij} = -2\alpha\bar{H}^\perp_{ij} + 2\alpha p^{-1}\delta_{ij}\left[\bar{H}_{\mu\nu}n^\mu n^\nu - 2\bar{H}_{0\mu}n^\mu + \bar{H}_{00} + \bar{H}\right] \tag{5.10}$$

$$\begin{aligned} &= -2\alpha\left(\bar{M}_i{}^{cde}\bar{M}_{jcde} + 2\bar{N}_i{}^{cd}\bar{N}_{jcd} + \bar{N}^{cd}{}_i\bar{N}_{cd\,j} + 2\bar{M}_i{}^d\bar{M}_{jd}\right) \\ &\quad + \alpha p^{-1}\delta_{ij}\Big[2\left(\bar{M}_0{}^{cde}\bar{M}_{0cde} + 2\bar{N}_0{}^{cd}\bar{N}_{0cd} + \bar{N}^{cd}{}_0\bar{N}_{cd0} + 2\bar{M}_0{}^d\bar{M}_{0d}\right) \\ &\quad + 4(\bar{M}_{0cde}\bar{N}^{dec} - 2\bar{M}^{cd}\bar{N}_{0cd}) + \left(\bar{M}^{cdef}\bar{M}_{cdef} + 6\bar{N}^{cde}\bar{N}_{cde} + 8\bar{M}^{cd}\bar{M}_{cd}\right)\Big]. \end{aligned} \tag{5.11}$$

The Hamiltonian constraint becomes

$$\mathrm{H} = \bar{\mathrm{H}} + \delta \mathrm{H}^{(\alpha)} + \delta\bar{\mathrm{H}}^{(H)} = 0, \tag{5.12}$$

$$\bar{\mathrm{H}} \equiv \bar{M}, \qquad \delta \mathrm{H}^{(\alpha)} \equiv \delta M, \tag{5.13}$$

with

$$\delta\bar{\mathrm{H}}^{(H)} \equiv -4\alpha \bar{H}_{\mu\nu} n^\mu n^\nu = \alpha\left(-4\bar{M}_{ab}\bar{M}^{ab} + \bar{M}_{abcd}\bar{M}^{abcd}\right). \tag{5.14}$$

Notice that the frame choice in (5.6) singles out a preferred time coordinate ∂_0 and thus breaks Lorentz invariance. It has been shown in [57] that with the frame (5.6), the Petrov type I condition for vacuum Einstein gravity $\bar{\mathrm{P}}_{ij} = 0$ is violated at order ϵ^4:

$$\bar{\mathrm{P}}^{(E)}_{ij} = \bar{\mathrm{P}}_{ij} = \frac{1}{2} r_c^{-3}\left[6 r_c v_k v_{(i}\omega^k{}_{j)} - 2r_c^2 v_{(i}\partial^2 v_{j)} - 4r_c^2 v^k\partial_{(i}\omega^k{}_{j)} + r_c^3\partial^2\sigma_{ij}\right] + O(\epsilon^6). \tag{5.15}$$

However, after straightforward calculations with the stress tensor (5.2) and (5.3), we find

$$\delta\bar{\mathrm{H}}^{(H)} = \delta \mathrm{H}^{(\alpha)} = O(\epsilon^6), \tag{5.16}$$

$$\delta\bar{\mathrm{P}}^{(H)}_{ij} = -\delta \mathrm{P}^{(\alpha)}_{ij} = -6\alpha r_c^{-2}\left(\omega_{ik}\omega^k{}_j + p^{-1}\delta_{ij}\omega_{kl}\omega^{kl}\right) + O(\epsilon^5). \tag{5.17}$$

Thus, we see that there are no Gauss-Bonnet corrections to the Hamiltonian constraint (5.12) and Petrov type I condition (5.8) up to the order ϵ^4 and up to the linear order in α. This implies that adding the Gauss-Bonnet term does not become worse for the violation of the Petrov type I condition up to the order ϵ^4. In the following subsection, we will show that either demand $\bar{\mathrm{P}}_{ij} = 0$ or (5.15) which lead to the stress tensor (5.2) of Rindler fluid in vacuum Einstein gravity, and impose

$$\delta \mathrm{H} = \delta \mathrm{H}^{(\alpha)} + \delta\bar{\mathrm{H}}^{(H)} = 0, \qquad \delta \mathrm{P}_{ij} = \delta \mathrm{P}^{(\alpha)}_{ij} + \delta\bar{\mathrm{P}}^{(H)}_{ij} = 0, \tag{5.18}$$

we can get exactly the contribution (5.3) of the Gauss-Bonnet term to the stress tensor of the dual fluid, without solving the Einstein-Gauss-Bonnet field equations.

5.2 Recover the Gauss-Bonnet corrections

If we pretend the Petrov type I condition $\bar{\mathrm{P}}_{ij} = 0$ holds in vacuum Einstein gravity, it has been shown in [57] that the stress tensor in (5.2) can be recovered up to an additional term at ϵ^4:

$$\delta\bar{T}^{(E)}_{ij} = r_c^{-5/2}\left[6 r_c v_k v_{(i}\omega^k{}_{j)} - 2r_c^2 v_{(i}\partial^2 v_{j)} - 4r_c^2 v^k\partial_{(i}\omega^k{}_{j)} + r_c^3\partial^2\sigma_{ij}\right] + O(\epsilon^6). \tag{5.19}$$

Then using $\bar{T}_{ab} + \delta\bar{T}^{(E)}_{ab}$ instead of $\bar{T}_{ab}$ in (5.2), we can obtain the extrinsic curvature $\bar{K}_{ab}$ from (3.28), and put them into (5.14) and (5.11), which lead to the same results in (5.16) and (5.17). This implies that

$$\delta\bar{\mathrm{H}}^{(H)} = O(\epsilon^6), \tag{5.20}$$

$$\delta\bar{\mathrm{P}}^{(H)}_{ij} = -6\alpha r_c^{-2}\left(\omega_{ik}\omega^k{}_j + p^{-1}\delta_{ij}\omega_{kl}\omega^{kl}\right) + O(\epsilon^5), \tag{5.21}$$

are not affected by the additional term $\delta\bar{T}^{(E)}_{ab}$. To cancel the non-vanishing $\delta\bar{\mathrm{P}}^{(H)}_{ij}$ at order ϵ^4 in (5.21), we assume $\delta T_{ab} \sim O(\epsilon^4)$ such that $\delta\mathrm{H}^{(\alpha)}$ in (5.13) and $\delta\mathrm{P}^{(\alpha)}_{ij}$ in (5.9) also appear at order ϵ^4. As $\bar{T}^{\tau}_{i}$ in (5.2) has been fixed through the frame choice of the velocity [57], we only need to set the Gauss-Bonnet correction $\delta T^{\tau}{}_{i} = O(\epsilon^5)$. Then put the relation (3.29) into (5.13) and (5.9), we obtain

$$\delta\mathrm{H}^{(\alpha)} = \frac{1}{2} r_c^{-1/2}\left[-\delta T^{\tau}{}_{\tau} + 4\alpha\left(\bar{J} - 3\bar{J}^{\tau}{}_{\tau}\right)\right], \tag{5.22}$$

$$\delta\mathrm{P}^{(\alpha)}_{ij} = \frac{1}{2} r_c^{-1/2}\left[-\delta T{}_{ij} - 4\alpha\left(3\bar{J}_{ij} - 2p^{-1}\bar{J}\delta_{ij}\right) + p^{-1}\delta T\delta_{ij}\right]. \tag{5.23}$$

With (2.7), (3.28) and (5.2), we have the non-zero components of $\bar{J}_{ab}$ as

$$\bar{J}^{\tau}{}_{\tau} = \frac{1}{6} r_c^{-3/2}(\sigma_{ij}\sigma^{ij}) + O(\epsilon^6), \quad \bar{J}_{ij} = \frac{1}{3} r_c^{-3/2}\sigma_{ik}\sigma^{k}{}_{j} + O(\epsilon^6), \quad \bar{J} = \bar{J}^{\tau}{}_{\tau} + \bar{J}^{i}{}_{i}\,. \tag{5.24}$$

Substituting them into (5.18), we finally obtain

$$\delta T^{\tau}{}_{\tau} = O(\epsilon^6), \tag{5.25}$$

$$\begin{aligned}\delta T{}_{ij} &= -4\alpha r_c^{-3/2}\left(\sigma_{ik}\sigma^{k}{}_{j} + 3\,\omega_{ik}\omega^{k}{}_{j}\right) \\ &\quad + p^{-1}\left[\delta T + 4\alpha r_c^{-3/2}\left(\sigma_{kl}\sigma^{kl} - 3\,\omega_{kl}\omega^{kl}\right)\right]\delta_{ij} + O(\epsilon^6).\end{aligned} \tag{5.26}$$

After choosing the isotropic gauge such that there are no corrections to the δ_{ij} part of the stress tensor at this order as in [44, 45], we have $\delta T = -4\alpha r_c^{-3/2}\left(\sigma_{ij}\sigma^{ij} - 3\,\omega_{ij}\omega^{ij}\right)$. These results exactly match with the Gauss-Bonnet corrections in the stress tensor of Rindler fluid dual to the vacuum Einstein-Gauss-Bonnet gravity, which are given in (5.3) and (5.4) from the fluid/gravity calculation.

Here we stress that even for the vacuum Einstein gravity, namely for the case with $\alpha = 0$, the deduced stress tensor $\bar{T}_{ab} + \delta\bar{T}^{(E)}_{ab}$ for the fluid by imposing Petrov type I condition and Hamiltonian constraint are different from those from the fluid/gravity duality calculation. The former is named as "Petrov type I fluid" in [57]. The main reason for this is that as checked in [57], the Petrov type I condition is violated at order ϵ^4 for the geometry of the vacuum Einstein gravity in the non-relativistic hydrodynamic expansion. In section 5.1, we have shown that adding the Gauss-Bonnet term, the violation for the condition does not become worse up to the order ϵ^4. Thus, if turn the logic around, imposing the Petrov type I condition which requires (5.15) and (5.18), as well as the Hamiltonian constraint (5.12), we can recover the stress tensor of the fluid dual to the vacuum Einstein-Gauss-Bonnet gravity up to the order ϵ^4 in the non-relativistic hydrodynamic expansion.

6 Conclusion

To summarize, we have checked the Petrov type I condition for the vacuum solutions of Einstein-Gauss-Bonnet gravity in both relativistic and non-relativistic hydrodynamic expansions. With the solution constructed in [48], we have shown that the spacetime is at least Petrov type I up to the second order in the relativistic hydrodynamic expansion. Turn the logic around, assuming the Hamiltonian constraint and Petrov type I condition on a finite cutoff hypersurface, we have shown that the dual stress tensor can be recovered with correct first and second order transport coefficients, up to the linear order of the Gauss-Bonnet coefficient. While in the non-relativistic hydrodynamic expansion [45], although the Petrov type I condition is violated at the order ϵ^4 in the vacuum Einstein gravity [57], we have found that the Gauss-Bonnet term does not contribute to the violation terms in the Petrov type I condition up to ϵ^4. Thus, given the stress tensor of the Rindler fluid in the vacuum Einstein gravity, we have shown that demanding the additional Gauss-Bonnet corrections to the Petrov type I condition and Hamiltonian constraint vanish at the linear order of α, the Gauss-Bonnet corrections to the stress tensor of dual fluid can also be recovered.

Note that one key step in [50, 57, 58] is to substitute the extrinsic curvature K_{ab} in terms of the stress tensor T_{ab} of the dual fluid into the Petrov type I condition. The Hamiltonian constraint gives the equation of state, and the Petrov type I condition leads to the constraints on the stress tensor. When the Gauss-Bonnet corrections or some counter terms appear in the Brown-York stress tensor $T_{ab}^{(BY)}$ [41], this step will increase the complexity. However, we have shown that this step is in fact not necessary in section 4.2. Even for the vacuum Einstein gravity case in section 4.1, once the initial zeroth order expressions have been fixed, we can expand K_{ab} first and obtain its higher order terms through imposing Hamiltonian constraint and Petrov type I condition in the derivative expansion. Then put the resulted K_{ab} into the definition of T_{ab}, we can also reach the final stress tensor directly. In addition, let us stress that writing the Petrov type I condition in terms of K_{ab} would be quite useful and it might be a promising way to build up a connection with the membrane paradigm [41].

In section 5.2, we still express K_{ab} in terms of T_{ab} in order to match the spirit in [50], and to obtain the relation (3.29), we only consider the linear order terms of α. The motivation for the linear approximation of α is to express the Petrov type I condition as a function of extrinsic curvature and other intrinsic quantities on the hypersurface. Only with this approximation we can recover the stress tensor of dual fluid from the Petrov type I condition in both relativistic and non-relativistic hydrodynamic expansions. However, note the fact that the Einstein-Gauss-Bonnet field equations are quasi-linear in terms of α [64, 65], and the dual stress tensor with Gauss-Bonnet corrections in (2.18) only contain linear order terms of α. It is not surprised that we can still recover the stress tensor (2.18) even when we only take into account of the linear order terms of α in the calculation.

So far most of studies on the Petrov type I condition has been focused on the case with asymptotically flat spacetimes. It is quite important to investigate corresponding ones for finite cutoff fluid in asymptotically AdS spacetimes [28, 30]. However, it has been pointed out in [58] that except in the near horizon region, the Petrov type I condition on a finite cutoff surface in asymptotically AdS spacetimes is violated at the first order in derivative expansion. This can also be seen from the result in [56] that the Einstein constraint equations can not be recovered correctly by imposing Petrov type I condition. While in the AdS/CFT correspondence, the regularity condition is necessary for the perturbations, and imposing the Petrov type I condition is mathematically much simpler than directly solving the gravitational field equations in order to find the stress tensor of dual fluid.

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Thus, the Petrov type I condition is expected to be equivalent to the regularity condition on the future horizon of the spacetime [50, 58]. It is quite interesting to further study the role of Perov type I condition in the asymptotically AdS case.

On the other hand, the KSS bound [5] states that the universal value of the ratio of shear viscosity over entropy density from the AdS/CFT calculation is always above $\eta/s = 1/4\pi$, while in the AdS gravity with curvature squared corrections, the bound is found to be violated by the Gauss-Bonnet term [61–63]. With the static black brane solution in [66], it has been shown that the universal value $\eta(r_c)/s(r_c) = \left[1 - 2(p+1)(p-2)\alpha/L^2\right]/4\pi$ does not run with the finite cutoff surface [28]. In a forthcoming work, we will show that this ratio at the horizon $\eta(r_h)/s(r_h)$ can also be recovered through imposing Petrov type I condition on the dual fluid on a finite cutoff hypersurface in the near horizon limit.

What's more, it would be much more interesting if one could find a system where the Petrov type I condition and all the gravitational field equations are compatible to arbitrary order away from the cutoff surface.

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A Classification of the Weyl tensor

In a four dimensional spacetime, tensor classification plays an important role in studying the exact solutions of Einstein field equations [67]. And in particular, the Petrov type classification of Weyl tensor has interesting physical applications. It has been generalized to the arbitrarily higher dimensional spacetimes in [68]. In this appendix, we briefly summarize these results based on [69, 70], which can also be reduced to the Petrov type classification in four dimensions.

Consider a $p+2$ dimensional Lorentz manifold ($p \geq 2$) with signature $(- + \ldots +)$ and choose a null frame $\boldsymbol{\ell}$, $\boldsymbol{k}$, $\boldsymbol{m}_i$, which satisfies the following orthogonal and normalization conditions

$$\boldsymbol{\ell}^2 = \boldsymbol{k}^2 = 0, \quad (\boldsymbol{k}, \boldsymbol{\ell}) = 1, \quad (\boldsymbol{m}_i, \boldsymbol{k}) = (\boldsymbol{m}_i, \boldsymbol{\ell}) = 0, \quad (\boldsymbol{m}_i, \boldsymbol{m}_j) = \delta_{ij}, \tag{A.1}$$

so that in this frame the metric of the manifold can be decomposed as

$$g_{\mu\nu} = 2\boldsymbol{\ell}_{(\mu}\boldsymbol{k}_{\nu)} + \delta_{ij}\boldsymbol{m}^i{}_\mu \boldsymbol{m}^j{}_\nu, \qquad g^{\mu\nu} = 2\boldsymbol{\ell}^{(\mu}\boldsymbol{k}^{\nu)} + \delta^{ij}\boldsymbol{m}_i{}^\mu \boldsymbol{m}_j{}^\nu. \tag{A.2}$$

The null frame is covariant under the following boost transformation,

$$\boldsymbol{\ell} \to \lambda\boldsymbol{\ell}, \quad \boldsymbol{k} \to \lambda^{-1}\boldsymbol{k}, \quad \boldsymbol{m}_i \to \boldsymbol{m}_i, \quad \lambda \neq 0. \tag{A.3}$$

For a rank q tensor T on the manifold, its components $T_{\mu_1\ldots\mu_q}$ with fixed list of indices are null frame scalars, and they transform under the boost transformation as

$$T_{\mu_1\ldots\mu_q} \to \lambda^{b_{\{\mu\}}} T_{\mu_1\ldots\mu_q}, \quad b_{\{\mu\}} = b_{\mu_1} + \ldots + b_{\mu_q},\ b_{(\boldsymbol{\ell})} = 1,\ b_i = 0,\ b_{(\boldsymbol{k})} = -1. \tag{A.4}$$

b is named as the boost-weight of the null-frame scalar $T_{\mu_1\ldots\mu_q}$. The boost order (along $\boldsymbol{\ell}$) of the tensor T is defined to be the largest value of $b_{\{\mu\}}$ among all the non-vanishing components $T_{\mu_1\ldots\mu_q}$. It is only a function of the null direction $\boldsymbol{\ell}$ and is denoted as $\mathcal{B}(\boldsymbol{\ell})$.

The Weyl tensor can be decomposed and sorted by the boost weight of its components,

$$C_{\alpha\beta\gamma\delta} = C^{[2]}_{\alpha\beta\gamma\delta} + C^{[1]}_{\alpha\beta\gamma\delta} + C^{[0]}_{\alpha\beta\gamma\delta} + C^{[-1]}_{\alpha\beta\gamma\delta} + C^{[-2]}_{\alpha\beta\gamma\delta}, \tag{A.5}$$

where the superscript index indicates the boost weight and

$$\begin{aligned}
C^{[2]}_{\alpha\beta\gamma\delta} &= 4C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta\boldsymbol{k}_\gamma\boldsymbol{m}^j{}_{\delta\}},\\
C^{[1]}_{\alpha\beta\gamma\delta} &= 8C_{(\boldsymbol{\ell})(\boldsymbol{k})(\boldsymbol{\ell})i}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta\boldsymbol{k}_\gamma\boldsymbol{m}^i{}_{\delta\}} + 4C_{(\boldsymbol{\ell})ijk}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta\boldsymbol{m}^j{}_\gamma\boldsymbol{m}^k{}_{\delta\}},\\
C^{[0]}_{\alpha\beta\gamma\delta} &= 4C_{(\boldsymbol{\ell})(\boldsymbol{k})(\boldsymbol{\ell})(\boldsymbol{k})}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta\boldsymbol{k}_\gamma\boldsymbol{\ell}_{\delta\}} + 4C_{(\boldsymbol{\ell})(\boldsymbol{k})ij}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta\boldsymbol{m}^i{}_\gamma\boldsymbol{m}^j{}_{\delta\}}\\
&\qquad + 8C_{(\boldsymbol{\ell})i(\boldsymbol{k})j}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta\boldsymbol{\ell}_\gamma\boldsymbol{m}^j{}_{\delta\}} + C_{ijkl}\boldsymbol{m}^i_{\{\alpha}\boldsymbol{m}^j{}_\beta\boldsymbol{m}^k{}_\gamma\boldsymbol{m}^l{}_{\delta\}},\\
C^{[-1]}_{\alpha\beta\gamma\delta} &= 8C_{(\boldsymbol{k})(\boldsymbol{\ell})(\boldsymbol{k})i}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{k}_\beta\boldsymbol{\ell}_\gamma\boldsymbol{m}^i{}_{\delta\}} + 4C_{(\boldsymbol{k})ijk}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{m}^i{}_\beta\boldsymbol{m}^j{}_\gamma\boldsymbol{m}^k{}_{\delta\}},\\
C^{[-2]}_{\alpha\beta\gamma\delta} &= 4C_{(\boldsymbol{k})i(\boldsymbol{k})j}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{m}^i{}_\beta\boldsymbol{\ell}_\gamma\boldsymbol{m}^j{}_{\delta\}}.
\end{aligned} \tag{A.6}$$

The notations $T_{\{\alpha\beta\gamma\delta\}} \equiv (T_{[\alpha\beta][\gamma\delta]} + T_{[\gamma\delta][\alpha\beta]})/2$, as well as $C_{(\boldsymbol{\ell})i(\boldsymbol{k})j} \equiv C_{\mu\alpha\nu\beta}\boldsymbol{\ell}^\mu\boldsymbol{m}_j{}^\alpha\boldsymbol{k}^\nu\boldsymbol{m}_j{}^\beta$ and so on, have been introduced. The Weyl tensor is generically of boost order $\mathcal{B}(\boldsymbol{\ell}) = 2$, and a null vector $\boldsymbol{\ell}$ is defined to be aligned with the Weyl tensor whenever $\mathcal{B}(\boldsymbol{\ell}) \le 1$. In this case, $\boldsymbol{\ell}$ is a Weyl aligned null direction, and $1 - \mathcal{B}(\boldsymbol{\ell}) \in \{0, 1, 2, 3\}$ is the order of alignment. It usually depends on the rank and symmetry properties of the tensors.

According to [68], the principal type of the Weyl tensor in a Lorentzian manifold is I, II, III, N according to whether there exists an aligned $\boldsymbol{\ell}$ of alignment order $0, 1, 2, 3$, respectively. If no aligned $\boldsymbol{\ell}$ exists, the manifold is of (general) type G, if the Weyl tensor vanishes the manifold is of type O. The algebraically special types with necessary condition are summarized as follows:

$$\begin{aligned}
&\text{Type I}: && C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j} = 0,\\
&\text{Type II}: && C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j} = C_{(\boldsymbol{\ell})ijk} = 0,\\
&\text{Type III}: && C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j} = C_{(\boldsymbol{\ell})ijk} = C_{ijkl} = C_{(\boldsymbol{\ell})(\boldsymbol{k})ij} = 0,\\
&\text{Type N}: && C_{(\boldsymbol{\ell})i(\boldsymbol{\ell})j} = C_{(\boldsymbol{\ell})ijk} = C_{ijkl} = C_{(\boldsymbol{\ell})(\boldsymbol{k})ij} = C_{(\boldsymbol{k})ijk} = 0.
\end{aligned} \tag{A.7}$$

Following the curvature tensor symmetries and the trace-free condition [69], one can reach some familiar Petrov types with the following properties,

$$\begin{aligned}
&\text{Type I}: && C^{[2]}_{\alpha\beta\gamma\delta} = 0,\\
&\text{Type II}: && C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = 0,\\
&\text{Type D}: && C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = C^{[-2]}_{\alpha\beta\gamma\delta} = 0,
\end{aligned}$$

$$
\begin{aligned}
&\text{Type III}: \ C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = 0, \\
&\text{Type N}: \ C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = 0, \\
&\text{Type O}: \ C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = C^{[-2]}_{\alpha\beta\gamma\delta} = 0.
\end{aligned} \tag{A.8}
$$

Further classifications in more detail can be found in [69, 70].

B Decomposition of the Riemann tensor

The Riemann tensor and its contractions can be decomposed along and perpendicular to a spacelike unit normal vector n,

$$
\begin{aligned}
g^{\alpha}_{\mu} g^{\beta}_{\nu} g^{\gamma}_{\sigma} g^{\delta}_{\lambda} R_{\alpha\beta\gamma\delta} = \ & M_{\mu\nu\sigma\lambda} - n_\mu N_{\sigma\lambda\nu} + n_\nu N_{\sigma\lambda\mu} - n_\sigma N_{\mu\nu\lambda} + n_\lambda N_{\mu\nu\sigma} \\
& + n_\mu n_\sigma Y_{\nu\lambda} - n_\mu n_\lambda Y_{\nu\sigma} + n_\nu n_\lambda Y_{\mu\sigma} - n_\nu n_\sigma Y_{\mu\lambda}, \\
g^{\alpha}_{\mu} g^{\beta}_{\nu} R_{\alpha\beta} = \ & M_{\mu\nu} + n_\mu N_\nu + n_\nu N_\mu + Y_{\mu\nu} + n_\mu n_\nu Y, \\
R = \ & M + 2Y = -M + 2\gamma^{\beta\delta} R_{\beta\delta},
\end{aligned} \tag{B.1}
$$

where we have defined the following notations with transverse tensor $\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$,

$$
\begin{aligned}
&M_{\mu\nu\sigma\lambda} \equiv \gamma^{\alpha}_{\mu} \gamma^{\beta}_{\nu} \gamma^{\gamma}_{\sigma} \gamma^{\delta}_{\lambda} R_{\alpha\beta\gamma\delta}, \quad N_{\mu\nu\sigma} \equiv \gamma^{\alpha}_{\mu} \gamma^{\beta}_{\nu} \gamma^{\gamma}_{\sigma} n^{\delta} R_{\alpha\beta\gamma\delta}, \quad Y_{\mu\nu} \equiv \gamma^{\alpha}_{\mu} n^{\beta} \gamma^{\gamma}_{\nu} n^{\delta} R_{\alpha\beta\gamma\delta}, \\
&M_{\mu\nu} \equiv \gamma^{\alpha\beta} M_{\mu\alpha\nu\beta}, \qquad M \equiv \gamma^{\alpha\beta} M_{\alpha\beta}, \qquad N_\mu \equiv \gamma^{\alpha\beta} N_{\alpha\mu\beta}, \qquad Y \equiv \gamma^{\alpha\beta} Y_{\alpha\beta}.
\end{aligned} \tag{B.2}
$$

One can also obtain the decomposition of their combinations, such as,

$$
\begin{aligned}
R_{\mu}{}^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} n^\mu n^\nu &= N^{cde} N_{cde} + 2Y^{cd} Y_{cd}, \\
R_{\mu}{}^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} n^\mu h^\nu_b &= -M_{bcde} N^{dec} - 2Y^{cd} N_{bcd}, \\
R_{\mu}{}^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} h^\mu_a h^\nu_b &= M_a{}^{cde} M_{bcde} + 2N_a{}^{cd} N_{bcd} + N^{cd}{}_a N_{cdb} + 2Y_a{}^c Y_{cb}, \\
R_{\mu}{}^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} g^{\mu\nu} &= M^{cdef} M_{cdef} + 4N^{cde} N_{cde} + 4Y^{cd} Y_{cd}.
\end{aligned} \tag{B.3}
$$

Then $\bar{H}_{\mu\nu} \equiv \bar{R}_{\mu}{}^{\sigma\lambda\rho} \bar{R}_{\nu\sigma\lambda\rho} - \frac{1}{4}\left(\bar{R}^{\kappa\sigma\lambda\rho}\bar{R}_{\kappa\sigma\lambda\rho}\right)\bar{g}_{\mu\nu}$ in (3.15) can be decomposed as

$$
\begin{aligned}
\bar{H}_{(n)(n)} &\equiv \bar{H}_{\mu\nu} n^\mu n^\nu = \bar{Y}^{cd}\bar{Y}_{cd} - \frac{1}{4}\bar{M}^{cdef}\bar{M}_{cdef}, \\
\bar{H}_{(n)(u)} &\equiv \bar{H}_{\mu\nu} n^\mu \gamma^\nu_b u^b = -\bar{M}_{(u)cde}\bar{N}^{dec} - 2\bar{Y}^{cd}\bar{N}_{(u)cd}, \\
\bar{H}_{(u)(u)} &\equiv \bar{H}_{\mu\nu}\gamma^\mu_a\gamma^\nu_b u^a u^b = \bar{M}_{(u)}{}^{cde}\bar{M}_{(u)cde} + 2\bar{N}_{(u)}{}^{cd}\bar{N}_{(u)cd} + \bar{N}^{cd}{}_{(u)}\bar{N}_{cd(u)} + 2\bar{Y}_{(u)}{}^{d}\bar{Y}_{(u)d} \\
&\qquad + \frac{1}{4}\left(\bar{M}^{cdef}\bar{M}_{cdef} + 4\bar{N}^{cde}\bar{N}_{cde} + 4\bar{Y}^{cd}\bar{Y}_{cd}\right), \\
\bar{H}^{\perp}_{ab} &\equiv \bar{H}_{\mu\nu}\gamma^\mu_c\gamma^\nu_d h^c_a h^d_b = h^m_a h^n_b\left(\bar{M}_m{}^{cde}\bar{M}_{ncde} + 2\bar{N}_m{}^{cd}\bar{N}_{ncd} + \bar{N}^{cd}{}_m\bar{N}_{cdn} + 2\bar{Y}_m{}^d\bar{Y}_{nd}\right) \\
&\qquad - \frac{1}{4}\left(\bar{M}^{cdef}\bar{M}_{cdef} + 4\bar{N}^{cde}\bar{N}_{cde} + 4\bar{Y}^{cd}\bar{Y}_{cd}\right)h_{ab}, \\
\bar{H} &\equiv \bar{H}_{\mu\nu}g^{\mu\nu} = -\frac{p-2}{4}\left(\bar{M}^{cdef}\bar{M}_{cdef} + 4\bar{N}^{cde}\bar{N}_{cde} + 4\bar{Y}^{cd}\bar{Y}_{cd}\right).
\end{aligned} \tag{B.4}
$$

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