

Kahler moduli inflation revisited

Jose J. Blanco-Pillado,^a Duncan Buck,^b Edmund J. Copeland,^b Marta Gomez-Reino^c
and Nelson J. Nunes^d

^a*Institute of Cosmology, Department of Physics and Astronomy,
Tufts University, Medford, MA 02155, U.S.A.*

^b*School of Physics and Astronomy, University of Nottingham
University Park, Nottingham, NG7 2RD, U.K.*

^c*Theory Division, Physics Department, CERN,
CH-1211, Geneva 23, Switzerland*

^d*Institute für Theoretische Physik,
Philosophenweg 16, D-69120 Heidelberg, Germany*

E-mail: jose@cosmos.phy.tufts.edu, ppxdb@nottingham.ac.uk,
ed.copeland@nottingham.ac.uk, marta.gomez-reino.perez@cern.ch,
n.nunes@thphys.uni-heidelberg.de

ABSTRACT: We perform a detailed numerical analysis of inflationary solutions in Kahler moduli of type IIB flux compactifications. We show that there are inflationary solutions even when all the fields play an important role in the overall shape of the scalar potential. Moreover, there exists a direction of attraction for the inflationary trajectories that correspond to the constant volume direction. This basin of attraction enables the system to have an island of stability in the set of initial conditions. We provide explicit examples of these trajectories, compute the corresponding tilt of the density perturbations power spectrum and show that they provide a robust prediction of $n_s \approx 0.96$ for 60 e-folds of inflation.

KEYWORDS: Cosmology of Theories beyond the SM, Flux compactifications

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1 Introduction

The theory of inflation has been very successful in resolving many of the most important puzzles in early universe cosmology. However, there is, at the moment, no compelling evidence as to what could actually produce this period of accelerated expansion. It is therefore interesting to look for ways to understand this period of cosmic evolution within the framework provided by a fundamental theory. String theory is, at present, one of the most promising candidates for a fundamental theory and has inspired many attempts to embed inflation within it (for reviews see [1–7]). One of the most important challenges that has faced string phenomenology for a long time has been the issue of moduli stabilization [8–11]. Any successful model of low energy string theory should somehow be able to fix all the moduli such that it would be compatible with our current observations. On the other hand, the universe is not a static place but dynamical, so one is also interested in learning how we reached this low energy state, making other regions of the moduli space, and not only the final minimum, important in order to confront theory with cosmological observations. Taking this into account it is not surprising that recent developments of general methods of moduli stabilisation [10], and in particular stabilisation techniques of KKLT [11], have led to a large number of new inflationary scenarios using either the open string moduli related to the position of a mobile D-brane [12–19] or the closed string moduli coming from the compactification [20–34] as the relevant scalar fields.

This plethora of models should not be taken as a sign that inflation is easy to achieve within string theory. In fact, it is probably safe to say that it is just the opposite since most of these models have some degree of fine tuning in them. Indeed some of these problems were already encountered in the early models of modular inflation [8, 35, 36]. The main reason for these difficulties is the fact that the majority of these models are based on $\mathcal{N} = 1$ supergravity theories that have notorious problems to overcome if they are to satisfy the slow roll conditions necessary to have a successful inflationary model, the so-called η problem [37]. It is therefore very interesting to look for models based within string theory that can somehow alleviate or ameliorate these difficulties.

In this paper we will focus on a particular model of modular inflation that makes use of the special form of the potential for the Kahler moduli [33] enabling it to avoid the η -problem. The model is embedded within the Large Volume scenario developed in [38–41] something that, as we will show, turns out to be an important ingredient for the arguments presented in [33]. These Large Volume Models have been extensively studied in the last few years, due to their phenomenological interest as an explicit example within string theory of the large extra dimensional scenarios envisioned by [42]. It is therefore very interesting to study the cosmological implications of these type of models since they could provide us with a way to select the correct properties of the compactification scenario that we would like to have.

The purpose of this work is two fold. Firstly we demonstrate that there are inflationary solutions consistent with current observational data even when all of the moduli fields are allowed to vary during the cosmological evolution. Secondly, we show with explicit examples that the set of initial conditions that lead to a stable evolution, i.e., that avoid a runaway in the decompactification direction, is fairly wide. This property results from the existence of a basin of attraction in field space. There is an overlap in places between our work and that of Bond et al. [34], and where appropriate we will compare our results with theirs.

The outline of this paper is as follows. In section 2 we introduce the models under study. In section 3 we briefly review the mechanism of Kahler moduli inflation. In section 4 we numerically investigate the parameter space of the model where one can obtain inflationary trajectories and illustrate our results through some examples. In section 5 we study the basin of attraction of the inflationary solutions, before concluding in section 6.

2 The Kahler moduli potential

Our inflationary scenario can be obtained within a class of Type IIB flux compactification models on a Calabi-Yau orientifold. In this context it has been shown in [10, 11] that the superpotentials generated by background fluxes and by non-perturbative effects like instantons or gaugino condensation may generate a scalar potential that stabilizes all the geometric moduli coming from the compactification. More concretely, the introduction of background fluxes in the model induces a superpotential that freezes the dilaton as well as the complex structure moduli to their values at their supersymmetric minimum [10]. The remaining moduli, that is, the Kahler moduli, could be then stabilized by non-perturbative contributions to the superpotential [11]. The resulting effective 4D description of the Kahler

moduli T_i is an $\mathcal{N} = 1$ supergravity theory with a superpotential of the type,

$$W = W_0 + \sum_{i=2}^n A_i e^{-a_i T_i}. \quad (2.1)$$

In this formula W_0 is the perturbative contribution coming from the fluxes, which depends only on the frozen dilaton and the complex structure moduli, and therefore we will take to be a constant. There is also a non-perturbative piece depending on the Kahler moduli T_i where A_i and a_i are model dependent constants.

The F -term scalar potential is then given by the standard $\mathcal{N} = 1$ formula

$$V(T_i) = e^{\mathcal{K}} [\mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2], \quad (2.2)$$

where $D_i W = \partial_i W + (\partial_i \mathcal{K}) W$ is the covariant derivative of the superpotential and \mathcal{K} is the Kahler potential for T_i . In this paper we will concentrate in the kind of type IIB models presented in [38–41] in which the α' corrections to the potential are taken into account. For these type IIB models the expression for the α' -corrected Kahler potential is given by [43]

$$\mathcal{K}_{\alpha'} = -2 \ln \left(\mathcal{V} + \frac{\xi}{2} \right), \quad (2.3)$$

where \mathcal{V} denotes the overall volume of the Calabi-Yau manifold in string units and $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$ is proportional to $\zeta(3) \approx 1.2$. The Euler characteristic of the compactification manifold M is given by $\chi(M) = 2(h^{(1,1)} - h^{(1,2)})$ where $h^{(1,1)}$ and $h^{(1,2)}$ are the Hodge numbers of the Calabi-Yau. We will concentrate on models for which $\xi > 0$ (or equivalently with more complex structure moduli than Kahler moduli, $h^{(1,2)} > h^{(1,1)}$). As was explained in [38–41, 44], the reason for this is that in order to have the non-supersymmetric minimum at large volume the leading contribution to the scalar potential coming from the α' correction should be positive.

Following [33] we will consider models for which the internal volume of the Calabi-Yau can be written in the form,

$$\mathcal{V} = \frac{\alpha}{2\sqrt{2}} \left[(T_1 + \bar{T}_1)^{\frac{3}{2}} - \sum_{i=2}^n \lambda_i (T_i + \bar{T}_i)^{\frac{3}{2}} \right] = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^n \lambda_i \tau_i^{3/2} \right), \quad (2.4)$$

where the complex Kahler moduli are given by $T_i = \tau_i + i\theta_i$, with τ_i describing the volume of the internal four cycles present in the Calabi-Yau and θ_i are their corresponding axionic partners. The parameters α and λ_i are model dependent constants that can be computed once we have identified a particular Calabi-Yau. These models correspond to compactifications for which only the diagonal intersection numbers of the Calabi-Yau are non-vanishing.

Taking into account the form of the Kahler function one can then easily compute the Kahler metric for an arbitrary number of moduli, namely,

$$\mathcal{K}_{1\bar{1}} = \frac{3\alpha^{4/3}(4\mathcal{V} - \xi + 6\alpha \sum_{k=2}^n \lambda_k \tau_k^{3/2})}{4(2\mathcal{V} + \xi)^2 (\mathcal{V} + \alpha \sum_{k=2}^n \lambda_k \tau_k^{3/2})^{1/3}}, \quad \mathcal{K}_{i\bar{j}} = \frac{9\alpha^2 \lambda_i \lambda_j \sqrt{\tau_i} \sqrt{\tau_j}}{2(2\mathcal{V} + \xi)^2}, \quad (2.5)$$

$$\mathcal{K}_{1\bar{j}} = -\frac{9\alpha^{5/3} \lambda_j \sqrt{\tau_j} (\mathcal{V} + \alpha \sum_{k=2}^n \lambda_k \tau_k^{3/2})^{1/3}}{2(2\mathcal{V} + \xi)^2}, \quad \mathcal{K}_{i\bar{i}} = \frac{3\alpha \lambda_i (2\mathcal{V} + \xi + 6\alpha \lambda_i \tau_i^{3/2})}{4(2\mathcal{V} + \xi)^2 \sqrt{\tau_i}}, \quad (2.6)$$

which can be inverted to give,

$$\begin{aligned} \mathcal{K}^{1\bar{1}} &= \frac{4(2\mathcal{V}+\xi)(\mathcal{V}+\alpha\sum_{k=2}^n\lambda_k\tau_k^{3/2})^{1/3}(2\mathcal{V}+\xi+6\alpha(\sum_{k=2}^n\lambda_k\tau_k^{3/2}))}{3\alpha^{4/3}(4\mathcal{V}-\xi)}, & \mathcal{K}^{i\bar{j}} &= \frac{8(2\mathcal{V}+\xi)\tau_i\tau_j}{4\mathcal{V}-\xi}, \\ \mathcal{K}^{1\bar{j}} &= \frac{8(2\mathcal{V}+\xi)\tau_j(\mathcal{V}+\alpha\sum_{k=2}^n\lambda_k\tau_k^{3/2})^{2/3}}{\alpha^{2/3}(4\mathcal{V}-\xi)}, & \mathcal{K}^{i\bar{i}} &= \frac{4(2\mathcal{V}+\xi)\sqrt{\tau_i}(4\mathcal{V}-\xi+6\alpha\lambda_i\tau_i^{3/2})}{3\alpha(4\mathcal{V}-\xi)\lambda_i}, \end{aligned} \quad (2.7)$$

where we have rewritten for later convenience τ_1 in terms of \mathcal{V} and τ_i , $i = 2 \dots n$. With all this information we can use (2.2) to obtain the F-term scalar potential for the moduli fields which we find to be,

$$\begin{aligned} V &= \sum_{\substack{i,j=2 \\ i < j}}^n \frac{A_i A_j \cos(a_i \theta_i - a_j \theta_j)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} e^{-(a_i \tau_i + a_j \tau_j)} (32(2\mathcal{V} + \xi)(a_i \tau_i + a_j \tau_j + 2a_i a_j \tau_i \tau_j) + 24\xi) \\ &+ \frac{12W_0^2 \xi}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \sum_{i=2}^n \left[\frac{12e^{-2a_i \tau_i} \xi A_i^2}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \frac{16(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3\alpha \lambda_i (2\mathcal{V} + \xi)} \right] \\ &+ \frac{32e^{-2a_i \tau_i} a_i A_i^2 \tau_i (1 + a_i \tau_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} + \frac{8W_0 A_i e^{-a_i \tau_i} \cos(a_i \theta_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} \left(\frac{3\xi}{2\mathcal{V} + \xi} + 4a_i \tau_i \right) + V_{uplift}. \end{aligned} \quad (2.8)$$

In this expression for the potential we have introduced an additional uplift term of the form V_{uplift} . The purpose of this term is to uplift the minima of the potential from an anti-de Sitter minimum to a nearly Minkowski vacuum. Its origin in model building has been the subject of some debate. It could be achieved by breaking explicitly supersymmetry through the introduction of anti-branes located in a region with strong red-shift, as suggested in [11], or in other alternative ways involving vector multiplets [45, 46]. Also, from a low-energy effective field theory point of view, it can in principle be implemented by using as an uplifting sector any kind of theory leading to spontaneous supersymmetry breaking, provided the supersymmetric sector is appropriately shielded from this uplifting sector [47]. Of course, the different ways that a term of this form can appear in the low energy description of the theory may lead to slightly different dependencies on the internal volume. For simplicity, as well as for the sake of comparison, we will take the same form as it was previously assumed in [33], namely $V_{uplift} = \frac{\beta}{\mathcal{V}^2}$. Nevertheless it is interesting to point out that in these Large Volume Models the presence of an uplifting sector is not necessary in order to break supersymmetry as a non-supersymmetric minimum is already present at large volume [38–41, 44]. This is in fact our case here as well.

3 Single field Kahler moduli inflation

In ref. [33] the authors argued that the scalar potential given by the expression (2.8) should be able to support a period of slow roll inflation without any fine tuning, making it a natural candidate to realize the idea of modular inflation. In this section we will briefly review their argument, and in the following sections we will proceed to test how general can this argument be made.

The first thing one should take into account is that the form of the potential (2.8) simplifies substantially in the limit in which $\mathcal{V} \gg 1$. In this limit, as can be inferred from (2.4), there should be one four-cycle (the one given by τ_1) much bigger than the rest,

$\tau_1 \gg \tau_i, i = 2, \dots, n$. Taking this into consideration one can approximate the full potential by the expression:

$$V_{LARGE} = \sum_{i=2}^n \frac{8(a_i A_i)^2}{3\alpha\lambda_i\mathcal{V}} \sqrt{\tau_i} e^{-2a_i\tau_i} + \sum_{i=2}^n \frac{4W_0 a_i A_i}{\mathcal{V}^2} \tau_i e^{-a_i\tau_i} \cos(a_i\theta_i) + \frac{3\xi W_0^2}{4\mathcal{V}^3} + \frac{\beta}{\mathcal{V}^2}, \quad (3.1)$$

where we have only included the leading terms up to $\mathcal{O}(\frac{1}{\mathcal{V}^3})$.

The basic idea now to have inflation in this model is to look for the possibility of having a flat enough potential by displacing one of the fields from its minimum value while keeping the others fixed at their global minimum values. It is reasonable to expect that this strategy would lead to a successful inflationary period since the potential is exponentially suppressed along the directions τ_i ($i = 2, \dots, n$). On the other hand, the authors in [33] also point out correctly that for this idea to work one should show that the whole inflationary evolution occurs along a single τ_i direction, otherwise one would not be able to draw conclusions by looking at that particular slice of the potential in field space.

The way they propose to enforce this constraint is the following: let us assume for concreteness that inflation happens along the τ_2 direction. Then in the limit in which $a_i\tau_i \gg 1$, for $i = 2, \dots, n$, the authors of [33] claim that by imposing that the parameters appearing in the potential satisfy the condition $\rho \ll 1$, where

$$\rho \equiv \frac{\lambda_2}{a_2^{3/2}} \left(\sum_{i=2}^n \frac{\lambda_i}{a_i^{3/2}} \right)^{-1}, \quad (3.2)$$

the minimum of the potential along the other field directions remain virtually unchanged even if one displaces τ_2 from its global minimum value. In other words, for small enough values of ρ , there exists a valley of the potential very much aligned with the direction of τ_2 and therefore one can assume that moving along that valley all the fields except τ_2 would stay in their global minimum.

Assuming that this is the case, one can then proceed to approximate the potential along the inflaton direction τ_2 as,

$$V_{LARGE} = \frac{BW_0^2}{\mathcal{V}^3} - \frac{4W_0 a_2 A_2 \tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2}, \quad (3.3)$$

where B includes several terms from eq. (3.1) that depend on the parameters of the potential as well as on the values of the other fields at their minimum. Also note that the axions θ_i have been set to their minimum, for which $\cos(a_i\theta_i) = -1$. This is needed in order for a minimum for all the fields τ_i at finite values to exist. Otherwise one would have a runaway behavior for some of them.

We can now obtain the values of the slow roll parameters for this potential at large values of τ_2 by using their conventional definitions in the single field inflation models, namely (we work in Planck units $M_P = 1$),

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = \left(\frac{V''}{V} \right), \quad (3.4)$$

where the primes denote derivatives with respect to the canonically normalised field ψ , defined by normalising the kinetic term for the inflaton. In the single field inflation approximation we discuss here and to leading order in the volume we see that,

$$\psi = \sqrt{\frac{4\alpha\lambda_2}{3\mathcal{V}}}\tau_2^{3/4}, \tag{3.5}$$

which in turn means that the slow roll parameters are given by [33],

$$\epsilon = \frac{32\mathcal{V}^3}{3\alpha B^2\lambda_2 W_0^2} a_2^2 A_2^2 \sqrt{\tau_2} (1 - a_2\tau_2)^2 e^{-2a_2\tau_2}, \tag{3.6}$$

$$\eta = -\frac{4a_2 A_2 \mathcal{V}^2}{3\alpha\lambda_2\sqrt{\tau_2} B W_0} (1 - 9a_2\tau_2 + 4(a_2\tau_2)^2) e^{-a_2\tau_2}, \tag{3.7}$$

and in the limit of slow roll, the associated scalar spectral index and tensor to scalar ratio r are given by

$$n_s - 1 = 2\eta - 6\epsilon, \tag{3.8}$$

$$r \sim 12.4\epsilon. \tag{3.9}$$

The number of e-foldings can be computed within this approximate potential by,

$$N_e = \int_{\psi_{end}}^{\psi} \frac{V}{V'} d\psi \approx \frac{-3BW_0\alpha\lambda_2}{16a_2 A_2 \mathcal{V}^2} \int_{\tau_2^{end}}^{\tau_2} \frac{e^{a_2\tau_2}}{\sqrt{\tau_2}(1 - a_2\tau_2)} d\tau_2, \tag{3.10}$$

where τ_2^{end} is taken to be the point in field space where the slow roll conditions break down i.e. when $\epsilon = \eta = \mathcal{O}(1)$. It is clear from the expressions (3.6)–(3.10) that one can get small enough slow-roll parameters as well as a large number of e-folds, just by starting at large enough values of τ_2 so that $\mathcal{V}^2 e^{-a_2\tau_2} \ll 1$. Taking into account that we are in the slow roll regime, we can then calculate the amplitude of the adiabatic scalar perturbations using the expression,

$$P = \frac{1}{150\pi^2} \left(\frac{V}{\epsilon}\right) \simeq \frac{1}{150\pi^2} \left(\frac{3\alpha B^3 W_0^4 \lambda_2 e^{2a_2\tau_2}}{32\mathcal{V}^6 a_2^2 A_2^2 \sqrt{\tau_2} (1 - a_2\tau_2)^2}\right) \tag{3.11}$$

In [33], the authors proposed a ‘footprint’ for their model of Kahler inflation. Normalising the density perturbations to COBE and seeking N_e efoldings of inflation (typically between 50-60) they obtained the results

$$\eta \simeq -\frac{1}{N_e}, \quad \epsilon < 10^{-12}, \tag{3.12}$$

$$0.960 < n_s < 0.967, \quad 0 < |r| < 10^{-10}. \tag{3.13}$$

Such a small value for ϵ at horizon exit implies that the inflationary energy scale is of order $V_{inf} \sim 10^{13}$ GeV, which in turn implies that tensor modes would be unobservable. A final point that they make is that for the model to work, the internal volume \mathcal{V} is found numerically to live within a range of values

$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6, \tag{3.14}$$

where $l_s = (2\pi)\sqrt{\alpha'}$. It is remarkable how narrow the range of n_s is in eq. (3.13) and how relatively restrictive the range of allowed volumes are eq. (3.14). One of the goals of this work will be to see whether these footprints really do define the model when we allow for the volume modulus and other moduli fields to evolve.

4 Full Kahler moduli Inflation

The discussion in the previous section suggests that inflation may be naturally realized in a large subset of string compactifications. This is an interesting claim so we would like to carefully study the validity of the approximations made as well as compare the observable quantities estimated earlier, such as the number of e-folds, the validity of the assumption $\rho \ll 1$, the constancy of the volume modulus and the scalar spectral index, with the more accurate results obtained by numerical integration of the full equations of motion using the full potential in (2.8) instead of the approximate large volume one in (3.3). Our approach differs in detail from that adopted by Bond et al. [34] in that we will be allowing for a number of the moduli fields to vary, including the volume modulus. This will allow us to fully explore the validity of the assumption that the volume remains effectively constant during inflation. In their approach, the volume modulus was kept constant and an analysis of the region of parameter space which led to inflation was based upon that assumption.

4.1 Numerical evolution

The equations of motion for our moduli fields can be obtained by varying the minimal $N = 1$, $d = 4$ effective SUGRA action of the form (in Planck units),

$$S = - \int d^4x \sqrt{-g} \left(\frac{1}{2}R + \mathcal{K}_{i\bar{j}} \partial_\mu T^i \partial^\mu \bar{T}^{\bar{j}} + V(T^m, \bar{T}^{\bar{m}}) \right) \quad (4.1)$$

where $\mathcal{K}_{i\bar{j}}$ is the Kahler metric, T^i and $\bar{T}^{\bar{j}}$ are the complex chiral fields. Considering a spatially flat FRW spacetime we get,

$$\begin{aligned} \ddot{T}^l + 3H\dot{T}^l + \Gamma_{ij}^l \dot{T}^i \dot{T}^j + \mathcal{K}^{l\bar{k}} \partial_{\bar{k}} V &= 0 \\ 3H^2 &= \left(\mathcal{K}_{i\bar{j}} \dot{T}^i \dot{\bar{T}}^{\bar{j}} + V \right) \end{aligned} \quad (4.2)$$

where we have used the definition of the connections of the Kahler metric $\Gamma_{ij}^l = \mathcal{K}^{l\bar{k}} \frac{\partial \mathcal{K}_{i\bar{k}}}{\partial T^j}$. Armed with the full equations of motion we can now explore numerically the evolution of all the fields and find out what regions of moduli space are suitable for inflation.

4.2 Example 1

Following [33] we first analyze the case where the parameters are such that $\mathcal{V} \gg 1$ and $\rho \ll 1$ and we only displace the inflaton (τ_2) from its global minimum value. Our numerical integration confirms the predictions of the previous analytic arguments. We observe that all the other fields remain nearly constant during the whole evolution while τ_2 slowly rolls down to its minimum, essentially reproducing the single field scenario discussed earlier.

An example with these properties can be obtained by taking the following set of parameters,

$$\begin{aligned} \xi = 24, \quad \alpha = 1, \quad \lambda_2 = \frac{1}{100}, \quad \lambda_3 = 1, \quad a_2 = 20\pi, \quad a_3 = \frac{\pi}{2} \\ A_2 = \frac{1}{300}, \quad A_3 = \frac{1}{300}, \quad \beta = 1.984002914 \times 10^{-6}, \quad W_0 = 2. \end{aligned} \quad (4.3)$$

We have chosen a viable example of this scenario with the minimal number of fields possible, which is three. We first obtain the global minimum of the potential, i.e. the minimum at zero cosmological constant,¹ finding it to be at,

$$\begin{aligned} \tau_1^f = 35189.343156992, \quad \tau_2^f = 0.302053449, \quad \tau_3^f = 5.886085128, \\ \mathcal{V}^f = 6.601 \times 10^6. \end{aligned} \quad (4.4)$$

We see that indeed this is a large volume compactification scenario so we should be well within the regime of applicability of the approximations that we indicated in the previous section. On the other hand, we have chosen these parameters to have

$$\rho \approx 10^{-5}, \quad (4.5)$$

so we expect that the value of the volume at the minimum should remain pretty much unaffected by the displacement of τ_2 . We choose the initial value of the inflaton to be $\tau_2^i = 0.8510534498$ and find numerically the new values of the local minima in the τ_1 and τ_3 directions for this case to be,

$$\tau_1^i = 35244.7673818281, \quad \tau_3^i = 5.887497350, \quad \mathcal{V}^i = 6.616 \times 10^6. \quad (4.6)$$

Comparing these values to ones obtained in the global minimum one can clearly see that the displacement of τ_2 does not have a big impact on the position of the local minima for the other fields in agreement with the analytic arguments given above.

We have chosen this particular value of τ_2^i to illustrate that it is straightforward to obtain sixty e-folds of inflation with this set of values. Similarly we have normalized the parameters in the potential namely, A_2 , A_3 and W_0 in (4.3) so that we obtain the correct magnitude of the perturbations for this particular solution.

We can now compute the observational signatures of this model within the analytic approximations described above. Using the expressions given in (3.3), (3.6), (3.7), (3.10) and (3.11) we find,²

$$\begin{aligned} N_e \simeq 61, \quad V_{inf} = 10^{13} \text{ GeV}, \quad P = 4 \times 10^{-10}, \\ \epsilon = 4 \times 10^{-17}, \quad \eta = -0.0165, \quad n_s = 0.967. \end{aligned} \quad (4.7)$$

Having identified a particular set of parameters that leads to a successful inflationary scenario within the approximations described in [33] we would now like to numerically

¹In all the examples in this paper we have adjusted β so that the overall minimum of the potential is located at vanishing value of the $4d$ cosmological constant. We tried to be as precise as possible for its value so the model can be explicitly reproduced by other people.

²Note that for this set of parameters one should take $B \approx 0.002$ and $\tau_2 \approx \tau_2^i$.

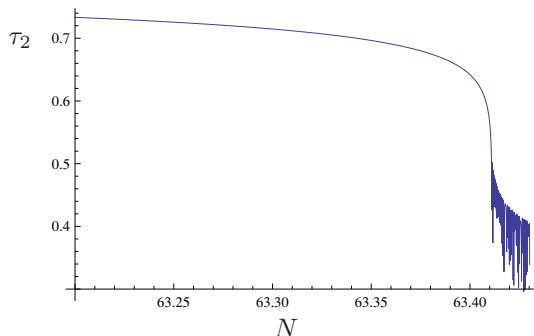


Figure 1. Evolution of the τ_2 field in the last few e-folds in Example 1.

investigate this example in detail to confirm its analytic predictions. We have evolved the system of equations presented in (4.2) considering the complete potential (2.8), in other words, without using any of the approximations we discussed earlier. This way we check that the fields behave as we expect them to do all the way to their global minimum even in the region of the potential that is not well approximated by the analytic expressions given above. We show in figure 1 the last period of the numerical evolution for τ_2 that starts at $(\tau_1^i, \tau_2^i, \tau_3^i)$. We only show the τ_2 trajectory since both τ_1 and τ_3 stay constant throughout the whole evolution until the last moment. This confirms that for these set of parameters we can regard the evolution as effectively a one dimensional problem.

Having found the solution numerically we can now calculate the amplitude of the adiabatic scalar density perturbations directly from the solutions by computing,

$$P(N) = \frac{1}{150\pi^2} \frac{V(N)}{\epsilon(N)}, \tag{4.8}$$

where N denotes the number of e-foldings along the numerical trajectory and $\epsilon(N)$ correspond to the slow-roll parameter which in terms of the Kahler metric and the potential takes the form

$$\epsilon(N) = \frac{\mathcal{K}^{i\bar{j}} \nabla_i V \nabla_{\bar{j}} V}{V^2}. \tag{4.9}$$

We can also extract the spectral index n_s from the expression of the form,

$$n_s = 1 + \frac{d \log P(N)}{dN}. \tag{4.10}$$

Putting all these expressions together we obtain the following results numerically,

$$\begin{aligned} N_e &\simeq 63, & V_{inf} &= 10^{13} \text{ GeV}, & P &= 4 \times 10^{-10}, \\ \epsilon &= 4 \times 10^{-17}, & n_s &= 0.963. \end{aligned} \tag{4.11}$$

which is in very good agreement with current observational data, and also with the analytic prediction of Conlon and Quevedo [33] given in this case by eqs. (4.7).

4.3 Example 2

It is interesting to note that we can still find a successful scenario for inflation within these type of models even when some of the approximations used in the previous analytic

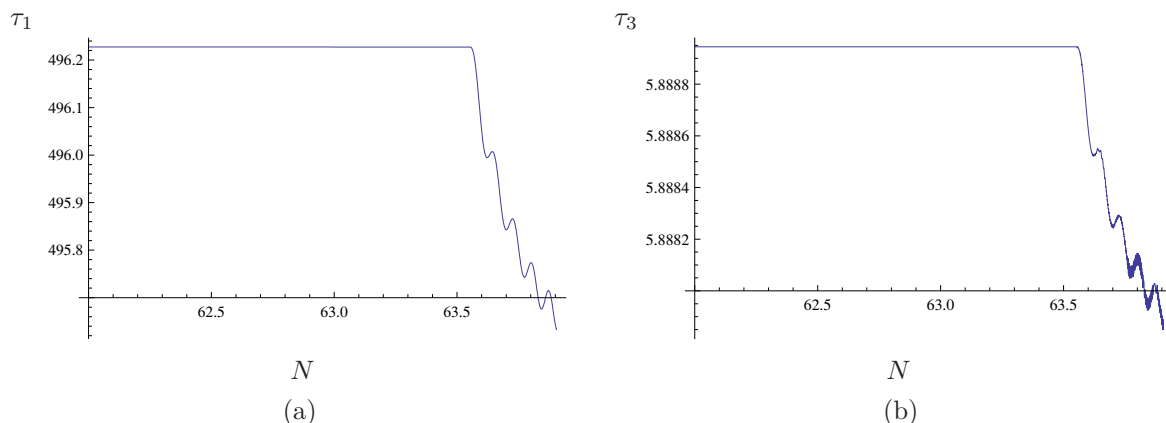


Figure 2. Evolution of the different moduli fields in the last few e-folds in Example 2. a) Evolution of the field τ_1 . b) Evolution of the field τ_3 .

arguments break down for a particular set of parameters. Let us consider for example what happens when one relaxes the constraint of considering a very large volume. We can accomplish this by just considering a smaller value of the W_0 , namely the following parameters,³

$$\begin{aligned} \xi = 24, \quad \alpha = 1, \quad \lambda_2 = \frac{1}{100}, \quad \lambda_3 = 1, \quad a_2 = 20\pi, \quad a_3 = \frac{\pi}{2} \\ A_2 = \frac{1}{300}, \quad A_3 = \frac{1}{300}, \quad \beta = 3.29801836 \times 10^{-9}, \quad W_0 = \frac{1}{300} \end{aligned} \quad (4.12)$$

where we have also changed the value of β to be able to set the global minimum at zero cosmological constant. In this case the global minimum becomes,

$$\tau_1^f = 495.4469043856, \quad \tau_2^f = 0.302090805, \quad \tau_3^f = 5.8875322868, \quad \mathcal{V}^f = 11013.6 \quad (4.13)$$

which has a much smaller value of the volume than the one obtained in the previous example and which is not within the range given in (3.14). In fact, one can check that in this case some of the expressions for the large volume limit give a poor approximation for the real values, due to the fact that the volume is not sufficiently large. Nevertheless one can still displace τ_2 without disturbing the values of the other fields in their minima. In particular, one can show that fixing $\tau_2^i = 0.747090805$ one changes the values of the local minimum of the potential along the other directions to,

$$\tau_1^i = 496.227462068, \quad \tau_3^i = 5.888944614, \quad \mathcal{V}^i = 11039.2. \quad (4.14)$$

³Note that from the form of the potential in (3.1) one can easily check that the value of the volume at the minimum is at leading order proportional to W_0 , while the values of the rest of the fields are proportional to $\frac{W_0}{\mathcal{V}}$ (see for example [38–41]). This implies in particular that by rescaling the parameter W_0 then one rescales the value of the volume at the minimum while leaving the minimum values of the rest of the moduli invariant.

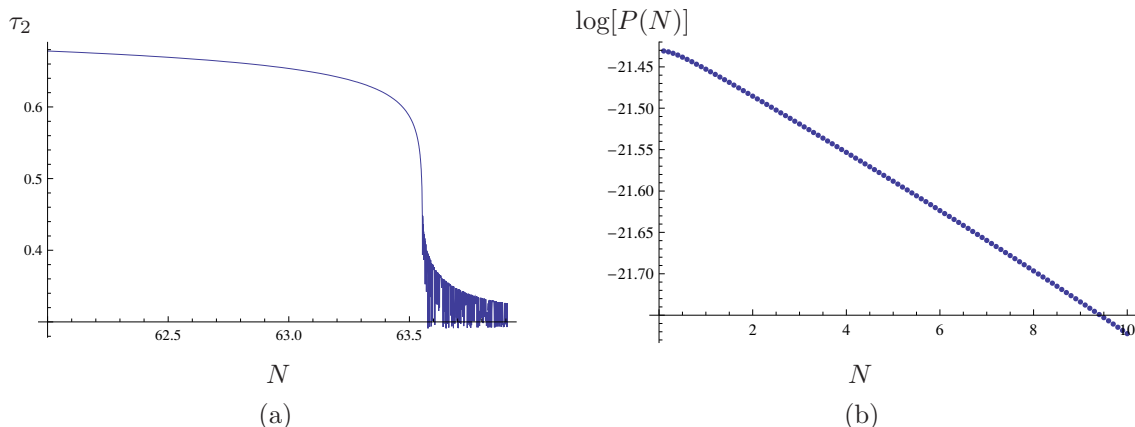


Figure 3. a) Evolution of the moduli field τ_2 (the inflaton) in the last few e-folds in Example 2. b) Amplitude of the density perturbations in the 10 observationally relevant e-foldings.

We notice that the change in the volume between these two points in field space is actually much less than 1%. In fact, even if we increase the value of τ_2^i considerably the situation will not really change, because the value of the volume or of the local minimum in which the fields τ_1^i, τ_3^i sit, remains relatively insensitive to the position of τ_2^i .

We show in figures 2–3 the last few e-folds of the numerical evolution that starts at $(\tau_1^i, \tau_2^i, \tau_3^i)$. We see that τ_1 and τ_3 stay constant through out the whole evolution until the last moment where their values drop abruptly to their global minimum values. So effectively our model is still a one dimensional inflationary model.

We can now use the expressions given above in eqs. (4.8) and (4.10) to get in this case,

$$n_s = 0.965 \tag{4.15}$$

where we have normalized the potential to obtain the correct magnitude of the perturbations within the cosmologically observable region. (See figure 3).

We conclude from this example that one can extend the region of the parameter space where a successful inflationary region can occur even when one can not use some of the large volume approximations presented in the previous section, but rather the expressions computed from the full potential. We will see in the following that this is also the case for some of the other assumptions made in [33].

4.4 Example 3

As we explained above, the value of the internal volume in the previous examples remains very much the same during inflation and it is almost exactly the same as the final value of the volume in the overall minimum of the potential. This seems to be a stronger requirement than necessary. In fact, we should only impose that the volume remains constant during the inflationary period but it is otherwise free to change substantially after that in its way to the global minimum. In the following, we will describe one such example where the

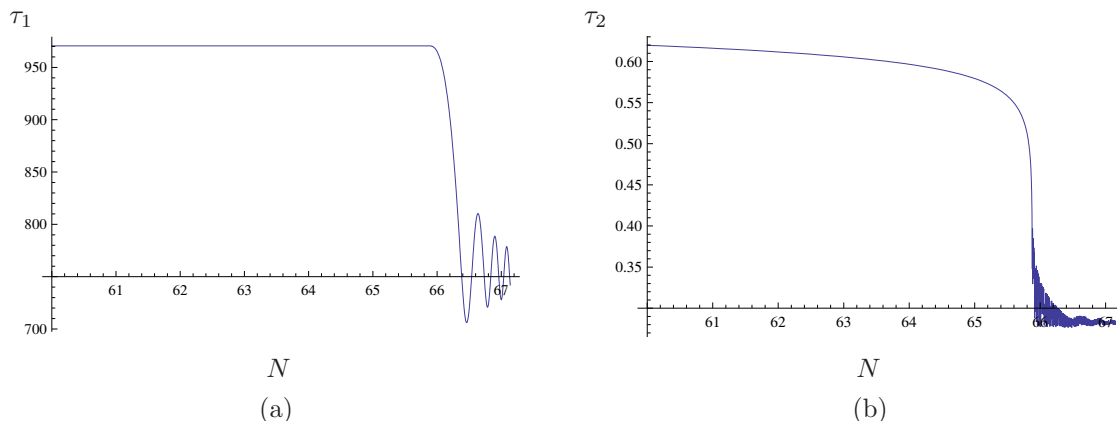


Figure 4. Evolution of the different moduli fields in the last few e-folds in Example 3. a) Evolution of the field τ_1 . b) Evolution of the field τ_2 .

volume varies by 45% from its value during inflation to the final value.⁴ To illustrate this point let us consider an example with the following values of the parameters

$$\begin{aligned} \xi = 24, \quad \alpha = 1, \quad \lambda_2 = 1, \quad \lambda_3 = 1, \quad a_2 = 20\pi, \quad a_3 = \frac{\pi}{2} \\ A_2 = \frac{3}{32}, \quad A_3 = \frac{1}{320}, \quad \beta = 6.213734280 \times 10^{-9}, \quad W_0 = \frac{1}{160} \end{aligned} \quad (4.16)$$

With these values, $\rho \sim 10^{-3}$, so we are again working in the regime considered in [33]. For this particular example, we can show that the global minimum of the potential is located at

$$\tau_1^f = 751.9457707162, \quad \tau_2^f = 0.2824390994, \quad \tau_3^f = 5.8472434856, \quad \mathcal{V}^f = 20605.289 \quad (4.17)$$

while by displacing the value of $\tau_2^i = 0.6624390994$ we see that the new minimum for the other fields is now found at,

$$\tau_1^i = 970.6098419930, \quad \tau_3^i = 6.0764936267, \quad \mathcal{V}^i = 30170.0176 \quad (4.18)$$

Once again we evolve the system of equations presented in (4.2) using the complete potential (2.8) and check what is the behaviour of the different fields. We plot in figure 4 and in part a) of figure 5 the results for evolution of the fields for this example. In part b) of figure 5 we show the evolution of the internal volume in the last few e-folds. We can clearly see there that the volume remains constant for the relevant period of inflation and only changes to its global minimum value within the last e-fold or so.

What can we conclude from this case compared to the previous examples? It is clear that although the set of parameters that we have used here represents a slightly different behaviour from the one described in [33], in particular the fact that the volume modulus can change quite considerably at the end of inflation, nevertheless, it still represents a

⁴There may be other regions of the parameter space where this change is in fact more drastic, however, we have restricted ourselves to this milder example for simplicity.

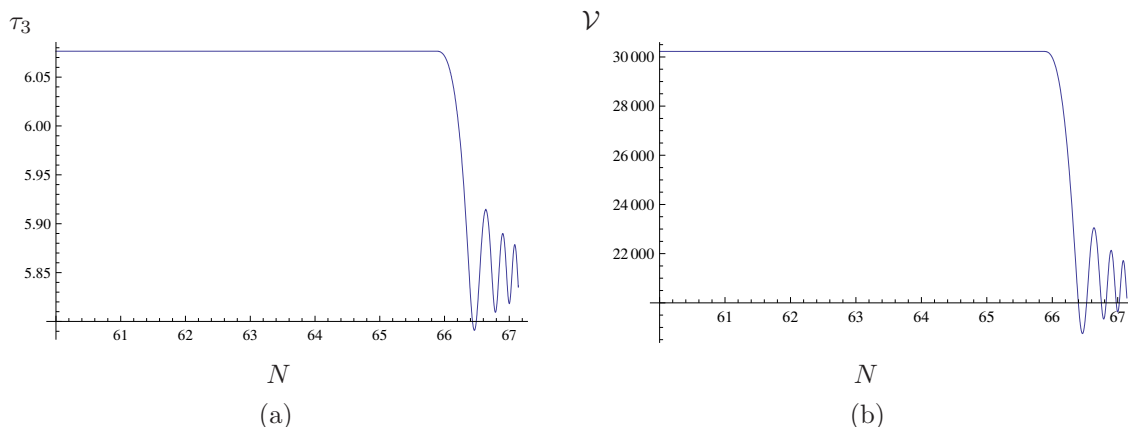


Figure 5. Evolution of the different moduli fields in the last few e-folds in Example 3. a) Evolution of the field τ_3 . b) Evolution of the field \mathcal{V} .

perfectly valid inflationary period regarding its observational signatures so once again this example increases the acceptable region of the parameter space within this kind of model. Actually we have again normalized the parameters in the potential so that we obtain the correct magnitude of the perturbations and therefore we can use the expressions (4.10) together with (4.8) to get in this case

$$n_s = 0.967, \tag{4.19}$$

once again perfectly consistent with the range predicted in [33].

We can therefore see from this example that the real condition in order for a successful period of inflation to take place is that the volume remains constant during inflation only, but not necessarily during the whole evolution of the fields.⁵

4.5 Example 4

As we have mentioned, the analytic estimates made in [33] for the spectral index n_s , are based on the assumption that during inflation $\rho \ll 1$. In this final example, we relax that condition, and address whether successful inflation still occurs in that situation (recall we are allowing all the fields to evolve). For this purpose let us consider the following values of the parameters,

$$\begin{aligned} \xi = \frac{1}{2}, \quad \alpha = \frac{1}{9\sqrt{2}}, \quad \lambda_2 = 10, \quad \lambda_3 = 1, \quad a_2 = \frac{2\pi}{30}, \quad a_3 = \frac{2\pi}{3}, \\ A_2 = \frac{1}{1.7 \times 10^6}, \quad A_3 = \frac{1}{425}, \quad \beta = 6.9468131457 \times 10^{-5}, \quad W_0 = \frac{40}{17}. \end{aligned} \tag{4.20}$$

which yields,

$$\rho \sim 0.99. \tag{4.21}$$

⁵This does not mean, of course, that there could not be successful models of inflation where the volume can change during the inflationary period but it is clear that in this case there should be another mechanism (or an additional fine-tuning) in play that makes it possible to avoid the η problem.

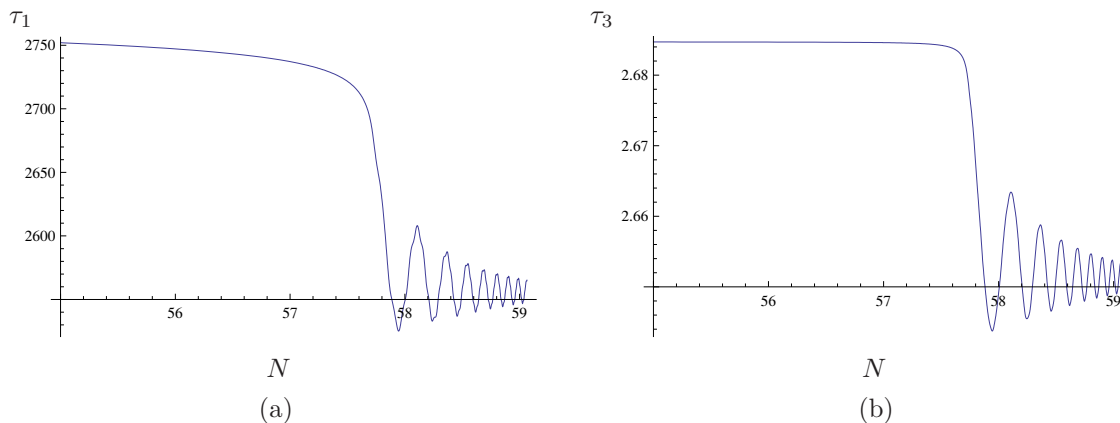


Figure 6. Evolution of the different moduli fields in the last few e-folds in Example 4. a) Evolution of the field τ_1 . b) Evolution of the field τ_3 .

The global minimum of the potential is now located at

$$\tau_1^f = 2555.95, \quad \tau_2^f = 4.7752, \quad \tau_3^f = 2.6512, \quad \mathcal{V}^f = 10143.94363. \quad (4.22)$$

Displacing the value of τ_2 to a substantially larger value, namely, $\tau_2^i = 78.7752067$ we see that the new minimum for the remaining fields is found at,

$$\tau_1^i = 2781.185086997, \quad \tau_3^i = 2.684717126, \quad \mathcal{V}^i = 10973.9. \quad (4.23)$$

As in the previous example we can now evolve again the system of equations presented in (4.2) using the complete potential (2.8) and check what is the behaviour of the different fields. We plot the results in figures 6 and 7. More concretely in figure 6 and in part a) of figure 7 the evolution of the fields τ_i and in part b) of figure 7 the evolution of the internal volume in the last few e-folds.

As before, we find a successful period of inflation with this new set of parameters, reproducing the correct amplitude of density perturbations and obtaining

$$n_s = 0.960, \quad (4.24)$$

which is very similar to the range predicted in [33].

The reason why this system of parameters works is that the initial value of τ_2 is large enough so that the exponential dependence of the potential with this field makes a negligible contribution to the calculation of the minima as a function of the other two degrees of freedom, namely, τ_3 and \mathcal{V} . This allows for the possibility of having an inflationary valley sitting at the minimum of the potential along those directions, even in cases where $\rho \sim 1$.

In summary, the examples shown above demonstrate the existence of a large region of parameter space within these models with inflationary solutions consistent with current cosmological observations even when one relaxes most of the constraints stated in [33].

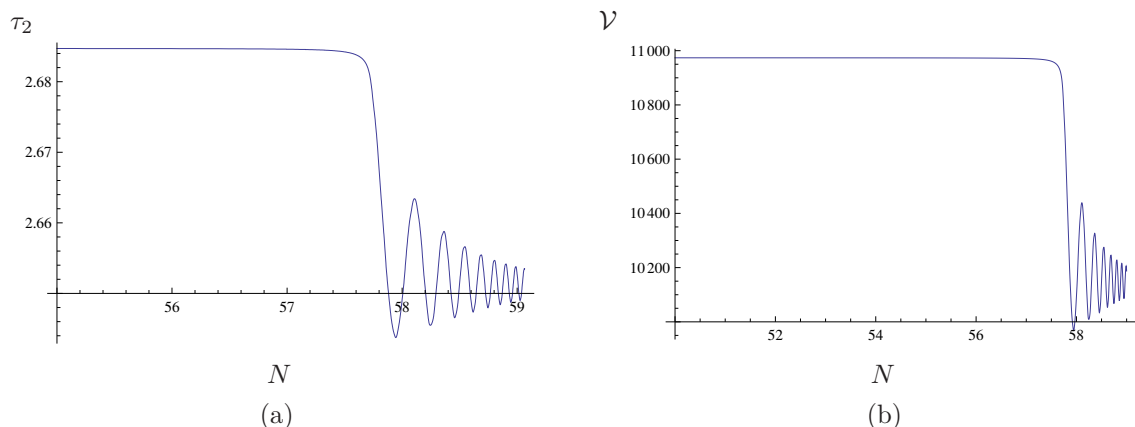


Figure 7. Evolution of the different moduli fields in the last few e-folds in Example 4. a) Evolution of the field τ_2 (the inflaton). b) Evolution of the field \mathcal{V} .

5 Basin of attraction

In the previous section, we have considered that the fields τ_1 and τ_3 were initially placed at the local minimum associated with the displacement of τ_2 from its global minimum, i.e. $\tau_1^i = \tau_1^{\text{local}}$ and $\tau_3^i = \tau_3^{\text{local}}$. In this section, we want to relax this assumption and verify whether the model allows for some freedom in the choice of initial conditions, namely we would like to see whether there is a region in the space of τ_1^i , τ_2^i and τ_3^i that leads to viable inflationary solutions as good as the ones presented above.

We note that the relative difference between τ_3^{local} and τ_3^f as given by eqs. (4.4), (4.6), (4.13), (4.14), (4.17), (4.18) and (4.22), (4.23), is at the most of only of a few percent. Hence, we can consider that for an initial condition in the vicinity of τ_3^{local} , τ_3 is nearly constant during inflation. This simplification allows us to illustrate the shape of the scalar potential during inflation, and in particular to show that there is a basin of attraction in the (τ_1, τ_2) plane. We show this plane in figures 8a and 8b as well as figure 9a and 9b corresponding to examples 1, 2, 3 and 4, respectively. The dashed line represents the direction of constant volume \mathcal{V} for fixed $\tau_3^i = \tau_{\text{local}}$. We also show in figures 8 and 9 the full numerical evolution of the fields with initial conditions slightly away from (4.6), (4.13), (4.18) and (4.23). This choice serves our purpose but initial conditions further away from the local minimum are also allowed and can in fact increase the number of e-folds of inflation as τ_2 can be displaced to higher values.

We see that the basin of attraction not only stabilises the evolution of the fields directing them towards the global minimum but also forces them to satisfy the essential condition $\mathcal{V} \approx \text{constant}$ which is established by the orientation of the basin itself in the (τ_1, τ_2) plane. The oscillations of the field trajectories occur as the fields bounce around the walls of the valley of the potential characterized by a constant value of the volume (\mathcal{V}) and therefore happen before the onset of inflation. When inflation terminates, the fields quickly evolve to the global minimum and the evolution departs from the trajectory $\mathcal{V} \approx \text{constant}$ represented in the figures.

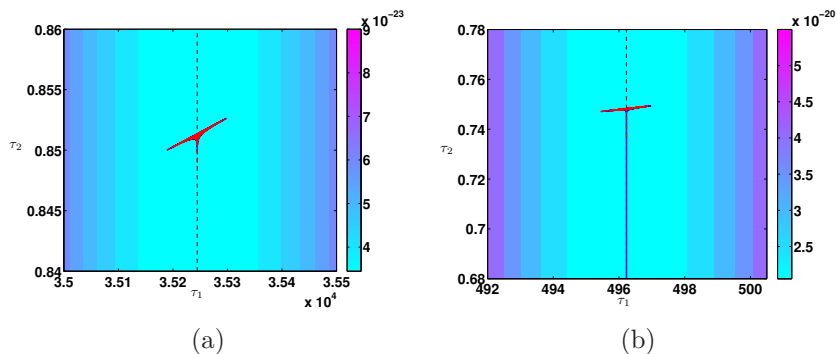


Figure 8. a) Contour plot of the scalar potential V in the (τ_1, τ_2) plane for fixed $\tau_3 = \tau_3^{\text{local}}$, for example 1. The dashed line shows the trajectory which maintains volume \mathcal{V} constant at this fixed τ_3 . b) Contour plot of the scalar potential V in the (τ_1, τ_2) plane for fixed $\tau_3 = \tau_3^{\text{local}}$, for example 2. The dashed line shows the trajectory which maintains volume \mathcal{V} constant at this fixed τ_3 . -Trajectories are $\theta_2^i = \theta_{2\text{min}}$ (blue solid line) and $\theta_2^i \neq \theta_{2\text{min}}$ (red dashed line).

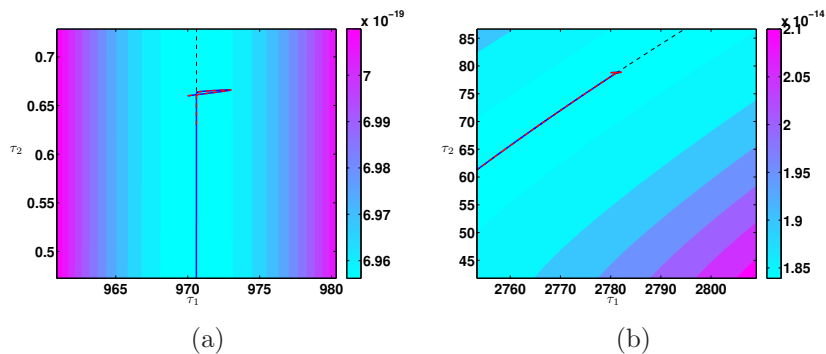


Figure 9. a) Contour plot of the scalar potential V in the (τ_1, τ_2) plane for fixed $\tau_3 = \tau_3^{\text{local}}$, for example 3. The dashed line shows the trajectory which maintains volume \mathcal{V} constant at this fixed τ_3 . b) Contour plot of the scalar potential V in the (τ_1, τ_2) plane for fixed $\tau_3 = \tau_3^{\text{local}}$, for example 4. The dashed line shows the trajectory which maintains volume \mathcal{V} constant at this fixed τ_3 . In this example, the inflationary trajectory is essentially given by a combination of two fields. Trajectories are $\theta_2^i = \theta_{2\text{min}}$ (blue solid line) and $\theta_2^i \neq \theta_{2\text{min}}$ (red dashed line).

Curiously, this variation of the internal manifold volume \mathcal{V} leads to the existence of two different scales for the gravitino mass. During inflation $\mathcal{V} \approx \mathcal{V}_i$ and once it ends, the fields fall to the global minimum where, at least in our examples, $\mathcal{V} = \mathcal{V}_f < \mathcal{V}_i$. Given that the gravitino mass is,

$$m_{3/2}^2 \sim e^K |W|^2 \sim W_0^2 / (\mathcal{V} + \xi/2)^2 \sim W_0^2 / \mathcal{V}^2, \tag{5.1}$$

we have $m_{3/2\text{inflation}}^2 \approx W_0^2 / \mathcal{V}_i^2$ and $m_{3/2f}^2 = W_0^2 / \mathcal{V}_f^2$, leading to a larger gravitino mass after inflation. For the sets of parameters in our examples, the gravitino masses during inflation are $m_{3/2} = 3 \times 10^{-7}$, 3×10^{-7} , 2×10^{-7} and 2×10^{-4} in Planck units, for examples 1, 2, 3 and 4, respectively. These scales are rather high and therefore not very

appealing phenomenologically. Actually this is typically the case in most of the inflationary models built from string theory. This follows from the fact that, as was argued in [48], the scale of inflation is generically bounded from above by the mass of the gravitino $H \lesssim m_{3/2}$. Therefore since these string inflationary models (in order to reproduce the correct amplitude for the density perturbations given by current observational data) predict a high scale of inflation, they also predict as well a high supersymmetry breaking scale. This is sometimes referred to as the gravitino mass problem. This feature is stronger in this class of inflationary models built from the α' -corrected Kahler potential [49, 50], where the scale of inflation that can be realised within these setups corresponds to $H \sim m_{3/2}/\mathcal{V}^{1/2}$ or $H \sim m_{3/2}^{3/2}$ using eq. (5.1), which will typically give rise to even higher supersymmetry breaking scales. Recall however that this mass corresponds to the gravitino mass during inflation, which does not have to be necessarily the same as the gravitino mass in the vacuum. This point has been used for example in [51–53] to propose a mechanism which can achieve low energy supersymmetry breaking scales, which consists in performing an extra fine-tuning in the models so that the gravitino mass during and at the end of inflation are substantially different. In this context it is interesting to note that the examples mentioned above also display a different gravitino mass during and after inflation. Unfortunately in our examples we have always found $m_{3/2}^i < m_{3/2}^f$ and therefore the gravitino mass at the vacuum is heavier than the one during inflation which is going in the wrong direction. The question is whether or not there are trajectories of the form described in the previous examples which can lead to the volume increasing immediately after inflation. There is no obvious reason why this can not happen but it remains a challenge to find an example.

5.1 Evolving the axions

We can also investigate what might happen if we allow the axions θ_i to evolve, as well as all the fields τ_i . As expected, if the initial conditions for the axions are such that they are placed at their minimal values the examples described above do not change, as the axions do not get displaced from their minimum. However the situation is modified when the initial conditions for the axions are such that they are perturbed from their minimal values. In such a situation two different scenarios emerge. In the case in which only the axion corresponding to the inflaton field (that is, the field θ_2) is perturbed, we see that the fields evolve in such a way as to reproduce the situation described in [34] (for the case $\rho \ll 1$). In particular, viable inflationary trajectories exist, but the new initial conditions allow for a greater variety of trajectories in which the rolling of the axion can increase the number of e-foldings over those trajectories restricted to lie only in the τ_2 direction. Such evolutions are the red dashed trajectories in figures 8, 9.

The second class of scenarios correspond to perturbing an axion which belongs to the same multiplet as the field which plays the dominant role in the stabilization of the volume, which in our examples would correspond to the field θ_3 . In this case one can show that the set of viable initial conditions for inflationary trajectories is restricted to small initial perturbations in the position of the axion ($\delta\theta_3 \ll 1$) away from its minimum. Any other significant perturbation of θ_3 leads to runaway non-inflationary trajectories.

The reason for these different behaviors can be understood as follows: the rolling of the axion field θ_2 does not have a noticeable impact on the position of the minimum of the volume modulus. On the other hand, a displacement of the axionic field θ_3 will instead have an effect on the position of the volume modulus, as can be easily read from (3.1). This means that almost any displacement on the field θ_3 will have the effect of displacing the volume modulus from its minimum and as a consequence of that the fields would tend to roll towards the decompactification limit.

In the previous sections, where the axion fields were frozen, we have computed the scalar power spectrum of adiabatic perturbations under the assumption that isocurvature perturbations do not feed into the curvature modes. This is a reasonable assumption provided that only one of the fields is light and that the remaining are heavy, in the sense that their effective mass is $m_{\text{eff}}^2 \gg H^2$, which is the case in the examples considered here. The axion fields, however, are light and their evolution can in fact lead to a large contribution to the curvature perturbations amplitude as was explicitly shown in ref. [54, 55] in considering the Roulette Inflation model of Bond et al. [34]. It would be interesting to see their influence in the full model we have been discussing here.

6 Conclusions

Realising inflation in the context of string theory has provided a number of difficulties from the onset. The former relies on scalar fields slowly evolving in an almost flat potential, whereas the natural scales for parameters in string theory tend to have potentials which are too steep to sustain an extended period of inflation. Moreover, the plethora of moduli fields arising in these models makes it difficult to have just one field evolving (the inflaton) whilst the others remain fixed in their minima. Therefore, when a model is proposed which appears to successfully reconcile these two important disciplines it deserves attention. The model proposed by Conlon and Quevedo [33] is one such example and has been the focus of this work.

We have performed a detailed numerical analysis of inflationary solutions in the Kahler moduli sector of the Large Volume Models built in the context of type IIB flux compactifications. Our investigations confirmed the key result of [33], namely that there are inflationary solutions where all but one of the moduli fields, (the inflaton), are stabilised to the local minima of the potential. We have provided explicit examples of these trajectories, and shown how the corresponding tilt of the density perturbations power spectrum leads to a robust prediction of $n_s \approx 0.96$ for 60 e-folds of inflation, in agreement with the analytic prediction. However, we have gone further and showed that even when all the moduli fields play an important role in the overall shape of the scalar potential, inflationary trajectories still exist. In particular, we have demonstrated that there exists a direction of attraction for the inflationary trajectories that correspond to the constant volume direction. It leads to a basin of attraction which enables the system to have an island of stability in the set of initial conditions leading to inflation.

Furthermore we were able to show, using the numerical evolution of the fields under the influence of the full potential, that there are still successful inflationary trajectories even

when one relaxes most of the assumptions made in the analytical approximations of [33]. This is an interesting point that makes the conclusions from these type of models much more robust. In particular, we considered a case in which the constraint of considering a very large volume is relaxed, a case where the volume varies by 45% when inflation terminates, and a case where $\rho \approx 1$.

Having looked at the evolution of the moduli fields with the axion fields restricted to be in their minima, we then extended the analysis to allow also the axions to be slightly displaced from their equilibrium position. Whereas a variation of the axion θ_2 (the partner of the inflaton) still led to a large basin of attraction for the inflationary trajectories (as in [34]), in the case of the axion θ_3 (the one that lives in the same multiplet as the moduli field responsible for stabilising the volume modulus), a small fluctuation of θ_3 from its true minimum value is enough to create runaway solutions where all the fields roll towards the decompactification limit. Hence we have a new restriction on these class of models, θ_3 needs to be very close to its minimum value for inflation to take place.

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