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Two-loop QCD corrections to Higgs $\rightarrow b + \bar{b} + g$ amplitude

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Abstract: Exclusive observables involving Higgs boson in association with jets are often well suited to study the Higgs boson properties. They are rates involving cuts on the final state jets or differential distributions of rapidity, transverse momentum of the observed Higgs boson. While they get dominant contributions from gluon initiated partonic subprocesses, it is important to include the subdominant ones coming from other channels. In this article, we study one such channel namely the Higgs production in association with a jet in bottom anti-bottom annihilation process. We compute relevant amplitude $H \rightarrow b+\bar{b}+g$ up to two loop level in QCD where Higgs couples to bottom quark through Yukawa coupling. We use projection operators to obtain the coefficients for each tensorial structure appearing in this process. We have demonstrated that the renormalized amplitudes do have the right infrared structure predicted by the QCD factorization in dimensional regularization. The finite parts of the one and two loop amplitudes are presented after subtracting the infrared poles using Catani's subtraction operators.

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Contents

1 Introduction

The tests of the Standard Model (SM) have been going on for several decades in various experiments and most of its predictions have been tested in an unprecedented accuracy. The recent discovery of Higgs boson by ATLAS [1] and CMS [2] collaborations at the Large Hadron Collider (LHC) puts the SM on firm footing. The Higgs boson results from Higgs mechanism that provides a framework for electroweak symmetry breaking. Elementary particles such as leptons, quarks, gauge bosons and Higgs boson acquire masses through the Higgs mechanism. The mass of the Higgs boson being a parameter of the theory can not be predicted by the SM and hence its discovery provides a valuable information on this. Results from Higgs searches at LEP [3] and Tevatron [4] were crucial ingredients to the recent discovery in narrowing down the search regions for the LHC collaborations. The direct searches at the LEP excluded Higgs of mass below 114.4 GeV and the precision electroweak measurements [5] hinted for Higgs boson in the mass less than 152 GeV at 95% confidence level (CL). Tevatron on the other hand excluded Higgs of mass in the range 162 – 166 GeV at 95% CL.

The dominant production mechanism for the Higgs production at the LHC is gluon gluon fusion through top quark loop. The subdominant ones come from vector boson fusion, associated production of Higgs with vector bosons and top anti-top pairs and bottom anti-bottom annihilation. The inclusive production cross section for the Higgs production is known to an unprecedented accuracy due to many breakthroughs in the computation of amplitudes, loop and phase space integrals. For gluon-gluon [6–14], vector boson fusion processes [15], and associated production with vector bosons [16, 17], the inclusive rates are known to NNLO accuracy in QCD. There are also studies related to the Higgs production in association with bottom quarks which were also motivated to study Higgs boson in certain SUSY models, namely MSSM. The coupling of bottom quarks become large in the large $\tan\beta$ region, where $\tan\beta$ is the vacuum expectation values of up and down type Higgs fields in the Higgs sector of MSSM. Such large couplings can enhance gluon fusion as well as bottom quark fusion subprocesses. Fully inclusive cross section for Higgs production in association with bottom quark to NNLO level accuracy is also known in the variable flavour scheme (VFS) [18–23], while it is known only up to NLO level in the fixed flavour scheme (FFS) [24–29]. In the VFS, one assumes the initial state bottom quarks inside the proton. They are there as a result of emission of collinear bottom anti-bottom states from the gluons intrinsically present inside the proton. They being collinear give large logs which need to be resummed. The resummed contribution is the source for non-vanishing bottom and anti-bottom parton distribution functions inside the proton in the VFS scheme.

The differential distributions for Higgs production and its decay to pair of photons [30] or massive vector bosons [31, 32] have also been known at NNLO level in QCD in the infinite top quark mass limit. Such exclusive observables allow direct comparison of theoretical predictions with experimental results which include kinematical cuts on the final state particles. In particular, observables with jet vetos enhance the significance of the signal considerably allowing us to study the properties of Higgs boson and its coupling to other SM particles. NNLO QCD prediction [33] for production of Higgs with one jet through effective gluon-gluon-higgs vertex in the infinite top quark mass limit is available, thanks to various ingredients that are computed to the required accuracy by different groups [34–37]. As the experimental accuracy improves, it will be important to include other subdominant production mechanisms. In this article, we provide the relevant one and two loop amplitudes for the process $H \to b + \bar{b} + g$ which is analytically continued also to obtain the production of Higgs boson with one jet in bottom anti-bottom annihilation, i.e., $b + \bar{b} \to H + g$, where Higgs couples to bottom quark through Yukawa coupling denoted by λ. We use VFS scheme throughout. This will be an important supplement to the Higgs boson with one jet at NNLO level as it includes the bottom quark effects in VFS scheme.

Beyond leading order in perturbation theory, one encounters large number of Feynman amplitudes with rich Lorentz and gauge structures. In addition, the loop integrals become increasingly complicated due to their multiple kinematic dependence. Generation of diagrams, simplification of Lorentz, Dirac and color indices can be done symbolically. Using integration by parts (IBP) and Lorentz invariant (LI) identities the large number of loop integrals can be reduced in a rather straight forward way to few master integrals (MI). The two loop MIs for four legs processes where all fields but one external leg are massless were solved by Gehrmann and Remiddi [38, 39] using an elegant method of differential equations.

In this article we present one and two loop QCD amplitudes for the process $H \to b + \bar{b} + g$ treating both bottom and other four light quarks massless. We do not include top quark in our analysis. To obtain infrared safe observables, we require, in addition to these two loop amplitudes, one loop corrected $H \to b + \bar{b} + 2$ partons and tree level $H \to b + \bar{b} + 3$ partons amplitudes. Note that they are individually infrared singular due to the presence of massless partons in the amplitudes. There exist several equally efficient frameworks which use these infrared sensitive contributions to combine them to obtain infrared safe observables. They go by the names sector decomposition [40–46], q_T-subtraction [47] and antenna subtraction [48–54] methods. More recently the method developed by Czakon using sector decomposition and FKS [55] phase space slicing, was applied to obtain top quark pair production [56–58] at NNLO level and NNLO QED corrections [59] to $Z \to e^+e^-$. Antenna subtraction was used to obtain NNLO QCD corrections to di-jet production at the LHC. The NNLO corrections to Higgs plus one jet resulting from only gluon-gluon-Higgs effective interaction are obtained recently in [37] making best use of the subtraction methods in an efficient way. The amplitudes presented in this article will constitute contributions coming from bottom-antibottom-higgs interactions to Higgs plus one jet observable at NNLO level. We have presented the amplitudes in the form suitable for easier implementation to study infra-red safe hadron level observables involving Higgs plus one jet at NNLO in QCD.

In the next section, we discuss the Lagrangian that describes coupling of Higgs boson with bottom quark, explain how the projector technique can be used to obtain the amplitudes and describe the renormalization and factorization properties of the amplitudes. Section 3 is dedicated to the computational details. Final results in compact form are given in section 4 and corresponding coefficients are given in the appendix. In section 5, we conclude with our findings.

2 Theory

The interaction part of the action involving bottom quarks and Higgs boson is given by

$$S_I^b = -\lambda \int d^4x\, \phi(x) \overline{\psi}_b(x) \psi_b(x) \tag{2.1}$$

where, $\psi_b(x)$ denotes the bottom quark field and $\phi(x)$ the scalar field. λ is the Yukawa coupling given by $\sqrt{2} m_b/v$, with the bottom quark mass m_b and the vacuum expectation value $v \approx 246\,\text{GeV}$. For the pseudoscalar Higgs of MSSM, we need to replace $\lambda\phi(x)\overline{\psi}_b(x)\psi_b(x)$ by $\tilde{\lambda}\tilde{\phi}(x)\overline{\psi}_b(x)\gamma_5\psi_b(x)$ in the above equation. The MSSM couplings are

$$\tilde{\lambda} = \begin{cases} -\frac{\sqrt{2} m_b \sin\alpha}{v \cos\beta}\,, & \tilde{\phi} = h\,, \\ \frac{\sqrt{2} m_b \cos\alpha}{v \cos\beta}\,, & \tilde{\phi} = H\,, \\ \frac{\sqrt{2} m_b \tan\beta}{v}\,, & \tilde{\phi} = A \end{cases}$$

respectively. The angle α is the measure of mixing of weak and mass eigenstates of neutral Higgs bosons. In the VFS scheme, except in the Yukawa coupling, m_b is taken to be zero like other light quarks in the theory. The number of active flavours is taken to be $n_f = 5$. We work in Feynman gauge throughout.

2.1 Notation and kinematics

We consider the decay of Higgs boson to a bottom quark, anti-bottom quark and a gluon

$$H(q) \longrightarrow b(p_1) + \bar{b}(p_2) + g(p_3)\,. \tag{2.2}$$

The associated Mandelstam variables are defined as

$$s \equiv (p_1 + p_2)^2, \qquad t \equiv (p_2 + p_3)^2, \qquad u \equiv (p_1 + p_3)^2 \tag{2.3}$$

which satisfy

$$s > 0,\ t > 0,\ u > 0,\quad s + t + u = M_H^2 \equiv Q^2 > 0 \tag{2.4}$$

where, M_H is the mass of the Higgs boson. We also define the following dimensionless invariants which appear in harmonic polylogarithms (HPL) [60] and 2dHPL [38, 39] as

$$x \equiv s/Q^2, \qquad y \equiv u/Q^2, \qquad z \equiv t/Q^2 \tag{2.5}$$

satisfying

$$0 < x < 1,\ 0 < y < 1,\ 0 < z < 1,\quad \text{and } x + y + z = 1. \tag{2.6}$$

Analytical continuation: In order to compute the Higgs + 1 jet production at hadron colliders, the decay amplitudes must be analytically continued to the appropriate kinematical regions. The corresponding processes are

$$\begin{aligned}
&1. \quad \bar{b}(-p_1) + b(-p_2) \to g(p_3) + H(p_4)\\
&2. \quad b(-p_2) + g(-p_3) \to b(p_1) + H(p_4)\\
&3. \quad \bar{b}(-p_1) + g(-p_3) \to \bar{b}(p_2) + H(p_4)
\end{aligned} \tag{2.7}$$

For the process 1, $Q^2 = M_H^2 > 0$, $s > 0$, $t < 0$ and $u < 0$. Hence we introduce the dimensionless parameters u_1 and v_1 with the following definitions

$$u_1 \equiv -\frac{u}{s}, \qquad v_1 \equiv \frac{Q^2}{s} \tag{2.8}$$

such that $0 < u_1 < 1$ and $0 < v_1 < 1$.

Similarly, for the process 2, $Q^2 = M_H^2 > 0$, $s < 0$, $t > 0$ and $u < 0$ and the dimensionless parameters are u_2 and v_2 with the following definitions

$$u_2 \equiv -\frac{u}{t}, \qquad v_2 \equiv \frac{Q^2}{t} \tag{2.9}$$

such that $0 < u_2 < 1$ and $0 < v_2 < 1$. The last one is trivially related to the second one.

2.2 The general structure of the amplitude

In this section, we describe how the amplitude for $H \to b + \bar{b} + g$ can be obtained using projector technique. Since the amplitude contains one external gluon, it can be expressed as

$$|\mathcal{M}\rangle = \mathcal{S}_\mu(b, \bar{b}; g)\varepsilon^\mu \tag{2.10}$$

where, ε^μ is the gluon polarization vector.

We observe the amplitude has the following general structure in terms of the coefficients A', A'' and A_2:

$$\mathcal{S}_\mu(b,\bar{b};g) = \bar{u}(p_1)\Big\{A'\ p_{1\mu} + A''\ p_{2\mu} + A_2\ \not{p}_3\gamma_\mu\Big\}v(p_2) \tag{2.11}$$

where, we have used $p_3.\varepsilon = 0$. QCD Ward identity gives

$$A'\ p_1.p_3 + A''\ p_2.p_3 = 0 \quad \Rightarrow \quad A' = -A''\ \frac{p_2.p_3}{p_1.p_3} \equiv A_1\ p_2.p_3\,. \tag{2.12}$$

Hence, the amplitude takes the following form:

$$\begin{aligned}\mathcal{S}_\mu(b,\bar{b};g)\ \varepsilon^\mu &= \bar{u}(p_1)\Big\{A_1\ (p_2.p_3\ p_{1\mu} - p_1.p_3\ p_{2\mu}) + A_2\ \not{p}_3\gamma_\mu\Big\}v(p_2)\ \varepsilon^\mu \\ &\equiv A_1\ \mathrm{T}_1 + A_2\ \mathrm{T}_2\,. \end{aligned}\tag{2.13}$$

The coefficients A_m $(m = 1, 2)$ can be obtained from the amplitude $|\mathcal{M}\rangle$ using appropriate projectors $\mathcal{P}(A_m)$

$$A_m = \sum_{\text{spins}} \mathcal{P}(A_m)\mathcal{S}_\mu(b,\bar{b};g)\ \varepsilon^\mu = \sum_{\text{spins}} \mathcal{P}(A_m)|\mathcal{M}\rangle \tag{2.14}$$

where, in d space-time dimensions, the projectors are found to be

$$\begin{aligned}\mathcal{P}(A_1) &= \frac{2(d-2)}{s^2\,t\,u\,(d-3)}{\mathrm{T}_1}^\dagger + \frac{1}{s\,t\,u\,(d-3)}{\mathrm{T}_2}^\dagger\,, \\ \mathcal{P}(A_2) &= \frac{1}{s\,t\,u\,(d-3)}{\mathrm{T}_1}^\dagger + \frac{1}{2\,t\,u\,(d-3)}{\mathrm{T}_2}^\dagger\,. \end{aligned}\tag{2.15}$$

Expanding the coefficients A_m in powers of strong coupling constant $a_s = g_s^2/16\pi^2$, we obtain

$$A_m = \frac{\lambda}{\mu_R^\epsilon}\ 4\pi\sqrt{a_s}T^a_{ij}\Big\{A_m^{(0)} + a_s A_m^{(1)} + a_s^2 A_m^{(2)} + \mathcal{O}(a_s^3)\Big\} \tag{2.16}$$

where, T^a are the Gell-Mann matrices, a is adjoint and i, j are fundamental indices of SU(3) and μ_R is the renormalization scale. These coefficients $A_m^{(l)}$ completely specify the amplitude order by order in perturbation theory.

As described in section 2.1, for Higgs + 1 jet production, the above amplitudes have to be suitably crossed and the coefficients A_m will be expressed in terms of corresponding u_i and v_i.

2.3 Ultraviolet renormalization

The Feynman amplitudes for the process $H \to b + \bar{b} + g$ beyond leading order develop ultraviolet divergences in QCD. We have used dimensional regularization to regulate them taking space-time dimension to be $d = 4 + \epsilon$. The scale μ_0 is introduced to scale the mass dimension of the dimension-full strong coupling constant in d dimensions. If we denote the dimensionless strong coupling constant by $\hat{g}_s$ in d dimensions, then the unrenormalized amplitude can be expanded in terms of $\hat{a}_s = \hat{g}_s^2/16\pi^2$ as

$$|\mathcal{M}\rangle = \frac{\hat{\lambda}}{\mu_0^\epsilon}S_\epsilon\left(\frac{\hat{a}_s}{\mu_0^\epsilon}S_\epsilon\right)^{\frac{1}{2}}\left\{|\hat{\mathcal{M}}^{(0)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon}S_\epsilon\right)|\hat{\mathcal{M}}^{(1)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon}S_\epsilon\right)^2|\hat{\mathcal{M}}^{(2)}\rangle + \mathcal{O}(\hat{a}_s^3)\right\} \tag{2.17}$$

where, $S_\epsilon = \exp[\frac{\epsilon}{2}(\gamma_E - \ln 4\pi)]$ with Euler constant $\gamma_E = 0.5772\ldots$, results from loop integrals beyond leading order. $|\hat{\mathcal{M}}^{(i)}\rangle$ is the unrenormalized color-space vector which represents the i^{th} loop amplitude. In $\overline{MS}$ scheme, the renormalized coupling constant $a_s \equiv a_s(\mu_R^2)$ at the renormalization scale μ_R is related to unrenormalized coupling constant $\hat{a}_s$ by

$$\begin{aligned}\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon &= \frac{a_s}{\mu_R^\epsilon} Z(\mu_R^2) \\ &= \frac{a_s}{\mu_R^\epsilon}\left[1 + a_s\left(\frac{1}{\epsilon} r_{a_{1;1}}\right) + a_s^2\left(\frac{1}{\epsilon^2} r_{a_{2;2}} + \frac{1}{\epsilon} r_{a_{2;1}}\right) + \mathcal{O}(a_s^3)\right]\end{aligned} \tag{2.18}$$

where,

$$r_{a_{1;1}} = 2\beta_0\,, \quad r_{a_{2;2}} = 4\beta_0^2\,, \quad r_{a_{2;1}} = \beta_1\,,$$

$$\beta_0 = \left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right), \quad \beta_1 = \left(\frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F n_f - 4C_F T_F n_f\right) \tag{2.19}$$

with $C_A = N$, $C_F = (N^2-1)/2N$, $T_F = 1/2$ and n_f is the number of active quark flavors. The bare coupling constant $\hat{\lambda}$ is renormalized using

$$\begin{aligned}\frac{\hat{\lambda}}{\mu_0^\epsilon} S_\epsilon &= \frac{\lambda}{\mu_R^\epsilon} Z_\lambda(\mu_R^2) \\ &= \frac{\lambda}{\mu_R^\epsilon}\left[1 + a_s\left(\frac{1}{\epsilon} r_{\lambda_{1;1}}\right) + a_s^2\left(\frac{1}{\epsilon^2} r_{\lambda_{2;2}} + \frac{1}{\epsilon} r_{\lambda_{2;1}}\right) + \mathcal{O}(a_s^3)\right],\end{aligned} \tag{2.20}$$

with $\lambda = \lambda(\mu_R^2)$ and

$$r_{\lambda_{1;1}} = 6C_F\,,\ r_{\lambda_{2;2}} = \left(18C_F^2 + 6\beta_0 C_F\right),\ r_{\lambda_{2,1}} = \left(\frac{3}{2}C_F^2 + \frac{97}{6}C_F C_A - \frac{10}{3}C_F T_F n_f\right). \tag{2.21}$$

Using the eq. (2.18) and eq. (2.20), we now can express $|\mathcal{M}\rangle$ (eq. (2.17)) in powers of renormalized a_s with UV finite matrix elements $|\mathcal{M}^{(i)}\rangle$

$$|\mathcal{M}\rangle = \frac{\lambda}{\mu_R^\epsilon}(a_s)^{\frac{1}{2}}\left(|\mathcal{M}^{(0)}\rangle + a_s|\mathcal{M}^{(1)}\rangle + a_s^2|\mathcal{M}^{(2)}\rangle + \mathcal{O}(a_s^3)\right) \tag{2.22}$$

where,

$$\begin{aligned}|\mathcal{M}^{(0)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{1}{2}}|\hat{\mathcal{M}}^{(0)}\rangle\,, \\ |\mathcal{M}^{(1)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{3}{2}}\left[\,|\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^\epsilon\left(\frac{r_{a_1}}{2} + r_{\lambda_1}\right)|\hat{\mathcal{M}}^{(0)}\rangle\,\right], \\ |\mathcal{M}^{(2)}\rangle &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{5}{2}}\left[\,|\hat{\mathcal{M}}^{(2)}\rangle + \mu_R^\epsilon\left(\frac{3r_{a_1}}{2} + r_{\lambda_1}\right)|\hat{\mathcal{M}}^{(1)}\rangle\right. \\ &\qquad\left. + \mu_R^{2\epsilon}\left(\frac{r_{a_2}}{2} - \frac{r_{a_1}^2}{8} + \frac{r_{a_1}}{2}r_{\lambda_1} + r_{\lambda_2}\right)|\hat{\mathcal{M}}^{(0)}\rangle\,\right]\end{aligned} \tag{2.23}$$

with

$$
\begin{aligned}
r_{a_1} &= \left(\frac{1}{\epsilon} r_{a_{1;1}}\right) , & r_{a_2} &= \left(\frac{1}{\epsilon^2} r_{a_{2;2}} + \frac{1}{\epsilon} r_{a_{2;1}}\right) , \\
r_{\lambda_1} &= \left(\frac{1}{\epsilon} r_{\lambda_{1;1}}\right) , & r_{\lambda_2} &= \left(\frac{1}{\epsilon^2} r_{\lambda_{2;2}} + \frac{1}{\epsilon} r_{\lambda_{2;1}}\right) .
\end{aligned} \tag{2.24}
$$

We describe the computation of unrenormalized amplitudes $|\hat{\mathcal{M}}^{(l)}\rangle, l = 0, 1, 2$ in section 3.

2.4 Infrared factorization

In addition to UV divergences, the amplitude suffers from soft and collinear divergences beyond leading order due to the presence of soft gluons and collinear massless partons in the loops. According to KLN theorem [61, 62], to obtain infrared safe observables, we need to include appropriate contributions coming from real emission processes along with mass factorization counter terms and to perform sum over degenerate configurations. Thanks to factorization properties of QCD amplitudes, the infrared divergence structure of the amplitudes is well understood. The earliest account on two loop QCD amplitudes was by Catani [63], who predicted the infrared poles in ϵ of multi-parton QCD amplitudes in dimensional regularization excluding two loop single pole. In [64], Sterman and Tejeda-Yeomans demonstrated the connection of single pole in ϵ to a soft anomalous dimension matrix, later computed in [65, 66] using factorization properties of the scattering amplitudes along with infrared evolution equations. The decomposition of single pole term into universal collinear and soft anomalous dimensions at two loop level in QCD was first observed in electromagnetic and Higgs form factors [67]. Becher and Neubert [68], using soft collinear effective theory, derived the exact formula for the infra-red divergences of scattering amplitudes with an arbitrary number of loops and legs in massless QCD including single pole in dimensional regularization. Gardi and Magnea also arrived at, a similar all order result [69] using Wilson lines for hard partons and soft and eikonal jet functions in dimensional regularization. Following Catani, we express the renormalized amplitudes $|\mathcal{M}^{(i)}\rangle$ in terms of the universal subtraction operators $\mathbf{I}_b^{(i)}(\epsilon)$ as follows[1]

$$
\begin{aligned}
|\mathcal{M}^{(1)}\rangle &= 2\, \mathbf{I}_b^{(1)}(\epsilon)\, |\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(1)fin}\rangle\,, \\
|\mathcal{M}^{(2)}\rangle &= 2\, \mathbf{I}_b^{(1)}(\epsilon)\, |\mathcal{M}^{(1)}\rangle + 4\, \mathbf{I}_b^{(2)}(\epsilon)\, |\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(2)fin}\rangle
\end{aligned} \tag{2.25}
$$

where,

$$
\begin{aligned}
\mathbf{I}_b^{(1)}(\epsilon) = \frac{1}{2}\frac{e^{-\frac{\epsilon}{2}\gamma_E}}{\Gamma(1+\frac{\epsilon}{2})}\Bigg\{ &\left(\frac{4}{\epsilon^2} - \frac{3}{\epsilon}\right)(C_A - 2C_F)\left[\left(-\frac{s}{\mu_R^2}\right)^{\frac{\epsilon}{2}}\right] \\
&+ \left(-\frac{4C_A}{\epsilon^2} + \frac{3C_A}{2\epsilon} + \frac{\beta_0}{2\epsilon}\right)\left[\left(-\frac{t}{\mu_R^2}\right)^{\frac{\epsilon}{2}} + \left(-\frac{u}{\mu_R^2}\right)^{\frac{\epsilon}{2}}\right]\Bigg\} ,
\end{aligned}
$$

[1]The numerical coefficients 2 and 4 with $\mathbf{I}^{(i)}$ come due to the different definition of a_s between ours and Catani.

$$\mathbf{I}_b^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}_b^{(1)}(\epsilon)\left[\mathbf{I}_b^{(1)}(\epsilon) - \frac{2\beta_0}{\epsilon}\right] + \frac{e^{\frac{\epsilon}{2}\gamma_E}\,\Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})}\left[-\frac{\beta_0}{\epsilon} + K\right]\mathbf{I}_b^{(1)}(2\epsilon)$$
$$+\left(2\mathbf{H}_q^{(2)}(\epsilon) + \mathbf{H}_g^{(2)}(\epsilon)\right) \tag{2.26}$$

with

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_F n_f\,, \tag{2.27}$$
$$\mathbf{H}_q^{(2)}(\epsilon) = \frac{1}{\epsilon}\Bigg\{C_A C_F\left(-\frac{245}{432} + \frac{23}{16}\zeta_2 - \frac{13}{4}\zeta_3\right) + C_F^2\left(\frac{3}{16} - \frac{3}{2}\zeta_2 + 3\zeta_3\right)$$
$$+ C_F n_f\left(\frac{25}{216} - \frac{1}{8}\zeta_2\right)\Bigg\}\,,$$
$$\mathbf{H}_g^{(2)}(\epsilon) = \frac{1}{\epsilon}\Bigg\{C_A^2\left(-\frac{5}{24} - \frac{11}{48}\zeta_2 - \frac{1}{4}\zeta_3\right) + C_A n_f\left(\frac{29}{54} + \frac{1}{24}\zeta_2\right) - \frac{1}{4}C_F n_f - \frac{5}{54}n_f^2\Bigg\}\,. \tag{2.28}$$

The born amplitude $|\mathcal{M}^{(0)}\rangle$ and the finite parts $|\mathcal{M}^{(l)fin}\rangle, l = 1, 2$ are process dependent and hence they are determined by explicit computation.

3 Calculation of the amplitudes

We now describe how we compute the coefficients A_m from the amplitudes $|\hat{\mathcal{M}}^{(l)}\rangle$ for the process $H \to b + \bar{b} + g$ up to two loop level in QCD perturbation theory. QGRAF [70] is used to generate the Feynman amplitudes for this process. There are 2 diagrams at tree level, 13 at one loop and 251 at two loops excluding tadpole and self energy corrections to the external legs.

Using FORM [71, 72] and Mathematica, output of the QGRAF is converted to a form suitable for further symbolic manipulation. Using the projectors given in eq. (2.15), we have projected out unrenormalized $\hat{A}_i$ from these amplitudes. They contain only scalar products among internal and external momenta. For the external on-shell gluon leg the physical polarization sum is done using

$$\sum_s \varepsilon^\mu(p_3, s)\varepsilon^{\nu*}(p_3, s) = -g^{\mu\nu} + \frac{p_3^\mu q^\nu + q^\mu p_3^\nu}{p_3 \cdot q} \tag{3.1}$$

where, p_3 is the gluon momentum and q is an arbitrary light-like 4-vector for which we choose $q = p_1$. The Lorentz contractions and Dirac algebra are done in $d = 4+\epsilon$ dimensions. The next step involves the evaluation of one and two loop tensor and scalar integrals. This is done by first reducing them to an irreducible set of MIs using IBP identities and LI identities and substituting the MIs evaluated to desired accuracy in ϵ. We have used a Mathematica package LiteRed [73, 74] to use IBP [75, 76] and LI identities [77] in an efficient manner. The MIs for the kinematic configuration of the problem at hand are analytically known from the seminal works of Gehrmann and Remiddi [38, 39]. We use them to obtain the unrenormalized coefficients in a Laurent series in ϵ. In order to optimize the use of LiteRed,

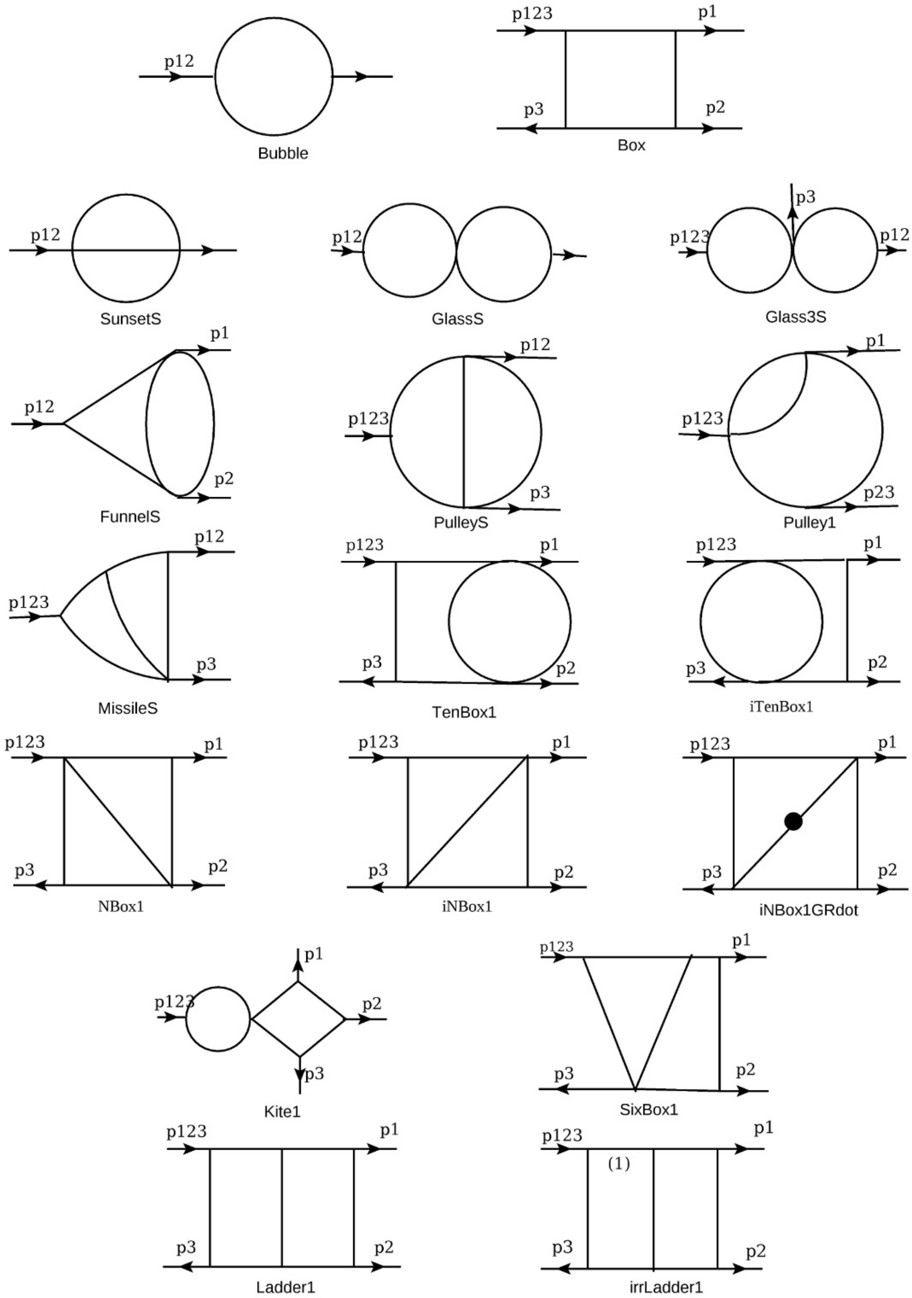

Figure 1. Planar topologies of master integrals.

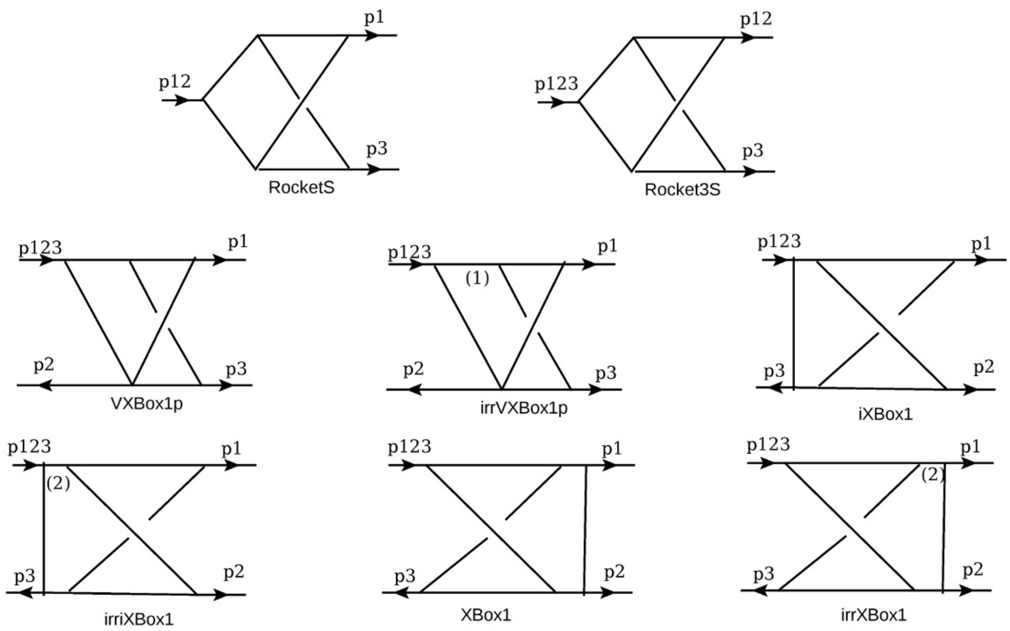

Figure 2. Non-planar topologies of master integrals.

we have reduced all the one and two loop integrals to belong to few integral sets. This is done by shifting the loop momenta suitably using an in-house algorithm which uses FORM. We find that the sets for both one and two loop integrals are exactly same as those given in [78] for the case of massive spin-2 resonance → 3 gluons. The topologies of the appearing planar and non-planar master integrals are shown in figure 1 and figure 2 respectively. For one-loop diagrams, the integral belongs to one of the following sets:

$$\{\mathcal{D},\, \mathcal{D}_1,\, \mathcal{D}_{12},\, \mathcal{D}_{123}\}\,, \{\mathcal{D},\, \mathcal{D}_2,\, \mathcal{D}_{23},\, \mathcal{D}_{123}\}\,, \{\mathcal{D},\, \mathcal{D}_3,\, \mathcal{D}_{31},\, \mathcal{D}_{123}\} \tag{3.2}$$

where,

$$\mathcal{D} = k_1^2,\; \mathcal{D}_i = (k_1 - p_i)^2,\; \mathcal{D}_{ij} = (k_1 - p_i - p_j)^2,\; \mathcal{D}_{ijk} = (k_1 - p_i - p_j - p_k)^2\,. \tag{3.3}$$

At two loops, we have nine independent Lorentz invariants involving loop momenta k_1 and k_2, namely $\{(k_\alpha \cdot k_\beta), (k_\alpha \cdot p_i)\}, \alpha, \beta = 1, 2;\; i = 1, \ldots, 3$. Shifting of loop momenta allows us to express each two loop Feynman integral to contain terms belonging to one of the following six sets:

$$\begin{aligned}
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;1},\, \mathcal{D}_{2;1},\, \mathcal{D}_{1;12},\, \mathcal{D}_{2;12},\, \mathcal{D}_{1;123},\, \mathcal{D}_{2;123}\}\,,\\
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;2},\, \mathcal{D}_{2;2},\, \mathcal{D}_{1;23},\, \mathcal{D}_{2;23},\, \mathcal{D}_{1;123},\, \mathcal{D}_{2;123}\}\,,\\
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;3},\, \mathcal{D}_{2;3},\, \mathcal{D}_{1;31},\, \mathcal{D}_{2;31},\, \mathcal{D}_{1;123},\, \mathcal{D}_{2;123}\}\,,\\
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;1},\, \mathcal{D}_{2;1},\, \mathcal{D}_{0;3},\, \mathcal{D}_{1;12},\, \mathcal{D}_{2;12},\, \mathcal{D}_{1;123}\}\,,\\
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;2},\, \mathcal{D}_{2;2},\, \mathcal{D}_{0;1},\, \mathcal{D}_{1;23},\, \mathcal{D}_{2;23},\, \mathcal{D}_{1;123}\}\,,\\
&\{\mathcal{D}_0,\, \mathcal{D}_1,\, \mathcal{D}_2,\, \mathcal{D}_{1;3},\, \mathcal{D}_{2;3},\, \mathcal{D}_{0;2},\, \mathcal{D}_{1;31},\, \mathcal{D}_{2;31},\, \mathcal{D}_{1;123}\}
\end{aligned} \tag{3.4}$$

where,

$$\mathcal{D}_0 = (k_1 - k_2)^2, \qquad \mathcal{D}_\alpha = k_\alpha^2, \qquad \mathcal{D}_{\alpha;i} = (k_\alpha - p_i)^2, \qquad \mathcal{D}_{\alpha;ij} = (k_\alpha - p_i - p_j)^2,$$
$$\mathcal{D}_{0;i} = (k_1 - k_2 - p_i)^2, \quad \mathcal{D}_{\alpha;ijk} = (k_\alpha - p_i - p_j - p_k)^2 \,. \tag{3.5}$$

The UV singularities present in the bare coefficients are systematically removed using eqs. (2.18) & (2.20). The resulting UV finite coefficients do contain divergences from soft and collinear partons. In the next section we will demonstrate that our results correctly reproduce divergences described in the section 2.4 at one and two loop level. We will also present the finite parts of the coefficients A_m up to two loop level.

4 Results

In this section we present the results up to two loop level in QCD for the amplitude $H \to b + \bar{b} + g$ in the $\overline{MS}$ scheme. The results are presented after subtracting the one and two loop universal subtraction operators $I_b^{(i)}(\epsilon), i = 1, 2$ as described in the section 2.4. Following the eqs. (2.13), (2.16) & (2.22), the l^{th} loop amplitude can be written as

$$|\mathcal{M}^{(l)}\rangle = 4\pi\, T^a_{ij} \Big\{ A_1^{(l)} \mathrm{T}_1 + A_2^{(l)} \mathrm{T}_2 \Big\} \,. \tag{4.1}$$

The renormalised coefficients $A_m^{(l)}$ are related to their bare counterparts $\hat{A}_m^{(l)}$ through (see eq. (2.23)):

$$\begin{aligned}
A_m^{(0)} &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{1}{2}} \hat{A}_m^{(0)} \,, \\
A_m^{(1)} &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{3}{2}} \left[\hat{A}_m^{(1)} + \mu_R^\epsilon \left(\frac{r_{a_1}}{2} + r_{\lambda_1}\right) \hat{A}_m^{(0)} \right] \,, \\
A_m^{(2)} &= \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{5}{2}} \left[\hat{A}_m^{(2)} + \mu_R^\epsilon \left(\frac{3 r_{a_1}}{2} + r_{\lambda_1}\right) \hat{A}_m^{(1)} \right. \\
&\qquad \left. + \mu_R^{2\epsilon} \left(\frac{r_{a_2}}{2} - \frac{r_{a_1}^2}{8} + \frac{r_{a_1}}{2} r_{\lambda_1} + r_{\lambda_2} \right) \hat{A}_m^{(0)} \right] \,.
\end{aligned} \tag{4.2}$$

Using the procedure discussed in the previous section, we first compute the bare coefficients $\hat{A}_m^{(l)}$ and the eqs. (4.2) give the renormalized coefficients. The finite parts of the coefficients $A_m^{(l)}$ are defined after subtracting terms proportional to universal subtraction terms $I_b^{(l)}$ as follows

$$\begin{aligned}
A_m^{(1)} &= 2\, \mathbf{I}_b^{(1)}(\epsilon)\, A_m^{(0)} + A_m^{(1)fin} \,, \\
A_m^{(2)} &= 2\, \mathbf{I}_b^{(1)}(\epsilon)\, A_m^{(1)} + 4\, \mathbf{I}_b^{(2)}(\epsilon)\, A_m^{(0)} + A_m^{(2)fin}
\end{aligned} \tag{4.3}$$

where, we have used eqs. (2.13) & (2.25).

Expanding the right sides of eqs. (4.2) & (4.3) in powers of ϵ, we find that the infrared poles agree exactly, providing a crucial test on the correctness of our computation. The

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finite parts of the coefficients have the following expansions:

$$A_m^{(l)fin} = \sum_{n=0}^{l} A_m^{(0)} \mathcal{B}_{m;n}^{(l)} \ln^n\Big(-\frac{Q^2}{\mu^2}\Big) \tag{4.4}$$

where,

$$A_1^{(0)} = -\frac{4i}{t\,u} \qquad \text{and} \qquad A_2^{(0)} = i\Big(\frac{1}{t}+\frac{1}{u}\Big) \tag{4.5}$$

and the remaining coefficients $\mathcal{B}_{m;n}^{(l)}$ are given in the appendix. We also performed an independent computation of $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(l)}\rangle$ for $l = 1, 2$ without using any projectors and then compared against one obtained using the projectors, i.e using the coefficients $A_m^{(l)}$. We find both give the same result, providing an independent check on our computation.

Following [79],[2] we have obtained results for the crossed reactions given in eq. (2.7) relevant for Higgs+1 jet production at hadron colliders. The corresponding finite coefficients $A_m^{(l)fin}$ are attached with the arXiv submission.

5 Conclusions

We have presented the amplitudes for the partonic subprocess $H \to b + \bar{b} + g$ and other subprocesses related by crossing, up to two loop level in QCD that contribute to exclusive observables involving Higgs boson and a jet. The dominant one is from gluon gluon fusion which is already known to this accuracy. We have used dimensional regularization to perform our computation. Using appropriate projectors, the amplitude is expressed in terms of two scalar coefficients A_m. We have found that the infrared structure of the amplitude is according to Catani's prediction on QCD amplitudes upto two loop level. Also, the coefficient of single pole term is found to be in agreement with predictions based on the observation of the universal behavior of poles in the multi-parton QCD amplitudes.

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A Harmonic polylogarithms

Here, we provide the definition of HPL and 2dHPL. HPL is represented by $H(\vec{m}_w; y)$ with a w-dimensional vector $\vec{m}_w$ of parameters and its argument y. The elements of $\vec{m}_w$ belong

[2] We thank T. Gehrmann for providing relevant analytically continued HPLs and 2d HPLs.

to $\{1, 0, -1\}$ through which we define the following rational functions

$$f(1; y) \equiv \frac{1}{1-y}, \qquad f(0; y) \equiv \frac{1}{y}, \qquad f(-1; y) \equiv \frac{1}{1+y}\,. \tag{A.1}$$

The weight 1 ($w = 1$) HPLs are

$$H(1, y) \equiv -\ln(1-y), \qquad H(0, y) \equiv \ln y, \qquad H(-1, y) \equiv \ln(1+y)\,. \tag{A.2}$$

For $w > 1$, the definition of $H(m, \vec{m}_w; y)$ is

$$H(m, \vec{m}_w; y) \equiv \int_0^y dx\; f(m, x)\; H(\vec{m}_w; x), \qquad\qquad m \in 0, \pm 1\,. \tag{A.3}$$

The 2dHPLs are defined in the same way as eq. (A.3) with the new elements $\{2, 3\}$ in $\vec{m}_w$ representing a new class of rational functions

$$f(2; y) \equiv f(1-z; y) \equiv \frac{1}{1-y-z}, \qquad f(3; y) \equiv f(z; y) \equiv \frac{1}{y+z} \tag{A.4}$$

and correspondingly with the weight 1 ($w = 1$) 2dHPLs

$$H(2, y) \equiv -\ln\left(1 - \frac{y}{1-z}\right), \qquad H(3, y) \equiv \ln\left(\frac{y+z}{z}\right). \tag{A.5}$$

B One-loop coefficients

$$\begin{aligned}
\mathcal{B}^{(1)}_{1;1} &= \frac{1}{6}(-11C_A - 18C_F + 2n_f) \\
\mathcal{B}^{(1)}_{1;0} &= \frac{C_A}{6}\Big(-6H(0,y)H(0,z) - 6H(0,y)H(1,z) - 6H(2,y)H(0,z) + 12H(3,y)\;H(1,z) - 10H(0,y) \\
&\quad -9H(2,y) - 6H(0,2,y) - 6H(2,0,y) + 12H(3,2,y) - 10H(0,z) - 9\;H(1,z) + 6H(0,1,z) \\
&\quad -6H(1,0,z) - 6\zeta_2\Big) + C_F\Big(2H(0,y)H(1,z) - 4H(3,y)H(1,z) + 2H(2,y)H(0,z) + 3H(2,y) \\
&\quad +12\;H(0,2,y) - 2H(1,0,y) + 2H(2,0,y) - 4H(3,2,y) + 3H(1,z) - 2H(0,1,z) - 2\Big) \\
&\quad +n_f\frac{1}{6}\Big(H(0,y) + H(0,z)\Big) \\
\mathcal{B}^{(1)}_{2;1} &= \frac{1}{6}(-11C_A - 18C_F + 2n_f) \\
\mathcal{B}^{(1)}_{2;0} &= \frac{C_A}{6}\Big(-6H(0,y)H(0,z) - 6H(0,y)H(1,z) - 6H(2,y)H(0,z) + 12H(3,y)\;H(1,z) - 10H(0,y) \\
&\quad -9H(2,y) - 6H(0,2,y) - 6H(2,0,y) + 12H(3,2,y) - 10H(0,z) - 9\;H(1,z) + 6H(0,1,z) \\
&\quad -6H(1,0,z) - 6\zeta_2 + 6\Big) + C_F\Big(2H(0,y)H(1,z) - 4H(3,y)H(1,z) + 2H(2,y)H(0,z) + 3H(2,y) \\
&\quad +2H(0,2,y) - 2\;H(1,0,y) + 2H(2,0,y) - 4H(3,2,y) + 3H(1,z) - 2H(0,1,z) - 3\Big) \\
&\quad +n_f\frac{1}{6}\Big(H(0,y) + H(0,z)\Big)
\end{aligned}$$

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C Two-loop coefficients

$$\mathcal{B}^{(2)}_{1;2} = \frac{1}{24}\Big(121C_A^2 + 44C_A(6C_F - n_f) + 4\Big(27C_F^2 - 12C_F n_f + n_f^2\Big)\Big)$$

$$\mathcal{B}^{(2)}_{1;1} = C_A^2\frac{1}{108}\Big(594H(0,y)H(0,z) + 594H(0,y)H(1,z) + 594H(2,y)H(0,z) - 1188\ H(3,y)H(1,z)$$
$$+990H(0,y) + 891H(2,y) + 594H(0,2,y) + 594H(2,0,y) - 1188\ H(3,2,y) + 990H(0,z)$$
$$+891H(1,z) - 594H(0,1,z) + 594H(1,0,z) + 495\ \zeta_2 - 108\zeta_3 - 702\Big)$$
$$+C_AC_F\frac{1}{108}\Big(- 1188H(0,y)H(1,z) + 54H(0,y)(6H(0,z) + 6H(1,z) + 10) - 1188\ H(2,y)H(0,z)$$
$$+54(6H(2,y) + 10)H(0,z) + 1728H(3,y)H(1,z) - 1296H(2,y) - 864\ H(0,2,y) + 1188H(1,0,y)$$
$$-864H(2,0,y) + 1728H(3,2,y) - 1296H(1,z) + 864H(0,1,z) + 324H(1,0,z) + 1566\zeta_2 - 2808\zeta_3 - 1048\Big)$$
$$+C_F^2\frac{1}{108}\Big(- 648H(0,y)H(1,z) - 648H(2,y)H(0,z) + 1296H(3,y)H(1,z) - 972H(2,y) - 648H(0,2,y)$$
$$+648H(1,0,y) - 648H(2,0,y) + 1296H(3,2,y) - 972H(1,z) + 648H(0,1,z) - 1296\zeta_2 + 2592\zeta_3 + 648\Big)$$
$$+C_An_f\frac{1}{108}\Big(- 2\Big(- 27(4H(3,y) - 3)H(1,z) + 81H(2,y) + 54H(0,2,y) + 54H(2,0,y) - 108H(3,2,y)$$
$$-54H(0,1,z) + 54H(1,0,z) + 45\zeta_2 - 206\Big) - 9H(0,y)\ (12H(0,z) + 12H(1,z) + 31) - 9(12H(2,y)$$
$$+31)H(0,z)\Big) + C_Fn_f\frac{1}{54}\Big(108H(0,y)H(1,z) + 108H(2,y)H(0,z) - 216H(3,y)H(1,z) - 27H(0,y)$$
$$+162\ H(2,y) + 108H(0,2,y) - 108H(1,0,y) + 108H(2,0,y) - 216H(3,2,y) - 27H(0,z)$$
$$+162\ H(1,z) - 108H(0,1,z) - 54\zeta_2 + 32\Big) + n_f^2\frac{1}{54}\Big(9H(0,y) + 9H(0,z) - 20\Big)$$

$$\mathcal{B}^{(2)}_{1;0} = C_A^2\Bigg\{\zeta_4\Big(39/8\Big) + \zeta_3\Bigg(- H(0,y) - 6H(1,y) + 11H(2,y) - H(0,z) + 5H(1,z) + 6(t+u)/s + 407/36\Bigg)$$
$$+\zeta_2\Bigg(108s^2H(0,y)H(0,z) + 72s^2H(1,y)H(0,z) + 108s^2H(0,y)H(1,z) + 72s^2\ H(1,y)H(1,z)$$
$$+36s^2H(2,y)H(0,z) - 72s^2H(3,y)H(1,z) + 147s^2H(0,y) + 453\ s^2H(2,y) + 108s^2H(0,2,y)$$
$$+72s^2H(1,2,y) + 36s^2H(2,0,y) - 72s^2\ H(3,2,y) + 147s^2H(0,z) + 237s^2H(1,z) + 36s^2H(0,1,z)$$
$$+36s^2H(1,0,z) + 72s^2H(1,1,z) - 216s(s+t)H(1,y) - 108stH(0,z) - 108suH(0,y) - 216suH(1,z)$$
$$-40s^2 + 96tu\Bigg)\Big/(36s^2) + \Bigg(- \frac{13}{4}H(2,y)H(1,z) + \frac{134}{9}H(3,y)H(1,z) + 7H(0,0,y)\ H(1,z)$$
$$-\frac{2}{3}H(0,2,y)H(1,z) - 7H(0,3,y)H(1,z) - \frac{2}{3}H(2,0,y)\ H(1,z) + \frac{13}{3}H(2,3,y)H(1,z)$$
$$-7H(3,0,y)H(1,z) + \frac{4}{3}H(3,2,y)\ H(1,z) + \frac{22}{3}H(3,3,y)H(1,z) + 2H(0,0,2,y)H(1,z)$$
$$+2H(0,2,0,y)\ H(1,z) + 4H(0,3,0,y)H(1,z) + 4H(0,3,3,y)H(1,z) + 2H(1,0,3,y)H(1,z)$$
$$-2\ H(1,2,3,y)H(1,z) + 2H(2,0,0,y)H(1,z) + 2H(2,0,3,y)H(1,z) + 2H(2,1,0,y)\ H(1,z)$$
$$+4H(2,2,3,y)H(1,z) + 2H(2,3,0,y)H(1,z) - 8H(2,3,3,y)H(1,z) + 4\ H(3,0,3,y)H(1,z)$$
$$+4H(3,3,0,y)H(1,z) - 16H(3,3,3,y)H(1,z) - \frac{361}{54}\ H(1,z) - \frac{361}{54}H(2,y) + \frac{80}{9}H(0,0,y)$$
$$+7H(2,y)H(0,0,z) + 2H(0,0,y)H(0,0,z) + \frac{80}{9}H(0,0,z) + \frac{11}{3}H(2,y)H(0,1,z) + \frac{1}{3}H(3,y)H(0,1,z)$$
$$+2H(0,0,y)H(0,1,z) + \frac{179}{18}\ H(0,1,z) + 2H(0,0,z)H(0,2,y) - 2H(0,1,z)H(0,2,y) - \frac{89}{18}H(0,2,y)$$
$$-2\ H(0,1,z)H(0,3,y) + \frac{8tuH(1,0,y)}{3s^2} + \frac{8tuH(1,0,z)}{3\ s^2} + 6H(2,y)H(1,0,z) - 7H(3,y)H(1,0,z)$$
$$+2H(0,0,y)H(1,0,z) - 6H(0,3,y)\ H(1,0,z) - \frac{89}{18}H(1,0,z) + \frac{4}{3}H(3,y)H(1,1,z)$$

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$$
\begin{aligned}
&+2H(0,0,y)\,H(1,1,z)-\frac{13}{4}H(1,1,z)-2H(0,1,z)H(1,2,y)+2H(1,0,z)H(1,2,y)\\
&+2\,H(0,0,z)H(2,0,y)+2H(0,1,z)H(2,0,y)+2H(1,0,z)H(2,0,y)-\frac{89}{18}\,H(2,0,y)\\
&+2H(0,0,z)H(2,2,y)+4H(0,1,z)H(2,2,y)+4H(1,0,z)\,H(2,2,y)-\frac{13}{4}H(2,2,y)\\
&-6H(0,1,z)H(2,3,y)+2H(1,0,z)H(2,3,y)+2\,H(0,1,z)H(3,0,y)-2H(1,0,z)H(3,0,y)\\
&+\frac{134}{9}H(3,2,y)-12H(0,1,z)\,H(3,3,y)+4H(1,0,z)H(3,3,y)-2H(1,y)H(0,0,1,z)\\
&-2H(2,y)H(0,0,1,z)-8\,H(3,y)H(0,0,1,z)+\frac{1}{3}H(0,0,1,z)+7H(0,0,2,y)+\frac{3u\,H(0,1,0,y)}{s}\\
&+\frac{3tH(0,1,0,z)}{s}+2H(1,y)H(0,1,0,z)+6H(2,y)\,H(0,1,0,z)+4H(3,y)H(0,1,0,z)\\
&+\frac{2}{3}H(0,1,1,z)+7\,H(0,2,0,y)-\frac{2}{3}H(0,2,2,y)-7H(0,3,2,y)+2H(2,y)H(1,0,0,z)\\
&+7\,H(1,0,0,z)-2H(1,y)H(1,0,1,z)+6H(2,y)H(1,0,1,z)+\frac{11}{3}\,H(1,0,1,z)\\
&-\frac{6(s+t)H(1,1,0,y)}{s}-\frac{6uH(1,1,0,z)}{s}+2\,H(1,y)H(1,1,0,z)+6H(2,y)H(1,1,0,z)\\
&+H(0,y)\bigg(-\frac{3H(1,0,z)u}{s}+\frac{8u}{3s}+\left(\frac{8tu}{3s^2}-\frac{14}{3}\right)H(0,z)-\frac{89}{18}H(1,z)+7H(0,0,z)-\frac{13}{6}H(0,1,z)\\
&+\frac{17}{2}H(1,0,z)-\frac{2}{3}H(1,1,z)-2H(0,0,1,z)-2H(0,1,1,z)+2H(1,0,0,z)+2H(1,1,0,z)-\frac{142}{27}\bigg)\\
&+7H(2,0,0,y)-\frac{2}{3}H(2,0,2,y)+\frac{20}{3}H(2,1,0,y)-\frac{2}{3}H(2,2,0,y)+\frac{13}{3}H(2,3,2,y)-7H(3,0,2,y)\\
&-7H(3,2,0,y)+\frac{4}{3}H(3,2,2,y)+\frac{22}{3}\,H(3,3,2,y)+H(0,z)\bigg(\frac{8t}{3s}-\frac{89}{18}H(2,y)+7\,H(0,0,y)\\
&+\frac{29}{6}H(0,2,y)-\frac{3(s+t)H(1,0,y)}{s}+\frac{23}{2}\,H(2,0,y)-\frac{2}{3}H(2,2,y)-7H(3,2,y)+2H(0,0,2,y)\\
&+2H(0,2,0,y)-2\,H(0,2,2,y)-6H(0,3,2,y)-2H(1,0,2,y)+2H(2,0,0,y)+2H(2,2,0,y)\\
&+2\,H(2,3,2,y)-2H(3,0,2,y)-2H(3,2,0,y)+4H(3,3,2,y)-\frac{142}{27}\bigg)+8\,H(0,0,1,0,z)\\
&+2H(0,0,1,1,z)+2H(0,0,2,2,y)+2H(0,1,0,1,z)+6H(0,1,1,0,z)+2\,H(0,2,0,2,y)\\
&+4H(0,2,1,0,y)+2H(0,2,2,0,y)+4H(0,3,0,2,y)+4H(0,3,2,0,y)+4\,H(0,3,3,2,y)\\
&+6H(1,0,1,0,z)+2H(1,0,3,2,y)+2H(1,1,0,0,z)+4H(1,1,0,1,z)+6\,H(1,1,1,0,z)\\
&-2H(1,2,3,2,y)+2H(2,0,0,2,y)+2H(2,0,1,0,y)+2H(2,0,2,0,y)+2\,H(2,0,3,2,y)\\
&+2H(2,1,0,2,y)+2H(2,1,2,0,y)+2H(2,2,0,0,y)+4H(2,2,1,0,y)+4\,H(2,2,3,2,y)\\
&+2H(2,3,0,2,y)+2H(2,3,2,0,y)-8H(2,3,3,2,y)+4H(3,0,3,2,y)+4\,H(3,3,0,2,y)\\
&+4H(3,3,2,0,y)-16H(3,3,3,2,y)-\frac{571}{81}\bigg)\bigg\}+C_AC_F\bigg\{\zeta_4\left(\frac{93}{4}\right)+\zeta_3\bigg(22H(1,y)-18H(2,y)\\
&+4H(1,z)+\frac{4t^2}{s^2}+\frac{4u^2}{s^2}-\frac{8\,(t+u)}{s}+\frac{335}{18}\bigg)+\zeta_2\bigg(-\frac{4t^2H(1,y)}{s^2}-\frac{2t^2H(0,z)}{s^2}-\frac{2u^2\,H(0,y)}{s^2}\\
&-\frac{4u^2H(1,z)}{s^2}+\frac{8tH(1,y)}{s}+\frac{4tH(0,z)}{s}+\frac{4uH(0,y)}{s}+\frac{8uH(1,z)}{s}-\frac{107t^2H(2,y)}{6(t+u)^2}-\frac{79t^2H(1,z)}{6(t+u)^2}\\
&-\frac{119tuH(2,y)}{3(t+u)^2}-\frac{91tuH(1,z)}{3(t+u)^2}-\frac{107u^2H(2,y)}{6(t+u)^2}-\frac{79u^2H(1,z)}{6(t+u)^2}-2H(1,y)H(0,z)-6H(0,y)H(1,z)\\
&-2H(1,y)H(1,z)-4H(2,y)H(0,z)+\frac{14}{3}H(1,y)+2H(0,2,y)+6\,H(1,0,y)-2H(1,2,y)\\
&-4H(2,0,y)-8H(2,1,y)+8H(2,2,y)+2H(0,1,z)+2H(1,0,z)-2H(1,1,z)-\frac{18tu}{s^2}-\frac{2t^2}{s(t+u)}
\end{aligned}
$$

$$-\frac{2u^2}{s\ (t+u)}-\frac{17}{6}\Bigg)+\Bigg(-\frac{2H(1,0,y)t^2}{s(t+u)}-\frac{5H(2,y)H(1,0,z)t^2}{3\ (t+u)^2}+\frac{2H(1,0,z)t^2}{s(t+u)}+\frac{10H(0,1,0,y)t^2}{3\ (t+u)^2}$$
$$+\frac{2H(0,1,0,z)t^2}{s^2}+\frac{28H(0,1,0,z)t^2}{3\ (t+u)^2}-\frac{4H(1,1,0,y)t^2}{s^2}+\frac{6H(1,1,0,z)\ t^2}{(t+u)^2}+\frac{3H(2,1,0,y)t^2}{(t+u)^2}$$
$$-\frac{3H(1,z)\ H(3,y)t}{u}+\frac{4uH(1,0,y)t}{s(t+u)}-\frac{18uH(1,0,y)\ t}{s^2}-\frac{152H(1,0,y)t}{9(t+u)}+\frac{4uH(1,0,z)t}{s\ (t+u)}$$
$$-\frac{18uH(1,0,z)t}{s^2}-\frac{22uH(2,y)H(1,0,z)t}{3\ (t+u)^2}-\frac{H(1,0,z)t}{(t+u)}-\frac{3H(3,2,y)t}{u}+\frac{32\ uH(0,1,0,y)t}{3(t+u)^2}$$
$$+\frac{68uH(0,1,0,z)t}{3\ (t+u)^2}-\frac{4H(0,1,0,z)t}{s}+\frac{8H(1,1,0,y)t}{s}+\frac{8u\ H(1,1,0,z)t}{(t+u)^2}+\frac{2uH(2,1,0,y)\ t}{(t+u)^2}$$
$$+\frac{604}{27}H(1,z)+2H(1,z)H(2,y)+\frac{604}{27}\ H(2,y)-\frac{304}{9}H(1,z)H(3,y)-14H(1,z)H(0,0,y)$$
$$-14H(2,y)\ H(0,0,z)-\frac{22}{3}H(2,y)H(0,1,z)-\frac{20}{3}H(3,y)H(0,1,z)-4\ H(0,0,y)H(0,1,z)$$
$$-\frac{197}{9}H(0,1,z)-\frac{14}{3}H(1,z)H(0,2,y)-4\ H(0,0,z)H(0,2,y)+6H(0,1,z)H(0,2,y)+\frac{107}{9}H(0,2,y)$$
$$+20H(1,z)\ H(0,3,y)+4H(0,1,z)H(0,3,y)+\frac{2u^2H(1,0,y)}{s(t+u)}-\frac{143u\ H(1,0,y)}{9(t+u)}+3H(1,z)H(1,0,y)$$
$$-2H(0,1,z)H(1,0,y)-\frac{2u^2\ H(1,0,z)}{s(t+u)}-\frac{5uH(1,0,z)}{(t+u)}-\frac{5u^2H(2,y)\ H(1,0,z)}{3(t+u)^2}+20H(3,y)H(1,0,z)$$
$$-4H(0,0,y)H(1,0,z)+8H(0,2,y)\ H(1,0,z)+16H(0,3,y)H(1,0,z)+2H(1,0,y)H(1,0,z)$$
$$+\frac{28}{3}H(3,y)H(1,1,z)-8H(0,0,y)H(1,1,z)+4H(0,3,y)H(1,1,z)+2H(1,1,z)+6H(0,1,z)H(1,2,y)$$
$$-2H(1,0,z)H(1,2,y)-\frac{14}{3}H(1,z)H(2,0,y)-4H(0,0,z)\ H(2,0,y)-2H(0,1,z)H(2,0,y)$$
$$-6H(1,0,z)H(2,0,y)+\frac{107}{9}H(2,0,y)-8\ H(0,0,z)H(2,2,y)-16H(0,1,z)H(2,2,y)$$
$$-8H(1,0,z)H(2,2,y)+2\ H(2,2,y)-\frac{8}{3}H(1,z)H(2,3,y)+12H(0,1,z)H(2,3,y)$$
$$-4H(1,0,z)\ H(2,3,y)+20H(1,z)H(3,0,y)-4H(0,1,z)H(3,0,y)+4H(1,1,z)\ H(3,0,y)$$
$$+\frac{28}{3}H(1,z)H(3,2,y)-4H(0,1,z)H(3,2,y)-4H(1,0,z)\ H(3,2,y)-\frac{304}{9}H(3,2,y)$$
$$-\frac{80}{3}H(1,z)H(3,3,y)+28H(0,1,z)\ H(3,3,y)-12H(1,0,z)H(3,3,y)-8H(1,1,z)H(3,3,y)$$
$$+6H(1,y)H(0,0,1,z)+16H(3,y)H(0,0,1,z)-\frac{2}{3}H(0,0,1,z)-8H(1,z)H(0,0,2,y)-14H(0,0,2,y)$$
$$+4\ H(1,z)H(0,0,3,y)+\frac{2u^2H(0,1,0,y)}{s^2}+\frac{10u^2H(0,1,0,y)}{3\ (t+u)^2}-\frac{4uH(0,1,0,y)}{s}+2H(1,z)H(0,1,0,y)$$
$$+\frac{28u^2\ H(0,1,0,z)}{3(t+u)^2}-2H(1,y)H(0,1,0,z)-12H(2,y)H(0,1,0,z)-4\ H(3,y)H(0,1,0,z)$$
$$-4H(3,y)H(0,1,1,z)+\frac{14}{3}H(0,1,1,z)-8H(1,z)\ H(0,2,0,y)-14H(0,2,0,y)-\frac{14}{3}H(0,2,2,y)$$
$$+2H(1,z)H(0,2,3,y)-8\ H(1,z)H(0,3,0,y)+4H(1,z)H(0,3,2,y)+20H(0,3,2,y)$$
$$-12H(1,z)H(0,3,3,y)+4\ H(1,z)H(1,0,0,y)+14H(1,0,0,y)-4H(2,y)H(1,0,0,z)$$
$$+6H(1,y)H(1,0,1,z)-18\ H(2,y)H(1,0,1,z)-4H(3,y)H(1,0,1,z)-\frac{13}{3}H(1,0,1,z)+3H(1,0,2,y)$$
$$-6\ H(1,z)H(1,0,3,y)+\frac{14}{3}H(1,1,0,y)+H(0,y)\Bigg(\Bigg(-\frac{18t\ u}{s^2}+\frac{4tu}{s(t+u)}+2\Bigg)H(0,z)$$
$$+\Bigg(\frac{3\ u}{t}+\frac{107}{9}\Bigg)H(1,z)+\frac{1}{3}\Bigg(-\frac{19H(1,0,z)\ t^2}{(t+u)^2}-\frac{54ut}{s(s+t)}-\frac{50uH(1,0,z)\ t}{(t+u)^2}+\frac{10t}{(s+t)}-\frac{36u}{(s+t)}$$
$$+13\ H(0,1,z)-\frac{6u^2H(1,0,z)}{s^2}-\frac{19u^2\ H(1,0,z)}{(t+u)^2}+\frac{12uH(1,0,z)}{s}-14H(1,1,z)+12H(0,0,1,z)$$

$$+6\ H(0,1,0,z)+24H(0,1,1,z)+6H(1,0,1,z)-12H(1,1,0,z)+\frac{10\ s}{(s+t)}\Big)\Big)-\frac{4u^2H(1,1,0,z)}{s^2}$$
$$+\frac{6u^2\ H(1,1,0,z)}{(t+u)^2}+\frac{8uH(1,1,0,z)}{s}-2H(1,y)H(1,1,0,z)-18\ H(2,y)H(1,1,0,z)$$
$$-4H(3,y)H(1,1,0,z)+3H(1,2,0,y)+6H(1,z)H(1,2,3,y)-8\ H(1,z)H(2,0,0,y)-14H(2,0,0,y)$$
$$-\frac{14}{3}H(2,0,2,y)-4H(1,z)\ H(2,0,3,y)+\frac{3u^2H(2,1,0,y)}{(t+u)^2}-2H(1,z)\ H(2,1,0,y)-\frac{14}{3}H(2,2,0,y)$$
$$-16H(1,z)H(2,2,3,y)-4H(1,z)\ H(2,3,0,y)-\frac{8}{3}H(2,3,2,y)+16H(1,z)H(2,3,3,y)$$
$$+4H(1,z)H(3,0,2,y)+20H(3,0,2,y)-12H(1,z)H(3,0,3,y)+4H(1,z)H(3,2,0,y)+20H(3,2,0,y)$$
$$+\frac{28}{3}H(3,2,2,y)-8H(1,z)H(3,2,3,y)-12H(1,z)\ H(3,3,0,y)+H(0,z)\bigg(\Big(\frac{3t}{u}+\frac{107}{9}\Big)\ H(2,y)$$
$$+\frac{1}{3}\bigg(-\frac{47H(2,0,y)t^2}{(t+u)^2}-\frac{54u\ t}{s(s+u)}-\frac{106uH(2,0,y)t}{(t+u)^2}-\frac{36\ t}{(s+u)}+\frac{10u}{(s+u)}-47H(0,2,y)$$
$$+\Big(-\frac{6\ t^2}{s^2}+\frac{12t}{s}+28\Big)H(1,0,y)-\frac{47u^2\ H(2,0,y)}{(t+u)^2}-14H(2,2,y)+60H(3,2,y)-24H(0,0,2,y)$$
$$+6\ H(0,1,0,y)+12H(0,2,2,y)+48H(0,3,2,y)+12H(1,0,0,y)+12H(1,0,2,y)+6\ H(1,2,0,y)$$
$$-12H(2,0,0,y)+6H(2,0,2,y)-6H(2,1,0,y)-12H(2,2,0,y)-12\ H(2,3,2,y)+24H(3,0,2,y)$$
$$+12H(3,2,2,y)-36H(3,3,2,y)+\frac{10\ s}{(s+u)}\bigg)\bigg)-8H(1,z)H(3,3,2,y)-\frac{80}{3}H(3,3,2,y)$$
$$+40\ H(1,z)H(3,3,3,y)+4H(0,0,1,0,y)-8H(0,0,1,0,z)-8H(0,0,1,1,z)-8\ H(0,0,2,2,y)$$
$$+4H(0,0,3,2,y)-8H(0,1,0,1,z)+2H(0,1,0,2,y)-12H(0,1,1,0,z)+2\ H(0,1,2,0,y)$$
$$-8H(0,2,0,2,y)-2H(0,2,1,0,y)-8H(0,2,2,0,y)+2H(0,2,3,2,y)-8\ H(0,3,0,2,y)$$
$$-8H(0,3,2,0,y)+4H(0,3,2,2,y)-12H(0,3,3,2,y)-2H(1,0,0,1,z)+4\ H(1,0,0,2,y)$$
$$+4H(1,0,1,0,y)-4H(1,0,1,0,z)+4H(1,0,2,0,y)-6H(1,0,3,2,y)-12\ H(1,1,0,1,z)$$
$$-14H(1,1,1,0,z)+4H(1,2,0,0,y)+4H(1,2,1,0,y)+6H(1,2,3,2,y)-8\ H(2,0,0,2,y)$$
$$-8H(2,0,2,0,y)-4H(2,0,3,2,y)+4H(2,1,0,0,y)-2H(2,1,0,2,y)-8\ H(2,1,1,0,y)$$
$$-2H(2,1,2,0,y)-8H(2,2,0,0,y)-8H(2,2,1,0,y)-16H(2,2,3,2,y)-4\ H(2,3,0,2,y)$$
$$-4H(2,3,2,0,y)+16H(2,3,3,2,y)+8H(3,0,1,0,y)+4H(3,0,2,2,y)-12\ H(3,0,3,2,y)$$
$$+4H(3,2,0,2,y)-8H(3,2,1,0,y)+4H(3,2,2,0,y)-8H(3,2,3,2,y)-12\ H(3,3,0,2,y)$$
$$-12H(3,3,2,0,y)-8H(3,3,2,2,y)+40\ H(3,3,3,2,y)-\frac{467}{81}-\frac{3uH(1,z)H(3,y)}{t}-\frac{3u\ H(0,1,z)}{t}$$
$$+\frac{3uH(0,2,y)}{t}-\frac{3u^2H(1,0,y)}{(t+u)\ t}+\frac{3uH(2,0,y)}{t}-\frac{3uH(3,2,y)}{t}\bigg)\bigg\}+C_F^2\bigg\{\zeta_4(-22)+\zeta_3\bigg(-16H(1,y)$$
$$+8H(2,y)-8H(1,z)-\frac{4t^2}{s^2}-\frac{4u^2}{s^2}+\frac{8\ (t+u)}{s}-30\bigg)+\zeta_2\bigg(\frac{4t^2H(1,y)}{s^2}+\frac{2t^2H(0,z)}{s^2}+\frac{2u^2\ H(0,y)}{s^2}$$
$$+\frac{4u^2H(1,z)}{s^2}-\frac{8tH(1,y)}{s}-\frac{4t\ H(0,z)}{s}-\frac{4uH(0,y)}{s}-\frac{8uH(1,z)}{s}+\frac{6t^2\ H(2,y)}{(t+u)^2}+\frac{6t^2H(1,z)}{(t+u)^2}$$
$$+\frac{16tu\ H(2,y)}{(t+u)^2}+\frac{16tuH(1,z)}{(t+u)^2}+\frac{6u^2\ H(2,y)}{(t+u)^2}+\frac{6u^2H(1,z)}{(t+u)^2}-8H(2,y)H(1,z)+8\ H(0,1,y)$$
$$-8H(0,2,y)+8H(1,1,y)+8H(2,1,y)-16H(2,2,y)+\frac{12t\ u}{s^2}+\frac{2t^2}{s(t+u)}+\frac{2u^2}{s(t+u)}+16\bigg)$$
$$+\bigg(\frac{2H(1,0,y)t^2}{s(t+u)}-\frac{2H(1,0,z)t^2}{s\ (t+u)}+\frac{12H(0,1,0,y)t^2}{(t+u)^2}-\frac{2H(0,1,0,z)\ t^2}{s^2}+\frac{4H(1,1,0,y)t^2}{s^2}$$
$$-\frac{6H(1,1,0,z)\ t^2}{(t+u)^2}-\frac{12H(2,1,0,y)t^2}{(t+u)^2}+\frac{6H(1,z)\ H(3,y)t}{u}-\frac{4uH(1,0,y)t}{s(t+u)}+\frac{12uH(1,0,y)\ t}{s^2}$$

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$$+\frac{6H(1,0,y)t}{(t+u)}-\frac{4uH(1,0,z)t}{s\,(t+u)}+\frac{12uH(1,0,z)t}{s^2}+\frac{4uH(2,y)H(1,0,z)\,t}{(t+u)^2}-\frac{2H(1,0,z)t}{(t+u)}$$
$$+\frac{6H(3,2,y)\,t}{u}+\frac{20uH(0,1,0,y)t}{(t+u)^2}-\frac{4uH(0,1,0,z)\,t}{(t+u)^2}+\frac{4H(0,1,0,z)t}{s}-\frac{8H(1,1,0,y)t}{s}$$
$$-\frac{8uH(1,1,0,z)t}{(t+u)^2}-\frac{20uH(2,1,0,y)t}{(t+u)^2}-18H(1,z)+9H(1,z)H(2,y)-18H(2,y)+8H(1,z)H(3,y)$$
$$+12H(3,y)H(0,1,z)+4\,H(0,1,z)+12H(1,z)H(0,2,y)-4H(0,1,z)H(0,2,y)-4H(0,2,y)$$
$$-12H(1,z)\,H(0,3,y)-\frac{2u^2H(1,0,y)}{s(t+u)}+\frac{8u\,H(1,0,y)}{(t+u)}-6H(1,z)H(1,0,y)+4H(0,1,z)H(1,0,y)$$
$$+\frac{2u^2H(1,0,z)}{s(t+u)}+\frac{2uH(1,0,z)}{(t+u)}-12H(3,y)H(1,0,z)-8H(0,2,y)H(1,0,z)-8H(0,3,y)H(1,0,z)$$
$$-24H(3,y)H(1,1,z)+8H(0,0,y)\,H(1,1,z)-8H(0,3,y)H(1,1,z)+9H(1,1,z)$$
$$-4H(0,1,z)H(1,2,y)+12H(1,z)\,H(2,0,y)-4H(0,1,z)H(2,0,y)-4H(2,0,y)$$
$$+8H(0,0,z)H(2,2,y)+16H(0,1,z)\,H(2,2,y)+9H(2,2,y)-12H(1,z)H(2,3,y)$$
$$-12H(1,z)H(3,0,y)-8H(1,1,z)\,H(3,0,y)-24H(1,z)H(3,2,y)+8H(0,1,z)H(3,2,y)$$
$$+8H(3,2,y)+24H(1,z)\,H(3,3,y)-8H(0,1,z)H(3,3,y)+8H(1,0,z)H(3,3,y)$$
$$+16H(1,1,z)H(3,3,y)-4\,H(1,y)H(0,0,1,z)+8H(2,y)H(0,0,1,z)+8H(1,z)H(0,0,2,y)$$
$$-8H(1,z)H(0,0,3,y)-\frac{2u^2H(0,1,0,y)}{s^2}+\frac{12u^2H(0,1,0,y)}{(t+u)^2}+\frac{4uH(0,1,0,y)}{s}-4H(1,z)H(0,1,0,y)$$
$$+4\,H(2,y)H(0,1,0,z)+8H(3,y)H(0,1,1,z)-12H(0,1,1,z)+8H(1,z)H(0,2,0,y)$$
$$+12H(0,2,2,y)-4H(1,z)H(0,2,3,y)-8H(1,z)H(0,3,2,y)-12H(0,3,2,y)+8H(1,z)H(0,3,3,y)$$
$$-8H(1,z)H(1,0,0,y)+2H(0,y)\biggl(\frac{H(1,0,z)\,u^2}{s^2}-\frac{2t(s-3(t+u))H(0,z)u}{s^2(t+u)}-\frac{2\,H(1,0,z)u}{s}$$
$$+\frac{2tH(1,0,z)u}{(t+u)^2}+\frac{6\,u}{s}+\left(-\frac{3u}{t}-2\right)H(1,z)+6H(1,1,z)-4H(0,1,1,z)-2\,H(1,0,1,z)\biggr)$$
$$-4H(1,y)H(1,0,1,z)+12H(2,y)H(1,0,1,z)+8H(3,y)H(1,0,1,z)-6H(1,0,1,z)-6H(1,0,2,y)$$
$$+4H(1,z)H(1,0,3,y)+\frac{4u^2H(1,1,0,z)}{s^2}-\frac{6u^2H(1,1,0,z)}{(t+u)^2}-\frac{8uH(1,1,0,z)}{s}+4H(2,y)H(1,1,0,z)$$
$$-6H(1,2,0,y)-4H(1,z)H(1,2,3,y)+8H(1,z)\,H(2,0,0,y)+12H(2,0,2,y)-\frac{12u^2H(2,1,0,y)}{(t+u)^2}$$
$$-4H(1,z)H(2,1,0,y)+12H(2,2,0,y)+16H(1,z)H(2,2,3,y)-12H(2,3,2,y)-8H(1,z)H(3,0,2,y)$$
$$-12H(3,0,2,y)+8H(1,z)H(3,0,3,y)-8H(1,z)H(3,2,0,y)-12\,H(3,2,0,y)-24H(3,2,2,y)$$
$$+16H(1,z)H(3,2,3,y)+8H(1,z)H(3,3,0,y)+16H(1,z)\,H(3,3,2,y)+24H(3,3,2,y)$$
$$+2H(0,z)\biggl(\frac{3H(2,0,y)\,t^2}{(t+u)^2}+\frac{(s+t)H(1,0,y)t}{s^2}+\frac{8uH(2,0,y)\,t}{(t+u)^2}+\frac{6t}{s}+\left(-\frac{3t}{u}-2\right)H(2,y)$$
$$+6\,H(0,2,y)-\frac{3(s+t)H(1,0,y)}{s}+\frac{3u^2\,H(2,0,y)}{(t+u)^2}+6H(2,2,y)-6H(3,2,y)+4H(0,0,2,y)$$
$$-4H(0,3,2,y)-2H(2,0,2,y)-4H(3,0,2,y)-4H(3,2,2,y)+4H(3,3,2,y)\biggr)-16H(1,z)H(3,3,3,y)$$
$$+8H(0,0,1,1,z)+8H(0,0,2,2,y)-8H(0,0,3,2,y)+8H(0,1,0,1,z)-4\,H(0,1,0,2,y)$$
$$+8H(0,1,1,0,y)+8H(0,1,1,0,z)-4H(0,1,2,0,y)+8H(0,2,0,2,y)-4\,H(0,2,1,0,y)$$
$$+8H(0,2,2,0,y)-4H(0,2,3,2,y)-8H(0,3,2,2,y)+8H(0,3,3,2,y)+4\,H(1,0,0,1,z)$$
$$-8H(1,0,0,2,y)+4H(1,0,1,0,y)-8H(1,0,2,0,y)+4H(1,0,3,2,y)+8\,H(1,1,0,0,y)$$
$$+8H(1,1,0,1,z)+8H(1,1,1,0,y)+8H(1,1,1,0,z)-8H(1,2,0,0,y)-4\,H(1,2,1,0,y)$$
$$-4H(1,2,3,2,y)+8H(2,0,0,2,y)-4H(2,0,1,0,y)+8H(2,0,2,0,y)-8\,H(2,1,0,0,y)$$
$$-4H(2,1,0,2,y)+8H(2,1,1,0,y)-4H(2,1,2,0,y)+8H(2,2,0,0,y)+16\,H(2,2,3,2,y)$$

$$
\begin{aligned}
&-8H(3,0,1,0,y)-8H(3,0,2,2,y)+8H(3,0,3,2,y)-8H(3,2,0,2,y)+8\ H(3,2,1,0,y)\\
&-8H(3,2,2,0,y)+16H(3,2,3,2,y)+8H(3,3,0,2,y)+8H(3,3,2,0,y)+16\ H(3,3,2,2,y)\\
&-16H(3,3,3,2,y)+6+\frac{6uH(1,z)H(3,y)}{t}+\frac{6u\ H(0,1,z)}{t}-\frac{6uH(0,2,y)}{t}+\frac{6u^2H(1,0,y)}{(t{+}u)\ t}\\
&-\frac{6uH(2,0,y)}{t}+\frac{6uH(3,2,y)}{t}\Bigg)\Bigg\}+C_An_f\Bigg\{\zeta_3\Bigg(-\frac{37}{18}\Bigg)+\zeta_2\Bigg(-\frac{1}{3}H(0,y)+\frac{1}{6}H(2,y)-\frac{1}{3}H(0,z)\\
&+\frac{1}{6}\ H(1,z)+\frac{tu}{3s^2}-\frac{7}{36}\Bigg)+\Bigg(\frac{1}{36}H(0,y)\Bigg(H(0,z)\Bigg(\frac{12tu}{s^2}+20\Bigg)+31\ H(1,z)-36H(0,0,z)\\
&+24H(0,1,z)-36H(1,0,z)+24H(1,1,z)+\frac{12\ u}{s}+86\Bigg)+\frac{tuH(1,0,y)}{3s^2}+\frac{tuH(1,0,z)}{3s^2}\\
&+H(0,z)\Bigg(\frac{31}{36}H(2,y)-H(0,0,y)-\frac{1}{3}H(0,2,y)-H(2,0,y)+\frac{2}{3}H(2,2,y)+H(3,2,y)+\frac{t}{3s}+\frac{43}{18}\Bigg)\\
&+H(2,y)H(1,z)-\frac{20}{9}H(3,y)H(1,z)-H(0,0,y)\ H(1,z)+\frac{2}{3}H(0,2,y)H(1,z)+H(0,3,y)H(1,z)\\
&+\frac{2}{3}H(2,0,y)\ H(1,z)-\frac{4}{3}H(2,3,y)H(1,z)+H(3,0,y)H(1,z)-\frac{4}{3}H(3,2,y)\ H(1,z)\\
&-\frac{4}{3}H(3,3,y)H(1,z)-H(2,y)H(0,0,z)-\frac{2}{3}H(2,y)\ H(0,1,z)-\frac{1}{3}H(3,y)H(0,1,z)\\
&+H(3,y)H(1,0,z)-\frac{4}{3}H(3,y)H(1,1,z)+\frac{17}{27}H(2,y)-\frac{41}{18}H(0,0,y)+\frac{31}{36}H(0,2,y)+\frac{31}{36}H(2,0,y)\\
&+H(2,2,y)-\frac{20}{9}\ H(3,2,y)-H(0,0,2,y)-H(0,2,0,y)+\frac{2}{3}\ H(0,2,2,y)+H(0,3,2,y)\\
&-H(2,0,0,y)+\frac{2}{3}H(2,0,2,y)-\frac{2}{3}\ H(2,1,0,y)+\frac{2}{3}H(2,2,0,y)-\frac{4}{3}\ H(2,3,2,y)+H(3,0,2,y)\\
&+H(3,2,0,y)-\frac{4}{3}H(3,2,2,y)-\frac{4}{3}\ H(3,3,2,y)+\frac{17}{27}H(1,z)-\frac{41}{18}H(0,0,z)-\frac{49}{36}\ H(0,1,z)\\
&+\frac{31}{36}H(1,0,z)+H(1,1,z)-\frac{1}{3}H(0,0,1,z)-\frac{2}{3}H(0,1,1,z)-H(1,0,0,z)-\frac{2}{3}H(1,0,1,z)+\frac{65}{162}\Bigg)\Bigg\}\\
&+C_Fn_f\Bigg\{\zeta_3\Bigg(-\frac{1}{9}\Bigg)+\zeta_2\Bigg(\frac{1}{3}(4H(1,y)-5H(2,y)-H(1,z)+1)\Bigg)+\Bigg(-\frac{279}{2}H(0,y)H(1,z)\\
&-108H(0,y)H(0,1,z)+27H(0,y)H(1,0,z)-108\ H(0,y)H(1,1,z)-\frac{279}{2}H(2,y)H(0,z)\\
&-162H(2,y)H(1,z)+360H(3,y)\ H(1,z)+162H(0,0,y)H(1,z)+162H(2,y)H(0,0,z)\\
&+108H(2,y)H(0,1,z)+54H(3,y)\ H(0,1,z)+54H(0,2,y)H(0,z)-108H(0,2,y)H(1,z)\\
&-162H(0,3,y)H(1,z)-27\ H(1,0,y)H(0,z)-108H(2,y)H(1,0,z)-162H(3,y)H(1,0,z)\\
&+216H(3,y)\ H(1,1,z)+54H(2,0,y)H(0,z)-108H(2,0,y)H(1,z)-108H(2,2,y)H(0,z)\\
&+216\ H(2,3,y)H(1,z)-162H(3,0,y)H(1,z)-162H(3,2,y)H(0,z)+216H(3,2,y)\ H(1,z)\\
&+216H(3,3,y)H(1,z)-27H(0,y)-102H(2,y)-\frac{279}{2}H(0,2,y)+180\ H(1,0,y)-\frac{279}{2}H(2,0,y)\\
&-162H(2,2,y)+360H(3,2,y)+162H(0,0,2,y)-27\ H(0,1,0,y)+162H(0,2,0,y)\\
&-108H(0,2,2,y)-162H(0,3,2,y)-162H(1,0,0,y)+108\ H(1,1,0,y)+162H(2,0,0,y)\\
&-108H(2,0,2,y)-108H(2,2,0,y)+216H(2,3,2,y)-162\ H(3,0,2,y)-162H(3,2,0,y)\\
&+216H(3,2,2,y)+216H(3,3,2,y)-27H(0,z)-102\ H(1,z)+\frac{441}{2}H(0,1,z)+\frac{81}{2}H(1,0,z)\\
&-162H(1,1,z)+54H(0,0,1,z)-27H(0,1,0,z)+108H(0,1,1,z)+108H(1,0,1,z)+200\Bigg)\Big/81\Bigg\}\\
&+n_f^2\Bigg\{\frac{1}{108}\Bigg(H(0,y)(3H(0,z)-20)+15H(0,0,y)-20H(0,z)+15H(0,0,z)+6\ \zeta_2\Bigg)\Bigg\}
\end{aligned}
$$

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$$\mathcal{B}_{2;2}^{(2)} = \frac{1}{24}\Big(121C_A{}^2 + 44C_A(6C_F - n_f) + 4\ \Big(27C_F{}^2 - 12C_F n_f + n_f{}^2\Big)\Big)$$

$$\mathcal{B}_{2;1}^{(2)} = C_A^2\Bigg\{\frac{1}{108}\Big(594H(0,y)H(0,z) + 594H(0,y)H(1,z) + 594H(2,y)H(0,z) - 1188\ H(3,y)H(1,z)$$
$$+990H(0,y) + 891H(2,y) + 594H(0,2,y) + 594H(2,0,y) - 1188\ H(3,2,y) + 990H(0,z)$$
$$+891H(1,z) - 594H(0,1,z) + 594H(1,0,z) + 495\ \zeta_2 - 108\zeta_3 - 1296\Big)\Bigg\}$$
$$+C_AC_F\Bigg\{\frac{1}{108}\Big(- 1188H(0,y)H(1,z)+54H(0,y)(6H(0,z)+6H(1,z)+10)-1188H(2,y)H(0,z)$$
$$+54(6H(2,y) + 10)H(0,z) + 1728H(3,y)H(1,z) - 1296H(2,y) - 864\ H(0,2,y) + 1188H(1,0,y)$$
$$-864H(2,0,y)+1728H(3,2,y)-1296H(1,z)+864H(0,1,z)+324H(1,0,z)+1566\zeta_2-2808\zeta_3-778\Big)\Bigg\}$$
$$+C_F^2\Bigg\{\frac{1}{108}\Big(- 648H(0,y)H(1,z) - 648H(2,y)H(0,z) + 1296H(3,y)H(1,z) - 972\ H(2,y)$$
$$-648H(0,2,y) + 648H(1,0,y) - 648H(2,0,y) + 1296H(3,2,y) - 972H(1,z) + 648\ H(0,1,z)$$
$$-1296\zeta_2 + 2592\zeta_3 + 972\Big)\Bigg\} + C_An_f\Bigg\{\frac{1}{108}\Big(- 2\Big(- 27(4H(3,y) - 3)H(1,z) + 81H(2,y)$$
$$+54H(0,2,y) + 54\ H(2,0,y) - 108H(3,2,y) - 54H(0,1,z) + 54H(1,0,z) + 45\zeta_2 - 260\Big)$$
$$-9H(0,y)\ (12H(0,z) + 12H(1,z) + 31) - 9(12H(2,y) + 31)H(0,z)\Big)\Bigg\}$$
$$+C_Fn_f\Bigg\{\frac{1}{108}\Big(- 2\Big(54(4H(3,y) - 3)H(1,z) - 162H(2,y) - 108H(0,2,y) + 108\ H(1,0,y)$$
$$-108H(2,0,y) + 216H(3,2,y) + 108H(0,1,z) + 54\zeta_2 + 22\Big) - 9H(0,y)\ (6 - 24H(1,z))$$
$$-9(6 - 24H(2,y))H(0,z)\Big)\Bigg\} + n_f^2\Bigg\{\frac{1}{6}H(0,y) + \frac{1}{6}H(0,z) - \frac{10}{27}\Bigg\}$$

$$\mathcal{B}_{2;0}^{(2)} = C_A^2\Bigg\{\zeta_4\left(\frac{39}{8}\right) + \zeta_3\Bigg(- H(0,y) - 6H(1,y) + 11H(2,y) - H(0,z) + 5H(1,z) - \Big(25s^4(t{+}u)$$
$$+2s^3\ \Big(79t^2 + 266tu + 79u^2\Big) + s^2\Big(133t^3 + 1097t^2u + 1097t\ u^2 + 133u^3\Big) + 2stu\Big(241t^2 + 806tu$$
$$+241u^2\Big) + 457t^2\ (t{+}u)u^2\Big)\Big/\Big(36(s{+}t)^2(s{+}u)^2(t{+}u)\Big)\Bigg) + \zeta_2\Bigg(\Big(H(0,z)\Big(s^2(109t + 49u)$$
$$+2st(115t + 61u) + t^2(121t + 85\ u)\Big)\Big)\Big/\Big(12(s{+}t)^2(t{+}u)\Big) + \Big(H(0,y)\Big(s^2(49t + 109u)$$
$$+2\ su(61t + 115u) + u^2(85t + 121u)\Big)\Big)\Big/\Big(12(s{+}u)^2(t{+}u)\Big) + \Big(H(2,y)\Big(- 144s^5 - 257s^4(t{+}u)$$
$$-2s^3\Big(35\ t^2 + 202tu + 35u^2\Big) + s^2\Big(43t^3 - 25t^2u - 25tu^2 + 43u^3\Big) + 2\ stu\Big(55t^2 + 74tu + 55u^2\Big)$$
$$+79t^2(t{+}u)u^2\Big)\Big)\Big/\Big(12(s{+}t)^2(s{+}u)^2(t{+}u)\Big) + \Big(H(1,z)\Big(- 72s^5 - s^4(113\ t + 41u)$$
$$+2s^3\Big(t^2 + 14tu + 73u^2\Big) + s^2\Big(43t^3 + 191t^2u + 407\ tu^2 + 115u^3\Big) + 2stu\Big(55t^2 + 182tu + 127u^2\Big)$$
$$+t^2u^2(79\ t + 151u)\Big)\Big)\Big/\Big(12(s{+}t)^2(s{+}u)^2(t{+}u)\Big) + \frac{6(s{+}t)\ H(1,y)}{(t{+}u)} + 3H(0,y)H(0,z)$$
$$+2H(1,y)H(0,z) + 3H(0,y)H(1,z) + 2H(1,y)\ H(1,z) + H(2,y)H(0,z) - 2H(3,y)H(1,z)$$
$$+3H(0,2,y) + 2H(1,2,y) + H(2,0,y) - 2\ H(3,2,y) + H(0,1,z) + H(1,0,z) + 2H(1,1,z)$$
$$-\Big(28s^3(t{+}u) + s^2\Big(28\ t^2 + 71tu + 28u^2\Big) + 34st(t{+}u)u - 3t^2u^2\Big)\Big/\Big(9s(s{+}t)(s{+}u)(t{+}u)\Big)\Bigg)$$

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$$+\Bigg(-\frac{2(143t+89u)H(0,y)}{27(t+u)}+\frac{1}{3}\Big(\frac{tu}{s\,(t+u)}-17\Big)H(0,z)H(0,y)-\Big(\Big(179u^2+161su+107tu$$

$$+107\,st\Big)H(1,z)H(0,y)\Big)\Big/\Big(18(s+u)(t+u)\Big)+7H(0,0,z)H(0,y)-\Big(\Big((13t+25u)s^2$$

$$+2u(19t+28u)s+31(t+u)\,u^2\Big)H(0,1,z)H(0,y)\Big)\Big/\Big(6(s+u)^2(t+u)\Big)$$

$$+\Big((17t+23u)\,H(1,0,z)H(0,y)\Big)\Big/\Big(2(t+u)\Big)-\frac{2}{3}H(1,1,z)H(0,y)-2H(0,0,1,z)\,H(0,y)$$

$$-2H(0,1,1,z)H(0,y)+2H(1,0,0,z)H(0,y)+2H(1,1,0,z)H(0,y)-\frac{2(89t+143u)H(0,z)}{27(t+u)}$$

$$-\frac{140}{27}H(1,z)-\Big(\Big(179t^2+161st+107ut+107su\Big)H(0,z)H(2,y)\Big)\Big/\Big(18(s+t)\,(t+u)\Big)$$

$$-\frac{13}{4}H(1,z)H(2,y)-\frac{140}{27}H(2,y)+\Big(\Big(188\,s^2+197(t+u)s+206tu\Big)H(1,z)H(3,y)\Big)\Big/$$

$$\Big(9(s+t)\,(s+u)\Big)+7H(0,z)H(0,0,y)+7H(1,z)H(0,0,y)+\frac{80}{9}H(0,0,y)+7\,H(2,y)H(0,0,z)$$

$$+2H(0,0,y)H(0,0,z)+\frac{80}{9}H(0,0,z)+\Big(\Big((215\,t+269u)s^2+\Big(215t^2+520ut+287u^2\Big)s$$

$$+tu(233t+305u)\Big)H(0,1,z)\Big)\Big/\Big(18(s+t)(s+u)(t+u)\Big)+\Big(\Big(18s^5+59(t+u)s^4+\Big(61t^2+152ut$$

$$+61u^2\Big)s^3+4\Big(5t^3+31ut^2+31u^2t+5u^3\Big)s^2+34t(t+u)^2us+11t^2(t+u)u^2\Big)H(2,y)H(0,1,z)\Big)\Big/$$

$$\Big(3(s+t)^2(s+u)^2(t+u)\Big)+\frac{1}{3}H(3,y)H(0,1,z)+2H(0,0,y)H(0,1,z)-\Big(\Big(179u^2+161su+107tu$$

$$+107st\Big)H(0,2,y)\Big)\Big/\Big(18(s+u)\,(t+u)\Big)+\Big(\Big((17t+29u)s^2+2t(14t+23u)s$$

$$+11t^2(t+u)\Big)H(0,z)H(0,2,y)\Big)\Big/\Big(6(s+t)^2(t+u)\Big)-\frac{2}{3}H(1,z)H(0,2,y)+2H(0,0,z)H(0,2,y)$$

$$-2H(0,1,z)H(0,2,y)-\Big(\Big((5t+9u)s^4+\Big(9t^2+28ut+19u^2\Big)s^3+2\Big(2t^3+14ut^2+21u^2t+5u^3\Big)s^2$$

$$+2tu\Big(5t^2+14ut+9u^2\Big)s+7t^2(t+u)\,u^2\Big)H(1,z)H(0,3,y)\Big)\Big/\Big((s+t)^2(s+u)^2(t+u)\Big)$$

$$-2H(0,1,z)\,H(0,3,y)+\Big(u\Big(6s^2-2ts+9us+tu\Big)H(1,0,y)\Big)\Big/\Big(3s(s+u)\,(t+u)\Big)$$

$$-\frac{3uH(0,z)H(1,0,y)}{(t+u)}-\Big(3(s+t+u)\,H(1,z)H(1,0,y)\Big)\Big/\Big((t+u)\Big)-\Big(\Big((125t+107u)s^2$$

$$+t(125t+119u)s-6t^2u\Big)H(1,0,z)\Big)\Big/\Big(18s(s+t)(t+u)\Big)+\Big(\Big(-3s^3-(t-3u)s^2+xs8t(t+u)s$$

$$+6t^2(t+u)\Big)H(2,y)\,H(1,0,z)\Big)\Big/\Big((s+t)^2(t+u)\Big)-7H(3,y)H(1,0,z)+2H(0,0,y)H(1,0,z)$$

$$-6\,H(0,3,y)H(1,0,z)+\frac{4}{3}H(3,y)H(1,1,z)+2H(0,0,y)\,H(1,1,z)-\frac{13}{4}H(1,1,z)$$

$$-2H(0,1,z)H(1,2,y)+2H(1,0,z)H(1,2,y)-\Big(\Big(179u^2+161su+107tu+107st\Big)H(2,0,y)\Big)\Big/$$

$$\Big(18\,(s+u)(t+u)\Big)+\frac{23}{2}H(0,z)H(2,0,y)-\frac{2}{3}H(1,z)\,H(2,0,y)+2H(0,0,z)H(2,0,y)$$

$$+2H(0,1,z)H(2,0,y)+2H(1,0,z)\,H(2,0,y)-\frac{2}{3}H(0,z)H(2,2,y)+2H(0,0,z)H(2,2,y)$$

$$+4H(0,1,z)\,H(2,2,y)+4H(1,0,z)H(2,2,y)-\frac{13}{4}H(2,2,y)+\Big(\Big(18s^5+61\,(t+u)s^4$$

$$+5\Big(13t^2+32ut+13u^2\Big)s^3+2\Big(11t^3+67u\,t^2+67u^2t+11u^3\Big)s^2+38t(t+u)^2us$$

$$+13t^2(t+u)\,u^2\Big)H(1,z)H(2,3,y)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big)-6\,H(0,1,z)H(2,3,y)$$

$$+2H(1,0,z)H(2,3,y)-7H(1,z)H(3,0,y)+2H(0,1,z)\,H(3,0,y)-2H(1,0,z)H(3,0,y)$$

$$+\Big(\Big(188s^2+197(t+u)s+206t\,u\Big)H(3,2,y)\Big)\Big/\Big(9(s+t)(s+u)\Big)-7H(0,z)H(3,2,y)$$

$$+\frac{4}{3}\,H(1,z)H(3,2,y)+\frac{22}{3}H(1,z)H(3,3,y)-12H(0,1,z)H(3,3,y)+4H(1,0,z)\,H(3,3,y)$$

$$+\Big(\Big((7u-5t)s^4+\Big(-13t^2+4ut+17u^2\Big)s^3-2\Big(4t^3+8ut^2-13u^2t-5u^3\Big)s^2$$
$$+2tu\Big(-5t^2+2ut+7u^2\Big)s+t^2(t+u)u^2\Big)H(0,0,1,z)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big)$$
$$-2H(1,y)H(0,0,1,z)-2H(2,y)H(0,0,1,z)-8H(3,y)H(0,0,1,z)+2H(0,z)H(0,0,2,y)$$
$$+2H(1,z)H(0,0,2,y)+7H(0,0,2,y)-\Big(u\Big(s^2+(u-2t)s-3tu\Big)H(0,1,0,y)\Big)\Big/\Big((s+u)^2(t+u)\Big)$$
$$-\Big(t\Big(s^2+(t-2u)s-3tu\Big)H(0,1,0,z)\Big)\Big/\Big((s+t)^2(t+u)\Big)+2H(1,y)H(0,1,0,z)$$
$$+6H(2,y)H(0,1,0,z)+4H(3,y)H(0,1,0,z)+\frac{2}{3}H(0,1,1,z)+2H(0,z)H(0,2,0,y)$$
$$+2H(1,z)H(0,2,0,y)+7H(0,2,0,y)-2H(0,z)H(0,2,2,y)-\frac{2}{3}H(0,2,2,y)$$
$$+4H(1,z)H(0,3,0,y)-\Big(\Big((5t+9u)s^4+\Big(9t^2+28ut+19u^2\Big)s^3+2\Big(2t^3+14ut^2+21u^2t+5u^3\Big)s^2$$
$$+2tu\Big(5t^2+14ut+9u^2\Big)s+7t^2(t+u)u^2\Big)H(0,3,2,y)\Big)\Big/\Big((s+t)^2(s+u)^2(t+u)\Big)$$
$$-6H(0,z)H(0,3,2,y)+4H(1,z)H(0,3,3,y)+2H(2,y)H(1,0,0,z)+7H(1,0,0,z)$$
$$+\Big(\Big(9s^5+32(t+u)s^4+\Big(34t^2+80ut+34u^2\Big)s^3+\Big(11t^3+61ut^2+61u^2t+11u^3\Big)s^2$$
$$+tu\Big(16t^2+23ut+16u^2\Big)s+2t^2(t+u)u^2\Big)H(1,0,1,z)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big)$$
$$-2H(1,y)H(1,0,1,z)+6H(2,y)H(1,0,1,z)-\Big(3(s+t+u)H(1,0,2,y)\Big)\Big/\Big((t+u)\Big)$$
$$-2H(0,z)H(1,0,2,y)+2H(1,z)H(1,0,3,y)+\Big(6(s+t)H(1,1,0,y)\Big)\Big/\Big((t+u)\Big)$$
$$+\Big(\Big(2(t+3u)s^2+t(5t+14u)s+3t^2(t+3u)\Big)H(1,1,0,z)\Big)\Big/\Big((s+t)^2(t+u)\Big)$$
$$+2H(1,y)H(1,1,0,z)+6H(2,y)H(1,1,0,z)-\frac{3(s+t+u)H(1,2,0,y)}{(t+u)}$$
$$-2H(1,z)H(1,2,3,y)+2H(0,z)H(2,0,0,y)+2H(1,z)H(2,0,0,y)+7H(2,0,0,y)$$
$$-\frac{2}{3}H(2,0,2,y)+2H(1,z)H(2,0,3,y)+\Big(\Big(-9s^3+(11t-u)s^2+28(t+u)us$$
$$+20(t+u)u^2\Big)H(2,1,0,y)\Big)\Big/\Big(3(s+u)^2(t+u)\Big)+2H(1,z)H(2,1,0,y)+2H(0,z)H(2,2,0,y)$$
$$-\frac{2}{3}H(2,2,0,y)+4H(1,z)H(2,2,3,y)+2H(1,z)H(2,3,0,y)+\Big(\Big(18s^5+61(t+u)s^4$$
$$+5\Big(13t^2+32ut+13u^2\Big)s^3+2\Big(11t^3+67ut^2+67u^2t+11u^3\Big)s^2+38t(t+u)^2us$$
$$+13t^2(t+u)u^2\Big)H(2,3,2,y)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big)+2H(0,z)H(2,3,2,y)$$
$$-8H(1,z)H(2,3,3,y)-2H(0,z)H(3,0,2,y)-7H(3,0,2,y)+4H(1,z)H(3,0,3,y)$$
$$-2H(0,z)H(3,2,0,y)-7H(3,2,0,y)+\frac{4}{3}H(3,2,2,y)+4H(1,z)H(3,3,0,y)$$
$$+4H(0,z)H(3,3,2,y)+\frac{22}{3}H(3,3,2,y)-16H(1,z)H(3,3,3,y)+8H(0,0,1,0,z)+2H(0,0,1,1,z)$$
$$+2H(0,0,2,2,y)+2H(0,1,0,1,z)+6H(0,1,1,0,z)+2H(0,2,0,2,y)+4H(0,2,1,0,y)$$
$$+2H(0,2,2,0,y)+4H(0,3,0,2,y)+4H(0,3,2,0,y)+4H(0,3,3,2,y)+6H(1,0,1,0,z)$$
$$+2H(1,0,3,2,y)+2H(1,1,0,0,z)+4H(1,1,0,1,z)+6H(1,1,1,0,z)-2H(1,2,3,2,y)$$
$$+2H(2,0,0,2,y)+2H(2,0,1,0,y)+2H(2,0,2,0,y)+2H(2,0,3,2,y)+2H(2,1,0,2,y)$$
$$+2H(2,1,2,0,y)+2H(2,2,0,0,y)+4H(2,2,1,0,y)+4H(2,2,3,2,y)+2H(2,3,0,2,y)$$
$$+2H(2,3,2,0,y)-8H(2,3,3,2,y)+4H(3,0,3,2,y)+4H(3,3,0,2,y)+4H(3,3,2,0,y)$$
$$-16H(3,3,3,2,y)+\frac{761}{81}\Big)\Big\}+C_AC_F\Big\{\zeta_4\Big(\frac{93}{4}\Big)+\zeta_3\Big(22H(1,y)-18H(2,y)+4H(1,z)$$
$$+\Big(803s^5(t+u)+4s^4\Big(424t^2+965tu+424u^2\Big)+s^3\Big(821t^3+5329t^2u+5329tu^2+821u^3\Big)$$

$$+s^2\Big(-72t^4+1966t^3u+5336t^2u^2+1966tu^3-72\,u^4\Big)+stu\Big(-144t^3+1307t^2u+1307tu^2$$
$$-144u^3\Big)-72t^2\,u^2\Big(t^2+u^2\Big)\Big)\Big/\Big(18s(s+t)^2(s+u)^2(t+u)\Big)\Bigg)+\zeta_2\Bigg(-\Big(2H(1,y)\Big(18s^2+23st$$
$$+11su-6t^2\Big)\Big)\Big/\Big(3s\,(t+u)\Big)+\Big(tH(0,z)\Big(-10s^3-3s^2(7t+2u)-9st(t+u)+2\,t^3\Big)\Big)\Big/$$
$$\Big(s(s+t)^2(t+u)\Big)+\Big(uH(0,y)\Big(-10s^3-3s^2(2\,t+7u)-9s(t+u)u+2u^3\Big)\Big)\Big/\Big(s(s+u)^2\,(t+u)\Big)$$
$$+\Big(H(2,y)\Big(144s^5(t+u)+s^4\Big(289t^2+554t\,u+289u^2\Big)+4s^3\Big(32t^3+147t^2u+147tu^2+32u^3\Big)$$
$$+s^2\,\Big(-17t^4+90t^3u+166t^2u^2+90tu^3-17u^4\Big)-2stu\Big(35\,t^3+111t^2u+111tu^2+35u^3\Big)$$
$$-t^2u^2\Big(71t^2+166tu+71\,u^2\Big)\Big)\Big)\Big/\Big(6(s+t)^2(s+u)^2(t+u)^2\Big)-\Big(H(1,z)\,\Big(-72s^6(t+u)^2$$
$$-s^5\Big(101t^3+231t^2u+183tu^2+53\,u^3\Big)+4s^4(t+u)^2\Big(8t^2+25tu+26u^2\Big)+s^3\Big(61\,t^5+427t^4u$$
$$+1120t^3u^2+1216t^2u^3+523tu^4+61u^5\Big)+2s^2\,(t+u)^2u\Big(79t^3+236t^2u+79tu^2-12u^3\Big)$$
$$+stu^2\Big(115t^4+369t^3u+321t^2u^2+19tu^3-48u^4\Big)-24t^2(t+u)^2u^4\Big)\Big)\Big/\Big(6s(s+t)^2(s+u)^2(t+u)^3\Big)$$
$$-2\,H(1,y)H(0,z)-6H(0,y)H(1,z)-2H(1,y)H(1,z)-4H(2,y)H(0,z)+2H(0,2,y)$$
$$+6\,H(1,0,y)-2H(1,2,y)-4H(2,0,y)-8H(2,1,y)+8H(2,2,y)+2H(0,1,z)+2H(1,0,z)$$
$$-2\,H(1,1,z)+\Big(19s^3(t+u)+s^2\Big(19t^2+182tu+19u^2\Big)+145\,st(t+u)u+108t^2u^2\Big)\Big/$$
$$\Big(6s(s+t)(s+u)(t+u)\Big)\Bigg)+\Bigg(\Big(\Big(26t^2+26st+33ut+15su\Big)H(0,y)\Big)\Big/\Big(3(s+t)\,(t+u)\Big)$$
$$+\Big(\Big(18tu\Big)\Big/\Big(s(t+u)\Big)+3\Big)H(0,z)H(0,y)+\Big(\Big(242u^2+215su+134tu+134st\Big)H(1,z)H(0,y)\Big)\Big/$$
$$\Big(9\,(s+u)(t+u)\Big)+\Big(\Big((13t+31u)s^2+u(44t+71u)s+40\,(t+u)u^2\Big)H(0,1,z)H(0,y)\Big)\Big/$$
$$\Big(3(s+u)^2\,(t+u)\Big)-\Big(\Big(19st^2+62sut+31su^2-6(t+u)u^2\Big)\,H(1,0,z)H(0,y)\Big)\Big/\Big(3s(t+u)^2\Big)$$
$$-\frac{14}{3}H(1,1,z)H(0,y)+4H(0,0,1,z)\,H(0,y)+2H(0,1,0,z)H(0,y)+8H(0,1,1,z)H(0,y)$$
$$+2H(1,0,1,z)H(0,y)-4\,H(1,1,0,z)H(0,y)+\Big(\Big(26u^2+26su+33tu+15st\Big)H(0,z)\Big)\Big/$$
$$\Big(3\,(s+u)(t+u)\Big)+\frac{803}{54}H(1,z)+\Big(\Big(242t^2+215st+134\,ut+134su\Big)H(0,z)H(2,y)\Big)\Big/$$
$$\Big(9(s+t)(t+u)\Big)+2H(1,z)\,H(2,y)+\frac{803}{54}H(2,y)-\Big(\Big(466s^2+493(t+u)s$$
$$+520t\,u\Big)H(1,z)H(3,y)\Big)\Big/\Big(9(s+t)(s+u)\Big)-14H(1,z)H(0,0,y)-14\,H(2,y)H(0,0,z)$$
$$-\Big(\Big((251t+332u)s^2+\Big(251t^2+637ut+359\,u^2\Big)s+2tu(139t+193u)\Big)H(0,1,z)\Big)\Big/$$
$$\Big(9(s+t)(s+u)\,(t+u)\Big)-\Big(\Big(36s^5+112(t+u)s^4+\Big(107t^2+268ut+107\,u^2\Big)s^3$$
$$+\Big(31t^3+191ut^2+191u^2t+31u^3\Big)s^2+2tu\Big(22t^2+35ut+22u^2\Big)s+4t^2(t+u)u^2\Big)H(2,y)H(0,1,z)\Big)\Big/$$
$$\Big(3(s+t)^2(s+u)^2(t+u)\Big)-\frac{20}{3}H(3,y)H(0,1,z)-4H(0,0,y)H(0,1,z)+\Big(\Big(242u^2+215su+134tu$$
$$+134s\,t\Big)H(0,2,y)\Big)\Big/\Big(9(s+u)(t+u)\Big)-\Big(\Big((29t+47u)s^2+t\,(49t+76u)s$$
$$+20t^2(t+u)\Big)H(0,z)H(0,2,y)\Big)\Big/\Big(3(s+t)^2\,(t+u)\Big)-\frac{14}{3}H(1,z)H(0,2,y)-4H(0,0,z)H(0,2,y)$$
$$+6H(0,1,z)\,H(0,2,y)+\Big(\Big(2(7t+13u)s^4+5\Big(5t^2+16ut+11u^2\Big)\,s^3+\Big(11t^3+79ut^2+121u^2t$$
$$+29u^3\Big)s^2+4tu\Big(7t^2+20ut+13u^2\Big)s+20t^2(t+u)u^2\Big)H(1,z)H(0,3,y)\Big)\Big/\Big((s+t)^2(s+u)^2(t+u)\Big)$$
$$+4H(0,1,z)H(0,3,y)+\Big(\Big(-2(76t+103u)\,s^2+(37t-233u)us+162tu^2\Big)H(1,0,y)\Big)\Big/$$

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$$\Big(9s(s+u)\ (t+u)\Big) + \Big(2\Big(3t^2 + 8st + 14su\Big)H(0,z)H(1,0,y)\Big)\Big/\Big(3s\ (t+u)\Big) + \Big(\frac{6s}{(t+u)}$$

$$+9\Big)H(1,z)H(1,0,y) - 2H(0,1,z)H(1,0,y) + \Big(\Big((t-2u)s^2 + t(t+19u)s + 18t^2u\Big)H(1,0,z)\Big)\Big/$$

$$\Big(s\ (s+t)(t+u)\Big) + \Big(\Big(18(t+u)s^3 + \Big(31t^2 + 32ut + 13\ u^2\Big)s^2 - t\Big(t^2 + 17ut - 8u^2\Big)s$$

$$-2t^2\Big(7t^2 + 20ut + 7\ u^2\Big)\Big)H(2,y)H(1,0,z)\Big)\Big/\Big(3(s+t)^2(t+u)^2\Big) + 20H(3,y)\ H(1,0,z)$$

$$-4H(0,0,y)H(1,0,z) + 8H(0,2,y)H(1,0,z) + 16H(0,3,y)H(1,0,z) + 2\ H(1,0,y)H(1,0,z)$$

$$+\frac{28}{3}H(3,y)H(1,1,z) - 8H(0,0,y)H(1,1,z) + 4\ H(0,3,y)H(1,1,z) + 2H(1,1,z)$$

$$+6H(0,1,z)H(1,2,y) - 2H(1,0,z)\ H(1,2,y) + \Big(\Big(242u^2 + 215su + 134tu + 134st\Big)H(2,0,y)\Big)\Big/$$

$$\Big(9\ (s+u)(t+u)\Big) - \Big(\Big(47t^3 + 153ut^2 + 153u^2t + 47u^3\Big)\ H(0,z)H(2,0,y)\Big)\Big/\Big(3(t+u)^3\Big)$$

$$-\frac{14}{3}H(1,z)H(2,0,y) - 4H(0,0,z)\ H(2,0,y) - 2H(0,1,z)H(2,0,y) - 6H(1,0,z)H(2,0,y)$$

$$-\frac{14}{3}H(0,z)\ H(2,2,y) - 8H(0,0,z)H(2,2,y) - 16H(0,1,z)H(2,2,y) - 8H(1,0,z)H(2,2,y)$$

$$+2\ H(2,2,y) - \Big(\Big(36s^5 + 98(t+u)s^4 + \Big(79t^2 + 212ut + 79\ u^2\Big)s^3 + \Big(17t^3 + 121ut^2 + 121u^2t$$

$$+17u^3\Big)s^2 + 2tu\ \Big(8t^2 + 7ut + 8u^2\Big)s - 10t^2(t+u)u^2\Big)H(1,z)\ H(2,3,y)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big)$$

$$+12H(0,1,z)H(2,3,y) - 4\ H(1,0,z)H(2,3,y) + 20H(1,z)H(3,0,y) - 4H(0,1,z)H(3,0,y)$$

$$+4H(1,1,z)\ H(3,0,y) - \Big(\Big(466s^2 + 493(t+u)s + 520tu\Big)H(3,2,y)\Big)\Big/\Big(9\ (s+t)(s+u)\Big)$$

$$+20H(0,z)H(3,2,y) + \frac{28}{3}H(1,z)H(3,2,y) - 4\ H(0,1,z)H(3,2,y) - 4H(1,0,z)H(3,2,y)$$

$$-\frac{80}{3}H(1,z)H(3,3,y) + 28\ H(0,1,z)H(3,3,y) - 12H(1,0,z)H(3,3,y) - 8H(1,1,z)H(3,3,y)$$

$$+\Big(\Big(4(4\ t-5u)s^4 + \Big(41t^2 - 8ut - 49u^2\Big)s^3 + \Big(25t^3 + 53ut^2 - 73u^2\ t - 29u^3\Big)s^2 + 8tu\Big(4t^2 - ut$$

$$-5u^2\Big)s - 2t^2(t+u)\ u^2\Big)H(0,0,1,z)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big) + 6H(1,y)\ H(0,0,1,z)$$

$$+16H(3,y)H(0,0,1,z) - 8H(0,z)H(0,0,2,y) - 8H(1,z)H(0,0,2,y) - 14\ H(0,0,2,y)$$

$$+4H(1,z)H(0,0,3,y) + \Big(\Big(-6(t+u)u^4 - s\Big(17\ t^2 + 22ut + 17u^2\Big)u^2 + s^2\Big(2t^2 + 19ut - 7u^2\Big)u$$

$$+2s^3\ \Big(5t^2 + 13ut + 2u^2\Big)\Big)H(0,1,0,y)\Big)\Big/\Big(3s(s+u)^2\ (t+u)^2\Big) + 2H(0,z)H(0,1,0,y)$$

$$+2H(1,z)H(0,1,0,y) + \Big(\Big(-6\ (t+u)t^4 + s\Big(t^2 + 14ut + u^2\Big)t^2 + s^2\Big(29t^2 + 91ut + 38\ u^2\Big)t$$

$$+s^3\Big(22t^2 + 62ut + 28u^2\Big)\Big)H(0,1,0,z)\Big)\Big/\Big(3s\ (s+t)^2(t+u)^2\Big) - 2H(1,y)H(0,1,0,z)$$

$$-12H(2,y)H(0,1,0,z) - 4H(3,y)\ H(0,1,0,z) - 4H(3,y)H(0,1,1,z) + \frac{14}{3}H(0,1,1,z)$$

$$-8H(1,z)\ H(0,2,0,y) - 14H(0,2,0,y) + 4H(0,z)H(0,2,2,y) - \frac{14}{3}H(0,2,2,y)$$

$$+2\ H(1,z)H(0,2,3,y) - 8H(1,z)H(0,3,0,y) + \Big(\Big(2(7t + 13u)s^4 + 5\Big(5\ t^2 + 16ut + 11u^2\Big)s^3$$

$$+\Big(11t^3 + 79ut^2 + 121u^2t + 29u^3\Big)s^2 + 4tu\Big(7t^2 + 20ut + 13u^2\Big)s + 20t^2(t+u)u^2\Big)H(0,3,2,y)\Big)\Big/$$

$$\Big((s+t)^2(s+u)^2(t+u)\Big) + 16H(0,z)H(0,3,2,y) + 4\ H(1,z)H(0,3,2,y) - 12H(1,z)H(0,3,3,y)$$

$$+4H(0,z)H(1,0,0,y) + 4H(1,z)\ H(1,0,0,y) + 14H(1,0,0,y) - 4H(2,y)H(1,0,0,z) - \Big(\Big(18s^5$$

$$+49(t+u)s^4 + \Big(35t^2 + 88ut + 35u^2\Big)s^3 + 4\Big(t^3 + 5ut^2 + 5u^2t + u^3\Big)s^2 - 2tu\Big(5t^2 + 28ut + 5u^2\Big)s$$

$$-23t^2\ (t+u)u^2\Big)H(1,0,1,z)\Big)\Big/\Big(3(s+t)^2(s+u)^2(t+u)\Big) + 6\ H(1,y)H(1,0,1,z)$$

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$$-18H(2,y)H(1,0,1,z)-4H(3,y)H(1,0,1,z)+\left(\frac{6s}{(t+u)}+9\right)H(1,0,2,y)+4H(0,z)H(1,0,2,y)$$
$$-6H(1,z)H(1,0,3,y)-\left(2\left(18s^2+23ts+11us-6t^2\right)H(1,1,0,y)\right)\Big/\left(3s(t+u)\right)-\Big(\Big(2u(3t+u)s^3$$
$$+\left(3t^3+21ut^2+6u^2t-4\ u^3\right)s^2+t\left(3t^3+18ut^2+3u^2t-8u^3\right)s-4t^2\ (t+u)u^2\Big)H(1,1,0,z)\Big)\Big/$$
$$\left(s(s+t)^2(t+u)^2\right)-2H(1,y)\ H(1,1,0,z)-18H(2,y)H(1,1,0,z)-4H(3,y)H(1,1,0,z)$$
$$+\left(\frac{6s}{(t+u)}+9\right)H(1,2,0,y)+2H(0,z)H(1,2,0,y)+6H(1,z)H(1,2,3,y)-4\ H(0,z)H(2,0,0,y)$$
$$-8H(1,z)H(2,0,0,y)-14H(2,0,0,y)+2H(0,z)H(2,0,2,y)-\frac{14}{3}H(2,0,2,y)-4H(1,z)H(2,0,3,y)$$
$$+\Big(\Big(6\ (t+u)s^3+\left(9t^2+20ut+15u^2\right)s^2+u\left(12t^2+13ut+9\ u^2\right)s-4tu^3\Big)H(2,1,0,y)\Big)\Big/$$
$$\left((s+u)^2(t+u)^2\right)-2H(0,z)\ H(2,1,0,y)-2H(1,z)H(2,1,0,y)-4H(0,z)H(2,2,0,y)$$
$$-\frac{14}{3}\ H(2,2,0,y)-16H(1,z)H(2,2,3,y)-4H(1,z)H(2,3,0,y)-\Big(\Big(36s^5+98\ (t+u)s^4+\Big(79t^2$$
$$+212ut+79u^2\Big)s^3+\left(17t^3+121u\ t^2+121u^2t+17u^3\right)s^2+2tu\left(8t^2+7ut+8u^2\right)s$$
$$-10\ t^2(t+u)u^2\Big)H(2,3,2,y)\Big)\Big/\left(3(s+t)^2(s+u)^2\ (t+u)\right)-4H(0,z)H(2,3,2,y)$$
$$+16H(1,z)H(2,3,3,y)+8H(0,z)H(3,0,2,y)+4\ H(1,z)H(3,0,2,y)+20H(3,0,2,y)$$
$$-12H(1,z)H(3,0,3,y)+4H(1,z)H(3,2,0,y)+20\ H(3,2,0,y)+4H(0,z)H(3,2,2,y)$$
$$+\frac{28}{3}H(3,2,2,y)-8H(1,z)\ H(3,2,3,y)-12H(1,z)H(3,3,0,y)-12H(0,z)H(3,3,2,y)$$
$$-8H(1,z)H(3,3,2,y)-\frac{80}{3}H(3,3,2,y)+40H(1,z)H(3,3,3,y)+4H(0,0,1,0,y)-8H(0,0,1,0,z)$$
$$-8H(0,0,1,1,z)-8H(0,0,2,2,y)+4H(0,0,3,2,y)-8H(0,1,0,1,z)+2\ H(0,1,0,2,y)$$
$$-12H(0,1,1,0,z)+2H(0,1,2,0,y)-8H(0,2,0,2,y)-2H(0,2,1,0,y)-8\ H(0,2,2,0,y)$$
$$+2H(0,2,3,2,y)-8H(0,3,0,2,y)-8H(0,3,2,0,y)+4H(0,3,2,2,y)-12\ H(0,3,3,2,y)$$
$$-2H(1,0,0,1,z)+4H(1,0,0,2,y)+4H(1,0,1,0,y)-4H(1,0,1,0,z)+4\ H(1,0,2,0,y)$$
$$-6H(1,0,3,2,y)-12H(1,1,0,1,z)-14H(1,1,1,0,z)+4H(1,2,0,0,y)+4\ H(1,2,1,0,y)$$
$$+6H(1,2,3,2,y)-8H(2,0,0,2,y)-8H(2,0,2,0,y)-4H(2,0,3,2,y)+4\ H(2,1,0,0,y)$$
$$-2H(2,1,0,2,y)-8H(2,1,1,0,y)-2H(2,1,2,0,y)-8H(2,2,0,0,y)-8\ H(2,2,1,0,y)$$
$$-16H(2,2,3,2,y)-4H(2,3,0,2,y)-4H(2,3,2,0,y)+16H(2,3,3,2,y)+8\ H(3,0,1,0,y)$$
$$+4H(3,0,2,2,y)-12H(3,0,3,2,y)+4H(3,2,0,2,y)-8H(3,2,1,0,y)+4\ H(3,2,2,0,y)$$
$$-8H(3,2,3,2,y)-12H(3,3,0,2,y)-12H(3,3,2,0,y)-8\ H(3,3,2,2,y)+40H(3,3,3,2,y)$$
$$-\frac{4003}{162}\Big)\Big\}+C_F^2\Big\{\zeta_4\Big(-22\Big)+\zeta_3\Big(-16H(1,y)+8H(2,y)-8H(1,z)-\Big(2\Big(25s^5(t+u)$$
$$+s^4\left(51\ t^2+112tu+51u^2\right)+4s^3\left(6t^3+37t^2u+37tu^2+6u^3\right)-2\ s^2\Big(t^4-27t^3u-69t^2u^2$$
$$-27tu^3+u^4\Big)+stu\left(-4t^3+33t^2u+33tu^2-4u^3\right)-2t^2u^2\left(t^2+u^2\right)\Big)\Big)\Big/\left(s(s+t)^2(s+u)^2(t+u)\right)\Big)$$
$$+\zeta_2\Bigg(\Big(2tH(0,z)\left(4s^3+2s^2(4t+u)+3st(t+u)-t^3\right)\Big)\Big/\left(s(s+t)^2(t+u)\right)+\Big(2uH(0,y)\Big(4s^3+2s^2(t$$
$$+4u)+3s(t+u)u-u^3\Big)\Big)\Big/\left(s(s+u)^2(t+u)\right)+\Big(2H(2,y)\Big(s^4\left(5t^2+12tu+5u^2\right)+s^3\Big(11t^3+39t^2u$$
$$+39tu^2+11\ u^3\Big)+s^2\left(6t^4+42t^3u+76t^2u^2+42tu^3+6u^4\right)+2stu\ \left(7t^3+26t^2u+26tu^2+7u^3\right)$$
$$+t^2u^2\left(9t^2+20tu+9\ u^2\right)\Big)\Big)\Big/\left((s+t)^2(s+u)^2(t+u)^2\right)+\Big(2H(1,z)\ \Big(s^5\left(5t^2+16tu+9u^2\right)$$

$$+s^4\left(11t^3+47t^2u+53tu^2+17u^3\right)+s^3\left(6t^4+46t^3u+92t^2u^2+54tu^3+6u^4\right)+2s^2u\left(7t^4+29t^3u\right.$$

$$\left.+29t^2u^2+6tu^3-u^4\right)+stu^2\left(9t^3+20t^2u+5tu^2-4u^3\right)-2t^2(t+u)u^4\Big)\Big)\Big/\left(s(s+t)^2(s+u)^2(t+u)^2\right)$$

$$+\left(4t(2s-t)H(1,y)\right)\Big/\left(s\ (t+u)\right)-8H(2,y)H(1,z)+8H(0,1,y)-8H(0,2,y)+8H(1,1,y)$$

$$+8H(2,1,y)-16\ H(2,2,y)+\Big(2\Big(6s^3(t+u)+s^2\left(6t^2+4tu+6u^2\right)-s\ t(t+u)u-6t^2u^2\Big)\Big)\Big/$$

$$\left(s(s+t)(s+u)(t+u)\right)\Bigg)+\Bigg(-\left(12tuH(0,y)H(0,z)\right)\Big/\left(s(t+u)\right)-\Big(2(3s(2t+u)+t(7t$$

$$+3u))H(2,y)H(0,z)\Big)\Big/\left((s+t)(t+u)\right)+\Big(2\left((4t+6u)s^2+t(7t+10u)s+3t^2(t+u)\right)H(0,2,y)H(0,z)\Big)\Big/$$

$$\left((s+t)^2\ (t+u)\right)-\Big(2\left(t^2+s(t+3u)\right)H(1,0,y)H(0,z)\Big)\Big/\left(s\ (t+u)\right)+\Big(\Big(6t^3+22ut^2+22u^2t$$

$$+6u^3\Big)H(2,0,y)\ H(0,z)\Big)\Big/\left((t+u)^3\right)+12H(2,2,y)H(0,z)-12H(3,2,y)H(0,z)$$

$$+8H(0,0,2,y)\ H(0,z)-8H(0,3,2,y)H(0,z)-4H(2,0,2,y)H(0,z)-8H(3,0,2,y)H(0,z)$$

$$-8\ H(3,2,2,y)H(0,z)+8H(3,3,2,y)H(0,z)-\Big(2(3s(t+2u)+u(3t+7u))\ H(0,y)H(1,z)\Big)\Big/$$

$$\left((s+u)(t+u)\right)-9H(1,z)+9H(1,z)H(2,y)-9\ H(2,y)+\Big(\Big(20s^2+22(t+u)s$$

$$+24tu\Big)H(1,z)\ H(3,y)\Big)\Big/\left((s+t)(s+u)\right)+\Big(2\Big((4t+7u)s^2+\left(4t^2+13u\ t+8u^2\right)s+tu(5t$$

$$+9u)\Big)H(0,1,z)\Big)\Big/\left((s+t)(s+u)\ (t+u)\right)-\Big(2u((s+t)+u)(2s+3u)H(0,y)H(0,1,z)\Big)\Big/$$

$$\left((s+u)^2\ (t+u)\right)-\Big(2\Big(2(t+u)s^4+\left(5t^2+12ut+5u^2\right)\ s^3+\left(3t^3+19ut^2+19u^2t+3u^3\right)s^2$$

$$+2tu\left(4t^2+11u\ t+4u^2\right)s+6t^2(t+u)u^2\Big)H(2,y)H(0,1,z)\Big)\Big/\left((s+t)^2\ (s+u)^2(t+u)\right)$$

$$+12H(3,y)H(0,1,z)-\Big(2(3s(t+2u)+u(3t+7\ u))H(0,2,y)\Big)\Big/\left((s+u)(t+u)\right)$$

$$+12H(1,z)H(0,2,y)-4H(0,1,z)\ H(0,2,y)-\Big(2\Big(4(t+2u)s^4+\left(7t^2+24ut+17u^2\right)\ s^3$$

$$+\left(3t^3+23ut^2+37u^2t+9u^3\right)s^2+8tu\left(t^2+3ut+2\ u^2\right)s+6t^2(t+u)u^2\Big)H(1,z)H(0,3,y)\Big)\Big/$$

$$\left((s+t)^2\ (s+u)^2(t+u)\right)+\Big(2\Big((3t+5u)s^2+\left(6u^2-4t\ u\right)s-6tu^2\Big)H(1,0,y)\Big)\Big/\left(s(s+u)(t+u)\right)$$

$$-6H(1,z)H(1,0,y)+4H(0,1,z)H(1,0,y)-\Big(2t\left(s^2+(t+7u)s+6tu\right)H(1,0,z)\Big)\Big/\left(s(s+t)(t+u)\right)$$

$$+\Big(2u(2s(2t+u)-(t+u)u)H(0,y)H(1,0,z)\Big)\Big/\left(s(t+u)^2\right)+\Big(2t\Big(2(t+2u)s^2+\left(5t^2+11ut+2u^2\right)s$$

$$+t\left(3t^2+8ut+3u^2\right)\Big)H(2,y)\ H(1,0,z)\Big)\Big/\left((s+t)^2(t+u)^2\right)-12H(3,y)H(1,0,z)$$

$$-8H(0,2,y)\ H(1,0,z)-8H(0,3,y)H(1,0,z)+12H(0,y)H(1,1,z)-24H(3,y)H(1,1,z)$$

$$+8\ H(0,0,y)H(1,1,z)-8H(0,3,y)H(1,1,z)+9H(1,1,z)-4H(0,1,z)H(1,2,y)-\Big(2\ (3s(t+2u)$$

$$+u(3t+7u))H(2,0,y)\Big)\Big/\left((s+u)(t+u)\right)+12H(1,z)\ H(2,0,y)-4H(0,1,z)H(2,0,y)$$

$$+8H(0,0,z)H(2,2,y)+16H(0,1,z)H(2,2,y)+9H(2,2,y)-\Big(2\Big(8(t+u)s^4+\left(17t^2+36ut+17u^2\right)s^3$$

$$+\left(9t^3+49ut^2+49u^2t+9u^3\right)s^2+2tu\left(10t^2+23u\ t+10u^2\right)s+12t^2(t+u)u^2\Big)H(1,z)H(2,3,y)\Big)\Big/$$

$$\left((s+t)^2\ (s+u)^2(t+u)\right)-12H(1,z)H(3,0,y)-8H(1,1,z)\ H(3,0,y)+\Big(\Big(20s^2+22(t+u)s$$

$$+24tu\Big)\ H(3,2,y)\Big)\Big/\left((s+t)(s+u)\right)-24H(1,z)H(3,2,y)+8H(0,1,z)H(3,2,y)$$

$$+24\ H(1,z)H(3,3,y)-8H(0,1,z)H(3,3,y)+8H(1,0,z)H(3,3,y)+16H(1,1,z)\ H(3,3,y)$$

$$-\Big(2s(t-u)\Big(2s^3+5(t+u)s^2+\left(3t^2+10ut+3\ u^2\right)s+4t(t+u)u\Big)H(0,0,1,z)\Big)\Big/$$

$$\left((s+t)^2(s+u)^2\ (t+u)\right)-4H(1,y)H(0,0,1,z)+8H(2,y)H(0,0,1,z)+8H(1,z)H(0,0,2,y)$$

$$-8 H(1,z)H(0,0,3,y)+\Big(2\Big((t+u)u^4+s\Big(9t^2+16ut+9u^2\Big)u^2+2s^2\Big(7t^2+12ut+7u^2\Big)u$$
$$+2s^3\Big(3t^2+5ut+3u^2\Big)\Big)H(0,1,0,y)\Big)\Big/\Big(s(s+u)^2(t+u)^2\Big)-4H(1,z)H(0,1,0,y)+\Big(2t\Big(-2us^3$$
$$+2\Big(t^2+u^2\Big)s^2+t\Big(3t^2+4ut+3u^2\Big)s+t^3(t+u)\Big)H(0,1,0,z)\Big)\Big/\Big(s(s+t)^2(t+u)^2\Big)$$
$$+4H(2,y)H(0,1,0,z)-8H(0,y)H(0,1,1,z)+8H(3,y)H(0,1,1,z)-12H(0,1,1,z)$$
$$+8H(1,z)H(0,2,0,y)+12H(0,2,2,y)-4H(1,z)H(0,2,3,y)-\Big(2\Big(4(t+2u)s^4+\Big(7t^2+24ut$$
$$+17u^2\Big)s^3+\Big(3t^3+23ut^2+37u^2t+9u^3\Big)s^2+8tu\Big(t^2+3ut+2u^2\Big)s+6t^2(t+u)u^2\Big)H(0,3,2,y)\Big)\Big/$$
$$\Big((s+t)^2(s+u)^2(t+u)\Big)-8H(1,z)H(0,3,2,y)+8H(1,z)H(0,3,3,y)-8H(1,z)H(1,0,0,y)$$
$$-\Big(2\Big(5(t+u)s^4+\Big(11t^2+24ut+11u^2\Big)s^3+\Big(6t^3+34ut^2+34u^2t+6u^3\Big)s^2+2tu\Big(7t^2+17ut$$
$$+7u^2\Big)s+9t^2(t+u)u^2\Big)H(1,0,1,z)\Big)\Big/\Big((s+t)^2(s+u)^2(t+u)\Big)-4H(0,y)H(1,0,1,z)$$
$$-4H(1,y)H(1,0,1,z)+12H(2,y)H(1,0,1,z)+8H(3,y)H(1,0,1,z)-6H(1,0,2,y)$$
$$+4H(1,z)H(1,0,3,y)+\Big(4(2s-t)tH(1,1,0,y)\Big)\Big/\Big(s(t+u)\Big)-\Big(2\Big(\Big(t^2-2ut-u^2\Big)s^3$$
$$+\Big(t^3-7ut^2-2u^2t+2u^3\Big)s^2+\Big(4tu^3-6t^3u\Big)s+2t^2(t+u)u^2\Big)H(1,1,0,z)\Big)\Big/\Big(s(s+t)^2(t+u)^2\Big)$$
$$+4H(2,y)H(1,1,0,z)-6H(1,2,0,y)-4H(1,z)H(1,2,3,y)+8H(1,z)H(2,0,0,y)+12H(2,0,2,y)$$
$$-\Big(2\Big(\Big(6t^2+8ut+4u^2\Big)s^2+u\Big(10t^2+13ut+7u^2\Big)s+u^2\Big(3t^2+4ut+3u^2\Big)\Big)H(2,1,0,y)\Big)\Big/$$
$$\Big((s+u)^2(t+u)^2\Big)-4H(1,z)H(2,1,0,y)+12H(2,2,0,y)+16H(1,z)H(2,2,3,y)-\Big(2\Big(8(t+u)s^4$$
$$+\Big(17t^2+36ut+17u^2\Big)s^3+\Big(9t^3+49ut^2+49u^2t+9u^3\Big)s^2+2tu\Big(10t^2+23ut+10u^2\Big)s$$
$$+12t^2(t+u)u^2\Big)H(2,3,2,y)\Big)\Big/\Big((s+t)^2(s+u)^2(t+u)\Big)-8H(1,z)H(3,0,2,y)-12H(3,0,2,y)$$
$$+8H(1,z)H(3,0,3,y)-8H(1,z)H(3,2,0,y)-12H(3,2,0,y)-24H(3,2,2,y)$$
$$+16H(1,z)H(3,2,3,y)+8H(1,z)H(3,3,0,y)+16H(1,z)H(3,3,2,y)+24H(3,3,2,y)$$
$$-16H(1,z)H(3,3,3,y)+8H(0,0,1,1,z)+8H(0,0,2,2,y)-8H(0,0,3,2,y)+8H(0,1,0,1,z)$$
$$-4H(0,1,0,2,y)+8H(0,1,1,0,y)+8H(0,1,1,0,z)-4H(0,1,2,0,y)+8H(0,2,0,2,y)$$
$$-4H(0,2,1,0,y)+8H(0,2,2,0,y)-4H(0,2,3,2,y)-8H(0,3,2,2,y)+8H(0,3,3,2,y)$$
$$+4H(1,0,0,1,z)-8H(1,0,0,2,y)+4H(1,0,1,0,y)-8H(1,0,2,0,y)+4H(1,0,3,2,y)$$
$$+8H(1,1,0,0,y)+8H(1,1,0,1,z)+8H(1,1,1,0,y)+8H(1,1,1,0,z)-8H(1,2,0,0,y)$$
$$-4H(1,2,1,0,y)-4H(1,2,3,2,y)+8H(2,0,0,2,y)-4H(2,0,1,0,y)+8H(2,0,2,0,y)$$
$$-8H(2,1,0,0,y)-4H(2,1,0,2,y)+8H(2,1,1,0,y)-4H(2,1,2,0,y)+8H(2,2,0,0,y)$$
$$+16H(2,2,3,2,y)-8H(3,0,1,0,y)-8H(3,0,2,2,y)+8H(3,0,3,2,y)-8H(3,2,0,2,y)$$
$$+8H(3,2,1,0,y)-8H(3,2,2,0,y)+16H(3,2,3,2,y)+8H(3,3,0,2,y)+8H(3,3,2,0,y)$$
$$+16H(3,3,2,2,y)-16H(3,3,3,2,y)+\frac{19}{2}\Big)\Big\}+C_An_f\Big\{\zeta_3\Big(-\frac{37}{18}\Big)+\zeta_2\Big(-\frac{1}{3}H(0,y)+\frac{1}{6}H(2,y)$$
$$-\frac{1}{3}H(0,z)+\frac{1}{6}H(1,z)-\frac{tu}{3s(t+u)}-\frac{7}{36}\Big)+\Big(H(0,y)H(0,z)\Big(\frac{5}{9}-\frac{tu}{3s(t+u)}\Big)-\frac{tu\,H(1,0,y)}{3s(t+u)}$$
$$+H(1,0,z)\Big(\frac{31}{36}-\frac{tu}{3s(t+u)}\Big)+\frac{(29t+20u)H(0,y)}{9(t+u)}+\frac{(20t+29u)H(0,z)}{9(t+u)}+\frac{31}{36}H(0,y)H(1,z)$$
$$-H(0,y)H(0,0,z)+\frac{2}{3}H(0,y)H(0,1,z)-H(0,y)H(1,0,z)+\frac{2}{3}H(0,y)H(1,1,z)+\frac{31}{36}H(2,y)H(0,z)$$
$$+H(2,y)H(1,z)-\frac{20}{9}H(3,y)H(1,z)-H(0,0,y)H(0,z)-H(0,0,y)H(1,z)-H(2,y)H(0,0,z)$$
$$-\frac{2}{3}H(2,y)H(0,1,z)-\frac{1}{3}H(3,y)H(0,1,z)-\frac{1}{3}H(0,2,y)H(0,z)+\frac{2}{3}H(0,2,y)H(1,z)$$

$$
\begin{aligned}
&+H(0,3,y)H(1,z)+H(3,y)H(1,0,z)-\frac{4}{3}H(3,y)H(1,1,z)-H(2,0,y)H(0,z)+\frac{2}{3}H(2,0,y)H(1,z)\\
&+\frac{2}{3}H(2,2,y)\ H(0,z)-\frac{4}{3}H(2,3,y)H(1,z)+H(3,0,y)H(1,z)+H(3,2,y)\ H(0,z)\\
&-\frac{4}{3}H(3,2,y)H(1,z)-\frac{4}{3}H(3,3,y)H(1,z)+\frac{17}{27}\ H(2,y)-\frac{41}{18}H(0,0,y)+\frac{31}{36}H(0,2,y)\\
&+\frac{31}{36}\ H(2,0,y)+H(2,2,y)-\frac{20}{9}H(3,2,y)-H(0,0,2,y)-H(0,2,0,y)+\frac{2}{3}\ H(0,2,2,y)\\
&+H(0,3,2,y)-H(2,0,0,y)+\frac{2}{3}H(2,0,2,y)-\frac{2}{3}\ H(2,1,0,y)+\frac{2}{3}H(2,2,0,y)-\frac{4}{3}\ H(2,3,2,y)\\
&+H(3,0,2,y)+H(3,2,0,y)-\frac{4}{3}H(3,2,2,y)-\frac{4}{3}\ H(3,3,2,y)+\frac{17}{27}H(1,z)-\frac{41}{18}H(0,0,z)\\
&-\frac{49}{36}H(0,1,z)+H(1,1,z)-\frac{1}{3}H(0,0,1,z)-\frac{2}{3}H(0,1,1,z)-H(1,0,0,z)-\frac{2}{3}H(1,0,1,z)-\frac{439}{162}\Bigg)\Bigg\}\\
&+C_F n_f\Bigg\{\zeta_3\left(-\frac{1}{9}\right)+\zeta_2\left(\frac{4}{3}H(1,y)-\frac{5}{3}H(2,y)-\frac{1}{3}H(1,z)+\frac{1}{3}\right)+\Bigg(-\frac{(7t+3u)H(0,y)}{6(t+u)}\\
&-\frac{(3t+7u)H(0,z)}{6\ (t+u)}-\frac{31}{18}H(0,y)H(1,z)-\frac{4}{3}H(0,y)\ H(0,1,z)+\frac{1}{3}H(0,y)H(1,0,z)\\
&-\frac{4}{3}H(0,y)H(1,1,z)-\frac{31}{18}H(2,y)H(0,z)-2H(2,y)H(1,z)+\frac{40}{9}H(3,y)H(1,z)+2H(0,0,y)H(1,z)\\
&+2H(2,y)H(0,0,z)+\frac{4}{3}H(2,y)\ H(0,1,z)+\frac{2}{3}H(3,y)H(0,1,z)+\frac{2}{3}H(0,2,y)H(0,z)\\
&-\frac{4}{3}\ H(0,2,y)H(1,z)-2H(0,3,y)H(1,z)-\frac{1}{3}H(1,0,y)H(0,z)-\frac{4}{3}\ H(2,y)H(1,0,z)\\
&-2H(3,y)H(1,0,z)+\frac{8}{3}H(3,y)H(1,1,z)+\frac{2}{3}\ H(2,0,y)H(0,z)-\frac{4}{3}H(2,0,y)H(1,z)\\
&-\frac{4}{3}H(2,2,y)\ H(0,z)+\frac{8}{3}H(2,3,y)H(1,z)-2H(3,0,y)H(1,z)-2H(3,2,y)\ H(0,z)\\
&+\frac{8}{3}H(3,2,y)H(1,z)+\frac{8}{3}H(3,3,y)H(1,z)-\frac{34}{27}\ H(2,y)-\frac{31}{18}H(0,2,y)+\frac{20}{9}H(1,0,y)\\
&-\frac{31}{18}H(2,0,y)-2\ H(2,2,y)+\frac{40}{9}H(3,2,y)+2H(0,0,2,y)-\frac{1}{3}H(0,1,0,y)+2\ H(0,2,0,y)\\
&-\frac{4}{3}H(0,2,2,y)-2H(0,3,2,y)-2H(1,0,0,y)+\frac{4}{3}\ H(1,1,0,y)+2H(2,0,0,y)-\frac{4}{3}H(2,0,2,y)\\
&-\frac{4}{3}\ H(2,2,0,y)+\frac{8}{3}H(2,3,2,y)-2H(3,0,2,y)-2H(3,2,0,y)+\frac{8}{3}\ H(3,2,2,y)+\frac{8}{3}H(3,3,2,y)\\
&-\frac{34}{27}H(1,z)+\frac{49}{18}\ H(0,1,z)+\frac{1}{2}H(1,0,z)-2H(1,1,z)+\frac{2}{3}H(0,0,1,z)-\frac{1}{3}\ H(0,1,0,z)\\
&+\frac{4}{3}H(0,1,1,z)+\frac{4}{3}H(1,0,1,z)+\frac{371}{81}\Bigg)\Bigg\}+n_f^2\Bigg\{\frac{1}{36}H(0,y)H(0,z)-\frac{5}{27}H(0,y)+\frac{5}{36}\ H(0,0,y)\\
&-\frac{5}{27}H(0,z)+\frac{5}{36}H(0,0,z)+\frac{\zeta_2}{18}\Bigg\}
\end{aligned}
$$

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