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Three- and four-point correlators of excited bosonic twist fields

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ABSTRACT: We compute three- and four-point correlation functions containing excited bosonic twist fields. Our results can be used to determine properties, such as lifetimes and production rates, of massive string excitations localised at D-brane intersections, which could be signatures of a low string scale even if the usual string resonances are inaccessible to the LHC.

KEYWORDS: D-branes, Conformal Field Models in String Theory

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1 Introduction

Orientifold compactifications has been proven to be a very fruitful framework for realistic model building. In those constructions the gauge symmetry does live on the world-volume of lower-dimensional hyperplanes, called D-branes, whereas the chiral matter is localised at the intersection of different D-brane stacks. For recent reviews on D-brane model building and concrete MSSM D-brane realizations, see [1–4] and references therein.

A particular intriguing subclass of D-brane realizations are string compactifications with a significantly lower string scale [5–7]. Such low string scale scenarios with a string scale M_s that can be as low as a few TeV provide a potential solution to the hierarchy and cosmological constant problems but may also lead to interesting signatures observable at the LHC. Among those signatures rank experimental signs of anomalous Z' physics as well as Kaluza Klein states (see, e.g. [8–38]).

Low string scale compactifications also allow for the direct detection of string excitations. In a series of papers [25, 29–31] the authors study tree-level string scattering amplitudes containing at most two chiral fermions. They show that these amplitudes exhibit a universal behaviour independently of the specifics of the compactification, which gives their results a predictive power. The observed poles correspond to the exchanges of Regge excitations of the standard model gauge bosons, whose mass scales like the string mass, and this has enabled the experiments to now limit the string scale to be above 5.08 TeV [39–41].

On the other hand at the intersection of two D-brane stacks exists a tower of stringy excitations, dubbed as *light stringy states* in the following, whose mass scales as $M^2 \sim \theta M_s^2$, where θ denotes the intersection angle. Thus for small intersection angles those states can be significantly lighter than the first Regge excitations of the gauge bosons and are expected to be observed prior to the latter. Therefore it is of utmost interest to study properties, such as decay rates and life time, of these light stringy states.

As recently discussed in [42] the vertex operators of such light stringy states¹ contain *excited* versions of the bosonic twist fields present in the operators for massless chiral matter states (see e.g. [43, 44] for recent work involving these). However, little is known about correlators involving such excited bosonic twist fields,² a crucial ingredient in the computation of scattering amplitudes involving light stringy states.

This work is dedicated to filling this gap. For simplicity and reasons of calculability we assume the compactification manifold to be a factorizable six-torus $T^6 = T^2 \times T^2 \times T^2$ wrapped by D6-branes. Such models are useful toys, but are still under active development (for original work, see e.g. [52–71], for recent work, see e.g. [72, 73]). In that case the internal part of the world sheet integrand of the scattering amplitudes split into three separate factorizable parts for each two-tori T^2 , for which one can apply the developed CFT techniques. Using the latter we derive various three- and four-point correlators containing (higher) excited bosonic twist fields, where the latter live on one of three two-tori. More precisely we derive correlators of the type

$$\begin{aligned}
 & \langle \tau_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\
 & \langle \tau_\alpha^+(x_1) \tau_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\
 & \langle \omega_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\
 & \langle \tau_\alpha^+(x_1) \tau_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \\
 & \langle \omega_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle ,
 \end{aligned} \tag{1.1}$$

where the σ fields are the usual bosonic twist fields while τ and ω denote the excited and double excited bosonic twist fields. The subscript denotes the intersection angle which is measured in units of π and ranges in the interval $[0, 1)$.

¹Here the authors assumed as compactification manifold a factorizable six-torus $T^6 = T^2 \times T^2 \times T^2$.

²For some preliminary work on excited bosonic twist field correlators, see [45–48]. For a different approach to obtain correlation functions containing excited twist fields, see [49–51]. There the authors identified the pp-wave current algebra in terms of twist fields and used the Knizhnik-Zamolodchikov equations to determine the desired bosonic twist field correlators.

Since the excited twist fields, τ and ω are not primary conformal fields we will take a detour to determine the correlators of type (1.1) by first evaluating higher point correlators containing solely the primary fields, namely solely bosonic twist fields σ as well as the conformal fields ∂Z and $\partial \bar{Z}$. Given those correlators we obtain the correlators displayed in (1.1) by applying various operator product expansions OPE's when performing appropriate limits of the higher point correlators.

This paper is organised as follows: in section 2 we illustrate the method which we use to evaluate the three and four-point correlators containing the excited bosonic twist fields. In section 3, we apply that method to compute the three-point correlators while in section 4 we derive the four-point correlators with one and two independent angles. The appendices A and B contain necessary material for the derivation of the correlators. In appendix C we demonstrate the vanishing of the quantum part of the correlators with only one excited field. In appendices D we display the remaining four-point correlators which were omitted in the main text in order to keep the paper in a more readable/compact form.

2 General considerations

As already mentioned in the introduction we assume the compactification manifold to be a factorizable six-torus $T^6 = T^2 \times T^2 \times T^2$. In terms of the compactified coordinates Z^i, \bar{Z}^i we have

$$Z^i = X^{2i+2} + iX^{2i+3}, \quad \bar{Z}^i = X^{2i+2} - iX^{2i+3} \tag{2.1}$$

with the index i denoting the three different two-tori. In the following, we will focus on a single two-torus T^2 . The analysis for the other two two-tori is analogous.

In [74–77]³ the authors determined the three and four-point bosonic twist field correlators in order to determine the physical Yukawa couplings as well as the Kähler metric for chiral matter.⁴ Here we will perform an analogous derivation for the three- and four-point correlators containing (higher) excited bosonic twist fields.

Let us illustrate the method which we will use to derive the above displayed correlators (1.1). Here we will focus on the three point correlators but the same procedure will apply to the four-point correlators. We start by determining the correlators containing three bosonic twist fields σ and up to two of the bosonic conformal fields ∂Z and $\partial \bar{Z}$. More explicitly we derive the four and five point correlators

$$\begin{aligned} & \langle \partial Z(z) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial \bar{Z}(z) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial Z(z) \partial Z(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial Z(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle . \end{aligned} \tag{2.2}$$

³For similar work on the T-dual side with magnetized branes, see [78].

⁴For a generalization to non-factorizable six-torus, see [79, 80].

We have to keep in mind that the conformal fields ∂Z and $\partial\bar{Z}$ do split into a classical piece ∂Z_{cl} and $\partial\bar{Z}_{cl}$, respectively, and a quantum piece denoted by ∂Z_{qu} and $\partial\bar{Z}_{qu}$

$$\partial Z = \partial Z_{cl} + \partial Z_{qu} \quad \partial\bar{Z} = \partial\bar{Z}_{cl} + \partial\bar{Z}_{qu} . \quad (2.3)$$

Here ∂Z_{cl} and $\partial\bar{Z}_{cl}$ satisfy the classical equation of motion with the boundary conditions set by the insertions of the bosonic twist fields. Thus the four- and five-point correlators split into a pure quantum part as well as a part which contains the classical piece, dubbed the *mixed part* in what follows.

For the derivation of the pure quantum correlator we will employ the energy momentum tensor method initially used in [81–83] in the context of closed string theory on orbifolds and more recently applied to open string theory [45, 75–77, 84, 85]. On the other hand the mixed part requires the knowledge of the classical solutions for ∂Z_{cl} and $\partial\bar{Z}_{cl}$ for the respective boundary conditions. Those were derived in [76, 77] for arbitrary number of bosonic twist field insertions as well as arbitrary intersection angles. Adopting their results for our specific setups in combination with known pure bosonic twist field three-point correlator [46, 75, 86] allows us to determine the mixed part.

Finally, any correlator is suppressed by the world-sheet instanton $e^{-S_{cl}}$, which is given by⁵

$$S_{cl} = \frac{1}{4\pi\alpha'} \int d^2z \left\{ \partial Z_{cl}(z) \partial Z_{cl}(z) + \partial\bar{Z}_{cl}(z) \partial\bar{Z}_{cl}(z) \right\} \quad (2.4)$$

with ∂Z_{cl} and $\partial\bar{Z}_{cl}$ being again the classical solutions to the equation of motion satisfying the boundary conditions set by the bosonic twist fields. Given those solutions for the two-torus one can easily determine (2.4) where the whole world-sheet instanton contribution is a sum over all possible closed triangles connecting the three intersection points [74–77] (see also [25]).⁶

After establishing the complete four- and five-point correlators (2.2), containing the pure quantum and the mixed part as well as the world-sheet contribution, we investigate various limits. Applying the OPE's

$$\begin{aligned} \partial Z(z) \sigma_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \tau_{\alpha}^{+}(w) & \partial Z(z) \sigma_{\alpha}^{-}(w) &\sim (z-w)^{-\alpha} \tau_{\alpha}^{-}(w) \\ \partial\bar{Z}(z) \sigma_{\alpha}^{+}(w) &\sim (z-w)^{-\alpha} \tilde{\tau}_{\alpha}^{+}(w) & \partial\bar{Z}(z) \sigma_{\alpha}^{-}(w) &\sim (z-w)^{\alpha-1} \tilde{\tau}_{\alpha}^{-}(w) \end{aligned} \quad (2.5)$$

will then give the three-point correlators containing the excited bosonic twist fields.

Even though, the procedure for the derivation of correlators containing exciting twist field correlators was laid out for three-point correlators it generalizes to four-point correlators as we will exemplify for four-point correlators containing excited twist fields with one and two independent angles.

⁵Note that we applied the doubling trick that extends the world-sheet from the upper half complex plane to the whole complex plane.

⁶For analogous results in the T-dual IIB framework with magnetized D-branes, see [78, 80].

3 Three-point correlators

In this section we apply the previously described procedure to obtain three-point bosonic twist field correlators that contain excited bosonic twist fields. As laid out above we take a detour by first deriving higher point correlators containing the bosonic twist fields σ_α , σ_β and σ_γ as well as the conformal fields ∂Z and $\partial\bar{Z}$. The insertions of the bosonic twist fields σ lead to the following boundary conditions

$$\begin{aligned}
 \partial Z - \partial\bar{Z} &= 0 & \text{for} & \quad (-\infty, x_1) \\
 e^{i\pi\alpha}\partial Z - e^{-i\pi\alpha}\partial\bar{Z} &= 0 & \text{for} & \quad (x_1, x_2) \\
 e^{i\pi(\alpha+\beta)}\partial Z - e^{-i\pi(\alpha+\beta)}\partial\bar{Z} &= 0 & \text{for} & \quad (x_2, x_3) \\
 e^{i\pi(\alpha+\beta+\gamma)}\partial Z - e^{-i\pi(\alpha+\beta+\gamma)}\partial\bar{Z} &= 0 & \text{for} & \quad (x_3, \infty) .
 \end{aligned} \tag{3.1}$$

In the following we assume that the sum of the intersection angles is 1 rather than 2, i.e. $\alpha + \beta + \gamma = 1$. The results for the latter setup can be easily obtained making the replacement $\alpha, \beta, \gamma \rightarrow 1 - \alpha, 1 - \beta, 1 - \gamma$.

3.1 Three-point correlators with one excited twist field

Let us start with the three-point correlators containing one excited twist field. Those arise from the four-point correlators

$$\begin{aligned}
 \langle \partial Z(z) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\
 \langle \partial\bar{Z}(z) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle ,
 \end{aligned} \tag{3.2}$$

where we focus on a setup for which the intersection angles add up to 1, i.e. $\alpha + \beta + \gamma = 1$. As indicated in section 2 the conformal fields ∂Z as well as $\partial\bar{Z}$ split into a classical and a quantum piece, see eq. (2.3). As we show in appendix C any bosonic twist field correlator containing an odd number of the conformal fields ∂Z_{qu} and $\partial\bar{Z}_{qu}$ vanishes. Thus the four-point correlators (3.2) consist of a purely mixed part that takes the form

$$\begin{aligned}
 \partial Z_{cl}(z) \langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\
 \partial\bar{Z}_{cl}(z) \langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle .
 \end{aligned} \tag{3.3}$$

The three-point bosonic twist field correlator is given by [46, 75, 86]⁷

$$\langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle = \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \right)^{\frac{1}{4}} x_{12}^{-\alpha\beta} x_{13}^{-\alpha\gamma} x_{23}^{-\beta\gamma} . \tag{3.4}$$

On the other hand the classical solutions are given by the asymptotic behaviour close to the insertions of the bosonic twist fields σ , which specifies the solutions up to normalizations

⁷In case the intersection angles add up to 2 the correlator takes the form

$$\langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle = \left(2\pi \frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)} \right)^{\frac{1}{4}} x_{12}^{-(1-\alpha)(1-\beta)} x_{13}^{-(1-\alpha)(1-\gamma)} x_{23}^{-(1-\beta)(1-\gamma)} .$$

that can be evaluated from the global monodromy conditions. This procedure has been applied to three bosonic twist field correlators in [74–76]⁸ where the authors derived the classical solutions for compactification manifold M , being planar $M = R^2$ and toroidal $M = T^2$. They take the form⁹

$$\begin{aligned} \partial Z_{cl}(z) &= \sqrt{\alpha'} e^{i\pi(\gamma-1)z^{\alpha-1}}(z-1)^{\beta-1} v_c \frac{\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)} \\ \partial \bar{Z}_{cl}(z) &= 0, \end{aligned} \tag{3.5}$$

where we used $SL(2, \mathbf{R})$ invariance to fix the bosonic twist field positions to $x_1 = 0$, $x_2 = 1$ and $x_3 = \infty$.

On the plane the vector v_a is simply given by the (dimensionless) distance between the two intersection points f_1 and f_2 of the D-brane stack a with the other two D-brane stacks b and c , thus $v_a = f_1 - f_2$.

However, on a two-torus T^2 one has to take into account that the action of the twist operator does not only rotate Z_{cl} but also may shift it by a lattice translation. That results into an infinite number of solutions with v_a given by

$$v_a[n] = f_1 - f_2 + n\tilde{L}_a. \tag{3.6}$$

where \tilde{L}_a takes the form

$$\tilde{L}_a = \frac{|I_{bc}|}{gcd(|I_{ab}|, |I_{bc}|, |I_{ca}|)} L_a. \tag{3.7}$$

In eq. (3.7) $I_{xy} = n_x m_y - n_y m_x$ is the intersection number between two D-brane stacks x and y , $L_x = R_1 \sqrt{n_x^2 + (m_x \mathcal{T})^2}$ the length of the D-brane stack x , with (n_x, m_x) being the wrapping numbers and \mathcal{T} denoting the complex structure of the two-torus. In the simple setup in which all three D-brane stacks intersect each other exactly once \tilde{L}_x coincides with the length L_x of the respective D-brane.

Let us exemplify the notion of the $v_c[n]$ with the explicit example depicted in figure 1. Here we have three stacks of D-branes, a , b and c , wrapping one-cycles on a T^2 . Their intersection numbers are given by $I_{ab} = 1$, $I_{bc} = 3$, $I_{ac} = 1$, where f_1 denotes the intersection between D-brane stack c and a while f_2 , \tilde{f}_2 and $\tilde{\tilde{f}}_2$ denote the three intersection points among the D-brane stacks b and c . The primed f 's are the corresponding lattice shifted intersection points. For this explicit example the vectors $v_c[0]$ and $v_c[1]$ take the form $v_c[0] = f_1 - f_2$ and $v_c[1] = f_1''' - f_2' = f_1 - f_2 + \tilde{L}_c$, respectively.

All correlators are suppressed by world-sheet instanton contributions which can be easily computed applying (2.4) as well as using the classical solutions (3.5). One obtains

$$S_{cl}[n] = \frac{1}{2\pi} \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\sin(\pi\gamma)} |v_c[n]|^2, \tag{3.8}$$

⁸For analogous results in the T-dual IIB framework with magnetized D-branes, see [78].

⁹Here we suppress the x_∞ dependence and assume $z < 0$.

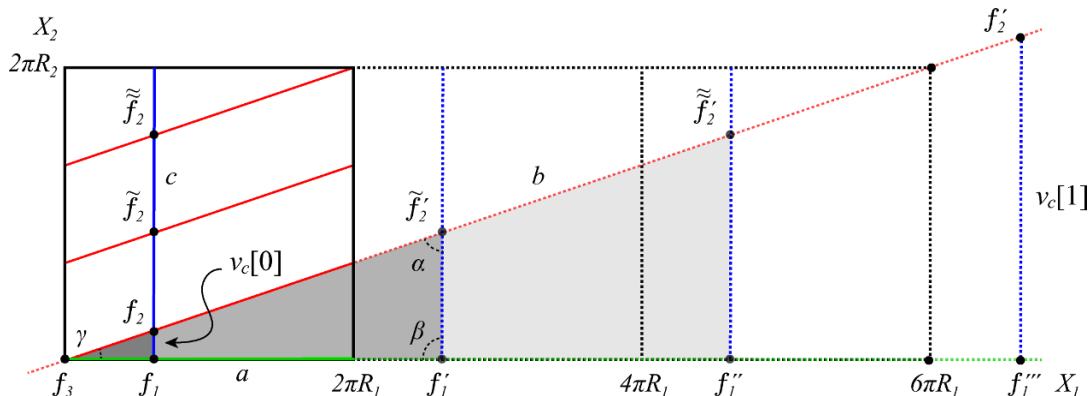


Figure 1. Let us consider the three intersecting D-brane stacks wrapping the one-cycles $a = (1, 0)$, $b = (3, 1)$, $c = (0, 1)$ on a rectangular two-torus.

Thus the whole four-point correlator $\langle \partial Z \sigma \sigma \sigma \rangle$ takes the form¹⁰

$$\begin{aligned} \langle \partial Z(z) \sigma_\alpha^+(0) \sigma_\beta^+(1) \sigma_\gamma^+(\infty) \rangle & \quad (3.9) \\ & = \sqrt{\alpha'} \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma^5(1-\gamma)}{\Gamma^5(\alpha)\Gamma^5(\beta)\Gamma(\gamma)} \right)^{\frac{1}{4}} e^{i\pi(\gamma-1)z^{\alpha-1}} (z-1)^{\beta-1} \sum_n v_c[n] e^{-S_{cl}[n]} \end{aligned}$$

with the world-sheet instanton given in eq. (3.8) and the sum is over all closed triangles, thus over all integers n . On the other hand the four-point correlator $\langle \partial \bar{Z} \sigma \sigma \sigma \rangle$ vanishes.

Given the correlator (3.10) we can now derive the three-point correlator containing an excited twist field. Performing the limit $z \rightarrow 0$ gives on

$$\langle \tau_\alpha^+(0) \sigma_\beta^+(1) \sigma_\gamma^+(\infty) \rangle = \sqrt{\alpha'} \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma^5(1-\gamma)}{\Gamma^5(\alpha)\Gamma^5(\beta)\Gamma(\gamma)} \right)^{\frac{1}{4}} \sum_n v_c[n] e^{-S_{cl}[n]} \quad (3.10)$$

where we used on the left-hand side the OPE's displayed in eq. (2.5). Reinstating the complete x_i dependence gives¹¹

$$\langle \tau_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle = \sqrt{\alpha'} \frac{\left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma^5(1-\gamma)}{\Gamma^5(\alpha)\Gamma^5(\beta)\Gamma(\gamma)} \right)^{\frac{1}{4}}}{x_{12}^{\alpha(1+\beta)} x_{13}^{\alpha(1+\gamma)} x_{23}^{\beta\gamma-\alpha}} \sum_n v_c[n] e^{-S_{cl}[n]} \quad (3.11)$$

with the world-sheet instanton contribution given by (3.8).

¹⁰In the following we will suppress the x_∞ factor, which we will recover when we reinstate the full x_i dependence.

¹¹Any 3-point correlator of quasi-primary fields takes the form

$$\langle \Phi_1(x_1) \Phi_2(x_2) \Phi_3(x_3) \rangle = \frac{C_{123}}{x_{12}^{h_1+h_2-h_3} x_{13}^{h_1-h_2+h_3} x_{23}^{-h_1+h_2+h_3}},$$

where h_i denote the conformal dimension of the field Φ_i .

3.2 Three-point correlators with two excited twist fields

Three-point correlators containing two excited bosonic twist field (or one double excited bosonic twist field) arise from the five point correlators of type

$$\begin{aligned} & \langle \partial Z(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial Z(z) \partial Z(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle \\ & \langle \partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle . \end{aligned} \tag{3.12}$$

In contrast to the four point correlators discussed in the previous subsection the quantum parts of the correlators (3.12) do not vanish. In order to determine those we define the following functions

$$g(z, w) = \frac{\langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle} \tag{3.13}$$

$$k(z, w) = \frac{\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle} \tag{3.14}$$

$$m(z, w) = \frac{\langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\beta^+(x_2) \sigma_\gamma^+(x_3) \rangle} \tag{3.15}$$

whose knowledge, together with the three-point correlator (3.4), allows us to derive the 5-point quantum correlator. With the local behaviour (2.5) as well as

$$\partial Z(z) \partial \bar{Z}(w) \sim \frac{\alpha'}{(z-w)^2} \quad \partial Z(z) \partial Z(w) \sim \text{regular} \quad \partial \bar{Z}(z) \partial \bar{Z}(w) \sim \text{regular} \tag{3.16}$$

the form of the functions g , k and m is given by

$$g(z, w) = \alpha' \omega_{1-\alpha, 1-\beta, 1-\gamma}(z) \omega_{\alpha, \beta, \gamma}(w) \frac{P}{(z-w)^2} \tag{3.17}$$

$$k(z, w) = 0 \tag{3.18}$$

$$m(z, w) = \alpha' \omega_{1-\alpha, 1-\beta, 1-\gamma}(z) \omega_{1-\alpha, 1-\beta, 1-\gamma}(w) C(\{x_i\}) \tag{3.19}$$

with

$$\omega_{\alpha, \beta, \gamma}(z) = (z-x_1)^{-\alpha} (z-x_2)^{-\beta} (z-x_3)^{-\gamma} \tag{3.20}$$

and

$$\begin{aligned} P = & \alpha (w-x_1)(z-x_2)(z-x_3) + \beta (z-x_1)(w-x_2)(z-x_3) \\ & + \gamma (z-x_1)(z-x_2)(w-x_3) . \end{aligned} \tag{3.21}$$

Here we use the fact that any correlation function that contains a conformal field of dimension h at insertion x behaves as x^{-2h} for $x \rightarrow \infty$. While the functions g and k are

fully determined by requiring the proper local behaviour for the full determination of m we need to impose the monodromy conditions (3.1), that results into the condition¹²

$$e^{i\pi\alpha} \int_0^1 m(z, w) dw - e^{-i\pi\alpha} \int_0^1 g(z, w) dw = 0 \quad (3.22)$$

which after dividing by $\omega_{1-\alpha, 1-\beta, 1-\gamma}(z)$, using $\text{SL}(2, \mathbf{R})$ symmetry to fix the bosonic twist field positions to

$$x_1 = 0 \quad x_2 = 1 \quad x_3 = \infty \quad (3.23)$$

and performing the limit $z \rightarrow \infty$ results into

$$e^{i\pi\alpha} \int_0^1 \tilde{m}(w) dw - e^{-i\pi\alpha} \int_0^1 \tilde{g}(w) dw = 0. \quad (3.24)$$

Here $\tilde{g}(w)$ and $\tilde{m}(w)$ take the form

$$\tilde{g}(z) = \alpha' \gamma z^{-\alpha} (z-1)^{-\beta} \quad \tilde{m}(z) = \alpha' \tilde{C} z^{-1+\alpha} (z-1)^{-1+\beta} \quad (3.25)$$

with \tilde{C} being $\frac{C(0,1,\infty)}{(-x_\infty)^{2\gamma-2}}$. Solving (3.24) allows to determine the missing piece of the function $m(z, w)$, \tilde{C} , to

$$\tilde{C} = -e^{-2\pi i(\alpha+\beta)} \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}. \quad (3.26)$$

Together with the three-point correlator (3.4) we obtain the following five-point quantum correlators¹³

$$\begin{aligned} \langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\beta^+(1) \sigma_\gamma^+(\infty) \rangle &= \alpha' \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \right)^{\frac{1}{4}} \\ &\times z^{\alpha-1} (z-1)^{\beta-1} w^{-\alpha} (w-1)^{-\beta} \left\{ \frac{\alpha w(z-1) + \beta z(w-1) + \gamma z(z-1)}{(z-w)^2} \right\} \end{aligned} \quad (3.27)$$

$$\begin{aligned} \langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_\alpha^+(0) \sigma_\beta^+(1) \sigma_\gamma^+(\infty) \rangle &= -\alpha' (2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \right)^{\frac{5}{4}} \\ &\times e^{2\pi i\gamma} z^{\alpha-1} (z-1)^{\beta-1} w^{\alpha-1} (w-1)^{\beta-1} \end{aligned} \quad (3.28)$$

$$\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\beta^+(1) \sigma_\gamma^+(\infty) \rangle = 0. \quad (3.29)$$

¹²The setup with three bosonic twist field insertions has only one independent world-sheet contour (see appendix C). Without loss of generality we choose the contour to be 0 to 1.

¹³Again we suppress the x_∞ factors.

Since $\partial\bar{Z}_{cl} = 0$ only $\langle\partial Z\partial Z\sigma_\alpha^+\sigma_\beta^+\sigma_\gamma^+\rangle$ gets contribution from the mixed part. Using (3.5) the full five-point correlators take the form

$$\langle\partial Z(z)\partial\bar{Z}(w)\sigma_\alpha^+(0)\sigma_\beta^+(1)\sigma_\gamma^+(\infty)\rangle = \alpha' \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{1}{4}} \quad (3.30)$$

$$\times z^{\alpha-1}(z-1)^{\beta-1}w^{-\alpha}(w-1)^{-\beta} \left\{ \frac{\alpha w(z-1) + \beta z(w-1) + \gamma z(z-1)}{(z-w)^2} \right\} \sum_n e^{-S_{cl}[n]}$$

$$\langle\partial Z(z)\partial Z(w)\sigma_\alpha^+(0)\sigma_\beta^+(1)\sigma_\gamma^+(\infty)\rangle = -\alpha'(2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{5}{4}} \quad (3.31)$$

$$\times e^{2\pi i\gamma} z^{\alpha-1}(z-1)^{\beta-1}w^{\alpha-1}(w-1)^{\beta-1} \sum_n \left\{ 1 - \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\pi\sin(\pi\gamma)} |v_c[n]|^2 \right\} e^{-S_{cl}[n]}$$

$$\langle\partial\bar{Z}(z)\partial\bar{Z}(w)\sigma_\alpha^+(0)\sigma_\beta^+(1)\sigma_\gamma^+(\infty)\rangle = 0, \quad (3.32)$$

where the world-sheet instanton suppression is given in (3.8) and the sum is over all closed triangles formed by the three D-brane intersections.

In order to derive the three-point correlators containing excited bosonic twist fields we take various limits of the five-point correlators, analogously to the derivation performed in the previous subsection. We illustrate this analysis with the example of the five-point correlator $\langle\partial Z\partial\bar{Z}\sigma_\alpha^+\sigma_\beta^+\sigma_\gamma^+\rangle$ and display the three-point correlators derived from the correlator $\langle\partial Z\partial Z\sigma_\alpha^+\sigma_\beta^+\sigma_\gamma^+\rangle$ at the end of this section.

From (3.30) we obtain in the limit $z \rightarrow 0, w \rightarrow 1$

$$\langle\tau_\alpha^+(0)\tilde{\tau}_\beta^+(1)\sigma_\gamma^+(\infty)\rangle = \alpha' \alpha \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{1}{4}} \sum_n e^{-S_{cl}[n]}. \quad (3.33)$$

After reinstating the x_i dependence one gets

$$\langle\tau_\alpha^+(x_1)\tilde{\tau}_\beta^+(x_2)\sigma_\gamma^+(x_3)\rangle = \alpha' \alpha \frac{\left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{1}{4}}}{x_{12}^{1+\alpha-(1-\alpha)\beta} x_{13}^{-(1-\alpha)\gamma} x_{23}^{(1+\beta)\gamma}} \sum_n e^{-S_{cl}[n]}. \quad (3.34)$$

where again the world-sheet instanton suppression is given in (3.8).

In an analogous fashion we derive from the correlator $\langle\partial Z\partial Z\sigma_\alpha^+\sigma_\beta^+\sigma_\gamma^+\rangle$ (see eq. (3.31)) the three-point correlators

$$\langle\tau_\alpha^+(x_1)\tau_\beta^+(x_2)\sigma_\gamma^+(x_3)\rangle = -\alpha'(2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{5}{4}} \quad (3.35)$$

$$\times x_{12}^{-\alpha\beta-\alpha-\beta} x_{13}^{-\alpha(2+\gamma)-\gamma+1} x_{23}^{-\beta(2+\gamma)-\gamma+1} \sum_n \left\{ 1 - \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\pi\sin(\pi\gamma)} |v_c[n]|^2 \right\} e^{-S_{cl}[n]}$$

and

$$\langle\omega_\alpha^+(x_1)\sigma_\beta^+(x_2)\sigma_\gamma^+(x_3)\rangle = -\alpha'(2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{5}{4}} \quad (3.36)$$

$$\times x_{12}^{-\alpha(2+\beta)} x_{13}^{-\alpha(2+\gamma)} x_{23}^{-\beta(2+\gamma)-2\gamma+2} \sum_n \left\{ 1 - \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\pi\sin(\pi\gamma)} |v_c[n]|^2 \right\} e^{-S_{cl}[n]}.$$

Again the world-sheet contributions are given by (3.8).

4 Four-point correlators

In this section we apply the same formalism as above to determine four-point correlators containing excited bosonic twist fields. We will discuss two different scenarios, namely bosonic twist field four-point correlators with one and two independent angles, starting with the case of one independent angle.

4.1 Four-point correlators with one independent angle

As for the three-point correlators we derive the four-point correlators by computing higher point correlators containing the bosonic twist fields σ_α^+ and σ_α^- as well as ∂Z and $\partial \bar{Z}$. The insertion of the bosonic twist fields $\langle \sigma_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle$ lead to the following boundary conditions on the world-sheet

$$\begin{aligned} \partial Z - \partial \bar{Z} &= 0 \quad \text{for } (-\infty, x_1) \cup (x_2, x_3) \cup (x_4, \infty) \\ e^{i\pi\alpha} \partial Z - e^{-i\pi\alpha} \partial \bar{Z} &= 0 \quad \text{for } (x_1, x_2) \cup (x_3, x_4) . \end{aligned} \quad (4.1)$$

Given those boundary conditions we will determine again the correlators containing one or two excited bosonic twist fields, starting with the correlators containing only one excited bosonic twist field.

4.1.1 Four-point correlators containing one excited twist field

Analogously to the three-point correlator derivation we determine the four-point correlators containing one excited bosonic twist field by first computing the 5-point correlators

$$\langle \partial Z(z) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \quad (4.2)$$

$$\langle \partial \bar{Z}(z) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle , \quad (4.3)$$

which get only a contribution from the classical part of ∂Z and $\partial \bar{Z}$, respectively.

Again the asymptotic behaviour close to the insertion of the bosonic twist fields specify the classical solutions ∂Z_{cl} and $\partial \bar{Z}_{cl}$ up to normalizations

$$\partial Z_{cl}(z) = \sqrt{\alpha'} \tilde{a} \omega_{1-\alpha, 1-\alpha}(z) \quad \partial \bar{Z}_{cl}(z) = \sqrt{\alpha'} \tilde{b} \omega_{\alpha, \alpha}(z) \quad (4.4)$$

with $\omega_{\alpha, \beta}$ being

$$\omega_{\alpha, \beta}(z) = (z - x_1)^{-\alpha} (z - x_2)^{-1+\alpha} (z - x_3)^{-\beta} (z - x_4)^{-1+\beta} . \quad (4.5)$$

The normalizations \tilde{a} and \tilde{b} are determined by satisfying the global monodromy conditions. One obtains [75, 77]¹⁴

$$\begin{aligned} \tilde{a} &= -\frac{\sin(\pi\alpha)}{2\pi} \frac{v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x)}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\ \tilde{b} &= -\frac{\sin(\pi\alpha)}{2\pi} \frac{v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x)}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]} , \end{aligned} \quad (4.6)$$

¹⁴Here we used $SL(2, \mathbf{R})$ invariance to fix the twist field insertions to $x_1 = 0$, $x_2 = x$, $x_3 = 1$ and $x_4 = x_\infty = \infty$. Furthermore we suppress all x_∞ dependence.

where $\tau(x)$ is given by the ratio

$$\tau(x) = \frac{{}_2F_1[\alpha, 1 - \alpha, 1, 1 - x]}{{}_2F_1[\alpha, 1 - \alpha, 1, x]} \quad (4.7)$$

and

$$v_a[n] = f_2 - f_1 + n\tilde{L}_a, \quad v_b[m] = f_3 - f_2 + m\tilde{L}_b \quad (4.8)$$

with n, m being integers and \tilde{L}_x given in eq. (3.7). Together with the four point correlator [45, 75–77, 84, 85, 87]

$$\begin{aligned} & \langle \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle \\ &= \frac{\sin^{\frac{1}{2}}(\pi\alpha)}{{}_2F_1[\alpha, 1 - \alpha, 1, x]^{\frac{1}{2}} {}_2F_1[\alpha, 1 - \alpha, 1, 1 - x]^{\frac{1}{2}}} [x(1-x)]^{-\alpha(1-\alpha)} \end{aligned} \quad (4.9)$$

one obtains for the 5-point correlators (4.2) and (4.3)

$$\begin{aligned} \langle \partial Z(z) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle &= -\sqrt{\alpha'} e^{-i\pi\alpha} z^{\alpha-1} (z-x)^{-\alpha} (z-1)^{\alpha-1} [x(1-x)]^{-\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1[\alpha, 1 - \alpha, 1, x]^{\frac{1}{2}} {}_2F_1[\alpha, 1 - \alpha, 1, 1 - x]^{\frac{3}{2}}} \sum_{n,m} (v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x)) e^{-S_{cl}[n,m]} \end{aligned} \quad (4.10)$$

$$\begin{aligned} \langle \partial \bar{Z}(z) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle &= -\sqrt{\alpha'} e^{i\pi(\alpha-1)} z^{-\alpha} (z-x)^{\alpha-1} (z-1)^{-\alpha} [x(1-x)]^{-\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1[\alpha, 1 - \alpha, 1, x]^{\frac{1}{2}} {}_2F_1[\alpha, 1 - \alpha, 1, 1 - x]^{\frac{3}{2}}} \sum_{n,m} (v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x)) e^{-S_{cl}[n,m]}. \end{aligned} \quad (4.11)$$

As already indicated both correlators are suppressed by the world-sheet instanton contributions $e^{-S_{cl}[n,m]}$ which we discuss momentarily. With (2.4) and (4.4) we can compute the classical action in the presence of the four bosonic twist fields [77] (see also [25])

$$S_{cl}^{T^2}[n, m] = \pi \sin(\pi\alpha) \left\{ |v_a[n]|^2 \tau(x) + |v_b[m]|^2 \tau(1-x) \right\}, \quad (4.12)$$

where $\tau(x)$ is given by (4.7). The complete world-sheet instanton contribution is the sum over all possible closed polygons connecting the four intersection points, thus we sum over all integers m, n appearing in $v_a[n]$ and $v_b[m]$.

In the following we will perform the limits $z \rightarrow 0$ and $z \rightarrow x$ in order to derive the four point correlators with one excited bosonic twist field. We will illustrate that with the example $\langle \partial Z \sigma^+ \sigma^- \sigma^+ \sigma^- \rangle$ and display the results arising from the five-point correlator $\langle \partial \bar{Z} \sigma^+ \sigma^- \sigma^+ \sigma^- \rangle$ in the appendix D.1. From the limit $z \rightarrow 0$ one obtains

$$\begin{aligned} \langle \tau_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle &= -\sqrt{\alpha'} x^{-\alpha(2-\alpha)} (1-x)^{-\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1[\alpha, 1 - \alpha, 1, x]^{\frac{1}{2}} {}_2F_1[\alpha, 1 - \alpha, 1, 1 - x]^{\frac{3}{2}}} \sum_{n,m} (v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x)) e^{-S_{cl}[n,m]}, \end{aligned} \quad (4.13)$$

which after reinstating all x_i dependence gives (recall $x = \frac{x_{12}x_{34}}{x_{13}x_{24}}$)¹⁵

$$\begin{aligned} \langle \tau_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle &= -\sqrt{\alpha'} \left(\frac{x_{24}}{x_{12}x_{14}} \right)^{\alpha(2-\alpha)} \left(\frac{x_{13}}{x_{23}x_{34}} \right)^{\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1^{\frac{1}{2}}[\alpha, 1-\alpha, 1, x] {}_2F_1^{\frac{3}{2}}[\alpha, 1-\alpha, 1, 1-x]} \sum_{n,m} (v_b[m] + e^{i\pi\alpha}v_a[n]\tau(x)) e^{-S_{cl}[n,m]} . \end{aligned} \quad (4.15)$$

Analogously we get for the limit $z \rightarrow x$

$$\begin{aligned} \langle \sigma_\alpha^+(x_1)\tau_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle &= -\sqrt{\alpha'} \left(\frac{x_{13}}{x_{12}x_{23}} \right)^{(1-\alpha)(1+\alpha)} \left(\frac{x_{24}}{x_{14}x_{34}} \right)^{\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1^{\frac{1}{2}}[\alpha, 1-\alpha, 1, x] {}_2F_1^{\frac{3}{2}}[\alpha, 1-\alpha, 1, 1-x]} \sum_{n,m} (v_b[m] + e^{i\pi\alpha}v_a[n]\tau(x)) e^{-S_{cl}[n,m]} . \end{aligned} \quad (4.16)$$

A comparison of (4.15) and (4.16) confirms the following identifications¹⁶

$$\tau_\alpha^- = \tau_{1-\alpha}^+ \quad (4.17)$$

that were already suggested in [42, 87] by studying the OPE's of such higher bosonic twist fields. Note that this identification uses the fact that $e^{i\pi\alpha}v_a[n] = e^{i\pi(1-\alpha)}v_a[n]$ and holds also true for $\tilde{\tau}_\alpha^- = \tilde{\tau}_{1-\alpha}^+$ and as we will see later also generalizes to higher excited bosonic twist fields. Let us point out that this identification among bosonic twist fields and its counterpart, the bosonic anti-twist fields, can be also checked by performing the same analysis with the bosonic twist field ordering of the type $\langle \sigma_\alpha^-(x_1)\sigma_\alpha^+(x_2)\sigma_\alpha^-(x_3)\sigma_\alpha^+(x_4) \rangle$ instead of the considered one here $\langle \sigma_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle$.

The four point-correlator arising from the five-point correlator (4.11) can be derived in an analogous fashion and is displayed in appendix D.1.

4.1.2 Four-point correlators containing two excited twist fields

Analogously to the three-point correlators containing two excited bosonic twist fields the four-point correlators containing two excited bosonic twist fields arise from six-point correlators containing two conformal fields ∂Z and $\partial \bar{Z}$. More specifically, they arise from the six-point correlators

$$\begin{aligned} &\langle \partial Z(z)\partial \bar{Z}(w)\sigma_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle \\ &\langle \partial \bar{Z}(z)\partial \bar{Z}(w)\sigma_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle \\ &\langle \partial Z(z)\partial Z(w)\sigma_\alpha^+(x_1)\sigma_\alpha^-(x_2)\sigma_\alpha^+(x_3)\sigma_\alpha^-(x_4) \rangle . \end{aligned} \quad (4.18)$$

¹⁵Here we use the fact that an open string four-point correlator is given by

$$\langle \phi_{h_1}(x_1)\phi_{h_2}(x_2)\phi_{h_3}(x_3)\phi_{h_4}(x_4) \rangle = \mathcal{F}(x) \prod_{i<j} x_{ij}^{-(h_i+h_j)+\frac{\Delta}{3}} , \quad (4.14)$$

where $\Delta = \sum_{i=1}^4 h_i$ and $x = \frac{x_{12}x_{34}}{x_{13}x_{24}}$. This expression allows us to reinstate the x_i dependence in all the four-point correlators.

¹⁶In order to check that identification one takes (4.16) and exchanges $\alpha \rightarrow 1-\alpha$ and simultaneously exchanges $x_1 \leftrightarrow x_2$ and $x_3 \leftrightarrow x_4$ and proves it gives the expression (4.15). Note that the latter replacement, $x_1 \leftrightarrow x_2$ and $x_3 \leftrightarrow x_4$, is in agreement with the non-vanishing of the trace over Chan-Paton factors accompanying each vertex operator, and thus each bosonic twist field correlator.

In contrast to the five-point correlators considered in the previous subsection (4.1.1) the pure quantum part of the correlators 4.18 does not vanish. We will determine it applying the same procedure performed in section 3.2 for the derivation of the pure quantum five-point correlators.

Let us define the following three functions

$$g(z, w) = \frac{\langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle}{\langle \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle} \quad (4.19)$$

$$k(z, w) = \frac{\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle}{\langle \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle} \quad (4.20)$$

$$m(z, w) = \frac{\langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle}{\langle \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle} \quad (4.21)$$

that will be determined momentarily using local behaviour of the bosonic twist fields as well as global monodromies. Together with the knowledge of the four-point correlator $\langle \sigma_{\alpha}^{+}(x_1) \sigma_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(x_3) \sigma_{\alpha}^{-}(x_4) \rangle$ the functions $g(z, w)$, $k(z, w)$ and $m(z, w)$ allow us to derive the requested six-point correlators.

Using the local behaviour of the bosonic twist fields (2.5) as well as (3.16) we obtain for the functions $g(z, w)$, $k(z, w)$ and $m(z, w)$

$$g(z, w) = \alpha' \omega_{1-\alpha, 1-\alpha}(z) \omega_{\alpha, \alpha}(w) \left\{ \frac{P}{(z-w)^2} + A(\{x_i\}) \right\} \quad (4.22)$$

$$k(z, w) = \alpha' \omega_{\alpha, \alpha}(z) \omega_{\alpha, \alpha}(w) B(\{x_i\}) \quad (4.23)$$

$$m(z, w) = \alpha' \omega_{1-\alpha, 1-\alpha}(z) \omega_{1-\alpha, 1-\alpha}(w) C(\{x_i\}) \quad (4.24)$$

where we use various symmetries of the functions under the exchanges of the x_i 's. Here $\omega_{\alpha, \beta}$ is given by eq. (4.5) and P takes the form

$$\begin{aligned} 2P = & (1 - \xi) (z - x_1)(z - x_2)(w - x_3)(w - x_4) \\ & + 2(-\alpha + \xi) (z - x_1)(w - x_2)(z - x_3)(w - x_4) \\ & + (1 - \xi) (z - x_1)(w - x_2)(w - x_3)(z - x_4) \\ & + (1 - \xi) (w - x_1)(z - x_2)(z - x_3)(w - x_4) \\ & + 2(-1 + \alpha + \xi) (w - x_1)(z - x_2)(w - x_3)(z - x_4) \\ & + (1 - \xi) (w - x_1)(w - x_2)(z - x_3)(z - x_4) . \end{aligned} \quad (4.25)$$

The ξ is a free parameter that is neither fixed by the local behavior nor by any symmetry of the functions. The functions $A(\{x_i\})$, $B(\{x_i\})$ and $C(\{x_i\})$ are functions on positions of the bosonic twist fields only and thus independent of z and w . Those functions will be determined below using constraints arising from global monodromies.

From the boundary conditions (4.1) one gets the monodromy constraints that result into the following set of equations¹⁷

$$\int_x^1 dz (g(z, w) - k(z, w)) = 0, \quad \int_0^x dz (e^{i\pi\alpha} g(z, w) - e^{-i\pi\alpha} k(z, w)) = 0 \quad (4.26)$$

$$\int_x^1 dw (m(z, w) - g(z, w)) = 0, \quad \int_0^x dw (e^{i\pi\alpha} m(z, w) - e^{-i\pi\alpha} g(z, w)) = 0 \quad (4.27)$$

Focusing on the derivation of $g(z, w)$ and $k(z, w)$ the constraints (4.26) lead after using $SL(2, \mathbf{R})$ symmetry to fix $x_1 = 0$, $x_2 = x$, $x_3 = 1$ and $x_4 = x_\infty$, dividing by $\omega_{\alpha, \alpha}(w)$ and taking the limit $w \rightarrow \infty$ to

$$\int_x^1 dz (\tilde{g}(z) - \tilde{k}(z)) = 0, \quad \int_0^x dz (e^{i\pi\alpha} \tilde{g}(z) - e^{-i\pi\alpha} \tilde{k}(z)) = 0 \quad (4.28)$$

where $\tilde{g}(z)$ and $\tilde{k}(z)$ are given by

$$\begin{aligned} \tilde{g}(z) &= x_\infty^{1+\alpha} \alpha' \tilde{\omega}_{1-\alpha, 1-\alpha}(z) \left\{ \frac{(1-\xi)}{2} z + \frac{(1-\xi)}{2} (z-1) - (1-\alpha-\xi)(z-x) + \tilde{A}(x) \right\} \\ \tilde{k}(z) &= x_\infty^{1+\alpha} \alpha' \tilde{\omega}_{\alpha, \alpha}(z) \tilde{B}(x) \end{aligned}$$

with

$$\tilde{A}(x) = \frac{-A(0, x, 1, x_\infty)}{x_\infty} \quad \tilde{B}(x) = \frac{-B(0, x, 1, x_\infty)}{x_\infty} \quad \tilde{\omega}_{\alpha, \beta}(z) = z^{-\alpha} (z-x)^{\alpha-1} (z-1)^{-\beta} \quad (4.29)$$

Solving the system of equations (4.28) by using the integrals displayed in appendix B we obtain for $\tilde{A}(x)$

$$\tilde{A}(x) = \frac{\alpha + \xi - 1}{2} (2x - 1) - \frac{\alpha}{2} \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} + \frac{\alpha}{2} \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \quad (4.30)$$

and for $\tilde{B}(x)$

$$\tilde{B}(x) = \frac{\alpha}{2} e^{2i\pi\alpha} \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right\}. \quad (4.31)$$

Together with (4.22) and (4.23) this determines the functions $g(z, w)$ and $k(z, w)$ at the positions $(z, w, 0, x, 1, \infty)$. Note that $g(z, w)$ and $k(z, w)$ are completely independent on the choice of ξ , as it should be. Thus for the function $g(z, w)$, \tilde{A} compensates for the ξ dependence in P .

In an analogous way one can determine $\tilde{C}(x)$ ¹⁸ by solving the system of equations (4.27) and obtain for $\tilde{A}(x)$ again (4.30) while for $\tilde{C}(x)$ one gets

$$\tilde{C}(z) = \frac{1-\alpha}{2} e^{-2\pi i\alpha} \left\{ 1 - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right\}. \quad (4.32)$$

¹⁷The setup with four bosonic twist field insertions has two independent world-sheet contours (see appendix C) which we choose without loss of generality to be 0 to x and x to 1.

¹⁸Here $\tilde{C}(x) = \frac{-C(0, x, 1, x_\infty)}{x_\infty}$.

The knowledge of the four-point correlator (4.9) then allows us to determine the pure quantum part of the six-point correlators $\langle \partial Z \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$, $\langle \partial \bar{Z} \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ and $\langle \partial Z \partial Z \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$. After a few manipulations one obtains

$$\begin{aligned}
 & \langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle \\
 &= \alpha' \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(w-x)(z-1)]^{\alpha-1} [w(z-x)(w-1)]^{-\alpha}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
 & \times \left\{ (1-\alpha) \frac{z(w-x)(z-1)}{(z-w)^2} + \alpha \frac{w(z-x)(w-1)}{(z-w)^2} \right. \\
 & \left. + \frac{1}{2} \alpha (1-\alpha) \left(x \frac{{}_2F_1[1-\alpha, \alpha, 2, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} - (1-x) \frac{{}_2F_1[1-\alpha, \alpha, 2, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} \right) \right\}.
 \end{aligned} \tag{4.33}$$

For $\langle \partial \bar{Z}_{qu} \partial \bar{Z}_{qu} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ one obtains

$$\begin{aligned}
 & \langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle \\
 &= \alpha' \alpha e^{2i\pi\alpha} \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(z-1)w(w-1)]^{-\alpha} [(z-x)(w-x)]^{\alpha-1}}{2\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
 & \times \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right\}
 \end{aligned} \tag{4.34}$$

while $\langle \partial Z_{qu} \partial Z_{qu} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ reads

$$\begin{aligned}
 & \langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle \\
 &= \alpha' (1-\alpha) e^{-2\pi i\alpha} \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(z-1)w(w-1)]^{-\alpha} [(z-x)(w-x)]^{\alpha-1}}{2\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
 & \times \left\{ 1 - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right\}.
 \end{aligned} \tag{4.35}$$

Adding the “mixed” part that contains the classical solutions ∂Z_{cl} and $\partial \bar{Z}_{cl}$, given by (4.4) as well as the world-sheet instanton contributions (4.12) we obtain for the three different six-point correlators:

$$\begin{aligned}
 & \langle \partial Z(z) \partial \bar{Z}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\alpha^+(1) \sigma_\alpha^-(\infty) \rangle \\
 &= \alpha' \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(w-x)(z-1)]^{\alpha-1} [w(z-x)(w-1)]^{-\alpha}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
 & \times \sum_{n,m} \left\{ (1-\alpha) \frac{z(w-x)(z-1)}{(z-w)^2} + \alpha \frac{w(z-x)(w-1)}{(z-w)^2} - \frac{\sin^2(\pi\alpha)}{4\pi^2} \frac{v_b^2[m] - e^{2i\pi\alpha} v_a^2[n] \tau^2(x)}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]^2} \right. \\
 & \left. + \frac{1}{2} \alpha (1-\alpha) \left(x \frac{{}_2F_1[1-\alpha, \alpha, 2, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} - (1-x) \frac{{}_2F_1[1-\alpha, \alpha, 2, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} \right) \right\} e^{-S_{cl}[n,m]}
 \end{aligned} \tag{4.36}$$

$$\begin{aligned}
& \langle \partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty) \rangle \tag{4.37} \\
&= \alpha' \alpha e^{2i\pi\alpha} \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(z-1)w(w-1)]^{-\alpha} [(z-x)(w-x)]^{\alpha-1}}{2\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
&\quad \times \sum_{n,m} \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\
&\quad \left. + \frac{\sin^2(\pi\alpha)}{2\pi^2\alpha} \frac{(v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

$$\begin{aligned}
& \langle \partial Z(z) \partial Z(w) \sigma_{\alpha}^{+}(0) \sigma_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty) \rangle \tag{4.38} \\
&= \alpha' (1-\alpha) e^{-2\pi i\alpha} \frac{\sqrt{\pi} [x(1-x)]^{-\alpha(1-\alpha)} [z(z-1)w(w-1)]^{\alpha-1} [(z-x)(w-x)]^{-\alpha}}{2\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[1-\alpha, \alpha, 1, x] {}_2F_1[1-\alpha, \alpha, 1, 1-x]} \\
&\quad \times \sum_{n,m} \left\{ 1 - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[1-\alpha, \alpha-1, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\
&\quad \left. + \frac{\sin^2(\pi\alpha)}{2\pi^2(1-\alpha)} \frac{(v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

with S_{cl} being the world-sheet instanton contribution given in equation (4.12).

Given these six-point correlators we can derive the various four-point correlators containing higher excited twist fields by taking the appropriate limits as we will exemplify momentarily. Taking the limit $z \rightarrow 0$ and $w \rightarrow x$ of (4.36) and using the OPE's (2.5) one gets

$$\begin{aligned}
& \langle \tau_{\alpha}^{+}(0) \tilde{\tau}_{\alpha}^{-}(x) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty) \rangle \tag{4.39} \\
&= \alpha' \frac{\sqrt{\pi} x^{-\alpha(3-\alpha)} (1-x)^{-\alpha(2-\alpha)}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\quad \times \sum_{n,m} \left\{ \alpha(1-x) + \frac{1}{2}\alpha(1-\alpha) \left(x \frac{{}_2F_1[1-\alpha, \alpha, 2, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} - (1-x) \frac{{}_2F_1[1-\alpha, \alpha, 2, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} \right) \right. \\
&\quad \left. - \frac{\sin^2(\pi\alpha)}{4\pi^2} \frac{v_b^2[m] - e^{2i\pi\alpha} v_a^2[n] \tau^2(x)}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

which after reinstating all x_i dependence gives (see, footnote 15 and recall $x = \frac{x_{12}x_{34}}{x_{13}x_{24}}$)

$$\begin{aligned}
& \langle \tau_{\alpha}^{+}(x_1) \tilde{\tau}_{\alpha}^{-}(x_2) \sigma_{\alpha}^{+}(1) \sigma_{\alpha}^{-}(\infty) \rangle \tag{4.40} \\
&= \alpha' \frac{\sqrt{\pi} x_{12}^{-\alpha(3-\alpha)} x_{34}^{-\alpha(1-\alpha)} \left(\frac{x_{13}x_{24}}{x_{14}x_{23}} \right)^{\alpha(2-\alpha)}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\quad \times \sum_{n,m} \left\{ \alpha(1-x) + \frac{1}{2}\alpha(1-\alpha) \left(x \frac{{}_2F_1[1-\alpha, \alpha, 2, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} - (1-x) \frac{{}_2F_1[1-\alpha, \alpha, 2, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} \right) \right. \\
&\quad \left. - \frac{\sin^2(\pi\alpha)}{4\pi^2} \frac{v_b^2[m] - e^{2i\pi\alpha} v_a^2[n] \tau^2(x)}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]} .
\end{aligned}$$

If we take the limit $z \rightarrow x$ and $w \rightarrow 0$ of (4.36) we get the four-point correlator $\langle \tilde{\tau}_\alpha^+ \tau_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$. Comparing this correlator to (4.39) confirms again the identifications

$$\tau_\alpha^- = \tau_{1-\alpha}^+ \quad \tilde{\tau}_\alpha^- = \tilde{\tau}_{1-\alpha}^+ \quad (4.41)$$

that were already suggested in section 4.1. Looking at other limits of the six-point correlators (4.36), (4.37) and (4.38) that give us four-point correlators containing excited bosonic twist fields we can generalize this identification to the higher excited twist fields ω^- and $\tilde{\omega}^-$. Thus we have

$$\omega_\alpha^- = \omega_{1-\alpha}^+ \quad \tilde{\omega}_\alpha^- = \tilde{\omega}_{1-\alpha}^+ . \quad (4.42)$$

In appendix D.1 we display all other four-point correlators arising from the six-point correlators (4.36), (4.37) and (4.38) by investigation of particular limits of z and w .

4.2 Four-point correlators with two independent angles

In this section we perform the same analysis as above, however, for the more generalized setup of two independent angles. The bosonic twist field insertions $\sigma_\alpha^+(x_1)$, $\sigma_\alpha^-(x_2)$, $\sigma_\beta^+(x_3)$ and $\sigma_\beta^-(x_4)$ lead to the following boundary conditions

$$\begin{aligned} \partial Z - \partial \bar{Z} &= 0 \quad \text{for } (-\infty, x_1) \cup (x_2, x_3) \cup (x_4, \infty) \\ e^{i\pi\alpha} \partial Z - e^{-i\pi\alpha} \partial \bar{Z} &= 0 \quad \text{for } (x_1, x_2) \\ e^{i\pi\beta} \partial Z - e^{-i\pi\beta} \partial \bar{Z} &= 0 \quad \text{for } (x_3, x_4) . \end{aligned} \quad (4.43)$$

Analogously to section 4.1 we will determine the correlators containing one or two excited bosonic twist fields via deriving five- and six-point correlators. Let us start with the derivation of the four-point correlators containing one excited bosonic twist field.

4.2.1 Four-point correlators containing one excited twist field

As in the subsection 4.1.1 the four-point correlators containing one excited bosonic twist field arise from the five-point correlators

$$\langle \partial Z(z) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \quad (4.44)$$

$$\langle \partial \bar{Z}(z) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle , \quad (4.45)$$

which has no purely quantum part, but is completely dictated by the classical part of ∂Z and $\partial \bar{Z}$. They have been computed in [77] (see also [75]) and take the form

$$\partial Z_{cl}(z) = \sqrt{\alpha'} \tilde{a} \omega_{1-\alpha, 1-\beta}(z) \quad \partial \bar{Z}_{cl}(z) = \sqrt{\alpha'} \tilde{b} \omega_{\alpha, \beta}(z) . \quad (4.46)$$

with $\omega_{\alpha, \beta}$ being (4.5) and \tilde{a} and \tilde{b} given by

$$\begin{aligned} \tilde{a} &= - \left(v_b[m] G_2[x] + \frac{\sin(\pi\alpha)}{\pi} e^{i\pi\alpha} v_a[n] (1-x)^{\alpha-\beta} B_1 H_1[1-x] \right) I^{-1}(x) \\ \tilde{b} &= - \left(v_b[m] G_1[x] - \frac{\sin(\pi\alpha)}{\pi} e^{i\pi\alpha} v_a[n] (1-x)^{\beta-\alpha} B_2 H_2[1-x] \right) I^{-1}(x) . \end{aligned} \quad (4.47)$$

Here $v_a[n]$ and $v_b[m]$ are given by (4.8) and we used the following definitions

$$\begin{aligned}
 B_1 &= \frac{\Gamma(\alpha)\Gamma(1-\beta)}{\Gamma(1+\alpha-\beta)} & B_2 &= \frac{\Gamma(\beta)\Gamma(1-\alpha)}{\Gamma(1+\beta-\alpha)} \\
 G_1[x] &= {}_2F_1[\alpha, 1-\beta, 1; x] & G_2[x] &= {}_2F_1[1-\alpha, \beta, 1; x] \\
 H_1[x] &= {}_2F_1[\alpha, 1-\beta, 1+\alpha-\beta; x] & H_2[x] &= {}_2F_1[1-\alpha, \beta, 1-\alpha+\beta; x]
 \end{aligned} \tag{4.48}$$

with which $I(x)$ takes the form

$$I(x) = B_1 G_2[x]H_1[1-x] + B_2 G_1[x]H_2[1-x]. \tag{4.49}$$

It is easy to check that at the limit $\beta \rightarrow \alpha$ the classical solution (4.46) simplifies to the one with just one independent angle (4.4). Moreover in the limit $x \rightarrow 1$ this solution goes over into the classical solution of three bosonic twist field insertions (3.5) discussed in section 3.

Together with the four point correlator [25, 46, 75, 77, 87]

$$\langle \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle = x^{-\alpha(1-\alpha)}(1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \sqrt{\frac{2\pi}{I(x)}} \tag{4.50}$$

we obtain for the two five point correlators

$$\begin{aligned}
 \langle \partial Z(z) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle &= -\sqrt{\alpha'} z^{-1+\alpha} (z-x)^{-\alpha} (z-1)^{-1+\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \\
 &\times \sqrt{2\pi} e^{-\pi i \beta} \sum_{n,m} \frac{\left(G_2[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} e^{i\pi\alpha} v_a[n] (1-x)^{\alpha-\beta} B_1 H_1[1-x] \right)}{I^{\frac{3}{2}}(x)} e^{-S_{cl}[n,m]} \tag{4.51}
 \end{aligned}$$

$$\begin{aligned}
 \langle \partial \bar{Z}(z) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle &= \sqrt{\alpha'} z^{-\alpha} (z-x)^{-1+\alpha} (z-1)^{-\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \\
 &\times \sqrt{2\pi} e^{\pi i \beta} \sum_{n,m} \frac{\left(G_1[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\beta-\alpha} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^{\frac{3}{2}}(x)} e^{-S_{cl}[n,m]}. \tag{4.52}
 \end{aligned}$$

The classical action can be computed in an analogous fashion as previously using (2.4) as well as (4.46) which gives [76] (see also [25])

$$S_{cl}^{T^2}[n, m] = \pi \sin(\pi\alpha) \left\{ |v_a[n]|^2 \left(\tau(x) + \frac{\rho}{2} \right) + |v_b[m] + \frac{\rho}{2} v_a[n]|^2 \left(\tau(x) + \frac{\rho}{2} \right)^{-1} \right\} \tag{4.53}$$

where $\tau(x)$ and ρ are

$$\tau(x) = \frac{1}{\pi} \sin(\pi\alpha) \frac{B_2 H_2[1-x]}{G_2[x]}, \quad \rho = -\frac{\sin(\pi(\alpha-\beta))}{\sin(\pi\beta)} \tag{4.54}$$

and $v_a[n]$ and $v_b[m]$ are defined as previously. Again we have to sum over all world-sheet instanton contribution, i.e. sum over all possible closed polygons connecting the four intersection points, which implies a sum over all integers p and q .

Given those five-point correlators (4.51) and (4.52) we can derive the desired four-point correlators. We illustrate this by applying this procedure to the five-point correlator (4.51). The limit $z \rightarrow 0$ gives

$$\langle \tau_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle = -\sqrt{2\pi\alpha'} e^{-i\pi\alpha} x^{-\alpha(2-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \quad (4.55)$$

$$\times \sum_{n,m} \frac{\left(G_2[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\alpha-\beta} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^{\frac{3}{2}}(x)} e^{-S_{cl}[n,m]}$$

which after reinstating the whole x_i dependence takes the form

$$\langle \tau_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle = -\sqrt{2\pi\alpha'} x_{12}^{-\alpha(2-\alpha)} x_{34}^{-\beta(1-\beta)} \left(\frac{x_{23}}{x_{13}} \right)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \left(\frac{x_{14}}{x_{24}} \right)^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}}$$

$$\times e^{-i\pi\alpha} \sum_{n,m} \frac{\left(G_2[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\alpha-\beta} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^{\frac{3}{2}}(x)} e^{-S_{cl}[n,m]} \quad (4.56)$$

where we used the definitions (4.48) as well as (4.49) and the world-sheet instanton contributions are given by (4.53). The other independent four-point correlator arising from the five-point correlator $\langle \partial \bar{Z} \sigma \sigma \sigma \sigma \rangle$ is displayed in appendix D.2.

4.2.2 Four-point correlators containing two excited twist fields

Four-point correlators containing two excited bosonic twist fields arise from the six-point correlators

$$\langle \partial Z(z) \partial Z(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \quad (4.57)$$

$$\langle \partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \quad (4.58)$$

$$\langle \partial Z(z) \partial \bar{Z}(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \quad (4.59)$$

which we derive momentarily, applying the same procedure as for the five-point correlators in section 3.2. Let us start again with the derivation of the pure quantum part, for which we define in a similar fashion to the case with one independent angle

$$g(z, w) = \frac{\langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle} \quad (4.60)$$

$$k(z, w) = \frac{\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle} \quad (4.61)$$

$$m(z, w) = \frac{\langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle}{\langle \sigma_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle} . \quad (4.62)$$

Applying the OPE's (2.5) the ansatz for the functions read

$$g(z, w) = \alpha' \omega_{1-\alpha, 1-\beta}(z) \omega_{\alpha, \beta}(w) \left\{ \frac{P}{(z-w)^2} + A(\{x_i\}) \right\} \quad (4.63)$$

$$k(z, w) = \alpha' \omega_{\alpha, \beta}(z) \omega_{\alpha, \beta}(w) B(\{x_i\}) \quad (4.64)$$

$$m(z, w) = \alpha' \omega_{1-\alpha, 1-\beta}(z) \omega_{1-\alpha, 1-\beta}(w) C(\{x_i\}) , \quad (4.65)$$

where $\omega_{\alpha,\beta}$ is given by (4.5) and P takes the form

$$\begin{aligned}
2P = & (1 - 4\xi)(z - x_1)(z - x_2)(w - x_3)(w - x_4) \\
& + (1 - \alpha - \beta + 2\xi)(z - x_1)(w - x_2)(z - x_3)(w - x_4) \\
& - (\alpha - \beta - 2\xi)(z - x_1)(w - x_2)(w - x_3)(z - x_4) \\
& + (\alpha - \beta + 2\xi)(w - x_1)(z - x_2)(z - x_3)(w - x_4) \\
& + (\alpha + \beta - 1 + 2\xi)(w - x_1)(z - x_2)(w - x_3)(z - x_4) \\
& + (1 - 4\xi)(w - x_1)(w - x_2)(z - x_3)(z - x_4) .
\end{aligned} \tag{4.66}$$

Here we again used various symmetries of the function $g(z, w)$ as well as the singular behavior (3.16). The parameter ξ is not fixed by any symmetry or the singular behavior, but as before in the one independent angle case we expect that $g(z, w)$ itself is independent of ξ . Thus we expect that the ξ dependence of P is compensated by $A(\{x_i\})$. The functions $A(\{x_i\})$, $B(\{x_i\})$ and $C(\{x_i\})$ will be determined momentarily using the monodromy constraints that read again (4.26) and (4.27). As before we fix the vertex operator positions to $x_1 = 0$, $x_2 = x$, $x_3 = 1$ and $x_4 = x_\infty$ using $SL(2, \mathbf{R})$ symmetry, divide the monodromy constraint (4.26) by $\omega_{\alpha,\beta}(w)$ and the monodromy constraint (4.27) by $\omega_{1-\alpha,1-\beta}(z)$ and eventually take the limit $w \rightarrow \infty$ and $z \rightarrow \infty$, respectively. This will lead to the monodromy constraints

$$\int_x^1 dz (\tilde{g}(z) - \tilde{k}(z)) = 0 , \quad \int_0^x dz (e^{i\pi\alpha}\tilde{g}(z) - e^{-i\pi\alpha}\tilde{k}(z)) = 0 \tag{4.67}$$

$$\int_x^1 dz (\tilde{g}'(z) - \tilde{m}(z)) = 0 , \quad \int_0^x dz (e^{i\pi\alpha}\tilde{g}'(z) - e^{-i\pi\alpha}\tilde{m}(z)) = 0 \tag{4.68}$$

with

$$\tilde{g}(z) = \alpha' \tilde{\omega}_{1-\alpha,1-\beta}(z) \left\{ \frac{4\xi - 1 + (1 - \alpha - \beta - 2\xi)x + 2\beta z}{2} + \tilde{A}(x) \right\} \tag{4.69}$$

$$\tilde{g}'(z) = \alpha' \tilde{\omega}_{\alpha,\beta}(z) \left\{ \frac{4\xi - 1 + (\alpha + \beta - 1 - 2\xi)x + 2(\beta - 1)z}{2} + \tilde{A}(x) \right\} \tag{4.70}$$

$$\tilde{k}(z) = \alpha' \tilde{B}(x)\tilde{\omega}_{\alpha,\beta}(z) \quad \tilde{m}(z) = \alpha' \tilde{C}(x)\tilde{\omega}_{1-\alpha,1-\beta}(z) . \tag{4.71}$$

Here \tilde{A} , \tilde{B} , \tilde{C} and $\tilde{\omega}$ are defined as in the previous subsection (see eq. (4.29) and footnote 18). Solving the system of equations (4.67) and (4.68) gives

$$\begin{aligned}
\tilde{A} = & \frac{1}{2}(1 - 4\xi) + \frac{1}{2}(\alpha + \beta - 1 + 2\xi)x - \alpha \\
& - \beta \frac{B_2 G_1[x]}{I(x)} {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1 - x] + \alpha \frac{B_2 G_1[x] H_2[1 - x]}{I(x)} \\
& - \alpha(1 - x)^{\beta-\alpha} \frac{B_2 H_2[1 - x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] + \alpha(1 - x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]}
\end{aligned} \tag{4.72}$$

$$\begin{aligned} \tilde{B} &= \beta e^{2\pi i \beta} (1-x)^{\beta-\alpha} \\ &\times \left\{ \frac{\alpha B_2 G_1[x] H_2[1-x]}{\beta I(x)} - \frac{B_2 G_1[x]}{I(x)} {}_2F_1[-\alpha, \beta, 1-\alpha+\beta, 1-x] \right. \\ &\quad \left. - \frac{\alpha}{\beta} (1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] \right\} \end{aligned} \quad (4.73)$$

$$\begin{aligned} \tilde{C} &= (1-\alpha) e^{-2\pi i \beta} (1-x)^{\alpha-\beta} \frac{B_1}{I(x)} \\ &\times \left\{ G_2[x] H_1[1-x] - \frac{1-\beta}{1-\alpha} x G_2[x] {}_2F_1[2-\beta, \alpha, 1+\alpha-\beta, 1-x] \right. \\ &\quad \left. - (1-x) H_1[1-x] {}_2F_1[2-\alpha, \beta, 1, x] \right\}, \end{aligned} \quad (4.74)$$

where we used the definitions of (4.48) and (4.49). Note that as required the functions $\tilde{g}(z)$ and $\tilde{g}'(z)$ are indeed independent of the choice of ξ .

Together with the expression for the four point correlator $\langle \sigma_\alpha^+ \sigma_\alpha^- \sigma_\beta^+ \sigma_\beta^- \rangle$ (4.50) one obtains for the quantum six-point correlators

$$\langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.75)$$

$$\begin{aligned} &= \alpha' \frac{\sqrt{2\pi} [z(w-x)]^{\alpha-1} [w(z-x)]^{-\alpha} (z-1)^{\beta-1} (w-1)^{-\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\ &\times \frac{1}{2} \left\{ \frac{z(z-x)(w-1) + (1-\alpha-\beta)z(w-x)(z-1) - (\alpha-\beta)z(w-x)(w-1)}{(z-w)^2} \right. \\ &\quad + \frac{(\alpha-\beta)w(z-x)(z-1) + (\alpha+\beta-1)w(z-x)(w-1) + w(w-x)(z-1)}{(z-w)^2} \\ &\quad + 1 - 2\alpha + (\alpha+\beta-1)x + 2 \frac{B_2 G_1[x]}{I(x)} (\alpha H_2[1-x] - \beta {}_2F_1[-\alpha, \beta, 1-\alpha+\beta, 1-x]) \\ &\quad \left. - 2\alpha(1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] + 2\alpha(1-x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]} \right\} \end{aligned}$$

$$\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.76)$$

$$\begin{aligned} &= \alpha' \frac{\sqrt{2\pi} [zw]^{-\alpha} [(z-x)(w-x)]^{\alpha-1} [(z-1)(w-1)]^{-\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\ &\times e^{2\pi i \beta} (1-x)^{\beta-\alpha} \frac{B_2}{I(x)} \left\{ \alpha G_1[x] H_2[1-x] - \beta G_1[x] {}_2F_1[-\alpha, \beta, 1-\alpha+\beta, 1-x] \right. \\ &\quad \left. - \alpha (1-x)^{\beta-\alpha} H_2[1-x] {}_2F_1[-\alpha, \beta, 1, x] \right\} \end{aligned}$$

$$\langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.77)$$

$$\begin{aligned} &= \alpha' \frac{\sqrt{2\pi} [zw]^{-\alpha-1} [(z-x)(w-x)]^{-\alpha} [(z-1)(w-1)]^{\beta-1} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\ &\times (1-\alpha) e^{-2\pi i \beta} (1-x)^{\alpha-\beta} \frac{B_1}{I(x)} \left\{ - (1-x) H_1[1-x] {}_2F_1[2-\alpha, \beta, 1, x] \right. \\ &\quad \left. + G_2[x] H_1[1-x] - \frac{1-\beta}{1-\alpha} x G_2[x] {}_2F_1[2-\beta, \alpha, 1+\alpha-\beta, 1-x] \right\}. \end{aligned}$$

Adding the mixed part containing the classical solutions ∂Z_{cl} and $\partial \bar{Z}_{cl}$ (4.46) as well as the world-sheet instantons (4.53)

$$\begin{aligned}
 & \langle \partial Z(z) \partial \bar{Z}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.78) \\
 &= \alpha' \frac{\sqrt{2\pi} [z(w-x)]^{\alpha-1} [w(z-x)]^{-\alpha} (z-1)^{\beta-1} (w-1)^{-\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\
 & \times \frac{1}{2} \sum_{n,m} e^{-S_{cl}[n,m]} \left\{ \frac{z(z-x)(w-1) + (1-\alpha-\beta)z(w-x)(z-1) - (\alpha-\beta)z(w-x)(w-1)}{(z-w)^2} \right. \\
 & + \frac{(\alpha-\beta)w(z-x)(z-1) + (\alpha+\beta-1)w(z-x)(w-1) + w(w-x)(z-1)}{(z-w)^2} \\
 & + 1 - 2\alpha + (\alpha+\beta-1)x + 2 \frac{B_2 G_1[x]}{I(x)} (\alpha H_2[1-x] - \beta {}_2F_1[-\alpha, \beta, 1-\alpha+b, 1-x]) \\
 & - 2\alpha(1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] + 2\alpha(1-x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]} \\
 & \left. - \frac{\left(G_1[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right) \left(G_2[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^2(x)} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \langle \partial \bar{Z}(z) \partial \bar{Z}(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.79) \\
 &= \alpha' \frac{\sqrt{2\pi} [z w]^{-\alpha} [(z-x)(w-x)]^{\alpha-1} [(z-1)(w-1)]^{-\beta} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\
 & \times e^{2\pi i \beta} \sum_{n,m} \left\{ (1-x)^{\beta-\alpha} \frac{B_2}{I(x)} \left\{ \alpha G_1[x] H_2[1-x] - \beta G_1[x] {}_2F_1[-\alpha, \beta, 1-\alpha+\beta, 1-x] \right. \right. \\
 & \left. \left. - \alpha (1-x)^{\beta-\alpha} H_2[1-x] {}_2F_1[-\alpha, \beta, 1, x] \right\} \right. \\
 & \left. + \frac{\left(G_1[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\beta-\alpha} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}
 \end{aligned}$$

$$\begin{aligned}
 & \langle \partial Z(z) \partial Z(w) \sigma_\alpha^+(0) \sigma_\alpha^-(x) \sigma_\beta^+(1) \sigma_\beta^-(\infty) \rangle \quad (4.80) \\
 &= \alpha' \frac{\sqrt{2\pi} [z w]^{\alpha-1} [(z-x)(w-x)]^{-\alpha} [(z-1)(w-1)]^{\beta-1} x^{-\alpha(1-\alpha)} (1-x)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}}{\sqrt{I(x)}} \\
 & \times e^{-2\pi i \beta} \sum_{n,m} \left\{ (1-\alpha)(1-x)^{\alpha-\beta} \frac{B_1}{I(x)} \left\{ - (1-x) H_1[1-x] {}_2F_1[2-\alpha, \beta, 1, x] \right. \right. \\
 & \left. \left. + G_2[x] H_1[1-x] - \frac{1-\beta}{1-\alpha} x G_2[x] {}_2F_1[2-\beta, \alpha, 1+\alpha-\beta, 1-x] \right\} \right. \\
 & \left. + \frac{\left(G_2[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\alpha-\beta} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]} .
 \end{aligned}$$

Note that taking the limits $x \rightarrow 1$ of the six-point functions (4.78), (4.79) and (4.80) as well as use the OPE

$$\sigma_{\alpha}^{-}(x)\sigma_{\beta}^{+}(1) \sim \left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1+\alpha-\beta)\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta-\alpha)\Gamma(1-\beta)}\right)^{\frac{1}{4}} (1-x)^{-\alpha(1-\beta)}\sigma_{\beta-\alpha}^{+}(1) \quad (4.81)$$

one obtains the five-point functions (3.30), (3.31) and (3.32), which were evaluated in the previous section, respectively.

Given all the six-point correlators we determine the four-point correlators containing the excited bosonic twist fields by taking various limits of the six-point correlators. Performing the limit $z \rightarrow 0, w \rightarrow x$ in (4.78) one obtains

$$\begin{aligned} &\langle \tau_{\alpha}^{+}(0)\tilde{\tau}_{\alpha}^{-}(x)\sigma_{\beta}^{+}(1)\sigma_{\beta}^{-}(\infty) \rangle \quad (4.82) \\ &= \alpha' \sqrt{\frac{2\pi}{I(x)}} x^{\alpha(\alpha-3)} (1-x)^{-\alpha(1-\beta) + \frac{\alpha}{2} - \frac{3\beta}{2}} \\ &\quad \times \sum_{n,m} e^{-S_{cl}[n,m]} \left\{ -\alpha(1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] \right. \\ &\quad + \frac{B_2 G_1[x]}{I(x)} (\alpha H_2[1-x] - \beta {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1-x]) + \alpha(1-x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]} \\ &\quad \left. - \frac{\left(G_1[x]v_b[m] + \frac{\sin(\pi\alpha)}{\pi} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right) \left(G_2[x]v_b[m] - \frac{\sin(\pi\alpha)}{\pi} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^2(x)} \right\} \end{aligned}$$

which after reinstalling all x_i dependence takes the form

$$\begin{aligned} &\langle \tau_{\alpha}^{+}(x_1)\tilde{\tau}_{\alpha}^{-}(x_2)\sigma_{\beta}^{+}(x_3)\sigma_{\beta}^{-}(x_4) \rangle \quad (4.83) \\ &= \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{\alpha(\alpha-3)} x_{34}^{-\beta(1-\beta)} \left(\frac{x_{14}x_{23}}{x_{13}x_{24}} \right)^{\alpha\beta - \frac{\alpha}{2} - \frac{3\beta}{2}} \\ &\quad \times \sum_{n,m} \left\{ -\alpha(1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] \right. \\ &\quad + \frac{B_2 G_1[x]}{I(x)} (\alpha H_2[1-x] - \beta {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1-x]) + \alpha(1-x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]} \\ &\quad \left. - \frac{\left(G_1[x]v_b[m] + \frac{\sin(\pi\alpha)}{\pi} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right) \left(G_2[x]v_b[m] - \frac{\sin(\pi\alpha)}{\pi} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^2(x)} \right\} \\ &\quad \times e^{-S_{cl}[n,m]}, \end{aligned}$$

where again $x = \frac{x_{12}x_{34}}{x_{13}x_{24}}$ and the world-sheet instanton contribution is given by (4.53). The remaining four-point correlator functions for two independent angles are displayed in appendix D.2.

5 Conclusions

In this paper we have derived the three- and four-point correlators containing excited bosonic twist fields. More specifically, we computed correlators that consist of regular bosonic twist fields σ as well as excited and doubly excited bosonic twist fields τ and ω , respectively. The knowledge of these correlators is required in the computation of lifetime and decay rates of string excitations that arise at intersections of D-branes. Assuming a low string scale scenario those stringy states can be significantly lighter than the first Regge excitations of the gauge bosons, and thus significantly lighter than the string scale and potentially observable at LHC.

In order to evaluate these correlators, we took a detour and determined five- and six-point correlators containing the conformal fields ∂Z and $\partial \bar{Z}$ as well as the regular bosonic twist fields σ . Given those five- and six-point correlators we performed various limits to derive the three- and four-point correlators containing higher excited bosonic twist fields.

The detailed computation of amplitudes that contain stringy states localized at the intersection of two D-brane stacks as well as their decay rates and lifetime is relegated to future work. However, let us already point out a few phenomenological implications of our results.

Our results reveal that the three-point correlator containing just one excited bosonic twist field τ does not contain a purely quantum part, but is dictated by the classical solution. That implies that the decay rate of the first excited stringy scalar state, localized at the intersection of D-brane stacks, into two massless fermions, depends crucially (not just via world-sheet instanton suppression) on the displacement in the compactification manifold between the stringy massive state and the massless states localized at different D-brane intersections.

On the other hand the displacement in the compactification manifold between different D-brane intersection points is related to the observed Yukawa coupling hierarchies of the massless fermions. This potentially allows one to obtain bounds on the decay rate of light stringy states in terms of observed mass hierarchies. We now have all of the components assembled to do such an analysis.

As already shown in [42], in $2 \rightarrow 2$ processes involving chiral superfields, only the *second* state in the tower is exchanged, and these processes would be suppressed by the Yukawa couplings - so the bounds on these states will be drastically weaker than those for string Regge excitations. Indeed, from dijet searches for related resonances [39–41], since the cross-section is so suppressed we can infer that the bounds on such states will be much less than a TeV. This raises the intriguing prospect that the string scale could be just out of reach of the LHC, but the light stringy states could be hiding in plain sight.

Finally, let us make some comments on how to generalize the results derived here.

The derivation procedure applied in this work can readily be extended to obtain also higher point correlators containing higher excited bosonic twist fields. Using the results of [77] for higher point correlators $\langle \sigma_{\alpha_1}^+(x_1) \dots \sigma_{\alpha_n}^+(x_n) \rangle$ we can analogously to the case of three and four bosonic twist field insertions, discussed in this work, derive the corresponding

correlators containing ∂Z and $\partial\bar{Z}$ and take the appropriate limits to obtain the correlators containing higher excited bosonic twist fields.

Moreover, one can also obtain in a similar fashion as laid out here the three and four-point correlators of higher excited bosonic twist fields, beyond the single and doubly excited fields τ , $\tilde{\tau}$, ω and $\tilde{\omega}$. In that case one first determines higher order correlators containing more than just two of the conformal fields ∂Z and $\partial\bar{Z}$ in an analogous way as done in this work and then performs the respective limits.

Making any concrete statements about the form of such correlators requires a detailed computation and is beyond the scope of this work. However, let us point out that the observation that the three-point correlator containing just one excited bosonic twist field τ does not contain a purely quantum part, but is dictated by the classical solution will generalize. Any N-point correlator that contains excited bosonic twist fields, whose order of excitation level adds up to an odd number will have no pure quantum part.

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A OPE’s of the twist fields

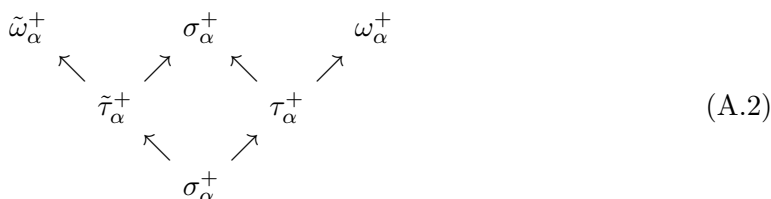
Here, we provide the OPE’s of $\partial Z(x)$ and $\partial\bar{Z}(x)$ on twisted fields:

$$\begin{aligned}
 \partial Z(z) \sigma_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \tau_{\alpha}^{+}(w) & \partial\bar{Z}(z) \sigma_{\alpha}^{+}(w) &\sim (z-w)^{-\alpha} \tilde{\tau}_{\alpha}^{+}(w) \\
 \partial Z(z) \tau_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \omega_{\alpha}^{+}(w) & \partial\bar{Z}(z) \tau_{\alpha}^{+}(w) &\sim (z-w)^{-\alpha-1} \sigma_{\alpha}^{+}(w) \\
 \partial Z(z) \omega_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \rho_{\alpha}^{+}(w) & \partial\bar{Z}(z) \omega_{\alpha}^{+}(w) &\sim (z-w)^{-\alpha-1} \tau_{\alpha}^{+}(w) \\
 \partial Z(z) \tilde{\tau}_{\alpha}^{+}(w) &\sim (z-w)^{-2+\alpha} \sigma_{\alpha}^{+}(w) & \partial\bar{Z}(z) \tilde{\tau}_{\alpha}^{+}(w) &\sim (z-w)^{-\alpha} \tilde{\omega}_{\alpha}^{+}(w) \\
 \partial Z(z) \sigma_{\alpha}^{-}(w) &\sim (z-w)^{-\alpha} \tau_{\alpha}^{-}(w) & \partial\bar{Z}(z) \sigma_{\alpha}^{-}(w) &\sim (z-w)^{\alpha-1} \tilde{\tau}_{\alpha}^{-}(w) \\
 \partial Z(z) \tau_{\alpha}^{-}(w) &\sim (z-w)^{-\alpha} \omega_{\alpha}^{-}(w) & \partial\bar{Z}(z) \tau_{\alpha}^{-}(w) &\sim (z-w)^{-2+\alpha} \sigma_{\alpha}^{-}(w) \\
 \partial Z(z) \tilde{\tau}_{\alpha}^{-}(w) &\sim (z-w)^{-1+\alpha} \sigma_{\alpha}^{-}(w) & \partial\bar{Z}(z) \tilde{\tau}_{\alpha}^{-}(w) &\sim (z-w)^{-1-\alpha} \tilde{\omega}_{\alpha}^{-}(w) \\
 \partial Z(z) \tilde{\omega}_{\alpha}^{-}(w) &\sim (z-w)^{-1+\alpha} \tilde{\tau}_{\alpha}^{-}(w) & \partial\bar{Z}(z) \tilde{\omega}_{\alpha}^{-}(w) &\sim (z-w)^{-1-\alpha} \tilde{\rho}_{\alpha}^{-}(w)
 \end{aligned} \tag{A.1}$$

Fields	conformal dimensions	Fields	conformal dimensions
σ_α^+	$\frac{1}{2}\alpha(1-\alpha)$	σ_α^-	$\frac{1}{2}\alpha(1-\alpha)$
τ_α^+	$\frac{1}{2}\alpha(3-\alpha)$	τ_α^-	$\frac{1}{2}(2+\alpha)(1-\alpha)$
ω_α^+	$\frac{1}{2}\alpha(5-\alpha)$	ω_α^-	$\frac{1}{2}(\alpha+4)(1-\alpha)$
$\tilde{\tau}_\alpha^+$	$\frac{1}{2}(\alpha+2)(1-\alpha)$	$\tilde{\tau}_\alpha^-$	$\frac{1}{2}\alpha(3-\alpha)$
$\tilde{\omega}_\alpha^+$	$\frac{1}{2}(\alpha+4)(1-\alpha)$	$\tilde{\omega}_\alpha^-$	$\frac{1}{2}\alpha(5-\alpha)$

Table 1. The conformal dimensions of bosonic twist fields.

Schematically, we can depict the above OPE's in the following diagram



where \nearrow, \swarrow denote the action of $\partial Z, \partial \bar{Z}$ on various twisted fields and can be easily extended to higher excited bosonic twist fields.

With these OPE's one can determine the conformal dimension of the respective twist fields. We summarize our findings in table 1. The above OPE's suggest the following identifications among twist- and anti-twist fields

$$\sigma_\alpha^-(z) = \sigma_{1-\alpha}^+(z) \quad \tau_\alpha^-(z) = \tau_{1-\alpha}^+(z) \quad \tilde{\tau}_\alpha^-(z) = \tilde{\tau}_{1-\alpha}^+(z) \tag{A.3}$$

which can be easily generalized to higher excited twist fields.

B Various integrals

Here we display all necessary integrals needed throughout this work.

$$\begin{aligned}
 \int_0^x dz \tilde{\omega}_{\alpha,\beta}(z) &= -e^{i\pi(\alpha-\beta)} B(1-\alpha, \alpha) {}_2F_1[\beta, 1-\alpha, 1, x] \\
 \int_x^1 dz \tilde{\omega}_{\alpha,\beta}(z) &= e^{-i\pi\beta} (1-x)^{\alpha-\beta} B(1-\beta, \alpha) {}_2F_1[\alpha, 1-\beta, 1+\alpha-\beta, 1-x] \\
 \int_0^x dz z \tilde{\omega}_{\alpha,\beta}(z) &= -e^{i\pi(\alpha-\beta)} x B(2-\alpha, \alpha) {}_2F_1[\beta, 2-\alpha, 2, x] \\
 \int_x^1 dz z \tilde{\omega}_{\alpha,\beta}(z) &= e^{-i\pi\beta} (1-x)^{\alpha-\beta} B(1-\beta, \alpha) {}_2F_1[-1+\alpha, 1-\beta, 1+\alpha-\beta, 1-x],
 \end{aligned}$$

where $B[\alpha, \beta]$ and ${}_2F_1[\alpha, \beta, \gamma, x]$ denote Euler-Beta - and the hypergeometric functions, respectively that take the form

$${}_2F_1[\alpha, \beta, \gamma, x] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dz z^{\beta-1} (1-z)^{\gamma-\beta-1} (1-xz)^{-\alpha} \tag{B.1}$$

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \int_0^1 dz z^{\alpha-1} (1-z)^{\beta-1} \tag{B.2}$$

with $\Gamma(\alpha)$ being $\Gamma(\alpha) = \int_0^\infty dz z^{\alpha-1} e^{-z}$.

C Vanishing of the correlators $\langle \partial Z_{qu} \sigma_\alpha \sigma_\beta \sigma_\gamma \rangle$

Here we demonstrate that the quantum part of amplitudes involving a single singly-excited twist field must vanish. To do this, we simply construct the function

$$f(z) \equiv \frac{\langle \partial Z_{qu}(z) \prod_i \sigma_{\alpha_i}^+(x_i) \rangle}{\langle \prod_i \sigma_{\alpha_i}^+(x_i) \rangle}, \quad \tilde{f}(z) \equiv \frac{\langle \partial \bar{Z}_{qu}(z) \prod_i \sigma_{\alpha_i}^+(x_i) \rangle}{\langle \prod_i \sigma_{\alpha_i}^+(x_i) \rangle} \quad (C.1)$$

where i runs from 0 to L . The functions $f(z)$ and $\tilde{f}(z)$ are holomorphic functions which do exhibit the poles

$$f(z) \sim (z - x_i)^{\alpha_i - 1}, \quad \tilde{f}(z) \sim (z - x_i)^{-\alpha_i}. \quad (C.2)$$

in agreement (2.5). Since $\partial Z, \partial \bar{Z}$ have unit conformal weight, we furthermore require $f(z), \tilde{f}(z) \sim z^{-2}$ as $z \rightarrow \infty$.

For L twist field insertions, there are $L - 2$ independent contours that we can choose for boundary conditions. There are $L - 1$ contours for L points, but the boundary must form a closed polygon, so one is not independent. We construct the cut differentials

$$\omega(z) \equiv \prod_i (z - x_i)^{\alpha_i - 1} \sim z^{M-L}, \quad \tilde{\omega}(z) \equiv \prod_i (z - x_i)^{-\alpha_i} \sim z^{-M} \quad (C.3)$$

where $M \equiv \sum \alpha_i$. Then we can write

$$f(z) \equiv \omega(z)p(z), \quad \tilde{f}(z) \equiv \tilde{\omega}(z)\tilde{p}(z) \quad (C.4)$$

so the number of differentials is given by the number of independent polynomials p, \tilde{p} which have order $L - M - 2, M - 2$, respectively. This is determined by counting the poles and allowing one parameter for the overall normalisation; the total number is thus $(L - M - 2 + 1) + (M - 2 + 1) = L - 2$. These are forced by the boundary conditions to be real parameters. We can then solve the boundary conditions for the quantum amplitudes

$$\int_{C_i} dZ = \int_{C_i} d\bar{Z} = 0. \quad (C.5)$$

These are a non-degenerate set of $L - 2$ equations in $L - 2$ real parameters, and hence all of the correlators containing just one ∂Z_{qu} are zero.

D Four-point correlators containing excited bosonic twist fields

D.1 Various four-point correlators with one independent angle

Here we display all the different four point correlators that can be derived from the six-point correlators (4.36), (4.37) and (4.38).¹⁹ From the correlator $\langle \partial \bar{Z} \sigma_\alpha^+ \sigma_{1-\alpha}^+ \sigma_\alpha^+ \sigma_{1-\alpha}^+ \rangle$ one obtains

$$\begin{aligned} \langle \tilde{\tau}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle &= -\sqrt{\alpha'} \left(\frac{x_{24}}{x_{12}x_{14}} \right)^{(1-\alpha)(1+\alpha)} \left(\frac{x_{13}}{x_{23}x_{34}} \right)^{\alpha(1-\alpha)} \\ &\times \frac{\sin^{\frac{3}{2}}(\pi\alpha)}{2\pi {}_2F_1^{\frac{1}{2}}[\alpha, 1-\alpha, 1, x] {}_2F_1^{\frac{3}{2}}[\alpha, 1-\alpha, 1, 1-x]} \sum_{n,m} (v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x)) e^{-S_{cl}[n,m]}. \end{aligned} \quad (D.1)$$

¹⁹Note that we do not display all possible limits since the missing correlators can be obtained by the identifications between boson twist and anti-twist fields.

From the correlator $\langle \partial Z \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ one gets

$$\begin{aligned}
& \langle \tau_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tilde{\tau}_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \tag{D.2} \\
&= -\alpha' \frac{\alpha(1-\alpha) \sqrt{\pi} (x_{12}x_{14})^{-\alpha(2-\alpha)} (x_{23}x_{34})^{-(1-\alpha)(1+\alpha)} x_{13}^{\alpha(1-\alpha)} x_{24}^{1+\alpha-\alpha^2}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\times \sum_{n,m} \left\{ x \frac{{}_2F_1[1-\alpha, \alpha, 2, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} - (1-x) \frac{{}_2F_1[1-\alpha, \alpha, 2, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} \right. \\
&\left. - \frac{\sin^2(\pi\alpha)}{2\pi^2\alpha(1-\alpha)} \frac{v_b^2[m] - e^{2i\pi\alpha} v_a^2[n] \tau^2(x)}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}.
\end{aligned}$$

From the correlator $\langle \partial \bar{Z} \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ one obtains

$$\begin{aligned}
& \langle \tilde{\omega}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \tag{D.3} \\
&= \alpha' \frac{\alpha \sqrt{\pi} \left(\frac{x_{24}}{x_{12}x_{14}}\right)^{(2+\alpha)(1-\alpha)} \left(\frac{x_{13}}{x_{23}x_{34}}\right)^{\alpha(1-\alpha)}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\times \sum_{n,m} \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\
&\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2\alpha} \frac{(v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

$$\begin{aligned}
& \langle \tilde{\tau}_\alpha^+(x_1) \tilde{\tau}_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \tag{D.4} \\
&= -\alpha' \frac{\alpha \sqrt{\pi} x_{12}^{-\alpha(1-\alpha)-1} \left(\frac{x_{13}}{x_{23}}\right)^{\alpha(2-\alpha)} \left(\frac{x_{24}}{x_{14}}\right)^{(1+\alpha)(1-\alpha)} x_{34}^{-\alpha(1-\alpha)}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\times \sum_{n,m} \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\
&\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2\alpha} \frac{(v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

$$\begin{aligned}
& \langle \tilde{\tau}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tilde{\tau}_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \tag{D.5} \\
&= \alpha' \frac{\alpha \sqrt{\pi} (x_{12}x_{14}x_{23}x_{34})^{-(\alpha+1)(1-\alpha)} x_{13}^{\alpha(1-\alpha)} x_{24}^{(1-\alpha)(\alpha+2)}}{2 \sqrt{\Gamma(\alpha) \Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\
&\times \sum_{n,m} \left\{ 1 - \frac{{}_2F_1[-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - \frac{{}_2F_1[-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\
&\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2\alpha} \frac{(v_b[m] - e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

while from the correlator $\langle \partial Z \partial Z \sigma_\alpha^+ \sigma_\alpha^- \sigma_\alpha^+ \sigma_\alpha^- \rangle$ one gets

$$\langle \omega_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \quad (\text{D.6})$$

$$\begin{aligned} &= \alpha' \frac{(1-\alpha)}{2} \frac{\sqrt{\pi} x_{12}^{-\alpha(3-\alpha)} \left(\frac{x_{13}}{x_{23}}\right)^{\alpha(1-\alpha)} \left(\frac{x_{24}}{x_{14}}\right)^{\alpha(3-\alpha)} x_{34}^{-\alpha(1-\alpha)}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\ &\times \sum_{n,m} \left\{ 1 - x \frac{{}_2F_1[2-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - (1-x) \frac{{}_2F_1[2-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\ &\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2(1-\alpha)} \frac{(v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]} \end{aligned}$$

$$\langle \tau_\alpha^+(x_1) \tau_\alpha^-(x_2) \sigma_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \quad (\text{D.7})$$

$$\begin{aligned} &= \alpha' \frac{(1-\alpha)}{2} \frac{\sqrt{\pi} x_{12}^{-\alpha(1-\alpha)-1} \left(\frac{x_{13}}{x_{23}}\right)^{(1+\alpha)(1-\alpha)} \left(\frac{x_{24}}{x_{14}}\right)^{\alpha(2-\alpha)} x_{34}^{-\alpha(1-\alpha)}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\ &\times \sum_{n,m} \left\{ 1 - x \frac{{}_2F_1[2-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - (1-x) \frac{{}_2F_1[2-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\ &\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2(1-\alpha)} \frac{(v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]} \end{aligned}$$

$$\langle \tau_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tau_\alpha^+(x_3) \sigma_\alpha^-(x_4) \rangle \quad (\text{D.8})$$

$$\begin{aligned} &= \alpha' \frac{(1-\alpha)}{2} \frac{\sqrt{\pi} (x_{12} x_{14} x_{23} x_{24})^{-\alpha(2-\alpha)} x_{13}^{\alpha(1-\alpha)} x_{24}^{\alpha(3-\alpha)}}{\sqrt{\Gamma(\alpha)\Gamma(1-\alpha)} {}_2F_1[\alpha, 1-\alpha, 1, x] {}_2F_1[\alpha, 1-\alpha, 1, 1-x]} \\ &\times \sum_{n,m} \left\{ 1 - x \frac{{}_2F_1[2-\alpha, \alpha, 1, 1-x]}{{}_2F_1[1-\alpha, \alpha, 1, 1-x]} - (1-x) \frac{{}_2F_1[2-\alpha, \alpha, 1, x]}{{}_2F_1[1-\alpha, \alpha, 1, x]} \right. \\ &\left. + \frac{\sin^2(\pi\alpha)}{2\pi^2(1-\alpha)} \frac{(v_b[m] + e^{i\pi\alpha} v_a[n] \tau(x))^2}{{}_2F_1[\alpha, 1-\alpha, 1, 1-x]^2} \right\} e^{-S_{cl}[n,m]} \end{aligned}$$

D.2 Various four-point correlators with two independent angles

Here we display all the different four point correlators that can be derived from the five point correlator (4.52) and the six-point correlators, (4.75), (4.76) and (4.77).

From the five-point correlator $\langle \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\beta^+ \sigma_\beta^- \rangle$ we obtain the four-point correlator

$$\langle \tilde{\tau}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \quad (\text{D.9})$$

$$\begin{aligned} &= \frac{\sqrt{2\pi\alpha'} x_{12}^{-1+\alpha^2} \left(\frac{x_{13}}{x_{23}}\right)^{-\alpha\beta+\frac{\alpha}{2}+\frac{\beta}{2}} \left(\frac{x_{14}}{x_{24}}\right)^{\alpha\beta+\frac{\alpha}{2}-\frac{\beta}{2}-1} x_{34}^{(\beta-1)\beta}}{\sqrt{I(x)}} \\ &\times \sum_{n,m} \frac{\left(G_1[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\beta-\alpha} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I(x)} e^{-S_{cl}[n,m]} \end{aligned}$$

From the six-point correlator $\langle \partial Z \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\beta^+ \sigma_\beta^- \rangle$ we obtain the four-point correlators

$$\begin{aligned}
 & \langle \tau_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tilde{\tau}_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.10} \\
 &= \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-\alpha(2-\alpha)} x_{13}^{-\alpha\beta-\frac{\alpha}{2}+\frac{3\beta}{2}} x_{14}^{\alpha\beta-\frac{\alpha}{2}-\frac{3\beta}{2}} x_{23}^{\alpha\beta+\frac{\alpha}{2}-\frac{\beta}{2}-1} x_{24}^{-\alpha\beta+\frac{\alpha}{2}+\frac{\beta}{2}+1} x_{34}^{-(1+\beta)(1-\beta)} \\
 & \times \sum_{n,m} e^{-S_{cl}[n,m]} \left\{ -a(1-x) + \alpha \frac{B_2 G_1[x] H_2[1-x]}{I(x)} - a(1-x)^{\beta-\alpha} \frac{B_2 H_2[1-x]}{I(x)} {}_2F_1[-\alpha, \beta, 1, x] \right. \\
 & - b \frac{B_2 G_1[x]}{I(x)} {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1 - x] + \alpha(1-x)^{\beta-\alpha} \frac{{}_2F_1[-\alpha, \beta, 1, x]}{G_1[x]} \\
 & \left. - \frac{\left(G_1[x] v_b[m] + \frac{\sin(\pi\alpha)}{\pi} B_1 H_1[1-x] e^{i\pi\alpha} v_a[n] \right) \left(G_2[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)}{I^2(x)} \right\}
 \end{aligned}$$

while from the six-point correlator $\langle \partial \bar{Z} \partial \bar{Z} \sigma_\alpha^+ \sigma_\alpha^- \sigma_\beta^+ \sigma_\beta^- \rangle$ we get the four-point correlators

$$\begin{aligned}
 & \langle \tilde{\omega}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.11} \\
 &= \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-(\alpha+2)(1-\alpha)} x_{13}^{-\alpha\beta+\frac{3\alpha}{2}-\frac{\beta}{2}} x_{14}^{\alpha\beta+\frac{\alpha}{2}+\frac{\beta}{2}-2} x_{23}^{\alpha\beta-\frac{3\alpha}{2}+\frac{\beta}{2}} x_{24}^{-\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}+2} x_{34}^{-\beta(1-\beta)} \\
 & \times \sum_{n,m} \left\{ \frac{B_2}{I(x)} \left(\alpha G_1[x] H_2[1-x] - \beta G_1[x] {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1 - x] \right. \right. \\
 & \left. \left. - \alpha(1-x)^{\beta-\alpha} H_2[1-x] {}_2F_1[-\alpha, \beta, 1, x] \right) \right. \\
 & \left. + \frac{\left(G_1[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\beta-\alpha} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}
 \end{aligned}$$

$$\begin{aligned}
 & \langle \tilde{\tau}_\alpha^+(x_1) \tilde{\tau}_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.12} \\
 &= -\alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-\alpha(1-\alpha)-1} x_{13}^{-\alpha\beta+\frac{3\alpha}{2}+\frac{\beta}{2}} x_{14}^{\alpha\beta+\frac{\alpha}{2}-\frac{\beta}{2}-1} x_{23}^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}} x_{24}^{-\alpha\beta-\frac{\alpha}{2}+\frac{\beta}{2}+1} x_{34}^{-\beta(1-\beta)} \\
 & \times \sum_{n,m} \left\{ \frac{B_2}{I(x)} \left(\alpha G_1[x] H_2[1-x] - \beta G_1[x] {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1 - x] \right. \right. \\
 & \left. \left. - \alpha(1-x)^{\beta-\alpha} H_2[1-x] {}_2F_1[-\alpha, \beta, 1, x] \right) \right. \\
 & \left. + \frac{\left(G_1[x] v_b[m] - \frac{\sin(\pi\alpha)}{\pi} (1-x)^{\beta-\alpha} B_2 H_2[1-x] e^{i\pi\alpha} v_a[n] \right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}
 \end{aligned}$$

$$\begin{aligned}
 & \langle \tilde{\tau}_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tilde{\tau}_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.13} \\
 &= \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-(\alpha+1)(1-\alpha)} x_{13}^{-\alpha\beta+\frac{\alpha}{2}+\frac{\beta}{2}} x_{14}^{\alpha\beta+\frac{\alpha}{2}-\frac{\beta}{2}-1} x_{23}^{\alpha\beta-\frac{\alpha}{2}+\frac{\beta}{2}-1} x_{24}^{-\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}+2} x_{34}^{-(\beta+1)(1-\beta)} \\
 & \times \sum_{n,m} \left\{ \frac{B_2}{I(x)} \left(\alpha G_1[x] H_2[1-x] - \beta G_1[x] {}_2F_1[-\alpha, \beta, 1 - \alpha + \beta, 1 - x] \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& -\alpha(1-x)^{\beta-\alpha}H_2[1-x]{}_2F_1[-\alpha,\beta,1,x]) \\
& + \frac{\left(G_1[x]v_b[m] - \frac{\sin(\pi\alpha)}{\pi}(1-x)^{\beta-\alpha}B_2H_2[1-x]e^{i\pi\alpha}v_a[n]\right)^2}{I^2(x)} \Big\} e^{-S_{cl}[n,m]}.
\end{aligned}$$

From the six-point correlator $\langle \partial Z \partial Z \sigma_\alpha^+ \sigma_\alpha^- \sigma_\beta^+ \sigma_\beta^- \rangle$ one can derive the following four-point correlators

$$\langle \omega_\alpha^+(x_1) \sigma_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.14}$$

$$\begin{aligned}
& = \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-\alpha(3-\alpha)} x_{13}^{-\alpha\beta-\frac{\alpha}{2}+\frac{3\beta}{2}} x_{14}^{\alpha\beta-\frac{3\alpha}{2}-\frac{3\beta}{2}} x_{23}^{\alpha\beta+\frac{\alpha}{2}-\frac{3\beta}{2}} x_{24}^{-\alpha\beta+\frac{3\alpha}{2}+\frac{3\beta}{2}} x_{34}^{-\beta(1-\beta)} \\
& \times \sum_{n,m} \left\{ (1-\alpha) \frac{B_1}{I(x)} \left\{ G_2[x]H_1[1-x] - (1-x)H_1[1-x]{}_2F_1[2-\alpha,\beta,1,x] \right. \right. \\
& \left. \left. - \frac{1-\beta}{1-\alpha} x G_2[x]{}_2F_1[2-\beta,\alpha,1+\alpha-\beta,1-x] \right\} \right. \\
& \left. + \frac{\left(G_2[x]v_b[m] + \frac{\sin(\pi\alpha)}{\pi}(1-x)^{\alpha-\beta}B_1H_1[1-x]e^{i\pi\alpha}v_a[n]\right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

$$\langle \tau_\alpha^+(x_1) \tau_\alpha^-(x_2) \sigma_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.15}$$

$$\begin{aligned}
& = -\alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-\alpha(1-\alpha)-1} x_{13}^{-\alpha\beta-\frac{\alpha}{2}+\frac{\beta}{2}+1} x_{14}^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}} x_{23}^{\alpha\beta+\frac{\alpha}{2}-\frac{\beta}{2}-1} x_{24}^{-\alpha\beta+\frac{3\alpha}{2}+\frac{\beta}{2}} x_{34}^{-\beta(1-\beta)} \\
& \times \sum_{n,m} \left\{ (1-\alpha) \frac{B_1}{I(x)} \left(G_2[x]H_1[1-x] - (1-x)H_1[1-x]{}_2F_1[2-\alpha,\beta,1,x] \right. \right. \\
& \left. \left. - \frac{1-\beta}{1-\alpha} x G_2[x]{}_2F_1[2-\beta,\alpha,1+\alpha-\beta,1-x] \right\} \right. \\
& \left. + \frac{\left(G_2[x]v_b[m] + \frac{\sin(\pi\alpha)}{\pi}(1-x)^{\alpha-\beta}B_1H_1[1-x]e^{i\pi\alpha}v_a[n]\right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}
\end{aligned}$$

$$\langle \tau_\alpha^+(x_1) \sigma_\alpha^-(x_2) \tau_\beta^+(x_3) \sigma_\beta^-(x_4) \rangle \tag{D.16}$$

$$\begin{aligned}
& = \alpha' \sqrt{\frac{2\pi}{I(x)}} x_{12}^{-\alpha(2-\alpha)} x_{13}^{-\alpha\beta+\frac{\alpha}{2}+\frac{\beta}{2}} x_{14}^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}} x_{23}^{\alpha\beta-\frac{\alpha}{2}-\frac{3\beta}{2}} x_{24}^{-\alpha\beta+\frac{3\alpha}{2}+\frac{3\beta}{2}} x_{34}^{-\beta(2-\beta)} \\
& \times \sum_{n,m} \left\{ (1-\alpha) \frac{B_1}{I(x)} \left\{ G_2[x]H_1[1-x] - (1-x)H_1[1-x]{}_2F_1[2-\alpha,\beta,1,x] \right. \right. \\
& \left. \left. - \frac{1-\beta}{1-\alpha} x G_2[x]{}_2F_1[2-\beta,\alpha,1+\alpha-\beta,1-x] \right\} \right. \\
& \left. + \frac{\left(G_2[x]v_b[m] + \frac{\sin(\pi\alpha)}{\pi}(1-x)^{\alpha-\beta}B_1H_1[1-x]e^{i\pi\alpha}v_a[n]\right)^2}{I^2(x)} \right\} e^{-S_{cl}[n,m]}.
\end{aligned}$$

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