

Electroweak vacuum stability and the seesaw mechanism revisited

J. N. Ng^{1,a}, Alejandro de la Puente^{1,2,b}

¹ Theory Department, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada

² Department of Physics, Carleton University, 1125 Colonel By Drive, Ottawa, ON K1S 5B6, Canada

Received: 8 December 2015 / Accepted: 25 February 2016 / Published online: 4 March 2016

© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract We study the electroweak vacuum stability in Type I seesaw models for three generations of neutrinos in scenarios where the right-handed neutrinos have explicit bare mass terms in the Lagrangian and where these are dynamically generated through the mechanism of spontaneous symmetry breaking. To best highlight the difference of the two cases we concentrate on the absolute stability of the scalar potential. We observe that for the first scenario, the scale at which the scalar potential becomes unstable is lower from that within the standard model. In addition the Yukawa couplings \mathbf{Y}_ν are constrained such that $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \lesssim 10^{-3}$. In the second scenario the electroweak stability can be improved in a large region of parameter space. However, we found that the scalar used to break the lepton number symmetry cannot be too light and have a large coupling to right-handed neutrinos in order for the seesaw mechanism to be a valid mechanism for neutrino mass generation. In this case we have $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \lesssim 0.01$.

1 Introduction

The discovery of the Higgs boson at the LHC completes the standard model (SM) of particle physics. The measured value of the Higgs boson mass, $m_h = 125.09 \pm 0.21_{\text{stat.}} \pm 0.11_{\text{sys.}}$ as given by the combined results of both ATLAS and CMS experiments [1–3] implies that the quartic coupling, λ_H , of the SM scalar potential, $V(H)$, is relatively small. Specifically, the scalar potential is defined by $V(H) = \frac{\lambda_H}{2} (H^\dagger H - v^2/2)^2$ where H is the SM doublet Higgs field and $v = 246$ GeV. When it is used for the boundary value of the renormalization group (RG) running of λ_H , it results in a negative value of λ_H below the Planck scale due to the large top quark Yukawa coupling [4–7]. In fact, the latest NNLO study of the vacuum stability requires that

$m_h > 129.4 \pm 1.8$ GeV [5] for absolute stability up to the Planck scale. As a result, the electroweak vacuum is unstable at high energies. However, the most up-to-date measurements of the top quark mass and the strong coupling constant result in a scalar potential that is metastable with a sufficiently long-lived electroweak vacuum [8, 9]. The exact value of the scale, which we denote by Λ_I^{SM} where λ_H becomes negative strongly depends on the value of the top quark mass and the strong coupling constant and it is well known that at the 2-loop level in the renormalization group equation evolution of SM couplings the value of λ_H can turn negative at scales in the range $\sim 10^{10}$ – 10^{13} GeV. These remarks hold if we assume that no new physics enters in extrapolating the SM to such high energies.

Below we study the electroweak vacuum stability in Type I seesaw models for three generations of neutrinos in scenarios where the right-handed neutrinos have explicit bare mass terms in the Lagrangian and where these are dynamically generated through the mechanism of spontaneous symmetry breaking. We compare the two scenarios for the case of absolute stability of the scalar potential.

2 Stability in the Type I seesaw

Despite its spectacular success the SM cannot be the complete theory of nature. We now have convincing evidence that neutrinos have small but finite masses. Within the SM this can arise by incorporating a dimension five effective operator $\kappa LLHH$, where L denotes a SM lepton doublet. After electroweak symmetry breaking neutrinos obtain a mass given by $\frac{\kappa v^2}{2}$. By dimensional arguments, κ scales as $1/M$ where M is the new physics scale where the above operator is generated. The seesaw mechanism is the simplest manifestation of the above idea. One adds two or more right-handed neutrinos, N_i , where $i > 2$ to the SM. In most scenarios, i is taken to be 3 for symmetry reasons and for concreteness this is what we assume. In this simplest version the Lagrangian that gives rise to neutrino masses is given by

^a e-mail: misery@triumf.ca

^b e-mail: apuate@physics.carleton.ca

$$\mathcal{L}_\nu = -\overline{N_R} \mathcal{M}_D \nu_L - \frac{1}{2} \overline{N_R} \mathcal{M}_M N_R^c + h.c. \tag{1}$$

where $\mathcal{M}_D = \frac{1}{\sqrt{2}} v \mathbf{Y}_\nu$ and \mathbf{Y}_ν is the 3×3 neutrino Yukawa coupling matrix. \mathcal{M}_M is the Majorana mass matrix for the right-handed N_i fields. Without loss of generality we shall take \mathcal{M}_M to be diagonal and we will work in the charged lepton mass basis. It is well known that for $M \gg v$ the masses of the light SM neutrinos are given by $m_\nu \simeq \frac{(y_\nu v)^2}{2M}$ as a result of integrating out the heavy right-handed neutrinos [10–15]. For energies below M , κ is a running parameter [16, 17] and its one-loop RGE is approximately given by

$$\beta_\kappa = \frac{1}{16\pi^2} (-3g^2 + 2\lambda_H + 6y_t^2) \kappa. \tag{2}$$

In the equation above, g denotes the $SU(2)_W$ gauge coupling and y_t the top Yukawa coupling. This is the only new parameter added to the SM and its value is very small since it has to yield the active neutrino masses. Its contribution to the running of λ_H is of order $v^2 \kappa^2$ and thus negligible. The coupling κ increases with energy due to y_t being large albeit slowly. Moreover, the rest of the SM couplings run undisturbed. However, above the scale M the neutrino Yukawa couplings start to run and affect the running of λ_H much like the top quark Yukawa coupling. Hence, the stability of the electroweak vacuum will set a limit on how large Y_ν can be if we demand that the presence of neutrinos does not make the electroweak vacuum unstable too soon. This was first studied in [18] and later in [19] with an emphasis on the Dirac Yukawa matrix dependence. Furthermore, the work in [20] covers in depth the case where the scalar potential is metastable with a lifetime longer than the age of the universe and in the presence of three degenerate right-handed neutrinos. In this work, we will extend their studies using the full 2-loop RG running of the SM couplings and \mathbf{Y}_ν and absolute stability in order to contrast with the next scenario. We also implement realistic neutrino mass matrices that encode the data from neutrino oscillations experiments [21]. For simplicity we take \mathcal{M}_M to be proportional to the unit matrix with scale given by M_N . To be consistent we require that $M_N < \Lambda_I$ where the electroweak vacuum becomes unstable.

An important quantity in our analysis is $\text{Tr}[\kappa(m_h)] = \frac{2}{v^2} \sum_i^3 m_i$ where m_i denotes the active neutrino masses. An upper limit on the above sum constrained by cosmology and astrophysics is given in [22]. We are mindful that the above is not without theoretical assumptions, i.e. the validity of the Λ_{CDM} cosmology model. For degenerate neutrino masses we have $m_{\text{cosmo}} = \sum_i m_i = 3m + \frac{\delta m^2}{2m} + \frac{\Delta m^2}{2m}$ where m is the common mass and δm^2 and Δm^2 are the solar and atmospheric neutrino oscillation frequencies, respectively. The most recent values for these parameters can be found in [23].

For a normal hierarchy $m_{\text{cosmo}} \simeq m_0 \left(1 + 2\sqrt{1 - \frac{\Delta m^2}{m_0^2}} \right)$

where m_0 is the heaviest of the three neutrinos. For the inverted hierarchy $m_{\text{cosmo}} \simeq m_0 \left(2 + \sqrt{1 - \frac{\Delta m^2}{m_0^2}} \right)$. In both cases the smaller δm^2 term is dropped. Furthermore, we use the neutrino mass matrix elements found in [21] and define at the Majorana mass scale $y_{\ell\ell}^2 = 2 \frac{M_N}{v^2} m_{\ell\ell}$ where $\ell = e, \mu, \tau$. In addition, our study is carried out using the RGEs outlined in [24] and where to one-loop order, the running of λ_H , $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]$ and the diagonal elements of the neutrino Yukawa coupling matrix are approximately given by

$$\begin{aligned} \frac{d\lambda_H}{dt} &\approx \frac{1}{(4\pi)^2} \left[\frac{9}{4} \left(\frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) \right. \\ &\quad - \left(\frac{9}{5} g_1^2 + 9g_2^2 \right) \lambda_H + 12y_t^2 \lambda_H - 12y_t^4 + 12\lambda_H^2 \\ &\quad \left. + \lambda_H \text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \cdot \theta_{M_N} - 4(y_{ee}^4 + y_{\mu\mu}^4 + y_{\tau\tau}^4) \cdot \theta_{M_N} \right] \\ \frac{dy_{ii}}{dt} &\approx \theta_{M_N} \cdot \frac{y_{ii}}{(4\pi)^2} \left[\frac{3}{2} y_{ii}^2 + (3y_t^2 + \text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]) \right. \\ &\quad \left. - \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) \right] \\ \frac{d\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]}{dt} &\approx \theta_{M_N} \cdot \frac{2}{(4\pi)^2} \left[(3y_t^2 + \text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]) \text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \right. \\ &\quad \left. + \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) \text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] + \frac{3}{2} (y_{ee}^4 + y_{\mu\mu}^4 + y_{\tau\tau}^4) \right], \tag{3} \end{aligned}$$

where $i = e, \mu, \tau$ and $\theta_{M_N} \equiv \theta(\mu - M_N)$ is a step function accounting for the threshold, M_N , at which the new neutrino Yukawa couplings begin to run. We have ignored the contributions from the bottom quark and τ lepton Yukawa couplings.

In Fig. 1a we show the value of Λ_I as a function of M_N using a value of the strong coupling constant $\alpha_s = 0.1186 \pm 0.0006$ [25] and a top quark mass of 173.21 GeV [26]. The green circles correspond to a light neutrino mass consistent with the cosmological upper bound of $\sum_i m_i < 0.23$ eV [27], while the red squares and brown diamonds correspond to the values of $m_0(m) = 2, 10$ eV, respectively. The latter value is ruled out by current experimental bounds from tritium decays [28, 29] and we intend it to be for illustration purposes only. For a 0.05–2 eV neutrino, the Majorana mass scale is bounded by 10^{10} GeV. Beyond this scale, the value of λ_H at the Majorana mass scale is negative. This places an upper bound on the value of $\text{Tr} \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ at the Majorana mass scale of $\sim 10^{-4}$ for $M_N \sim 10^{10}$ GeV and $m_0(m) = 0.07$ eV. We show this in Fig. 1b. Our results are in general agreement with that of [20]. However, we used realistic neutrino mass matrices and did not assume degenerate active neutrinos.

Beyond a Majorana mass scale of $\sim 10^{10}$ GeV the value of $\lambda_H(M_N)$ is negative and high-scale seesaw models break down if the scalar potential were to remain stable as we require. We also see that the absolute value of \mathbf{Y}_ν is of order

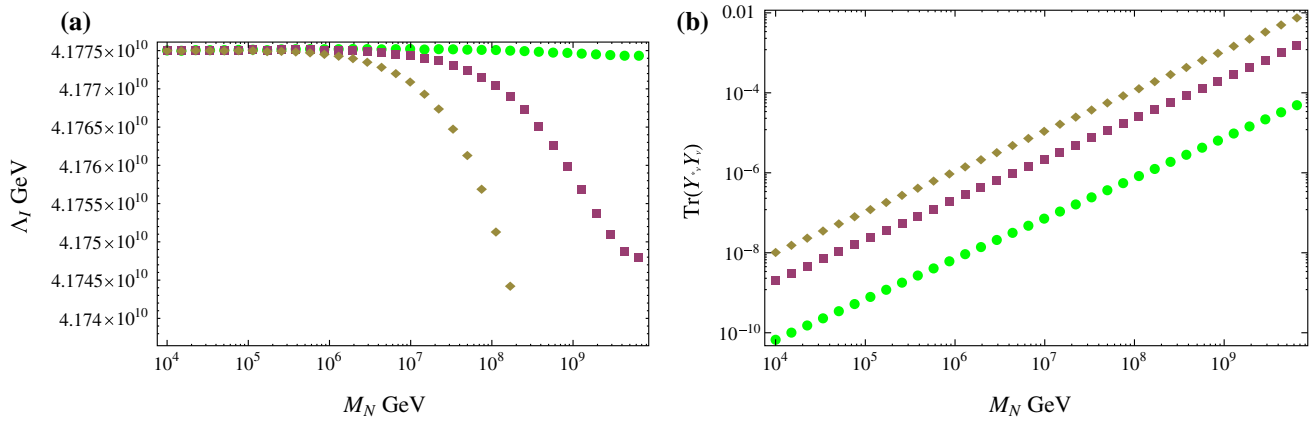


Fig. 1 On the left **a** we plot the scale of instability, Λ_I as a function of the bare mass M_N using values for the sum of the active neutrino masses of 0.23 eV (green), 2 eV (red), and 10 eV (brown). On the right **b** we show the value of $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]$ as a function of M_N

0.01. This is three orders of magnitude lower than the perturbative bound. This is valuable information since we know of no direct measurement of \mathbf{Y}_ν that can be made due to the very massive nature of the N_R 's. This is a well known problem of the high-scale seesaw model. Yet, neutrino Yukawa couplings are vital for Type I seesaw models. In attempts to circumvent this, more elaborate schemes such as low-scale seesaw models [30–36], inverse seesaw models [37,38] and also the left-right symmetric models [40] have been introduced. The latter gives rise to signatures with same sign leptons plus jets that can be searched for at LHC. A detailed study using a simplified model approach was recently given in [41] where many references can be found. As for the high-scale case, which has the virtue of being simple, one can only rely on theoretical studies. Indeed we conclude that for the current preferred value of α_S , electroweak stability would lead to seesaw scales approximately six orders of magnitude lower than the Grand Unified theory scale with neutrino Yukawa couplings of order $\mathcal{O}(10^{-2})$. Recently, the authors in [39] have analyzed the stability of the electroweak vacuum in the presence of a low-scale seesaw model. They find that low-scale seesaw models are viable and do not disrupt the stability of the Higgs potential for $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] < 0.4$. Their results are complementary to our findings.

3 Stability in the Type I seesaw with a complex electroweak singlet scalar

In Eq. (1) the heavy right-handed neutrino masses are introduced by hand and represents the case of explicit lepton number breaking. However, these masses can also be generated by spontaneous symmetry breaking of lepton number. The simplest model that achieves this involves adding a complex SM gauge singlet scalar field, S . It has a Yukawa coupling to right-handed neutrinos given by $Y_N \overline{N}_R N_R^c S$ and as such preserves a global $U(1)_L$ symmetry responsible for lepton num-

ber if S has a global lepton number charge of two units. Upon breaking this symmetry a Goldstone boson, the majoron, will emerge [42,43] which can serve as a candidate for dark radiation [44,45]. This scalar couples to the Higgs field via the term $\lambda_{HS} S^\dagger S H^\dagger H$. One thus expects the running of the SM couplings and the stability of the scalar potential to be different. In the following we will address these issues and present as a detailed RG analysis of this model.

The embedding of the majoron model into a more complete model is not the purpose of this work. Indeed the $U(1)_L$ can be replaced by any $U(1)_X$. A well known example is a gauged $X = B - L$ symmetry where the singlet scalar can serve as the inflaton. Here we focus on the effects of a complex scalar gauge singlet on the electroweak vacuum stability captured by the simple majoron model and compare the results with the explicit mass case studied above. The majoron plays no role in this investigation.

The addition of a complex scalar S , singlet under the SM gauge symmetries and charged under a global $U(1)_L$ lepton number symmetry, can be parametrized by a scalar potential given by

$$V_{H,S} = \frac{\lambda_H}{2} (H^\dagger H - v^2/2)^2 + \frac{\lambda_S}{2} (S^\dagger S - w^2/2)^2 + \lambda_{HS} (H^\dagger H - v^2/2) (S^\dagger S - w^2/2). \tag{4}$$

Within this framework, for $\lambda_H, \lambda_S > 0$ and $\lambda_{HS}^2 < \lambda_H \lambda_S$, the minimum of the potential is at $\langle H \rangle = v/\sqrt{2}$ and $\langle S \rangle = w/\sqrt{2}$. The mixing between the two neutral CP even scalars is given by

$$\tan \theta = \frac{\lambda_{HS} v w}{\lambda_S w^2 - \lambda_H v^2}. \tag{5}$$

In this work, we are interested in the limit where the lightest scalar is the SM-like Higgs boson. This limit is characterized

by a large singlet vacuum expectation value, $w \gg v$, and masses for the two CP even scalars given by

$$m_{h_0}^2 \simeq \lambda_H v^2, \quad m_S^2 \simeq \lambda_S w^2. \tag{6}$$

This also implies a small mixing of the SM-like Higgs with the heavy scalar and hence only small corrections to the Higgs couplings to other SM particles. However, the presence of a heavy scalar, coupling at tree-level to the SM-like Higgs boson, may lead to a large positive contribution to the RGE for λ_H [46–53] as well as a tree-level threshold effect [54] that arises from the matching effect of the singlet at the energy $Q \sim m_S$. Both are known to affect the stability of the Higgs potential. The authors in [54] have studied the effects from a threshold corrections to the Higgs quartic coupling, $\delta\lambda = \lambda_{HS}^2/\lambda_S$ against those that arise from positive contributions in the running of λ_H . They studied two interesting cases: One where $\lambda_{HS} > 0$, where large threshold corrections increase the scale at which the Higgs potential develops an instability and the second where $\lambda_{HS} < 0$. In the latter, the tree-level threshold effect is not sufficient to increase the instability scale and RG effects become important since the new stability condition, $\lambda_H > \delta\lambda$ must be satisfied at all scales.

We wish to study the effects on the instability scale of the SM scalar potential in the presence of a new heavy scalar, as the one discussed above, and also incorporate three Majorana right-handed neutrinos. Within this framework, a Majorana mass is dynamically generated once the new scalar develops a vev with the coupling $Y_N \overline{N_R} N_R^c S$. For simplicity we will take Y_N to be real. The RG evolution of λ_H will be modified at the scales M_N and m_S due to the presence of new fermion and scalar degrees of freedom, the former which will tend to destabilize the scalar potential. We analyze both the case where $\lambda_{HS} > 0$ and where $\lambda_{HS} < 0$ and compare our results to a model with only a scalar gauge singlet.

The scale of instability can be calculated using the RG equations for the scalar and new Yukawa couplings which at one-loop order are given by

$$\begin{aligned} \frac{d\lambda_H}{dt} &\approx \frac{1}{(4\pi)^2} \left[\frac{9}{4} \left(\frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) \right. \\ &\quad - \left(\frac{9}{5} g_1^2 + 9g_2^2 \right) \lambda_H + 12y_t^2 \lambda_H - 12y_t^4 + 12\lambda_H^2 \\ &\quad + \left(2\lambda_{HS}^2 + \lambda_H \text{Tr} \left[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \right] \right. \\ &\quad \left. \left. - 4(y_{ee}^4 + y_{\mu\mu}^4 + y_{\tau\tau}^4) \right) \cdot \theta_{M_{\text{high}}} \right] \\ \frac{d\lambda_{HS}}{dt} &\approx \theta_{M_{\text{high}}} \cdot \frac{1}{(4\pi)^2} \left[\frac{1}{2} \left(12y_t^2 - \frac{9}{5} g_1^2 - 9g_2^2 \right) \lambda_{HS} \right. \\ &\quad + 4\lambda_{HS} \left(\frac{3}{2} \lambda_H + \lambda_S \right) + 4\lambda_{HS}^2 \\ &\quad \left. + 4(Y_1^2 + Y_2^2 + Y_3^2) \lambda_{HS} \right] \end{aligned}$$

$$\begin{aligned} \frac{d\lambda_S}{dt} &\approx \theta_{M_{\text{high}}} \cdot \frac{1}{(4\pi)^2} \left[4\lambda_{HS}^2 + 10\lambda_S^2 - 2(Y_1^4 + Y_2^4 + Y_3^4) \right. \\ &\quad \left. + 4(Y_1^2 + Y_2^2 + Y_3^2) \lambda_S^2 \right] \\ \frac{dY_i}{dt} &\approx \theta_{M_{\text{high}}} \cdot \frac{Y_i}{(4\pi)^2} \left[4Y_i^2 + 2(Y_1^2 + Y_2^2 + Y_3^2) \right], \tag{7} \end{aligned}$$

where $\theta_{M_{\text{high}}}$ is a step function accounting for the threshold, $M_{\text{high}} = \text{Min}\{M_N, m_S\}$, at which the new Yukawa coupling the singlet to the right-handed Majorana neutrinos and scalar potential parameters begin to run. We assume that S couples diagonally to all three right-handed neutrinos with universal strength $Y_i = Y_N$ for all i , and the vev of S sets the Majorana mass scale given by $M_{N_i} = 2Y_N \langle S \rangle = (2/\sqrt{2})Y_N w$. The running of the Yukawa couplings between left- and right-handed neutrinos to the SM-like Higgs, Y_ν , are given as Eq. (3) with $M_N \rightarrow M_{\text{high}}$. The instability scale for $\lambda_{HS} > 0$ is defined as the energy scale where either λ_H or λ_S first vanish, rendering the scalar potential unstable and invalidating the seesaw mechanism in models were one demands absolute stability of the scalar potential. For $\lambda_{HS} < 0$, the condition $\lambda_H > \delta\lambda$ must also be met for all scales together with $\lambda_S > 0$. By inspection of Eq. (7) we expect that for large values of $Y_N (M_{\text{high}})$, λ_S will be driven toward negative values very rapidly given that it runs with the fourth power of Y_N . In fact, for $\lambda_{HS} > 0$, three effects define the scale where the scalar potential becomes unstable:

- A potentially large tree-level threshold effect to the SM Higgs quartic coupling. This tends to increase the instability scale of the SM Higgs.
- A large value of $\lambda_S (M_{\text{high}})$ which provides a large positive contribution to the running of λ_H which also improves the stability of the SM Higgs.
- A large negative contribution to the running of λ_S from large values of $Y_N (M_{\text{high}})$. This tends to drive λ_S toward zero at scales near large M_{high} .

There is a second effect that disfavors large values of Y_N . The last equation in Eq. (7) show that these couplings hit a Landau pole very quickly. If this happens within the stability region of the scalar potential, the theory would become strongly coupling. Although a very interesting scenario, it is beyond the scope of this work. For small values of Y_N , λ_S can also arrive at the Landau pole too soon. These conspire to limit the range of interesting values of Y_N and λ_S .

A qualitative picture of these effects for $\lambda_{HS} > 0$ is depicted in Fig. 2a–c where we plot the scale of instability, Λ_I , as a function of $\delta\lambda (M_{\text{high}})$ for values of $Y_N (M_{\text{high}}) = 0.01, 0.1, 0.5$, respectively. In the figures, the dashed lines correspond to $\lambda_S (M_{\text{high}}) = 0.4$, while the solid lines correspond to $\lambda_S (M_{\text{high}}) = 0.0001$. The solid dots correspond to the complex scalar extension of the SM parametrized by Eq. (4); i.e. without the Majorana neutrino effects. The

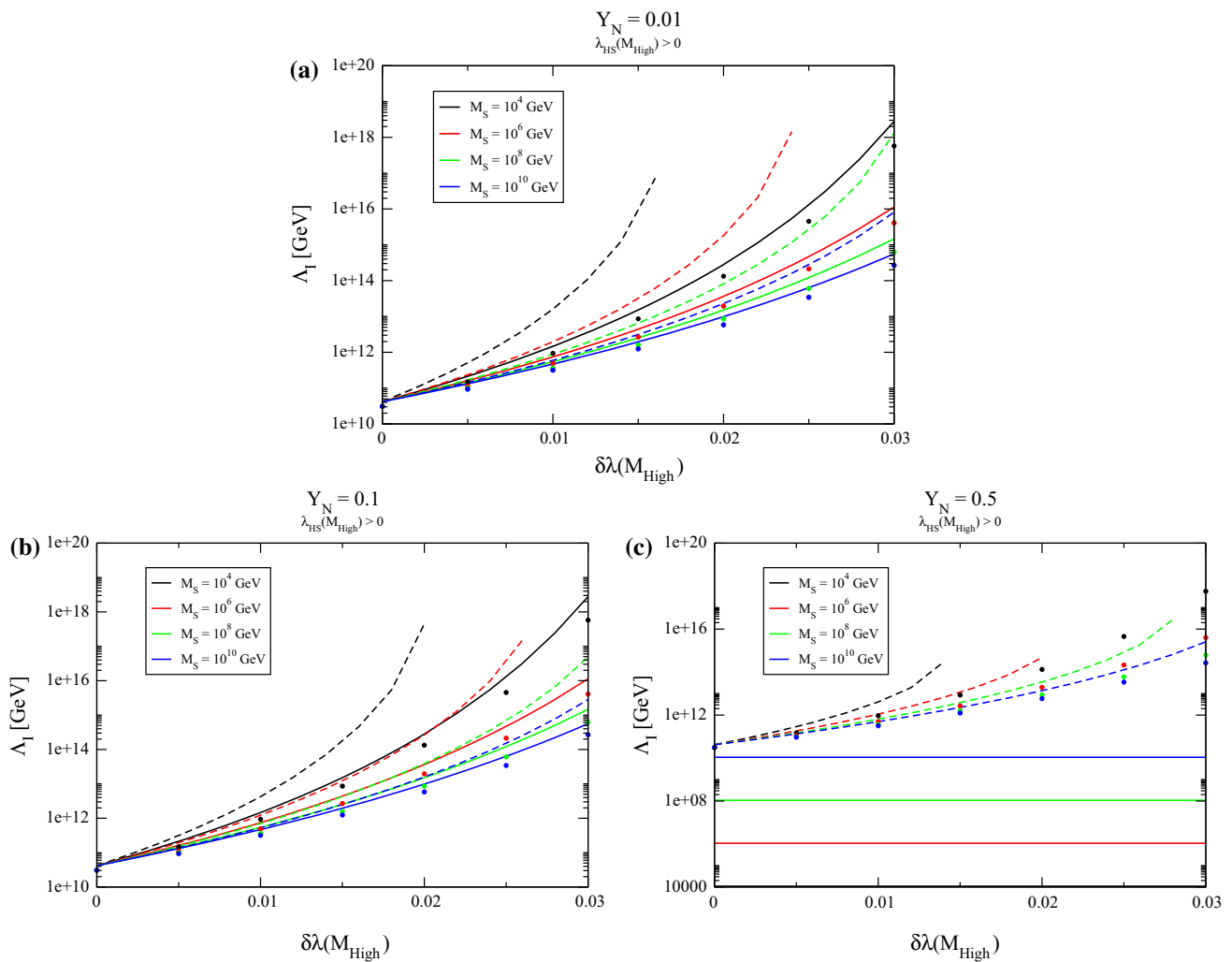


Fig. 2 The scale of instability, Λ_I within Type I seesaw models extended by a complex scalar gauge singlet as a function of the threshold contribution to λ_H , $\delta\lambda$ defined at M_{high} for different values of m_S .

The figure on the top panel, **a**, corresponds to $\lambda_{HS} > 0$ and $Y_N = 0.01$ while the bottom panel, **b**, **c** correspond to $Y_N = 0.1, 0.5$, respectively

finite nature of the various curves denote the following: If $\delta\lambda$ increased either we obtain stability up to the Planck scale with couplings either perturbative or non-perturbative at the Planck scale, or one of the quartics goes to zero and then hits a Landau Pole driving the other couplings toward a Landau Pole. Basically, a larger $\delta\lambda$ leads to a larger λ_{HS} , which also affects the running. From the figures it is clear that a large $\lambda_S(M_{high})$ aids a lot at stabilizing the Higgs potential for small values of $Y_N(M_{high})$. This is due to the positive contribution to the running of λ_H , which together with the tree-level threshold effect help increase the stability of the scalar potential compared to $\Lambda_I^{SM} \approx 10^{10}$ GeV. However, as $Y_N(M_{high})$ increases, the effect from the tree-level threshold effect shift remains fairly constant since λ_S goes to zero near M_{high} for small $\lambda_S(M_{high})$. For larger $\lambda_S(M_{high})$, the contributions from λ_S to the running of λ_S for a large range of energies and it is eventually overtaken by the contribution from Y_N . This aids at stabilizing the scalar potential at high

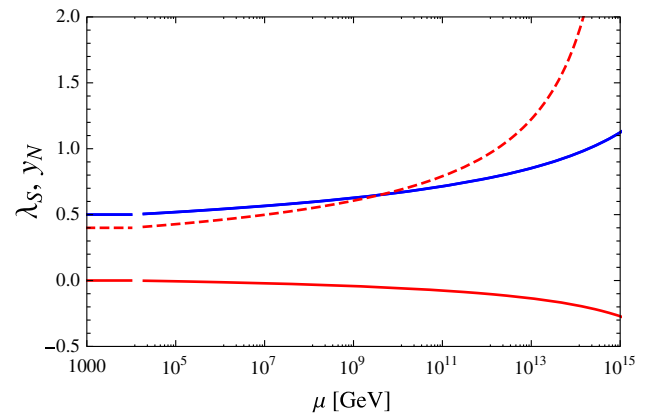


Fig. 3 Running of Y_N with $Y(M_{high}) = 0.5$ denoted by the blue solid line while in red we show the running of λ_S with $\lambda_S(M_{high}) = 10^{-4}$ (solid) $\lambda_S(M_{high}) = 0.4$ (dashed)

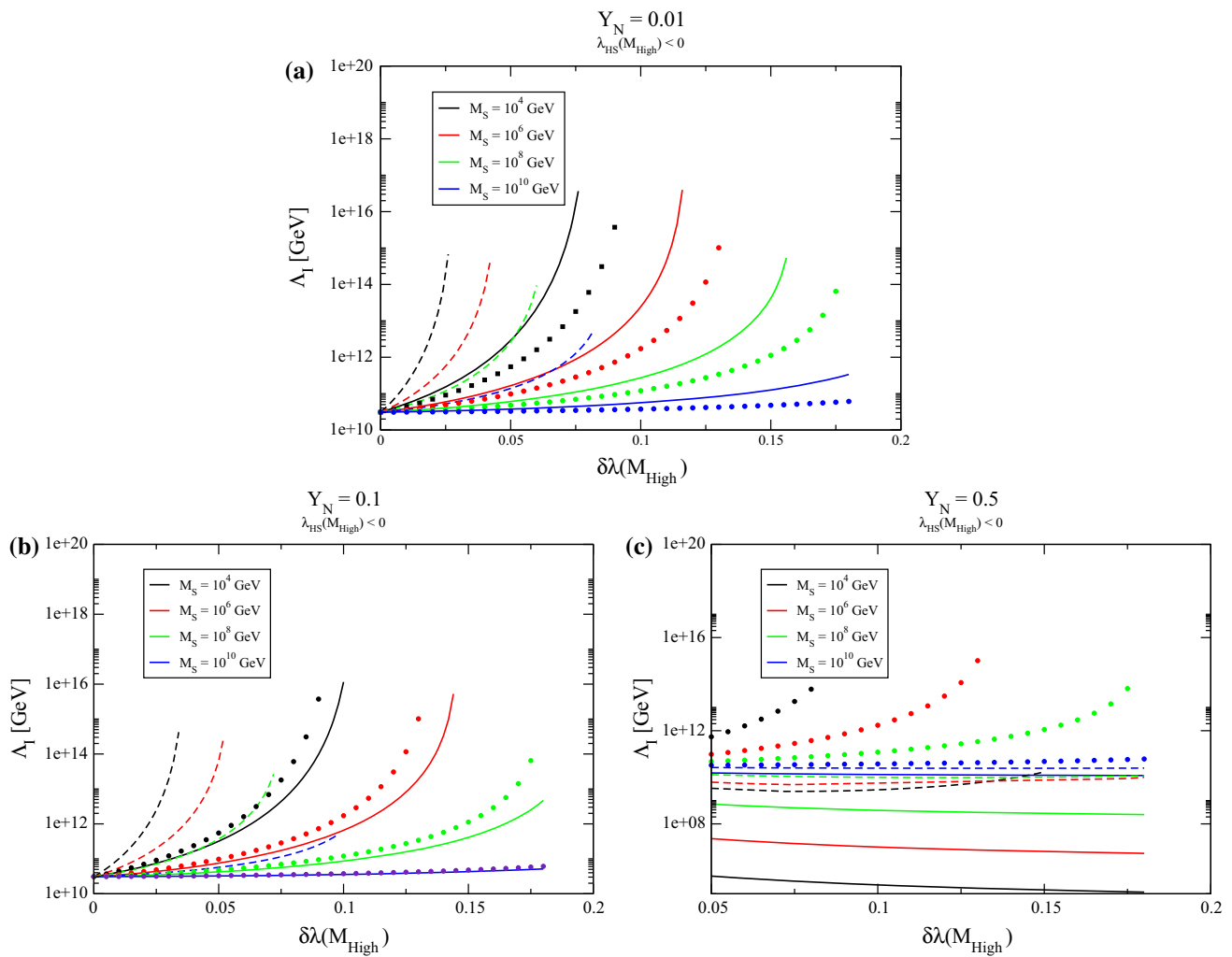


Fig. 4 The scale of instability, Λ_I within Type I seesaw models extended by a complex scalar gauge singlet as a function of the threshold contribution to λ_H , $\delta\lambda$ defined at M_{high} for different values of m_S .

The figure on the *top panel*, **a**, corresponds to $\lambda_{HS} < 0$ and $Y_N = 0.01$ while the *bottom panel*, **b**, **c** correspond to $Y_N = 0.1, 0.5$, respectively

energies. We can observe this effect in Fig. 3 for $m_S = 10^4$ GeV. In the figure, we denote with the solid blue line the running of Y_N with $Y_N(M_{\text{high}}) = 0.5$ and in solid and dashed red the running of λ_S for $\lambda_S(M_{\text{high}}) = 10^{-4}, 0.4$, respectively. The stabilization of the scalar potential is helped by the large value of λ_S at M_{high} and this effect becomes more relevant for higher M_{high} since the new couplings run for a smaller range up to the Planck scale.

For $\lambda_{HS} < 0$, the new stability condition, $\lambda_H > \lambda_{HS}^2/\lambda_S$ together with $\lambda_S > 0$ must be satisfied. The new condition on λ_H has the effect of destabilizing the scalar potential at lower energies for small values of $\lambda_S(M_{\text{high}})$ since a positive contribution to λ_H from λ_S does not help to drive λ_H above $\delta\lambda$ at scales above M_{high} . For large values of $\lambda_S(M_{\text{high}})$ an additional positive contribution to the running of λ_H raises the scale of instability for smaller values of $\delta\lambda(M_{\text{high}})$. In Fig. 4a–c we plot the scale of instability, Λ_I , as a function

of $\delta\lambda(M_{\text{high}})$ for values of $Y_N(M_{\text{high}}) = 0.01, 0.1, 0.5$, respectively. In the figures, the dashed lines correspond to $\lambda_S(M_{\text{high}}) = 0.4$ while the solid lines correspond to $\lambda_S(M_{\text{high}}) = 0.02$. The solid dots correspond to the complex scalar extension of the SM parametrized by Eq. (4).

We note that our results suggest that λ_S cannot be too small unless Y_N is simultaneously small since otherwise λ_S will be driven negative very quickly. To see this we scan over $\lambda_S(M_{\text{high}})$ and $Y_N(M_{\text{high}})$ and calculate the scale of instability. Our results are shown in Fig. 5 for $\delta\lambda = 0.015$ and $\lambda_{HS} > 0$. The gray region corresponds to regions of parameter space where $\Lambda_I \gtrsim 10^{10}$ GeV for $m_S = 10^4$ GeV. The regions in light red, green and blue are stacked behind the gray region and correspond to $m_S = 10^6, 10^8, 10^{10}$ GeV, respectively, with the light blue region extending over the entire grid.

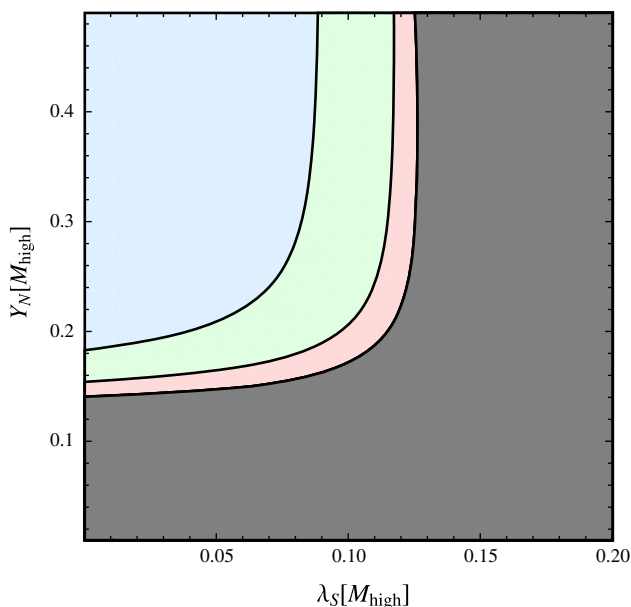


Fig. 5 Regions of parameter space where $\Delta_I \gtrsim 10^{10}$ GeV for $m_S = 10^4$ GeV shown in gray. The regions in light red, green, and blue are stacked being the gray region and correspond to $m_S = 10^6, 10^8, 10^{10}$ GeV, respectively, with the light blue region extending over the entire grid

Next we turn our attention to \mathbf{Y}_ν . In the above we have seen that spontaneously generating heavy Majorana neutrino masses can raise the instability scale of the electroweak vacuum with the help of the added scalars; especially for $\lambda_{HS} > 0$. One can then increase the upper bound on $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]$ compared to the case of explicit mass terms. This can be seen in Fig. 6 with $Y_N = 0.1$. Indeed the bound is $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \lesssim 0.01$, higher than before but is only for a relatively large M_{high} . In this figure, only points which lead to an instability scale below or at the Planck scale with perturbative couplings up to the Planck scale are shown. Regions of parameter space for which couplings are not perturbative below the Planck scale are not shown. These points tend to be for low M_{high} . For very large M_{high} , the couplings run as in the SM and the instability scale is below M_{high} . This region of parameter space corresponds to points where $M_{\text{high}} \gtrsim 10^{10}$ GeV, and are not accepted either. The window of allowed values of M_{high} gets smaller for larger values of λ_S . This can be seen in Fig. 7a, b with $\lambda_S = 0.6$ and 0.8 , respectively. Values of λ_S beyond ~ 0.9 do not lead to a viable seesaw. Furthermore, increasing Y_N only decreases the upper bound on λ_S .

4 Discussion

We conclude that in the two heavy Majorana mass generation mechanisms we have studied, the neutrino Yukawa couplings will remain perturbative. We find that for high-scale

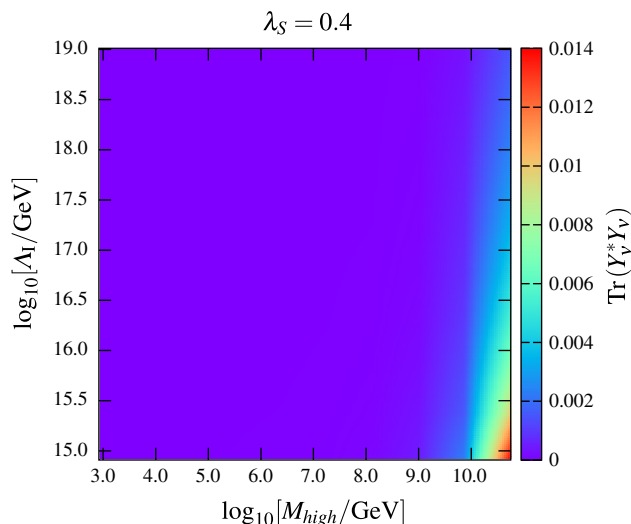


Fig. 6 The value of $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu](M_{\text{high}})$ in the $\log[\Delta_I/\text{GeV}] - \log[M_{\text{high}}/\text{GeV}]$ plane. In this figure we choose $\lambda_{HS} > 0, \delta\lambda = 0.030, \lambda_S = 0.4$ and $Y_N = 0.1$ defined at the scale M_{high}

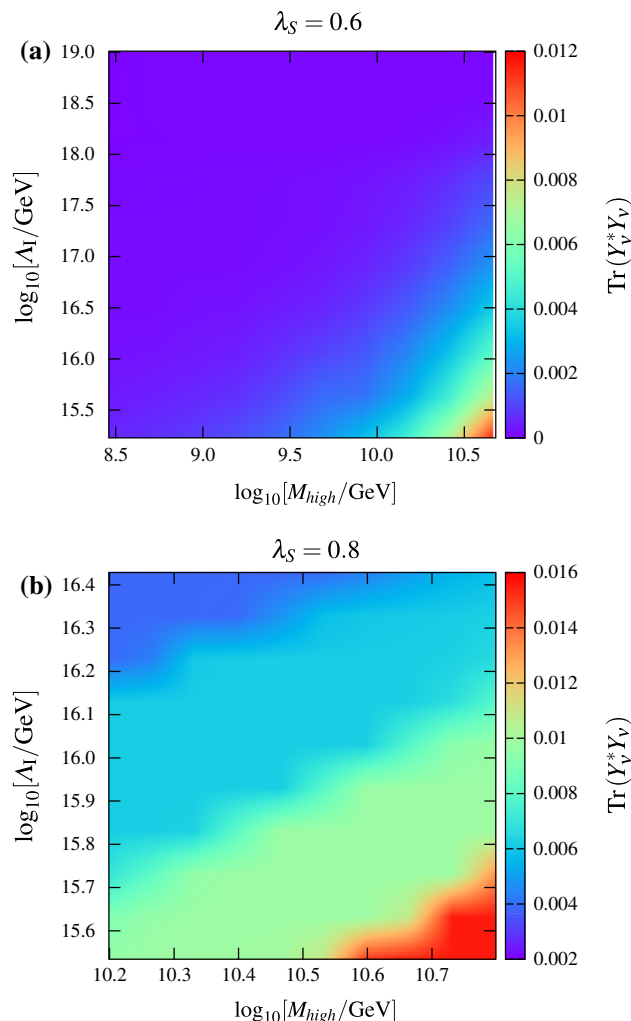


Fig. 7 Same as in Fig. 6 with $\lambda_S = 0.6$ (top a) and $\lambda_S = 0.8$ (bottom b)

Type I seesaw models in which lepton number is explicitly broken by Majorana bare mass terms the stability is lowered due to the Dirac Yukawa couplings, \mathbf{Y}_ν . We also find that $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \lesssim 10^{-3}$ in order for the SM electroweak vacuum stability not be worsened. In the scenario where right-handed neutrinos masses are generated spontaneously through a singlet scalar vev , the electroweak vacuum can be stable up to the Planck scale through threshold effects from decoupling a heavy scalar and positive contributions to the running of λ_H , which ultimately depends on the scalar mixing parameter λ_{HS} . We find that the scale at which the scalar potential becomes unstable is not improved over Λ_I^{SM} if the quartic coupling of the singlet is small while Y_N is large. This result is worsened for light singlet scalar masses; since λ_S will be driven negative at very low energy scales. We also set the limit $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \lesssim 0.01$. This is because both Y_N and λ_S must not be too large as not to hit the Landau pole within stability region. Thus, the viable boundary values of M_N are low and consequently small values of $\text{Tr}[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]$ from κ .

Acknowledgments This work is supported in parts by the National Science and Engineering Council of Canada.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

- G. Aad et al. [ATLAS, CMS], Phys. Rev. Lett. **114**, 191803 (2015) [[arXiv:1503.07589](https://arxiv.org/abs/1503.07589)]
- G. Aad et al., [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [[arXiv:1207.7214](https://arxiv.org/abs/1207.7214)] [hep-ex]
- S. Chatrchyan et al., [CMS Collaboration], Phys. Lett. B **716**, 30 (2012). [[arXiv:1207.7235](https://arxiv.org/abs/1207.7235)] [hep-ex]
- F. Bezrukov, M.Y. Kalmykov, B.A. Kniehl, M. Shaposhnikov, Higgs boson mass and new physics. JHEP **1210**, 140 (2012). doi:[10.1007/JHEP10\(2012\)140](https://doi.org/10.1007/JHEP10(2012)140). [[arXiv:1205.2893](https://arxiv.org/abs/1205.2893)] [hep-ph]
- G. Degrandi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia, Higgs mass and vacuum stability in the Standard Model at NNLO. JHEP **1208**, 098 (2012). [[arXiv:1205.6497](https://arxiv.org/abs/1205.6497)] [hep-ph]
- S. Alekhin, A. Djouadi, S. Moch, The top quark and Higgs boson masses and the stability of the electroweak vacuum. Phys. Lett. B **716**, 214 (2012). doi:[10.1016/j.physletb.2012.08.024](https://doi.org/10.1016/j.physletb.2012.08.024). [[arXiv:1207.0980](https://arxiv.org/abs/1207.0980)] [hep-ph]
- D. Buttazzo, G. Degrandi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio, A. Strumia, Investigating the near-criticality of the Higgs boson. JHEP **1312**, 089 (2013). doi:[10.1007/JHEP12\(2013\)089](https://doi.org/10.1007/JHEP12(2013)089). [[arXiv:1307.3536](https://arxiv.org/abs/1307.3536)] [hep-ph]
- G. Isidori, G. Ridolfi, A. Strumia, On the metastability of the standard model vacuum. Nucl. Phys. B **609**, 387 (2001). [[arXiv:hep-ph/0104016](https://arxiv.org/abs/hep-ph/0104016)]
- J. Ellis, J.R. Espinosa, G.F. Giudice, A. Hoecker, A. Riotto, The probable fate of the standard model. Phys. Lett. B **679**, 369 (2009). [[arXiv:0906.0954](https://arxiv.org/abs/0906.0954)] [hep-ph]
- P. Minkowsky, Phys. Lett. B **67**, 421 (1977)
- T. Yanagida, *Workshop on Unified Theories*, KEK rept, vol. 95, p. 79–18 (1979)
- M. Gell-Mann, P. Ramond, R. Slansky *Supergravity*, ed. by P. van Nieuwenhuizen, D. Freedman, vol. 315 (North Holland, Amsterdam, 1979)
- S. L. Glashow *Cargese Summer Institute on Quarks and Leptons*, ed. M. Levy, p. 687 (Plenum Press, N.Y. 1980)
- R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980)
- J. Schechter, J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980)
- K.S. Babu, C.N. Leung, J.T. Pantaleone, Renormalization of the neutrino mass operator. Phys. Lett. B **319**, 191 (1993). [[arXiv:hep-ph/9309223](https://arxiv.org/abs/hep-ph/9309223)]
- S. Antusch, M. Drees, J. Kersten, M. Lindner, M. Ratz, Neutrino mass operator renormalization revisited. Phys. Lett. B **519**, 238 (2001). [[arXiv:hep-ph/0108005](https://arxiv.org/abs/hep-ph/0108005)]
- J.A. Casas, V. Di Clemente, A. Ibarra, M. Quiros, Phys. Rev. D **62**, 053005 (2000)
- W. Rodejohann, H. Zhang, Impact of massive neutrinos on the Higgs self-coupling and electroweak vacuum stability. JHEP **1206**, 022 (2012). [[arXiv:1203.3825](https://arxiv.org/abs/1203.3825)] [hep-ph]
- J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Riotto, A. Strumia, Higgs mass implications on the stability of the electroweak vacuum. Phys. Lett. B **709**, 222 (2012). [[arXiv:1112.3022](https://arxiv.org/abs/1112.3022)] [hep-ph]
- E. Bertuzzo, P.A.N. Machado, R.Z. Funchal, Neutrino mass matrix textures: a data-driven approach. JHEP **1306**, 097 (2013). [[arXiv:1302.0653](https://arxiv.org/abs/1302.0653)] [hep-ph]
- K.A. Olive et al., Review of particle physics. Chin. Phys. C **38**, 090001 (2014)
- F. Capozzi, G.L. Fogli, E. Lisi, Q.A. Marrone, D. Montanino, A. Palazzo, Phys. Rev. D **89**, 093018 (2014)
- Y.F. Pirogov, O.V. Zenin, Two loop renormalization group restrictions on the standard model and the fourth chiral family. Eur. Phys. J. C **10**, 629 (1999). [[arXiv:hep-ph/9808396](https://arxiv.org/abs/hep-ph/9808396)]
- J. de Blas, M. Chala, M. Perez-Victoria, J. Santiago, Observable effects of general new scalar particles. JHEP **1504**, 078 (2015). [[arXiv:1412.8480](https://arxiv.org/abs/1412.8480)] [hep-ph]
- K.A. Olive et al., [Particle Data Group Collaboration], Review of particle physics. Chin. Phys. C **38**, 090001 (2014)
- P.A.R. Ade et al., [Planck Collaboration], Planck 2015 results. XIII. Cosmological parameters. [[arXiv:1502.01589](https://arxiv.org/abs/1502.01589)] [astro-ph.CO]
- Ch. Kraus et al., Eur. Phys. J. C **40**, 447 (2005)
- V.M. Lobashev, Phys. Lett. B **460**, 227 (1995)
- A. Ibarra, E. Molinaro, S.T. Petcov, TeV Scale see-saw mechanisms of neutrino mass generation, the majorana nature of the heavy singlet neutrinos and $(\beta\beta)_{0\nu}$ -decay. JHEP **1009**, 108 (2010). [[arXiv:1007.2378](https://arxiv.org/abs/1007.2378)] [hep-ph]
- J. Chakraborty, TeV scale double seesaw in left-right symmetric theories. [[arXiv:1003.3154](https://arxiv.org/abs/1003.3154)] [hep-ph]
- W. Chao, Neutrino masses and a TeV scale seesaw mechanism. Phys. Rev. D **82**, 016008 (2010). [[arXiv:1003.1468](https://arxiv.org/abs/1003.1468)] [hep-ph]
- F.M.L. de Almeida, Y.A. Coutinho, J.A. Martins, Simoes, A. J. Ramalho, L. Ribeiro Pinto, S. Wulck, M.A.B. do Vale, Double seesaw mechanism in a left-right symmetric model with TeV neutrinos. Phys. Rev. D **81**, 053005 (2010). [[arXiv:1001.2162](https://arxiv.org/abs/1001.2162)] [hep-ph]
- H. Zhang, S. Zhou, The minimal seesaw model at the TeV scale. Phys. Lett. B **685**, 297 (2010). [[arXiv:0912.2661](https://arxiv.org/abs/0912.2661)] [hep-ph]
- P.S.B. Dev, R.N. Mohapatra, TeV scale inverse seesaw in SO(10) and leptonic non-unitarity effects. Phys. Rev. D **81**, 013001 (2010). [[arXiv:0910.3924](https://arxiv.org/abs/0910.3924)] [hep-ph]

36. Z.Z. Xing, S. Zhou, Multiple seesaw mechanisms of neutrino masses at the TeV scale, *Phys. Lett. B* **679**, 249 (2009). [arXiv:0906.1757](#) [hep-ph]
37. R.N. Mohapatra, Mechanism for understanding small neutrino mass in superstring theories. *Phys. Rev. Lett.* **56**, 561 (1986)
38. R.N. Mohapatra, J.W.F. Valle, Neutrino mass and baryon number nonconservation in superstring models. *Phys. Rev. D* **34**, 1642 (1986)
39. L. Delle Rose, C. Marzo, A. Urbano, On the stability of the electroweak vacuum in the presence of low-scale seesaw models. [arXiv:1506.03360](#) [hep-ph]
40. R.N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980)
41. J.N. Ng, A. de la Puente, B.W-P. Pan, [arXiv:1505.01934](#) [hep-ph]
42. Y. Chikashige, R.N. Mohapatra, R.D. Peccei, Are there real goldstone bosons associated with broken lepton number? *Phys. Lett. B* **98**, 265 (1981)
43. J. Schechter, J.W.F. Valle, Neutrino decay and spontaneous violation of lepton number. *Phys. Rev. D* **25**, 774 (1982)
44. W.F. Chang, J.N. Ng, J.M.S. Wu, *Phys. Lett. B* **730**, 347 (2014)
45. W.F. Chang, J.N. Ng, *Phys. Rev. D* **90**, 065034 (2014)
46. M. Gonderinger, Y. Li, H. Patel, M.J. Ramsey-Musolf, Vacuum stability, perturbativity, and scalar singlet dark matter, *JHEP* **1001**, 053 (2010). [arXiv:0910.3167](#) [hep-ph]
47. O. Lebedev, H.M. Lee, Higgs portal inflation. *Eur. Phys. J. C* **71**, 1821 (2011). [arXiv:1105.2284](#) [hep-ph]
48. M. Kadastik, K. Kannike, A. Racioppi, M. Raidal, Implications of the 125 GeV Higgs boson for scalar dark matter and for the CMSSM phenomenology. *JHEP* **1205**, 061 (2012). [arXiv:1112.3647](#) [hep-ph]
49. M. Gonderinger, H. Lim, M.J. Ramsey-Musolf, Complex scalar singlet dark matter: vacuum stability and phenomenology. *Phys. Rev. D* **86**, 043511 (2012). [arXiv:1202.1316](#) [hep-ph]
50. C.S. Chen, Y. Tang, Vacuum stability, neutrinos, and dark matter. *JHEP* **1204**, 019 (2012). [arXiv:1202.5717](#) [hep-ph]
51. R. Costa, A.P. Morais, M.O.P. Sampaio, R. Santos, Two-loop stability of a complex singlet extended standard model. [arXiv:1411.4048](#) [hep-ph]
52. R. Costa, A.P. Morais, M.O.P. Sampaio, R. Santos, Two-loop stability of singlet extensions of the SM with dark matter. [arXiv:1504.06421](#) [hep-ph]
53. A. Falkowski, C. Gross, O. Lebedev, A second Higgs from the Higgs portal. *JHEP* **1505**, 057 (2015). [arXiv:1502.01361](#) [hep-ph]
54. J. Elias-Miro, J.R. Espinosa, G.F. Giudice, H.M. Lee, A. Strumia, Stabilization of the electroweak vacuum by a scalar threshold effect. *JHEP* **1206**, 031 (2012). [arXiv:1203.0237](#) [hep-ph]