

Observations on fluxes near anti-branes

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ABSTRACT: We revisit necessary conditions for gluing local (anti-)D3 throats into flux throats with opposite charge. These consistency conditions typically reveal singularities in the 3-form fluxes whose meaning is being debated. In this note we prove, under well-motivated assumptions, that unphysical singularities can potentially be avoided when the anti-branes polarise into spherical NS5 branes, with a specific radius. If a consistent solution can then indeed be found, our analysis seems to suggest a rather large correction to the radius of the polarization sphere compared to the probe result. We furthermore comment on the gluing conditions at finite temperature and point out that one specific assumption of a recent no-go theorem can be broken if anti-branes are indeed to polarise into spherical NS5 branes at zero temperature.

KEYWORDS: Flux compactifications, Supersymmetry Breaking, dS vacua in string theory, D-branes

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1 Introduction

Supersymmetric throat geometries supported by fluxes are stable string theory solutions with important applications for holography, flux compactifications and black holes. One explicit method to break the supersymmetry in the throat while preserving classical stability is adding branes to the flux background that carry charges opposite to the charge dissolved in flux [1–5], from here on referred to as “anti-branes”. We focus on anti-D3 branes in the Klebanov-Strassler (KS) throat as first studied in [2]. If the anti-brane charge p is small compared to the background RR-flux M , then their addition can be seen as a small perturbation of the KS throat. At the same time, the limit of small anti-brane charge is what guarantees metastability, at least at probe level [2]. The decay channel is the annihilation of the p anti-D3 branes with some of the background NSNS flux. If the corrections to the probe result come with positive powers of p/M it guarantees the self-consistency of the approach as long as $p/M \ll 1$.

If the above reasoning is to hold, then the results of [2] suggest that the supergravity solution describing the metastable state should be spherical NS5 branes wrapping a contractible two-cycle of finite size. These spherical NS5 branes carry no monopole NS5 brane

charge but the original p anti-D3 charges instead. This supergravity picture should hold in the following limit:

$$g_s \ll 1, \quad g_s p \gg 1, \quad g_s M \gg 1, \quad (1.1)$$

and at the same time $p/M \ll 1$ as required for stability. The above limit ensures that string loop corrections can be ignored and that the length scales of the tip of the KS throat and of the contractible 2-cycle are large in string units such that higher derivate corrections are also suppressed. Recently interesting arguments have been presented that for $p = 1$ the metastability is not threatened by backreaction [6, 7]. A single anti-brane is however beyond the scope of this paper since it cannot be understood within the 10D supergravity limit. The complementary regime of p/M of order unity or larger cannot be regarded as a perturbation to the KS throat and the would-be supergravity solution in this case can better be thought of as being $\text{AdS}_5 \times \text{S}^5$ perturbed by M units of three-form flux [8]. We do not go into that limit in this work since it does not correspond to meta-stable supersymmetry breaking by perturbatively small amounts as originally intended in [2].

The first attempts for constructing the supergravity solutions (in certain approximate schemes) revealed singularities which were claimed to be different from the usual singularities associated to brane sources [9, 10]. The singularity is such that the scalar $e^{-\phi}|H_3|^2$, which gives the contribution of the H_3 flux to the energy-momentum tensor in Einstein frame, diverges near the sources. Although it is tempting to interpret this divergence as the self-energy of the NS5 brane, it was claimed this is not the correct interpretation, because the orientation of the fluxes and the magnitude of the divergence was not right for NS5 branes. One might worry that this mismatch is inherent to the approximations of the original papers [9, 10], but in fact the singularity was shown to persist beyond those approximations [11, 12]. Nonetheless, that interpretation of the singularity as unphysical is incomplete since the computations of [11, 12] assume genuine anti-D3 branes instead of spherical NS5 branes carrying anti-D3 charges. One could therefore speculate that when one looks instead for supergravity solutions describing spherical NS5 branes at the tip of the throat, one finds acceptable singularities. This is the first point we investigate in this paper and we find that, under well motivated assumptions, certain fields can be tuned near the horizon such that the singularity corresponds to the usual divergent self-energy of the NS5 brane. There is no guarantee that this tuning is possible in the sense that the UV can be glued consistently to the IR, but at least we find that all earlier no-go theorems against that gluing can be circumvented.

Second we investigate the configuration at finite temperature [13]. The temperature acts as an IR cut-off in field theory. If there is a mechanism in string theory that can resolve the singularity or turn it into a physical divergence (such as with brane polarisation), one expects the singularity to be shielded at finite temperature. Numerical evidence has shown that this hope was in vain [14, 15] at least for anti-D3 branes that are smeared over the finite tip at the bottom of the throat or for localised anti-D6 branes [16]. In addition, a no-go theorem was found for localised anti-D p branes with $p \leq 6$ [12]: the would-be supergravity solution will develop a divergent flux density $e^{-\phi}|H_3|^2$ at the horizon. However, as all no-go results, the theorem is only as strong as its assumptions. In this paper we argue that NS5

polarization is a priori consistent with relaxing one assumption in [12] on the boundary conditions. If finite T solutions exist, our results should then be a useful lead on how to pick boundary conditions near the brane sources. In fact, some progress on the existence of smooth finite T solutions was reported in [17] and we verify that indeed this boundary condition was used. Still the construction of [17] misses a compact ‘A’-cycle, which is crucial for the physics of anti-brane metastable states. Without such a compact A-cycle the construction of smooth finite temperature solutions was also argued earlier in [18].

We start the remainder of this note with a review of the main results of [12]. Then we analyse the H_3 flux density. At zero temperature, we review the singular flux for anti-D3 brane boundary conditions in the IR region of the flux throat and extend the analysis to spherical NS5-branes carrying anti-D3 charge. We show the flux energy density is again singular, but it is possible to obtain the appropriate power of divergence expected for a local NS5-source. This turns out to be only possible at a NS5 radius which scales as $R \sim \sqrt{p/M}$ for small p/M , which differs from the probe prediction $R \sim p/M$. We then heat up the background, and elaborate on a caveat in the arguments of [12] and discuss under which conditions the singularity can be cloaked by a smooth horizon. Finally, we discuss and interpret our results.

2 Gluing IR to UV at $T = 0$

We recall the formalism of [12]. This formalism extends the results in [11] and relates the boundary conditions near the anti-brane in the IR to the generalized ADM mass, which is measured in the UV. These consistency relations are necessary (but not sufficient) for the existence of well-behaved interpolating solutions. They have proven useful to demonstrate the absence of solutions in many non-trivial circumstances, which are otherwise only amenable to heavy numerical techniques.

We focus the discussion on throat geometries supported by 3-form fluxes in which anti-D3 branes are added at the tip. The example to have in mind is the Klebanov-Strassler throat [19]. The would-be solution takes the following form in 10D Einstein frame:

$$\begin{aligned}
 ds_{10}^2 &= \mathcal{E}^{2A} \left(-e^{2f} dt^2 + \delta_{ij} dx^i dx^j \right) + ds_6^2, \\
 C_4 &= \tilde{\star}_4 \alpha, \\
 H_3 &= -\mathcal{E}^{\phi-4A-f} \star_6 ((\alpha + \alpha_0)F_3 + X_3).
 \end{aligned}
 \tag{2.1}$$

The functions ϕ, A, f, α only depend on the internal coordinates and α_0 is a constant. The horizon is located at $e^{2f} = 0$. At this point the throat metric ds_6^2 is completely general. We use notation with tildes for metrics and Hodge duals without any warp factor e^{2A} nor e^{2f} . For instance $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$ is the Minkowski metric.

We require the throat geometries to have a 3-dimensional subspace non-trivial in homology, which we call the A-cycle, and F_3 to have a fixed quantised flux $\int_A F_3 = 4\pi^2 M$. This piece of information is crucial to describe the backreaction of anti-branes in the Klebanov-Strassler background. The A-cycle is present in the UV limit of the supergravity solution [9] and was essential in proving the presence of the original non-perturbative

brane-flux instabilities [2] and the singularity [12]. For a relatively low number of anti-branes p such that $p/M \ll 1$ we expect the topology to remain the same and the A-cycle to be present.

The 3-form fluxes H_3 and F_3 are closed and take values inside the 6-dimensional throat geometry. Then H_3 is the most general form that solves the B_2 equation of motion

$$d(\mathcal{E}^{-\phi} \star_{10} H_3) = -F_5 \wedge F_3, \tag{2.2}$$

provided that X_3 is closed as well. The ansatz for the H_3 flux (2.1) seems to have a redundancy, as any shift in X_3 along F_3 shifts the constant α_0 . We fix this redundancy by demanding that $\int_A X_3 = 0$. We will furthermore fix the gauge for C_4 (and hence α) such that $\alpha_0 = 0$. This is the gauge used in [9, 20], whose results we use below.

The key observation of [12], which we repeat in the appendix, is that the following 9-form

$$\mathcal{B} = -C_4 \wedge F_5 - \tilde{\star}_4 1 \wedge B_2 \wedge X_3 + \star_{10} d(\phi - 4A - f) \tag{2.3}$$

integrates over a spacetime volume \mathcal{M} to

$$\frac{1}{\tilde{v}_4} \oint_{\partial\mathcal{M}} \mathcal{B} = M_{\text{ADM}}, \tag{2.4}$$

if the closed surface $\partial\mathcal{M}$ encapsulates the anti-brane sources; otherwise the integral vanishes. In this expression, \tilde{v}_4 is the four-dimensional volume. This is very much like Gauss' law in electrodynamics with the role of the electric charge played by the generalized ADM mass. When supersymmetry is broken by p anti-branes, M_{ADM} is proportional to p and positive [12]. This condition can then be used to constrain the behavior of the supergravity fields near the source by letting the closed surface approach the anti-brane horizon. We want to stress that this integral has only been shown to reproduce the ADM mass for the perturbative solution of [9]. We come back to more general asymptotics allowing $p/M \gg 1$ in section 5.3.

The above formalism needs to be altered in order to apply it to spherical NS5 branes. The reason is that we intend to integrate the 9-form (2.3) on a surface $\partial\mathcal{M}_{\text{IR}}$ infinitesimally close to the spherical NS5 horizon. But along this surface B_2 is not everywhere defined since an NS5 acts as a magnetic source for B_2 . This would imply that we need to compute the contribution from a surface of Dirac “strings” stretching from one side of the spherical NS5 to the opposite side, as depicted in figure 1.¹

Luckily this complication can be avoided by using B_6 as the fundamental potential instead of B_2 . Going through an analogous computation as in [12], but using B_6 , one finds:

$$\mathcal{B} = -C_4 \wedge F_5 - B_6 \wedge H_3 + \star_{10} d(\phi - 4A - f), \tag{2.5}$$

where we made use of

$$\mathcal{E}^{-\phi} \star H_3 \equiv dB_6 - C_4 \wedge F_3, \tag{2.6}$$

¹In a first version of this paper we missed that contribution and we are grateful to J. Polchinski for pointing that out. See also comments in [21].

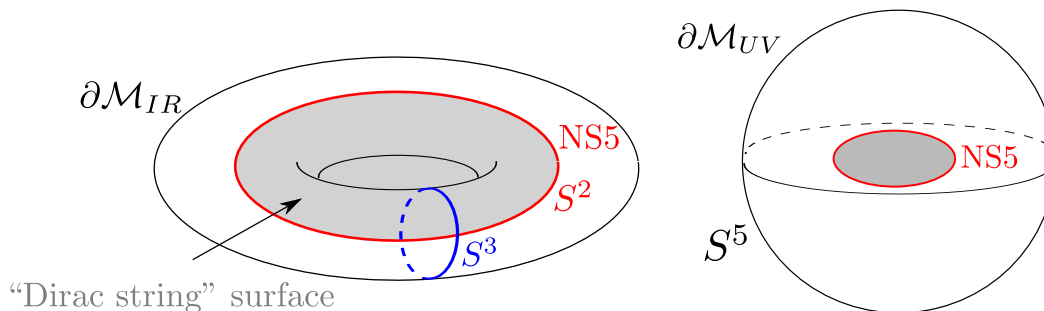


Figure 1. Boundaries $\partial\mathcal{M}$, with 4d Minkowski directions suppressed. Left: the IR surface that encapsulates the NS5 corresponds to an $S^3 \times S^2$ close to the brane. The spherical NS5 is drawn symbolically as a red circle. We can choose a patch such that the B_2 gauge field is well-defined everywhere except in they gray surface stretching from one side of the NS5-brane to another. Right: the UV boundary can be taken to be S^5 .

such that

$$dB_6 = -\tilde{\star}_4 1 \wedge X_3. \tag{2.7}$$

Note that we ignore a possible closed but not exact piece in X_3 . Such a harmonic piece would give a non-zero bulk contribution $\int_{\mathcal{M}} H_3 \wedge \tilde{\star}_4 1 \wedge X_3$ to the ADM mass (see appendix A.1 for more information). This harmonic contribution to the ADM mass would be similar to the mechanism to smooth support black hole microstate geometries [22]. We will explore this connection in more detail in a forthcoming publication.

3 Flux divergences

We focus on the density of the NS-NS 3-form flux as it appears in the energy-momentum tensor. From the ansatz (2.1) we find

$$\mathcal{E}^{-\phi} |H_3|^2 = e^{\phi-8A-2f} |\alpha F_3 + X_3|^2. \tag{3.1}$$

It is the aim of this paper to infer whether this quantity stays finite near the source or not, by using consistency relations for gluing the IR solution to the UV solution.

3.1 Anti-D3 boundary condition

Consider eq. (3.1) and take zero temperature ($e^{2f} = 1$). Close to the anti-D3 brane, we can use the near-D3 solution of the appendix, eq. (A.16). The factor e^{-8A} is expected to blow up whereas ϕ remains finite. The metric transverse to the brane scales as $ds_6^2 = e^{-2A} \tilde{ds}_6^2$, such that

$$\mathcal{E}^{-\phi} |H_3|^2 \sim e^{-2A} |\alpha F_3 + X_3|_{\tilde{ds}_6^2}^2, \tag{3.2}$$

where the subscript indicates we take the contraction without warp factors using \tilde{ds}_6^2 . Hence unless the combination $\alpha F_3 + X_3$ vanishes at the position of the brane, we find a singular H_3 density scaling. This is the infamous 3-form singularity encountered in many places in the literature.

One can argue that the combination $\alpha F_3 + X_3$ indeed does not vanish at the anti-D3-brane position using eq. (2.4). As we are dealing with anti-D3 branes, we are free to use the (2.3) boundary term, since we do not expect a contribution from Dirac strings. For the standard (anti-)D3 boundary condition, the last term (whole bracket) in (2.3) does not contribute. Hence X_3 and α cannot both vanish since $M_{\text{ADM}} > 0$. As F_3 has a non-vanishing component along the A-cycle, the combination $\alpha F_3 + X_3$ is nonzero by construction and hence introduces a three-form singularity. Note that for a D3-brane boundary condition with $M_{\text{ADM}} = 0$, this issue is not present.

A possible way out would be that F_3 and X_3 vanish at the position of the anti-D3 brane, while α is non-zero (see the discussion in [11]). This is a priori not impossible since only the integral along the A-cycle of F_3 equals $4\pi^2 M$ and the flux could be distributed inhomogeneously along that cycle. If one tracks down what is needed for a regular H -flux density, one finds that the charge density in the fluxes has to scale to zero near the source as

$$H_3 \wedge F_3 \sim e^{4A} \star_6 1. \tag{3.3}$$

We consider this to be an unphysical boundary condition, since the anti-brane attracts both gravitationally and electromagnetically the charges dissolved in flux; therefore one expects that the maximal value for $H_3 \wedge F_3$ is reached near the source, instead of going to zero. We leave it for further research to find a full mathematical proof of this.

3.2 NS5 boundary condition

In the probe limit, the anti-D3 brane is unstable towards puffing up into an NS5-brane that wraps a contractible 2-cycle inside the A-cycle [2]. This NS5 brane carries no NS5 monopole charge, but rather anti-D3 charge through flux on its worldvolume. Since the probe NS5-branes can sit at a classically stable position, one expects a consistent supergravity solution with an NS5 brane boundary condition in the IR. We argue now that this possibility can indeed not be excluded from the gluing conditions.

With NS5 boundary conditions at the brane position, the four-dimensional transverse metric scales as e^{-6A} , while e^ϕ scales as e^{-4A} . Three non-trivial conditions have to be met in order for the H_3 density (3.1) to be consistent with an NS5 brane (the interested reader can corroborate them using the expressions in appendix A.2):

1. Near the NS5 source, the H_3 density should be singular of a specific degree: $e^{-\phi}|H_3|^2 \sim \mathcal{E}^{-2A}$.
2. Near the source, the dominating legs of H_3 are perpendicular to the NS5 worldvolume.
3. $\int_{\partial\mathcal{M}} \mathcal{B}$ has to be finite and positive by (2.4), with \mathcal{M} a spacetime volume encapsulating the source.

The ansatz (2.1) and the second requirement imply that $\alpha F_3 + X_3$ must have two legs on the NS5 worldvolume and one transverse leg. Together with the local metric scaling of an NS5 brane this implies

$$\mathcal{E}^{-\phi}|H_3|^2 = \mathcal{E}^{-10A}|\alpha F_3 + X_3|_{\widetilde{\text{d}s}_6}^2. \tag{3.4}$$

Then condition 1 requires that $\alpha F_3 + X_3$ scales as \mathcal{E}^{4A} . F_3 indeed has two legs on the NS5 when it threads the A -cycle, but we assume it cannot scale to zero near the NS5 brane for exactly the same reasons mentioned around equation (3.3).

Hence either α and X_3 scale as \mathcal{E}^{4A} , or one does and the other one vanishes still more rapidly. We now argue that neither of these two possibilities can be excluded since sending α and X_3 to zero can still be consistent with a positive $1/\tilde{v}_4 \oint_{\partial\mathcal{M}} \mathcal{B}$ integral.

Consider (2.5) and its integral over the 9-surface in the IR along the NS5 horizon. Again one finds that the last term does not contribute by comparing with the known flat space solution. Interestingly the first two terms can be integrated explicitly, such that we find the equality

$$M_{\text{ADM}} = \text{Vol}_4 \left(\alpha_H Q_3 + b_H Q_5 \text{Vol}_2 \right), \tag{3.5}$$

where Q_3 is the *monopole* anti-D3 charge and Q_5 the *dipole* NS5 charge carried by the spherical NS5 brane defined as

$$Q_3 = \int_{S^5} F_5, \quad Q_5 = \int_{S^3} H_3, \tag{3.6}$$

for an S^5 surrounding the D3-branes and an S^3 linking the NS5-brane, as in figure 1.

The values of the gauge potentials near the horizon are denoted

$$C_4 = \alpha_H \tilde{\star}_4 1, \quad B_6 = b_H \tilde{\star}_4 1 \wedge \epsilon_2, \tag{3.7}$$

with ϵ_2 the volume-form of the 2-cycle wrapped by the NS5 brane, whose integrated value is denoted Vol_2 . The above value formula for the ADM mass computed near the horizon coincides exactly with what would have been found using the formalism of [11] (and [12]) that relates the ADM mass to the on-shell brane action $S_{\text{DBI}} + S_{\text{WZ}}$. Indeed, the above expression is the on-shell Wess-Zumino term for a spherical NS5 brane carrying anti-D3 charge. The DBI probe action always vanishes on-shell since the warp-factor vanishes near the horizon. This is a nice consistency check of our results. It is now clear that we can take α_H to vanish as long as b_H remains finite to account for the ADM mass. At the same time this is consistent with X_3 scaling down as \mathcal{E}^{4A} such that the expression for the flux divergence (3.4) is consistent with a spherical NS5 brane. The way X_3 scales down is not affected by the finite value of b_H (needed to account for the ADM mass) since X_3 relates to the derivatives of b_H but not b_H itself (2.7).

We can glean interesting quantitative information from our analysis concerning the NS5 position as well. Consider the 2-cycle wrapped by the NS5. Its volume scales as $g_s M$

$$\text{Vol}_2 = \int \epsilon_2 = g_s M R^2, \tag{3.8}$$

where R is the radius of the S^2 on the A -cycle wrapped by the NS5 brane, with the overall $g_s M$ scaling taken out and we ignored numerical prefactors.² Since the ADM mass is proportional to the number of anti-branes p [20, 23]

$$M_{\text{ADM}} = 2 \text{Vol}_4 \mathcal{E}^{4A_{\text{tip}}} p, \tag{3.9}$$

²This is a consequence of the fact that the S^3 tip of the KS cone has a radius set by $\sqrt{g_s M}$.

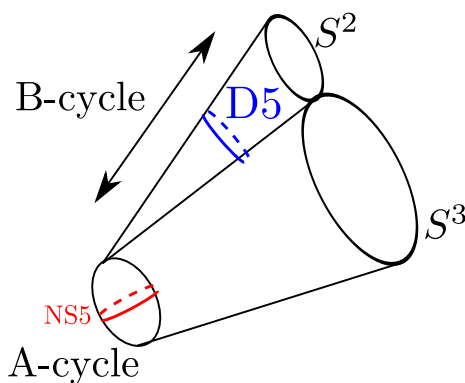


Figure 2. Pictorial representation of the Klebanov-Strassler geometry. There are a priori two polarization channels for anti-D3 branes: NS5 polarization on an S^2 inside the A-cycle, or D5 polarization on the S^2 of the B-cycle.

we find from combining the last two equations with (3.5) that

$$R^2 \sim \frac{p}{M}. \tag{3.10}$$

This differs from the probe level, where it was found that for small p/M [2]

$$R \sim \frac{p}{M}. \tag{3.11}$$

In the discussion we comment on the interpretation of this mismatch.

Finally a word on the assumption of having a spherical NS5 brane as an IR configuration. This is clearly motivated by the probe analysis of [2]. Nevertheless, it could be that the NS5 polarisation channel comes with partner five-brane polarisation channels when backreaction is taken into account, similar to the Polchinski-Strassler (PS) background [8]. Therefore, if the supergravity solution locally is a non-supersymmetric version of PS, there could be a rather involved web of (p, q) 5-branes spanning different directions. The natural direction for D5 polarisation in the KS throat is the contractible B-cycle as drawn schematically in figure 2. If the D5 channel is also present, our computations are not valid for at least two reasons. First, having two intersecting branes around complicates the metric scaling such that our above arguments are invalidated. Second, we have relied on a trivial Bianchi identity for F_3 , which is altered in the presence of D5 charges. It has been argued in the regime when $p/M \gg 1$ that D5 polarisation does not occur [24, 25]. This seems to be a peculiar property of anti-branes in long warped throats. For the purpose of this paper, the absence of a D5-polarization channel also for small values of p/M would imply a big simplification: we would then simply discard the analysis of boundary conditions where a spherical D5 polarization accompanies the NS5 brane.

However the full story seems more intricate. As suggested in [25] in the regime $p/M \gg 1$, the preferred channel might well be an oblique phase, a combination of D5 and NS5 polarisation along resp. B- and A-cycles. We leave a full treatment of possible oblique phases and more general asymptotics that allow to go beyond the limit $p/M \ll 1$ to future work.

4 Revisiting $T > 0$ no-go claims

We go further in our investigation of the near brane behavior of anti-branes by heating up the background. If the metastable state persists in the supergravity regime, one expects to cover all the involved 5-brane polarisation processes behind a smooth horizon [18]. For that reason we ignore the issue of not having a well defined B_2 field along the IR surface.

Once we introduce temperature in the form of a non-zero blackening factor, we find

$$e^{-\phi}|H_3|^2 \sim e^{-2f}|\alpha F_3 + X_3|^2, \tag{4.1}$$

where we assume that \mathcal{E}^{2A} and ϕ are finite at the horizon. It was shown in [12] that also at finite T one can argue that α and X_3 cannot simultaneously vanish at the horizon: this is necessary in order to have a non-zero finite boundary integral $\oint \mathcal{B}$ at the horizon. Hence we seem to have a singular H_3 density since e^{-2f} becomes infinite at the horizon.

There is however one assumption that went into the no-go result of [12] that should perhaps be relaxed. Five assumptions were made explicitly. Let us take r to be the coordinate perpendicular to the horizon:

1. The temperature shows up under the form of $g_{tt} \sim e^{2f}$, $g_{rr} \sim e^{-2f}$, with $e^{2f} \sim r - r_h$.
2. \mathcal{E}^{2A} , e^{2f} , ϕ depend only on r near the horizon r_h .
3. The ansatz (2.1) for metric and form fields.
4. The component of F_3 with all legs along the horizon is non-zero.
5. The relevant part of the boundary term,

$$\oint_{\partial\mathcal{M}_{\text{horizon}}} \tilde{\star}_4 1 \wedge [\alpha F_3 + B_2 \wedge X_3], \tag{4.2}$$

is finite and non-zero.

These assumptions are rather solid, following either from black brane solutions (points 1 and 2), the expectation of the backreaction in the KS background (points 3, 4) or the study of the boundary term (5). Condition 4 however was used in a stronger form: not only F_3 , but $\alpha F_3 + X_3$ was assumed to always have a component *along* the horizon. This was partially inspired by former smeared branes setups in which this was necessary for preserving the symmetries [14]. We think that this requirement might be too strong, and indeed some comments about this were already present in [12]. Our aim here is to further clarify this issue.

First, one of the distinguishing features is that F_3 has flux along the topological A-cycle. This leads to $\star_6 H_3 = e^{\phi-4A-f}(\alpha F_3 + X_3)$ also having flux through this cycle. This H_3 flux is the cause of the Myers effect that polarizes the probe branes into NS5-branes and we expect it to be present in the backreaction as well. In general, the radial coordinate along the A-cycle will not be identified with the radial component r orthogonal to the

horizon. Nevertheless, these coordinates will have a non-trivial relation, leading to the near-horizon behaviour:

$$\alpha F_3 + X_3 = dr \wedge \omega_2 + \dots, \tag{4.3}$$

for some two-form ω_2 . In case the terms \dots vanish we find that the H_3 density remains finite since $g_{rr} \sim e^{-2f}$:

$$e^{-\phi} |H_3|^2 \sim |\alpha F_3 + X_3|_{ds_6}^2, \tag{4.4}$$

where now the contraction does not include e^{2A} factors nor blackening factors e^{2f} . Intriguingly, this is exactly the mechanism in [17] that provides a finite solution. However, it is unclear whether this extends to a viable finite T version of anti-D3 branes in KS backgrounds. In [17], there was no topological compact A-cycle. Rather the simplification was made that the six-dimensional transverse space was \mathbb{R}^6 , the role of the A-cycle being played by an \mathbb{R}^3 factor. Also, the work of [17] included the first order backreaction of the fluxes on the black D3-brane geometry, but did not include the backreaction of the metric nor second order effects that can turn up in the flux density. We believe that this issue still has to be settled.

5 Conclusions and outlook

In this paper we emphasized the importance of exploring the boundary term that evaluates to the generalized ADM mass to shed new light on the back-reaction of anti-branes in flux throats. This method is very powerful, as it gives strong constraints on the back-reacted supergravity solutions without having to construct them explicitly. Our main result is the application of this method to the polarized NS5 state of supersymmetry-breaking anti-D3 branes in warped throats. Our method allows for the first time to study the polarized NS5 state in the regime where supergravity and probe limits are both expected to be applicable ($g_s p \gg 1$ and $p/M \ll 1$, with p the anti-brane charge and M the 3-form flux through the A-cycle of the KS throat).

5.1 Summary of results

Our main result is the observation that the 3-form flux divergence, typical to anti-brane solutions, can potentially be made physical by polarising the anti-D3 branes into spherical NS5 branes. This polarisation process is expected from probe computations [2] and the presence of local NS5 sources offers a natural explanation for the presence of singular 3-form fluxes. We found that the conditions for gluing the IR geometry to the UV geometry do not forbid such a 3-form singularity. Furthermore the computation suggests that there is a unique radius R for the spherical NS5 brane and that the radius scales as $R \sim \sqrt{p/M}$. The probe analysis however suggests that $R \sim p/M$. So for small values of p/M , which is the regime in which one can expect meta-stable SUSY-breaking, the radius of the NS5 brane is much bigger than the radius predicted by the probe computation. This is a clear indication that, if the full back-reacted supergravity solution exists, the flux clumping process described in [26, 27] indeed significantly pushes the NS5 brane towards the equator.

However, in contrast with [26, 27] we cannot conclude that it actually goes over the equator nor that the system becomes locally unstable.

Our result rests on the absence of D5 polarisation in the B-cycle of the KS throat, which was argued based on the computations carried out in [24, 25]. This turns out to be the simplification needed to apply the techniques of [11, 12] to compute the 3-form flux density near the source without the need for the full supergravity solution. Our proof does not depend on any details of the background (aside the absence of D5 polarisation): the only requirement is that supersymmetry is broken such that the (generalised) ADM mass is positive. But as mentioned in [25] there can be oblique D5/NS5 polarisation channels and we consider it an interesting challenge to extend our result to that case.

As an aside, we reinvestigated the assumptions that went into the no-go theorem for the existence of smooth finite temperature anti-brane solutions [12]. In view of the possible NS5 polarisation channel we argued that a specific assumption about the directions of the 3-form flux near the horizon could be relaxed. If the 3-form $\star_6 H_3$ near the horizon is of the form $dr \wedge \omega_2$, with ω_2 a two-form and r the local coordinate transverse to the horizon, the flux density at the horizon will be smooth. This condition cannot be satisfied for smeared anti-D3 solutions [14] or localised anti-D6 branes [16]. Hence if smooth finite T solutions exist, then their construction will necessarily involve the boundary condition $\star_6 H = dr \wedge \omega_2$ at the horizon, which can be natural for NS5 branes.

We used that at large enough T we expect that the details of the NS5 polarization are hidden behind the horizon. One can still try to heat up the NS5-polarization itself to see the effect of small temperature. We leave this for future work.

5.2 Numerical analysis

We hope that our analysis can be the starting point for a numerical investigation of fully backreacted NS5 solutions. Our results could be useful for choosing boundary conditions near the horizon in order to find a solution. We have shown that boundary conditions exist, consistent with the ADM mass, that evade unphysical singularities. Having certainty that anti-brane supersymmetry breaking is meta-stable as indicated by probe computations [2] requires that a well-behaved supergravity solution can be found. At least this statement is true for large flux numbers and charges such that all typical length scales are within the classical gravity level. For small charges, arguments beyond the probe approximation have been suggested in [6, 21].

If a numerical study suggests that our IR boundary conditions cannot be chosen, then an unphysical 3-form singularity remains that can only be interpreted as a fatal attraction of the D3 charges dissolved in flux towards the anti-D3 brane. In that case we speculate that this could be an explanation for the tachyon found in the analysis of Bena et al. [25]. The tachyon corresponds to a force on anti-D3 branes that has a non-zero projection towards the top of the A-cycle. The same should hence apply to spherical NS5 branes carrying anti-D3 charge [28]. If the picture of [26, 27, 29] is correct, then one expects exactly a tachyonic mode that pushes spherical NS5 branes towards the North Pole of the S^3 A-cycle, consistent with [28].

5.3 Connection to Smarr relations and black hole physics

In [12], the boundary term (2.3) has only been related to the ADM mass for KS-like throats with added anti-brane charge of [20] based on the perturbation of [9]. However, we believe that it can be applied much more widely. The expression for the mass in terms of a boundary term that we used in this paper is a special case of a Smarr relation. For instance in five-dimensional supergravity, the generalized ADM mass for asymptotically flat or AdS solutions with horizons (black holes, black rings) can be derived from a boundary term that evaluates to (see [30, 31] and references therein):

$$M_{\text{ADM}} = TS + \Phi Q + \phi q + V\Lambda. \tag{5.1}$$

The parameters in this expression are charges and dual potentials: entropy S and temperature T , electric monopole charge Q and electrostatic potential Φ , magnetic dipole charge q and its magnetic potential ϕ , and (minus) the effective inside the horizon and the cosmological constant Λ . The appearance of the dipole charge might be surprising: it is not a global conserved charge, but can only be measured locally for instance for dipole black rings. Still, it contributes to the Smarr relation because it is impossible to define the dipole potential ϕ using only a single patch, as explained in great detail in [32]. Recently the Smarr relation was shown to allow also for a bulk contribution (not a boundary term), that is non-zero only for non-trivial topology, which was previously overlooked. This topological contribution makes the construction of stationary spacetimes possible even in absence of horizons and underpins the black hole microstate geometry programme [22].

The form of the Smarr relation (5.1) is not restricted to asymptotically flat or AdS spaces, but to more general asymptotics. The prescription for the conserved energy (mass) in generic spacetimes with a timelike Killing vector has been discussed by Hawking and Horowitz in a Hamiltonian formalism [33]. Its application to flat space or AdS reduces to the known form of the Smarr formula in terms of Komar integrals, while for the anti-branes in KS it should become our boundary term (2.3). We sketch the analogy between Smarr relations for black holes and warped throats in [34] and we will come back to this in great detail in a forthcoming publication [35].

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A Technical details

This appendix has three sections with more technical details. We rederive the boundary term that integrates to the ADM mass in A.1, give a quick review of p -brane solutions in flat space in appendix A.2 and in appendix A.3 we conclude with the details for extending our finite temperature results from anti-D3 to anti-D p branes for any $p \leq 6$.

A.1 Boundary term

Here we rederive the boundary term from the Einstein equations and other equations of motion. First we choose a shorthand notation X_7 in the ansatz that solves the B_2 equation of motion (2.2):

$$e^{-\phi} \star_{10} H_3 = -C_4 \wedge F_3 + X_7, \quad dX_7 = 0. \quad (\text{A.1})$$

In the main body of the text we used Poincaré invariance in four dimensions to write $X_7 = \tilde{\star}_4 1 \wedge X_3$ with X_3 closed.

With the ansatz we describe in section 2, the Einstein equations can be massaged to [12]

$$R_4 = -\nabla^2 \phi - e^{-\phi} |H_3|^2 - |F_5|^2 - \mu_3 \delta(\Sigma). \quad (\text{A.2})$$

Here, R_4 is the trace of the Ricci tensor along the four macroscopic dimensions and Σ denotes the brane world-volume. The ansatz (2.1) together with the equations of motion for the form fields also imply that

$$\star_{10} e^{-\phi} |H_3|^2 = -C_4 \wedge F_3 \wedge H_3 + X_7 \wedge H_3, \quad (\text{A.3})$$

$$\star_{10} |F_5|^2 = d(C_4 \wedge F_5) + C_4 \wedge F_3 \wedge H_3 - \mu_3 \delta(\Sigma) C_4 \wedge \star_6 1, \quad (\text{A.4})$$

$$C_4 = \star_4(e^{-4A} \alpha). \quad (\text{A.5})$$

From these equations, it follows that

$$\star_{10} R_4 = d \star_{10} d\phi - d(C_4 \wedge F_5) - X_7 \wedge H_3 + \star_{10}(\alpha e^{-4A} - 1) \mu_3 \delta(\Sigma). \quad (\text{A.6})$$

In order to get a still more suggestive form, we use the following relation between the Ricci scalars of the metrics with and without warp factors:

$$R_4 = e^{-2A} \tilde{R}_4 + \star_{10} d \star_{10} d(4A). \quad (\text{A.7})$$

Then with (2.6) and $\tilde{R}_4 = 0$ (Minkowski space), we are left with

$$\star_{10} (1 - \alpha e^{-4A}) \mu_3 \delta(\Sigma) = d \star_{10} d(\phi - 4A) - d(C_4 \wedge F_5) - X_7 \wedge H_3. \quad (\text{A.8})$$

The first two terms on the right-hand side are total derivatives, but not the last one. However, we can remark that since X_7 is closed we can write it as

$$X_7 = dB_6 + X_7^{\text{harm}}, \quad (\text{A.9})$$

where B_6 is globally well-defined and X_7^{harm} is harmonic. From now on we assume that $X_7^{\text{harm}} = 0$. We will come back to the contribution of such harmonic terms to the ADM

mass and the comparison to black hole microstate geometries and fuzzballs in future work. Then we can write

$$X_7 \wedge H_3 = d(B_6 \wedge H_3) - B_6 \wedge dH_3. \quad (\text{A.10})$$

Now, we integrate both sides of (A.8) along a region of spacetime \mathcal{M} not containing the source. The region \mathcal{M} we have in mind has two boundaries: one IR boundary surrounding the branes at a small distance, which we will let go to zero; and another one far in the UV of the KS throat. In the region \mathcal{M} , there is no NS5 charge such that $dH_3 = 0$ and we get

$$\int_{\partial\mathcal{M}_{\text{UV}}} \mathcal{B} = \int_{\partial\mathcal{M}_{\text{IR}}} \mathcal{B}, \quad (\text{A.11})$$

where

$$\mathcal{B} = -C_4 \wedge F_5 - B_6 \wedge H_3 + \star_{10} d(\phi - 4A). \quad (\text{A.12})$$

In [12] it was shown that in the UV the integral

$$\frac{1}{\tilde{v}_4} \int_{\partial\mathcal{M}_{\text{UV}}} (-C_4 \wedge F_5 + dB_6 \wedge B_2 + \star_{10} d(\phi - 4A)) = M_{\text{ADM}} \quad (\text{A.13})$$

is equal to the ADM mass M_{ADM} , where \tilde{v}_4 is a volume factor accounting for the integration along the Minkowski directions. This ADM mass is non-vanishing whenever supersymmetry is broken by (anti-)D3 branes, as it is in the KPV set-up for metastable states. In the UV, B_2 can be integrated over $\partial\mathcal{M}_{\text{UV}}$. By partial integration and taking into account the fact that we integrate along a boundary, we see that

$$\frac{1}{\tilde{v}_4} \int_{\partial\mathcal{M}_{\text{UV}}} \mathcal{B} = M_{\text{ADM}} \neq 0. \quad (\text{A.14})$$

When we combine this with (A.11) and realize that the dilaton and the warp factor do not contribute at the IR, we obtain the remarkable result

$$\frac{1}{\tilde{v}_4} \int_{\partial\mathcal{M}_{\text{IR}}} (C_4 \wedge F_5 + B_6 \wedge H_3) = M_{\text{ADM}} \neq 0. \quad (\text{A.15})$$

This is what we need to argue for the singularities in section 3.

A.2 Supergravity brane solutions

Near the branes sources (D3 or NS5), we approximate the geometry by a p brane metric:

$$ds^2 = e^{2A} (-e^{-2f} dt^2 + d\vec{x}_p^2) + e^{2\frac{p+1}{p-7}A} (dr^2 + r^2 d\Omega_{8-p}^2), \quad (\text{A.16})$$

where the dilaton and sourced field strength for a Dp brane are

$$e^\phi = e^{4\frac{3-p}{p-7}A}, \quad F_{8-p} = e^{2f} \coth \beta Q_p d\Omega_{8-p} \quad (\text{A.17})$$

and for an NS5/F1 are

$$e^\phi = e^{-4\frac{3-p}{p-7}A}, \quad H_{8-p} = e^{2f} \coth \beta Q_p d\Omega_{8-p}. \quad (\text{A.18})$$

Also,

$$e^{\frac{16}{p-7}A} = 1 + \sinh^2 \beta r_0^{7-p} / r^{7-p}, \quad e^{2f} = 1 - \frac{r_0^{7-p}}{r^{7-p}}. \quad (\text{A.19})$$

The $T \rightarrow 0$ limit is $\beta \rightarrow \infty$, $r_0 \rightarrow 0$ while keeping $Q_p \equiv \sinh^2 \beta r_0^{7-p}$ fixed.

Finally we note that at $T = 0$, the flux density near p -branes scales with the warp factor A as:

$$NS5/F1: \quad e^{(3-p)\phi/2} |H_{8-p}|^2 \sim e^{-2\frac{(p-3)^2}{(7-p)^2}A}, \quad (\text{A.20})$$

$$Dp: \quad e^{-(3-p)\phi/2} |F_{8-p}|^2 \sim e^{-2\frac{(p-3)^2}{(7-p)^2}A}. \quad (\text{A.21})$$

A.3 Anti-D p branes at finite T

The extension of the finite temperature results to anti-D p branes inserted in throat geometries that carry D p brane charges dissolved in fluxes is immediate [12]. The ansatz generalises to

$$\begin{aligned} ds_{10}^2 &= \mathcal{E}^{2A} g_{\mu\nu} dx^\mu dx^\nu + ds_{9-p}^2, \quad (\text{A.22}) \\ C_{p+1} &= \tilde{x}_{p+1} \alpha, \\ H_3 &= \mathcal{E}^{\phi-(p+1)A} \star_{9-p} \left(\alpha F_{6-p} + X_{6-p} \right). \end{aligned}$$

Such backgrounds exist up to $p = 6$, which describes an anti-D6 brane in a background with Romans mass and H_3 flux carrying D6 charges. As with anti-D3 branes, there is a conserved current that entails a non-trivial gluing condition between the IR and the UV. Near the horizon in the IR the conserved charge is given by

$$\oint_{\text{IR}} \frac{2}{1-p} \star_{p+1} 1 \wedge [F_{8-p} + B_2 \wedge X_3] + \frac{4}{p+1} \star_{10} df, \quad (\text{A.23})$$

and as before this integral has to be positive and finite.

In complete analogy with anti-D3 branes we can investigate necessary gluing conditions to obtain the same conclusions, except for anti-D6 branes where we find a different result. When $p = 6$, the three-form H_3 is always proportional to the volume form in the three-dimensional transverse space (there is no A-cycle) and there is no X_0 term: $H_3 = e^{\phi-7A} \star_3 F_0$. For finite T solutions we furthermore cannot evade the no-go result as sketched earlier, since $\star_3 H_3 \sim F_0$ is a zero-form and hence cannot have a direction along dr . Therefore we still find a singular horizon for anti-D6 branes in flux background and their T-dual equivalents such as anti-D3 branes smeared on the tip of the Klebanov-Strassler throat.

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