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Reply to ‘Comment on “Helmholtz Theorem and the V-Gauge in the Problem of Superluminal and Instantaneous Signals in Classical Electrodynamics” by A. Chubykalo et al.’ by J.A. Heras [Found. Phys. Lett. vol. 19(6) p. 579 (2006)]

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José A. Heras has recently raised interesting criticism [1] regarding the conclusions to our paper [2]. His main point is his assertion that the equations we derived for the solenoidal (\mathbf{E}_s) and the irrotational (\mathbf{E}_i) components of the electric field \mathbf{E} are a pair of coupled equations that can be reduced to one equation. If this is the case, our claim that there exists two mechanisms of energy and momentum propagation is incorrect and our example using a bounded oscillating charge on the x -axis must be incorrect and Heras attempts to find contradictions in our focus. Therefore in this paper we will show that the equations for \mathbf{E}_i and \mathbf{E}_s are not coupled equations. We indicate the kind of problems that classical electrodynamics in its standard form cannot explain or predict, however, such problems can be solved using Helmholtz’ theorem and our interpretation of its use. Hence our claim that there exist two mechanisms of energy and momentum transfer, is, to our knowledge and according to our interpretation, correct.

We will now demonstrate that the equations for the solenoidal and the irrotational components of the electric field are not coupled equations.

We start from Maxwell’s standard equations

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$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{4}$$

From (1–3) we get the following wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 4\pi \left(\nabla \rho + \frac{1}{c^2} \frac{\partial \mathbf{j}}{\partial t} \right). \tag{5}$$

If we apply Helmholtz’ theorem and (4) we obtain three more equations

$$\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s, \quad \mathbf{j} = \mathbf{j}_i + \mathbf{j}_s, \quad \mathbf{B} = \mathbf{B}_s. \tag{6}$$

Using (6), Maxwell’s equations become

$$\nabla \times \mathbf{B}_s = \frac{4\pi}{c} \mathbf{j}_s + \frac{4\pi}{c} \mathbf{j}_i + \frac{1}{c} \frac{\partial \mathbf{E}_s}{\partial t} + \frac{1}{c} \frac{\partial \mathbf{E}_i}{\partial t}, \tag{7}$$

$$\nabla \times \mathbf{E}_s = -\frac{1}{c} \frac{\partial \mathbf{B}_s}{\partial t}, \tag{8}$$

$$\nabla \cdot \mathbf{E}_i = 4\pi \rho, \tag{9}$$

$$\nabla \cdot \mathbf{B}_s = 0. \tag{10}$$

A time differentiation of (7) and the use of (8) give us the equation

$$\nabla^2 \mathbf{E}_s - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial \mathbf{j}_s}{\partial t} + \frac{4\pi}{c^2} \frac{\partial \mathbf{j}_i}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_i}{\partial t^2}. \tag{11}$$

Equation (11) is Heras’ key equation. Here it seems that the solenoidal and the irrotational components of the electric field are coupled. If (7) and (8) were our only equations at hand undoubtedly Heras would be right. However we have (9). This equation is independent of (7) and (8), so we can use it to decouple the irrotational and solenoidal components in (11).

We can prove [2] the equation

$$-\frac{\partial \mathbf{E}_i}{\partial t} = 4\pi \mathbf{j}_i \tag{12}$$

from (9) directly by using the continuity equation, Helmholtz’ theorem and the asymptotic behavior of harmonic functions, as follows:

$$\begin{aligned} \mathbf{E}_i &= -\nabla \int \frac{\nabla' \cdot \mathbf{E}(\mathbf{r}', t)}{4\pi |\mathbf{r} - \mathbf{r}'|} dV' = -\nabla \int \frac{\rho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV' \Rightarrow \\ \frac{\partial \mathbf{E}_i}{\partial t} &= -\nabla \int \frac{\frac{\partial \rho(\mathbf{r}', t)}{\partial t}}{|\mathbf{r} - \mathbf{r}'|} dV' = \nabla \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV' = -4\pi \mathbf{j}_i \end{aligned}$$

If we combine (11) and (12) we get for \mathbf{E}_s

$$\nabla^2 \mathbf{E}_s - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial \mathbf{j}_s}{\partial t}. \quad (13)$$

For \mathbf{E}_i we must get a differential equation too. To achieve this we use (6) at (5), then we get for the irrotational component the equation

$$\Delta \mathbf{E}_i = 4\pi \nabla \varrho. \quad (14)$$

Equations (13) and (14) are the basic equations of our paper [2], and they are a direct consequence of Maxwell's equations and Helmholtz' theorem.

It is not correct to claim that (14) follows from (1) and (5), according to [1]. We have seen that (14) follows from (11), the continuity equation, and the asymptotic behavior of harmonic functions at spatial infinity. Indeed, we must remember that the term $\frac{\partial \mathbf{E}}{\partial t}$ was included by Maxwell himself on the right hand side of (1) to avoid any contradiction with the continuity equation. For this reason, Maxwell considered the charge conservation as a fundamental law that must be satisfied by Ampère law. In this sense, (14) is independent of (1) because we do not need (1) to deduce (14). Therefore we have proved that our basic equations (13) and (14) are independent, so the irrotational and the solenoidal components of the electric field are determined by independent differential equations. This is *the most important achievement* of our paper [2].

However we must point out that Heras' criticism [1] is, as it were, twofold: a mathematical technique that supports a physical situation. When he says that the components of the electric field are coupled, the underlying physical intuition and philosophical commitment is that in fact the irrotational component is not detectable and cannot mediate the electromagnetic interaction. So, in some sense, the non-physical components must be related to the physical ones and this relation must be expressed mathematically. For this reason the supposition that (11) relates both components is a natural one; *however*, our critical analysis shows that *this is not the case*: both components satisfy *different* uncoupled equations. Now we must show that a simple physical situation exists where the components may have a separate physical meaning.

We must show now that both fields can be realized as independent physical quantities. First of all, we point out in detail a problem that classical electrodynamics in its standard form cannot solve.

We start with the following question: Is it possible for the retarded solutions of the wave equation to explain all electromagnetic phenomena? We claim that this is not the case, as the following simple example (which is no other than the one used in [2]) shows.

Consider a point charge at rest at the origin of an inertial Cartesian coordinate system. If it begins to move along the x -axis there is a brief interval of time when the charge is accelerated, and the electromagnetic field of the moving charge is given by the retarded fields $\mathbf{E}_{\text{ret}}, \mathbf{B}_{\text{ret}} = \frac{\mathbf{R}}{R} \times \mathbf{E}_{\text{ret}}$ where \mathbf{R} is the position vector of the particle, in this case along the x -axis. The question is now: *Is there energy and momentum propagation along the x -axis?* Because along this axis the velocity \mathbf{V} and the acceleration \mathbf{a} of the particle are parallel to \mathbf{R} , the acceleration field is clearly zero. Hence

in the retarded electric field only the velocity field survives

$$\mathbf{E}_{\text{ret}} = \frac{q(\frac{\mathbf{R}}{R} - \frac{\mathbf{V}}{c})}{\gamma^2(1 - \frac{\mathbf{R}\cdot\mathbf{V}}{Rc})^3 R^2} \Big|_{\text{ret}}, \tag{15}$$

so we obtain for the retarded magnetic field $\mathbf{B}_{\text{ret}} = \frac{\mathbf{R}}{R} \times \mathbf{E}_{\text{ret}} = 0$.

Along the x -axis the Poynting vector $\mathbf{S} = 0$. Therefore along the x -axis there is no transmission of energy and momentum, and any test charge in this axis does not feel the presence of the moving charge initially at rest at the origin. But this contradicts experience, because if the charge initially at rest begins to move, any test charge along the x -axis begins to move too, so, a transmission of energy and momentum from one charge to the other *intervenes!* How is this so? Obviously through the Poynting vector this is not possible because *there is no energy flow* along the x -axis. Hence the obvious logical conclusion is that there must be another mechanism of energy transmission from an accelerated charge to other charges. In our original paper [2] we used an example where \mathbf{E}_s is zero along the line of action, and for this reason it is not possible to find any contradiction with the retarded solutions.

Now that our motivation is clear we must return to Heras' arguments. He claims that when $\mathbf{E} = \mathbf{E}_i$, Faraday's law is violated. For our case it is clear that $\mathbf{B}_s = 0$, so any violation of Faraday's law is not possible. However, even in general terms, it is not the case that when the electric field is purely irrotational Faraday's law is violated, because when we write Maxwell's equations in solenoidal form, Faraday's law becomes

$$\nabla \times \mathbf{E}_s = -\frac{1}{c} \frac{\partial \mathbf{B}_s}{\partial t}$$

as can be seen from (8). Now, if $\mathbf{B}_s = 0$ the vector field becomes purely irrotational as can be deduced from (7–10), and if $\mathbf{E}_s = 0$ then any time-independent magnetic field would satisfy Faraday's law. So, from Faraday's law Heras cannot deduce any contradiction.

We have expounded our motivation for the use of Helmholtz theorem: it is necessary to explain energy and momentum transmission in some simple cases where the retarded solutions are impotent, and we believe that the use of Helmholtz theorem is a way to achieve this goal using Maxwell's standard equations. We have replied to Heras too, showing that his supposed violation of Faraday's law is a misunderstanding. In conclusion we still claim that there are two mechanisms of transmission.

In this reply we have shown in detail that the criticism raised by J.A. Heras to our paper [2] is the result of a misunderstanding of the fundamental idea of our work. For that reason we explained the kind of problems that motivate our theory of the two mechanisms of transmission of energy and momentum and we deduced, once again and with logical detail, the basic equations of our theory. To summarize, one can say that our theory is quite simple: we have two *independent* equations for the irrotational and solenoidal components of the electric field, *but this is not enough for a physical theory* (that would be a point in favor of Heras), and for this reason, we must show that there are physical situations where the influence of the electric field

can be reduced to the influence of its irrotational component. We think that we have achieved this goal in our paper [2].

References

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