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Split nonconvex variational inequality problem

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Abstract

In this paper, we propose a split nonconvex variational inequality problem which is a natural extension of split convex variational inequality problem in two different Hilbert spaces. Relying on the prox-regularity notion, we introduce and establish the convergence of an iterative method for the new split nonconvex variational inequality problem. Further, we also establish the convergence of an iterative method for the split convex variational inequality problem. The results presented in this paper are new and different form the previously known results for nonconvex (convex) variational inequality problems. These results also generalize, unify, and improve the previously known results of this area.

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Introduction

Recently, Censor et al. [1] introduced and studied the following *split convex variational inequality problem* (SCVIP): Let H_1 and H_2 be real Hilbert spaces with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let C and Q be nonempty, closed, and convex subsets of H_1 and H_2, respectively. Let $f : H_1 \to H_1$ and $g : H_2 \to H_2$ be nonlinear mappings and $A : H_1 \to H_2$ be a bounded linear operator with its adjoint operator A^*. Then, the SCVIP is to find $x^* \in C$ such that

$$\langle f(x^*), x - x^* \rangle \geq 0, \quad \forall x \in C \tag{1a}$$

and such that

$$y^* = Ax^* \in Q \text{ solves } \langle g(y^*), y - y^* \rangle \geq 0, \quad \forall y \in Q. \tag{1b}$$

SCVIP amount to saying: find a solution of variational inequality problem (VIP) (1a), the image of which under a given bounded linear operator is a solution of VIP (1b). It is worth mentioning that SCVIP is quite general and permits split minimization between two spaces, so the image of a minimizer of a given function, under a bounded linear operator, is a minimizer of another function. The special cases of SCVIP are split zero problem and split feasibility problem which has already been studied and used in practice as a model in intensity-modulated radiation therapy treatment planning. This formulation is also at the core of the modeling of many inverse problems arising for phase retrieval and other real-world problems; for instance, in sensor networks in computerized tomography and data compression; see [2-5].

In this paper, we intend to generalize SCVIP (1a,b) to take into account of nonconvexity of the subsets C and Q. This new nonconvex problem is called split nonconvex variational inequality problem (SNVIP). To overcome the difficulties that arise from the nonconvexity of C and Q, we will consider the class of uniform prox-regular sets, which is sufficiently large to include the class of convex sets, p-convex sets, $C^{1,1}$ submanifolds, and many other nonconvex sets; see [6]. Using the properties of projection operator over uniformly prox-regular sets, we establish the convergence of an iterative method for SNVIP. Further, we also establish the convergence of an iterative method for the split convex variational inequality problem. The results presented in this paper are new and different form the previously known results for nonconvex (convex) variational inequality problems. These results also generalize, unify, and improve the previously known results of this area.

To begin with, let us recall the following concepts which are of common use in the context of nonsmooth analysis; see [6-9].

Throughout the rest of the paper unless otherwise stated, let C and Q be nonempty closed subsets of H_1 and H_2, respectively, not necessarily convex.

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Definition 1. The *proximal normal cone* of C at $x \in H_1$ is given by

$$N_C^P(x) := \{\xi \in H_1 : x \in P_C(x + \alpha\xi)\},$$

where $\alpha > 0$ is a constant and P_C is projection operator of H_1 onto C, that is,

$$P_C(x) = \{x^* \in C : d_C(x) = \|x - x^*\|\},$$

where $d_C(x)$ is the usual distance function to the subset C, that is,

$$d_C(x) = \inf_{\hat{x} \in C} \|\hat{x} - x\|.$$

The proximal normal cone $N_C^P(x)$ has the following characterization.

Lemma 1. Let C be a nonempty closed subset of H_1. Then, $\xi \in N_C^P(x)$ if and only if there exists a constant $\alpha := \alpha(\xi, x) > 0$ such that

$$\langle \xi, \hat{x} - x\rangle \le \alpha \|\hat{x} - x\|^2, \quad \forall \hat{x} \in C.$$

Definition 2. The *Clarke normal cone*, denoted by $N_C^{cl}(x)$, is defined as

$$N_C^{cl}(x) = \bar{\text{co}}[N_C^P(x)],$$

where $\bar{\text{co}}A$ means the closure of the convex hull of A.

Poliquin and Rockafellar [7] and Clarke et al. [8] have introduced and studied a class of nonconvex sets, which are called uniformly prox-regular sets. This class of uniformly prox-regular sets has played an important role in many nonconvex applications such as optimization, dynamic systems, and differential inclusions. In particular, we have

Definition 3. For a given $r \in (0, \infty]$, a subset C_r of H_1 is said to be *normalized uniformly prox-regular* (*or uniformly r-prox-regular*) if and only if every nonzero proximal normal to C_r can be realized by any r-ball, that is, $\forall x \in C_r$ and $0 \neq \xi \in N_{C_r}^P(x)$, one has

$$\left\langle \frac{\xi}{\|\xi\|}, \hat{x} - x \right\rangle \le \frac{1}{2r} \|\hat{x} - x\|^2, \ \forall \hat{x} \in C_r.$$

It is known that if C_r is a uniformly r-prox-regular set, the proximal normal cone $N_{C_r}^P(x)$ is closed as a set-valued mapping. Thus, we have $N_{C_r}^{cl}(x) = N_{C_r}^P(x)$. We make the convention $\frac{1}{r} = 0$ for $r = +\infty$. If $r = +\infty$, then uniformly r-prox-regularity of C_r reduces to its convexity; see [6].

Now, let us state the following proposition which summarizes some important consequences of the uniformly prox-regularities:

Proposition 1. Let $r > 0$ and let C_r be a nonempty, closed, and uniformly r-prox-regular subset of H_1. Set $U_r = \{x \in H_1 : d(x, C_r) < r\}$.

(1) For all $x \in U_r$, $P_{C_r}(x) \neq \emptyset$;
(2) For all $r' \in (0, r)$, P_{C_r} is Lipschitz continuous with constant $\frac{r}{r-r'}$ on $U_{r'} = \{x \in H_1 : d(x, C_r) < r'\}$.

Split nonconvex variational inequality problem

Throughout the paper unless otherwise stated, we assume that for given $r, s \in (0, +\infty)$, C_r and Q_s are uniformly prox-regular subsets of H_1 and H_2, respectively. The SNVIP is formulated as follows:

find $x^* \in C_r$ such that

$$\langle f(x^*), x - x^*\rangle + \left(\frac{\|f(x^*)\|}{2r}\right) \|x - x^*\|^2 \ge 0, \forall x \in C_r \quad (2a)$$

and such that

$$y^* = Ax^* \in Q_s \text{ solves } \langle g(y^*), y - y^*\rangle + \left(\frac{\|g(y^*)\|}{2s}\right) \|y - y^*\|^2 \ge 0, \ \forall y \in Q_s. \quad (2b)$$

By making use of Definition 3 and Lemma 1, SNVIP (2a,b) can be reformulated as follows:

find $(x^*, y^*) \in C_r \times Q_s$ with $y^* = Ax^*$ such that

$$\mathbf{0} \in \rho f(x^*) + N_{C_r}^P(x^*)$$
$$\mathbf{0} \in \lambda g(y^*) + N_{Q_s}^P(y^*),$$

where ρ and λ are parameters with positive values and $\mathbf{0}$ denotes the zero vectors of H_1 and H_2, which in turn, since $\text{Proj}_{C_r} = (I + N_{C_r}^P)^{-1}$ and $\text{Proj}_{Q_s} = (I + N_{Q_s}^P)^{-1}$, is equivalent to find $(x^*, y^*) \in C_r \times Q_s$ with $y^* = Ax^*$ such that

$$x^* = \text{Proj}_{C_r}(x^* - \rho f(x^*))$$
$$y^* = \text{Proj}_{Q_s}(y^* - \lambda g(y^*)),$$

where $0 < \rho < \frac{r}{1+\|f(x^*)\|}$, $0 < \lambda < \frac{s}{1+\|g(y^*)\|}$, and Proj_{C_r} and Proj_{Q_s} are, respectively, projection onto C_r and Q_s.

If $r, s = +\infty$, then $C_r = C$ and $Q_s = Q$, the closed convex subsets of H_1 and H_2, respectively, and hence, SNVIP (2a,b) reduces to SCVIP (1a,b) which is equivalent to find $(x^*, y^*) \in C \times Q$ with $y^* = Ax^*$ such that

$$x^* = \text{Proj}_C(x^* - \rho f(x^*))$$
$$y^* = \text{Proj}_Q(y^* - \lambda g(y^*)),$$

where Proj_C and Proj_Q are, respectively, projection onto C and Q.

If $H_1 = H_2$, $f = g$, $A = I$, identity operator, $\rho = \lambda$, $r = s$, and $C_r = Q_s$, then SNVIP (1a,b) reduces to the nonconvex variational inequality problems (NVIP):
find $x^* \in C_r$ such that

$$\langle f(x^*), x - x^*\rangle + \left(\frac{\|f(x^*)\|}{2r}\right) \|x - x^*\|^2 \ge 0, \ \forall x \in C_r.$$

A number of authors developed and studied iterative methods for various classes of NVIPs; see for instance [10-12] and the references therein.

Definition 4. A nonlinear mapping $f : H_1 \to H_1$ is said to be

(1) *monotone*, if

$$\langle f(x) - f(\hat{x}), x - \hat{x}\rangle \geq 0, \quad \forall x, \hat{x} \in H_1;$$

(2) α*-strongly monotone*, if there exists a constant $\alpha > 0$ such that

$$\langle f(x) - f(\hat{x}), x - \hat{x}\rangle \geq \alpha \|x - \hat{x}\|^2, \quad \forall x, \hat{x} \in H_1;$$

(3) k*-inverse strongly monotone*, if there exists a constant $k > 0$ such that

$$\langle f(x) - f(\hat{x}), x - \hat{x}\rangle \geq k\|f(x) - f(\hat{x})\|^2, \quad \forall x, \hat{x} \in H_1;$$

(4) β*-Lipschitz continuous*, if there exists a constant $k > 0$ such that

$$\|f(x) - f(\hat{x})\| \leq \beta \|x - \hat{x}\|, \quad \forall x, \hat{x} \in H_1.$$

It is easy to observe that every k-inverse strongly monotone mapping f is monotone and $\frac{1}{k}$-Lipschitz continuous.

Based on above arguments, we propose the following iterative method for approximating a solution to SNVIP (2a,b).

Algorithm 1. Given $x_0 \in C_r$, compute the iterative sequence $\{x_n\}$ defined by the iterative schemes:

$$y_n = \mathrm{Proj}_{C_r}(x_n - \rho f(x_n)) \tag{3a}$$

$$z_n = \mathrm{Proj}_{Q_s}(Ay_n - \lambda g(Ay_n)) \tag{3b}$$

$$x_{n+1} = \mathrm{Proj}_{C_r}[y_n + \gamma A^*(z_n - Ay_n)] \tag{3c}$$

for all $n = 0, 1, 2, ...$, $0 < \rho < \frac{r}{1+\|f(x_n)\|}$, $0 < \lambda < \frac{s}{1+\|g(Ay_n)\|}$ and $0 < \gamma < \frac{r}{1+\|A^*(z_n - Ay_n)\|}$.

As a particular case of Algorithm 1, we have the following algorithm for approximating a solution to SCVIP (1a,b).

Algorithm 2. Given $x_0 \in C$, compute the iterative sequence $\{x_n\}$ defined by the iterative schemes:

$$y_n = \mathrm{Proj}_{C}(x_n - \rho f(x_n)) \tag{4a}$$

$$z_n = \mathrm{Proj}_{Q}(Ay_n - \lambda g(Ay_n)) \tag{4b}$$

$$x_{n+1} = \mathrm{Proj}_{C}[y_n + \gamma A^*(z_n - Ay_n)] \tag{4c}$$

for all $n = 0, 1, 2, ...$.

Further, we propose the following iterative method for SCVIP (1a,b) which is more general than Algorithm 2.

Let $\{\alpha_n\} \subseteq (0, 1)$ be a sequence such that $\sum_{n=1}^{\infty} \alpha_n = +\infty$, and let ρ, λ, γ are parameters with positive values.

Algorithm 3. Given $x_0 \in H_1$, compute the iterative sequence $\{x_n\}$ defined by the iterative schemes:

$$y_n = \mathrm{Proj}_{C}(x_n - \rho f(x_n)) \tag{5a}$$

$$z_n = \mathrm{Proj}_{Q}(Ay_n - \lambda g(Ay_n)) \tag{5b}$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n[y_n + \gamma A^*(z_n - Ay_n)] \tag{5c}$$

for all $n = 0, 1, 2, ...$.

We remark that Algorithms 2 and 3 are different from Algorithm 5.1 [1].

Results

Now, we study the convergence analysis of the Algorithm 1.

Assume that $r' \in (0, r)$, $s' \in (0, s)$ and denote $\delta = \frac{r}{r-r'}$ and $\eta = \frac{s}{s-s'}$.

Theorem 1. For given $r, s \in (0, +\infty)$, let C_r and Q_s be uniformly prox-regular subsets of H_1 and H_2, respectively. Let $f : H_1 \to H_1$ be α-strongly monotone and β-Lipschitz continuous and let $g : H_2 \to H_2$ be σ-strongly monotone and μ-Lipschitz continuous. Let $A : H_1 \to H_2$ be a bounded linear operator such that $A(C_r) \subseteq Q_s$ and A^* be its adjoint operator. Suppose $x^* \in C_r$ is a solution to SNVIP (2a,b), then the sequence $\{x_n\}$ generated by Algorithm 1 strongly converges to x^* provided that ρ, λ, and γ satisfy the following conditions:

$$\frac{2\alpha}{\beta^2} - \Delta < \rho < \min\left\{\frac{2\alpha}{\beta^2} + \Delta, \frac{r'}{1 + \|f(x_n)\|}, \frac{r'}{1 + \|f(x^*)\|}\right\}, \tag{6a}$$

$$0 < \lambda < \min\left\{\frac{s'}{1 + \|g(Ay_n)\|}, \frac{s'}{1 + \|g(Ax^*)\|}\right\}, \tag{6b}$$

$$0 < \gamma < \min\left\{\frac{2}{\|A\|^2}, \frac{r'}{1 + \|A * (z_n - Ay_n)\|}\right\}, \tag{6c}$$

where

$$\Delta := \frac{1}{\beta^2}\left(\sqrt{4\alpha^2 - \beta^2(1 - d^2)}\right) \text{ with}$$

$$2\alpha > \beta\sqrt{1 - d^2}; \; 0 < r' < r, \; 0 < s' < s;$$

$$d := [\delta^2(1 + 2\eta\theta_2)]^{-1}; \quad \theta_1 := \sqrt{1 - 2\rho\alpha + \rho^2\beta^2};$$
$$\theta_2 := \sqrt{1 - 2\lambda\sigma + \lambda^2\mu^2}.$$

Proof. Since $x^* \in C_r$ is a solution to SNVIP (2a,b) and the parameters ρ, λ satisfy the conditions (6a,b), then we have

$$x^* = \mathrm{Proj}_{C_r}(x^* - \rho f(x^*))$$
$$Ax^* = \mathrm{Proj}_{Q_s}(Ax^* - \lambda g(Ax^*)).$$

From Algorithm 1 (3a) and condition (6a) on ρ, we have

$$\|y_n - x^*\| = \|\mathrm{Proj}_{C_r}(x_n - \rho f(x_n)) - \mathrm{Proj}_{C_r}(x^* - \rho f(x^*))\| \leq \delta \|(x_n - x^* - \rho(f(x_n) - f(x^*))\|.$$

Now, using the fact that f is α-strongly monotone and β-Lipschitz continuous, we have

$$\begin{aligned}\|(x_n - x^* - \rho(f(x_n) - f(x^*))\|^2 &= \|(x_n - x^*\|^2 - 2\rho\langle f(x_n) - f(x^*), x_n - x^*\rangle \\ &\quad + \rho^2\|f(x_n) - f(x^*)\|^2 \\ &\leq (1 - 2\rho\alpha + \rho^2\beta^2)\|(x_n - x^*\|^2.\end{aligned}$$

As a result, we obtain

$$\|y_n - x^*\| \leq \delta\theta_1\|x_n - x^*\|, \tag{7}$$

where $\theta_1 = \sqrt{1 - 2\rho\alpha + \rho^2\beta^2}$.

Similarly, from Algorithm 1 (3b), condition (6b) on parameter λ and using the fact that g is σ-strongly monotone and μ-Lipschitz continuous, and $A(C_r) \subseteq Q_s$, we have

$$\begin{aligned}\|z_n - Ax^*\| &= \|\mathrm{Proj}_{Q_s}(Ay_n - \lambda g(Ay_n)) \\ &\quad - \mathrm{Proj}_{Q_s}(Ax^* - \lambda g(Ax^*))\| \\ &\leq \eta\|Ay_n - x^* - \lambda(g(Ay_n) - g(Ax^*))\| \\ &\leq \eta\theta_2\|Ay_n - Ax^*\|,\end{aligned} \tag{8}$$

where $\theta_2 = \sqrt{1 - 2\lambda\sigma + \lambda^2\mu^2}$.

Next from Algorithm 1 (3c) and condition (6c) on γ, we have

$$\begin{aligned}\|x_{n+1} - x^*\| &= \|\mathrm{Proj}_{C_r}[y_n + \gamma A^*(z_n - Ay_n)] \\ &\quad - \mathrm{Proj}_{C_r}[x^* + \gamma A^*(Ax^* - Ax^*)]\| \\ &\leq \delta[\|y_n - x^* - \gamma A^*(Ay_n - Ax^*)\| \\ &\quad + \gamma\|A^*(z_n - Ax^*)\|].\end{aligned} \tag{9}$$

Further, using the definition of A^*, the fact that A^* is a bounded linear operator with $\|A^*\| = \|A\|$, and condition (6c), we have

$$\begin{aligned}\|y_n - x^* - \gamma A^*(Ay_n - Ax^*)\|^2 &= \|y_n - x^*\|^2 - 2\gamma\langle y_n - x^*, A^*(Ay_n - Ax^*)\rangle \\ &\quad + \gamma^2\|A^*(Ay_n - Ax^*)\|^2 \\ &\leq \|y_n - x^*\|^2 - \gamma(2 - \gamma\|A\|^2)\|Ay_n - Ax^*\|^2 \\ &\leq \|y_n - x^*\|^2,\end{aligned} \tag{10}$$

and using Eq. (8), we have

$$\begin{aligned}\|A^*(z_n - Ax^*)\| &\leq \|A\|\|z_n - Ax^*\| \\ &\leq \eta\theta_2\|A\|\|Ay_n - Ax^*\| \\ &\leq \eta\theta_2\|A\|^2\|y_n - x^*\|.\end{aligned} \tag{11}$$

Combining Eqs. (10) and (11) with inequality (9), as a result, we obtain

$$\|x_{n+1} - x^*\| \leq \delta[\|y_n - x^*\| + \gamma\eta\theta_2\|A\|^2\|y_n - x^*\|].$$

Using Eq. (7), we have

$$\|x_{n+1} - x^*\| \leq \theta\|x_n - x^*\|,$$

where $\theta = \delta^2\theta_1(1 + \gamma\|A\|^2\eta\theta_2)$.

Thus, we obtain

$$\|x_{n+1} - x^*\| \leq \theta^n\|x_0 - x^*\|. \tag{12}$$

Since $\gamma\|A\|^2 < 2$, hence the maximum value of $(1 + \gamma\|A\|^2\eta\theta_2)$ is $(1 + 2\eta\theta_2)$. Further, $\theta \in (0, 1)$ if and only if

$$\theta_1 < [\delta^2(1 + 2\eta\theta_2)]^{-1} =: d. \tag{13}$$

We also observe that $d \in (0, 1)$ since $\delta, \eta > 1$. Finally, the inequality (13) holds from condition (6a). Thus, it follows from Eq. (12) that $\{x_n\}$ strongly converges to x^* as $n \to +\infty$. Since A is continuous, it follows from Eqs. (7) and (8) that $y_n \to x^*$, $Ay_n \to Ax^*$ and $z_n \to Ax^*$ as $n \to +\infty$. This completes the proof. □

It is worth mentioning that in the particular case where $r = +\infty$, $s = +\infty$, one has $\delta = \eta = 1$ and we get the convergence result for Algorithm 3 to solve SCVIP (1a,b).

Theorem 2. Let C and Q be nonempty closed and convex subsets of H_1 and H_2, respectively. Let $f : H_1 \to H_1$ be α-strongly monotone and β-Lipschitz continuous and let $g : H_2 \to H_2$ be σ-strongly monotone and μ-Lipschitz continuous. Let $A : H_1 \to H_2$ be a bounded linear operator and A^* be its adjoint operator. Suppose $x^* \in C$ is a solution to SCVIP (1a,b), then the sequence $\{x_n\}$ generated by Algorithm 3 strongly converges to x^* provided that ρ, λ, and γ satisfy the following conditions:

$$\frac{2\alpha}{\beta^2} - \Delta < \rho < \frac{2\alpha}{\beta^2} + \Delta, \tag{14a}$$

$$\gamma \in \left(0, \frac{2}{\|A\|^2}\right), \tag{14b}$$

where

$$\Delta := \frac{1}{\beta^2}\left(\sqrt{4\alpha^2 - \beta^2(1 - d^2)}\right) \text{ with } 2\alpha > \beta\sqrt{1 - d^2};$$
$$d := [\delta^2(1 + 2\theta_2)]^{-1}; \quad \theta_2 := \sqrt{1 - 2\lambda\sigma + \lambda^2\mu^2}; \lambda > 0.$$

Proof. Since $x^* \in C$ is a solution to SCVIP (1a,b), then for $\rho, \lambda > 0$, we have

$$x^* = \text{Proj}_C(x^* - \rho f(x^*))$$
$$Ax^* = \text{Proj}_Q(Ax^* - \lambda g(Ax^*)).$$

Using the same arguments used in proof of Theorem 1, we obtain

$$\|y_n - x^*\| \leq \delta\theta_1 \|x_n - x^*\|,$$

where $\theta_1 = \sqrt{1 - 2\rho\alpha + \rho^2\beta^2}$, and

$$\|z_n - Ax^*\| \leq \theta_2 \|Ay_n - Ax^*\|,$$

where $\theta_2 = \sqrt{1 - 2\lambda\sigma + \lambda^2\mu^2}$.

Next from Algorithm 3 (5c), we have

$$\begin{aligned}\|x_{n+1} - x^*\| &\leq (1 - \alpha_n)\|x_n - x^*\| \\ &+\alpha_n[\|y_n - x^* - \gamma A^*(Ay_n - Ax^*)\| + \gamma\|A^*(z_n - Ax^*)\|] \\ &\leq [1 - \alpha_n(1 - \theta)]\, \|x_n - x^*\|,\end{aligned}$$

where $\theta = \theta_1(1 + \gamma\|A\|^2\theta_2)$.

Thus, we obtain

$$\|x_{n+1} - x^*\| \leq \prod_{j=1}^{n}[1 - \alpha_j(1 - \theta)]\, \|x_0 - x^*\|. \tag{15}$$

It follows from condition (14a) that $\theta \in (0,1)$. Since $\sum_{n=1}^{\infty} \alpha_n = +\infty$ and $\theta \in (0,1)$, it implies in the light of [13] that

$$\lim_{n\to+\infty} \prod_{j=1}^{n}[1 - \alpha_j(1 - \theta)] = 0.$$

The rest of the proof is the same as the proof of Theorem 1. This completes the proof. □

Remark 1. (1) We would like to stress that SNVIP (2a,b) can be viewed as the following split nonconvex common fixed point problem:

$$\text{find} \quad x^* \in U := \text{Fix}(\text{Proj}_{C_r}(I - \rho f)) \quad \text{such that}$$
$$Ax^* \in V := \text{Fix}(\text{Proj}_{Q_s}(I - \lambda g)),$$

where $\text{Fix}(T)$ denotes the set of fixed points of mapping T.

(2) It is of further research effort to extend the iterative methods presented in this paper for solving the split variational inclusions [2] and the split equilibrium problem [14].

Competing interests
The author declares that he has no competing interests.

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