



PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: December 31, 2013

ACCEPTED: January 10, 2014

PUBLISHED: January 27, 2014

An $sl(2, \mathbb{R})$ current algebra from AdS_3 gravity

Steven G. Avery, Rohan R. Poojary and Nemani V. Suryanarayana

*Institute of Mathematical Sciences, CIT Campus,
Taramani, Chennai 600113, India*

E-mail: savery@imsc.res.in, ronp@imsc.res.in, nemani@imsc.res.in

ABSTRACT: We provide a set of chiral boundary conditions for three-dimensional gravity that allow for asymptotic symmetries identical to those of two-dimensional induced gravity in light-cone gauge considered by Polyakov. These are the most general boundary conditions consistent with the boundary terms introduced by Compère, Song and Strominger recently. We show that the asymptotic symmetry algebra of our boundary conditions is a Virasoro algebra with Brown-Henneaux central charge c and an $sl(2, \mathbb{R})$ current algebra with level given by $c/6$. The fully non-linear solution in Fefferman-Graham coordinates is also provided along with its charges.

KEYWORDS: AdS-CFT Correspondence, 2D Gravity, Gauge Symmetry, Space-Time Symmetries

ARXIV EPRINT: [1304.4252](https://arxiv.org/abs/1304.4252)

Contents

1	Introduction	1
2	Chiral boundary conditions in AdS_3 gravity	1
2.1	New boundary conditions	3
2.2	The non-linear solution	3
3	Charges, algebra and central charges	4
4	Conclusion	6

1 Introduction

In his seminal 1987 paper [1], Polyakov provides a solution to the two-dimensional induced gravity theory [2],

$$S = \frac{c}{96\pi} \int d^2x \sqrt{-g} R \frac{1}{\nabla^2} R, \tag{1.1}$$

by working in a light-cone gauge. The gauge choice puts the metric into the form

$$ds^2 = -dx^+ dx^- + F(x^+, x^-) (dx^+)^2. \tag{1.2}$$

Polyakov shows that the quantum theory for the dynamical field $F(x^+, x^-)$ admits an $sl(2, \mathbb{R})$ current algebra symmetry with level $k = c/6$. In this note, we present the three-dimensional bulk theory that is dual to this two-dimensional theory.

2 Chiral boundary conditions in AdS_3 gravity

The action of three-dimensional gravity with negative cosmological constant [3] is given by

$$S = -\frac{1}{16\pi G} \int_{\mathcal{M}} d^3x \sqrt{-g} \left(R + \frac{2}{l^2} \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} \Theta + \frac{1}{8\pi G} S_{ct}(\gamma_{\mu\nu}), \tag{2.1}$$

where $\gamma_{\mu\nu}$ is the induced metric and Θ is trace of the extrinsic curvature of the boundary. Varying the action yields

$$\delta S = \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} \frac{1}{2} T^{\mu\nu} \delta\gamma_{\mu\nu}, \tag{2.2}$$

on the space of solutions where

$$T^{\mu\nu} = \frac{1}{8\pi G} \left[\Theta^{\mu\nu} - \Theta \gamma^{\mu\nu} + \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{ct}}{\delta\gamma_{\mu\nu}} \right]. \tag{2.3}$$

The variational principle is made well-defined by imposing $\delta\gamma_{\mu\nu} = 0$ (Dirichlet) or $T^{\mu\nu} = 0$ (Neumann) at the boundary (see [4] for a recent discussion).

Recently Compère, Song and Strominger (CSS) [5, 6] and Troessaert [7] proposed new sets of boundary conditions for three-dimensional gravity, which differ from the well-known Dirichlet-type Brown-Henneaux boundary conditions [8].¹ Before delving into specifics, let us discuss the general strategy employed by [6]. One begins by adding a term of the type

$$S' = -\frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} \frac{1}{2} \mathcal{T}^{\mu\nu} \gamma_{\mu\nu} \quad (2.4)$$

for a fixed ($\gamma_{\mu\nu}$ -independent) symmetric boundary tensor $\mathcal{T}^{\mu\nu}$. The variation of this term is

$$\delta S' = -\frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} \tilde{\mathcal{T}}^{\mu\nu} \delta\gamma_{\mu\nu}, \quad (2.5)$$

where $\tilde{\mathcal{T}}^{\mu\nu} = \mathcal{T}^{\mu\nu} - \frac{1}{2}(\mathcal{T}^{\alpha\beta} \gamma_{\alpha\beta}) \gamma^{\mu\nu}$. The variation of the total action then gives

$$\delta S + \delta S' = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} (T^{\mu\nu} - \tilde{\mathcal{T}}^{\mu\nu}) \delta\gamma_{\mu\nu}. \quad (2.6)$$

Now the boundary conditions consistent with the variational principle depend on $\tilde{\mathcal{T}}^{\mu\nu}$. Generically, this leads to “mixed” type boundary conditions. If for a given class of boundary conditions some particular component of $T^{\alpha\beta} - \tilde{\mathcal{T}}^{\alpha\beta}$ vanishes sufficiently fast in the boundary limit such that its contribution to the integrand in (2.6) vanishes, then the corresponding component of $\gamma_{\alpha\beta}$ can be allowed to fluctuate. Since we want the boundary metric to match (1.2), we would like Neumann boundary conditions for γ_{++} . Therefore we choose $\mathcal{T}^{\mu\nu}$ such that the leading term of T^{++} equals $\tilde{\mathcal{T}}^{++}$ in the boundary limit.

Such a mixed boundary condition has been imposed recently in [6], with the addition of an extra boundary term (2.4) with²

$$\mathcal{T}^{\mu\nu} = -\frac{1}{2r^4} N^2 l \delta_+^\mu \delta_+^\nu. \quad (2.7)$$

They proposed the following asymptotic fall-off conditions on the metric:

$$\begin{aligned} g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right), & g_{r\pm} &= \mathcal{O}\left(\frac{1}{r^3}\right), \\ g_{+-} &= -\frac{r^2}{2} + \mathcal{O}(r^0), & g_{++} &= r^2 f(x^+) + \mathcal{O}(r^0), & g_{--} &= -\frac{l^2}{4} N^2 + \mathcal{O}\left(\frac{1}{r}\right), \end{aligned} \quad (2.8)$$

where $f(x^+)$ is a dynamical field and N^2 is fixed constant.³ These boundary conditions give rise to an asymptotic symmetry algebra: a chiral U(1) current algebra with level determined by N . These also ensure that T_{--} is held fixed in the variational problem, whereas g_{++} is allowed to fluctuate as long as its boundary value is independent of x^- .

In what follows, we show that (2.8) are not the most general asymptotic fall-off conditions consistent with the variational principle and the extra boundary term given by (2.7). For this, we introduce a weaker set of consistent boundary conditions that enhance the asymptotic symmetry algebra to an $sl(2, \mathbb{R})$ current algebra whose level is independent of N .

¹In fact, the boundary conditions of [7] subsume those of [8].

²The induced metric $\gamma_{\mu\nu}$ differs from $g_{\mu\nu}^{(0)}$ of [6] by a factor of r^2 .

³To relate to the notation in [6], set $N^2 = -\frac{16G\Delta}{l}$ and $f(x^+) = l^2 \partial_+ \bar{P}(x^+)$.

2.1 New boundary conditions

In the new boundary conditions, the class of allowed boundary metrics coincides with that of (1.2). Since we want to allow γ_{++} to fluctuate, we keep T_{--} fixed in our asymptotically locally AdS_3 metrics. Therefore, we propose the following boundary conditions:

$$\begin{aligned}
 g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right), & g_{r+} &= \mathcal{O}\left(\frac{1}{r}\right), & g_{r-} &= \mathcal{O}\left(\frac{1}{r^3}\right), \\
 g_{+-} &= -\frac{r^2}{2} + \mathcal{O}(r^0), & g_{--} &= -\frac{l^2 N^2}{4} + \mathcal{O}\left(\frac{1}{r}\right), \\
 g_{++} &= r^2 F(x^+, x^-) + \mathcal{O}(r^0),
 \end{aligned}
 \tag{2.9}$$

where, as above, we take $F(x^+, x^-)$ to be a dynamical field and N fixed. The crucial difference between these boundary conditions and those in (2.8) is the different fall-off condition for g_{r+} which allows for the boundary component of g_{++} to depend on x^- as well. One must, of course, check the consistency of these conditions with the equations of motion. This involves constructing the non-linear solution in an expansion in inverse powers of r . Working to the first non-trivial order, one finds the following condition on $F(x^+, x^-)$:

$$N^2 \partial_- F(x^+, x^-) + \partial_-^3 F(x^+, x^-) = 0, \tag{2.10}$$

which forces $F(x^+, x^-)$ to take the form

$$F(x^+, x^-) = f(x^+) + g(x^+)e^{iNx^-} + \bar{g}(x^+)e^{-iNx^-} \tag{2.11}$$

where $f(x^+)$ is a real function and $\bar{g}(x^+)$ is the complex conjugate of $g(x^+)$.

Let us note that this is directly analogous to the form of $F(x^+, x^-)$ derived in [1]. Throughout our discussion we think of $\phi = \frac{x^+ - x^-}{2}$ as 2π -periodic (and $\tau = \frac{x^+ + x^-}{2}$ as the time coordinate), and therefore we restrict our consideration to $N \in \mathbb{Z}$. Similarly, we impose periodic boundary conditions on $f(x^+)$ and $g(x^+)$. If one takes the spatial part of the boundary to be \mathbb{R} instead of S^1 , there are no such restrictions and one may even consider $N^2 < 0$ like in [6].

2.2 The non-linear solution

One can write a general non-linear solution of AdS_3 gravity in Fefferman-Graham coordinates [9] as:

$$ds^2 = l^2 \frac{dr^2}{r^2} + r^2 \left[g_{ab}^{(0)} + \frac{l^2}{r^2} g_{ab}^{(2)} + \frac{l^4}{r^4} g_{ab}^{(4)} \right] dx^a dx^b. \tag{2.12}$$

The full non-linear solution with our boundary conditions is obtained when

$$\begin{aligned}
 g_{++}^{(0)} &= f(x^+) + g(x^+) e^{iNx^-} + \bar{g}(x^+) e^{-iNx^-}, & g_{+-}^{(0)} &= -\frac{1}{2}, \quad g_{--}^{(0)} = 0, \\
 g_{++}^{(2)} &= \kappa(x^+) + \frac{1}{2}N^2 \left[g^2(x^+) e^{2iNx^-} + \bar{g}^2(x^+) e^{-2iNx^-} \right] \\
 &\quad + \frac{i}{2}N \left[g'(x^+) e^{iNx^-} - \bar{g}'(x^+) e^{-iNx^-} \right], & (2.13) \\
 g_{+-}^{(2)} &= \frac{1}{4}N^2 \left[f(x^+) - g(x^+) e^{iNx^-} - \bar{g}(x^+) e^{-iNx^-} \right], & g_{--}^{(2)} &= -\frac{1}{4}N^2, \\
 g_{ab}^{(4)} &= \frac{1}{4}g_{ac}^{(2)}g_{(0)}^{cd}g_{db}^{(2)},
 \end{aligned}$$

where in the last line $g_{(0)}^{cd}$ is $g_{cd}^{(0)}$ inverse. As above, demanding that the solution respects the periodicity of ϕ -direction requires N to be an integer and the functions $f(x^+)$, $g(x^+)$ and $\kappa(x^+)$ to be periodic. This solution reduces to the one given in [6] when $g(x^+) = \bar{g}(x^+) = 0$, $N^2 \rightarrow -\frac{16G\Delta}{l}$ and $f(x^+) \rightarrow l^2\partial_+\bar{P}(x^+)$.⁴

Finally note that the global AdS_3 metric can be recovered by setting $f = g = \bar{g} = 0$, $N^2 = 1$ and $\kappa = -\frac{1}{4}$. Our solutions do not include the regular BTZ solutions. However by setting $f = g = \bar{g} = 0$, $N^2 = 1$ and $\kappa = -\frac{1}{4} + \Delta$ one can recover an extremal BTZ (with the horizon located at $r^2 = \Delta - 1/4$) solution.

3 Charges, algebra and central charges

It is easy to see that vectors of the form

$$\begin{aligned}
 \xi^r &= -\frac{1}{2} \left[B'(x^+) + iNA(x^+)e^{iNx^-} - iN\bar{A}(x^+)e^{-iNx^-} \right] r + \mathcal{O}(r^0) \\
 \xi^+ &= B(x^+) - \frac{l^2N^2}{2r^2} \left[A(x^+)e^{iNx^-} + \bar{A}(x^+)e^{-iNx^-} \right] + \mathcal{O}\left(\frac{1}{r^3}\right) \\
 \xi^- &= A_0(x^+) + A(x^+)e^{iNx^-} + \bar{A}(x^+)e^{-iNx^-} + \mathcal{O}\left(\frac{1}{r}\right)
 \end{aligned} \tag{3.1}$$

satisfy the criteria of [10], which allow us to construct corresponding asymptotic charges. If, on the other hand, one demands that the asymptotic symmetry generators ξ leave the space of boundary conditions invariant, one finds the same vectors but with the first subleading terms appearing at one higher order for each component. For either set of vectors, the Lie bracket algebra closes to the same order as one has defined the vectors.

Here, $B(x^+)$ and $A_0(x^+)$ are real and $A(x^+)$ is complex; therefore, there are four real, periodic functions of x^+ that specify this asymptotic vector. We take the following basis

⁴As mentioned in the previous subsection one can take N to be purely imaginary when the boundary spatial coordinate is not periodic. In this case too the non-linear solution (2.12) continues to be a valid solution with $g(x^+)$ and $\bar{g}(x^+)$ treated as two real and independent functions. However, we will not consider this case further here.

for the modes of the vector fields:

$$\begin{aligned}
 L_n &= i e^{i n x^+} \left[\partial_+ - \frac{i}{2} n r \partial_r \right] + \dots \\
 T_n^{(0)} &= \frac{i}{N} e^{i n x^+} \partial_- + \dots \\
 T_n^{(+)} &= \frac{i}{N} e^{i(n x^+ + N x^-)} \left[\partial_- - \frac{i}{2} N r \partial_r - \frac{N^2}{2r^2} \partial_+ \right] + \dots \\
 T_n^{(-)} &= \frac{i}{N} e^{i(n x^+ - N x^-)} \left[\partial_- + \frac{i}{2} N r \partial_r - \frac{N^2}{2r^2} \partial_+ \right] + \dots,
 \end{aligned} \tag{3.2}$$

which satisfy the Lie bracket algebra

$$\begin{aligned}
 [L_m, L_n] &= (m - n) L_{m+n}, & [L_m, T_n^{(a)}] &= -n T_{m+n}^{(a)}, \\
 [T_m^{(0)}, T_n^{(\pm)}] &= \mp T_{m+n}^{(\pm)}, & [T_m^{(+)}, T_n^{(-)}] &= 2 T_{m+n}^{(0)}.
 \end{aligned} \tag{3.3}$$

Thus, the classical asymptotic symmetry algebra is a Witt algebra and an $sl(2, \mathbb{R})$ current algebra.

We use the Brandt-Barnich-Compère (BBC) formulation [10, 11] for computing the corresponding charges of our geometry. We find that the charges are integrable over the solution space if $\delta N = 0$ with

$$\begin{aligned}
 \delta Q_\xi &= \frac{l}{8\pi G} \delta \int_0^{2\pi} d\phi \left\{ B(x^+) \left[\kappa(x^+) + N^2 \left(\frac{1}{2} f^2(x^+) - g(x^+) \bar{g}(x^+) \right) \right. \right. \\
 &\quad \left. \left. + \frac{N^2}{2} (e^{i N x^-} g(x^+) + e^{-i N x^-} \bar{g}(x^+)) \right] \right. \\
 &\quad \left. + \frac{i}{2} N \partial_+ [B(x^+) (e^{i N x^-} g(x^+) - e^{-i N x^-} \bar{g}(x^+))] \right\} \\
 &\quad - \frac{l}{8\pi G} \delta \int_0^{2\pi} d\phi N^2 \left[\frac{1}{2} A_0(x^+) f(x^+) - (g(x^+) \bar{A}(x^+) + \bar{g}(x^+) A(x^+)) \right].
 \end{aligned} \tag{3.4}$$

These can be integrated between the configurations trivially in the solution space from $f(x^+) = g(x^+) = \kappa(x^+) = 0$ to general values of these fields to write down the charges

$$\begin{aligned}
 Q_B &= \frac{l}{8\pi G} \int_0^{2\pi} d\phi \left[B(x^+) \left(\kappa(x^+) + \frac{N^2}{2} (f^2(x^+) - 2g(x^+) \bar{g}(x^+)) \right) \right. \\
 &\quad \left. + \frac{1}{2} (\partial_+ - \partial_-) \partial_- [e^{i N x^-} g(x^+) + e^{-i N x^-} \bar{g}(x^+)] \right] \\
 &= \frac{l}{8\pi G} \int_0^{2\pi} d\phi \left[B(x^+) \left[\kappa(x^+) + \frac{N^2}{2} (f^2(x^+) - 2g(x^+) \bar{g}(x^+)) \right] \right. \\
 &\quad \left. + \frac{1}{32\pi G} \partial_- [e^{i N x^-} g(x^+) + e^{-i N x^-} \bar{g}(x^+)] \Big|_{\phi=0}^{\phi=2\pi} \right], \tag{3.5}
 \end{aligned}$$

$$Q_A = -\frac{l}{8\pi G} N^2 \int_0^{2\pi} d\phi \left[\frac{1}{2} A_0(x^+) f(x^+) - (g(x^+) \bar{A}(x^+) + \bar{g}(x^+) A(x^+)) \right]. \tag{3.6}$$

The boundary term in (3.5) vanishes as we assumed $g(x^+)$ to be periodic and N to be an integer. The algebra of these charges admits central charges. We find that the central term

in the commutation relation between charges corresponding to two asymptotic symmetry vectors ξ and $\tilde{\xi}$ is given by

$$(-i)\frac{l}{32\pi G}\int_0^{2\pi}d\phi\left[B'(x^+)\tilde{B}''(x^+)-B(x^+)\tilde{B}'''(x^+)+2N^2A_0(x^+)\tilde{A}'_0(x^+)-4N^2\left(A(x^+)\tilde{A}'(x^+)+\bar{A}(x^+)\tilde{A}'(x^+)\right)\right]. \quad (3.7)$$

These give rise to the following algebra for the charges⁵

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}m^3\delta_{m+n,0}, \\ [L_m, T_n^a] &= -nT_{m+n}^a, \\ [T_m^a, T_n^b] &= f^{ab}{}_c T_{m+n}^c - \frac{k}{2}\eta^{ab}m\delta_{m+n,0} \end{aligned} \quad (3.8)$$

with

$$c = \frac{3l}{2G}, \quad k = \frac{c}{6}, \quad f^{0+}_+ = -1, \quad f^{0-}_- = 1, \quad f^{+-}_0 = 2, \quad \eta^{00} = -1, \quad \eta^{+-} = 2. \quad (3.9)$$

This is precisely the $sl(2, \mathbb{R})$ current algebra found in [1].

4 Conclusion

In this note we have provided boundary conditions for 3-dimensional gravity with negative cosmological constant such that the algebra of asymptotic symmetries is an $sl(2, \mathbb{R})$ current algebra. In the process we showed that the boundary term proposed by CSS [6] admits a more general set of boundary conditions, which enables our result.

It should be noted that our asymptotic symmetry algebra does contain the full isometry algebra of the global AdS_3 solution. This feature is similar to Brown-Henneaux [8] though one does not demand that the asymptotic vector fields of interest be asymptotically Killing; instead one uses the more general notion of asymptotic symmetries advocated by BBC [10, 11]. Using the BBC formulation, we computed the algebra of charges and found the level k to be $c/6$, independent of the parameter N .

To understand the relation to 2-dimensional induced gravity of Polyakov in light-cone gauge [1] further, it will be interesting to see if the correlation functions of the boundary currents, and the effective action for the dynamical fields of the boundary can also be recovered from the gravity side. See [12] for a discussion on the latter issue. Of course, connections between 3-dimensional gravity with negative cosmological constant and Liouville theory, which arises as a different gauge-fixing of (1.1), are well-known (see e.g. [13], and the recently proposed boundary conditions in [7]).

It will be interesting to see how adding matter to AdS_3 gravity would generalize our analysis. The boundary conditions of [6] have been found to be related to string theory solutions of [14] with a warped AdS_3 factor. It will be interesting to explore whether the boundary conditions in (2.9) also play a role in some string theory context.

⁵The bracket in (3.8) is i times the Dirac bracket.

The non-linear solution in (2.12), (2.13) does not contain the conventional positive mass BTZ [15] black hole, except for the extremal one. The special case of vanishing charges is given by $f(x^+) = g(x^+) = \bar{g}(x^+) = \kappa(x^+) = 0$. Another choice of vacuum is $f(x^+) = g(x^+) = 0$, $\kappa(x^+) = -1/4$ and $N = \pm 1$ - which simply is the global AdS_3 . In this case the expression for the charge Q_A remains as it is in (3.6) and the charge Q_B is obtained by replacing $\kappa(x^+)$ in (3.5) by $\kappa(x^+) + \frac{1}{4}$. One can show that the only modification of the algebra in (3.8) is in the central term in the Virasoro which now reads $\frac{c}{12}m(m^2 - 1)\delta_{m+n,0}$.

One important difference between CSS and our case is that the Ricci scalar of the boundary metric is not necessarily zero and therefore means that there is Weyl anomaly for the boundary theory. This has been taken care of by the equations of motion and results in the particular structure of x^- -dependence. One may wonder if one needs to add holographic counter terms that go as logarithm of the radial co-ordinate and if so that would modify the BBC prescription for computing the central charges etc.⁶ The log counter terms are usually used in the partition function computation (euclidean action) and one does not have to add this term to see the Weyl anomaly at all as it directly appears on holographic stress tensor.

Finally, it is intriguing that different ways of gauge-fixing the induced gravity [1] lead to different boundary conditions in the bulk and therefore apparently different holographic duals. This may be so because the diffeomorphisms that connect different gauge choices of the induced gravity theory are not small diffeomorphisms in two dimensions. It will be important to understand the class of physical theories one can obtain this way and how they are related to each other.

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⁶We thank Compère for a helpful correspondence on this aspect.

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