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Study on radiative decays of $D^*_{sJ}(2860)$ and $D^*_{s1}(2710)$ into D_s by means of LFQM

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Abstract The observed resonance peak around 2.86 GeV has been carefully reexamined by the LHCb Collaboration and it is found that under the peak there reside two states, $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$, which are considered as $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$ with slightly different masses and total widths. Thus, the earlier assumption that the resonance $D^*_{s1}(2710)$ was a $1D$ state should be reconsidered. We suggest to measure the partial widths of radiative decays of $D^*_{s1}(2860)$, $D^*_{s3}(2860)$, and $D^*_{s1}(2710)$ to confirm their quantum numbers. We would consider $D^*_{s1}(2710)$ as 2^3S_1 or a pure 1^3D_1 state, or their mixture and, respectively, calculate the corresponding branching ratios as well as those of $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$. A future precise measurement would provide us information to help identifying the structures of those resonances.

1 Introduction

Resonance $D^*_{sJ}(2860)$ was experimentally observed [1–4], but its quantum number is still to be eventually identified because the ratio $\Gamma(D^*_{sJ}(2860) \to D^*K)/\Gamma(D^*_{sJ}(2860) \to DK)$ is not well understood [5,6]. A careful reexamination on the spectrum peak around 2.86 GeV recently has been carried out by the LHCb Collaboration and it is found that a spin-1 state and a spin-3 state overlap under the peak. They are $D^*_{s1}(2860)$ with mass and width $M(D^*_{s1}(2860)) = (2859 \pm 12 \pm 6 \pm 23)$ MeV, $\Gamma(D^*_{s1}(2860)) = (159 \pm 23 \pm 27 \pm 72)$ MeV [7] and $D^*_{s3}(2860)$ with mass and width $M(D^*_{s3}(2860)) = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0)$ MeV, $\Gamma(D^*_{s3}(2860)) = (53 \pm 7 \pm 4 \pm 6)$ MeV [8]. Based on the new data Godfrey and Moats suggest that [5] $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ should be identified as $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$. Previously $D^*_{s1}(2710)$ [2] was measured and its mass and width are $M(D^*_{s1}(2710)) = (2709 \pm 4)$ MeV, $\Gamma(D^*_{s1}(2710)) = (117 \pm 13)$ MeV. It was assigned to be 1^3D_1 or 2^3S_1 or a mixture [5,9–11]. Obviously the pure 1^3D_1 state can only accommodate one physical particle, so if the 1^3D_1 state of $c\bar{s}$ is occupied by $D^*_{s1}(2860)$ the assignment of $D^*_{s1}(2710)$ should be a 2^3S_1 state or others. Since all resonances $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ and $D^*_{s1}(2710)$ have been undoubtedly reconstructed in the hadronic processes under investigation, the best channels to determine their quantum identities are their respective strong decays [5,12–14], which are in fact the dominant ones. However, on other aspect, one still has a chance to observe the resonances in their electromagnetic decays where excited states transit into ground states by emitting a photon. Especially the calculation on the electromagnetic decays is more reliable. In Ref. [15] the authors study the radiative decays of $D^*_{s1}(2710)$ and $D^*_{s1}(2860)$ into a P-wave $c\bar{s}$ meson. In this paper we will study the radiative decay of a D-wave meson into an S-wave $c\bar{s}$ meson. The results may help us to determine the quantum number of these particles in addition to the studies via strong processes.

In this work, we employ the light-front quark model (LFQM) to estimate the branching ratios. This relativistic model has been thoroughly discussed in the literature [16,17] and applied to the study of hadronic transition processes [18–20]. The results obtained in this framework qualitatively agree with the data for all the concerned processes.

In conventional LFQM the radiative decay of a 1^{--} (S-wave) meson into a 0^{-+} meson was evaluated [21,22] and the same formula can also be generalized to the covariant LFQM [23]. In our earlier papers [24–26] we studied radiative decays of some mesons such as $\chi_{c0}, h_c, D_s(2317), \Upsilon(2S)$ in covariant LFQM and now we will concentrate our attention to the radiative decays of 1^{--} (D-wave) mesons to 0^{-+} mesons. The results would be useful for confirming the identities of the aforementioned mesons. Since the Lorentz structure of the vertex functions of D-wave is the same as that of

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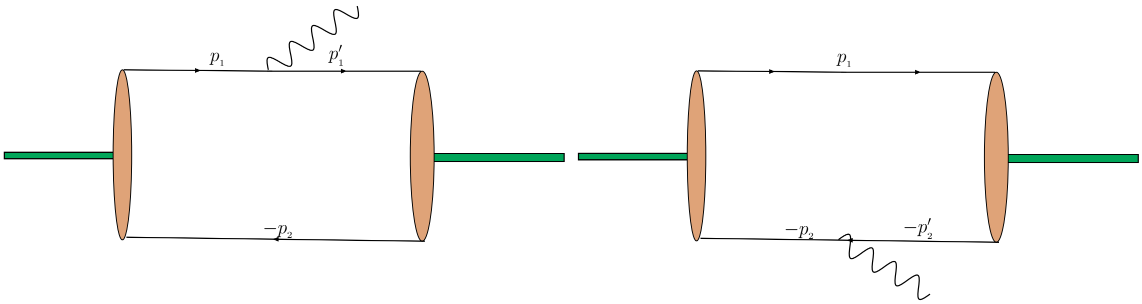

Fig. 1 Feynman diagrams depicting the radiative decay

S-wave [27], the formulas for the decays of the 1^{--} D-wave mesons can be simply obtained by replacing several functions which were used for the decays of the 1^{--} S-wave mesons.

This paper is organized as follows: after this introduction, we derive the theoretical formulas in the next section where we also present relevant formulas given in the literature, and then in Sect. 3, we present our numerical results along with all inputs which are needed for the numerical computations. In the last section we draw our conclusion and present a brief discussion.

2 The formulas for the radiative decay of 1^{--} meson in LFQM

In the light-front quark model, the transition matrix elements for the decay of $1^{--}(V) \to 0^{-+}(P)\gamma$ were examined (Fig. 1) and the form factor $\mathcal{F}_{V\to P}(q^2)$ can be expressed as [21,22]:

$$\mathcal{F}_{V\to P}(q^2) = e_1 I(m_1, m_2, q^2) + e_2 I(m_2, m_1, q^2), \tag{1}$$

where e_1 and e_2 are the electrical charges of the charm and strange quarks, $m_1 = m_c$, $m_2 = m_s$, and

$$\begin{aligned} I(m_1, m_2, q^2) &= \int_0^1 \frac{\mathrm{d}x}{8\pi^3} \int d^2\mathbf{p}_\perp \frac{\phi\phi'\left\{\mathcal{A} + \frac{2}{w_V}[\mathbf{p}_\perp^2 - \frac{(\mathbf{p}_\perp\cdot\mathbf{q}_\perp)^2}{\mathbf{q}_\perp^2}]\right\}}{x_1 \tilde{M}_0 \tilde{M}_0'} \\ &= N_c \int_0^1 \frac{\mathrm{d}x}{4\pi^3} \int d^2\mathbf{p}_\perp \frac{h_{^3S_1} h'_P \left\{\mathcal{A} + \frac{2}{w_{^3S_1}}[\mathbf{p}_\perp^2 - \frac{(\mathbf{p}_\perp\cdot\mathbf{q}_\perp)^2}{\mathbf{q}_\perp^2}]\right\}}{x_1^2 x_2 (M^2 - M_0^2)(M'^2 - M_0'^2)}, \end{aligned} \tag{2}$$

where $h_{^3S_1} = h_P = (M^2 - M_0^2)\sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2}\tilde{M}_0}\phi$, $w_{^3S_1} = M_0 + m_1 + m_2$, $\mathcal{A} = x_2 m_1 + x_1 m_2$, and $x = x_1$. It is noted that the 1^{--} meson in Refs. [21–23] just refers to the $^3S_1$ state. The other variables in Eq. (2) are presented in the appendix.

Obviously, a 1^{--} meson may be in a $^3D_1$ state or a $^3S_1$ state or a mixture.

In Ref. [27] the vertex function for $^3D_1$ states was deduced and its Lorentz structure is the same as that of $^3S_1$ state, so Eq. (2) is also valid for the radiative decay of $^3D_1$ through replacing the functions $h_{^3S_1}$ and $w_{^3S_1}$ by

$$\begin{aligned} h_{(^3D_1)} &= -(M^2 - M_0^2)\sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2}\tilde{M}_0} \frac{\sqrt{6}}{12\sqrt{5}M_0^2\beta^2} \\ &\quad \times [M_0^2 - (m_1 - m_2)^2][M_0^2 - (m_1 + m_2)^2]\phi, \\ w_{(^3D_1)} &= \frac{(m_1 + m_2)^2 - M_0^2}{2M_0 + m_1 + m_2}. \end{aligned}$$

The decay width is [21,22]

$$\Gamma(V \to P + \gamma) = \frac{\alpha}{3}\left[\frac{m_V^2 - m_P^2}{2m_V}\right]^3 \mathcal{F}^2_{V\to P}(0), \tag{3}$$

where V represents $D_{s1}(2860)$ or $D_{s3}(2860)$ or $D_{s1}(2710)$, P denotes D_s, α is the fine-structure constant and $\mathcal{F}_{V\to P}(0)$ is the form factor for the radiative decay present in Eq. (1) with $q^2 = 0$.

3 Numerical results

Before we carry out our numerical computations for evaluating the branching ratios of the D-wave mesons, we need to determine the nonperturbative parameter β, which exists in the wavefunction, in a proper way. In Ref. [17] the authors suggested that via calculating the decay constant of the ground state one can determine β. Alternatively, we also can get the value of β by fitting the spectra of the relevant mesons as done in Refs. [21,22]. In this work we follow the first scheme. With the averaged decay branching ratio of $D_s \to \mu\nu_\mu$ $(5.56 \pm 0.25) \times 10^{-3}$ [29] one obtains its decay constant as $f_{D_s} = (247 \pm 6)$ MeV. Then using Eq. (6) in Ref. [23] β is fixed as (0.534 ± 0.015) GeV^{-1} when we set $m_c = 1.4$ GeV, $m_s = 0.37$ GeV [17], and $m_{D_s} = 1.9685$ GeV [29].

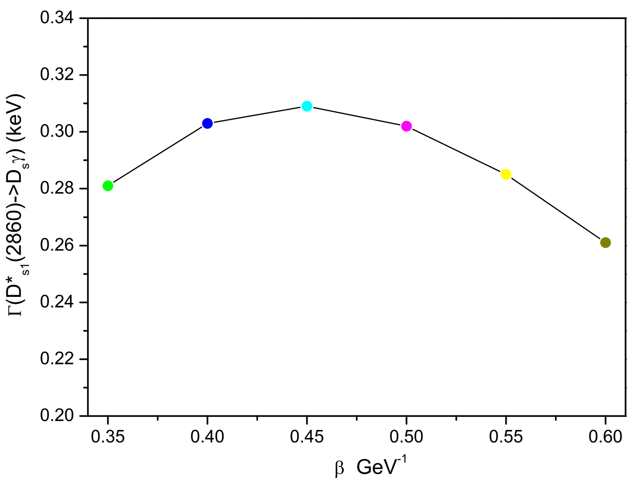

Fig. 2 $\Gamma(D^*_{s1}(2860) \to D_s\gamma)$ dependence on β

3.1 The radiative decays of $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$

In our numerical computations we adopt the assumption that $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ are $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$, respectively.

Using the parameters we calculate the form factor $\mathcal{F}(0)$ for $D^*_{s1}(2860) \to D_s\gamma$ which is (0.0168 ± 0.0002) GeV^{-1}. The decay width $\Gamma(D^*_{s1}(2860) \to D_s\gamma)$ is (0.291 ± 0.006) keV. Comparing with the total width of 159 MeV the estimated central value is rather small, namely the branching ratio is as small as about only 1.9×10^{-6}, even so one still has a chance to measure it in more accurate experiments. To explore its dependence on the parameter β we vary β from 0.35 to 0.6 GeV^{-1}. The results are depicted in Fig. 2. One can notice that the result is not sensitive to the value of β after all.

Since the vertex function of the $^3D_3$ state is more complicated we are not going to directly deduce the transition matrix elements for the radiative decays in this framework. Instead, we would take an approximate but reasonable scheme to estimate the radiative decay width of $^3D_3$. Namely, one obtains the rate of $^3D_3$ radiative decay in terms of that of the $^3D_1$ radiative decay. Under the nonrelativistic approximation the authors of Ref. [28] presented a formula to calculate the widths for the $M1$ transition as

$$\Gamma(i \to f\gamma) = \frac{\alpha}{3}\left(\frac{e_c}{m_c} - \frac{e_{\bar{s}}}{m_s}\right)^2 \times E_\gamma{}^3(2J_f + 1)|\langle f|j_0(kr/2)|i\rangle|^2. \quad (4)$$

If we ignore the spin–orbit coupling term in the potential which results in the fine-structure of the spectra, the wavefunctions of $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ obtained by solving the Schördinger equation would be the same because they have the same orbital angular momentum and intrinsic spin, thus we would naturally get $\langle D_s|j_0(kr/2)|D^*_{s1}(2860)\rangle = \langle D_s|j_0(kr/2)|D^*_{s3}(2860)\rangle$. Since the mass of $D^*_{s1}(2860)$ is close to that of $D^*_{s1}(2860)$, it hints that the contributions of the spin–orbit coupling term to the spectra and wavefunction are less important. By including all factors, it is straightforward to estimate $\Gamma(D^*_{s3}(2860) \to D_s\gamma) \approx \Gamma(D^*_{s1}(2860) \to D_s\gamma)$.

3.2 The radiative decay of $D^*_{s1}(2710)$

After $D^*_{s1}(2710)$ was found, a lot of work has been done to investigate its identity. In Ref. [11] the authors suggested that $D^*_{s1}(2710)$ should be a 2^3S_1 state, rather than a 1^3D_1. To be more open, here let us assume $D^*_{s1}(2710)$ to be, respectively, a 2^3S_1 state or a 1^3D_1 state and under the different assumptions, we calculate its radiative decay width. The results are listed in Table 1. For the S-wave state (2^3S_1) we employ the conventional wavefunction [S-wave(1)] and modified wavefunction [S-wave(2)] which was discussed in Ref. [24]. Then we continue to calculate the rate of radiative decay of the D-wave state in the aforementioned approximation.

One would notice that there exists a huge gap between the S-wave and D-wave cases.

If we assume that $D^*_{s1}(2710)$ is the mixture of 2^3S_1 and 1^3D_1 i.e. $|D^*_{s1}(2710)\rangle = \cos\theta|2^3S_1\rangle - \sin\theta|1^3D_1\rangle$ [15], using the values of $\mathcal{F}(0)$ given in Table 1, the corresponding radiative decay width is re-calculated. In Fig. 3 the dependence of the decay width on the mixing angle θ is depicted where the modified wavefunction is used for the 2S state.

In Ref. [15] the authors studied $\Gamma(D^*_{s1}(2710) \to D_{s2}(2573)\gamma)$, $\Gamma(D^*_{s1}(2710) \to D_{s0}(2317)\gamma)$, $\Gamma(D^*_{s1}(2710) \to D_{s1}(2460)\gamma)$, and $\Gamma(D^*_{s1}(2710) \to D_{s1}(2536)\gamma)$ which are 0.09–0.12, 7.80–7.97, 1.47–1.56, and 0.27–0.29 keV, respectively. The above cited estimates are concerned with radiative decays of $D^*_{s1}(2710)$ into a P-wave meson plus a photon, as we noted that for finally identifying the quantum numbers of $D^*_{s1}(2710)$, the decay mode under investigation: $D^*_{s1}(2710) \to D_s\gamma$, which is $D^*_{s1}(2710)$ decaying into a S-wave meson plus a photon, is not less important.

4 Summary

In this work we study the radiative decay of $D^*_{s1}(2860)$, $D^*_{s3}(2860)$, and $D^*_{s1}(2710)$, respectively, in terms of LFQM. Assuming $D^*_{s1}(2860)$, $D^*_{s3}(2860)$ to be 1^3D_1 and 1^3D_3 states, we obtain their partial widths. Our estimates on $\Gamma(D^*_{s1}(2860) \to D_s\gamma)$ and $\Gamma(D^*_{s3}(2860) \to D_s\gamma)$ are approximately 0.291 keV. The estimated branching ratios of the radiative decays of $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ are about 1.9×10^{-6} and 5.8×10^{-6}. By the achieved integrated luminosity at LHCb (3.0 fb^{-1}), the LHCb Collaboration [8] collected 12450 $B^0_s \to \bar{D}^0K^-\pi^+$ samples where only a part of the events concern $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$. Their radiative decays have not been observed yet due to the small database

Table 1 The form factor for $D^*_{s1}(2710) \to D_s$

	D-wave	S-wave(1)	S-wave(2)
$\mathcal{F}(0)$ (GeV^{-1})	-0.0168 ± 0.0002	0.099 ± 0.001	0.112 ± 0.001
Γ (keV)	0.179 ± 0.004	6.18 ± 0.07	8.00 ± 0.02
Branching ratio	$(1.53 \pm 0.17) \times 10^{-6}$	$(5.28 \pm 0.59) \times 10^{-5}$	$(6.84 \pm 0.76) \times 10^{-5}$

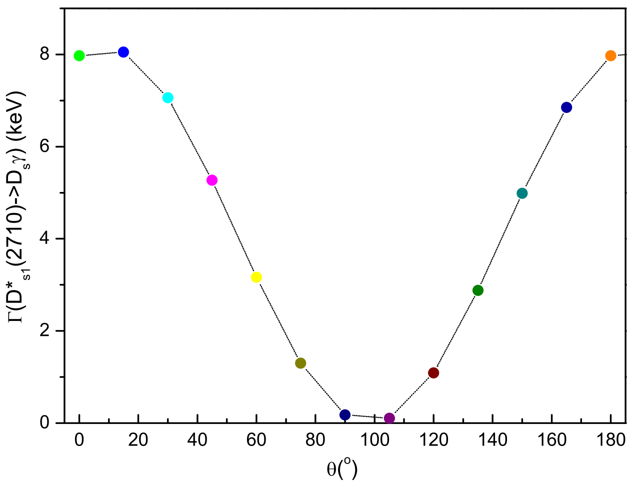

Fig. 3 Dependence of $\Gamma(D^*_{s1}(2710) \to D_s\gamma)$ on the mixing angle θ

for $D^*_s(2860)$. Indeed we need longer time and higher luminosity to observe the radiative decays $\Gamma(D^*_{s1}(2860) \to D_s\gamma)$ and $\Gamma(D^*_{s3}(2860) \to D_s\gamma)$.

Though the fractions of the radiative decays are small, they have a clear signal to be observed against the background, therefore the advantage of detecting those modes is obvious. Thus we expect our experimental colleagues to carry out accurate experiments to measure them.

Concerning $D^*_{s1}(2710)$, as discussed in the introduction, if $D^*_{s1}(2860)$ and $D^*_{s3}(2860)$ are confirmed to be the D-wave D_s meson, $D^*_{s1}(2710)$ cannot be a pure 1D-wave $c\bar{s}$ system, we calculate its radiative decay rate by assuming two possible assignments: 2^3S_1 or 1^3D_1, respectively. Our numerical results show that if it is a 2^3S_1 state the corresponding branching ratio is about 5.28×10^{-5}–6.84×10^{-5}, instead, while it is 1^3D_1, the corresponding rate is around 1.5×10^{-6}. There is an obvious gap between the estimated rates for the two assignments.

Because the LFQM is a relativistic model and its validity is widely recognized due to its success for explaining the available data for hadronic decays of heavy mesons, we may believe that the numerical results obtained in this framework is trustworthy, at most they could only decline from the real values by a small factor less than 2 which was confirmed by other phenomenological studies in terms of the same model. The possible uncertainties are incurred by the inputs. Even so, the results could help identifying the quantum numbers since in the two cases the resultant ratios of $\Gamma(D^*_{s1}(2710) \to D_s\gamma)$ are apparently apart.

No doubt, the final decision will be made by the future precise measurements. Our work only indicates the importance of studying the radiative decays because of their obvious advantage and it strongly suggests one to search such decay modes at the coming super-BELLE or next run of LHCb, even the expected ILC.

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Appendix A: Notations

Here we list some variables appearing in the context. The incoming meson in Fig. 1 has the momentum $P = p_1 + p_2$ where p_1 and p_2 are the momenta of the off-shell quark and antiquark and

$$p_1^+ = x_1 P^+, \quad p_2^+ = x_2 P^+,$$
$$p_{1\perp} = x_1 P_\perp + p_\perp, \quad p_{2\perp} = x_2 P_\perp - p_\perp, \tag{5}$$

where x_i and $p_\perp$ are internal variables and $x_1 + x_2 = 1$.

The variables M_0 and $\tilde{M}_0$ are defined as

$$M_0^2 = \frac{p_\perp^2 + m_1^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2},$$
$$\tilde{M}_0 = \sqrt{M_0^2 - (m_1 - m_2)^2},$$
$$\phi(1S) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp\left(-\frac{p_z^2 + p_\perp^2}{2\beta^2}\right),$$
$$\phi(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{\partial p_z}{\partial x}} \exp\left(-\frac{1}{2}\frac{p_z^2 + p_\perp^2}{\beta^2}\right) \times \left(3 - 2\frac{p_z^2 + p_\perp^2}{\beta^2}\right)\frac{1}{\sqrt{6}}$$
$$\phi_M(2S) = 4\Big(\frac{\pi}{\beta^2}\Big)^{3/4} \sqrt{\frac{\partial p_z}{\partial x_2}} \exp\Big(-\frac{2^\delta}{2}\frac{p_z^2 + p_\perp^2}{\beta^2}\Big) \times \left(a_2 - b_2\frac{p_z^2 + p_\perp^2}{\beta^2}\right). \tag{6}$$

with $p_z = \frac{x_2 M_0}{2} - \frac{m_2^2 + p_\perp^2}{2x_2 M_0}$, $\delta = 1/1.82$, $a_2 = 1.88684$, and $b_2 = 1.54943$.

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